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Abstract

We study the influence of reason and intuition on decision making over time. Facing a sequence of similar problems, agents can either decide rationally according to expected utility theory or intuitively according to case-based decision theory. Rational decisions are more precise but create higher costs, though these costs may decrease over time. We find that intuition will outperform reason in the long run if individuals are sufficiently ambitious. Moreover, intuitive decisions are prevalent in early and late stages of a learning process, whereas reason governs decisions in intermediate stages. Examples range from playing behavior in games like Chess to professional decisions during a manager's career.

JEL-Code: D810, D830, C630.

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1 Introduction

This paper studies the influence of reason and intuition on decision making over time.

When we were children and started playing Chess, we had no idea how to evaluate a certain position and move appropriately. We just played on a gut level and sometimes tried to mimic the moves of our more experienced opponents. Nevertheless, we made some progress on this trial and error basis. As we advanced, we started to use our brain. We increasedly tried to compute moves in advance and consult books in order to inform ourselves about promising variations and strategic concepts. While the first author of this paper has never left this stage of the learning process, the second author has become a chess grandmaster. As an experienced master, now again, his play is largely based on intuition. Recognizing patterns and using the corresponding best practices in order to decide on most of the moves allows for saving time and cognitive capacities.

You may have made similar experiences as a player of games like Chess or as a professional decision maker in the role of an author, editor, or manager. Human decision behavior often exhibits the following time pattern: Intuitive decisions are prevalent in both early and late stages of a learning process, whereas reason governs decisions in intermediate stages. The model developed here explains this pattern. Moreover, it allows to analyze how the prevalent use of reason or intuition reacts to variations in the complexity of the decision problem on the one hand and the decision maker's cognitive abilities on the other hand.[1]

In our setup, people face a sequence of similar decision problems. They can decide either rationally or intuitively. Rational decisions are modeled – according to expected utility theory (EUT, von Neumann and Morgenstern, 1944) – as the result of expected utility maximization. In contrast, intuitive decisions are modeled – according to case-based decision theory (CBDT, Gilboa and Schmeidler, 1995) – as the result of case based optimization (Gilboa and Schmeidler, 1996). In the latter case, the agent bases the decision on a comparison between alternatives from his memory set (case record) trying to satisfice a certain aspiration level. While the expected payoff from such behavior is lower, our simulations show that it converges to expected utility as the number of cases encountered increases and the aspiration level is adjusted in a realistic but ambitious way. Put differently, rational decisions are more precise than intuitive ones initially, but this advantage vanishes

[1] We are aware of the fact that reason and intuition are correlated in some way. This aspect is taken up in a heuristic analysis in von Weizsäcker (2010); see also Kindermann and von Weizsäcker (2013).

with a growing stock of experience.

In order to combine both theories, EUT and CBDT, within a single framework, we follow a transaction costs approach. Rational decision making creates higher costs than the use of intuition. Though this cost differential may be decreasing over time, it does not vanish. Under these assumptions, we compare the expected utilities net of costs from EUT and CBDT in order to assess whether a rational or intuitive decision is preferable at a given point in time. Depending on the cost structure, which reflects both, the complexity of the decision problem and the decision maker's cognitive abilities, we identify three typical scenarios:

First, if costs are initially high and slowly decreasing, i.e. if the complexity of the problem is high and the decision maker's learning curve is flat, it never pays to engage in reasoning. In this case, decisions will always be based on intuition.

Second, if costs are initially high but decreasing fast, i.e. if the complexity of the problem is high but the decision maker's learning curve is steep, we recognize the pattern described above for the example of Chess. After having experimented on a trial and error basis for a number of times, one has collected enough information to gain from reasoning. However, since reasoning remains costly whereas intuition further improves as the number of problems encountered increases, from some point on it will be beneficial to save on reasoning costs and rely on the growing experience and pattern recognition capability instead.

Third, if costs are low even initially, i.e. if the complexity of the problem is low, no initial stage of experimentation occurs. As an example among board games, think of Tic Tac Toe, the complexity of which is very low compared to Chess. It pays to engage in reasoning right from the start till the point at which, again, the agent finds it preferable to save on the related costs and rely on his vast experience instead.

Moreover, these time patterns exhibit a common feature that constitutes our main result: In the long run, intuitive decision making will always yield a higher net expected utility than reasoning if the decision maker is sufficiently ambitious.[2]

2 Related Literature

In this paper, we adopt expected utility theory (EUT) as the standard model of rational choice (von Neumann and Morgenstern, 1944). Over the last

[2]This result also relates to the question under which circumstances heuristic decisions are preferable to rational decisions; see e.g. Gigerenzer and Gaissmaier (2011).

decades, however, the classical view of purely rational economic agents has been challenged by behavioral models of bounded rationality. From its outset, the psycho-economic literature has discussed the role of reason and intuition in decision making.

Simon (1955, 1987) refers to rational decision making as some kind of *optimizing* behavior that is purely logical and consciously analytic. By contrast he describes intuition as subconscious pattern recognition, and intuitive decision making as a judgement that is based on some form of *satisficing* behavior (Frantz, 2003). Unlike classical economics, he proposes a behavioral model of rational choice in which agents do not maximize expected utility but try to reach some satisfactory aspiration level (Simon, 1955).

Gilboa and Schmeidler (1995) refer to such satisficing behavior when they present their case based decision theory (CBDT). The respective decision rule is based on a certain mode of pattern recognition and may be interpreted as a best practice approach: Faced with a specific problem, people choose the action that has proved best in similar situations in the past. To this end, the performance of each action is evaluated by the utility levels that resulted from using this action in past cases, each weighted by the similarity of that past case to the actual one.

In a dynamic context, the case based decision maker usually does not come close to the action that maximizes expected utility but is satisfied with any action that meets some aspiration level. However, Gilboa and Schmeidler (1996) show that if the aspiration level is adjusted over time in a realistic-but-ambitious way, agents will asymptotically choose almost only actions that maximize expected utility when faced with an infinite sequence of identical problems.[3] The adjustment is called realistic if the aspiration level relates to the average of its previous value and the best average performance so far. It is called ambitious if it is higher than the maximum average performance sufficiently often. This procedure guarantees that the decision maker initially experiments often enough to learn the optimal action in the sense of EUT. Gilboa and Schmeidler (1996) hence refer to it as case based optimization.

In our setup, we choose CBDT to model intuitive decisions and case based optimization to describe the evolution of intuition over time. Doing so we embody not only the above ideas of pattern recognition and satisficing behavior. We also prepare the ground for a fair comparison of rational and intuitive decision making. Unlike other economic models of learning, case based optimization eventually leads to actions that maximize expected util-

[3]Guerdjikova (2008) specifies conditions under which this result also holds for a sequence of problems with a lower degree of similarity.

ity.[4] Hence, at least in the long run, intuitive decisions based on CBDT do not per se fall short of rational decisions based on EUT.

Gilboa and Schmeidler (1995) discuss at length the relation between EUT and CBDT. [5] In particular, they state the following conjecture with respect to the applicability of the two models:

> "Classifying problems based on their novelty, one may consider three categories. We suggest that CBDT is useful at the extremes of the novelty scale, and EUT in the middle." (Gilboa and Schmeidler, 1995, page 622)

We reinterpret the "novelty scale" as the time line and measure time by the number of similar problems the decision maker has encountered so far. In order to verify their conjecture, we incorporate both EUT and CBDT into a single model that links the two theories by means of a transaction cost approach. The corresponding cost structure reflects the psycho-economic characterization of rational and intuitive decision making, respectively. Describing the cognitive systems, Kahneman (2003, Figure 1) assigns seven attributes to both, the process of decision making based on intuition (system 1) and reasoning (system 2).

On the one hand, he characterizes intuition as "fast", "parallel", "automatic", and "effortless", whereas reasoning is specified as "slow", "serial", "controlled", and "effortful". A comparison of this first group of attributes mirrors the common view of psychology that reasoning consumes more time and effort for collecting and processing information than intuition. In our model, we account for this agreement assuming that rational decisions according to EUT always create substantially higher costs than intuitive decisions according to CBDT. Put differently, the cost differential between rational decisions according to EUT and intuitive decisions according to CBDT is always strictly positive.

On the other hand, intuition is also found to be "associative", "slow-learning", and "emotional", whereas reasoning is characterized as "rule-governed", "flexible", and "neutral". Comparing this second group of attributes highlights the psychological finding that it is much harder to control or modify the process of intuitive decision making than that of rational decision making. While reasoning may be adjusted almost instantaneously, modifications of intuitive processes seem to result from evolution and take

[4] Sarin and Vahid (1999) and Börgers and Sarin (2000) provide similar models of (reinforcement) learning that do not necessarily lead to expected utility maximizing decisions. Sobel (2000) offers a comprehensive overview of economic learning models.

[5] Matsui (2000) proves an equivalence result between EUT and a modified version of CBDT.

much more time. We take account for this fact in our model assuming that the cost differential between rational decisions according to EUT and intuitive decisions according to CBDT is (weakly) decreasing over time.

The well established idea that reasoning resources are scarce and that people rather base their decisions on analogies in order to save cognitive costs is also a basic ingredient in the work of Samuelson (2001). Though he uses a game theoretic setup of strategic interaction within a static environment, the aim of his paper is similar to ours. He also asks the question under what circumstances more costly rational behavior is preferred over decision making based on simple analogies. He shows that this will be the case if the interaction is sufficiently important, distinct, and frequently encountered. By contrast, in our model, we focus on the specific circumstance of familiarity with the decision problem in the course of an agent's learning process. Doing so we ask how the preferability of one mode of decision making over the other evolves over time and how this time pattern depends on the complexity of the problem as well as the cognitive skills of the decision maker.

3 The Model

We build on the model of Gilboa and Schmeidler (1996). The decision maker faces a sequence of isomorphic problems.[6] In each period $t \in \mathbb{N}_0$ he chooses an act a from a finite set of alternative decisions $A = \{1, \ldots, n\}$. The utility resulting from act $a \in A$ in period $t \in \mathbb{N}$ is a random variable $X_{a,t}$ which is independent and identically distributed over time according to the distribution function F_a on $\mathbb{R}$. We assume that each F_a has finite first and second moments, which we refer to as the expected utility

$$\mu_a = E(X_{a,t}) = \int X_{a,t} dF_a = \int u_t dF_a(u_t)$$

and utility variance σ_a of act a, respectively. Here, u_t denotes the realization of $X_{a,t}$.

3.1 Rational decisions

We call a decision *rational* if it is based on the maximization of expected utility. Since the problem encountered is isomorphic in each period $t \in \mathbb{N}_0$, a rational decision maker always chooses some act $a_t \in \arg\max_{a \in A} \mu_a$ that

[6] I.e. essentially identical problems or, more formally, problems of similarity 1 in the language of Gilboa and Schmeidler (1995).

maximizes expected utility. Hence, the expected utility of a rational decision in period $t \in \mathbb{N}_0$ is constant over time:

$$EU(t) = \max_{a \in A} \mu_a =: EU.$$

3.2 Intuitive decisions

3.2.1 Case based decisions

We call a decision *intuitive* if it is case based in the sense of Gilboa and Schmeidler (1995, 1996). Since in our model the decision problems are isomorphic over time, a case is fully characterized by a pair of an act chosen and a utility realized. The cases actually encountered by the decision maker until period t are collected in his memory $M_t := \{(a_1, u_1), \dots, (a_{t-1}, u_{t-1})\}$. In each period t, the case based decision rule is choosing the act with the best cumulative performance so far. To be more precise, let $T_t(a) := \{\tau < t | a_\tau = a\}$ denote the set of periods preceding t in which a has been chosen. Moreover, define the cumulative performance of act a at period t by

$$U_t(a) = \sum_{\tau \in T_t(a)} (u_\tau - h_t),$$

where $h_t \in \mathbb{R}$ denotes the agent's aspiration level in period t.[7] Hence, in each period t, the case based decision maker chooses $a_t \in \arg\max_{a \in A} U_t(a)$. If $\arg\max_{a \in A} U_t(a)$ contains more than one element, we will assume that any $a_t \in \arg\max_{a \in A} U_t(a)$ is chosen with the same probability.

3.2.2 Case based optimization

We interpret the process of case based optimization (Gilboa and Schmeidler, 1996) as the evolution of intuition over time. It can be understood as a form of reinforcement learning that consists of two basic elements: A growing memory set and the adaption of aspiration levels. We characterize them formally describing the corresponding decision paths as appropriate subsets of $S_0 := (\mathbb{R} \times A \times \mathbb{R})^{\mathbb{N}}$. First, taking into account that the case based decision maker chooses $a_t \in \arg\max_{a \in A} U_t(a)$ in each period, the relevant paths form a subset of

$$S_1 := \{\omega = (h_t, a_t, u_t)_{t \in \mathbb{N}} \in S_0 | (\forall t \in \mathbb{N}) : a_t \in \arg\max_{a \in A} U_t(a)\}.$$

[7]Note that with $\sum_{\varnothing} = 0$, the cumulative performance of any act that has not been tried before is zero.

Second, in order to describe the adaption of aspiration levels, let

$$\bar{u}_t(a) := \frac{\sum_{\tau \in T_t(a)} u_\tau}{|T_t(a)|}$$

denote the average utility in period t obtained from past choices of a if $T_t(a) \neq \varnothing$, and

$$\bar{u}_t := \max_{a \in A} \{\bar{u}_t(a) | T_t(a) \neq \varnothing\}$$

the maximum average utility in period t from acts tried so far. Gilboa and Schmeidler (1996) propose two different rules for a realistic but ambitious adjustment of aspiration levels leading to the following sets of relevant paths:

$$S := \left\{ \omega \in S_1 \,\middle|\, \begin{array}{l} h_1 = h^o \\ h_t = \alpha h_{t-1} + (1-\alpha)\bar{u}_t \quad \text{for} \quad t \geq 2 \end{array} \right\}, \tag{1}$$

$$S' := \left\{ \omega \in S_1 \,\middle|\, \begin{array}{lll} h_1 = h^o, & & \\ h_t = \bar{u}_t + h & \text{if} & t \geq 2, t \in N \\ h_t = \alpha h_{t-1} + (1-\alpha)\bar{u}_t & \text{if} & t \geq 2, t \notin N \end{array} \right\}. \tag{2}$$

Ambition of the decision maker is expressed by the initial aspiration level $h^o \in \mathbb{R}$ and, for the learning process in (2), additionally by a positive constant $h > 0$ by which the aspiration level is increased in any period of ambition $t \in N$, where $N \subset \mathbb{N}$ is a sparse set. Realism of the decision maker is expressed by the speed of adaption $\alpha \in (0, 1)$ at wich the actual aspiration level is adjusted to past performances.

3.2.3 Convergence

Gilboa and Schmeidler (1996) show that realistic but ambitious adjustment of aspiration levels as specified in (1) or (2) eventually lead to optimal decisions in the following sense: Let P and P' be probability measures on S and S', respectively, which are consistent with $(F_a)_{a \in A}$. Denote by

$$\pi(a) := \lim_{t \to \infty} \frac{|T_t(a)|}{t}$$

the frequency at wich act a is chosen on a certain decision path[8] if the limit on the right hand side exists. Then, for any $\varepsilon > 0$ there is some $\bar{h}$ such that for any $h^o \geq \bar{h}$, the probability of being on a path $\omega \in S$ on which almost only acts that maximize expected utility are chosen exceeds

[8]Note that each of the variables $a_t, u_t, h_t, T_t(a), \bar{u}_t(a), \bar{u}_t, \pi(a)$ depends on the actual path w. To simplify the notation, we omit this dependence here.

$1-\varepsilon$ (Gilboa and Schmeidler, 1996, Theorem 1). Moreover, on almost any path $\omega \in S'$, almost only acts that maximize expected utility are chosen, i.e. $\pi(\arg\max_{a\in A}\mu_a) = 1$ (Gilboa and Schmeidler, 1996, Theorem 2). Loosely speaking, these results show that one can be arbitrarily sure to eventually pick only acts that maximize expected utility.

The notion of convergence underlying these limit considerations is peculiar in the sense that it takes a vertical view: It randomly picks a path and traces the frequency at which expected utility maximizing acts are chosen along this path. Though interesting as such, this measure of convergence will be of no avail if we want to compare the performances of rational and intuitive decisions over time. Instead, for such a comparison, we have to adopt a more standard notion of convergence that rests upon a horizontal view allowing to answer questions like: Looking across all possible paths, what is the probability of choosing expected utility maximizing acts at a given point in time t, and how does this probability evolve as t increases? Or, closely related, does the expected utility from an intuitive decision in period t, denoted by

$$CBU(t) := E_P(X_t) = \int X_t dP = \int u_t dP(u_t),$$

converge to the maximum expected utility, i.e. the expected utility from a rational decision EU? Here, $X_t : \Sigma \to \mathbb{R}$ with $\Sigma \in \{S, S'\}$ is the random variable that maps each path $w = (h_\tau, a_\tau, u_\tau)_{\tau\in\mathbb{N}}$ to its corresponding utility u_t in period t. Indeed, we conjecture that besides vertical convergence in the sense of Gilboa and Schmeidler (1996) there is also horizontal convergence in the following sense:

Conjecture 1 *Suppose that the process of case based optimization is characterized by*

(a) (1). Then for all $\alpha \in (0,1)$ and for all $\varepsilon > 0$ there is some $\bar{h} \in \mathbb{R}$ such that for any $h^o \geq \bar{h}$ there exists $t_{\varepsilon,h^o} \in \mathbb{N}$ such that $EU - CBU(t) < \varepsilon$ for all $t \geq t_{\varepsilon,h^o}$.

(b) (2). Then $\lim_{n\to\infty} \frac{1}{n}\sum_{t=1}^{n} CBU(t) = EU$ for all $\alpha \in (0,1)$, $h^o \in \mathbb{R}$, and $h > 0$.

Suppose the process of case based optimization is characterized by (1). If the agent is sufficiently ambitious, i.e. her initial aspiration level is sufficiently large, she will experiment sufficiently often so as to end up with expected utility maximizing acts on sufficiently many paths. Hence, the expected performance of an intuitive decision gets sufficiently close to the performance of a rational decision.

If instead the process of case based optimization is characterized by (2), part (a) of Conjecture 1 does not hold. Since the agent will restart to experiment in every period $t \in N$, one always finds some $\varepsilon > 0$ such that for infinitely many points in time $EU - CBU(t) > \varepsilon$. Therefore, we have to formulate the convergence in part (b) of Conjecture 1 in terms of average expected utility from intuitive decisions till period n. The difference between EU and $\frac{1}{n}\sum_{t=1}^{n} CBU(t)$ then measures the expected performance gap between rational and intuitive decision making on average till period n.

Though we have not yet been able to formally prove the respective statements, we have been running a large number of simulations all of which confirm them. In order to illustrate the process of case based optimization, some of our simulation results can be found in Appendix A.

Loosely speaking, Conjecture 1 states that in the long run, an intuitive decision maker will asymptotically perform as well as a rational decision maker in terms of expected utility if he is sufficiently ambitious. In order to be able to use the expected utility from an intuitive decision in period t as the respective performance measure instead of its average, in what follows we will rely on (1) for characterizing the process of case based optimization.

Assumption 1 *The process of case based optimization is characterized by the paths of S as given in (1).*

3.3 Cost of decisions

So far we have not taken into account the costs of decision making. Such costs might stem from the opportunity costs of the time spent during the decision process, the disutility of cognitive effort, or the physical costs of collecting the required information. As stated above, the common view of psychology is twofold. On the one hand reasoning consumes more time and effort for collecting and processing information than intuition. Hence, the costs of rational decision making are higher than the costs of intuitive decision making. On the other hand, it is much easier to control or modify the cognitive system that governs reason than the cognitive system that governs intuition. Therefore, the cost differential between rational and intuitive decisions may shrink throughout the learning process. Nevertheless, the speed of intuition will always be significantly higher than the speed of reasoning, i.e. the cost differential is bounded away from zero. The following assumptions account for these findings. Let $C(t)$ denote the cost differential between rational and intuitive decision making at time t, also referred to as the (net) costs of reasoning.

Assumption 2 *$C : \mathbb{N} \to \mathbb{R}$ is a non-increasing, convex function of t with $\underline{c} := \lim_{t\to\infty} C(t) > 0$.*

The particular shape of the cost function C will crucially depend on both the structure of the decision problem and the cognitive characteristics of the decision maker. For the sake of concreteness, we refer to the final costs of reasoning $\underline{c}$ as a measure of the decision maker's cognitive capacity (relative to the complexity of the problem) and to the initial costs of reasoning $\overline{c} := C(0)$ as the complexity of the problem (relative to the agent's innate cognitive skills). Moreover, we refer to the absolute value of the slope of C as the agent's speed of (rational) learning. In this context, the assumption of a convex C reads as a non-increasing speed of learning over time. Note that the cost differential between rational and intuitive decision making at time t does not depend on whether past decisions have been based on reasoning or intuition. Put differently, the speed of (rational) learning is independent of the mode of decision.

Finally, we refer to $NEU(t) := EU(t) - C(t)$ as the net expected utility of a rational decision in period t. In the following, we compare $NEU(t)$ with $CBU(t)$ to identify the periods in which rational decision making is preferable to intuition and vice versa.

4 The Results

4.1 Prevalence of intuition in the long run

Under Assumptions 1 and 2, Conjecture 1 (a) implies that for any $\underline{c} = \lim_{t\to\infty} C(t) > 0$ there is some $\bar{h} \in \mathbb{R}$ such that for any $h^o \geq \bar{h}$ there exists $t_{\underline{c},h^o} \in \mathbb{N}$ such that $EU - CBU(t) < \underline{c}$ for all $t \geq t_{\underline{c},h^o}$. Since $\underline{c} \leq C(t)$ for all $t \in \mathbb{N}$ this yields

Proposition 1 *In the long run, intuitive decision making will always yield a higher net expected utility than reasoning if the decision maker is sufficiently ambitious.*

Proposition 1 highlights the relative importance of ambition h^o and cognitive capacity $\underline{c}$ for success in the long run. The higher the cognitive skills of an agent, the smaller the lower bound of the cost differential $\underline{c}$ and the larger his net utility from a rational decision in the long run. As a consequence, the more precise an intuitive decision has to be to become superior. However, to reach a sufficiently high level of accuracy, the agent has to experiment sufficiently often. He will do so if he is sufficiently ambitious. As a result, the intuitive decision maker will eventually reach a higher net expected utility than the rational decision maker if he is sufficiently ambitious compared to the latter's cognitive capacity. Moreover, the result confirms the impression that the decisions of experienced leaders like managers, lawyers, physicians, or professional athletes often seem to be based on intuition since they come along with effortlessness and speed.

4.2 The performance of rational and intuitive decisions over time

In this subsection, we investigate the time pattern of the relative performance of rational and intuitive decision making.

4.2.1 An illustrative example

In order to illustrate the basic idea of our model, we start with the example of deterministic decisions. In this case, whenever some act a is chosen, the decision maker realizes the corresponding utility level of μ_a with certainty, i.e. $X_{a,t} = \mu_a$ for all $a \in A$ and $t \in \mathbb{N}$. However, the expected utilities from intuitive decisions $(CBU(t))_{t\in\mathbb{N}}$ depend on the exact specification of admissible paths S, i.e. on h^o and α.

For example, suppose that there is a single best act $a = n$ and, without loss of generality, $\mu_1 \leq \ldots \leq \mu_{n-1} < \mu_n$. If, moreover, $\mu_{n-1} < h^o < \mu_n$ and $\alpha = 1$, then $CBU(1) = \frac{1}{n}\sum_{k=1}^{n}\mu_k$, $CBU(t) = \mu_n$ for all $t \geq n$ and $CBU(t)$ linearly increasing for all $2 \leq t \leq n-1$. In this example, Conjecture 1 obviously holds and CBU is increasing and concave in t.

Whenever Conjecture 1 holds and CBU is increasing and concave in t – including the example just mentioned – we can distinguish between three cases.

Proposition 2 *Suppose that Conjecture 1 holds and CBU is increasing and concave in t.*

(a) If $\bar{c}$ is sufficiently high and C is slowly decreasing, $CBU(t) > NEU(t)$ for all $t \in \mathbb{N}$.

(b) If $\bar{c}$ is sufficiently high and C is decreasing fast, there will exist $\underline{t}, \bar{t} \in \mathbb{N}$ such that $CBU(t) \leq NEU(t)$ if and only if $t \in [\underline{t}, \bar{t}]$.

(c) If $\bar{c}$ is sufficiently low, there will exist $\bar{t} \in \mathbb{N}$ such that $CBU(t) \leq NEU(t)$ if and only if $t \leq \bar{t}$.

First, if cognitive costs are initially high and slowly decreasing, i.e. if the complexity of the problem is high and the decision maker's learning curve is flat, it never pays to engage in reasoning. In this case, decisions will always be based on intuition. The corresponding situation is depicted in Figure 1.

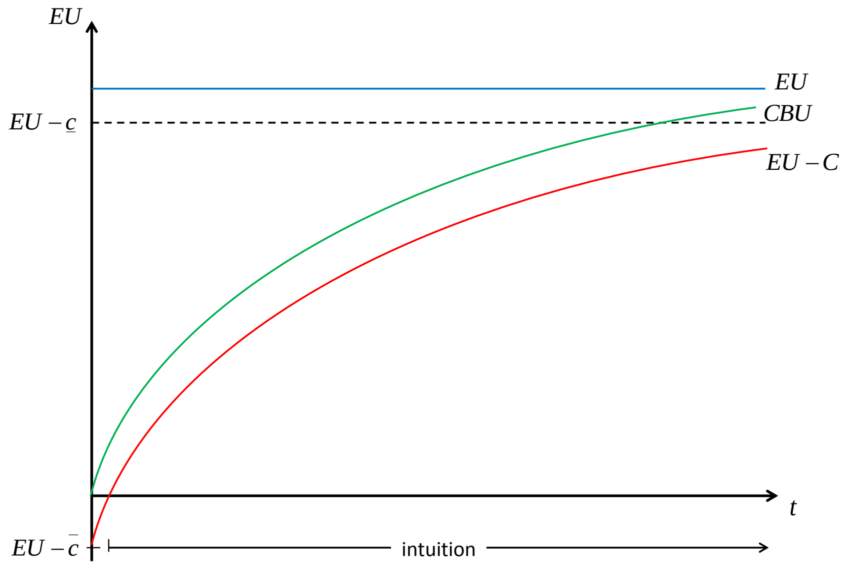

Figure 1: High complexity, slow learning

Second, if cognitive costs are initially high but decreasing fast, i.e. if the complexity of the problem is high but the decision maker's learning curve is steep, we recognize a pattern where intuitive decisions are prevalent in early and late stages of a learning process, whereas reason governs decisions in intermediate stages. The corresponding situation is depicted in Figure 2. After having experimented on a trial and error basis for a number of periods $\underline{t}$, one has collected enough information to gain from reasoning. However, since reasoning remains costly whereas intuition further improves as the number of problems encountered increases, from some period $\bar{t}$ on it will be beneficial to save on cognitive costs and rely on the growing experience instead.

Third, if cognitive costs are low even initially, i.e. if the complexity of the problem is low, no initial stage of experimentation occurs. The corresponding situation is depicted in Figure 3. It pays to engage in reasoning right from the start till the period $\bar{t}$ at which, again, the agent finds it preferable to save on cognitive costs and rely on his vast experience instead.

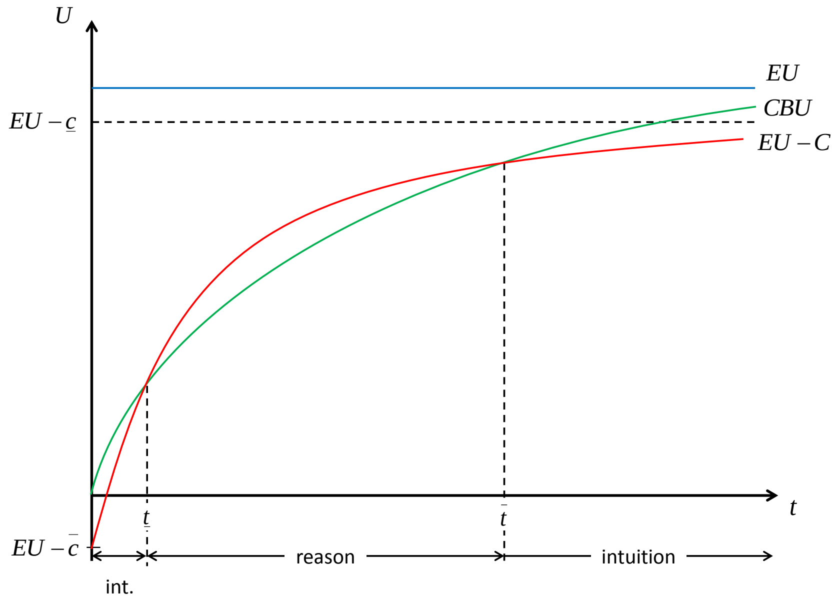

Figure 2: High complexity, fast learning

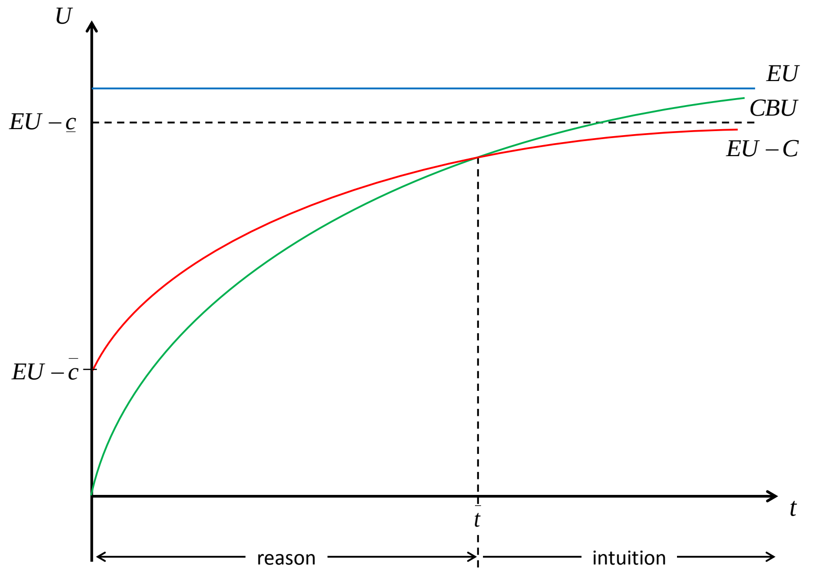

Figure 3: Low complexity

4.2.2 The general case

As the simulations in the Appendix illustrate, in general, CBU is neither concave nor monotonic in t, not even for deterministic decisions. However, since the expected utility μ_a from any action $a \in A$ is finite, $CBU(t)$ is bounded, and under Conjecture 1 we find increasing and concave functions $\underline{CBU}, \overline{CBU} : \mathbb{N} \to \mathbb{R}$ such that $\underline{CBU}(t) \leq CBU(t) \leq \overline{CBU}(t)$ and $\lim_{t\to\infty} \underline{CBU}(t) = \lim_{t\to\infty} \overline{CBU}(t) = EU$. Hence, we find similar time patterns of prevalent intuitive or rational decisions as for the cases in which CBU is increasing and concave in t. For example, Figure 4 depicts a situation in which the complexity of the problem is high and the decision maker is learning fast. The corresponding learning process can be divided into five stages.

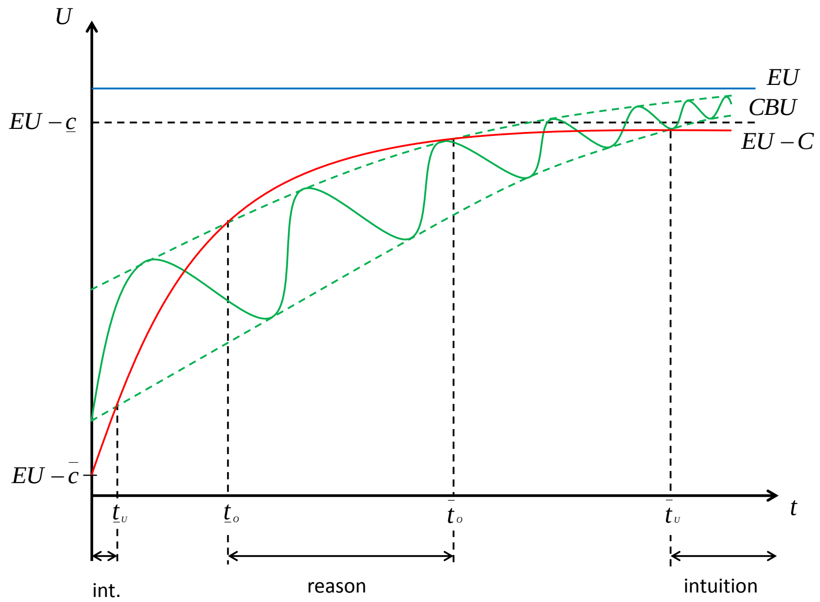

Figure 4: High complexity, fast learning, non monotonic CBU

Again, in the very early and very late stages 1 and 5 intuitive decisions are prevalent. The intermediate stage 3 is governed by rational decisions. Within the transitional stages 2 and 4 we can find both, phases in which reasoning and phases in which intuition is preferable.

5 Conclusion

We have studied the performance of rational and intuitive decision making over time. Facing a sequence of similar problems, agents can either decide rationally according to expected utility theory (EUT) or intuitively according to case-based decision theory (CBDT). Rational decisions are more precise but create higher costs, though these costs may decrease over time. Our simulations confirm the conjecture that the expected utility from case-based optimization converges to the maximum expected utility. As a consequence, we observe that, in the long run, intuitive decision making will yield a higher expected net utility than reasoning if the agent is sufficiently ambitious.

Moreover, for a plausible range of parameters, we derive the following result: In early and late stages of the learning process, an intuitive decision maker reaches a higher level of expected net utility than a rational decision maker – and vice versa in intermediate stages. The additional assumption that rational and intuitive decisions equally contribute to an agent's case base (memory set) allows for a more individualistic reinterpretation. The above result then explains the commonly observed pattern that intuitive decisions are prevalent in early and late stages of a learning process, whereas reason governs decisions in intermediate stages.

In our model, time measures the agent's experience or familiarity with a recurrent problem. This is only one relevant dimension of time in its relation to reason and intuition. Other important dimensions in this relation – e.g. the time the agent has at her disposal in order to reach a decision or the period of time to which the related decision refers – are beyond the scope of this paper.[9]

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[9] von Weizsäcker (2010) incorporates some of these aspects in a heuristic way.

A Appendix: Simulations

In order to confirm Conjecture 1 we have used a software tool called *Simulation Container*[10] to simulate the process of case based optimization as described by (1) and (2), respectively (Gilboa and Schmeidler, 1996). For illustrative purposes, below we present several simulation results for the following simple decision problem: There are only two acts one can choose from, A and B. The outcome of A is deterministic yielding a utility of $\mu_A = 1$. The outcome of B is stochastic yielding a utility of either 0 or 4 with equal probabilities. Hence, B is the act that maximizes expected utility with $\mu_B = EU = 2$.

Example A

We first consider the process of case based optimization as described by (1). The following tables show the simulation results for different values of the initial aspiration level h^o and the speed of adaption α.

From Table 1 to 2 we fix $\alpha = 0.5$ and increase h^o. This extends the phase of experimentation and has two consequences for the process of case based optimization: On the one hand, the number of periods for which we observe alternating majorities of the relative frequencies at which A and B are chosen across paths increases (from $t = 5$ in Table 1 to $t = 47$ in Table 2), i.e. convergence is reached at a later point in time. On the other hand, however, the relative frequency at which the optimal act B is chosen after this phase of experimentation increases, i.e. the level of convergence rises.

As the comparison of Tables 1 and 3 shows, similar observations can be made if we fix $h^o = 100$ and increase α, i.e. reduce the speed at which the aspiration level is adjusted to the utility levels realized so far: Convergence is reached later but at a higher level.

[10]This tool has been developed in collaboration with Stephan da Costa Ribeiro at Technische Universität München and will be provided by the authors upon request.

Table 1: Example A with $h^o = 100$, $\alpha = 0.5$

t	CBU(t)	Relative frequency A	Relative frequency B	Lowest aspiration level
0	0	0	0	100
1	1,5	0,5	0,5	50
2	1,5	0,5	0,5	25,5
3	1,5	0,5	0,5	13,25
4	1,5	0,5	0,5	7,125
5	1,75	0,75	0,25	4,0625
6	1,625	0,625	0,375	2,53125
7	1,734375	0,734375	0,265625	1,765625
8	1,6484375	0,6484375	0,3515625	1,3828125
9	1,75	0,75	0,25	1,19140625
10	1,740234375	0,740234375	0,259765625	1,095703125
11	1,765625	0,765625	0,234375	1,047851563
12	1,743164063	0,743164063	0,256835938	1,023925781
13	1,732421875	0,732421875	0,267578125	1,011962891
14	1,740722656	0,740722656	0,259277344	1,005981445
15	1,733947754	0,733947754	0,266052246	1,002990723
16	1,718139648	0,718139648	0,281860352	1,001495361
17	1,755401611	0,755401611	0,244598389	1,000747681
18	1,753204346	0,753204346	0,246795654	1,00037384
19	1,740345001	0,740345001	0,259654999	1,00018692
20	1,751146317	0,751146317	0,248853683	1,00009346
21	1,738828659	0,738828659	0,261171341	1,00004673
22	1,743149757	0,743149757	0,256850243	1,000023365
23	1,745584011	0,745584011	0,254415989	1,000011683
24	1,739115953	0,739115953	0,260884047	1,000005841
25	1,74928838	0,74928838	0,25071162	1,000002921
26	1,750645369	0,750645369	0,249354631	1,00000146
27	1,748244971	0,748244971	0,251755029	1,00000073
28	1,748856299	0,748856299	0,251143701	1,000000365
29	1,74791164	0,74791164	0,25208836	1,000000183
30	1,748292979	0,748292979	0,251707021	1,000000091
31	1,747418432	0,747418432	0,252581568	1,000000046
32	1,749664855	0,749664855	0,250335145	1,000000023
33	1,750387764	0,750387764	0,249612236	1,000000011
34	1,751172218	0,751172218	0,248827782	1,000000006
35	1,751323611	0,751323611	0,248676389	1,000000003
36	1,750962991	0,750962991	0,249037009	1,000000001
37	1,750409053	0,750409053	0,249590947	1,000000001
38	1,751266185	0,751266185	0,248733815	1
39	1,75100176	0,75100176	0,24899824	1
40	1,750574453	0,750574453	0,249425547	1

Table 2: Example A with $h^o = 10^{15}$, $\alpha = 0.5$

t	CBU(t)	Relative frequency A	Relative frequency B	Lowest aspiration level
0	0	0	0	1,00E+15
1	1,5	0,5	0,5	5,00E+14
2	1,5	0,5	0,5	2,50E+14
3	1,5	0,5	0,5	1,25E+14
4	1,5	0,5	0,5	6,25E+13
5	1,75	0,25	0,75	3,13E+13
6	1,25	0,75	0,25	1,56E+13
7	1,875	0,125	0,875	7,81E+12
8	1,125	0,875	0,125	3,91E+12
9	1,8125	0,1875	0,8125	1,95E+12
10	1,1875	0,8125	0,1875	9,77E+11
11	1,8125	0,1875	0,8125	4,88E+11
12	1,1875	0,8125	0,1875	2,44E+11
13	1,890625	0,109375	0,890625	1,22E+11
14	1,109375	0,890625	0,109375	6,10E+10
15	1,9375	0,0625	0,9375	3,05E+10
16	1,0625	0,9375	0,0625	1,53E+10
17	1,91015625	0,08984375	0,91015625	7,63E+09
18	1,08984375	0,91015625	0,08984375	3,81E+09
19	1,91015625	0,08984375	0,91015625	1,91E+09
20	1,08984375	0,91015625	0,08984375	9,54E+08
21	1,9453125	0,0546875	0,9453125	4,77E+08
22	1,0546875	0,9453125	0,0546875	2,38E+08
23	1,967285156	0,032714844	0,967285156	1,19E+08
24	1,032714844	0,967285156	0,032714844	5,96E+07
25	1,953857422	0,046142578	0,953857422	2,98E+07
26	1,046142578	0,953857422	0,046142578	1,49E+07
27	1,953857422	0,046142578	0,953857422	7450581,597
28	1,046142578	0,953857422	0,046142578	3725291,298
29	1,971313477	0,028686523	0,971313477	1862646,149
30	1,028686523	0,971313477	0,028686523	931323,5746
31	1,982421875	0,017578125	0,982421875	465662,2873
32	1,017578125	0,982421875	0,017578125	232831,6437
33	1,975479126	0,024520874	0,975479126	116416,3218
34	1,024520874	0,975479126	0,024520874	58208,66091
35	1,975479126	0,024520874	0,975479126	29104,83046
36	1,024520874	0,975479126	0,024520874	14552,91523
37	1,984558105	0,015441895	0,984558105	7276,957614
38	1,015441895	0,984558105	0,015441895	3638,978807
39	1,990394592	0,009605408	0,990394592	1819,989404
40	1,009605408	0,990394592	0,009605408	910,4947018
41	1,986698151	0,013301849	0,986698151	455,7473509
42	1,013301849	0,986698151	0,013301849	228,3736754
43	1,986698151	0,013301849	0,986698151	114,6868377
44	1,013301849	0,986698151	0,013301849	57,84341886
45	1,99154973	0,00845027	0,99154973	29,42170943
46	1,210886717	0,789113283	0,210886717	15,21085472
47	1,994688988	0,005311012	0,994688988	8,105427358
48	1,973109543	0,026890457	0,973109543	4,552713679
49	1,970764607	0,029235393	0,970764607	2,776356839
50	1,994688988	0,005311012	0,994688988	1,88817842
51	1,990142047	0,009857953	0,990142047	1,44408921
52	1,990873918	0,009126082	0,990873918	1,222044605

Table 3: Example A with $h^o = 100$, $\alpha = 0.9$

t	CBU(t)	Relative frequency A	Relative frequency B	Lowest aspiration level
0	0	0	0	100
1	1,5	0,5	0,5	90
2	1,5	0,5	0,5	81,1
3	1,5	0,5	0,5	73,09
4	1,5	0,5	0,5	65,881
5	1,75	0,25	0,75	59,3929
6	1,25	0,75	0,25	53,55361
7	1,875	0,125	0,875	48,298249
8	1,125	0,875	0,125	43,5684241
9	1,8125	0,1875	0,8125	39,31158169
10	1,1875	0,8125	0,1875	35,48042352
11	1,8125	0,1875	0,8125	32,03238117
12	1,1875	0,8125	0,1875	28,92914305
13	1,890625	0,109375	0,890625	26,13622875
14	1,109375	0,890625	0,109375	23,62260587
15	1,9375	0,0625	0,9375	21,36034528
16	1,06640625	0,93359375	0,06640625	19,32431076
17	1,90625	0,09375	0,90625	17,49187968
18	1,1796875	0,8203125	0,1796875	15,84269171
19	1,8203125	0,1796875	0,8203125	14,35842254
20	1,266357422	0,733642578	0,266357422	13,02258029
21	1,7734375	0,2265625	0,7734375	11,82032226
22	1,550048828	0,449951172	0,550048828	10,73829003
23	1,538452148	0,461547852	0,538452148	9,764461029
24	1,575561523	0,424438477	0,575561523	8,888014926
25	1,604187012	0,395812988	0,604187012	8,099213434
26	1,534088135	0,465911865	0,534088135	7,38929209
27	1,624717712	0,375282288	0,624717712	6,750362881
28	1,646995544	0,353004456	0,646995544	6,175326593
29	1,730239868	0,269760132	0,730239868	5,657793934
30	1,595653534	0,404346466	0,595653534	5,192014541
31	1,716667175	0,283332825	0,716667175	4,772813086
32	1,727862835	0,272137165	0,727862835	4,395531778
33	1,687292814	0,312707186	0,687292814	4,0559786
34	1,714115977	0,285884023	0,714115977	3,75038074
35	1,750372231	0,249627769	0,750372231	3,475342666
36	1,780712992	0,219287008	0,780712992	3,227808399
37	1,812078193	0,187921807	0,812078193	3,005027559
38	1,839079514	0,160920486	0,839079514	2,804524804
39	1,831856772	0,168143228	0,831856772	2,624072323
40	1,849466432	0,150533568	0,849466432	2,461665091
41	1,856421662	0,143578338	0,856421662	2,315498582
42	1,90397102	0,09602898	0,90397102	2,183948724
43	1,888951536	0,111048464	0,888951536	2,065553851
44	1,923810822	0,076189178	0,923810822	1,958998466
45	1,911053362	0,088946638	0,911053362	1,86309862

Example B

We now consider the process of case based optimization as described by (2). We fix the initial aspiration level $h^o = 2$, the speed of adaption $\alpha = 0.5$, and focus on a variation of the increment $h > 0$ by which the aspiration level is increased in any period of ambition $t \in \{n \in \mathbb{N} \mid \exists k \in \mathbb{N} : n = 2^k + 1\}$. As Tables 4 and 5 show, in each such period of ambition there starts a new phase of experimentation that temporarily leads to a sharp decline in the expected utility of intuitive decisions. Those phases of experimentation are the longer the larger the increment h is. Since the following increases in $CBU(t)$ are also larger, a larger increment h results in larger initial fluctuations in the expected utility of intuitive decisions.

Table 4: Example B with $h^o = 2$, $\alpha = 0.5$, $h = 1$

t	CBU(t)	Relative frequency A	Relative frequency B	Lowest aspiration level	Average CBU(t)
0	0	0	0	2	
1	1,5	0,5	0,5	1	1,5
2	1,75	0,25	0,75	2	1,625
3	1,25	0,75	0,25	1,5	1,5
4	1,375	0,625	0,375	2	1,46875
5	1,75	0,25	0,75	1,5	1,525
6	1,625	0,375	0,625	1,25	1,541666667
7	1,6875	0,3125	0,6875	1,125	1,5625
8	1,5625	0,4375	0,5625	2	1,5625
9	1,515625	0,484375	0,515625	1,5	1,557291667
10	1,8359375	0,1640625	0,8359375	1,25	1,58515625
11	1,796875	0,203125	0,796875	1,125	1,604403409
12	1,6953125	0,3046875	0,6953125	1,0625	1,611979167
13	1,78125	0,21875	0,78125	1,03125	1,625
14	1,7509766	0,249023438	0,750976563	1,015625	1,633998326
15	1,7172852	0,282714844	0,717285156	1,0078125	1,639550781
16	1,7406006	0,259399414	0,740600586	2	1,645866394
17	1,2970581	0,702941895	0,297058105	1,5	1,625348259
18	1,7769165	0,223083496	0,776916504	1,25	1,633768717
19	1,9130859	0,086914063	0,913085938	1,125	1,648469624
20	1,8904419	0,109558105	0,890441895	1,0625	1,660568237
21	1,868927	0,131072998	0,868927002	1,03125	1,670490083
22	1,8735352	0,126464844	0,873535156	1,015625	1,679719405
23	1,8614044	0,138595581	0,861404419	1,0078125	1,687618753
24	1,8635254	0,136474609	0,863525391	1,00390625	1,694948196
25	1,865591	0,134408951	0,865591049	1,001953125	1,701773911
26	1,8702726	0,129727364	0,870272636	1,000976563	1,708254631
27	1,8644495	0,135550499	0,864449501	1,000488281	1,714039626
28	1,8638992	0,136100769	0,863899231	1,000244141	1,719391755
29	1,8632988	0,136701226	0,863298774	1,00012207	1,724354066
30	1,8617415	0,138258547	0,861741453	1,000061035	1,728933645
31	1,8616427	0,138357259	0,861642741	1,000030518	1,733214584
32	1,86131	0,13869001	0,86130999	2	1,737217565
33	1,2057803	0,794219717	0,205780283	1,5	1,721113405
34	1,2826186	0,717381372	0,282618628	1,25	1,7082165
35	1,8483672	0,151632839	0,848367161	1,125	1,712220805

Table 5: Example B with $h^o = 2$, $\alpha = 0.5$, $h = 10000$

t	CBU(t)	Relative frequency A	Relative frequency B	Lowest aspiration level	Average CBU(t)
0	0	0	0	2	
1	1,5	0,5	0,5	1	1,5
2	1,75	0,25	0,75	10001	1,625
3	1,25	0,75	0,25	5001	1,5
4	1,5	0,5	0,5	10001	1,5
5	1,75	0,25	0,75	5001	1,55
6	1,25	0,75	0,25	2501	1,5
7	1,875	0,125	0,875	1251	1,553571429
8	1,125	0,875	0,125	10001	1,5
9	1,8125	0,1875	0,8125	5001	1,534722222
10	1,1875	0,8125	0,1875	2501	1,5
11	1,8125	0,1875	0,8125	1251	1,528409091
12	1,1875	0,8125	0,1875	626	1,5
13	1,890625	0,109375	0,890625	313,5	1,530048077
14	1,109375	0,890625	0,109375	157,25	1,5
15	1,9375	0,0625	0,9375	79,125	1,529166667
16	1,0625	0,9375	0,0625	10001	1,5
17	1,91015625	0,08984375	0,91015625	5001	1,524126838
18	1,08984375	0,91015625	0,08984375	2501	1,5
19	1,91015625	0,08984375	0,91015625	1251	1,521587171
20	1,08984375	0,91015625	0,08984375	626	1,5
21	1,9453125	0,0546875	0,9453125	313,5	1,521205357
22	1,0546875	0,9453125	0,0546875	157,25	1,5
23	1,967285156	0,032714844	0,967285156	79,125	1,520316746
24	1,032714844	0,967285156	0,032714844	40,0625	1,5
25	1,953857422	0,046142578	0,953857422	20,53125	1,518154297
26	1,17956543	0,82043457	0,17956543	10,765625	1,505131648
27	1,910217285	0,089782715	0,910217285	5,8828125	1,52013482
28	1,945129395	0,054870605	0,945129395	3,44140625	1,535313198
29	1,910217285	0,089782715	0,910217285	2,220703125	1,548240925
30	1,940765381	0,059234619	0,940765381	1,610351563	1,561325073
31	1,971313477	0,028686523	0,971313477	1,305175781	1,574550506
32	1,956039429	0,043960571	0,956039429	10001	1,586472034
33	1,036323547	0,963676453	0,036323547	5001	1,569800868
34	1,036323547	0,963676453	0,036323547	2501	1,554110359
35	1,051597595	0,948402405	0,051597595	1251	1,539752851
36	1,036323547	0,963676453	0,036323547	626	1,525768704
37	1,141605377	0,858394623	0,141605377	313,5	1,515385911
38	1,020503998	0,979496002	0,020503998	157,25	1,502362703
39	1,900611877	0,099388123	0,900611877	79,125	1,51257422
40	1,009605408	0,990394592	0,009605408	40,0625	1,5
41	1,986698151	0,013301849	0,986698151	20,53125	1,511870687
42	1,513300896	0,486699104	0,513300896	10,765625	1,511904739
43	1,986698151	0,013301849	0,986698151	5,8828125	1,522946446
44	1,983463764	0,016536236	0,983463764	3,44140625	1,533412749
45	1,971219301	0,028780699	0,971219301	2,220703125	1,543141784
46	1,982655168	0,017344832	0,982655168	1,610351563	1,552696422
47	1,99154973	0,00845027	0,99154973	1,305175781	1,562033727
48	1,987102449	0,012897551	0,987102449	1,152587891	1,570889325
49	1,98932609	0,01067391	0,98932609	1,076293945	1,579428851
50	1,99043791	0,00956209	0,99043791	1,038146973	1,587649032
51	1,988412809	0,011587191	0,988412809	1,019073486	1,595507145
52	1,988134854	0,011865146	0,988134854	1,009536743	1,603057678
53	1,987995876	0,012004124	0,987995876	1,004768372	1,610320663
54	1,987742738	0,012257262	0,987742738	1,002384186	1,617309961
55	1,987457338	0,012542662	0,987457338	1,001192093	1,624039913

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