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CESIFO WORKING PAPER NO. 5041  
CATEGORY 9: RESOURCE AND ENVIRONMENT ECONOMICS  
OCTOBER 2014

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# Battle for Climate and Scarcity Rents: Beyond the Linear-Quadratic Case

## Abstract

The nature of oil demand influences the oil extraction rate and hence has implications for both the timing of oil exhaustion and optimal climate policy. We analyse what role oil demand specification plays in strategic interactions between an oil-importing country producing final goods and wishing to mitigate global warming (Industria) and an oil-exporting country (Oilrabia) who buys final goods from the other country. Industria uses the carbon tax to impose an import tariff on oil and steal some of Oilrabia's scarcity rent. We derive subgame-perfect and open-loop Nash equilibrium outcomes and obtain results about the relative speeds of oil extraction and carbon accumulation and compare these with the efficient and competitive outcomes. We show that for the most typical demand functions, open-loop oil price will always be initially higher resulting in delayed extraction. However, we demonstrate that for certain more complex demand specification, Oilrabia has an incentive to initially price oil lower than the efficient level, resulting in more oil extraction and more climate damages. We further show that using the carbon tax as a tariff may not be as beneficial as suggested by previous studies. For certain demand functions, Industria sets the tariff too high leading to a decrease in the consumers welfare that isn't compensated by the higher tariff revenues.

JEL-Code: C730, H300, Q320, Q370, Q540.

Keywords: exhaustible resources, Hotelling rule, efficiency, carbon tax, climate rent, differential game, subgame-perfect Nash equilibrium.

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We are grateful to the ERC Advanced Grant 'Political Economy of Green Paradoxes' for financial support (FP-IDEAS-ERC Grant 269788), and to Florian Wagener, Matti Liski and Peter Neary for useful feedback.

## 1. Introduction

Oil exporters such as the OPEC can exert monopoly power on the world market. This monopoly power can result in significantly different oil extraction patterns relative to a perfectly competitive market. A key factor which determines the effect of the exporter's monopolist power is the nature of the importer's oil demand. However, many studies when modeling oil extractions choose the demand function for reasons of convenience/simplicity to make the models tractable. The goal of this study is to demonstrate that modelling the oil demand structure correctly is of high importance, by breaking down the way various demand specifications affect the bias of the exporter's monopolist power on the oil extraction rate.

A key result in the literature on oil extraction is that with zero extraction costs and isoelastic demand, the monopolist oil extraction is efficient and coincides with what would prevail in a competitive market (Stiglitz, 1976). In that case, the oil price increases according to the Hotelling rule at a rate equal to the market rate of interest. For non-isoelastic demand, the oil price path can be steeper or flatter than the Hotelling rate, depending on the demand's functional form. However, with zero extraction cost, a demand that increases faster than the Hotelling rent is ruled out by arbitrage arguments (Dasgupta and Heal, 1979).

The monopolist's oil extraction rate also plays a large role in the context of climate policies. As there is no global agreement on battling climate change, increasingly the developed countries are beginning to implement their own emission reduction policies, while the oil exporting nations are opposed to such measures. There is a range of studies modelling strategic interaction between a monopolist oil exporter who sets the oil price and an importer who combats climate change by setting a carbon tax (Rubio and Escriche 2001, Tahvonen 1995, 1996, Wirl 1995). In the equilibrium the importer uses the carbon tax as an import tariff to capture some of the monopolist's rents, while the exporter marks up his price so that he can capture a part of the carbon tax revenue.

Liski and Tahvonen (2004) explicitly analyze such strategic interactions. Their main results are that the subgame-perfect oil extraction path is flatter than in the efficient outcome, and also flatter than in the pure cartel outcome where no carbon tax is levied. Furthermore, they find that the level of damages due to global warming has a significant effect on equilibrium dynamics. The level of damages determines whether the import tariff or the Pigouvian environmental tax is the dominant component of the importer's carbon tax, and hence whether the carbon tax decreases or increases over time. The authors even find that for very high damages the tariff component may be negative – a subsidy. However, Liski and Tahvonen (2004) restrict their analysis to linear demand functions and parameterizations which lead to interior solutions (some oil still left in situ). As we will see these assumptions do end up affecting their result.

Our main contribution is to investigate the generality of these results to a variety of specifications of the oil demand. Similar in spirit to Mrazova and Neary (2013) and Xie (2000) we attempt to identify which demand function features have an effect on the monopolist's price mark up and the importer's carbon tax. We show that unlike the zero extraction cost case examined in Dasgupta and Heal (1979), if there is a stock dependent oil extraction cost the monopolist extraction is never efficient. When accounting for strategic interaction between the importer and the exporter we find that for the most common demand functions used in the economic literature, both open-loop and feedback equilibria lead to flatter oil price paths than in the efficient outcome. However, for some more unusual demand functions, the open-loop and cartel equilibrium oil price rises steeper than the efficient pace without violating any arbitrage conditions. This result is similar to Wirl (2007) who demonstrates that decreasing marginal elasticity of utility can lead to an increase in emissions in an international pollution control game, though in his setup there is no exhaustible resource, and both players are affected by climate change.

We further show that for demand specifications which asymptotically lead to full oil exhaustion, the level of carbon damages does not influence the carbon tax dynamics.<sup>3</sup> For such demand functions, the monopolist exporter earns significant amounts of scarcity rent, and thus the carbon tax is used primarily as a tariff so that the importer can capture some of these rents back. Lastly, we contribute to the literature by loosely calibrating our model to real-world values. Our calibration leads us to discard certain equilibrium scenarios. We demonstrate that for calibrated preference and extraction cost functions, no matter how high the damage intensity, the oil importer will always impose a positive import tariff on top of the carbon tax.

Our framework of analysis is general equilibrium, but we abstract from saving, investment and capital accumulation.<sup>4</sup> We also abstract from political economy issues in the oil-importing and oil-exporting countries. For example, oil exporters such as Saudi Arabia may have a desire to have a higher export price of oil if their populations are sizeable and need to be pacified with transfers. Oil importers may have to consider the income distributions of regressive carbon taxes, but we abstract from these.

The outline of the paper is as follows. Section 2 sets up the model of the oil-importing and oil-exporting blocks of countries and discusses the differential game that is played between them. Section 3 derives the open-loop Nash equilibrium outcome, compares this with the efficient and competitive outcomes and outlines the difference between equilibria in HARA and non-HARA class production functions. Section 4 derives the subgame-perfect Nash equilibrium outcome (also known as the

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<sup>3</sup> In most cases, studies restrict their attention to interior solutions to make them tractable, so we provide a numerical algorithm which can be used to solve such problems

<sup>4</sup> Van Wijnbergen (1985) and Van der Meijden et al. (2014) analyse two-country general equilibrium models with capital accumulation and international capital flows, but do not look at the strategic interactions of policies in a differential-game framework. Jaakkola (2013) does do this, but focuses at whether taxation of foreign interest income on oil exporters' assets helps to overcome the Green Paradox.

feedback Nash equilibrium) and shows that the resulting oil price path is always flatter than that of the open-loop Nash equilibrium outcome. Section 5 offers some illustrative simulations to highlight our results, and provides an example of demand for which the monopolist initially extracts less than the efficient rate. Section 6 concludes with a summary of results and suggestions for further research.

## 2. The Model

Our model describes two countries or blocks of countries. One country called “Industria” does not have oil, but imports  $R$  units of oil from the other country called “Oilrabia” and uses it to produce final goods according to the concave production function  $F(R)$  with  $F'(R) > 0$  and  $F''(R) < 0$ . All other factors of production (labour, land, etc.) are fixed and subsumed in the production function.  $C$  units of final goods are bought by Industria and  $C^*$  units by Oilrabia for consumption purposes. A further  $G(S)R$  units of final goods are bought by Oilrabia for extraction purposes, where  $S$  is the stock of oil reserves. Oil extraction costs rise as less oil reserves are left and less accessible fields have to be explored, so  $G'(S) < 0$ . Furthermore, we suppose that oil extraction costs are convex,  $G''(S) > 0$ .

World goods market equilibrium requires that

$$(1) \quad F(R) = C + C^* + G(S)R.$$

Oilrabia lives from its oil wealth, has monopoly power in the world market for oil, and has utilitarian preferences<sup>5</sup>. The government of Oilrabia has a given initial stock of oil  $S_0$ , which it manages optimally to maximize the present discounted value of its consumption stream which corresponds to the present value of its oil profits. We abstract from private oil extraction in Oilrabia. Industria also has utilitarian preferences and is thus concerned with the present value of its consumption stream. It also levies a specific carbon tax  $\tau$  to limit the damages from global warming. We are concerned with the strategic interactions between Industria and Oilrabia, where the former sets the carbon tax and the latter sets the oil price.

### 2.1. Demand for oil and private consumption in Industria

Firms in Industria operate under perfect competition and maximize profits  $\Pi \equiv F(R) - (p + \tau)R$ , taking the before-tax price of oil  $p$  and the specific carbon tax  $\tau$  as given. Here profits include wage income and land rental income. We measure oil in tonnes of carbon, which implies a carbon -emission coefficient of unity. The efficiency condition  $F'(R) = p + \tau$ , where  $F'(R)$  is the marginal product of oil, gives oil demand as a decreasing function of the user cost of oil,  $R = R(p + \tau) \geq 0$ . Consumers in

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<sup>5</sup> While utilitarian preferences may imply a variety of utility functions, for computational simplicity we model the consumers' preferences with linear utility functions.

Industria obtains income from final production and get carbon tax revenues rebated in lump-sum fashion, so that consumption is

$$(2) \quad C = \Pi + \tau R = F(R(p + \tau)) - pR(p + \tau) \equiv C(p, \tau).$$

### 2.2. The government of Industria

The stock of carbon in the atmosphere,  $E$ , increases with carbon emissions, so that

$$(3) \quad \dot{E} = R(p + \tau), \quad E(0) = E_0 \text{ given,}$$

where we abstract from natural decay of the stock of atmospheric carbon. Global warming damages are given by the quasi-convex function  $D(E)$  with  $D' > 0, D'' \geq 0$ . The objective of the government of Industria at time  $t$  is to choose the carbon tax to maximize the present discounted value of private consumption of different generations of households minus global warming damages, i.e.,

$$(4) \quad V(t) = \int_t^{\infty} e^{-\rho(s-t)} [C(p(s), \tau(s)) - D(E(s))] ds,$$

subject to the dynamics of the atmospheric stock of carbon (3), where  $\rho > 0$  is the constant rate of time preference. Linear preferences imply zero intergenerational inequality aversion. The social rate of discount is used by the government of Industria to discount welfare of future generations.<sup>6</sup>

Although all oil extraction takes place outside of Industria's borders, Industria's government is able to observe the remaining oil stock  $S$ . The dynamics of oil extraction follow from:

$$(5) \quad \dot{S} = -R(p + \tau) \geq 0, \quad S(t) \geq 0, \quad S(0) = S_0 \text{ given.}$$

The stock of carbon in the atmosphere is the initial stock plus cumulative oil depletion,

$E(t) = E_0 + S_0 - S(t), t \geq 0$ , so that, formally, only the stock of atmospheric carbon or the stock of in situ oil needs to be used as a state variable.

### 2.3. The government of Oilrabia

The government of Oilrabia takes account of oil demand from Industria and chooses oil prices to maximize the present discounted value from the stream of current and future oil profits,

$$(6) \quad V^*(t) = \int_t^{\infty} e^{-\rho^*(s-t)} [p(s)R(p(s) + \tau(s)) - G(S(s))R(p(s) + \tau(s))] ds,$$

subject to the dynamics of oil depletion (5), where  $\rho^* > 0$  is Oilrabia's rate of time preference. We will suppose that Industria and Oilrabia employ the same rate of time preference:  $\rho^* = \rho$ . From (1) and (2) we get  $C^* = F(R) - C - G(S)R = pR - G(S)R$ , so that the government maximizes (6) or

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<sup>6</sup> Due to our use of linear utility functions the social rate of discount is equal to the rate of pure time preference  $\rho$

equivalently the present value of Oilrabia's stream of current and future consumption levels,

$V^*(t) = \int_t^{\infty} e^{-\rho^*(s-t)} C(s) ds$ . Our general equilibrium model is equivalent to the two-country model of an oil exporter and an oil importer analysed by Liski and Tahvonen (2004) if the production function  $F(R)$  is replaced by the utility function  $U(R)$  with  $U' > 0$  and  $U'' < 0$ . In their model demand for oil comes from households setting  $U'(R) = p + \tau$  and in our model it comes from firms producing final goods setting  $F'(R) = p + \tau$ .

### 3. Open-Loop and Efficient Equilibria

We first consider the efficient equilibrium where the social planner maximizes the welfare sum of both Oilrabia and Industria. We then compare it to the open-loop equilibrium where each of the countries takes the time path of the other country's policy as given when setting its policy. This is distinct from the feedback Nash equilibrium where the countries cannot pre-commit to a time path, but instead base their strategies on the remaining oil stock  $S$  and pollution stock  $E$ , which we cover in section 4.

#### 3.1. Efficient equilibrium

In the efficient equilibrium, the social planner sets the user cost of oil, which is defined as  $q \equiv p + \tau$ , such that it maximizes total global consumption less the environmental damages, subject to the resource extraction constraint in (2). The Hamiltonian for the social planner is then  $H \equiv F(R(q)) - G(S)R - D(E) - \mu R(q)$  where the co-state variable  $\mu$  corresponds to the sum of the social cost of carbon and the Hotelling rent. The optimality conditions for this problem are:

$$(7) \quad \frac{dH}{dq} = F'(R(q)) - G(S) - \mu = 0, \quad \rho\mu - \dot{\mu} = \frac{dH}{dS} = D'(E) - G'(S)R.$$

Simplifying the optimality conditions, we express them as a partial differential equation of the user cost of oil  $q$ :

$$(7') \quad \dot{q} = \rho(q - G(S) - D'(E) / \rho).$$

The efficient user cost of oil can be decomposed into two components – the producer price (which increases according to Hotelling rule) and the Pigouvian tax (which is equal to the sum of discounted marginal damages). This result is fairly straightforward and has been shown in many previous studies but we reproduce it here for ease of comparison with open-loop and feedback equilibrium results.

#### 3.2. Open-loop Nash equilibrium: carbon taxation in Industria

We now move on to calculating the open-loop Nash equilibrium, first looking at Industria's problem. Industria maximizes its welfare (4) subject to (2). Its Hamiltonian therefore reads

$H \equiv F(R(q)) - G(S)R - D(E) - \mu R(q)$  where  $\mu \geq 0$  is Industria's social cost of carbon. The

optimality conditions for this problem are

$$(8) \quad \frac{\partial H}{\partial \tau} = (F'(R(p+\tau)) - p - \mu)R'(p+\tau) = 0, \quad \rho\mu - \dot{\mu} = \frac{\partial H}{\partial E} = D'(E), \quad \lim_{t \rightarrow \infty} e^{-\rho t} \mu(t)E(t) = 0.$$

The first part of (8) tells us that the optimal carbon tax must be set to Industria's social cost of carbon,  $\tau = \mu$ . It follows that the second part of (8) can be integrated to give the optimal carbon tax as the discounted sum of all future marginal damages from global warming:

$$(9) \quad \tau(t) = \int_t^{\infty} e^{-\rho(s-t)} D'(E(s)) ds, \quad t \geq 0.$$

The third part of (8) is the transversality condition and will be satisfied as  $S_0$  is finite and thus the maximum value of the carbon stock  $E_0 + S_0$  is finite too.

### 3.3 Open-loop Nash equilibrium: oil pricing by Oilrabia

Oilrabia's optimization problem is to maximize (6) subject to (5). Its Hamiltonian reads

$H^* \equiv [p - G(S) - \lambda^*]R(p+\tau)$ , where  $\lambda^* \geq 0$  is the marginal value of oil reserves to Oilrabia (also called the scarcity rent). The optimality and transversality conditions for this problem are

$$(10) \quad \frac{\partial H^*}{\partial p} = R + [p - G(S) - \lambda^*]R'(p+\tau) = 0, \quad \rho\lambda^* - \dot{\lambda}^* = \frac{\partial H^*}{\partial S} = -G'(S)R, \\ \lim_{t \rightarrow \infty} e^{-\rho t} \lambda^*(t)S(t) = 0.$$

The first part of (10) implies that the optimal price must equal the sum of oil extraction cost, the scarcity rent of oil and the carbon tax all multiplied by a monopoly mark-up:

$$(11) \quad p + \tau = \frac{G(S) + \lambda^* + \tau}{1 - 1/\varepsilon}.$$

where  $\varepsilon$  is the price elasticity of resource demand, defined by:  $\varepsilon(p+\tau) = \frac{(p+\tau)R'(p+\tau)}{R(p+\tau)}$ . In a

competitive market ( $\varepsilon \rightarrow \infty$ ) the oil price is simply set to the sum of the extraction cost and the scarcity rent, so (10) becomes  $p = G(S) + \lambda^*$ . An alternative way of writing (11) gives the market price of oil as the monopoly mark-up on the extraction cost and the scarcity rent plus the capture of the climate rent:

$$(11') \quad p = \frac{G(S) + \lambda^*}{1 - 1/\varepsilon} + \frac{\tau}{\varepsilon - 1}.$$

Capture of climate rent is more substantial if Oilrabia has more monopoly power on the oil market, which is the case if the price elasticity of oil demand  $\varepsilon$  is relatively low. Integrating the second part



of (10) gives the scarcity rent as the present value of all future reductions extraction costs resulting from keeping an extra unit of oil in the earth:

$$(12) \quad \lambda^*(t) = -\int_t^{\infty} e^{-\rho(s-t)} G'(S(s)) R(s) ds.$$

Having derived dynamics for Oilrabia's price and Industria's tax we now combine them to find equilibrium conditions.

### 3.4 Open-loop Nash equilibrium outcome

It is easiest to analyse the open-loop Nash equilibrium outcome in terms of oil reserves and the user cost of oil  $q$ , Combining (10) and (8), this outcome is described by the dynamic system described by (5) and

$$(13) \quad \dot{q} = \frac{\rho \left[ \left( 1 - \frac{1}{\varepsilon(q)} \right) q - G(S) \right] - D'(E_0 + S_0 - S)}{2 - \frac{R''(q)R(q)}{R'(q)^2}}.$$

The efficient outcome is described by (5) and

$$(14) \quad \dot{q} = \rho [q - G(S)] - D'(E_0 + S_0 - S),$$

and the competitive market outcome by (5) and

$$(15) \quad \dot{q} = \rho [q - G(S)].$$

The cartel or monopolist outcome (where Industria cannot set a carbon tax) follows from (5). Note that in any equilibrium we must have

$$(16) \quad G(S) < q, \quad D'(E) / \rho \leq \tau.$$

The first inequality is needed to have non-negative profits from oil extraction. The second inequality follows from the fact that the carbon tax is monotonically non-decreasing, because if it were decreasing, it would become negative eventually, which contradicts that there is a cost rather than a benefit associated with carbon accumulation. Hence, in each of the efficient, monopolistic and competitive outcomes the user cost of oil is increasing at a rate no greater than the social rate of discount. Another reason for considering only such price paths is to exclude arbitrage, as pointed at by Dasgupta and Heal (1979) for the case without extraction cost and global warming damages.

To provide some initial intuition to the result in (13) consider first the case of isoelastic demand,

$$R = p^{-1/\sigma}. \text{ Isoelastic demand follows from a CRRA production function } F(R) = \frac{R^{1-\sigma}}{1-\sigma}.$$

For this isoelastic demand specification, the elasticity is constant  $\varepsilon(p + \tau) = 1/\sigma$  and

$2 - R''(p)R(p) / R'(p)^2 = 1 + \sigma = 1 - 1/\varepsilon(p)$ . The user oil cost dynamics then become

$$(17) \quad \dot{q} = \frac{\rho[(1-1/\varepsilon)q - G(S) - D'(E_0 + S_0 - S)/\rho]}{1-1/\varepsilon}.$$

In (8) we have shown that the open-loop Nash equilibrium tax is the Pigouvian tax which evolves according to  $\dot{\tau} = \rho\tau - D'(E_0 + S_0 - S)$ . Hence from (17) we can obtain an expression for the dynamics of the oil price:

$$(17') \quad \dot{p} = \rho \left( p - \frac{G(S)}{1-1/\varepsilon} \right) - \frac{D'(E_0 + S_0 - S)}{\varepsilon - 1}$$

For the competitive market,  $\varepsilon \rightarrow \infty$  so (17') reduces to the Hotelling dynamics  $\dot{p} = \rho(p - G(S))$ .

This rule implies that the return on leaving an extra unit of oil in the ground (the capital gains) equals the interest on the net revenue from extracting and selling an extra unit of oil. For the special case of isoelastic demand, zero extraction cost and no climate damages (13) reduces to the efficient outcome which then coincides with the competitive outcome. The last term in (17) indicates that with monopoly power on the oil market the oil price captures part of the climate rent. If oil demand is not

isoelastic, we see from (13) that the solution becomes less trivial. The term  $\frac{R''(q)R(q)}{R'(q)^2}$  can be re-

interpreted as the measure of how much the elasticity of demand changes over time (the elasticity of the elasticity of demand). The term determines to what degree the consumers can substitute away from oil as the price rises, and thus how much scarcity rent the monopolist can capture. We spend the remainder of the paper determining the way in which various preference specifications affect equilibrium dynamics.

### 3.5 Comparing the open-loop Nash and efficient equilibria

#### 3.5.1 HARA Class Demand

Let us consider the class of HARA demand functions. This class of production functions is given by

$$F(R) = \frac{1-\varphi}{\varphi} \left[ \left( \frac{\psi R}{1-\varphi} + \chi \right)^\varphi - \chi^\varphi \right], \text{ where } \psi > 0. \text{ In Appendix A1, we examine four special cases of}$$

the HARA class function: quadratic, power, logarithmic and exponential production functions. We prove that for every one of these cases, as long as the necessary conditions are satisfied, the open-loop equilibrium initial extraction is lower than the efficient level. Thus we can tentatively conclude that for HARA production functions Oilrabia will initially extract less and the path of oil prices is flatter than in the efficient outcome. In that sense monopolistic oil barons are the conservationist's best friend.

#### 3.5.2 Non-HARA class demand

We now examine non-HARA class production to show that it is possible that monopolist extraction is bad for the environment. An example of a non-HARA class demand function is the shifted loglinear oil demand curve,  $R = (q - \chi)^{-\varphi}$  with  $q > \chi$ . This follows from the production function

$$F(R) = \chi R + \frac{R^{1-1/\varphi}}{1-1/\varphi}. \text{ It gives an elasticity of } \varepsilon = \left( \frac{q}{q - \chi} \right) \varphi. \text{ The elasticity has to exceed unity, so}$$

that we must have  $\varphi > \frac{q - \chi}{q}$ . Equation (13) becomes

$$\dot{q} = \frac{\rho[(\varphi - 1)q + \chi - \varphi G(S)] - \varphi D'(E_0 + S_0 - S)}{\varphi - 1}.$$

Without extraction costs and climate damages and with  $\chi > 0$ , we have  $\dot{q} = \rho q + \frac{\rho \chi}{\varphi - 1} > \rho q$ . Since

$\dot{q} > \rho q$  is ruled out on the basis of arbitrage arguments (Dasgupta and Heal, 1979), we must have

$\frac{q - \chi}{q} < \varphi < 1$  if  $\chi > 0$ . However, if  $G(S) + D'(E_0 + S_0 - S) / \rho$  is large enough, we can work with a

wider range of values of  $\varphi$ . We know that in equilibrium  $G(S) + D'(E_0 + S_0 - S) / \rho < q$  must hold and thus

$$\dot{q} = \rho q + \frac{\rho \chi}{\varphi - 1} - \frac{\varphi \rho [G(S) + D'(E_0 + S_0 - S) / \rho]}{\varphi - 1} > \frac{\rho}{1 - \varphi} (q - \chi) > 0.$$

A necessary condition for  $\dot{q} < \rho q$  to hold is  $\varphi > 1 - (q - \chi)$ . Thus we find that for demand functions outside of the HARA class it is possible that in the open-loop Nash equilibrium Oilrabia initially extracts more than in the efficient case. It is not possible to derive precise analytical conditions for when the open-loop Nash equilibrium is initially less conservative. Note however that using conditions (13) and (5) one can numerically solve for time paths of  $S$  and  $q$ , and then use (9) to find the equilibrium tax and price. In section 5 we perform such numerical analysis for a variety of demand functions and use simulations to demonstrate that the monopolist can be less conservationist than the efficient outcome.

#### 4. Feedback Nash Equilibrium

We assume that both Industria and Oilrabia have perfect information on both remaining oil reserves and the atmospheric carbon stock at each instant of time.<sup>7</sup> Hence, the optimal carbon tax imposed by Industria and the world market price of oil determined by Oilrabia can both be represented as a function of merely remaining oil reserves, since the carbon stock is the sum of the initial carbon stock

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<sup>7</sup> Other informational assumptions are feasible (e.g., Industria does not have knowledge of Oilrabia's stock of remaining oil reserves), but we will abstract from them here.

and all oil burnt so far. We denote equilibrium price and tax as  $p(S)$  and  $\tau(S)$  with derivatives denoted by  $p_S(S)$  and  $\tau_S(S)$ .

#### 4.1. The optimal feedback rule for the carbon tax

The Hamilton-Jacobi-Bellman (HJB) equation for Industria is

$$(18) \quad \rho V(S) = \max_{\tau} [F(R(p + \tau)) - p(S)R(p + \tau) - D(S_0 + E_0 - S) - V_S(S)R(p + \tau)],$$

where  $V(S)$  is Industria's value function. In a feedback Nash equilibrium Industria takes the price of oil as a given function of oil reserves and atmospheric carbon  $p(S)$ . The optimality condition

$(F'(R(p + \tau)) - p(S) = V_S(S))R'(p + \tau)$  gives Industria's feedback rule for the optimal carbon tax:

$$(19) \quad \tau = V_S(S) = \tau(S).$$

The carbon tax  $\tau$  is thus set to the social cost of carbon. Substituting (19) into (18) and differentiating of both sides with respect to the oil stock, we obtain the condition describing the dynamics of the carbon tax:

$$(20) \quad \tau_S R(p + \tau) = -\rho \tau^T - p_S(S)R(p + \tau) + D'(S_0 + E_0 - S).$$

The carbon tax can be then decomposed as a sum of the environmental tax  $\tau^E$  and the import tariff  $\tau^T$ . The environmental tax is the classic Pigouvian tax set to the total discounted marginal damages from pollution while the import tariff is the strategic component which Industria uses to steal Oilrabria's monopoly rent. The dynamics of these components follow from:

$$(20') \quad \tau_S^E R(p + \tau) = -\rho \tau^E + D'(S_0 + E_0 - S), \quad \tau_S^T R(p + \tau) = -\rho \tau^T - p_S(S)R(p + \tau).$$

#### 4.2. The optimal feedback rule for the oil price

Oilrabria maximizes the present discounted value of profits from oil extraction (6) subject to oil demand, carbon accumulation (3) and oil depletion (5) and given the policy rule for Industria's carbon tax  $\tau(S)$ . The HJB equation for Oilrabria is

$$(21) \quad \rho V^*(S) = \max_p \{ [p - G(S)]R(p + \tau) - V_S^*(S)R(p + \tau) \},$$

where  $V^*(S)$  is the value function for Oilrabria. The first-order condition gives the optimal oil price:

$$(22) \quad p = \frac{G(S) - V_S^*(S)}{1 - 1/\varepsilon(p + \tau)} + \frac{\tau}{\varepsilon(p + \tau) - 1}.$$

The first term in expression (22) is the monopoly mark-up on the sum of extraction cost and the scarcity rent. The mark-up increases if oil demand is less elastic. The second term in (22) reflects the extent to which Oilrabia can capture part of Industria's climate rent, which is easier if Oilrabia has more monopoly power on the world oil market.

To derive the dynamics of Industria's oil price we substitute (22) into (21) and differentiate both sides with respect to  $S$ , leading to the following condition:

$$(23) \quad (p_S + \tau_S)R(p + \tau) \left( \frac{R(p + \tau)R''(p + \tau)}{R'(p + \tau)^2} - 2 \right) = \rho \left[ p - \frac{p + \tau}{\varepsilon(p + \tau)} - G(S) \right].$$

### 4.3. Equilibrium

The feedback Nash equilibrium solves for the optimal rule for Industria (20) and Oilrabia (23). Combining them we get the equilibrium condition for the user cost of oil

$$(24) \quad q_S R(q) \left( \frac{R(q)R''(q)}{R'(q)^2} - 3 \right) = \rho \left[ q \left( 1 - \frac{1}{\varepsilon(q)} \right) - D(E_0 + S_0 - S) - G(S) \right].$$

As can be seen, similar to the open-loop Nash equilibrium, the demand function specification plays a key role in the nature of the equilibrium. An analytic solution is feasible if preferences are quadratic and extraction cost is linear by guessing that value functions are quadratic and solving with the method of undetermined coefficients. For more sophisticated demand specifications, normally one needs to find a fixed point in the policy reaction functions and value functions. However for a problem with a single state variable there are easier solution methods.

### 4.4. Time-domain representation and comparison with the open-loop Nash equilibrium outcome

We begin by demonstrating that we can express  $p(S)$  and  $\tau(S)$  as optimal paths  $p(t)$  and  $\tau(t)$ , without any information loss. Formally, the feedback equilibrium implies that the players do not pre-commit to optimal policy paths and instead they represent functions of the state variable. However recall that an interior solution is characterized by positive oil demand, i.e.  $R(t) > 0$  for all  $t \geq 0$ . This implies that the oil stock  $S$  is a monotonically decreasing function of time  $\dot{S} = -R < 0$ . Therefore for every time  $t$  there is a corresponding unique value of  $S(t) \in (S_0, 0)$ . Thus the solutions to the feedback equilibrium  $p(S(t))$  and  $\tau(S(t))$  can also be expressed simply as optimal equilibrium paths:  $p(t)$  and  $\tau(t)$ .

We now present the time-domain representation of the feedback equilibrium conditions (20) and (23). In addition to simplifying the numerical solution of the feedback equilibrium, expressing the optimal conditions in the time domain allows for easier comparison with the open-loop Nash equilibrium.

Utilizing  $-\tau_S R = \dot{\tau}$  and  $-p_S R = \dot{p}$  the feedback equilibrium carbon tax dynamics in (20) can be re-written as:

$$(25) \quad \dot{\tau} = \rho\tau - D'(E_0 + S_0 - S) - \dot{p}.$$

Compared to the open-loop Nash equilibrium carbon tax in (9) an extra term is added to the feedback equilibrium expression - the change in the oil price  $-\dot{p} < 0$ . Hence, the feedback carbon tax path is flatter than the open-loop one. The tax can be then decomposed into the environmental tax  $\tau^E$  and an import tariff  $\tau^T$ :

$$(25') \quad \dot{\tau}^E = \rho\tau^E - D'(E_0 + S_0 - S), \dot{\tau}^T = \rho\tau^T - \dot{p}.$$

Note that the open-loop Nash equilibrium carbon tax is the same as the environmental component of the feedback Nash carbon tax. In the open-loop Nash equilibrium Industria has to pre-commit to a tax and does not consider the response of Oilrabia – thus simply setting the carbon tax to the sum of the discounted marginal damages. In the feedback equilibrium Industria adds a tariff component to steal some of the scarcity rent from Oilrabia.

We can also re-write the feedback equilibrium condition of Oilrabia as

$$(26) \quad \dot{p} + \dot{\tau} = \frac{\rho((p + \tau)(1 - 1/\varepsilon(p + \tau)) - \tau - G(S))}{2 - R(p + \tau)R''(p + \tau) / (R'(p + \tau))^2}.$$

This expression is similar to the open-loop Nash equilibrium dynamics in (13). Since in equilibrium  $\rho\tau > D'(E)$ , we can conclude the time path for the user cost of oil is indeed flatter in subgame-perfect than in the open-loop Nash equilibrium outcome. To make solving the equilibrium easier we combine (25) and (26), representing the equilibrium through the dynamics of the combined user cost of oil.

$$(27) \quad \dot{q} = \frac{\rho(q(1 - 1/\varepsilon(q)) - D(E_0 + S_0 - S) / \rho - G(S))}{3 - R(q)R''(q) / (R'(q))^2}.$$

The subgame-perfect Nash equilibrium solution can now be obtained by numerically solving (5) and (27) for the time paths of  $S$  and  $q$ . From these one can calculate the time paths for the carbon stock  $E = E_0 + S_0 - S$ , the carbon tax  $\tau = [\dot{q} + D'(E)] / \rho$  (from (27)), and the market price of oil  $p = q - \tau$ .

We can calculate the pure Pigouvian component of the carbon tax as  $\tau^P(t) \equiv \int_t^\infty e^{-\rho(s-t)} D'(E(s)) ds$ ,

upon which the import tariff component can be found as  $\tau(t) - \tau^P(t)$ . The export mark-up for oil follows from  $p - G(S) - \lambda^{**}$ , where the scarcity rent is calculated as

$\lambda^{**}(t) = -\int_t^\infty e^{-\rho(s-t)} G'(S(s)) R(s) ds$ . These price and tax components can then be used to demonstrate

how the demand specification influences the amount of monopolist rent that can be stolen by the importer and vice versa.

Solving this feedback Nash equilibrium problem is relatively straightforward, due to the fact that our model collapses to only one state variable (the oil stock). If instead two or more state variables were required to describe the state of the world, we would have to explicitly solve for optimal strategies as functions of the states. For example, if we had included atmospheric decay of carbon in our model, the atmospheric carbon stock would no longer be the complement of the oil stock, and the two countries would base their strategies on both stocks. Similarly, if we would have allowed for capital accumulation in the model, the optimal strategies would also be a function of the capital stock.

## 5. Numerical Illustrations

We supplement our analytical results by illustrating the efficient, the open-loop and feedback Nash equilibrium outcomes with some numerical solutions. Furthermore, we perform additional analyses of carbon tax dynamics which were too complex to do analytically for non-linear demand. For that purpose, we use the functional forms and calibration reported in table 1. All of our emission variables are expressed in 1000 Gigaton Carbon (GtC) and prices/costs are expressed in \$1000 USD/ton Carbon (tC). We adopt a linear extraction cost function with an initial stock of oil reserves of 10. Extraction costs rise from an initial value of \$300 to \$1150 per tC when reserves are exhausted (corresponding to \$30 and \$115 per barrel of oil, following the IEA (2008) long term cost curve). The initial carbon stock is set to its preindustrial level of 800 GtC. We consider quadratic, CARA and Cobb-Douglas production functions, which are all members of the HARA class. They lead to, respectively, linear, semi-loglinear and loglinear oil demand functions. Calibration details are reported in the appendix (A2).

**Table 1: Calibration**

	Functional form	Parameter values
Extraction cost	$G(S) = \gamma_1 + \gamma_2(S_0 - S)$	$\gamma_1 = 0.3, \gamma_2 = 0.085, S_0 = 10$
Damages	$D(E) = \kappa E^2$	$E_0 = 8, \kappa = 0.00004$
Quadratic production	$F(R) = \alpha R - \beta R^2 / 2$	$\alpha = 0.732, \beta = 2.23$
CARA production	$F(R) = \frac{\omega_2}{\omega_1} (1 - e^{-\omega_1 R})$	$\omega_1 = 3.672, \omega_2 = .7430$
Cobb-Douglas production	$F(R) = \frac{R^{1-\sigma_1}}{\sigma_2(1-\sigma_1)}$	$\sigma_1 = 0.65, \sigma_2 = 5.43$

**Table 2: Welfare in the Various Outcomes**

Oil demand	Industria's welfare			Oilrabia's profit		
	Efficient	Open-loop	Feedback	Efficient	Open-loop	Feedback
Linear	-0.188	-0.242	-0.195	0.276	0.309	0.250
Loglinear	-0.183	-0.237	-0.189	0.279	0.313	0.252
Semi-loglinear	10.887	8.797	8.075	2.802	3.552	2.938

**Table 3: Time to Extract All Oil Reserves and Untapped Oil in the Various Outcomes**

Oil demand	Extraction Time			Final Oil Stock
	Efficient	Open-loop	Feedback	
Linear	506.1	776.3	857	6.295
Loglinear	580.4	939.2	986	6.179

In the remainder of this section we present the numerical solutions for various demand specifications. We begin in section 5.1 by replicating Liski and Tahvonen (2004), solving the model for the linear oil demand and showing that results do not change for loglinear oil demand. We proceed to solve the model for various levels of the damage intensity parameter,  $\kappa$ , demonstrating that given a reasonable calibration, the damage parameter has a minor effect on the resource extraction rate. Section 5.2 presents the results for the semi-loglinear oil demand specification, where the oil importer's government in the desire to capture the exporter's scarcity rent raises the tariff too high, causing more damage than benefit to the consumer. Finally, in section 5.3 we turn to the most interesting case: deriving the conditions and providing a numerical example of a non HARA class demand specification, for which the monopolist initially extracts more oil than the efficient rate.

### **5.1. Linear and loglinear demand: Liski and Tahvonen case**

We first consider the case of a quadratic production function and a linear oil demand function, the functional form used by Liski and Tahvonen (2004). The first set of panels of figure 1 compares time paths for oil reserves, the rate of oil extraction and the user cost of oil under the various outcomes for this case, figure 2 presents the subgame-perfect Nash equilibrium time paths in more detail, and figure 3 decomposes the equilibrium paths for the optimal carbon tax into its Pigouvian and its import tariff component. The Pigouvian tax is equal to the marginal environmental damages, while the import tariff is the remainder of the carbon tax. We decompose the optimal oil price into its extraction cost, scarcity rent (shadow price  $\lambda$ ) and export price mark-up (the remainder of the price) components.



The results confirm the results of Liski and Tahvonen (2004). First, the top-left panel of figure 1 confirms that in the open-loop Nash equilibrium the cartel initially extracts oil slower than the efficient rate of extraction but slower than in the Feedback Nash equilibrium. However, in the long run, the rate of oil depletion converges for these three outcomes. Figure 4 confirms that the competitive oil extraction path is initially flatter than the pure cartel outcome. The calibration we used to produce Figures 1-4 assumes fairly mild damages. Thus the import tariff component of the carbon tax falls over time and the Pigouvian component rises over time. Industria uses its monopsony market power to extract rents from Oilrabia with a high initial import tariff component that decreases over time. Both the scarcity rent and the export price mark-up decline over time, as the extraction cost rises and the cartel loses its monopoly power.

Next, we consider the case of a CARA production function and log-linear demand. Figure 5 compares the efficient, open-loop and Nash equilibria for log-linear demand, figure 6 presents the feedback Nash equilibrium carbon tax and oil price and figure 7 decomposes the feedback Nash carbon tax into the environmental and trade components. The resulting equilibria paths are similar to the linear demand case. As can be seen in figure 6 the feedback Nash equilibrium user cost of oil path is flatter than the open-loop Nash equilibrium path which in return is flatter than the efficient path. Similarly, for our mild damage calibration as Industria extracts oil rents from Oilrabia, there is a high tariff which decreases over time. Thus log-linear and linear demand lead to fairly similar equilibrium extraction paths. The key difference is quantitative. While (due to our calibration) initial equilibrium oil extraction and oil price are the same for both demand specification, for the log-linear demand more is extracted and the extraction time is longer,

How does the severity of damages affect the components of the equilibrium? The results for high-severity climate damages and linear demand are reported in Figure 8-9. Similar to Liski and Tahvonen (2004) with medium high damages the Pigouvian carbon tax component is fairly large. Industria still has a little market power to extract some rents from Oilrabia but the import tariff component is dominated by the Pigouvian component. Thus, as can be seen in the bottom right panel of figure 8, the carbon tax increases over time for high damages. This stands in contrast to the decreasing carbon tax for figure 1 when climate damage intensity is lower.

However, we do not find a damage intensity high enough such that the optimal feedback Nash carbon tax becomes smaller than the Pigouvian tax. Instead, for a very high damage intensity oil extraction becomes too harmful, and Industria sets a prohibitive carbon tax such that no oil is extracted. This result differs from Liski and Tahvonen (2004). Recall that in that study, for very high values of climate damage intensity, Industria sets the carbon tax below the optimal Pigouvian level, thus imposing an import subsidy (i.e., a negative tariff). This contradiction is mainly driven by our calibration. As we have loosely calibrated our model to real-world values, while Liski and Tahvonen (2004) parameter choice is fairly arbitrary, we argue that our result is more valid. Thus we can

conclude that no matter how high the damages, the importer will always add an import tariff on top of the Pigouvian carbon tax.

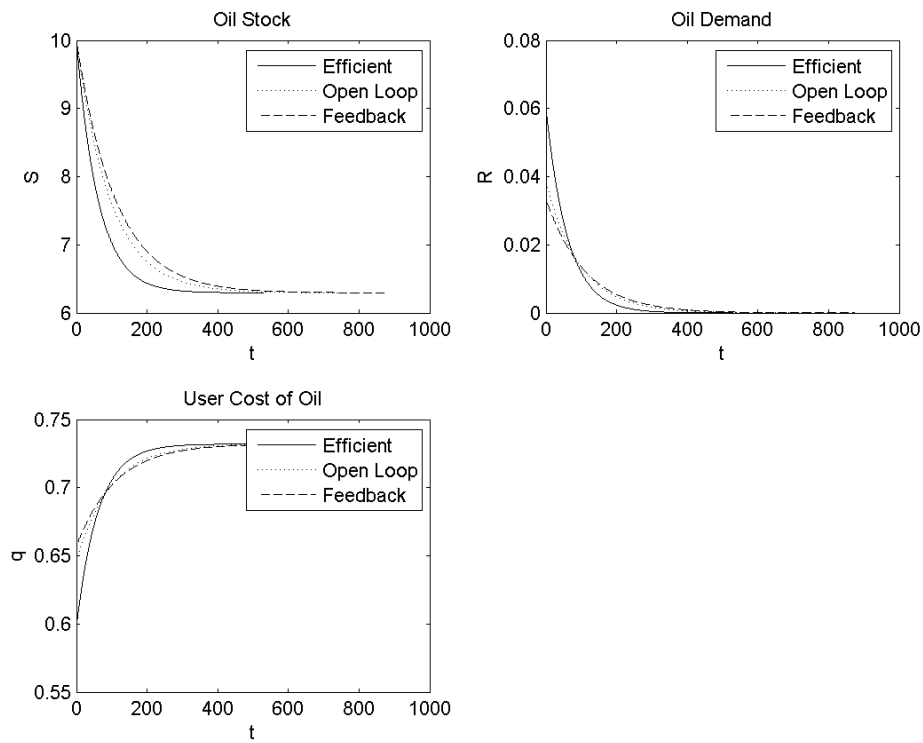


Figure 1: Results for Linear Oil demand - Comparison between equilibria

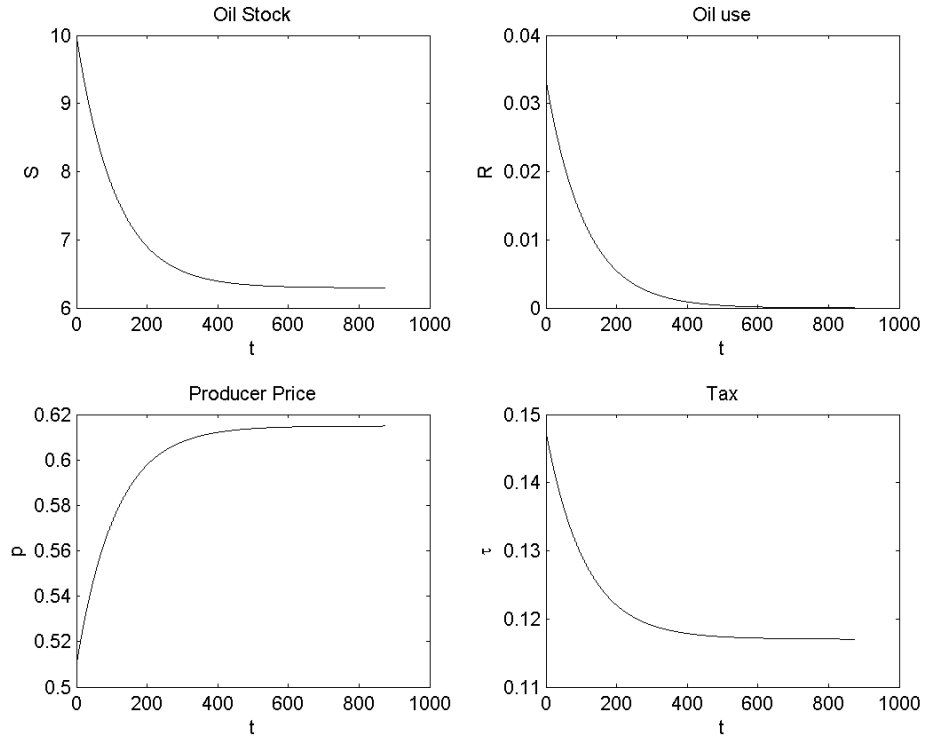


Figure 2: Result for Linear Oil demand- Feedback equilibrium

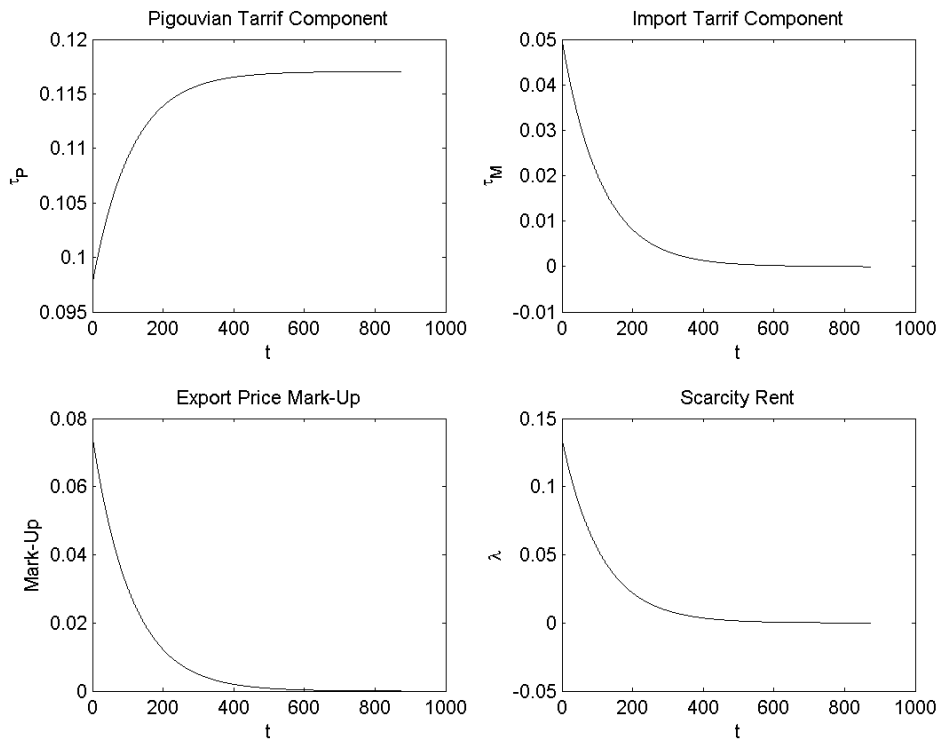


Figure 3: Result for Linear Oil demand- Price and tax components in the feedback equilibrium

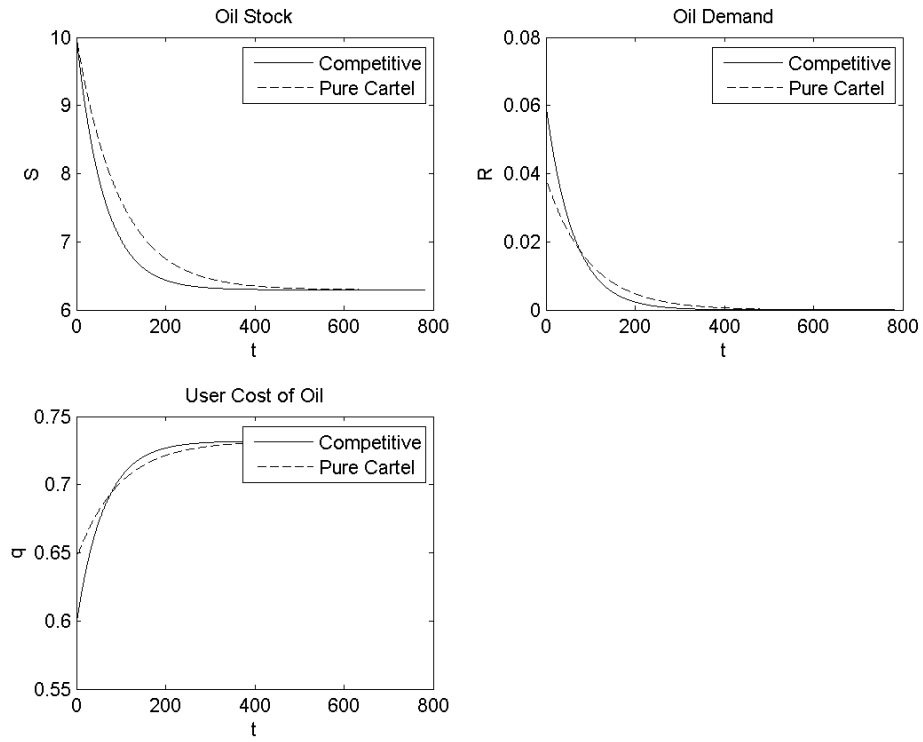


Figure 4: Results for Linear Oil demand - Comparison between the competitive and pure cartel outcome

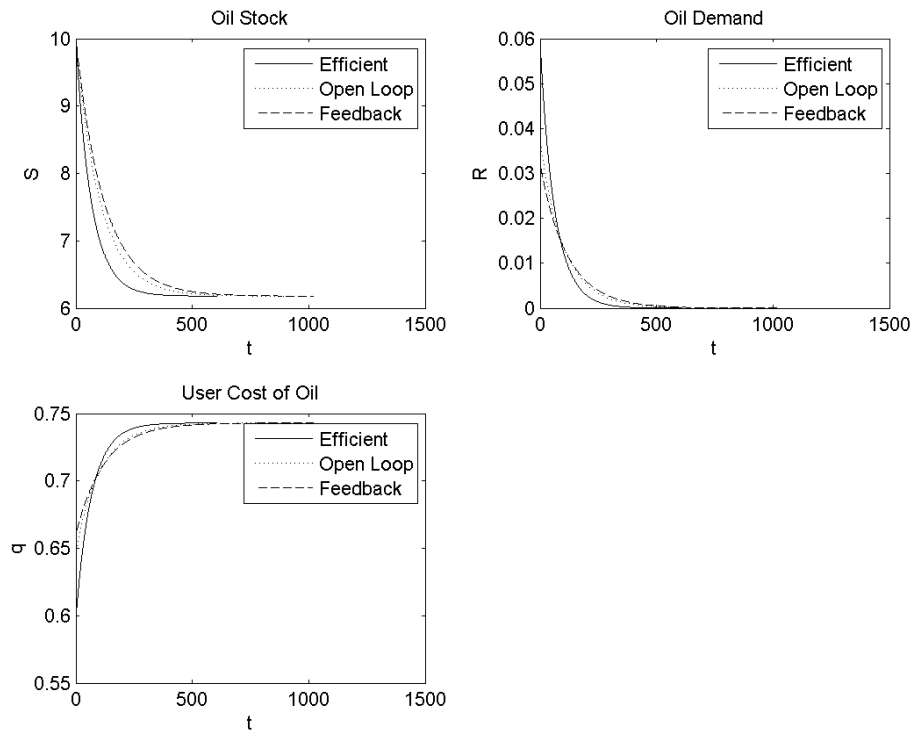


Figure 5: Results for loglinear Oil demand - Comparison between equilibria

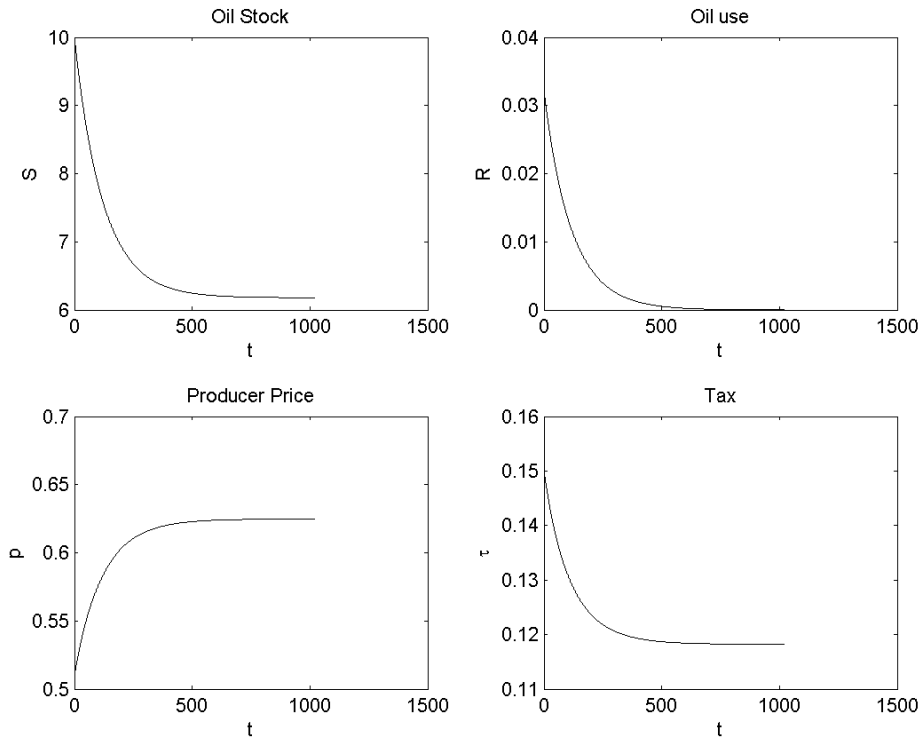


Figure 6: Result for Log linear Oil demand- Feedback equilibrium

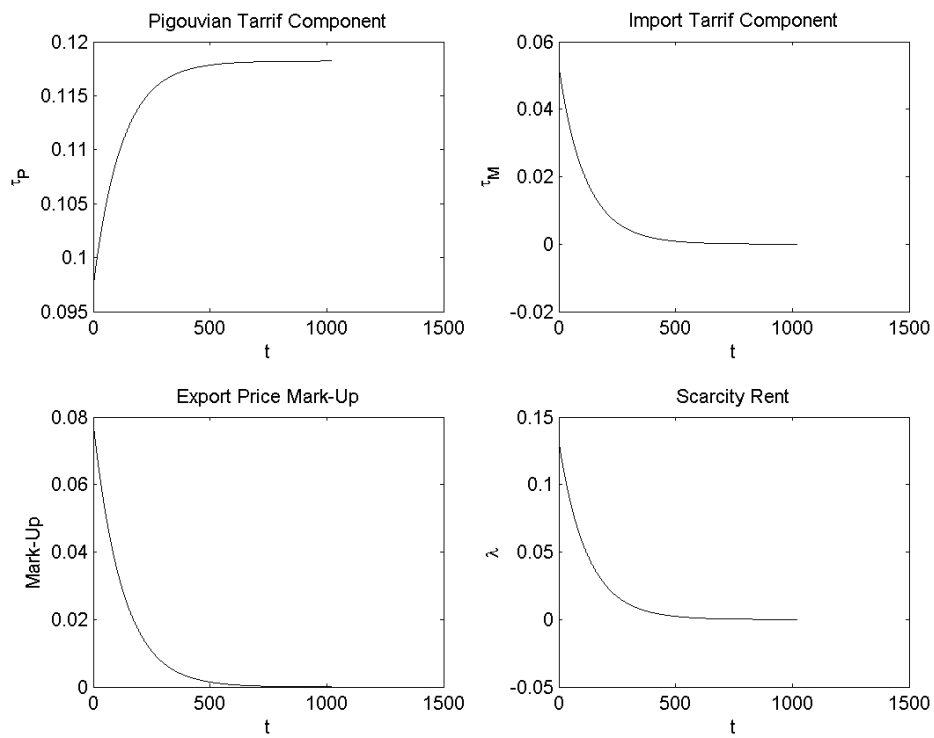


Figure 7: Results for loglinear Oil demand - Price and Tax Components in the Feedback Equilibrium

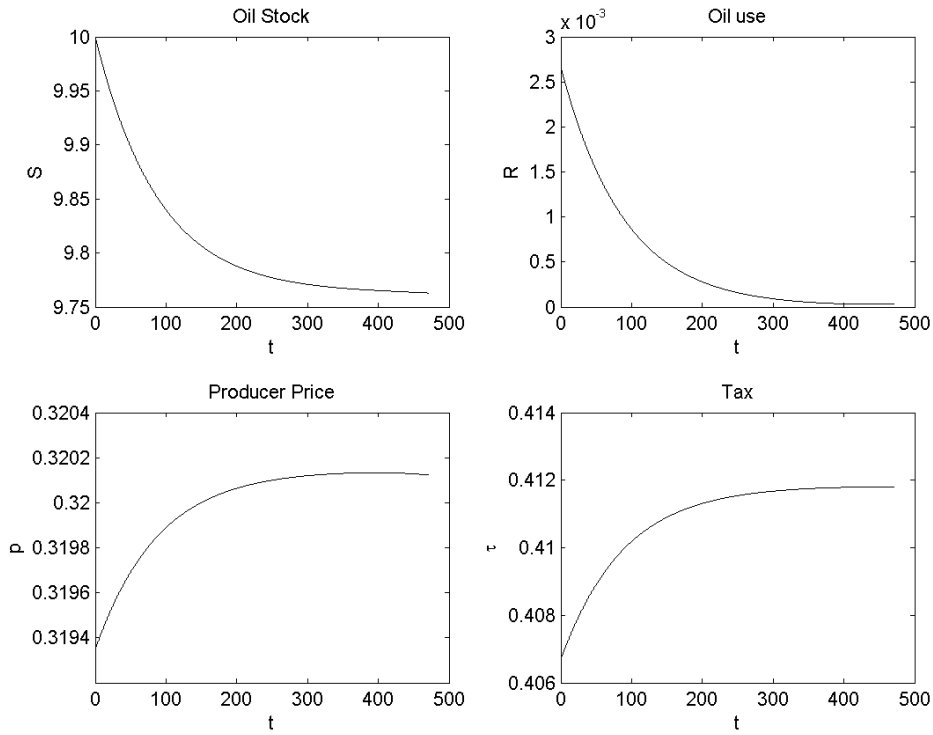


Figure 8: Simulation results for linear oil demand for high severity damages – Feedback Equilibrium

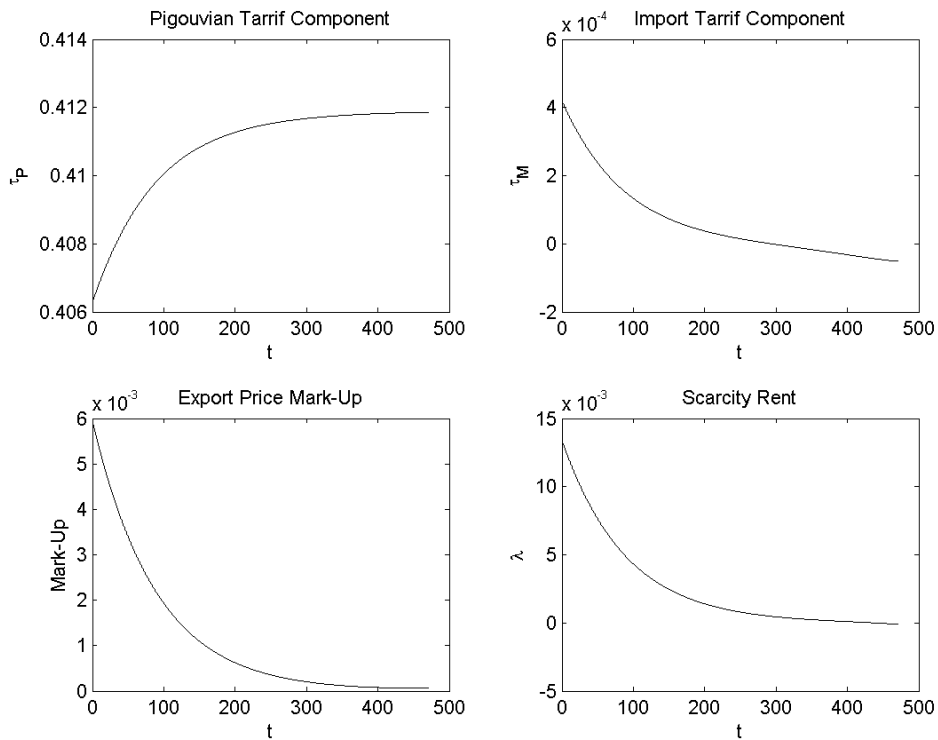


Figure 9: Simulation results for linear oil demand for high severity damages –tax components for the feedback equilibrium

## 5.2. Semi-loglinear Demand

We now examine the case of semi-log linear demand (isoelastic production). Note that solving the case of the semi-log linear demand is less straightforward than linear and log-linear. Because marginal productivity at 0 is infinite, full exhaustion of the oil stock occurs asymptotically. It is for this reason that many studies avoid studying semi-log linear oil demand, sticking to functional forms which lead to partial exhaustion and hence an interior solution. We resolve with a slightly more complicated numerical procedure (See Appendix A.3.1)

The simulations for semi log-linear demand are presented in figures 10 -11. The main result of Liski and Tahvonen (2004) holds – the path of the user cost of oil is less steep for both the feedback Nash equilibrium than for the efficient case. The results differ in the redistribution of rents. As mentioned earlier, for isoelastic production marginal productivity is infinite at 0, hence in the long run the price goes to infinity instead of converging to a finite level. As the oil stock vanishes Oilrabia increases the price of oil, and there are always incentives for Industria to increase the tariff on oil. Hence no matter how severe the damages, the tariff component always increases over time and always dominates the Pigouvian tax component. (see top right panel of figure 10). Thus, for semi log linear demand Industria always steals a significant amount of the Hotelling rent even for low-intensity damages.

The implications for Industria's welfare, however, are ambiguous. As can be seen in Table 2. For semi-loglinear demand, Industria's welfare for feedback Nash equilibrium is lower than the open-loop Nash equilibrium level of welfare (for linear and loglinear demand in the feedback Nash equilibrium outcome this is not the case). The intuition is as follows: the semi log linear demand does not set any upper bound on the reservation price. Thus as Oilrabia and Industria compete to maximize their rent, they set the price too high and end up harming Industria's consumers, decreasing Industria's welfare. This conclusion implies that the use of the carbon tax to steal rents from the importer, is not as beneficial as it has been presented in the literature as for certain demand specifications it can harm the consumers in the importing nation.

## 5.3 Non HARA Class Demand

The numerical simulations have so far confirmed our analytical conclusions: for HARA class production functions the initial extraction rate is smaller in the open-loop Nash equilibrium, and is even smaller in the feedback equilibrium. We now examine a non-HARA class demand specification to determine whether it is possible for the bias to go in the other direction: oil extraction rate being initially higher for the open-loop Nash than for efficient equilibrium. The functional form we use is a shifted semi-loglinear demand function following from the production function:

$$F(R) = \chi R + \frac{R^{1-1/\phi}}{1-1/\phi}. \text{ We begin by determining the conditions which must hold for parameters } \chi$$

and  $\phi$  in order for initial open-loop Nash extraction to be faster than efficient. Recall that in order for

the monopoly to be profitable elasticity must be greater than unity. For the shifted semi-loglinear demand  $\lim_{t \rightarrow \infty} \varepsilon(t) = \phi$ . Therefore,  $\phi > 1$ . Define  $Z(t) = G(S(t)) + D'(E_0 + S_0 - S(t)) / \rho$ . From the first order conditions we obtain the following differential equations. In the open-loop Nash equilibrium:

$$\dot{q}_{OL} = \rho \left[ q_{OL} + \frac{\chi - \phi Z}{\phi - 1} \right].$$

In the efficient outcome:

$$\dot{q}_{EFF} = \rho [q_{EFF} - Z].$$

In order to avoid arbitrage, we need  $\dot{q}_{OL} < \rho q_{OL}$ , corresponding to  $\frac{\chi - \phi Z}{\phi - 1} < 0$ . Consider the

steepness at time 0. Using the above first order conditions we find that the open-loop Nash path satisfies the arbitrage condition and is steeper at time 0 if and only if

$$\frac{\chi - \phi Z(0)}{\phi - 1} > -Z(0) + q_{EFF}(0) - q_{OL}(0).$$

Simplifying this expression, and combining it with the no arbitrage constraint, we obtain the following condition

$$Z(0) < \chi < \phi Z(0) + (\phi - 1)(q_{EFF}(0) - q_{OL}(0)).$$

We now numerically solve the open-loop Nash and efficient equilibrium problem for a range of parameter values with  $\phi > 1$  and  $Z(0) < \chi < \phi Z(0)$  to find such values for which satisfy the above condition. One example of such parameter values is  $\phi = 2.7$ ,  $\chi = 1$ . This parameterization leads to:

$q_{EFF}(0) = 2.571$ ,  $q_{OL}(0) = 2.558$  (note the initial user cost of oil in the open-loop Nash is lower than the efficient one). The paths for the user cost of oil, for initial and long-run extraction periods for both equilibria are presented in Figures 12 and 13, respectively (we present a separate plot for the initial extraction period as this is when monopolist behaves differently than in the HARA class demand case). In the results for the HARA class demand, the open-loop Nash path crossed the efficient path only once. For the shifted semi-loglinear demand however, there are multiple crossing points. The open-loop Nash equilibrium path starts out steeper than the efficient and then flattens out, crossing the efficient path again.

Thus initially the open-loop Nash equilibrium price is lower than the efficient level. This result may seem counter-intuitive: why would the monopolist price oil be lower than the competitive market? The intuition lies in the behaviour of the price elasticity. Imagine that Industria's population consists of two groups – the first has a fairly low reservation price for oil and can substitute away easily while the second has higher reservation price and cannot substitute away from oil. For example, suppose



that Industria consists of city dwellers that can switch to public transport, and country dwellers that have to drive to work. Thus for lower prices, demand is fairly elastic (city dwellers can stop using oil and switch to public transport) but beyond a certain threshold, the elasticity of demand decreases. The monopolist then initially sets the price slightly lower to capture some of the elastic demand (while the extraction cost is low), and then once all the cheap oil is extracted, increases the price to serve exclusively the more inelastic portion of demand. As a result of these strategic dynamics, more oil is extracted initially than in the efficient case. Ergo in the short run the monopolistic power of Oilrabia is bad for the environment. In the long run however the open loop user cost of oil path is flatter than efficient just like for HARA preferences.

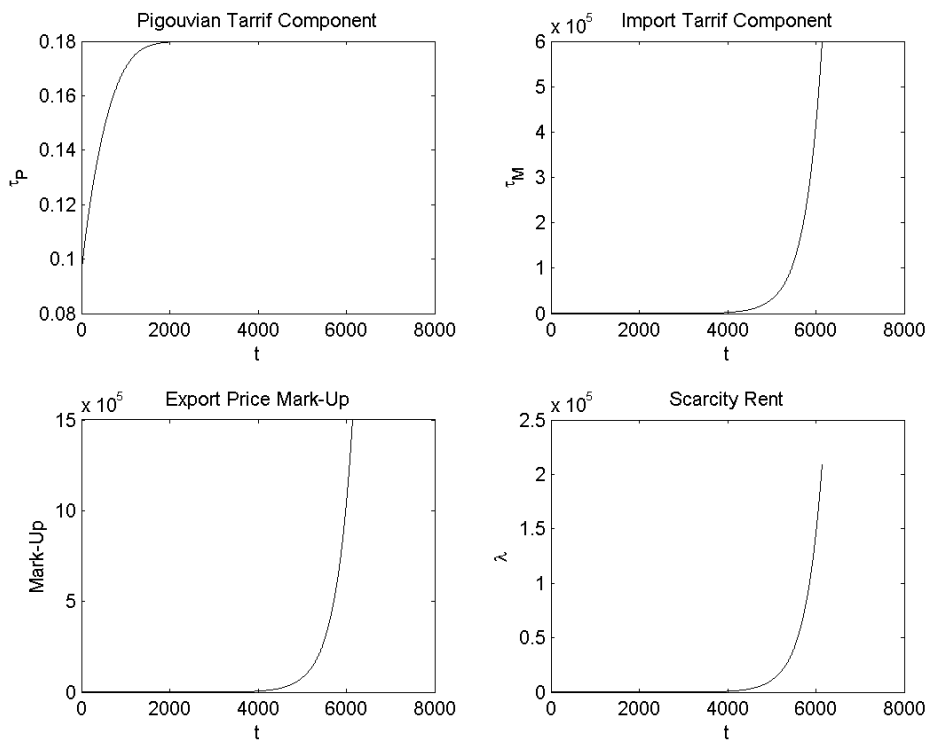


Figure 10: Results for semi-loglinear oil demand for high severity damages. Tax Components in the Feedback Equilibrium

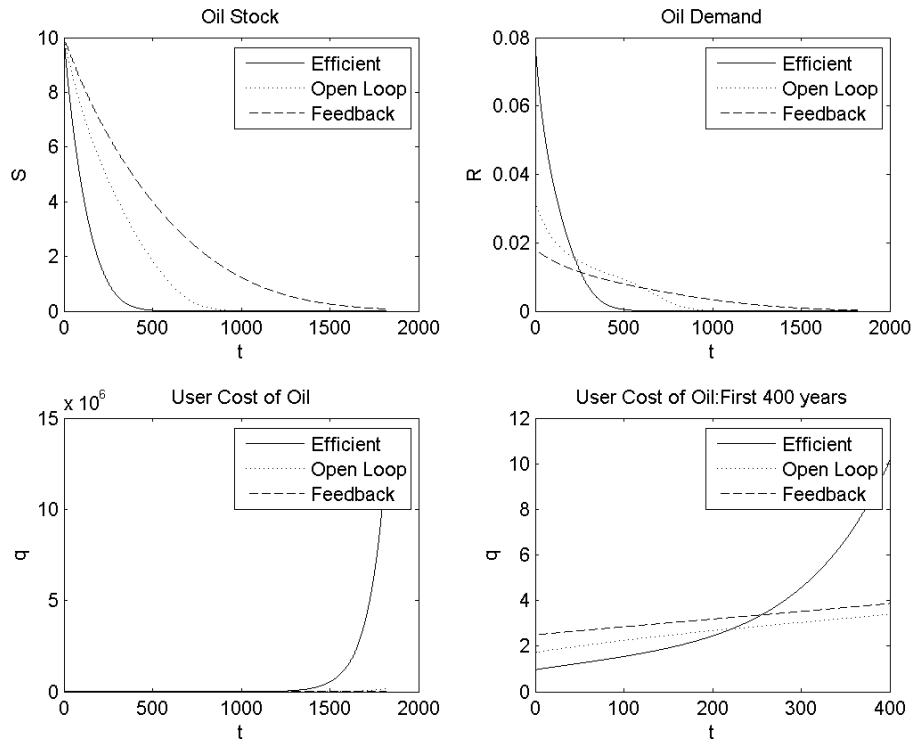


Figure 11: Results for semi-loglinear oil demand – comparison between equilibria.

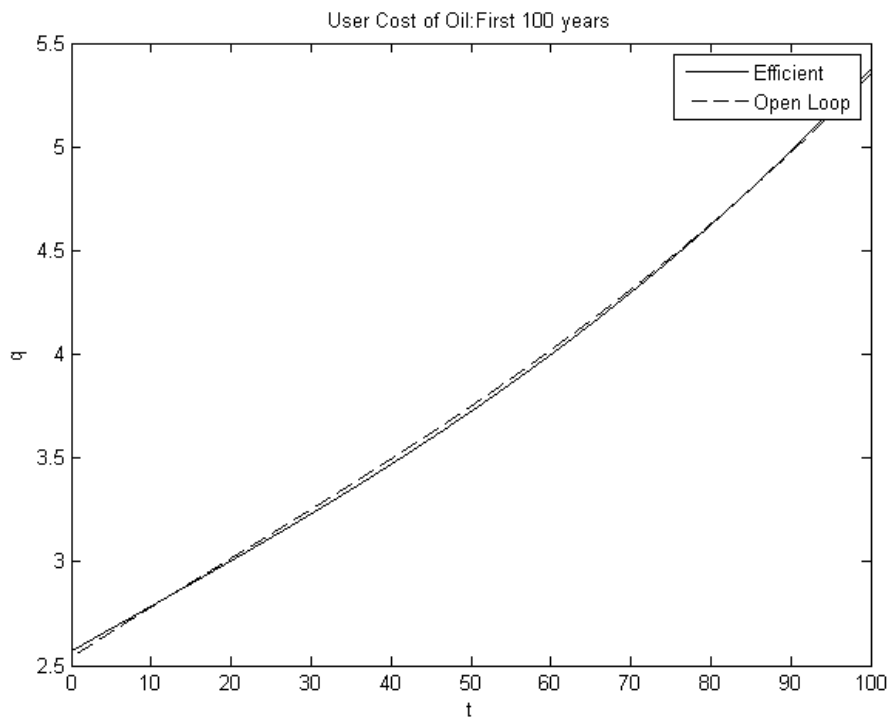


Figure 12: User cost of oil for shifted semi-loglinear oil demand, first 100 years – Comparison between equilibria.

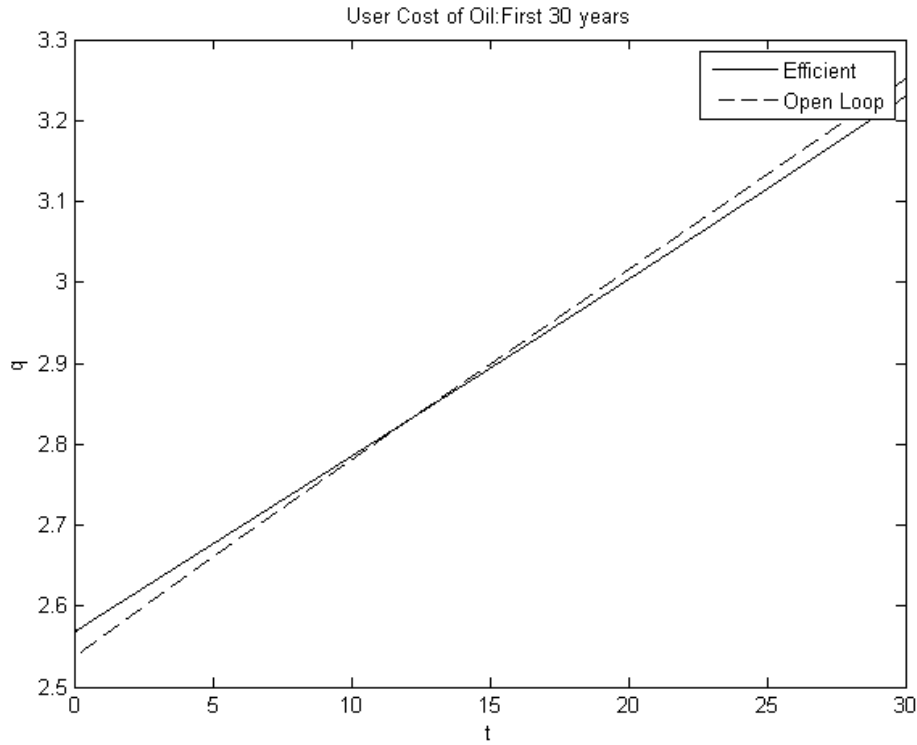


Figure 13: User cost of oil for shifted semi-loglinear oil demand, first 30 years – Comparison between equilibria.

## 6. Concluding Remarks

Curbing fossil fuel use is essential to prevent further global warming. This goal is complicated by the fact that reserves of fossil fuels are highly concentrated in a few non-western oil-and gas-rich nations with considerable monopolistic power. Previous studies (Liski and Tahvonen, 2004, Tahvonen 1995, Tahvonen 1996) have argued that western oil importers might nevertheless successfully limit fossil fuel consumption via a carbon tax. However, monopolistic exporters are likely to respond to the carbon tax by increasing the oil price more than they would have done in a competitive oil market, which would result in even fewer emissions. This way oil exporters can try to capture some of the climate rent. Additionally, the carbon tax can be used as an import tariff to extract scarcity rents from monopolistic oil exporters. This might further benefit the oil importing nation, but the ability of the importer to capture the monopolist’s scarcity rents depends on the intensity of carbon damages. However, the aforementioned studies limit their attention to linear oil demand in order to keep the analysis tractable. The nature of demand for oil can of course have an important effect on how consumers respond to an increase in the carbon tax. The objective of this paper is to examine how the specific nature of oil demand affects the oil importer’s efforts to prevent climate change and capture the monopolist’s rents.

We analyse the problem in the framework of a dynamic game between the monopolist oil exporter and the environmental oil importer and use numerical methods to solve the model for more complex oil demand specifications. We find that for the most common demand functions used in the economic literature the monopolist responds to the carbon tax by raising the price further to extract more rents. Hence, in line with the literature, the monopolist is the conservationist's best friend. Still certain demand specifications change the dynamics of the carbon tax. For example, the popular iso-elastic demand function requires the importer to levy a steadily increasing import tariff to capture more and more of the monopolist's rents. Thus the tariff becomes the main driving force behind the carbon tax, and in the long run leads to a prohibitive total oil price. In the long run such a tariff causes more disutility to the consumer than the amount of rents captured from the monopolist exporters, making the carbon tax less beneficial than Liski and Tahvonen (2004) would lead us to believe. Lastly, we find that for certain non-standard demand specifications, the monopolist will have an incentive to extract more oil initially, to capture the more elastic portion of the demand before the extraction cost and carbon tax increase. Hence, OPEC can sometimes be the environmentalist's worst enemy depending on the nature of the demand for oil in the developed world.

Our main conclusion is that the demand structure plays a significant role when determining the optimal carbon tax or import tariff for foreign oil. Given the number of papers which choose their demand function based on computational convenience these results serve as a cautionary tale: demand specifications chosen out of simplicity rather than reflecting reality may make a carbon tax seem more or less beneficial in the context of strategic interactions in regards to exhaustible resources.

While this study generalizes the results of previous studies by examining a range of demand specifications it remains limited in its scope. For instance, the model we examine excludes the possibility of saving and capital accumulation and a less stylized carbon cycle dynamics. Without the capital stock the economy collapses after oil runs out, a highly unrealistic outcome. Meanwhile, without a more sophisticated model of the carbon cycle, all CO<sub>2</sub> that is emitted into the atmosphere stays there forever, thus over-estimating the amount of climate change. To remedy these problems we would have to construct a model with additional state variables (for example, using the two reservoirs that Golosov et al. (2014) uses to model the carbon cycle). The player's strategies would then be based on the full state space the two CO<sub>2</sub> reservoirs and the oil stock. Finding the feedback equilibrium in this more complex setup would not only make our conclusions more realistic but also provide a contribution to the dynamic game literature through solving for a subgame-perfect Nash equilibrium with multiple state variables, one of which represents a fully-exhaustible resource. Another interesting extension would be to introduce multiple oil exporters into the model, similar to Groot et al. (2000, 2003) and Benchekroun et al. (2009, 2010). Adding a fringe in addition to the cartel exporter is a better model of reality and would lead to interesting dynamics with the cartel and

the fringe responding differently to the importer's carbon tax. We leave these extensions of our work for further research.

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## A. Appendix

### A.1 Comparing Open-loop Nash and efficient equilibria: HARA class production functions

Let us consider the class of HARA demand functions. This class of production functions is given by

$$F(R) = \frac{1-\varphi}{\varphi} \left[ \left( \frac{\psi R}{1-\varphi} + \chi \right)^\varphi - \chi^\varphi \right], \text{ where } \psi > 0. \text{ We restrict the analysis to the area of the demand}$$

where  $\varepsilon > 1$ , otherwise it is unprofitable for the monopolist to enter the market. We now examine how the open-loop Nash equilibrium dynamics compare with the efficient equilibrium for a few special cases of this class.

1. *Quadratic production function:*  $\varphi = 2$ , so  $F(R) = -\frac{1}{2}(\chi - \psi R)^2 + \frac{1}{2}\chi^2$  with  $\chi > 0$ . This

gives linear demand  $q = \chi\psi - \psi^2 R$  where  $\frac{1}{2}\chi\psi < q < \chi\psi$ , so that  $\varepsilon = \frac{q}{\chi\psi - q} > 1$ . and  $R'' = 0$ .

Equation (13) becomes  $\dot{q} = \frac{1}{2}\rho[2q - \chi\psi - G(S)] - \frac{1}{2}D'(E)$ . It follows from (16) and  $q < \chi\psi$  that the path for the consumer price of oil under the open-loop Nash equilibrium outcome is flatter and oil extraction is smaller initially than in the efficient outcome (7). This was known for the case of no extraction cost and no climate damages. But, it also holds more generally, under quadratic preferences.

2. *Power production function:*  $\varphi < 1, \chi > 0$  and  $\psi = 1 - \varphi$ , so  $F(R) = \frac{1-\varphi}{\varphi} \{(R + \chi)^\varphi - \chi^\varphi\}$ .

Defining  $y \equiv \left(\frac{q}{1-\varphi}\right)^{\frac{1}{\varphi-1}}$ , oil demand is  $R = y - \chi$  and thus  $\varepsilon = \frac{y}{(1-\varphi)(y-\chi)} > 1$  and

$R''(q)R(q) / R'(q)^2 = (2-\varphi)(y-\chi) / y$ . Equation (13) becomes

$$\dot{q} = \rho q \left[ \frac{\varphi y + (1-\varphi)\chi - y \{G(S) + D'(E) / \rho\}}{\varphi y - (\varphi-2)\chi} \right]. \text{ Using (16), } 0 < \varphi < 1 \text{ and } \chi \geq 0 \text{ we find that the price}$$

path in open-loop Nash equilibrium is flatter than in the efficient outcome (7). The case  $\varphi < 0$  with

$\chi = 0$  is excluded, because then  $q = \rho q \left[ 1 - \frac{1}{\varphi} \{G(S) + D'(E) / \rho\} \right] > \rho q$  can be ruled out on arbitrage arguments.

3. *Logarithmic production function:*  $\varphi \rightarrow 0$  and  $\psi = 1$ , so  $F(R) = \log(R + \chi) - \log(\chi)$ . Hence, oil demand is  $R = \frac{1}{q} - \chi$ . Positive oil use requires  $\chi q < 1$ , so have  $\varepsilon = \frac{1}{1 - \chi q} > 1$  provided  $\chi > 0$  and

$R''(q)R(q) / R'(q)^2 = 2(1 - \chi q)$ . Equation (13) gives  $\dot{q} = \frac{\rho[\chi q^2 - G(S)] - D'(E)}{2\chi q}$ . Now, the open-

loop Nash equilibrium oil price path is flatter than in the efficient outcome (7) if and only if

$\left(2 - \frac{1}{\chi q}\right) \frac{G(S) + D'(E) / \rho}{q} < 1$ . This inequality holds in view of (16) and  $\chi q < 1$ . Clearly, the

monopolist outcome also leads to flatter price paths than the competitive market.

4. *Exponential production function:*  $\varphi \rightarrow -\infty$  and  $\chi = 1$ , so that  $F(R) = 1 - e^{-\psi R}$ . Oil demand is  $R = -\frac{1}{\psi} \ln\left(\frac{q}{\psi}\right)$ . We thus get  $\varepsilon = -1 / \ln\left(\frac{q}{\psi}\right) > 1$  and  $R''(q)R(q) / R'(q)^2 = -\ln(p / \psi)$ . Equation

(13) becomes  $\dot{q} = \frac{\rho \left[ \left(1 + \ln\left(\frac{q}{\psi}\right)\right) q - G(S) \right] - D'(E)}{2 + \ln\left(\frac{q}{\psi}\right)}$ . The oil price path is flatter than in the efficient

outcome (15) if and only if  $\left[1 + \ln\left(\frac{q}{\psi}\right)\right] \frac{G(S) + D'(E) / \rho}{q} < 1$ . This inequality holds in view of (16) and

since  $\ln\left(\frac{q}{\psi}\right) < 0$  holds to have positive oil demand.

## A.2 Calibration details

The IEA (2008) long-term cost curve for oil puts oil reserves at around 10,000 billion barrels, with initial extraction cost between \$5 and \$30 per barrel and the highest cost reserves at \$115 per barrel. A barrel of oil contains 5.80 mmbtu and each mmbtu is equal to 20.31 kg CO<sub>2</sub> (EPA, 2013), so a barrel of oil is 115 kg of carbon or around 1/10 ton of Carbon (TC). All of our emission variables are expressed in 1000 Gigaton Carbon (GtC) and prices/costs are expressed in \$1000 USD/TC. Initial CO<sub>2</sub> concentration is set to approximately 800 GtC (EPA, 2013).

We then calibrate the production functions by solving the model for a variety of parameters and checking so that they correspond to real-world data. We calibrate the CARA and Quadratic utility functions so that for zero damages, initial prices and emissions are equal to their 2005 levels. We

calibrate the iso-elastic production function so that initial prices and emissions are equal to their 2010 levels (demand amounts are re-scaled to adjust for the population growth between 2005 and 2010). We set the interest rate and the rate of time preference at 1%.

### ***A.3 Solving for equilibria numerically***

The dynamic equilibrium conditions is described by differential equation (13) for the open loop equilibrium and (27) for the feedback Nash equilibrium. Because we have only one state variable, both problems collapse into a set of differential equations with respect to time, so hence we can use the same numerical method for both equilibria. We use a Runge-Kutta shooting algorithm, simulating the open loop and feedback equilibrium user cost of oil using (13) and (27) respectively and (5) to simulate the path of the oil stock. We do not know the initial value for the user cost of oil  $q$ , so to find it we introduce a final condition, which must hold when the oil extraction is finished.

For the quadratic and exponential production function we assume that the marginal productivity at zero is bounded at a low enough value such that some oil is left in situ. Thus the final time condition for the oil stock is:

$$(28) \quad G(S(T)) + D(E_0 + S_0 - S(T)) / \rho = U'(0)$$

(For the isoelastic and non-HARA production case where the marginal productivity of oil is infinite, the final condition becomes more complex. We describe how to derive the final condition in subsection A.3.1 below). Hence (28) is the final condition which must hold for both the open loop and feedback equilibria. We use a binary search algorithm to find initial price  $q(0)$ , which produces a extraction path that satisfies (28). We then use (9) or (25) to back out the open loop or feedback Nash equilibrium carbon tax respectively.

#### ***A.3.1 Solving for equilibria with infinite marginal utility at zero***

If marginal utility at zero is infinite, then asymptotically oil is fully exhausted, but we never actually get to a point where  $R = 0, S = 0$ . Thus we must find a way to derive a different final condition.

Let oil demand growth be  $g(t) = \frac{\dot{R}(t)}{R(t)}$ . Note that as the oil price increases demand decreases so

demand growth is actually negative. Now assume that as time goes to infinity, the demand growth rate converges to some finite value  $\lim_{t \rightarrow \infty} g(t) = \bar{g}$  (while we don't have a final proof of this conjecture, we can show that numerically this is the case). Let  $T$  be the time such that  $\dot{g} = 0$  i.e.,  $g(T) = \bar{g}$ .

Then it must hold that for all  $t > T$

$$(29) \quad R(t) = e^{\bar{g}(t-T)} R(T).$$



Substituting (29) in the differential equation for the oil stock (5) we get the condition for the stock of oil at time  $T$

$$(30) \quad S(T) = R(T) \int_{t=T}^{\infty} e^{\bar{g}(t-T)} dt = \frac{R(T)}{\bar{g}}$$

We can now use the same Runge-Kutta algorithm that we used for quadratic and exponential production function. We find time  $T$  numerically by using the Runge-Kutta method to shoot until the growth rate of demand converges, and use (30) as the final condition to check whether the oil extraction path satisfies the equilibrium conditions.