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Abstract

Regional productivity differences provide scope for productivity-enhancing labor mobility. Redistribution reduces relocation incentives through higher taxes or lower transfers. Combining an intensive labor supply margin with an extensive, productivity-enhancing migration margin, we determine how regional inequality and labor mobility affect optimal redistribution. Simulations using the productivity differences between rural and urban regions in the US indicate that optimal marginal tax rates are reduced by several percentage points if productivity enhancing migration is taken into account. Additionally, we study optimal regionally differentiated taxation.

JEL-Code: H110, J450, R120.

Keywords: optimal taxation, redistribution, regional inequality, migration, multi-dimensional screening, delayed optimal control.

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1 Introduction

Regional productivity differences are large in many countries. In the US, real per capita GDP in the New England Region was 40% higher than in the Southeast Region in 2013 (BEA, 2014), and, in Italy, 2011 real per capita GDP of the Northern and Central Regions was even 71% higher than in the Southern and Islands Region (ISTAT 2013), for example. The spatial dispersion of wages and incomes is well documented and the underlying causes are still subject to debate (Barro and Sala-i-Martin (1991), Ciccone and Hall (1996), Kanbur and Venables (2005), Acemoglu and Dell (2010) and Young (2013), among others). Given such productivity differences, the efficiency-enhancing potential of inter-regional mobility is substantial, and increases in personal income are key drivers of this mobility, see Kennan and Walker (2011) for a recent study for the US. Centralized redistribution schemes such as a federal income tax or federal social transfers reduce inter-regional migration incentives, since an individual who migrates from a low to a high productivity area has to share the realized productivity gains with the government through higher taxes or lower transfers. This generates a trade-off for an inequality-averse policy maker between redistribution and efficiency-enhancing inter-regional migration. Contrary to the emigration of high-income earners to low-tax countries or the immigration of welfare recipients from less generous jurisdictions, the role of internal migration for optimal federal tax policy has been mostly neglected.[1] We develop a conceptual framework to analyze the implications of internal migration for an optimal tax-transfer policy and assess its quantitative importance. While our focus is on efficiency-enhancing migration between regions with permanent productivity differences, our approach may also be used to address the related optimal taxation problem that arises with respect to efficiency-enhancing migration in response to idiosyncratic shocks to regional labor markets, as discussed by Blanchard and Katz (1992), or, more recently, Yagan (2014).

We propose a two-dimensional optimal taxation model which combines an extensive, inter-regional migration decision with an intensive labor supply decision. Our key innovation is the productivity-enhancing nature of the migration margin. The actual or realized productivity of individuals of any given innate productivity is location-dependent, such that individuals can increase their actual productivity by migrating from a low to a high productivity region. Thus, the extensive migration margin also affects the intensive labor supply decision, since productivity and, typically, the relevant marginal tax rate change whenever an individual decides to migrate, even though the same tax schedule applies nationwide.

[1]Studies addressing external migration include Mirrlees (1982), Wildasin (1991), Wilson (1992), Lehmann et al. (2014) and others.

This framework allows us to determine the optimal federal tax schedule as a function of the government's redistributive preferences, the observed regional earnings distributions, the earnings elasticity, and the inter-regional migration elasticity. Our analysis shows that regional disparities and the possibility of efficiency-enhancing inter-regional labor mobility can be important determinants of the optimal tax schedule. Optimal marginal taxes tend to be below the benchmark without regional inequality, since the decision to migrate to an area with higher productivity implies a fiscal externality that needs to be taken into account in tax policy design. The size of the fiscal externality depends on the migration elasticity and on the inter-regional tax differential, which is itself a function of regional productivity differences and the tax schedule. If marginal tax rates are positive throughout the tax schedule, the fiscal migration externality is always positive, such that optimal marginal tax rates are lower compared to a situation with the same nationwide posterior productivity distribution but without migration. Moreover, for some subset of the productivity distribution, negative marginal tax rates are possible. This latter result is similar to other studies that have analyzed the optimal tax-transfer schedule with an intensive labor supply decision and the participation decision (see e.g. Saez 2002 and Jacquet et al. 2013).

Our framework provides a methodological contribution to the theory of optimal taxation. Making the intensive margin dependent on the extensive margin is a useful extension of the class of multi-dimensional screening models, originally discussed by Rochet and Choné (1998) and Armstrong (1996). We argue that this class of models can be fruitfully studied using the delayed optimal control approach as recently formally analyzed by Göllmann et al. (2008) in its entire generality. The approach is suitable to address a range of other multi-dimensional screening problems, where an extensive margin directly affects an intensive margin. Several other optimal taxation problems are characterized by a similar structure. The discrete decision whether to participate in the labor market or not, for example, affects productivity endogenously, given that non-participation tends to result in the depreciation of human capital. Similarly, discrete education decisions also determine productivity and interact with marginal tax rates and the intensive labor supply margin, such that our framework may also be applied to optimal taxation problems with endogenous education decisions.

We additionally study regionally differentiated tax-transfer schemes. To the extent that such schemes are explicit, they are often difficult to enforce in practice, given the challenge to monitor the actual place of residence of individuals, and may also be challenged on the grounds of the violation of horizontal equity. Moreover, such schemes are pointless if migration costs are sufficiently low. Despite these caveats, regional differentiation can be an element of real world tax systems. From 1971 to 1994, the German tax system, for example, treated residents in West-Berlin differently from people in the rest of the country. Another example is the current path towards a more fiscally integrated Europe. As the Eurozone is moving towards deeper fiscal integration, it faces the choice between a system of explicit and implicit transfers between Member States combined with a different tax-transfer scheme within each Member State and the alternative of moving to an integrated Eurozone-wide tax-transfer scheme. This decision requires an understanding of the advantages and the challenges of a differentiated system vis-a-vis an integrated system.[2] Finally, nominally non-differentiated federal income taxation amounts to regionally differentiated taxation in real terms due to cost of living differences (Albouy 2009). Our analysis allows to compare such implicit regional differentiation with the optimal regionally differentiated tax-transfer scheme for redistribution.

Conceptually, regionally differentiated tax-transfer schemes use the region of residence as an additional tag in the design of tax transfer schemes. We add to the debate on tagging in optimal taxation by considering the region of residence as an endogenous tag. Obviously, if the characteristic used as a tag can be changed at zero cost, then the tag loses its value for government policy. If it can be changed at some cost, the tag-induced changes create additional efficiency costs that must be taken into account. At the same time, changing the region of residence from a low to a high productivity region increases actual productivity and, potentially, tax revenues. This gives rise to an interesting trade-off under differentiated taxation: On the one hand, the government can use the additional information to increase the amount of redistribution by reducing the efficiency costs of the system. On the other hand, differentiated taxes can be used to encourage efficiency-enhancing migration.

To get an idea about the quantitative importance of the additional constraint of internal migration, we apply our framework to the US. We focus on the productivity difference between urban and rural areas, which is a relevant application of our analysis in many countries. Using micro data from the Panel Study of Income Dynamics (PSID) on rural and urban areas along with the actual tax-transfer treatment, we retrieve the underlying regional productivity distributions and establish the productivity gain from relocating from rural to urban areas for each productivity level. Taking into account the empirical evidence on labor supply and migration elasticities, we then simulate the optimal

[2]To the extent that the Member States are unrestricted by the center to decide on their own tax-transfer scheme, additional considerations of tax competition have to be additionally taken into account. See Lehmann et al. (2014) for the analysis of such considerations in the optimal taxation framework. Bargain et al. (2013) have contrasted a Member-States-based redistribution schemes to an integrated scheme in Europe. However, they address the implications for macroeconomic stabilization, whereas we study the efficiency of redistribution.

non-differentiated tax schedule that takes potential efficiency-enhancing migration into account. We contrast this schedule with the optimal tax schedule for the posterior spatial distribution of individuals, but without migration. The results show that efficiency enhancing-mobility reduces optimal marginal tax rates by up to several percentage points. Efficiency-enhancing internal migration does not eliminate the case for progressive taxation, but it constitutes a quantitatively important constraint on redistribution.

2 Related literature

The normative implications of efficiency-enhancing internal migration for optimal redistribution have, to the best of our knowledge, not been studied to date. The constraint of inter-jurisdictional or international mobility for the redistribution policy of a single jurisdiction or country, however, has received considerable attention within the optimal taxation literature and beyond, see, in particular, Mirrlees (1982), Wildasin (1991), Wilson (1992), Lipatov and Weichenrieder (2010), Simula and Trannoy (2011), Lehmann et al. (2014). Our analysis reveals that labor mobility within a sufficiently large jurisdiction or between regions within a country can be a similarly important constraint to redistribution.

Conceptually, our analysis belongs to a class of two-dimensional screening models that have been recently used to analyze a range of tax policy questions. Lehmann et al. (2014) combine the intensive labor supply margin with an extensive migration margin. However, their focus is on independent governments competing for internationally mobile high productivity individuals, and it is therefore complementary to our analysis of regionally non-differentiated and differentiated taxation by a single government. Moreover, individual productivity is not location-dependent in their analysis, and they only focus on the threat of migration, whereas actual efficiency-enhancing migration is at the heart of our approach. Gordon and Cullen (2012) also use an optimal taxation approach to study inter-regional migration in a model with several states. However, they focus on the assignment problem of whether redistribution should be carried out at the national or the subnational level and also do not consider productivity differences. Jacquet et al. (2013) also study a two-dimensional optimal taxation model but focus on the participation decision at the extensive margin.

The structure of our approach owes much to Kleven et al. (2006, 2009) who study the optimal taxation of couples with cooperative households. Their analysis combines the intensive labor supply decision with the household's choice to become a single or a double earner household. However, our analysis differs in several important ways from their framework. First, we consider individuals and not households consisting of two persons whose respective incomes may be taxed separately. Secondly, in our approach individuals originally reside in different regions, such that the tax units not only differ among themselves regarding their costs to change their location, but also differ by the group they originally belong to. Finally and most importantly, we introduce an explicit consideration of endogenous individual productivity as a function of the extensive margin.

Rothschild and Scheuer (2014), and Rothschild and Scheuer (2013) and Gomes et al. (2014) also study optimal taxation of rent seeking activities and optimal taxation in the Roy model, respectively, using two-dimensional screening approaches. Wages are endogenously determined in their work, either by total labor supply in a given sector, or by total rent-seeking activities. Similarly, Scheuer (2014) studies entrepreneurial taxation with an endogenous decision, of whether to become an entrepreneur or a worker, where these decisions determine relative compensation in the aggregate. In our study individual productivity and thus market compensation, however, depends directly on the discrete decision of individuals and not on aggregate outcomes. Accordingly, our argument for optimally adjusting marginal tax rates is not based on the attempt to manipulate relative wages but to encourage efficiency-enhancing regional mobility.

Our analysis of the potential benefits of differentiated taxation relates to the increased interest in tagging in the design of tax-transfer-schemes. The idea that the government's information problem can be relaxed by using additional observable characteristics ("tags") that are correlated with the individual productivity goes back to Akerlof (1978) and has recently been discussed intensively in the optimal taxation literature, see Immonen et al. (1998), Weinzierl (2012), Mankiw and Weinzierl (2011), Boadway and Pestieau (2005), Cremer et al. (2010) and Best and Kleven (2013). We add to this literature in two ways. First, we consider the region of residence as a potential tag. Secondly, we explicitly study a tag that is endogenous and can be adjusted by individuals subject to some cost. In this respect, our paper is related to the literature that studies the interplay between human capital formation and optimal taxation, where the former shapes the productivity distribution and the latter influences incentives for human capital formation, see Stantcheva (2014) and the references therein. The endogeneity of productivity also relates our work to Best and Kleven (2013) who consider a dynamic setting where individual productivity intertemporally depends on the previous intensive labor supply decisions.

Albouy (2009) has argued that non-differentiated nominal federal taxation effectively implies de facto regionally differentiated taxation due to cost-of-living differences. He reasons that differential taxation distorts the spatial allocation in the economy and analyzes the associated efficiency costs and the implied interregional redistribution, but his analysis does not consider the question of optimal redistribution between heterogenous individuals. Our normative approach to regionally differential taxation can be regarded as complementary to his work, since we ask the question whether and to what extent federal taxes should be regionally differentiated for redistribution purposes, if such differentiation were possible. Finally, Eeckhout and Guner (2014) also study the effects of a progressive federal income tax on the spatial allocation of economic activity with a heterogenous population, and also consider regionally differentiated taxation, but they do not use a Mirrleesian optimal taxation framework and do not consider jointly the interaction of the intensive labor supply decision and the inter-regional migration decision.

3 The framework

We consider two sources of heterogeneity across workers: *innate* productivity n and migration costs q. These original individual characteristics are distributed over $[n_{\min}, n_{\max}] \times [0, +\infty)$, and the government can neither observe productivity nor migration costs. There are two regions, $i = A, B$, with total population normalized to two. Originally, half of the population resides in each region, but the endogenous migration decisions of individuals change these population shares. Our key assumption is that the regions differ in their productivity. An individual's *actual* or *realized* productivity n_i is a function of her innate productivity and her region of residence $n_i = \omega(n, i) = \omega_i(n)$, where ω_i is strictly increasing in n. We normalize $n_A = \omega_A(n) = n$. Accordingly, the function $n_B = \omega_B(n) = \omega(n)$ not only assigns the actual productivity to all original residents of region B, but also indicates the transformation of productivity for individuals who migrate from A to B. Without loss of generality we assume that region B is the more productive region, so that $\omega(n) > n$. Innate productivity is distributed in each region i according to the unconditional probability distribution $f(n)$ on $[n_{\min}, n_{\max}]$.[3] As in most of the optimal taxation literature, we treat wages as exogenous and independent of individual labor supply and aggregate migration decisions. Accordingly, the analysis applies to a situation where the effect of migration flows on wages is negligible. The empirical evidence supports the view that, for sufficiently large regions, the effects of internal migration on wages are rather small, see, for the US, Boustan et al. (2010) together with D'Amuri et al. (2010), and Frank (2009) for evidence from the German reunification.[4]

[3]It is straightforward to extend the analysis to the case in which regions also differ in their distribution of innate productivity. Similarly, we could allow for negative migration costs for some subset of individuals at each innate productivity level without affecting the results qualitatively. The latter can generate migration in both directions. For clarity, we abstract from these further aspects.

[4]For similar findings in case of immigration of foreigners see Borjas (1994) and Ottaviano and Peri (2007, 2008).

Following Diamond (1998), we use preferences that are separable in consumption and labor. The utility function of a worker of type (n, q) is similar to the formulation in Kleven et al. (2009), but depends on the region of residence,

$$u(c, z, l) = c_i - n_i h\left(\frac{z_i}{n_i}\right) - q^c l + q^h (1 - l), \tag{1}$$

where l is an indicator variable that takes the value of 1 in case of migration. The function $h(\cdot)$ is increasing, convex and twice-differentiable. It is normalized such that $h'(1) = 1$ and $h(0) = 0$. The other variables have standard interpretations. Consumption c_i equals gross income z_i minus taxes T_i, which itself depend on gross income, $c_i = z_i - T_i(z_i)$. Total migration costs are potentially made up of two components, $q = q^c + q^h$, where q^c is the cost of moving (the need to adapt to new conditions, to learn new language in case of mobility between regions where different languages are spoken, the transaction costs of selling your old house and buying a new one, etc.), and q^h is the utility derived from being at home and benefitting from the existing social networks. To isolate the impacts of the two types of heterogeneity, it is useful to consider them separately. The pure *cost of moving* model sets $q = q^c$ and $q^h = 0$; the pure *home attachment* model uses $q = q^h$ and $q^c = 0$. Ex post, i.e. after migration has taken place, heterogeneity in q^c reflects the differences between individuals who migrate, whereas heterogeneity in q^h reflects the differences between individuals who stay in their home region. In what follows we focus on the cost of moving case, but, with some minor modifications, the home attachment case is quite analogous. However, our optimal tax schedules and their derivations are sufficiently general to encompass both cases.

Each individual chooses l and z_i to maximize (1) for a given tax schedule, i.e. she decides whether to move or not and determines her gross earnings, given that she resides in region i. The first order condition for gross earnings is

$$h'\left(\frac{z_i}{n_i}\right) = 1 - \tau_i(z_i), \tag{2}$$

where τ_i is the marginal tax rate. Accordingly, n_i can be interpreted as potential income, given that individuals facing a marginal tax rate of zero would realize this level of gross earnings. The elasticity of gross earnings with respect to net-of tax-rate as a function of gross earnings and the region of residence is defined as

$$\varepsilon_i \equiv \frac{1 - \tau_i}{z_i} \frac{\partial z_i}{\partial (1 - \tau_i)} = \frac{n_i h'\left(\frac{z_i}{n_i}\right)}{z_i h''\left(\frac{z_i}{n_i}\right)}.$$

To focus on regional productivity differences, we assume that $\varepsilon_i = \varepsilon$ for all individuals and independent of the region. This simple benchmark arises with an iso-elastic formulation, i.e. $h\left(\frac{z_i}{n_i}\right) = \frac{1}{1+\epsilon}\left(\frac{z_i}{n_i}\right)^{1+\epsilon}$, such that $\varepsilon_A = \varepsilon_B = 1/\epsilon$, for example. Furthermore, we require the following property.

Assumption The function $x \to \frac{1-h'(x)}{xh''(x)}$ is decreasing.

Consider now the migration decision. We denote by $p\left(q|\, n\right)$ the density of q conditional on n, and by $P\left(q|\, n\right)$ the cumulated distribution of q conditional on n. Conditional on residing in region i, the individuals' choice of gross earnings is determined by (2), which allows to define indirect utility conditional on the place of residence and net of the costs of moving or the benefits of residing in one's home region as

$$V_i\left(n_i\right) = z_i - T_i\left(z_i\right) - n_i h\left(\frac{z_i}{n_i}\right).$$

Individuals will move from i to $j, j = A, B, i \neq j$, whenever their migration costs are below the net gain from moving, such that $\bar{q}_i \equiv \max\left\{V_j\left(n_j\right) - V_i\left(n_i\right), 0\right\}$ is the critical level of migration costs that determines the actual number of migrants for any innate productivity level.

3.1 The government's optimal tax problem

The government wants to maximize the social welfare function

$$\sum_i \int_{n_{\min}}^{n_{\max}} \int_0^{+\infty} \Psi\left(V_i\left(n\right) - q^c l + q^h\left(1 - l\right)\right) p\left(q, n\right) f(n) dq dn, \tag{3}$$

where $\Psi(.)$ is a concave and increasing transformation of individual utilities. Denoting by E the exogenous expenditure requirements, it needs to respect the budget constraint

$$\sum_i \int_{n_{\min}}^{n_{\max}} \int_0^{+\infty} T_i(z_i) p\left(q, n\right) f(n) dq dn \geq E. \tag{4}$$

Moreover, the government's tax schedule needs to be incentive compatible. This implies

$$\dot{V}\left(n\right) = \left[-h\left(\frac{z_i}{n_i}\right) + \frac{z_i}{n_i} h'\left(\frac{z_i}{n_i}\right)\right] \omega_i'\left(n\right) \geq 0, \tag{5}$$

where the dot above a variable denotes its derivative with respect to n. Moreover, in case of non-differentiated taxation, $T_A\left(z\right) = T_B\left(z\right)$. We show in the appendix that a path for z_A and z_B can be truthfully implemented by the government using a non-linear tax schedule.

Let $\lambda > 0$ be the multiplier associated with the budget constraint (4). The government's redistributive tastes may be represented by region-dependent social marginal welfare weights. In terms of income, our welfare weights will take the form of

$$g_i(z) = \frac{\Psi'(V_i(z))(1 - P(\bar{q}_i|z)) + \int_0^{\bar{q}_j} \Psi'(V_i(z) - q^c) p(q|z) dq}{\lambda(1 + P(\bar{q}_j|z) - P(\bar{q}_i|z))},$$

for the cost of moving model, where $\bar{q}_i(z) \equiv \max\{V_j(z) - V_i(z), 0\}$.

4 Optimal unified taxation

We first investigate the optimal non-differentiated tax-transfer system. The government maximizes (3) subject to (4) and (5) through its choice of $T(z)$. This problem formally amounts to a delayed optimal control problem as has been analyzed by Göllmann et al. (2008) in its entire generality. In our model, the delay is a non-fixed lag, though, given that we do not restrict the productivity gain from moving to be constant but treat it as a function of the innate productivity. The necessary conditions for optimal control in such a setting are presented in Abdeljawad et al. (2009). While we explicitly solve the problem in the Appendix to derive all our results rigorously, we first follow here the intuitive perturbation approach pioneered by Piketty (1997) and Saez (2001) to derive the optimal tax scheme. This heuristic derivation allows to disentangle the economic forces that determine the shape of marginal tax rates along the optimal tax schedule, including the effects generated by the possibility of efficiency-enhancing migration.

We use the endogenously realized distribution of gross incomes in both regions denoted by $v_i(z_i)$, and we denote by k the endogenously defined, strictly increasing function that maps gross income in the low productivity region to the gross income this individual would earn in the high productivity region, given his innate productivity and the respective tax treatment, i.e. $z_B = k(z_A)$.[5] We consider an increase in taxes for all individuals above gross income z. The increase is engineered through an increase in the marginal tax rate $d\tau$ in the small band $(z, z + dz)$, such that for all individuals with gross earnings above z the tax payments increase by $dzd\tau$. This tax increase gives rise to three different effects.

Revenue effect All taxpayers in either region with gross incomes above z pay additional taxes of $dzd\tau$. The net welfare effect of this tax payment for an individual in region

[5] In terms of our previous formulation, an individual of ability n receives gross income $z_A = z(n)$ in region A and gross income $z_B = z(\omega(n))$ in region B, where this notation abstracts from the fact that the gross income also depends on the tax schedule.

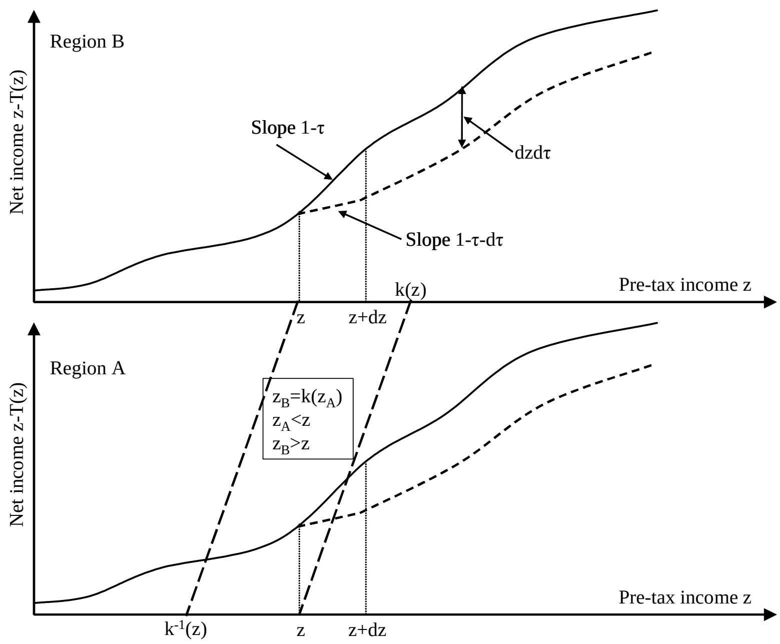

Figure 1: The migration effect comes into play for individuals for which $z'_A < z$ and $z'_B \geq z$.

i with gross earnings z' is given by $dzd\tau\,(1 - g_i\,(z'))$, and the total effect is then

$$R = dzd\tau \int_z^\infty \{[1 - g_A\,(z')]\, v_A\,(z')\, s_A\,(z') + [1 - g_B\,(z')]\, v_B\,(z')\, s_B\,(z')\}\, dz',$$

where $s_A\,(z) \equiv 1 - P\,(\bar{q}_A|\, z)$ and $s_B\,(z) \equiv 1 + P\,(\bar{q}_A|\, k^{-1}(z))$.

Behavioral effect Individuals in the band $(z, z + dz)$ will change their labor supply in response to the increase in the marginal tax rate. Given that $\varepsilon \equiv \frac{1-\tau}{z}\frac{dz}{d(1-\tau)}$, each individual in the band will reduce its income by $-d\tau\varepsilon\frac{z}{1-\tau}$. There are approximately $dz\,[v_A(z)s_A(z) + v_B(z)s_B(z)]$ of these individuals. The total effect on tax revenue is

$$L = -d\tau dz \frac{\tau z \varepsilon}{1-\tau}\,[v_A(z)s_A(z) + v_B(z)s_B(z)]\,.$$

Migration effect An increase in taxes for all individuals above gross income z does not affect the migration decision of individuals with gross income $z'_A \geq z$, and accordingly also $z'_B > z$, such that the tax increase affects them in both regions alike. The same holds true for all individuals for which $z'_B = k(z'_A) < z$ and accordingly $z'_A = k^{-1}(z'_B) < z$. However, as illustrated in Figure 1, for all individuals for which $z'_A < z$ and $z_B \geq z$ the migration decision is negatively affected. In this range, all individuals whose cost of moving is between $\bar{q}$ and $\bar{q} - dzd\tau$ will now decide not to migrate. There are $p\,(\bar{q}|\, z)\, v_A(z)dzd\tau$ affected individuals at any appropriate level of income z with a resulting tax effect of

$T_A(z) - T_B(k(z))$ for each of them. The total migration effect is thus

$$M = d\tau dz \int_{\tilde{z}}^{z} [T(z') - T(k(z'))] \, p\left(\bar{q} | \, z'\right) v_A(z') dz',$$

where $\tilde{z} \equiv k^{-1}(z)$. Note that there is an endogenous effect on the income distribution in each region. This affect does not come into play explicitly here, since we express the effects in terms of the posterior distribution.

The three effects must balance out in the optimum: $R + L + M = 0$. From this we have our first result.

Proposition 1 *The optimal unified tax schedule is characterized by*

$$\frac{\tau}{1-\tau} = \mathfrak{A}(z)\mathfrak{B}(z)\left[\mathfrak{C}(z) + \mathfrak{D}(z)\right], \tag{7}$$

where

$$\begin{aligned}
\mathfrak{A}(z) &\equiv \frac{1}{\varepsilon}, \; \mathfrak{B}(z) \equiv \frac{1}{z\left(v_A(z)s_A(z) + v_B(z)s_B(z)\right)}, \\
\mathfrak{C}(z) &\equiv \int_{z}^{\infty} \left\{[1 - g_A(z')] \, v_A(z') \, s_A + [1 - g_B(z')] \, v_B(z') \, s_B\right\} dz', \\
\mathfrak{D}(z) &\equiv \int_{\tilde{z}}^{z} [T(z') - T(k(z'))] \, p\left(\bar{q} | \, z'\right) v_A(z') dz'.
\end{aligned}$$

Proof. This follows from the exposition above. The equivalence to the optimal tax formula formally derived by using the delayed optimal control technique is presented in the Appendix C. ■

It is straightforward to compare the result with the alternative benchmark without migration. The optimal tax schedule then follows the usual Diamond (1998) and Saez (2001) results for the earnings distribution in the entire country without a migration effect. In this case, optimal marginal tax rates are determined by

$$\frac{\tau}{1-\tau} = \mathfrak{A}(z)\mathfrak{B}(z)\mathfrak{C}(z). \tag{8}$$

With $\mathfrak{D}(z) < 0$, the disincentive effects of higher tax rates on productivity-increasing mobility tend to reduce marginal tax rates, but note that, in general, $\mathfrak{B}(z)$ and $\mathfrak{C}(z)$ are endogenously determined by the migration flows. To make this formal, we consider the benchmark in which the government faces the same distribution of realized productivity v and of population shares s as in the posterior situation generated by the optimal tax schedule with migration. Given this posterior distribution assume that there is, or the government believes so, no reaction in terms of location choice from the tax system, i.e.

that the posterior distribution is fixed and individuals only react to the taxation through their intensive labor supply margin. In this case the optimal tax follows the formula (8) with terms $\mathfrak{A}(z) > 0, \mathfrak{B}(z) > 0, \mathfrak{C}(z) > 0$ identical to the ones in (7). In this case, for $\mathfrak{D}(z) < 0$, we have $\tau_{\text{migration}} < \tau_{\text{no migration}}$. This allows us to formulate the following proposition:

Proposition 2 *A government neglecting the effect of taxes on the migration decision, but facing the distribution of realized productivity generated by migration, should set higher marginal tax rates than a government taking the migration decision into account, if marginal tax rates are positive.*

Proof. Going through the derivation of the optimal tax formula in the Appendix A under the assumption that the effect of tax on migration decision is neglected, i.e. $\frac{\partial \bar{q}}{\partial z} = \frac{\partial \bar{q}}{\partial V} = 0$, we arrive at the optimal tax formula (12) short of the term

$$-\int_{\omega^{-1}(n)}^{n} \left(T\left(\omega(n')\right) - T(n')\right) p(\bar{q}|n') f\left(n'\right) dn'.$$

If this is non-positive (that is equivalent to $D(z) \leq 0$), the result immediately follows. If marginal tax rates are positive, this condition is always fulfilled. ■

Note that positive marginal tax rates are a sufficient but not a necessary condition for this result. Whenever $\mathfrak{D} < 0$ for any given level of gross income z, marginal tax rates are lower with migration relative to the no-migration benchmark with the posterior distribution. Thus, whenever efficiency-enhancing migration implies a positive fiscal externality at a given innate productivity level, marginal tax rates should be reduced to take the marginal fiscal externality of inter-regional migration appropriately into account. This constrains optimal redistribution beyond the classic adverse labor supply responses.

Another direct implication of the optimal unified taxation formula (7) is stated in the following proposition.

Proposition 3 *Optimal marginal tax rates can be negative.*

Proof. For $\mathfrak{D}(z) < 0$, it is possible that $\mathfrak{C}(z) + \mathfrak{D}(z) < 0$, and thus $\tau < 0$.[6] ■

Similar to the findings of other studies that combine an extensive participation decision with the intensive labor supply decision also endogenous mobility between regions of different productivity can give raise to negative marginal tax rates.

Using the posterior distribution as in Proposition 2 is our preferred benchmark as it allows switching migration on and off while keeping the productivity distribution fixed.

[6] By simulative example (available upon request) it can be illustrated that this is the case for certain parameter values.

This benchmark also corresponds directly to the empirically observed spatial distribution of individuals and productivity at a given point in time. Accordingly, we also focus on it in our simulations in Section 6. However, for completeness, another benchmark to compare our optimal solution to is an economy with the ex ante distribution of productivity and without internal migration. As we show in the Appendix, the comparison of optimal marginal tax rates is less clear cut in this case given the endogeneity of the posterior productivity distribution, when allowing for migration. This also impacts on $\mathfrak{B}(z)$ and $\mathfrak{C}(z)$, and these effects may drive optimal marginal tax rates in the opposite direction. Formally, we provide a sufficient condition for mobility to decrease the marginal tax rates for this alternative benchmark in the Appendix.

Finally, we make the following remark about the welfare comparison in the unified taxation case.

Remark The welfare achieved with unified taxation in the no migration case is not higher than the welfare achieved with migration.

Proof. Consider the tax schedule that maximizes welfare if migration is not allowed. Migration brings a Pareto improvement, because individuals move only if they find themselves better-off. Furthermore, with migration to the richer region only, the government budget constraint will not be violated, if the tax is nondecreasing in income. Thus, under the same tax schedule the welfare may not decrease with the introduction of a migration possibility. Finally, the government will change the tax schedule only if it brings further increase in welfare. Thus, the welfare with migration may not be lower than welfare with no migration, Q.E.D. ■

5 Optimal differentiated taxation

We now consider the possibility that the central government can choose differentiated tax schedules for both regions. If there were regional productivity differences but no migration, this setting would correspond to the analysis of an optimal tax scheme with tagging on the region of residence. However, we continue to assume that migration between the regions is possible and that individuals are heterogenous with respect to their migration costs, which are unobservable by the government.

We first study the optimal tax schedule in the low productivity region. Consider an increase of taxes in Region A for all individuals above gross income z_A. The increase is engineered through an increase in the marginal tax rate $d\tau_A$ in the small band $(z_A, z_A + dz_A)$, such that all individuals with gross earnings above z_A increase their tax payments by $dz_A d\tau_A$.

Revenue effect: All taxpayers in A pay additional taxes of $dz_A d\tau_A$. The net welfare effect of this tax payment for an individual with gross earnings z'_A is given by $dz_A d\tau_A \left(1 - g_A\left(z'_A\right)\right)$ and the total effect is

$$R_A = dz_A d\tau_A \int_{z_A}^{+\infty} \left[1 - g_A\left(z'_A\right)\right] v_A\left(z'_A\right) s_A\left(z'_A\right) dz'_A.$$

Behavioral effect: Individuals in the band $(z_A, z_A + dz_A)$ will change their labor supply in response to the increase in the marginal tax rate. Given that $\varepsilon \equiv \frac{1-\tau_i}{z_i} \frac{dz_i}{d(1-\tau_i)}$, each individual in the band will reduce its income by $-d\tau_A \varepsilon \frac{z_A}{1-\tau_A}$. There are approximately $dz_A v_A(z_A) s_A(z_A)$ of these individuals, such that the total effect on tax revenue is

$$L_A = -d\tau_A dz \varepsilon \frac{\tau_A}{1-\tau_A} z_A v_A(z_A) s_A(z_A).$$

Migration effect: An increase in taxes for all individuals above gross income z_A affects the migration decision of individuals with gross income in Region A above this level. At any ability level $z \geq z_A$ individuals whose cost of moving is between $\bar{q}$ and $\bar{q} + dz d\tau_A$ will now decide to migrate. There are $p\left(\bar{q}\right| z_A) v_A(z_A) dz d\tau_A$ affected individuals with a resulting tax effect of $T_B(k(z_A)) - T_A(z_A)$ for each of them. If the schedule results in migration from Region B for people of income z, the argument is analogous, as we show formally in the Appendix. The total effect is thus

$$M_A = dz d\tau_A \int_{z_A}^{\infty} \left[T_B(k(z'_A)) - T_A(z'_A)\right] p\left(\bar{q}\right| z'_A) v_A(z'_A) dz'_A.$$

In the optimum, these effects should cancel out such that optimal marginal tax rates can be characterized by

$$\begin{aligned} \frac{\tau_A}{1-\tau_A} &= \frac{1}{\varepsilon} \frac{1}{z_A v_A(z_A) s_A(z_A)} \\ &\times \int_{z_A}^{\infty} \left\{\left[1 - g_A\left(z'_A\right)\right] s_A(z'_A) + \left[T_B(k(z'_A)) - T_A(z'_A)\right] p\left(\bar{q}\right| z'_A)\right\} v_A(z'_A) dz'_A \end{aligned} \tag{9}$$

We turn now to the optimal tax schedule in the high productivity region. We consider a small increase in taxes by $dz_B d\tau_B$ for all individuals above z_B in Region B. This again generates three effects, which must balance out along the optimal tax schedule, such that

$$\begin{aligned} \frac{\tau_B}{1-\tau_B} &= \frac{1}{\varepsilon} \frac{1}{z_B v_B(z_B) s_B(z_B)} \\ &\times \int_{z_B}^{+\infty} \left\{\left[1 - g_B\left(z'_B\right)\right] s_B(z'_B) - \left[T_B(z'_B) - T_A(k^{-1}(z'_B)\right] p\left(\bar{q}\right| z'_B)\right\} v_B\left(z'_B\right) dz'_B. \end{aligned} \tag{10}$$

Both optimal tax schedules are derived rigorously in the Appendix. The optimal tax formulae not only differ by the different average welfare weights and the respective productivity distribution above the gross income level for which taxes are increased, but they also take the fiscal externality from the effect on migration into account. Typically, this externality will be negative for the high productivity region and positive for the low productivity area. Accordingly, from the optimal tax schedules under differentiated taxation (9) and (10) we have the following result.

Proposition 4 *For all levels of innate productivity and the corresponding gross incomes the marginal tax rate in the low productivity region τ_A is increasing in the difference in total tax liability between the high and the low productivity regions, and the marginal tax rate in the high productivity region τ_B is decreasing in this difference in total tax liability.*

Proof. The result follows directly from (9) and (10). ■

Intuitively, the larger the potential fiscal gains are from working in the high productivity region instead of working in the low productivity region, the more the government distorts labor supply in the low productivity region and the less it distorts labor supply in the high productivity region. This indicates that the marginal tax rates are used to steer migration flows. Differences in the demogrant may be used instead to target redistribution by using the region as a productivity tag. In the Appendix we additionally rearrange the optimal taxation formulae (9) and (10) to show how the regional semi-elasticities of migration act as a correction factor to the region-dependent marginal social welfare weights in the determination optimal marginal tax rates.

5.1 Asymptotic properties with differentiated taxation

Suppose the distribution of innate ability $f(n)$ has an infinite tail ($n_{\max} = \infty$). As is standard in the literature, we assume that $f(n)$ has a Pareto tail with parameter $a > 1$ ($f(n) = C/n^{1+a}$). Moreover, we also assume that $P(q|n)$, $T_B - T_A, \tau_A, \tau_B$, $\bar{q}_A, \bar{q}_B$ converge to $P^\infty(q)$, $\Delta T^\infty, \tau_A^\infty < 1, \tau_B^\infty < 1$, $\bar{q}_A^\infty, \bar{q}_B^\infty$ as $n \to \infty$. We assume that for sufficiently large n, $\omega(n) = n + c$, where $c \geq 0$ is a finite constant. In this case, the following proposition arises:

Proposition 5 *Under the assumptions on convergence formulated above, (i) average marginal social welfare weights in two regions converge to the same value $\bar{\psi}/\lambda \geq 0$; (ii) the difference between taxes in two regions converges to zero, $\Delta T^\infty = 0$; and (iii) the marginal tax rate in both regions converges to τ^∞ with*

$$\frac{1}{a\varepsilon^\infty}\left(1 - \frac{\bar{\psi}}{\lambda}\right) = \frac{\tau^\infty}{1 - \tau^\infty}. \tag{11}$$

Proof. *The proof is left to the Appendix AA.* ■

The intuition for zero difference of top taxes is similar to that in Kleven et al. (2009). Namely, starting from a wedge between T_B and T_A, welfare could be increased by marginally reducing this wedge due to the migration effect. If T_B is decreased, some people move to Region B and pay higher taxes; if T_A is increased, some people move to Region B and pay higher taxes. Thus, though there are substantial differences in differentiated vs. unified tax schedules, they disappear in the limit of the higher top of the ability distribution.[7]

5.2 Marginal differentiation of the tax schedule

Once we allow for differences in the tax schedules of two regions, it makes sense to ask how different the schedules should be, starting from a situation with undifferentiated marginal tax rates. In particular, in the following we show that (i) starting from identical tax schedules and not allowing different marginal taxes on income, it is optimal to make a transfer to the more productive region; (ii) starting from different tax schedules with the same marginal taxes for the same ability, it is optimal to lower marginal taxes in the high productivity region while raising them in the low productivity region.

5.2.1 On desirability of transfers given equal marginal taxes

Consider a tax system that is separable in the sense that the same wage in two regions faces the same marginal tax. This is very similar to uniform taxation, but now we are allowed to charge the same incomes different taxes. The maximization problem of the government is the same apart from the feature that instead of the restriction that $\Delta T = 0$ we have the restriction $\Delta T = C$, where C is constant in n and $\Delta T := T_B(n) - T_A(n)$. For this setting, we can formulate the following proposition:

Proposition 6 *Starting from a unified taxation schedule in the two regions, if the government is allowed to make a lump-sum transfer between regions it will choose to make a transfer from the less productive to the more productive region.*
Proof. *The proof is left to Appendix AA.* ■

[7] The optimal tax formula for the uniform case simplifies to

$$\frac{1 - F(n)}{nf(n)} \left(2 - \frac{\bar{\psi}}{\lambda}\right) = 2\frac{\tau^{\infty}}{1 - \tau^{\infty}}\varepsilon^{\infty},$$

which is identical to (11) under the Pareto distribution and proper rescaling of the Lagrange multiplier.

While this result may appear surprising there is a clear economic intuition behind it. The tax on the poor region has to be higher in order to induce extra migration, which is productivity-enhancing. The extensive margin is used to increase efficiency via increased labor mobility, whereas redistribution is engineered through the intensive margin. This result is independent of the interpretation of migration costs.

5.2.2 On suboptimality of equal marginal taxes for the same ability

Consider a tax schedule that is separable in the sense that $\tau_A = \tau_B$. Starting from this schedule, the following proposition shows that in the cost of moving model decreasing the marginal tax in region B and increasing it in region A would be desirable:

Proposition 7 *If Ψ' is convex, q and n are independently distributed and $\omega'(n) \geq 1$, it is optimal to introduce some wedge in marginal taxes to the system of separable taxation of the two regions. In particular, in the cost of moving model it is optimal to decrease the marginal tax in the high productivity region and increase it in the low productivity region.*
Proof. *The proof is left to the appendix AA.* ■

The proof is quite intuitive: it is based on the fact that in the cost of moving model the difference in marginal welfare weights of residents of regions A and B is decreasing with productivity, if the social welfare exhibits prudence (marginal social welfare is convex). Thus, it makes sense to make the lower part of the productivity distribution in region A marginally happier than in region B, while making the upper part of productivity distribution in region B marginally happier than in region A. Hence, lower marginal tax rates in region B are optimal. The productivity transformation function $\omega(n)$ may however reverse this finding, if migration in the lower part of distribution is related to substantially larger productivity gains than migration in the upper part of distribution, i.e. $\omega'(n) < 1$, hence the condition on this function.

6 Simulation and Calibration

In this section we provide numerical simulations for the US to gain insights into the quantitative importance of efficiency-enhancing migration for the design of tax policy and optimal redistribution.[8] We focus on the difference between an optimal unified tax schedule with and without migration for a given posterior spatial productivity distribution as in Propositions 1 and 2. To implement our framework empirically, we divide the US

[8] Our simulations use a modified and extended version of the code developed by Kleven et al. (2009). We would like to express our gratitude to them for providing their original code to us.

into rural (low productivity) and urban (high productivity) areas. We use the empirically observable income distribution to recover the underlying productivity distributions in both regions, as well as the implied migration gains for workers of different innate productivity. We then simulate the standard optimal tax formula with and without efficiency-enhancing migration for the posterior productivity distribution to gauge the difference between them. In what follows, we first specify functional forms and parameters used in the simulations and then describe the calibration procedure.

6.1 Simulation specification

For simulations we use iso-elastic utility $h(\frac{z}{n}) = (\frac{z}{n})^{1+\epsilon}/\,(1+\epsilon)$ with a constant earnings elasticity $\varepsilon = \frac{1}{\epsilon}$ as in Saez (2001). Paralleling our theoretical derivations, we concentrate on the cost of moving model, hence $q = q^c$. Moreover, we follow Kleven et al. (2009) by assuming a power law distribution for the costs at the extensive margin on the interval $[0, q_{max}]$ with $P(q) = (q/q_{max})^{\eta}$ and $p(q) = \eta/q_{max} \cdot (q/q_{max})^{\eta-1}$. This distribution of q is the same in each region and independent of n, that is $\partial q_{max}/\partial n = 0$. The parameter η may be interpreted as a migration elasticity of the form $\eta = \frac{\bar{q}}{P(\bar{q}|n)} \frac{\partial P(\bar{q}|n)}{\partial \bar{q}}$. Empirically, the migration elasticity is an issue of recently ongoing research and our simulations confirm the importance of this parameter. As the social objective we use the constant rate of risk aversion (CRRA) function $\Psi\,(V) = V^{1-\gamma}/(1-\gamma)$, where the parameter γ measures the government's preference for equity. We choose $\gamma = 1$, hence $\Psi\,(V) = \log(V)$ in line with Chetty (2006). Finally, the simulation is done in a way that, with optimal tax rates obtained, the ratio of exogenous budget expenditures E to aggregate production is .25 as in Saez (2001). All incomes and potential incomes (abilities) are expressed in \$1,000 per year.

6.2 Calibration to the US

We proceed with the calibration of our economy to the US in four steps. First, we choose regions by focussing on the considerable productivity discrepency between rural and urban regions in the US.[9] To do this, we draw on the Rural Urban Continuum Code (RUCC, also known as the Beale code) that is provided by the US Department of Agriculture. The RUCC assigns each county to one of 9 classes. Starting with highly urban counties central in a metropolitan area and with a population of more than 1 million (class 1), the code goes up to 9 for completely rural counties that are not adjacent to a metropolitan area and/or exhibit a population of less than 2,500. The PSID data provides the RUCC

[9] This procedure specifies regions in an economic rather than administrative or purely geographical sense and is applicable to most countries.

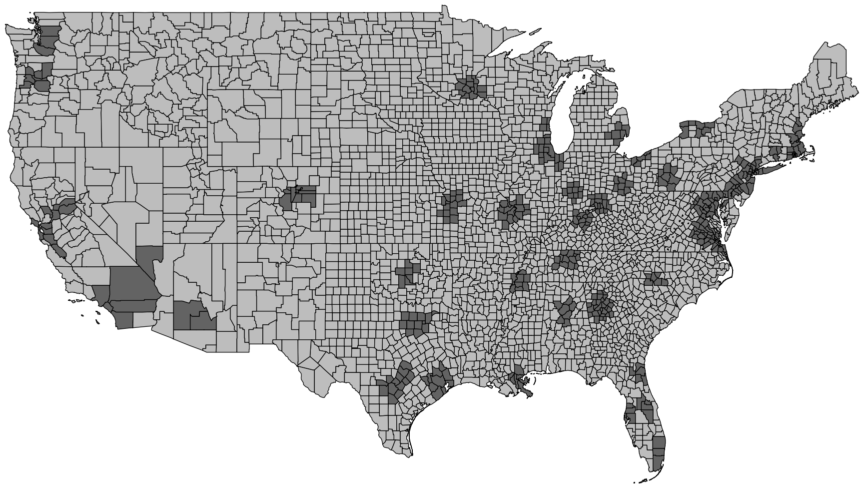

Figure 2: Split of US districts into two regions: Region A consists of rural counties (light), region B of urban ones (dark). Boundaries taken from US Census Bureau (census.gov: Cartographic Boundary Shapefiles).

for each individual's county of residence. We treat all counties belonging to class 1 to be the urban region (Region B), and counties of classes 2 through 9 to be the rural region (Region A) as illustrated by Figure 2.

Second, we recover the ability distributions for these regions using individuals' maximization as given by Equation (2) with earning elasticity $\varepsilon = 0.25$ as suggested by Saez (2001). Specifically, we combine the 2006 individual gross labor income data from the 2007 PSID for unmarried, working individuals with no children under 18 years with the corresponding marginal tax rate from the NBER TAXSIM model. This procedure is similar to Best and Kleven (2013), who differentiate individuals by age, whereas we use a regional distinction. As suggested by Diamond (1998) and Saez (2001), very high incomes are well approximated by a Pareto distribution. Therefore, the skill distributions are modified by assuming a Paretian shape for higher gross incomes, $z > \$150,000$, which amounts roughly to the top 3%. This parallels the assumptions used in Jacquet et al. (2013) or Best and Kleven (2013). This Paretian addition exhibits a reasonable coefficient for the US. We estimate the specific Pareto parameter for each of our regions by regression of the gross income ratio z_m/z between \$100,000-\$150,000 where z_m is the average of earnings above z. Note that for a Pareto distribution with parameter a, $z_m/z = a/(a-1)$. The recovering procedure is as follows: each individual's ability is computed from individual maximization using its income data from the PSID, the actual marginal tax rate corresponding to this income level from TAXSIM together with the earning elasticity ε and the functional assumption for $h(\frac{z}{n})$. For all incomes above \$150,000 the computed ability is then substituted by the respective Pareto value. This procedure is applied to both

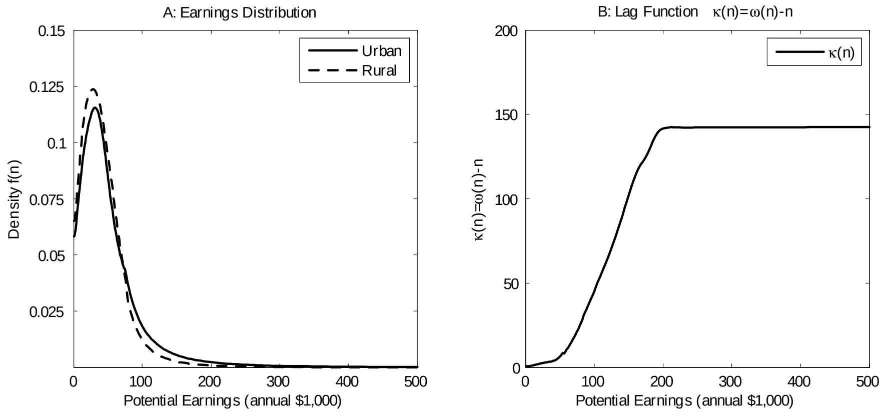

Figure 3: Revealed true abilities (plot A) and revealed lag function $\kappa(n)$ (plot B) for the two chosen regions of the US based on data from PSID 2008/09 and TAXSIM.

regions. Figure 3A depicts the computed skill distributions in regions A and B, where, for visualization, the discrete values are smoothed continuously using kernel estimation. The resulting descriptive statistics for both areas exhibit a (median) mean ability difference, that amounts to, in terms of our theory framework, a difference in potential income of (11.7%) 32.8% between the rural and urban areas.

Third, we estimate the lag function $\kappa(n) = \omega(n) - n$ from the data by using the differences in mean ability of each frequency percentile of the two regions. This difference is then assumed to be the productivity increase for the mean person (sampling point) of each percentile. The function is estimated by linear interpolation using these sampling points and smoothed afterwards.[10] We obtain an increasing nonlinear function presented in Figure 3B.[11] We find a substantial productivity increase from migration from a rural to an urban district, in particular for the types with potential annual incomes of around $50,000-$200,000 at their origin.

Fourth, the migration cost distribution is calibrated by choosing the migration elastic-

[10]In detail, Matlab routine 'interp1' is applied.

[11]Our empirical construction of ω may be justified as follows. By the definition of the transformation function ω the cumulative distribution functions of ability in two regions are related as $F_B(n) = F_A(\omega(n)) \forall n \in [n_{\min}, n_{\max}]$. Then, at each α-percentile it is true that $F_B(n_\alpha) = F_A(\omega(n_\alpha)) = \alpha$. From the properties of cdfs assumed (strictly increasing) it follows then that function ω can be reconstructed from F_A and F_B at any point $n_\alpha \forall \alpha \in [0,1]$ or, equivalently, $\forall n \in [n_{\min}, n_{\max}]$. In our simulation, we do not observe the true cdfs, but only their empirical counterparts, $\hat{F}_A$ and $\hat{F}_B$. The proof of statistical properties of our approach is beyond the scope of this paper. Note however that under the assumption that we actually observe the true cdfs at a limited number of data points m, a smooth interpolation is the best way to fill in the missing values in the estimates of F_A and F_B, because cdfs are smooth by continuity of the pdfs. Once we have the estimates of cdfs defined over the whole domain, we can recover the function ω for any point in the domain. The only remaining problem then are the corners. Whereas theoretically we should observe abilities starting from $n_{\min}$ in one region and $\omega(n_{\min})$ in another region, empirically we observe only the lowest income cathegory and hence $n_A(z_{\min}) = n_B(z_{\min})$. For high ability levels we assume that $\kappa(n)$ is constant.

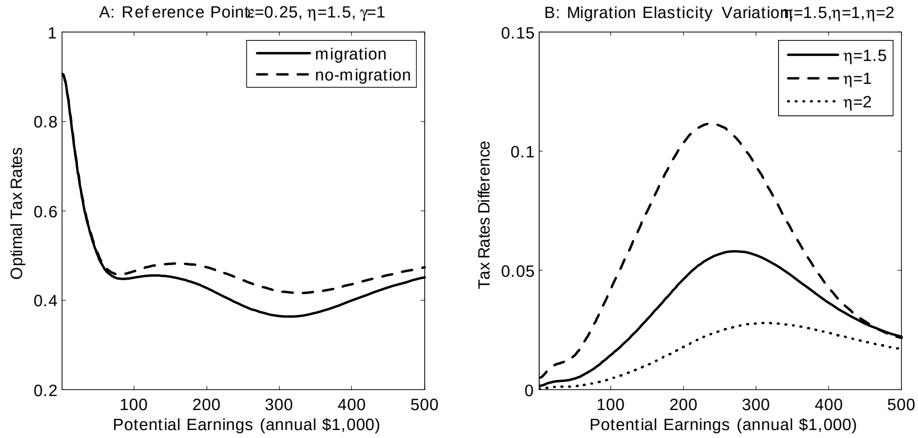

Figure 4: Optimal uniform tax simulations for the US based on data from PSID 2008/09 and TAXSIM.

ity η and the parameter $q_{\max}$. We use $\eta = 1.5$ for the migration elasticity, and additionally consider $\eta = 1$ and $\eta = 2$ to assess the sensitivity of the results with respect to this parameter. By choosing $q_{\max}$ we then calibrate the migration costs such that the typical move costs around $\$34,000$ which is the value estimated by Bayer and Juessen (2012) and, likewise, is in the range obtained by Kennan and Walker (2011).

6.3 Results for the US

Figure 4 illustrates the simulation outcomes for incomes up to $\$500,000$ under common parameter settings. As standard in the optimal tax literature following Diamond (1998) a U-shaped pattern appears. In the no-migration case (dashed line), which uses the posterior productivity distribution outcome with migration, the government chooses a higher marginal tax rate compared to the migration case (solid line) as stated in Proposition 2. Apparently, from Figure 4A, the migration effect on taxes leads to a marginal tax rate difference up to 6 percentage points in the most relevant level of mid-abilities. Moreover, productivity increasing migration seems to smooth the U-shaped pattern (Figure 4A). Interestingly, the feature of smoothing the U-shaped pattern is likewise obtained by Best and Kleven (2013) in their setting. Finally, using alternative values for the migration elasticity shows that, a higher migration elasticity reduces the marginal tax rate differences between the migration and the no-migration case as depicted in Figure 4B.[12]

[12]Our robustness checks indicate that this relationship is not monotone. As is also immediate from theory, the differences in marginal tax rates are also reduced for relatively low values of η, and completely disappear for $\eta \to 0$.

7 Concluding remarks

Regional inequality and the corresponding possibility of efficiency-enhancing migration can be an important determinant of the optimal redistributive tax-transfer scheme. A government that is constrained to use a unified redistribution policy faces an additional equity-efficiency trade-off beyond the intensive labor supply margin. Optimal taxation needs to be modified to take the fiscal migration externality into account, and our simulations indicate that the additional constaint to redistribution is quantitatively important. In our analysis we have abstracted from some aspects that are relevant in practice. First, in our analysis, the central government is restricted to use a unified or regionally differentiated tax scheme, but is not allowed to use targeted subsidies to migrants only. If such targeted transfers were available to the government, they could potentially loosen the trade-off between redistribution and internal migration. However, a fixed migration subsidy typically cannot eliminate the problem completely, since the fiscal migration externality differs by earnings level. Even if the migration subsidy could be adjusted by earnings level, these need to be incentive compatible, and this may imply an additional constraint for tax policy. Secondly, regional productivity differences are partly reflected in local prices of non-tradable goods, rents and house prices, which also reduces migration incentives. While this an important additional aspect, it does not challenge our basic intuition for the modification of the optimal tax schedule. The fiscal migration externality still exists in this case and should be taken into account accordingly. Thirdly, we have also abstracted from redistributive taxation and welfare programs at the state level, which are an important component in some countries, including the US. Such state-level policies additionally affect migration incentives. Depending on whether these policies increase or decrease the fiscal migration externalities, they may strengthen or weaken the constraint of productivity-enhancing inter-regional migration for federal redistribution.

Our results have immediate implications for redistributive tax policy. Policy makers should not only worry about external migration, but also need to consider the role of internal migration. This constraint is less important for countries that are characterized by low regional inequality. However, in countries where regional inequality is substantial, policy makers should carefully assess how tax progressivity may hurt productivity-enhancing inter-regional migration.

8 Appendix A: Formal derivation of the optimal tax formulae

We now show that the optimal tax formulae (7) and (9) and (10) can also be rigorously derived by standard optimal control techniques. The equivalence of the expression in terms of n and z is shown in Appendix D. We start with the differentiated case, since the unified case can be interpreted as the same problem with the additional constraint of the tax schedules to be identical in both regions.

8.1 Regionally differentiated taxation

The government maximizes

$$\begin{aligned} W &= \int_{n_{\min}}^{n_{\max}} [\int_{\bar{q}_B}^{+\infty} \Psi\left(V_B(\omega(n)) + q^h\right) p(q|n)dq + \int_0^{\bar{q}_A} \Psi\left(V_B(\omega(n)) - q^c\right) p(q|n)dq \\ &+ \int_{\bar{q}_A}^{+\infty} \Psi\left(V_A(n) + q^h\right) p(q|n)dq + \int_0^{\bar{q}_B} \Psi\left(V_A(n) - q^c\right) p(q|n)dq] f(n)dn, \end{aligned}$$

where $\bar{q}_A = \max\{V_B(\omega(n)) - V_A(n), 0\}$, $\bar{q}_B = \max\{V_A(n) - V_B(\omega(n)), 0\}$, $q = q^c + q^h$, and either $q^c = 0$ or $q^h = 0$. The first term in this expression stands for the social welfare from the population of region B who did not move, the second term stands for that of the population moved from A to B, the third term is for those who stayed in A, and the fourth term is for those who moved from B to A. Note that either the second or the fourth term is equal to zero, because migration in both direction at the same ability level is not possible.

The maximization is subject to

$$\begin{aligned} &\int_{n_{\min}}^{n_{\max}} [\left(z_B - \omega(n) h\left(\frac{z_B}{\omega(n)}\right) - V_B(\omega(n))\right)(1 + P(\bar{q}_A|n) - P(\bar{q}_B|n)) \\ &+ \left(z_A - nh\left(\frac{z_A}{n}\right) - V_A\right)(1 + P(\bar{q}_B|n) - P(\bar{q}_A|n))] f(n)dn \geq E \end{aligned}$$

and the corresponding incentive compatibility constraints. Note that either $P(\bar{q}_A|n)$ or $P(\bar{q}_B|n)$ is zero for the same reason as discussed above.

Let the Hamiltonian be $H(z_A, z_B, V_A, V_B, \lambda, \mu_A, \mu_B, n)$. The necessary conditions are

1. There exist absolutely continuous multipliers $\mu_A(n), \mu_B(n)$ such that on $(n_{\min}, n_{\max})$ $\dot{\mu}_B(n) = -\frac{\partial H(n)}{\partial V_B(n)}$, $\dot{\mu}_A(n) = -\frac{\partial H(n)}{\partial V_A(n)}$ almost everywhere with $\mu_i(n_{\min}) = \mu_i(n_{\max}) = 0$.

2. We have $H(z_i(n), V_i, \lambda, \mu_i, n) > H(z_i, V_i, \lambda, \mu_i, n)$ almost everywhere in n for all z. The first order conditions are $\frac{\partial H}{\partial z_A} = 0$, $\frac{\partial H}{\partial z_B} = 0$.

Uniqueness of z_A and z_B that solve the equations above can be established in the similar way to Kleven et al. (2009), using the assumption that $\varphi(x) = (1 - h'(x)) / (xh''(x))$ is decreasing in x. Indeed, the FOCs can be rewritten as

$$\begin{aligned} \frac{\mu_A(n)}{n}\frac{z_A}{n} h''\left(\frac{z_A}{n}\right) + \lambda\left(1 - h'\left(\frac{z_A}{n}\right)\right)(1 + P(\bar{q}_B|n) - P(\bar{q}_A|n)) f(n) = 0, \\ \varphi\left(\frac{z_A}{n}\right) = -\frac{\mu_A(n)}{\lambda n f(n)(1 + P(\bar{q}_B|n) - P(\bar{q}_A|n))} \end{aligned}$$

for region A and

$$\frac{\mu_B(n)}{\omega(n)}\frac{z_B}{\omega(n)}h''\left(\frac{z_B}{\omega(n)}\right)+\lambda\left(1-h'\left(\frac{z_B}{\omega(n)}\right)\right)\left(1+P(\bar{q}_A|n)-P(\bar{q}_B|n)\right)f(n)=0$$

$$\varphi\left(\frac{z_B}{\omega(n)}\right)=-\frac{\mu_B(n)}{\lambda\omega(n)f(n)\left(1+P(\bar{q}_A|n)-P(\bar{q}_B|n)\right)}$$

for region B. In both cases, LHS is decreasing in z_A/n $(z_B/\omega(n))$ whereas RHS is constant, which implies that $z_i(n)$ is a unique solution and a global maximum indeed. Continuity can be then established in a way similar to Kleven et al (2009).

The conditions for $\dot{\mu}_i(n)$ imply

$$\begin{aligned}
-\dot{\mu}_A(n) &= f(n)[\int_{\bar{q}_A}^{+\infty}\Psi'\left(V_A(n)+q^h\right)p(q|n)dq+\int_0^{\bar{q}_B}\Psi'\left(V_A(n)-q^c\right)p(q|n)dq \\
&\quad -\lambda\left(1+P(\bar{q}_B|n)-P(\bar{q}_A|n)\right) \\
&\quad +\lambda\left(T_A-T_B\right)\left(p(\bar{q}_B|n)+p(\bar{q}_A|n)\right)],
\end{aligned}$$

and

$$\begin{aligned}
-\dot{\mu}_B(n) &= f(n)[\int_{\bar{q}_B}^{+\infty}\Psi'\left(V_B(\omega(n))+q^h\right)p(q|n)dq+\int_0^{\bar{q}_A}\Psi'\left(V_B(\omega(n))-q^c\right)p(q|n)dq \\
&\quad -\lambda\left(1+P(\bar{q}_A|n)-P(\bar{q}_B|n)\right) \\
&\quad +\lambda\left(T_B-T_A\right)\left(p(\bar{q}_B|n)+p(\bar{q}_A|n)\right)],
\end{aligned}$$

Integrating this, we have

$$\begin{aligned}
-\frac{\mu_A(n)}{\lambda} &= \int_n^{n_{\max}}[-\frac{1}{\lambda}\left(\int_{\bar{q}_A}^{+\infty}\Psi'\left(V_A(n)+q^h\right)p(q|n')dq+\int_0^{\bar{q}_B}\Psi'\left(V_B\left(\omega(n')\right)-q^c\right)p(q|n')dq\right) \\
&\quad +1+P(\bar{q}_B|n')-P(\bar{q}_A|n') \\
&\quad -\left(T_A-T_B\right)\left(p(\bar{q}_B|n')+p(\bar{q}_A|n')\right)]f(n')dn',
\end{aligned}$$

Analogously, for region B we get

$$\begin{aligned}
-\frac{\mu_B(n)}{\lambda} &= \int_n^{n_{\max}}[-\frac{1}{\lambda}\left(\int_{\bar{q}_B}^{+\infty}\Psi'\left(V_B\left(\omega(n')\right)+q^h\right)p(q|n')dq+\int_0^{\bar{q}_A}\Psi'\left(V_B\left(\omega(n')\right)-q^c\right)p(q|n')dq\right) \\
&\quad +\left(1+P(\bar{q}_A|n')-P(\bar{q}_B|n')\right) \\
&\quad -\left(T_B-T_A\right)\left(p(\bar{q}_B|n')+p(\bar{q}_A|n')\right)]dn'.
\end{aligned}$$

Defining by $g_A(n)$ the average marginal social welfare weight of the region A residents with inborn ability n, by $g_B(n)$ the average marginal social welfare weight of the region B initial residents with inborn ability n, we have

$$\begin{aligned}
g_A(n) &= \frac{\int_{\bar{q}_A}^{+\infty}\Psi'\left(V_A(n)+q^h\right)p(q|n)dq+\int_0^{\bar{q}_B}\Psi'\left(V_A(n)-q^c\right)p(q|n)dq}{\lambda\left(1+P(\bar{q}_B|n)-P(\bar{q}_A|n)\right)}, \\
g_B(n) &= \frac{\int_{\bar{q}_B}^{+\infty}\Psi'\left(V_B(\omega(n))+q^h\right)p(q|n)dq+\int_0^{\bar{q}_A}\Psi'\left(V_B(\omega(n))-q^c\right)p(q|n)dq}{\lambda\left(1+P(\bar{q}_A|n)-P(\bar{q}_B|n)\right)}.
\end{aligned}$$

Using these, we can rewrite the optimality conditions as

$$
\begin{aligned}
-\frac{\mu_A(n)}{\lambda} &= \int_n^{n_{\max}} [\left(1 - g_A(n')\right)\left(1 + P(\bar{q}_B|n') - P(\bar{q}_A|n')\right) \\
&\quad - (T_A - T_B)\left(p(\bar{q}_B|n') + p(\bar{q}_A|n')\right)] f(n') dn',
\end{aligned}
$$

$$
\begin{aligned}
-\frac{\mu_B(n)}{\lambda} &= \int_n^{n_{\max}} [\left(1 - g_B(n')\right)\left(1 + P(\bar{q}_A|n') - P(\bar{q}_B|n')\right) \\
&\quad - (T_B - T_A)\left(p(\bar{q}_B|n') + p(\bar{q}_A|n')\right)] dn'.
\end{aligned}
$$

Inserting into the FOCs, we get

$$
\begin{aligned}
&\frac{1}{nf(n)\varepsilon_A\left(1 + P(\bar{q}_B|n) - P(\bar{q}_A|n)\right)} \int_n^{n_{\max}} [\left(1 - g_A(n')\right)\left(1 + P(\bar{q}_B|n') - P(\bar{q}_A|n')\right) \\
&\qquad - (T_A - T_B)\left(p(\bar{q}_B|n') + p(\bar{q}_A|n')\right)] f(n') dn' = \frac{\tau_A}{1 - \tau_A},
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1}{\omega(n)f(n)\varepsilon_B\left(1 + P(\bar{q}_A|n) - P(\bar{q}_B|n)\right)} \int_n^{n_{\max}} [\left(1 - g_B(n')\right)\left(1 + P(\bar{q}_A|n') - P(\bar{q}_B|n')\right) \\
&\qquad - (T_B - T_A)\left(p(\bar{q}_B|n') + p(\bar{q}_A|n')\right)] f(n') dn' = \frac{\tau_B}{1 - \tau_B}
\end{aligned}
$$

for the marginal rates in region A and in region B, respectively. The formulae are similar to Kleven et al (2009) except that two terms (rather than one) reflect the possibility of either immigration to or emigration from the given region. Note that for each n, there are two mutually exclusive scenarios: either there is migration from A to B (and $V_B(\omega(n)) > V_A(n)$) so that the formulae take the form

$$
\begin{aligned}
&\frac{1}{nf(n)\varepsilon_A\left(1 - P(\bar{q}_A|n)\right)} \int_n^{n_{\max}} [\left(1 - g_A(n')\right)\left(1 - P(\bar{q}_A|n')\right) \\
&\qquad - (T_A - T_B)\, p(\bar{q}_A|n')] f(n') dn' = \frac{\tau_A}{1 - \tau_A},
\end{aligned}
$$

and

$$
\begin{aligned}
&\frac{1}{\omega(n)f(n)\varepsilon_B\left(1 + P(\bar{q}_A|n)\right)} \int_n^{n_{\max}} [\left(1 - g_B(n')\right)\left(1 + P(\bar{q}_A|n')\right) \\
&\qquad - (T_B - T_A)\, p(\bar{q}_A|n')] f(n') dn' = \frac{\tau_B}{1 - \tau_B},
\end{aligned}
$$

or there is migration from B to A (and $V_B(\omega(n)) < V_A(n)$) so that the formulae turn to

$$
\begin{aligned}
&\frac{1}{nf(n)\varepsilon_A\left(1 + P(\bar{q}_B|n)\right)} \int_n^{n_{\max}} [\left(1 - g_A(n')\right)\left(1 + P(\bar{q}_B|n')\right) \\
&\qquad - (T_A - T_B)\, p(\bar{q}_B|n')] f(n') dn' = \frac{\tau_A}{1 - \tau_A}
\end{aligned}
$$

and

$$
\begin{aligned}
&\frac{1}{\omega(n)f(n)\varepsilon_B\left(1 - P(\bar{q}_B|n)\right)} \int_n^{n_{\max}} [\left(1 - g_B(n')\right)\left(1 - P(\bar{q}_B|n')\right) \\
&\qquad - (T_B - T_A)\, p(\bar{q}_B|n')] f(n') dn' = \frac{\tau_B}{1 - \tau_B}.
\end{aligned}
$$

The average marginal tax rate is then

$$f(n)[n\frac{\tau_A}{1-\tau_A}\varepsilon_A\left(1+P(\bar{q}_B|n)-P(\bar{q}_A|n)\right)$$
$$+\omega(n)\frac{\tau_B}{1-\tau_B}\varepsilon_B\left(1+P(\bar{q}_A|n)-P(\bar{q}_B|n)\right)]=\int_n^{n_{\max}}\left(2-\bar{g}\left(n'\right)\right)f(n')dn',$$

where
$$\bar{g}\left(n\right):=g_A(n)\left(1+P(\bar{q}_B|n)-P(\bar{q}_A|n)\right)+g_B(n)\left(1+P(\bar{q}_A|n)-P(\bar{q}_B|n)\right)$$

is the average social marginal welfare weight of the individuals with inborn ability n and we have 2 instead of 1 simply because our total population in two regions is of measure 2. Clearly, $\tau_A\left(n_{\max}\right)=0=\tau_B\left(\omega\left(n_{\max}\right)\right)$ and $\tau_A\left(n_{\min}\right)=0=\tau_B\left(\omega\left(n_{\min}\right)\right)$ from the transversality conditions.

Define migration semi-elasticities $\mu_i^+\left(n\right):=\frac{1}{1+P(\bar{q}_i|n)}\frac{\partial P(\bar{q}_i|n)}{\partial\bar{q}}=\frac{p(\bar{q}_i|n)}{1+P(\bar{q}_i|n)}$ for the region with inflow of population and $\mu_i^-\left(n\right):=\frac{1}{1-P(\bar{q}_i|n)}\frac{\partial P(\bar{q}_i|n)}{\partial\bar{q}}=\frac{p(\bar{q}_i|n)}{1-P(\bar{q}_i|n)}$ for the region with outflow of population. Define migration elasticity as $\nu_i:=\mu_i\left(T_A-T_B\right)$, whereby normalizing in terms of tax differential rather than utility differential V_B-V_A is for notational conviniency. We have

$$\frac{1}{nf(n)\varepsilon_A\left(1-P(\bar{q}_A|n)\right)}\int_n^{n_{\max}}\left[1-g_A(n')-\nu_A^-\left(n'\right)\right]\left(1-P(\bar{q}_A|n')\right)f(n')dn'=\frac{\tau_A}{1-\tau_A}$$

and

$$\frac{1}{\omega(n)f(n)\varepsilon_B\left(1+P(\bar{q}_A|n)\right)}\int_n^{n_{\max}}\left[1-g_B(n')+\nu_A^+\left(n'\right)\right]\left(1+P(\bar{q}_A|n')\right)f(n')dn'=\frac{\tau_B}{1-\tau_B}.$$

The effect of the migration elasticity as a top-up to the marginal social welfare weight is evident from the resulting formulae. Indeed, the marginal tax rate in region A (source region) is reduced by the migration elasticity in the same way it is reduced by the welfare weight of region A citizens. Conversely, the marginal tax rate in region B (recepient region) is increased by migration elasticity in the same way it is reduced by the welfare weight of region B citizens. Intuitively, marginal increase of tax for all skill levels above n in region A will lead to outflow of people resulting in the loss of revenue differential T_A-T_B between two regions, properly accounted for by the term $\nu_A^-\left(n'\right)$ at each skill level n'. In region B, the same mechanism is in action, only the loss of revenue differential is properly accounted for by the term $-\nu_A^+\left(n'\right)$. From the formulae above we can also see that more elastic migration response leads to higher marginal tax rates in region A and lower marginal tax rates in region B (migration elasticity is negative whenever $T_A<T_B$).

8.2 Non-differentiated tax schedule

In this case the tax schedules in two regions must be identical, and hence the indirect utilities also are (there are no differences in preferences). The government problem is to maximize

$$\begin{aligned} W &= \int_{n_{\min}}^{n_{\max}}[\int_0^{+\infty}\Psi\left(V\left(\omega(n)\right)+q^h\right)p(q|n)dq+\int_0^{\bar{q}}\Psi\left(V\left(\omega(n)\right)-q^c\right)p(q|n)dq \\ &\quad+\int_{\bar{q}}^{+\infty}\Psi\left(V\left(n\right)+q^h\right)p(q|n)dq]f(n)dn, \end{aligned}$$

where $\bar{q}=V\left(\omega(n)\right)-V\left(n\right)$, and either q^h or q^c is equal to zero. We have also dropped the subscript B from the omega function for more parsimonious notation. The maximization is subject to

$$\int_{n_{\min}}^{n_{\max}}[\left(z\left(\omega(n)\right)-\omega(n)h\left(\frac{z\left(\omega(n)\right)}{\omega(n)}\right)-V\left(\omega(n)\right)\right)(1+P(\bar{q}|n))$$
$$+\left(z-nh\left(\frac{z}{n}\right)-V\right)(1-P(\bar{q}|n))]f(n)dn\geq E,$$

where the superscript w stands for the individuals with productivity $\omega(n)$. Note that in the uniform case there cannot be migration from B to A, as this would imply $V\left(\omega(n)\right)<V\left(n\right)$ that contradicts incentive compatibility (the productivity type $\omega(n)$ can pretend to have productivity n without any costs).

Let the Hamiltonian be $H(z,z^w,V,V^w,\lambda,\mu,n)$. This is a delayed optimal control problem analogous to the one formally analyzed by Göllmann et al. (2008) in its entire generality. The difference is that whereas Göllmann et al. have a lag of fixed size over the whole domain of their functions, our lag is a smooth increasing function of n, namely $\omega(n)-n$. The necessary conditions for optimal control in such a setting is presented in Abdeljawad et al (2009). Namely, in our context the necessary conditions for the maximum are:

1. There exist absolutely continuous multipliers $\mu(n)$ such that on $(n_{\min},n_{\max})$ $\dot{\mu}(n)=-\frac{\partial H(n)}{\partial V_B(n)}-\mathbb{I}_{[\omega(n_{\min}),\omega(n_{\max})]}\frac{\partial H\left(\omega^{-1}(n)\right)}{\partial V^w(n)}$, $\dot{\mu}_A(n)=-\frac{\partial H(n)}{\partial V_A(n)}$ almost everywhere with $\mu(n_{\min})=\mu(n_{\max})=0$.

2. We have $H(z(n),z^w(n),V,V^w,\lambda,\mu,n)>H(z,z^w,V,V^w,\lambda,\mu,n)$ almost everywhere in n for all z. The first order condition is

$$\frac{\partial H}{\partial z}+I_{[\omega(n_{\min}),\omega(n_{\max})]}\frac{\partial H\left(\omega^{-1}(n)\right)}{\partial z_B^w}=0$$

The fact that this condition describes a global maximum can be established in the way similar to Kleven et al. (2009), using the assumption that $\varphi\left(x\right)=\left(1-h'\left(x\right)\right)/\left(xh''\left(x\right)\right)$ is decreasing in x.

$$\frac{\mu}{n}\frac{z}{n}h''\left(\frac{z}{n}\right)+\lambda\left(1-h'\left(\frac{z}{n}\right)\right)\left(\left(1-P(\bar{q}|n)\right)f(n)+\left(1+P(\bar{q}_1|\omega^{-1}(n))\right)f\left(\omega^{-1}(n)\right)\right)=0$$
$$\varphi\left(\frac{z}{n}\right)=-\frac{\mu(n)}{\lambda n\left(\left(1-P(\bar{q}|n)\right)f(n)+\left(1+P(\bar{q}_1|\omega^{-1}(n))\right)f\left(\omega^{-1}(n)\right)\right)}$$

LHS is decreasing in z_i/n whereas RHS is constant, which implies that $z_i(n)$ is a unique solution and a global maximum indeed. Continuity can be then established in a way similar to Kleven et al (2009). Further,

$$\begin{aligned}
-\dot{\mu}(n) &= f(\omega^{-1}(n))\int_0^{+\infty}\Psi'\left(V\left(n\right)+q^h\right)p(q|\omega^{-1}(n))dq\\
&+f\left(\omega^{-1}(n)\right)\int_0^{\bar{q}_1}\Psi'\left(V\left(n\right)-q^c\right)p(q|\omega^{-1}(n))dq\\
&+f(n)\int_{\bar{q}}^{+\infty}\Psi'\left(V\left(n\right)+q^h\right)p(q|n)dq\\
&+\lambda[-\left(1-P(\bar{q}|n)\right)f(n)-\left(1+P(\bar{q}_1|\omega^{-1}(n))\right)f(\omega^{-1}(n))\\
&-\left(z^w-n^wh\left(\frac{z^w}{n^w}\right)-V^w-\left(z-nh\left(\frac{z}{n}\right)-V\right)\right)p(\bar{q}|n)f(n)\\
&+\left(z-nh\left(\frac{z}{n}\right)-V-\left(z^{-w}-n^{-w}h\left(\frac{z^{-w}}{n^{-w}}\right)-V^{-w}\right)\right)p(\bar{q}_1|\omega^{-1}(n))f(\omega^{-1}(n))],
\end{aligned}$$

where $\bar{q}_1 = V(n) - V\left(\omega^{-1}(n)\right)$ and $\bar{q} = V(\omega(n)) - V(n)$. Rewriting in terms of taxes, we have

$$\begin{aligned}
-\dot{\mu}(n) &= f(\omega^{-1}(n))\left(\int_0^{+\infty}\Psi'\left(V(n)+q^h\right)p(q|\omega^{-1}(n))dq + \int_0^{\bar{q}_1}\Psi'\left(V(n)-q^c\right)p(q|\omega^{-1}(n))dq\right)\\
&\quad + f(n)\int_{\bar{q}}^{+\infty}\Psi'\left(V(n)+q^h\right)p(q|n)dq\\
&\quad + \lambda[-\left(1-P(\bar{q}|n)\right)f(n) - \left(1+P(\bar{q}_1|\omega^{-1}(n))\right)f(\omega^{-1}(n))\\
&\quad - \left(T(\omega(n)) - T(n)\right)p(\bar{q}|n)f(n)\\
&\quad + \left(T(n) - T(\omega^{-1}(n))\right)p(\bar{q}_1|\omega^{-1}(n))f(\omega^{-1}(n))]
\end{aligned}$$

Defining by $g_i(n)$ the average marginal social welfare weight of the region i residents with inborn ability n, we have

$$\begin{aligned}
g_A(n) &= \frac{1}{\lambda}\frac{\int_{\bar{q}}^{+\infty}\Psi'\left(V_A(n)+q^h\right)p(q|n)dq}{1-P(\bar{q}|n)},\\
g_B\left(\omega^{-1}(n)\right) &= \frac{\frac{1}{\lambda}\left(\int_0^{\bar{q}_1}\Psi'\left(V_B(n)-q^c\right)p(q|\omega^{-1}(n))dq + \int_0^{+\infty}\Psi'\left(V(n)+q^h\right)p(q|\omega^{-1}(n))dq\right)}{1+P(\bar{q}_1|\omega^{-1}(n))}.
\end{aligned}$$

Thus, we can write

$$\begin{aligned}
-\frac{\dot{\mu}(n)}{\lambda} &= \left(g_B\left(\omega^{-1}(n)\right)-1\right)\left(1+P(\bar{q}_1|\omega^{-1}(n))\right)f\left(\omega^{-1}(n)\right)\\
&\quad + \left(g_A(n)-1\right)\left(1-P(\bar{q}|n)\right)f(n)\\
&\quad - \left(T(\omega(n)) - T(n)\right)p(\bar{q}|n)f(n)\\
&\quad + \left(T(n) - T(\omega^{-1}(n))\right)p(\bar{q}_1|\omega^{-1}(n))f(\omega^{-1}(n))
\end{aligned}$$

and integrating

$$\begin{aligned}
-\frac{\mu(n)}{\lambda} &= \int_n^{n_{\max}}[\left(1-g_B\left(\omega^{-1}(n')\right)\right)\left(1+P(\bar{q}_1|\omega^{-1}(n'))\right)f\left(\omega^{-1}(n')\right)\\
&\quad + \left(1-g_A(n')\right)\left(1-P(\bar{q}|n')\right)f(n')\\
&\quad + \left(T\left(\omega(n')\right) - T(n')\right)p(\bar{q}|n')f(n')\\
&\quad - \left(T\left(n'\right) - T(\omega^{-1}(n'))\right)p(\bar{q}_1|\omega^{-1}(n'))f(\omega^{-1}(n'))]dn'
\end{aligned}$$

and substituting into the FOC (using the definition of elasticity $\varepsilon = nh'/zh''$),

$$\begin{aligned}
&\frac{1}{n\varepsilon}\frac{1}{\left(1-P(\bar{q}|n)\right)f(n) + \left(1+P(\bar{q}_1|\omega^{-1}(n))\right)f\left(\omega^{-1}(n)\right)}\times\\
&\int_n^{n_{\max}}[\left(1-g_B\left(\omega^{-1}(n')\right)\right)\left(1+P(\bar{q}_1|\omega^{-1}(n'))\right)f\left(\omega^{-1}(n')\right) + \left(1-g_A(n')\right)\left(1-P(\bar{q}|n')\right)f(n')\\
&+\left(T\left(\omega(n')\right) - T(n')\right)p(\bar{q}|n')f(n') - \left(T\left(n'\right) - T(\omega^{-1}(n'))\right)p(\bar{q}_1|\omega^{-1}(n'))f(\omega^{-1}(n'))]dn' = \frac{\tau}{1-\tau}.
\end{aligned}$$

Simplifying the integral expression, we get

$$\int_{\omega^{-1}(n)}^{n_{\max}} \left(1 - g_B\left(n'\right)\right)\right) \left(1 + P(\bar{q}|n')\right) f\left(n'\right) dn' + \int_{n}^{n_{\max}} \left(1 - g_A(n')\right) \left(1 - P(\bar{q}|n')\right) f(n') dn'$$

$$- \int_{\omega^{-1}(n)}^{n} \left(T\left(\omega(n')\right) - T(n')\right) p(\bar{q}|n') f\left(n'\right) dn' = \frac{\tau}{1-\tau} \times$$

$$\times n\varepsilon \left((1 - P(\bar{q}|n)) f(n) + \left(1 + P(\bar{q}_1|\omega^{-1}(n))\right) f\left(\omega^{-1}(n)\right)\right)$$

Defining by $\bar{g}(n)$ the average marginal social welfare weight of the people with observed productivity n as

$$\bar{g}(n) := g_A(n)\left(1 - P(\bar{q}|n)\right) + g_B(n)\left(1 + P(\bar{q}|n)\right),$$

we can rewrite the optimal tax formula as

$$\int_{n}^{n_{\max}} \left(2 - \bar{g}(n')\right) f(n') dn' + \int_{\omega^{-1}(n)}^{n} \left(\left(1 - g_B(n')\right) \left(1 + P(\bar{q}|n')\right) - \left(T\left(\omega(n')\right) - T(n')\right) p(\bar{q}|n')\right) f\left(n'\right) dn' \tag{12}$$

$$= \frac{\tau}{1-\tau} n\varepsilon \left((1 - P(\bar{q}|n)) f(n) + \left(1 + P(\bar{q}_1|\omega^{-1}(n))\right) f\left(\omega^{-1}(n)\right)\right)$$

which is analogous to the celebrated Mirrlees formula apart from the integral from $\omega^{-1}(n)$ to n that takes care of the revenue effect $((1 - g_B(n))(1 + P(\bar{q}|n))$ term) and migration effect $((T(\omega(n')) - T(n'))\, p(\bar{q}|n'))$. Clearly, when $n \in [n_{\min}, \omega(n_{\min})]$, only non-migrated region A inhabitants have this productivity, so the formula becomes

$$\int_{n}^{n_{\max}} [1 - g_A\left(n'\right)] f(n') dn' = \varepsilon \frac{\tau}{1-\tau} n f(n),$$

which is exactly the Mirrleesean formula. Note that the additional terms admit straightforward interpretation: $\int_{\omega^{-1}(n)}^{n} \left(\left(T\left(\omega(n')\right) - T(n')\right) p(\bar{q}|n')\right) f\left(n'\right) dn'$ is the tax paid by all migrants with skill from $\omega^{-1}(n)$ to n over and above the tax they would have paid if remaining in their home region. This characterizes a distortion that the government creates on extensive margin, stimulating $(T(\omega(n)) < T(n))$ or discouraging $(T(\omega(n)) < T(n))$ migration. The need for distortion comes from differences in social marginal welfare weights; its magnitude is determined, among other things, by the shape of the transformation function $\omega(n)$.

The other additional term, $\int_{\omega^{-1}(n)}^{n} \left(1 - g_B(n')\right) \left(1 + P(\bar{q}|n')\right) f\left(n'\right) dn'$, stands for the welfare effect of marginally increasing the tax for all productivity levels between n and $\omega(n)$ who migrate from region B to region A becasue of this increase (and thus realize productivity from $\omega^{-1}(n)$ to n). Using the elasticity defined as $\nu(n) := \left(T\left(\omega(n)\right) - T(n)\right) \frac{p(\bar{q}|n)}{1+P(\bar{q}|n)}$, we can rewrite the optimal tax formula as

$$\int_{\omega^{-1}(n)}^{n_{\max}} \left(1 - g_B\left(n'\right)\right)\right) (1 + P(\bar{q}|n)) f\left(n'\right) dn' + \int_{n}^{n_{\max}} \left(1 - g_A(n')\right) \left(1 - P(\bar{q}|n')\right) f(n') dn'$$

$$- \int_{\omega^{-1}(n)}^{n} \nu(n') \left(1 + P(\bar{q}|n')\right) f\left(n'\right) dn' = \frac{\tau}{1-\tau} \times$$

$$\times n\varepsilon \left((1 - P(\bar{q}|n)) f(n) + \left(1 + P(\bar{q}_1|\omega^{-1}(n))\right) f\left(\omega^{-1}(n)\right)\right).$$

We see that more elastic migration puts downward pressure on marginal tax rates whenever the migration elasticity is positive on $[\omega^{-1}(n), n]$.

9 Appendix AA: Further propositions and proofs

9.1 Two propositions using the alternative benchmark of the ex ante productivity distribution

Proposition 8 *In a country with regional productivity differences and internal migration, the optimal non-differentiated marginal tax rates may be higher or lower relative to a benchmark without internal migration and the ex ante productivity distribution. Assuming exogenous marginal welfare weights, they are lower if*

$$\begin{aligned}\int_{\omega^{-1}(n)}^{n} (1-g_B(n'))\, P(\bar{q}|n')f\left(n'\right)dn' - \int_{\omega^{-1}(n)}^{n} \left(T\left(\omega(n')\right)-T(n')\right)p(\bar{q}|n')f\left(n'\right)dn' \quad (13)\\ < \frac{P(\bar{q}_1|\omega^{-1}(n))f\left(\omega^{-1}(n)\right)-P(\bar{q}|n)f(n)}{f(n)+f\left(\omega^{-1}(n)\right)}\int_{n}^{n_{\max}}[1-g_A\left(n'\right)]f(n')dn'\\ +\int_{n}^{n_{\max}}\left[g_B(n')-g_A\left(n'\right)\right]P(\bar{q}|n')f(n')dn'\end{aligned}$$

Proof. *The optimal tax formula in case of unified taxation is presented by (12). For a government that does not take into account the possibility of migration, optimal marginal tax rates are implicitly defined by*

$$\begin{aligned}&\int_{n}^{n_{\max}}[2-g_A\left(n'\right)-g_B(n')]f(n')dn' + \int_{\omega^{-1}(n)}^{n}(1-g_B(n'))\,f\left(n'\right)dn'\\ =\;& \varepsilon\frac{\tau}{1-\tau}\left(f(n)+f\left(\omega^{-1}(n)\right)\right).\end{aligned}$$

Comparing the two expressions, we arrive at the condition (13). ■

Intuitively, there are three channels through which migration affects the magnitude of the marginal tax. The right hand side of (13) reflects how migration affects the revenue effect of a marginal change in the tax schedule. The first term on the right hand side is the difference of the revenue effects for migrants of productivity n and the migrants of productivity $\omega^{-1}(n)$, normalized by the total mass of people with such productivity. The second term takes care of the difference in social marginal welfare weights that all migrants of productivity n and above get upon migration from A to B. Loosely speaking, the possibility of migration enhances the revenue effect for region B and weakens it for region A simply because the migration flow is from A to B. The total change in the revenue depends on the difference in the migration flows at the initial and the new productivity levels, $P(\bar{q}_1|\omega^{-1}(n))f\left(\omega^{-1}(n)\right)$ and $P(\bar{q}|n)f(n)$, as well as on the difference in the social weights g_B and g_A. Each difference contributes to lowering the marginal tax in case of migration.

Another channel also works through altering the revenue effect, but only for the migrants between productivity levels $\omega^{-1}(n)$ and n. This effect is positive, it is represented by the first term on the left hand side of (13). Finally, the third channel is through the migration effect as the difference between the new and the old tax on migrants between productivity $\omega^{-1}(n)$ and n. This effect is negative as long as the tax schedule on the appropriate productivity interval is increasing.

Since, for this alternative benchmark using the ex ante distribution there is no unambiguous answer as to whether migration decreases or increases optimal marginal tax rates, we may ask whether it does so marginally. To answer this question we study what happens if, starting from two identical regions, we introduce a marginal productivity differencem. Formally, assume $\omega(n) = n + \Delta$, where Δ is infinitely small. It turns out that even in this case the effect on the optimal marginal tax is theoretically ambiguous and, as the following proposition tells us, in general depends on the shape of the ability distribution and the migration costs distribution.

Proposition 9 *Starting from two identical regions, introducing a marginal difference in productivity distribution lowers optimal marginal tax if and only if*

$$\frac{1-g_B(n)}{2} + \frac{\int_n^{n_{\max}} \left[2 - g_A(n') - g_B(n')\right] f(n')dn'}{4\left(f\left(n\right)\right)^2} \left(\frac{P(\bar{q}|n)}{\partial n} f\left(n\right) + f'\left(n\right)\right) < 0. \tag{14}$$

Proof. *We express the marginal tax rate $\frac{\tau}{1-\tau}$ from the optimal tax formula (12) under the assumption that $\omega(n) = n + \Delta$, take a derivative of it with respect to Δ, and evaluate it at $\Delta = 0$, keeping in mind that there is no migration at this point. The resulting expression is proportional to the left hand side of (14). Correspondingly, the marginal tax rate decreases with the introduction of marginal productivity differences if this expression is negative and it increases in case it is positive.* ∎

We can see that the terms in (14) related to the revenue effect are always positive. Thus, a sufficient condition for increase in marginal tax is that $\frac{P(\bar{q}|n)}{\partial n} f\left(n\right) + f'\left(n\right) \geq 0$. This is satisfied for independent distribution of costs ($\frac{P(\bar{q}|n)}{\partial n} = 0$) and a uniform distribution of ability. On the other hand, if the distribution of ability is sufficiently "decreasing", like the Pareto distribution, for example, then introducing marginal productivity differences puts downward pressure on marginal taxes.

9.2 Proof of Proposition 5

Under the assumptions formulated in the text, $V_A(n)$ and $V_B(n)$ are increasing in n without bound, because $\tau_A^\infty < 1, \tau_B^\infty < 1$. As $\Psi' > 0$ is decreasing, it converges to some $\bar{\psi} \geq 0$. Then, we have

$$g_A(n) = \frac{\int_{\bar{q}_A}^{+\infty} \Psi'\left(V_A\left(n\right) + q^h\right) p(q|n')dq + \int_0^{\bar{q}_B} \Psi'\left(V_A\left(n\right) - q^c\right) p(q|n)dq}{\lambda\left(1 + P(\bar{q}_B|n) - P(\bar{q}_A|n)\right)}, \tag{15a}$$

$$g_B\left(n\right) = \frac{\int_{\bar{q}_B}^{+\infty} \Psi'\left(V_B\left(\omega(n')\right) + q^h\right) p(q|n')dq + \int_0^{\bar{q}_A} \Psi'\left(V_B\left(\omega(n)\right) - q^c\right) p(q|n)dq}{\lambda\left(1 + P(\bar{q}_A|n) - P(\bar{q}_B|n)\right)} \tag{15b}$$

which converge to

$$g_A^\infty = g_B^\infty = \frac{\bar{\psi}}{\lambda}. \tag{16a}$$

If $T_B - T_A$ converges, it must be that $\tau_A^\infty = \tau_B^\infty = \tau^\infty$. But since

$$h'\left(\frac{z_i}{n_i}\right) = 1 - \tau_i\left(z_i\right), \tag{17}$$

z_i/n_i converges and hence elasticities converge to the same limit ε^∞. Moreover,

$$\lim_{n\to\infty} \frac{z_A}{n} = \lim_{n\to\infty} \frac{z_B}{\omega(n)}.$$

Because $P(q|n)$ and $\bar{q}_A, \bar{q}_B$ converge, $P(\bar{q}_i|n)$ and $p(\bar{q}_i|n)$ converge to $P^{\infty}(\bar{q}_i^{\infty})$ and $p^{\infty}(\bar{q}_i^{\infty})$. The Pareto distribution implies that $(1-F(n))/(nf(n)) = 1/a$ in the tail. Take the limit of our optimal tax formulae to get

$$\frac{1}{a\varepsilon^{\infty}}[1-\frac{\bar{\psi}}{\lambda}+\frac{\Delta T^{\infty}\left(p^{\infty}(\bar{q}_B^{\infty})+p^{\infty}(\bar{q}_A^{\infty})\right)}{1+P^{\infty}(\bar{q}_B^{\infty})-P^{\infty}(\bar{q}_A^{\infty})}]=\frac{\tau^{\infty}}{1-\tau^{\infty}}$$

for the marginal rates in region A and

$$\frac{1}{a\varepsilon^{\infty}}[1-\frac{\bar{\psi}}{\lambda}-\frac{\Delta T^{\infty}\left(p^{\infty}(\bar{q}_B^{\infty})+p^{\infty}(\bar{q}_A^{\infty})\right)}{1+P^{\infty}(\bar{q}_A^{\infty})-P^{\infty}(\bar{q}_B^{\infty})}]=\frac{\tau^{\infty}}{1-\tau^{\infty}}$$

for the region B. The right hand sides are equal, so we need $\Delta T^{\infty}=0$ for the left hand sides to be equal as well.

9.3 Proof of Proposition 6

The maximization problem of the government with the restriction that $\Delta T = C$ is constant in n is

$$\begin{aligned} W &= \int_{n_{\min}}^{n_{\max}}[\int_0^{+\infty}\Psi\left(V\left(\omega(n)\right)+q^h\right)p(q|n)dq+\int_0^{\bar{q}}\Psi\left(V\left(\omega(n)\right)-q^c\right)p(q|n)dq \\ &\quad +\int_{\bar{q}}^{+\infty}\Psi\left(V\left(n\right)+q^h+C\right)p(q|n)dq]f(n)dn, \end{aligned}$$

where $\bar{q}=V\left(\omega(n)\right)-V\left(n\right)-C$, and either q^h or q^c is equal to zero, and we assume C is small enough not to induce "reverse" migration (to the low productivity region). The maximization is subject to

$$\begin{aligned} &\int_{n_{\min}}^{n_{\max}}[\left(z\left(\omega(n)\right)-\omega(n)h\left(\frac{z\left(\omega(n)\right)}{\omega(n)}\right)-V\left(\omega(n)\right)\right)(1+P(\bar{q}|n)) \\ &+\left(z-nh\left(\frac{z}{n}\right)-V-C\right)(1-P(\bar{q}|n))]f(n)dn\geq E. \end{aligned}$$

Note that we express everything here in terms of region B taxes - that is why C appears in the expressions for region A as a correction term to increase indirect utility (in the objective function) or to reduce the tax revenue (in the government budget constraint). By the envelope theorem,

$$\begin{aligned} \frac{\partial W^*}{\partial C} &= \int_{n_{\min}}^{n_{\max}}[\left[\Psi\left(V\left(n\right)+\bar{q}^h+C\right)-\Psi\left(V\left(\omega(n)\right)-\bar{q}^c\right)\right]p(\bar{q}|n) \\ &\quad +\int_{\bar{q}}^{+\infty}\Psi'\left(V\left(n\right)+q^h\right)p(q|n)dq-\lambda\left(1-P(\bar{q}|n)\right) \\ &\quad -\lambda\left(T\left(\omega(n)\right)-T\left(n\right)+C\right)p(\bar{q}|n)]f(n)dn. \end{aligned}$$

$$\frac{\partial W^*}{\partial C}|_{C=0}=\int_{n_{\min}}^{n_{\max}}[\int_{\bar{q}}^{+\infty}\left(\Psi'\left(V\left(n\right)+q^h\right)-\lambda\right)p(q|n)dq-\lambda\left(T\left(\omega(n)\right)-T\left(n\right)\right)p(\bar{q}|n)]f(n)dn,$$

which is negative, if $\Psi'\left(V\left(n\right)+q^h\right)/\lambda=g_A\leq 1$ and $T\left(\omega(n)\right)>T\left(n\right)$ (a sufficient condition is that the marginal tax rate is positive everywhere).

9.4 Proof of Proposition 7

A separable tax schedule implies that $T_B - T_A$ is constant. Since $z_A/n = z_B/\omega(n)$, we have

$$\bar{q}_A = V_B - V_A = (\omega(n) - n)\left(\frac{z}{n} - h\left(\frac{z}{n}\right)\right) - (T_B - T_A),$$

so we can write

$$\dot{q}_A = \left(\omega'(n) - 1\right)\left(\frac{z}{n} - h\left(\frac{z}{n}\right)\right) - \left(T'_B - T'_A\right).$$

In particular, under separable taxation and $\omega'(n) = 1$, we have $\dot{q}_A = 0$. At that point, for the cost-of-moving model

$$\frac{d\left(g_A - g_B\right)}{dn} = \left[\frac{\Psi''\left(V_A\left(n\right)\right)}{\lambda} - \frac{\Psi''\left(V_A + \bar{q}_A\right) + \int_0^{\bar{q}_A}\Psi''\left(V_A + \bar{q}_A - q^c\right)p(q)dq}{\lambda\left(1 + P(\bar{q}_A)\right)}\right]\dot{V}_A < 0$$

iff Ψ' is convex.

Similar to Kleven et al. (2006, 2009) we can consider a tax reform introducing a little bit of "negative jointness" (a lower marginal tax for higher productivity region). This reform has two components. Above ability level n, we increase the tax in region A and decrease the tax in region B. Below ability level n, we decrease the tax in region A and increase the tax in region B. These tax burden changes are associated with changes in the marginal tax rates on earners around n. The direct welfare effect created by redistribution across regions at each income level:

$$\begin{aligned} dW &= \frac{dT}{F(n)}\int_{n_{\min}}^{n}\left(g_A(n') - g_B(n')\right)f(n')dn' \\ &\quad - \frac{dT}{1 - F(n)}\int_{n}^{n_{\max}}\left(g_A(n') - g_B(n')\right)f(n')dn'. \end{aligned}$$

Because $g_A - g_B$ is decreasing, $dW > 0$.

Second, there are fiscal effects associated with earnings responses induced by the changes in τ_A and τ_B around n. Since the reform increases the marginal tax rate in region A around n and reduces it in region B, the earnings responses are opposite. As we start from separable taxation, $\tau_A = \tau_B$, and hence identical elasticities, $\varepsilon_A = \varepsilon_B$, the fiscal effects of earning responses cancel out exactly.

Finally, the reform creates migration responses. Above n, migration to B will be induced. Below n, migration to B will be inhibited. The fiscal implications of these responses cancel out exactly only if $\omega'(n) = 1$. The elasticity η is constant in this case and since initial difference $T_A - T_B$ is constant, the gain in revenue from migrants above n will be compensated by the loss in revenue from migrants below n. By the same logic, for $\omega'(n) > 1$ the gain from migration will be stronger then the loss from it, so we will have another positive effect. With $\omega'(n) < 1$ the revenue gain from migration is smaller than the loss, so a bit of negative jointness is not necessarily optimal.

To complete the proof, we need that our reasoning holds for $\omega'(n) > 1$, i.e. $\dot{g}_A - \dot{g}_B < 0$ also for this case. Differentiating $g_A - g_B$ in this case, we have

$$\begin{aligned} \dot{g}_A - \dot{g}_B &= \dot{V}_A\frac{\Psi''\left(V_A\right)}{\lambda} - \omega'(n)\dot{V}_A\frac{\Psi''\left(V_A + \bar{q}\right) + \int_0^{\bar{q}}\Psi''\left(V_A + \bar{q} - q^c\right)p(q)dq}{\lambda\left(1 + P(\bar{q})\right)} \\ &\quad - \frac{g_A - g_B}{1 + P(\bar{q})}p(\bar{q})\left(\omega'(n) - 1\right)\dot{V}_A, \end{aligned}$$

The first two terms are negative, because Ψ' is convex by assumption. The second term is negative, because $g_A > g_B$ (which follows from concavity of Ψ).

10 Appendix B: Implementability

We follow the supplementary material to Kleven et al. (2009). The same reasoning applies. We have to show that the mechanism $(z_i(n), c_i(n))_{n\in[n_0,\bar{n}]}$ is actually truthful . We have

$$\begin{aligned} V_B(n) &= c_B - nh\left(\frac{z_B}{n}\right), \\ \bar{q}_A &= \max\{V_B(\omega(n)) - V_A(n), 0\}, \\ \bar{q}_B &= \max\{V_A(n) - V_B(\omega(n)), 0\} \end{aligned}$$

In case $\bar{q}_A > 0$, for all $n, n', q \geq \bar{q}_A(n)$ we have

$$u_A(z_A(n), c_A(n), 0, (n,q)) = V_A(n) \geq V_B(\omega(n)) - q \geq u_B\left(z_B(n'), c_B(n'), 1, (n,q)\right);$$

for all $n, n', q \leq \bar{q}_A(n)$ we have

$$u_B(z_B(\omega(n)), c_B(\omega(n)), 1, (n,q)) = V_B(\omega(n)) - q \geq V_A(n) \geq u_A\left(z_A(n'), c_A(n'), 0, (n,q)\right).$$

In case $\bar{q}_B > 0$, for all $n, n', q \geq \bar{q}_B(n)$ we have

$$u_B(z_B(\omega(n)), c_B(\omega(n)), 0, (n,q)) = V_B(\omega(n)) \geq V_A(n) - q \geq u_A\left(z_A(n'), c_A(n'), 1, (n,q)\right);$$

for all $n, n', q \leq \bar{q}_B(n)$ we have

$$u_A(z_A(n), c_A(n), 1, (n,q)) = V_A(n) - q \geq V_B(\omega(n)) \geq u_B\left(z_B(\omega(n')), c_B(\omega(n')), 0, (n,q)\right).$$

As in Kleven et al (2009), it is the separability of q in the utility specification that allows us to get these simple results. The proof for the uniform tax is analogous, dropping subscripts A and B.

11 Appendix C: on the equivalence of representations via income and via ability

Here we show that the optimal tax formulae obtained in the text are equivalent to those in the appendix. Consider the formula for non-differentiated taxation in the text:

$$\begin{aligned} \frac{\tau}{1-\tau} &= \frac{1}{z\varepsilon\left(v_A(z)s_A(z) + v_B(z)s_B(z)\right)} \times \\ &\quad [\int_z^\infty \left\{\left[1 - g_A(z')\right] v_A(z')\, s_A + \left[1 - g_B(z')\right] v_B(z')\, s_B\right\} dz' \\ &\quad + \int_{\tilde{z}}^{z} \left[T(z') - T(k(z'))\right] p\left(\bar{q}|\, z'\right) v_A(z') dz'], \end{aligned}$$

where $s_A(z) \equiv 1 - P(\bar{q}|\, z(n)) = 1 - P(\bar{q}|\, n)$ and $s_B \equiv 1 + P(\bar{q}|\, z') = 1 + P\left(\bar{q}_1|\, \omega^{-1}(n)\right)$ and $v_A(z\,(n)) = f(n)/z'(n)$, $v_B(z\,(n)) = f(\omega^{-1}(n))/z'(n)$. Further, $T(z\,(n)) = T(n)$, $T(k(z(n))) = T(\omega(n))$ and $\tilde{z} = k^{-1}(z(n)) = z\left(\omega^{-1}(n)\right)$; $g_i\,(z(n)) = g_i(n)$. Plugging into the expression above, we get

$$\begin{aligned}\frac{\tau}{1-\tau} &= \frac{1}{\frac{z(n)}{z'(n)}\varepsilon\left(\left(1-P(\bar{q}|n)\right)f(n) + \left(1+P(\bar{q}_1|\omega^{-1}(n))\right)f\left(\omega^{-1}(n)\right)\right)} \\ &\quad [\int_n^{n_{\max}} \{\left[1-g_A\left(n'\right)\right]\frac{f(n')}{z'(n')}\left(1-P\left(\bar{q}|\, n'\right)\right) \\ &\quad + \left[1-g_B\left(n'\right)\right]\frac{f(\omega^{-1}(n))}{z'(n')}\left(1+P\left(\bar{q}_1|\,\omega^{-1}(n)\right)\right)\}z'(n')dn' \\ &\quad + \int_{\omega^{-1}(n)}^{n}\left[T(n') - T(\omega(n'))\right]p\left(\bar{q}|\, n'\right)\frac{f(n')}{z'(n')}z'(n')dn'],\end{aligned}$$

compared to

$$\frac{1}{n\varepsilon}\frac{1}{\left(1-P(\bar{q}|n)\right)f(n) + \left(1+P(\bar{q}_1|\omega^{-1}(n))\right)f\left(\omega^{-1}(n)\right)}\times$$

$$\int_n^{n_{\max}}[\left(1-g_B\left(\omega^{-1}(n')\right)\right)\left(1+P(\bar{q}_1|\omega^{-1}(n'))\right)f\left(\omega^{-1}(n')\right) + \left(1-g_A(n')\right)\left(1-P(\bar{q}|n')\right)f(n')]dn'$$

$$-\int_{\omega^{-1}(n)}^{n}\left(T\left(\omega(n')\right) - T(n')\right)p(\bar{q}|n')f\left(n'\right)dn' = \frac{\tau}{1-\tau}.$$

The expressions are identical, if $\frac{z(n)}{z'(n)} = n$. To prove that this is indeed the case for the tax schedule linearized around the optimum in our model, simply totally differentiate the first order condition (2):

$$h''\left(\frac{z}{n}\right)\frac{nz'\,(n) - z}{n^2} = -T''\,(z)\,z'\,(n)\,.$$

For a linear approximation, $T''\,(z) = 0$, so we get $nz'\,(n) = z(n)$ indeed. This completes the proof of equivalence for the case of non-differentiated taxation. The derivation for differentiated taxation is analogous.

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