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Burkhard Heer

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Abstract

İmrohoroğlu, İmrohoroğlu and Joines [1995, A life-cycle analysis of Social Security, Economic Theory, vol. 6, 83-114] show that the optimal replacement ratio of the payas-you-go public pension system in the US economy amounts to 30%. We extend their analysis to a model that 1) replicates the empirical wage heterogeneity, 2) endogenizes the individual's labor supply decision and 3) accounts for contributions-defined pensions of the US social security system. With these more realistic modifications, the optimal replacement ratio is found to amount to approximately 5% and to be insensitive with regard to the aging of the US population; however, lower productivity growth would result in higher optimal pension payments. In addition, the optimal pension scheme is found to be more progressive than the present US pension system.

JEL-Code: C680, D310, D910, H550, J110, J260.

Keywords: optimal social security, progressive pensions, income and wealth distribution, demographic transition.

Burkhard Heer
University of Augsburg
Department of Economics
Universitätsstraße 16
Germany – 86159 Augsburg
burkhard.heer@wiwi.uni-augsburg.de

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1 Introduction

İmrohoroğlu, İmrohoroğlu, and Joines (1995) shows that the optimal unfunded pay-as-you-go public pension system in the US should be characterized by a replacement ratio of 30%. Even though the contributions to the pension systems distort the labor supply and reduce capital accumulation, pensions helps to insure households against income risk and, therefore, might be welfare-increasing. In our model, we extend the analysis of İmrohoroğlu, İmrohoroğlu, and Joines (1995) along multiple dimensions: First, we assume a more realistic distribution of wages. In İmrohoroğlu, İmrohoroğlu, and Joines (1995), all employed households of a cohort receive the same wage and, therefore, heterogeneity in income is only caused by periods of unemployment. Second, we assume that the worker is able to adjust its labor supply along the intensive margin. In the presence of contributions to a pay-as-you-go pension system, the worker will decrease his labor supply in our model. Third, we account for the fact that pensions in the US economy depend on contributions of the individual during his working life. As a consequence, the labor supply is less distorted, but the public pension system also redistributes less from the income-rich to the income-poor. Fourth, we allow for exogenous technological progress in our benchmark.[1] As a consequence, the return from the public pension contribution is increased. Fifth, a government also imposes an income tax that distorts labor supply additionally. Since the welfare costs of a distortionary tax increase non-linearly with the tax rate, a pension contribution rate that acts like a tax on wage income becomes more welfare-reducing. Accounting for all these effects, we find, as our main result, that the optimal replacement rate of pensions is much lower than that found by İmrohoroğlu, İmrohoroğlu, and Joines (1995).

In addition, we study two related questions: 1) What is the optimal progression of the public pension? For this reason, we study a linear pension scheme that consists of a lump-sum component and a component that is proportional to contributions. Should pensions redistribute with a lump-sum component so that workers are better

[1] İmrohoroğlu, İmrohoroğlu, and Joines (1995) only consider technological growth in their sensitivity analysis.

insured against negative income shocks or rather try to reduce distortionary effects on the labor supply and be contributions-related? 2) We study how the optimal pension scheme should change if the population ages due to the demographic change.

With regard to the first question, we find that pensions should be provided as a completely lump-sum payment to the retirees so that the earnings-related component is equal to zero. While this benefit scheme implies the maximum distortion with regard to the labor supply, the insurance affect against negative income shocks outweighs the welfare costs of such a policy. With regard to the second question, our results show that the optimal pension contribution rate is significantly higher in an older society than in a younger society. Between 2013 and 2050, the optimal stationary state contribution rate increases from 0.7% to 1.4%. As a consequence of the higher dependency ratio in 2050, however, the optimal replacement ratio of pensions relative to average wage earnings remains fairly constant around 4%-5%.

Our research is closely related to other studies on the welfare effects of public pensions. Fehr, Kallweit, and Kindermann (2013) compute the optimal mix between flat and earnings-related pensions for the German pension system. They find that the flat-rate pension share should equal 30% in total pensions in order to optimize the trade-off between the increased labor supply distortion and the benefit from increased insurance provision against labor market risk. In addition to our model, Fehr, Kallweit, and Kindermann (2013) endogenize the decision on the retirement age and also allow for disability risk reflecting the fact that 20% of new entries into the German pension system are due to disability. Different from our analysis, however, the contribution rate τ_b is set constant so that these authors do not study the optimal amount of pensions.[2]

Storesletten, Telmer, and Yaron (1999) also conduct a welfare analysis of the social security system in a large-scale Overlapping Generations (OLG) model, but focus on the distortion of social security contributions on the accumulation of capital. The main channel emphasized in their model is the financing of pensions with a distortionary

[2] Moreover, they abstract from population growth.

income tax that is levied on labor and capital income. Since labor supply is exogenous, the only distortion is on capital accumulation. The authors compare the current US pension system (as of 1996) to alternative scenarios including the abolition of the social security system and a system that is partially pay-as-you-go and partially fully-funded. They find the considered alternatives to imply significant welfare gains if general equilibrium effects are taken into account.

While we study the optimal pension policy and search for the optimal (non-negative) replacement ratios and the optimal contribution tariff, De Nardi, Imrohoroğlu, and Sargent (1999) concentrate their analysis on 8 different specific public policy measures in a similar OLG model. In order to finance additional expenditures on pensions due to the demographic transition, they consider policies that raise different taxes (on consumption and labor income), reduce pension benefits, or increase the mandatory retirement age. The authors also account for the welfare of the cohorts during the transition. In order to keep the model tractable and computable, they assume a special functional form of utility from consumption and disutility of labor (both quadratic and additive). In addition, the insurance properties of the social security are not motivated by a temporary shock on individual labor productivity, but rather a shock to the individual's wealth endowment. As a consequence of these assumptions, individual policy functions (e.g. individual consumption) are a linear function of individual state variables (in particular, wealth) so that aggregation is straightforward and does not depend on the distribution of wealth (different from our model). De Nardi, Imrohoroğlu, and Sargent (1999) find that the only policy of those considered in the paper that raises the welfare of all generations is one that switches to a purely defined contribution system.

The paper is organized as follows. The next section describes the model that we use to derive quantitative policy implications. Section 3 discusses the calibration of the model, while Section 4 presents our simulation results. Section 5 concludes. In the Appendix, we sketch the equilibrium properties of the benchmark equilibrium and the computational method, and present some additional sensitivity analysis of our results.

2 The Model

2.1 Demographics and Timing

A period, t, corresponds to one year. At each t, a new generation of households is born. Newborns have a real life age of 20 denoted by $s = 1$. All generations retire at age $s = R = 46$ (corresponding to real life age 65) and live up to a maximum age of $s = J = 75$ (real life age 94).

Let $N_t(s)$ denote the number of agents of age s at t. We denote total population at t by N_t. At t, all agents of age s survive until age $s+1$ with probability $\phi_{t,s}$ where $\phi_{t,0} = 1$ and $\phi_{t,J} = 0$.

2.2 Households

Each household comprises one (possibly retired) worker. Households maximize expected intertemporal utility at the beginning of age 1 in period t

$$\max E_t \sum_{s=1}^{J} \beta^{s-1} \left(\Pi_{j=1}^{s} \phi_{t+j-1,j-1} \right) \; u(c_{t+s-1}(s), l_{t+s-1}(s)), \tag{2.1}$$

where $\beta > 0$ denotes the discount factor, and per-period utility $u(c,l)$ is a function of consumption c and labor supply l

$$u(c,l) = \frac{\left(c^{\gamma}(1-l)^{1-\gamma}\right)^{1-\sigma} - 1}{1-\sigma}, \quad \sigma > 0, \quad \gamma \in (0,1). \tag{2.2}$$

Households are heterogeneous with regard to their age, s, their individual labor efficiency, $\eta \epsilon_j \bar{y}_s$, and their wealth, $k_t(s)$. We stipulate that an agent's efficiency depends on its age, $s \in \mathcal{S} \equiv \{1, 2, ..., 75\}$, and its efficiency type, $\epsilon_j \in \mathcal{E} \equiv \{\varepsilon_1, \varepsilon_2\}$. We choose the age-efficiency profile, $\{\bar{y}_s\}$, in accordance with the US wage profile. The permanent efficiency types ϵ_1 and ϵ_2 are meant to capture differences in education and ability. In addition, we follow Krueger and Ludwig (2007) and assume that a household's labor productivity is affected by an idiosyncratic shock, $\eta \in \Gamma \equiv \{\eta_1, \eta_2\}$, that follows a time-invariant Markov chain with transition probabilities

$$\pi(\eta'|\eta) = \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix}. \tag{2.3}$$

The net wage income in period t of an s-year old household with efficiency type $\eta\epsilon$ is given by $(1-\tau_w-\tau_b)w_t\eta\epsilon\bar{y}_s l_t(s)$, where w_t denotes the wage rate per efficiency unit in period t. The wage income is taxed at the constant rate τ_w. Furthermore, the worker has to pay contributions to the pension system at rate τ_b and accumulates the beginning-of period average contributions $x_t(s)$ according to:

$$x_{t+1}(s+1) = \begin{cases} 0 & \text{for } s=0 \\ x_t(s)\cdot\frac{s-1}{s} + \frac{\tau_b w_t \eta\epsilon\bar{y}_s l_t(s)}{s} & \text{for } 1 \le s \le 45 \\ \\ x_t(s)\cdot(1+g) & \text{for } s > 45 \end{cases} \tag{2.4}$$

The household is born without any accumulated contributions to the pension system, $x_t(1)=0$. During working life, $x_t(s)$ is adjusted so that it represents the average of all contributions. During retirement, $s > 45$, the government increases the contributions by the rate of exogenous labor productivity growth g. A retired worker receives pensions $b(x_t(s))$ that depends on both his contributions, $x_t(s)$, and a lump-sum component, pen_{min}, that is paid irrespective of former contributions. The details of the pension scheme are described below when we characterize the stationary equilibrium.

Households are born without assets at the beginning of age $s=1$, hence $k_t(1)=0$. In addition, household are not allowed to borrow so that $k_t(s) \ge 0$ for all ages s. Parents do not leave bequests to their children, and all accidental bequests are confiscated by the government. The household earns interest r_t on his wealth $k_t \in \mathbb{R}^+$. Capital income is taxed at the constant rate τ_r. In addition, households receive lump-sum transfers tr_t from the government. As a result, the budget constraint of an s-year old household with productivity type $\eta\epsilon$ and wealth k_t in period t is presented by:

$$
\begin{aligned}
& c_t(s) + k_{t+1}(s+1) \\
& = \begin{cases} (1-\tau_w-\tau_{b,t}) w_t \eta \epsilon \bar{y}_s l_t(s) + \left[1+(1-\tau_r) r_t\right] k_t(s) + tr_t, \\ \text{for } s \leq 45, \\ \\ b_t(x_t(s)) + \left[1+(1-\tau_r) r_t\right] k_t(s) + tr_t, \ \text{for } s > 45. \end{cases}
\end{aligned} \tag{2.5}
$$

2.3 Production

Production is characterized by constant returns to scale and assumed to be described by a Cobb-Douglas function:

$$
Y_t = K_t^{\alpha} \left(A_t L_t\right)^{1-\alpha},
$$

where labor-augmenting technological progress A_t grows at the exogenous rate g:

$$
A_t = (1+g) A_{t-1}. \tag{2.6}
$$

L_t denotes aggregate efficient labor and will be defined below for the stationary equilibrium.

Firms maximize profits:

$$
\Pi_t = K_t^{\alpha} \left(A_t L_t\right)^{1-\alpha} - w_t A_t L_t - r_t K_t - \delta K_t,
$$

implying the first-order conditions

$$
w_t = (1-\alpha) \left(\frac{K_t}{A_t L_t}\right)^{\alpha} = \tilde{k}_t^{\alpha} \tilde{L}_t^{-\alpha}, \tag{2.7a}
$$

$$
r_t = \alpha \tilde{k}_t^{\alpha-1} \tilde{L}_t^{1-\alpha} - \delta, \tag{2.7b}
$$

where $\tilde{k} \equiv K/(AN)$ is defined as capital per efficiency population and $\tilde{L} \equiv L/N$. For convenience, we will also refer to $\tilde{k}$ as capital.

2.4 Government

The government collects income taxes T_t in order to finance its expenditure on government consumption G_t and transfers Tr_t. In addition, it confiscates all accidental bequests Beq_t. The government budget is balanced in every period t, i. e.,

$$G_t + Tr_t = T_t + Beq_t. \tag{2.8}$$

In view of the tax rates τ_w and τ_r, the government's tax revenues are given by

$$T_t = \tau_w w_t L_t + \tau_r r_t K_t. \tag{2.9}$$

Government spending is exogenous and grows at the rate of labor augmenting-technological progress g and the population growth rate n_t:

$$G_t = G_{t-1}(1+g)(1+n_t). \tag{2.10}$$

2.5 Social Security

The social security system is a pay-as-you-go system. The social security authority collects contributions from the workers to finance its pension payments to the retired agents. In the individual retirement account, the contributions are just added over time and do not pay any interest which is the case for the most PAYG-pension systems in OECD countries. In addition, former payments are not adjusted for wage or productivity growth. Therefore, the individual with permanent productivity type ϵ at the beginning of age $s = 46$ in period t has accumulated the following contributions:

$$x_t(46) = \sum_{s=1}^{45} \frac{\tau_b w_{t-46+s} \eta_{t-46+s} \bar{y}_s \epsilon l_{t-46+s}(s)}{45}.$$

Division by A_t results in

$$\tilde{x}_t(46) \equiv \frac{x_t(46)}{A_t} = \sum_{s=1}^{45} \frac{\tau_b \tilde{w}_{t-46+s} \eta_{t-46+s} \bar{y}_s \epsilon l_{t-46+s}(s)}{45(1+g)^{46-s}},$$

with $\tilde{w} \equiv \frac{w_t}{A_t}$. As we have assumed that pension contributions during retirement are adjusted each year for productivity growth, $\tilde{x}_t(s) = \tilde{x}_t(46)$ for $s \geq 46$. The pension scale is progressive:

$$\tilde{b}_t(\tilde{x}_t(s)) = \tilde{b}_{min} + \rho_b \tilde{x}_t(s), \tag{2.11}$$

with $\tilde{b}_t = \frac{b_t}{A_t}$ and $\tilde{b}_{min}, \rho_b \geq 0$.

In equilibrium, the social security budget is balanced so that total expenditures on pensions, Pen_t, are equal to total contributions:

$$Pen_t = \tau_b w_t L_t. \tag{2.12}$$

2.6 Stationary Equilibrium

In the stationary equilibrium, individual behavior is consistent with the aggregate behavior of the economy, firms maximize profits, households maximize intertemporal utility, and factor and goods' markets clear. To express the equilibrium in terms of stationary variables only, we have to divide aggregate quantities by $A_t N_t$ (with the exception of aggregate labor supply L_t) and individual variables and prices by A_t. Therefore, we define the following stationary aggregate variables:

$$\tilde{Beq}_t \equiv \frac{Beq_t}{A_t N_t}, \quad \tilde{T}_t = \frac{T_t}{A_t N_t}, \quad \tilde{G}_t = \frac{G_t}{A_t N_t}, \quad \tilde{L}_t = \frac{L_t}{N_t},$$
$$\tilde{C}_t = \frac{C_t}{A_t N_t}, \quad \tilde{Y}_t = \frac{Y_t}{A_t N_t},$$

and stationary individual variables:

$$\tilde{c}_t \equiv \frac{c_t}{A_t}, \quad \tilde{w}_t \equiv \frac{w_t}{A_t}, \quad \tilde{b}_t \equiv \frac{b_t}{A_t}, \quad \tilde{k}_t \equiv \frac{k_t}{A_t}, \quad \tilde{tr}_t \equiv \frac{tr_t}{A_t}.$$

Notice that we divide aggregate labor supply L_t by total population, N_t, in period t to get a stationary variable. The mass of all individuals in our economy, therefore, is normalized to one in every period t.

Let f_t denote the cross-section measure of households in period t. The household's policy functions depend on his individual wealth $\tilde{k}_t$, his accumulated contributions $\tilde{x}_t$, his age s, his permanent efficiency type ϵ, and his idiosyncratic productivity η.

A *stationary equilibrium* for a constant government policy $\{\tau_r, \tau_w, \tau_b, \rho_b, \tilde{pen}_{min}, \tilde{G}, \tilde{tr}\}$ corresponds to a price system, an allocation, and a sequence of aggregate productivity indicators $\{A_t\}$ that satisfy the following conditions:

1. Population grows at the constant rate $n = \frac{N_{t+1}}{N_t} - 1$.

2. The aggregate productivity indicator, A_t, evolves according to (2.6).

3. The individual productivity shock, η, follows the Markov transition matrix (2.3).

4. Individual pension contributions during working life accumulate according to

$$\tilde{x}_{t+1}(s+1) = \begin{cases} 0 & \text{for } s = 0 \\ \tilde{x}_t(s) \cdot \frac{s-1}{s(1+g)} + \frac{\tau_b \tilde{w}_t \eta \epsilon \bar{y}_s l_t(s)}{s(1+g)} & \text{for } 1 \leq s \leq 45 \\ \\ \tilde{x}_t(s) & \text{for } s > 45 \end{cases} \tag{2.13}$$

5. Households maximize intertemporal utility (2.1) subject to the budget constraint (2.2), the accumulation of the pension contributions (2.13), $l_t(s) \in [0,1]$, and $\tilde{k}_t(s) \geq 0$. In stationary variables, the budget constraint is presented by

$$\begin{aligned} &\tilde{c}_t(s) + (1+g)\tilde{k}_{t+1}(s+1) \\ &= \begin{cases} (1 - \tau_w - \tau_{b,t})\tilde{w}_t \eta \epsilon \bar{y}_s l_t(s) + [1 + (1-\tau_r) r_t]\, \tilde{k}_t(s) + \tilde{tr}_t, \\ \text{for } s \leq 45, \\ \\ \tilde{b}_t(\tilde{x}_t(s)) + [1 + (1-\tau_r) r_t]\, \tilde{k}_t(s) + \tilde{tr}_t, \ \text{for } s > 45. \end{cases} \end{aligned} \tag{2.14}$$

   Moreover, there is a transversality condition requiring $\tilde{k}_t(71) = 0$.

   As a result, for each period t, individual labor supply $l_t(\tilde{k}, \tilde{x}, s, \epsilon, \eta)$, consumption $\tilde{c}_t(\tilde{k}, \tilde{x}, s, \epsilon, \eta)$, and optimal next period assets $\tilde{k}'_t(\tilde{k}, \tilde{x}, s, \epsilon, \eta)$ are functions of the individual state variables $\tilde{k} \in \tilde{\mathcal{K}}$, $\tilde{x} \in \tilde{\mathcal{X}}$, $s \in \mathcal{S}$, $\epsilon \in \mathcal{E}$, and $\eta \in \Gamma$ and are constant over time in the stationary equilibrium. Moreover, individual labor supply $l_t(\tilde{k}, \tilde{x}, s, \epsilon, \eta)$ implies the next-period (beginning-of-period) accumulated contributions $\tilde{x}'(\tilde{k}, \tilde{x}, s, \epsilon, \eta)$ by (2.13).

6. Firms maximize profits satisfying (2.7a) and (2.7b). In equilibrium, firm profits are zero.

7. Aggregate variables are equal to the sum of the individual variables:

$$\tilde{L}_t = \int \eta\epsilon\bar{y}_s l_t(\tilde{k}, \tilde{x}, s, \epsilon, \eta)\ f_t(d\tilde{k} \times d\tilde{x} \times ds \times d\epsilon \times d\eta), \tag{2.15a}$$

$$\tilde{K}_t = \int \tilde{k} f_t(d\tilde{k} \times d\tilde{x} \times ds \times d\epsilon \times d\eta), \tag{2.15b}$$

$$\tilde{B}eq_{t+1} = \int (1-\phi_{t+1,s+1})(1+r_{t+1}(1-\tau_r))\tilde{k}'_t(\tilde{k}, \tilde{x}, s, \epsilon, \eta)\ f_t(d\tilde{k} \times d\tilde{x} \times ds \times d\epsilon \times d\eta), \tag{2.15c}$$

$$\tilde{C}_t = \int \tilde{c}_t(\tilde{k}, \tilde{x}, s, \epsilon, \eta)\ f_t(d\tilde{k} \times d\tilde{x} \times ds \times d\epsilon \times d\eta), \tag{2.15d}$$

$$\tilde{T}_t = \tau_w \tilde{w}_t \tilde{L}_t + \tau_r r_t \tilde{K}_t, \tag{2.15e}$$

$$\tilde{P}en_t = \int \tilde{b}_t(\tilde{x}(s))\ f_t(d\tilde{k} \times d\tilde{x} \times ds \times d\epsilon \times d\eta). \tag{2.15f}$$

8. The government budget is balanced:

$$\tilde{G} + \tilde{t}r_t = \tilde{T}_t + \tilde{B}eq_t. \tag{2.16}$$

9. The budget of the social security system is balanced:

$$\tilde{P}en_t = \tau_b \tilde{w}_t \tilde{L}_t. \tag{2.17}$$

10. The market for the final good clears:

$$\tilde{Y}_t = \tilde{K}_t^{\alpha}\tilde{L}_t^{1-\alpha} = \tilde{C}_t + \tilde{G}_t + \delta\tilde{K}_t. \tag{2.18}$$

11. The cross-sectional measure f_t evolves as

$$f_{t+1}(\tilde{\mathcal{K}}\times\mathcal{X}\times\mathcal{S}\times\mathcal{E}\times\Gamma) = \int P_t\left((\tilde{k}, \tilde{x}, s, \epsilon, \eta), \tilde{\mathcal{K}} \times \mathcal{X} \times \mathcal{S} \times \mathcal{E} \times \Gamma\right)\ f_t(d\tilde{k}\times d\tilde{x}\times ds\times d\epsilon\times d\eta)$$

for all sets $\tilde{\mathcal{K}}$, $\tilde{\mathcal{X}}$, $\mathcal{S}$, $\mathcal{E}$, and Γ where the Markov transition function P_t is given by

$$P_t\left((\tilde{k}, \tilde{x}, s, \epsilon, \eta), \tilde{\mathcal{K}} \times \mathcal{X} \times \mathcal{S} \times \mathcal{E} \times \Gamma\right) = \begin{cases} \phi_{t,s}\pi(\eta'|\eta) & \text{if } \tilde{k}'_t(\tilde{k}, \tilde{x}, s, \epsilon, \eta) \in \tilde{\mathcal{K}} \text{ and} \\ & \tilde{x}'_t(\tilde{k}, \tilde{x}, s, \epsilon, \eta) \in \tilde{\mathcal{X}} \\ & \text{for } \epsilon \in \mathcal{E}, s+1 \in \mathcal{S}, \eta' \in \Gamma, \\ & \\ 0 & \textit{else}, \end{cases}$$

and for the newborns

$$f_{t+1}(\tilde{\mathcal{K}} \times \mathcal{X} \times 1 \times \mathcal{E} \times \Gamma) = \frac{N_{t+1}(1)}{N_{t+1}} \cdot \begin{cases} \Upsilon_1 & \text{if } 0 \in \tilde{\mathcal{K}},\, 0 \in \tilde{\mathcal{X}} \\ \\ 0 & else. \end{cases}$$

The initial distribution $\Upsilon_1(\epsilon, \eta)$ of $\epsilon \in \mathcal{E} = \{\epsilon_1, \epsilon_2\}$ and $\eta \in \Gamma = \{\eta_1, \eta_2\}$ among the one-year old is chosen to be uniform: $\Upsilon_1(\epsilon_1, \eta_1) = \Upsilon_1(\epsilon_1, \eta_2) = \Upsilon_1(\epsilon_2, \eta_1) = \Upsilon_1(\epsilon_2, \eta_2) = 1/4$.

3 Calibration

3.1 Demographics

We calibrate the parameters of the model in accordance with the US economy. The forecast for the US population development until 2050 is taken from United Nations (2013). We assume that the demographic transition is complete in 2050 and population is constant with a growth rate equal to $n = 0.0\%$. We use the two sets of the survival probabilities, $\{\phi_{t,s}\}_{s=1}^{75}$, $t \in \{2013, 2050\}$, and the corresponding population growth rates, $n = 1.1\%$ and $n = 0\%$, to study the optimal public pension policy. For simplification, we assume that the economy is in stationary steady state in 2013 and 2050, respectively.

3.2 Preference and Production Parameters

The intertemporal elasticity of substitution, $1/\sigma$, is chosen with $\sigma = 2.0$ in accordance with İmrohoroğlu, İmrohoroğlu, and Joines (1995). A sensitivity analysis for $1/\sigma = 1/4$ is also reported in Section 4. The parameter γ, which reflects the relative weight of consumption and leisure in utility, is set equal to 0.33 so that the average working hours $\bar{l}$ amount to approximately 0.30 in the benchmark equilibrium for the year 2013. We choose the discount factor $\beta = 1.011$ in accordance with the empirical estimates of Hurd (1989) who explicitly accounts for mortality risk.[3] Our calibration parameters are summarized in Table 3.1.

The elasticity of production with respect to capital is set equal to $\alpha = 0.36$, and capital depreciates at the rate of $\delta = 8.0\%$ annually. In stationary state, output per capita grows at the growth rate g. We set $g = 2.00\%$ corresponding to the average growth rate of US GDP per capita during 1960-2011 (using data provided by the Federal Reserve Bank at St. Louis at the link 'http://research.stlouisfed.org/fred2').

3.3 Individual Productivity

The s-year old household of type j has the productivity $\eta\epsilon\bar{y}_s$. The age-efficiency profile $\{\bar{y}_s\}_{s=1}^{45}$ is taken from Hansen (1993), interpolated to in-between years and normalized to one. The set of the equally distributed productivity types $\{\epsilon_1, \epsilon_2\} = \{0.57, 1.43\}$ is taken from Storesletten, Telmer, and Yaron (2004). Our choice of the stochastic individual productivity component, $\eta \in \{\eta_1, \eta_2\}$, is also motivated by Storesletten, Telmer, and Yaron (2004). In particular, the two state Markov chain is calibrated so that the annual persistence amounts to 0.98 with an implied conditional variance of 8%. Accordingly, $\{\eta_1, \eta_2\} = \{0.727, 1.273\}$ and

$$\pi(\eta'|\eta) = \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix} = \begin{pmatrix} 0.98 & 0.02 \\ 0.02 & 0.98 \end{pmatrix}.$$

With this calibration, we are able to replicate the empirical distribution of US wages. In our model, the Gini coefficient of the wage income distribution is equal to 0.388 which compares favorably with empirical values reported by, e. g., Díaz-Giménez, Quadrini, and Ríos-Rull (1997).[4]

[3]Related research that uses such a value for β includes İmrohoroğlu, İmrohoroğlu, and Joines (1995) and Huggett (1996). With this value of β, the effective time discount factor displays an increasing weight to instantaneous utility until real lifetime age 67, before they decline again and even fall below one after the real lifetime age 87 (for the survival probabilities of the year 2013).

[4]In Appendix A.1, the distributions of wage, income, and wealth in the model are compared to the empirical ones.

Table 3.1: Calibration of Parameters

Parameter	Value	Description
β	1.011	subjective discount factor
$1/\sigma$	$\{1/2,\ 1/4\}$	intertemporal elasticity of substitution
ψ_1	0.30	Frisch elasticity of labor supply
$\bar{l}$	0.3	steady state labor supply
α	0.36	share of capital income
δ	0.08	rate of capital depreciation
g	2.0%	growth rate
$\{\epsilon_1, \epsilon_2\}$	$\{0.57, 1.43\}$	permanent productivity types
$\{\eta_1, \eta_2\}$	$\{0.727, 1.273\}$	stochastic individual productivity
G/Y	0.195	share of government spending in steady state production
τ_w	24.8%	wage income tax
τ_r	42.9%	capital income tax
τ_b	12.4%	pension contribution rate
ρ_b	50%	marginal pension rate
$\pi_{11} = \pi_{22}$	0.98	persistence of idiosyncratic productivity shock

3.4 Government Policy

Government expenditures $\tilde{G}$ are set so that the government share G/Y is equal to the average ratio of government consumption in GDP, $G/Y = 19.5\%$, in the US economy during 1959-93 according to the Economic Report of the President (1994). The tax rates $\tau_w = 24.8\%$ and $\tau_r = 42.9\%$ are computed as the average values of the effective US tax rates over the time period 1965-88 that are reported by Mendoza, Razin, and Tesar (1994). Government transfers, tr, are computed using the equilibrium condition of the government budget (2.8).

The tax rate on wage income τ_b is set to 12.4%. During 1990-2014, the employer and employee had to pay 6.2% each to public old-age and survivors insurance and the disability insurance.[5] Huggett and Ventura (2000) study the effect of a progressive pension scheme on the distribution of income and wealth. Let x and $\bar{x}$ denote the average earnings of individual and the average earnings of all workers, respectively. Depending on which earnings bracket the retired agent's average earnings x were situated, he received 90% of the first 20% of x, 32% of the next 104% of x, and 15% of the remaining earnings ($x > 1.24\bar{x}$) in 1994. Our schedule is linear and we will choose a value of $\rho_b = 50\%$ as an initial value to target a mean value of the marginal rates reported by Huggett and Ventura (2000). In the benchmark case, the replacement ratio of average pension with respect to average wage income amounts to 71.2%. In our policy analysis below, we study how a change in ρ_b and τ_b affects the equilibrium allocation and welfare. We will speak of a more progressive pension system if the lump-sum component b_{min} rises.

The properties of the benchmark equilibrium and the computation of the model are described in the Appendices A.1 and A.2. As our measure of welfare change, we use the consumption equivalent change Δ that is computed as the percentage by which we need to increase (or reduce) the consumption in the case without pensions (with $\tau_b = 0\%$) to get the same welfare as under the policy $\{\tau_b, \rho_b\}$. Noticing the functional form of our utility function, Δ can be computed with the help of:

$$(1+\Delta)^{\gamma(1-\sigma)} W(0,0) = W(\tau_b, \rho_b), \tag{3.1}$$

[5]The value for τ_b is taken from the NIPA data (Table 6.3) of the Bureau of Economic Analysis.

where $W(\tau_b, \rho_b)$ denotes average stationary lifetime-utility for pension policy $\{\tau_b, \rho_b\}$.

4 Results

In this section, we present our results. In Section 4.1, we point out the distortion of the present US public pension system by comparing it to the case without public pensions. Abolishing public pensions from the present contribution rate $\tau_b = 12.4\%$ to $\tau_b = 0\%$ results in a welfare gain of about 13% of total consumption in stationary state. Next, we show our main result that the optimal replacement ratio of pensions should be approximately equal to 5% and pension should be provided lump-sum. In Section 4.2, we study the effect of aging on optimal pensions and compute the optimal pension for the (projected) population in the year 2050. In Section 4.3, we study the sensitivity of our results with respect to the assumptions on preferences and exogenous technological growth.

4.1 Optimal Amount of Public Pensions

Comparison to the Case without Social Security. In Table 4.1, the stationary-state allocation of the benchmark (with $\tau_b = 12.4\%$) is compared to the case without social security ($\tau_b = 0\%$). The abolition of social security increases savings for old age considerably so that the aggregate capital stock $\tilde{K}$ rises by 46.6%, from $\tilde{K} = 0.912$ to $\tilde{K} = 1.347$. In addition, the aboliton of distortionary pension contributions τ_b increases the labor supply (which is also augmented because of the rise in the marginal product of labor) so that the average working hours rises by 5.2%, from $\bar{l} = 0.295$ to $\bar{l} = 0.310$. As a consequence, equilibrium output $\tilde{Y}$ increases by 20.7% in response to the abolition of social security.

Without social security, wage income is less concentrated because the substitution effect of a higher net wage rate affects the labor supply of the low-efficiency workers to a larger extent than that of the high-efficiency workers. However, gross income is nevertheless more concentrated than in the case with social security because retired households with only interest income do not receive any income from pensions payments. Therefore, the Gini coefficient of gross income increases from 0.356 to 0.388 if pensions are abolished. The inequality of the wealth distribution decreases without pensions because, in this case, many low-income workers have to save in order to provide for old age and the number of households without any savings decreases from 32.3% to 26.9%.[6] Accordingly, the Gini coefficient of wealth decreases from 0.649 to 0.612 if social security is abolished. Even though the social security system redistributes from the income-rich to the income-poor, the distortionary effect of public pensions dominates, and welfare increases significantly by a consumption equivalent of 13.1% in the case without social security.

Optimal Benefit Level and Progressivity of the Pension Scheme. The optimal amount of the pension or, equally, the optimal contribution rate depend on the progressivity of the pension system as measured by the two parameters pen_{min} and ρ_b of the pension benefit scheme. The welfare changes associated with the different contribution rates τ_b for the two progression levels $\rho_b \in \{0\%, 50\%\}$ are presented in Fig. 4.1. We find that the optimal benefit scheme $\{\tau_b, \rho_b\}$ is a lump-sum pension benefit with $\rho_b = 0\%$. Higher values of ρ_b result in a smaller distortion effect on labor supply as future pensions are tied to the present labor supply. However, the insurance effect of pensions is reduced and dominates the reduction in the labor supply distortion for higher values of ρ_b.[7]

For the optimal progressivity of the pension system, $\rho_b = 0\%$, the equilibrium values

[6] Budría Rodriguez, Díaz-Giménez, Quadrini, and Ríos-Rull (2002) report that 2.5% of the households have zero wealth, and even 7.4% have negative wealth in the 1998 Survey of Consumer Finances.

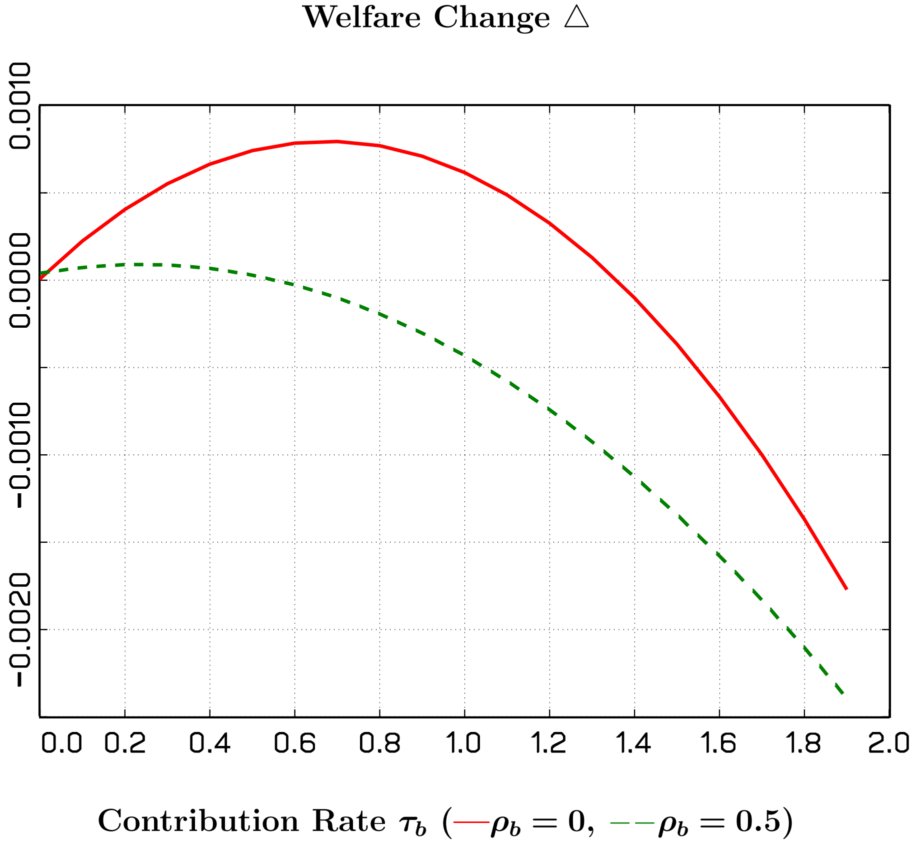
[7] At this point, let us mention one word of caution. The optimality result with respect to a flat-rate pension ($\rho_b = 0$) might be sensitive with regard to the assumption that there are no other redistributive government policies in the economy except for the (small) lump-sum transfers $\tilde{tr}$. Fehr, Kallweit, and Kindermann (2013), for example, find in their model of the German economy that the optimal share of flat-rate only amounts to 30% relative to the earnings-related component. Different from our model, they assume that the income tax scheme is progressive and redistributes from those with positive income (shocks) to those with negative income (shocks). Therefore, the optimal degree of progressivity is lower in their model simulation.

Table 4.1: Allocation Effects of Social Security

	$\tau_b = 12.4\%$ $\rho_b = 50.0\%$	$\tau_b = 0.0\%$
$\tilde{Y}$	0.407	0.491
$\tilde{K}$	0.912	1.347
$\tilde{L}$	0.258	0.278
$\bar{l}$	0.295	0.310
Gini coefficient		
Wage income	0.388	0.376
Gross income	0.356	0.388
Wealth	0.649	0.612
Liquidity-constrained	32.3%	26.9%
Welfare	-63.331	-60.463
Δ	-13.1%	0%

Note: Welfare is measured by the average lifetime utility of the newborn generation in stationary state. The welfare change Δ is computed as the consumption equivalent change relative to the case without social security ($\tau_b = 0\%$).

Figure 4.1: Welfare Effects of Pension Policies $\{\tau_b, \rho_b\}$



are presented in Table 4.2. Evidently, savings and, consequently, the capital stock $\tilde{K}$ decline with higher τ_b. Similarly, average labor supply $\bar{l}$ and, therefore, aggregate efficient labor $\tilde{L}$ also decrease. The trade-off between higher utility from insurance against negative income shocks versus the welfare losses from higher labor supply and savings distortion is a concave function of the contribution rate and welfare is reduced for contribution rates τ_b exceeding 0.7%. The optimal contribution rate implies an optimal replacement ratio of pensions relative to average wage earnings equal to 4.6% and a lump-sum component of pensions equal to 3.0% of GDP. In this case, welfare increases by 0.086% compared to the case without pensions. Obviously, the optimal replacement ratio of pensions is much smaller than the 30% found by İmrohoroğlu, İmrohoroğlu, and Joines (1995).

Table 4.2: Allocation Effects of Social Security for $\rho_b = 0\%$

τ_b	0.0%	0.7%	1.0%	2.0%
$\tilde{Y}$	0.491	0.482	0.478	0.468
$\tilde{K}$	1.347	1.298	1.278	1.220
$\tilde{L}$	0.278	0.276	0.275	0.273
$\bar{l}$	0.310	0.308	0.307	0.304
replacement ratio	0%	4.6%	6.5%	13.2%
Δ	0%	**0.085%**	0.055%	-0.19%

Notes: The welfare change Δ is computed relative to the case without social security ($\tau_b = 0\%$).

4.2 The Effect of Aging

How does a greyer population affect the optimal amount of pension payments? On the one hand, an increase of the old-age dependency ratio reduces the rate of return from the pension system. For given contribution rate τ_b, pensions *pen* will be lower. On the other hand, retirees are getting older on average so that the (discounted) loss from old-age utility as a consequence of possible negative income shocks is decreased to a larger extent. The overall effect can only be computed numerically.

For the population in 2050 that is characterized by a higher old-age dependency ratio, the optimal pension policy consists of a contribution rate that is still low, but about twice as high as in the case of the benchmark with the population data from the year 2013. If pensions would be abolished, the stationary welfare gain amounts to 8.5% and is reported in the second row of Table 4.3 (in the column entitled "Δ"). The optimal pension scheme is characterized by a contribution rate $\tau_b = 1.35\%$, and the optimal pension is again provided at a flat-rate, $\rho_b = 0\%$. In this case, welfare would decrease by 0.64% of total consumption if pensions were abolished.[8] The replacement ratio of pensions with respect to average earnings that corresponds to the optimal policy, however, is almost unchanged compared to the situation in 2013 and amounts to 4.2%. For the higher contribution rate in 2050, pensions are not significantly different from those in 2013 because the higher contributions are distributed among a larger share of retirees.[9]

4.3 Sensitivity Analysis

Our results are sensitive with regard to the choice of preferences and the growth rate. In particular, two parameters of the utility function that are crucial for quantitative

[8] We refrain from comparing the welfare of the generations born in 2013 with that of the generation born in 2050 because, due to the different lifetime expectations, lifetime utility of the two generations born in 2013 and 2050 would be different even if the individual cohorts would consume the same.

[9] The equilibrium values for the stationary state and $\rho_b = 0$ are presented in Table A.3.1 in the Appendix A.3.

results are the 1) intertemporal elasticity of substitution, $1/\sigma$, that determines the utility-costs of the variation of income over the life-cycle (and, therefore, consumption and instantaneous utility in each period) and 2) the utility parameter γ that determines the Frisch elasticity of labor supply and, therefore, the utility costs of substituting consumption by leisure in times of negative income shocks. In addition, we study the sensitivity of our results with respect to 3) the assumption on the type of the utility function introduced by Epstein and Zin (1989) that allows for a separate consideration of (relative) risk aversion and the intertemporal elasticity of substitution. Finally, we consider 4) the case without labor-augmenting technological growth. The sensitivity of our results on the optimal pension policy is summarized in the bottom four rows of Table 4.3. In essence, in all cases considered the optimal replacement ratio ranges between 0% and 11% and the optimal pension tariff has a stronger flat-rate component than the present one of the US pension system.

1. Intertemporal Elasticity of Substitution. In our sensitivity analysis, we present results for $\sigma = 4.0$ which is usually considered an upper value for σ in related studies, e.g. in İmrohoroğlu, İmrohoroğlu, and Joines (1995). For a lower intertemporal elasticity of substitution, $1/\sigma = 1/4$, precautionary savings increase to a larger extent if pensions are abolished. As a consequence, aggregate savings even increase by 61.2% relative to the benchmark case if the pension contribution rate τ_b drops from 12.4% to 0.0%. Therefore, the change in output is also quantitatively more significant compared to the case with $1/\sigma = 1/2$ and output increases by 24.9%. The most marked change, however, is the one on welfare. Abolishing social security results in stationary-state welfare gains equal to $\Delta = 35.6\%$ of total consumption. In addition, it is even optimal to abolish taxes completely and set $\tau_b = 0\%$.

2. Frisch Elasticity of Labor Supply. For our choice of the functional form of instantaneous utility (2.2), we cannot calibrate separately for the steady-state value labor supply, $\bar{l}$, and the Frisch elasticity of labor supply, $\eta_{l,w}$, since the parameter γ of the utility function determines both values. In the stationary equilibrium without

Table 4.3: Optimal Pension Policies

case	τ_b	ρ_b	repl. ratio	Δ
Benchmark	0.7%	0%	4.6%	13.1%
Year 2050	1.4%	0%	4.2%	8.5%
Sensitivity Analysis				
1. $\sigma = 4.0$	0%	0%	0%	35.6%
2. $\eta_{l,w} = 0.3$	0%	0%	0%	15.9%
3. Recursive preferences	1.3%	0%	6.8%	3.4%
4. No growth, $g = 0\%$	2.0%	20%	11.3%	12.7%

Notes: Cases 1 and 2 correspond to the sensitivity analysis with respect to a lower intertemporal elasticicty of substitution, $1/\sigma = 1/2$, and a lower Frisch labor supply elasticity, $\eta_{l,w} = 0.3$. The optimal pension policies for the sensitivity cases (1)-(4) are computed for the demographics prevailing in the year 2013. In the column entitled by "Δ", the stationary state welfare change from abolishing pensions from the present US pension system is reported.

pensions, the Frisch labor supply elasticity is given by

$$\eta_{l,w} = \frac{1-\gamma(1-\sigma)}{\sigma}\frac{1-l}{l}.$$

Therefore, for the case $\tau_b = 0$, the Frisch elasticity is equal to $\eta_{l,w} = 1.55$ for a household with labor supply $l = 0.30$.

In the following, we choose a different functional form for instantaneous utility that allows for the separate calibration of $\bar{l}$ and $\eta_{l,w}$:[10]

$$u(\tilde{c}, l) = \frac{\tilde{c}^{1-\sigma}}{1-\sigma} - \psi_0 \frac{l^{1+1/\psi_1}}{1+1/\psi_1}. \tag{4.1}$$

In the case without pensions, the Frisch intertemporal labor supply elasticity $\eta_{l,w}$ is equal to ψ_1. Estimates of $\eta_{l,w}$ implied by microeconometric studies vary considerably. MaCurdy (1981) and Altonij (1986) both use PSID data in order to estimate values of 0.23 and 0.28, respectively, while Killingsworth (1983) finds an US labor supply elasticity equal to $\eta_{l,w} = 0.4$.[11] We will use the conservative estimate $\eta_{l,w} = 0.3$ and choose $\psi_1 = 0.30$. We calibrate $\psi_0 = 885$ so that equilibrium labor supply is equal to 30% of available time, $\bar{l} = 0.30$, in the benchmark equilibrium with $\tau_b = 12.4$ and $\rho_b = 0.50$.

In this case, it is again optimal to abolish pensions in the stationary state. The associated welfare gain from reducing τ_b from 12.4% to 0% amounts to $\Delta = 15.9\%$ of total consumption.[12] The changes of aggregate savings and output from abolishing pensions are comparable to those of the benchmark case amounting to 63.5% and 18.6% for $\tilde{K}$ and $\tilde{Y}$, respectively.

[10]Notice that we used $\tilde{c} \equiv c/A$ as an argument of the utility function in (4.1). If we had used c instead of $\tilde{c}$, utility would not be stationary (labor l would converge to zero in the long-run for a growth rate $g_A > 0$ and $\sigma > 1$).

[11]Domeij and Floden (2006) argue that these estimates are biased downward due to the omission of borrowing constraints.

[12]Notice that we cannot use (3.1) in order to compute the consumption equivalent welfare change because the function is no longer multiplicatively, but additively separable in the utility from consumption and leisure, and the lifetime profiles of leisure depend on the pension policies. Instead, we

3. Recursive Preferences. As a third sensitivity analysis of our preferences, we consider a utility function that allows to separately study attitudes towards risk and intertemporal subsitution. We use Epstein-Zin preferences which are given by the following expression:[13]

$$\begin{aligned} V_t(k_t(s), s, \epsilon, \eta) = & \max_{k(s+1), c(s), l_t(s)} \Big\{ u\left(c_t(s), l_t(s)\right)^{1-\sigma} + \\ & \beta \phi_{t,s}\ E_t \left[V_{t+1}(k_{t+1}(s+1), s+1, \epsilon, \eta')^{1-\mu} \right]^{\frac{1-\sigma}{1-\mu}} \Big\}^{\frac{1}{1-\sigma}} , \end{aligned} \tag{4.2}$$

where $V_t(.)$ is the value function of the s-year old with individual productivity parameters ϵ and η and capital stock $k_t(s)$. The parameters $1/\sigma$ and μ denote the intertemporal elasticity of substitution and the coefficient of relative risk aversion. In the case $\sigma = \mu$, we are back to the benchmark case characterized by the time-separable expected utility specification (2.1).

In the calibration, we use the instantaneous utility function $u(c, l) = c^\gamma l^{1-\gamma}$ with $\gamma = 0.33$ in accordance with the utility function (2.2) of the benchmark equilibrium.[14]

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compute the average value of the discounted lifetime (dis)utility from consumption (labor)

$$W_c(\tau_b, \rho_b) = E_t \sum_{s=1}^{J} \beta^{s-1} \left(\Pi_{j=1}^{s} \phi_{t+j-1, j-1}\right) \frac{c(s)^{1-\sigma}}{1-\sigma},$$

$$W_l(\tau_b, \rho_b) = -E_t \sum_{s=1}^{J} \beta^{s-1} \left(\Pi_{j=1}^{s} \phi_{t+j-1, j-1}\right)\ \psi_0 \frac{l(s)^{1+1/\psi_1}}{1+1/\psi_1},$$

in our computation. The consumption equivalent change Δ from a change of the tax policy $\tau_b = 0$ (and $\rho_b = 0\%$) to $\{\tau_b, \rho_b\}$ can then be computed with the help of:

$$(1+\Delta)^{1-\sigma} = \frac{W(0,0) - W_l(\tau_b, \rho_b)}{W_c(\tau_b, \rho_b)}.$$

[13]These preferences were introduced by Epstein and Zin (1989). Epstein and Zin (1991) uses time series data on consumption and asset returns to test the representative-agent model. The preferences allow, among others, for a better explanation of some asset-price puzzles (see also Bansal and Yaron (2004)).

[14]We also conducted an additional sensitivity analysis using the utility function employed by Fehr, Kallweit, and Kindermann (2013):

$$u(c, l) = \left[c^{1-1/\rho} + \kappa l^{1-1/\rho} \right]^{\frac{1}{1-1/\rho}},$$

where we set the intra-temporal elasticity between consumption and leisure, ρ, equal to 0.6, and

As in the benchmark case, the intertemporal elasticity of substitution amounts to $1/\sigma = 1/2$. For the relative risk aversion μ, DSGE studies commonly consider a wide range of values. For example, Caldara, Fernández-Villaverde, and Rubio-Ramiírez (2012) consider values between 2 and 40. We will use a conservative intermediate estimate $\mu = 4.0$ as in Fehr, Kallweit, and Kindermann (2013).

We find that our optimality result with respect to zero pensions (or close to zero) to be insensitive with respect to the assumption of recursive utility. If risk aversion μ increases relative to the inverse of the intertemporal elasticity of substitution, σ, the utility costs of income uncertainty increases for the individual. Accordingly, a higher pension helps to increase (average) lifetime utility (4.2). However, the general equilibrium effects on aggregate savings and, therefore, income dominate this positive welfare effect, and we find that it is optimal to drastically reduce pensions in this case as well with the optimal contribution rate amounting to $\tau_b = 1.3\%$ (implying an optimal replacement ratio of pensions relative to average wage income of 6.8%). Again, the pension should be provided lump-sum with $\rho_b = 0\%$. The welfare gain in stationary state from abolishing pensions from the present level characterized by $\tau_b = 12.4\%$ to $\tau_b = 0\%$, however, is found to be considerably smaller due to the higher uncertainty costs than in the benchmark case and only amounts to 3.4% of total consumption.

4. No Growth. Our results with regard to the optimal amount of pensions, however, are sensitive with regard to the growth rate g of the economy. For the case of no growth, $g = 0\%$, the optimal amount of pensions almost triples so that the optimal contribution rate τ_b amounts to 2.0% implying an optimal replacement ratio equal to 11.3%.[15] The optimal flat-rate pen_{min} falls relative to the earnings-related component so that the optimal marginal rate ρ_b amounts to 20%. Abolishing pensions in the optimal case with $\{\tau_b, \rho_b\} = \{0.02, 0.20\}$ would result in welfare losses equal to 0.45%

$\kappa = 4.75$ is calibrated so that the average labor supply is equal to 0.30. In this case, both the optimal contribution rate τ_b and the replacement ratio of pensions with respect to wage income amount to 0%.

[15]In accordance with our results, İmrohoroğlu, İmrohoroğlu, and Joines (1995) find that the optimal pension replacement ratio falls significantly in the case of exogenous growth.

of total consumption. Moreover, our results indicate that the rate of progressivity in the pension system has little effect on welfare in the case of exogenous growth. For example, switching from the optimal regime with $\rho_b = 20.0\%$ to the flat-rate regime with $\rho_b = 0\%$ (and keeping $\tau_b = 2.0\%$ constant) only introduces welfare losses of 0.014% of total consumption.

5 Conclusion

We find that the optimal pension replacement ratio relative to gross wage income should be much lower than found by İmrohoroğlu, İmrohoroğlu, and Joines (1995) and should be equal to 0%-10% rather than 30%, both for the present US population and the projected US population in 2050. Our result is explained by the fact that, different from İmrohoroğlu, İmrohoroğlu, and Joines (1995), we consider endogenous labor supply and wage heterogeneity among and between cohorts. For the case of a low intertemporal elasticity of substitution or a utility function that is additively separable in the utility from consumption and leisure, we even find that pension should be abolished in the long run. A small public pension up to a threshold of approximately 10% of average wage income is optimal if the elasticity of intertemporal elasticity is high and growth is absent. In this case, pensions should ideally be more progressive than in the present US pension system and be provided lump-sum rather than earnings-dependent.

In conclusion, we would like to point out the direction for our future research. In the present paper, we analyse the insurance effect of redistributive pensions if individuals are subject to idiosyncratic income risk. However, the financing of the pensions in a pay-as-you-go system with the help of a tax on wage income also introduces distortions on the labor supply. In essence, the social security tax on wage income redistributes income from the young to the old and, depending on the progressivity of the pension system, from the income-rich to the income-poor households. In related research, Grant, Koulovatianos, Michaelides, and Padula (2010) have analyzed the empirical magnitude of the insurance versus the distortionary effect of the US income tax system.

They find strong evidence for the former, but milder evidence for the latter.[16] In future work, we would like to incorporate these findings into out model so that we are able to consider both lump-sum redistribution to all agents and redistribution that is specifically targeted at certain age or income groups in order to determine the optimal redistributive policy.

[16]In Grant, Koulovatianos, Michaelides, and Padula (2006), these authors also find that the optimal income tax rate amounts to 16% in the Aiyagari (1994) model with idiosyncratic income shocks.

Appendix

A.1 Properties of the Benchmark Equilibrium

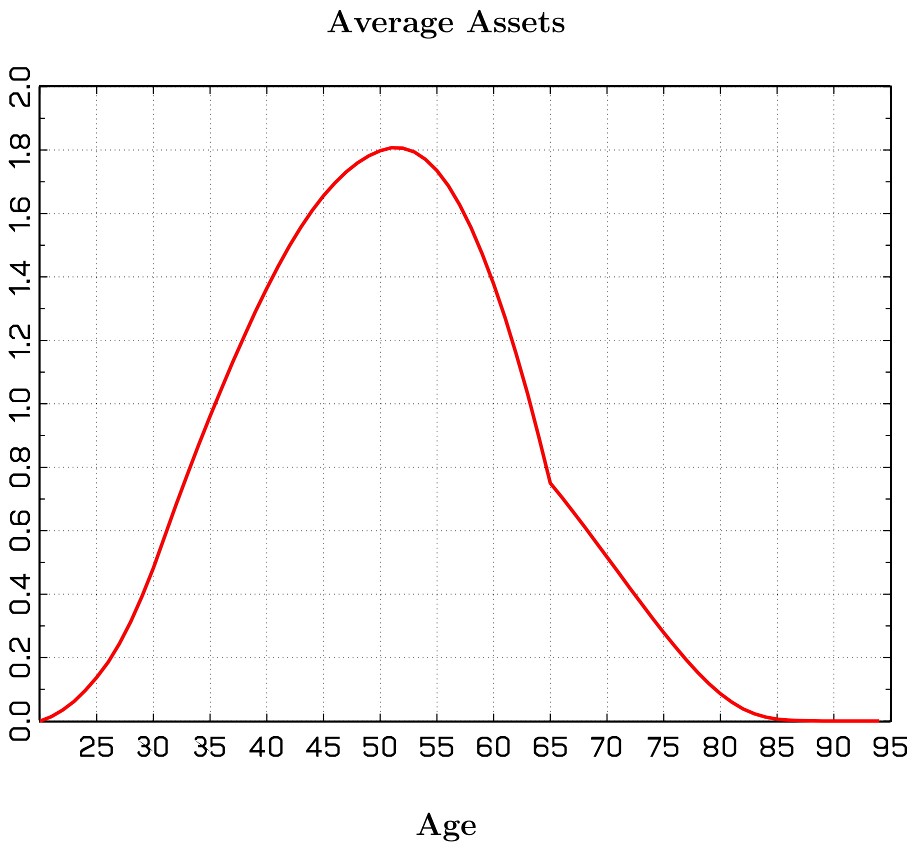
In stationary equilibrium of the benchmark case, the average wealth $\tilde{k}(s)$ and working hours $l(s)$ of the s-year-old cohort over the life cycle (or working life respectively) are graphed in Figs. A.1.1 and A.1.2. Households accumulate savings until the age of 51 before they start to dissave. In their effort to smooth consumption over their lifetime, households start to consume part of their savings as their income drops. The drop in income from wages is caused by the decrease of age-dependent efficiency $\bar{y}_s$ which also peaks at age 50. The decline in wealth is accelerated as soon as the households retire because pensions are below the former wage income. The profile of working hours in Fig. A.1.2 also mirrors the age-productivity profile because the substitution effect of higher wages dominates the income effect. However, the peak of working hours (at age 30) takes place prior to the peak in age-dependent efficiency $\bar{y}_s$ because of increasing wealth (prior to age 51) which reduces the labor supply.

The labor supply and wealth also depend on the permanent and temporary productivity types $\{\epsilon, \eta\}$. Both variables increase with higher productivity $\epsilon = \epsilon_2$ and $\eta = \eta_2$. The household with $\epsilon = \epsilon_1$ who experiences a negative productivity shock, $\eta = \eta_1$, is also liquidity-constrained, $\tilde{k} = 0$, if he has not accumulated sufficient savings in former periods. In fact, the percentage of households without savings amounts to 32.3% in our benchmark calibration.

The heterogeneity with regard to individual productivity, $\epsilon\eta\bar{y}_s$, results in inequality with regard to wages, income, and wealth. The Gini coefficient of wage income amounts to 0.388 and implies inequality in income and wealth that are characterized by Gini coefficients of 0.325 (net income after taxes), 0.356 (gross income before taxes), and 0.649 (wealth). Notice that the OLG model is able to generate much more inequality in wealth than in income as observed empirically.[17] However, all our inequality

[17]One of the first studies that pointed out the role of the OLG model to account for observed wealth

Figure A.1.1: Wealth-age profile

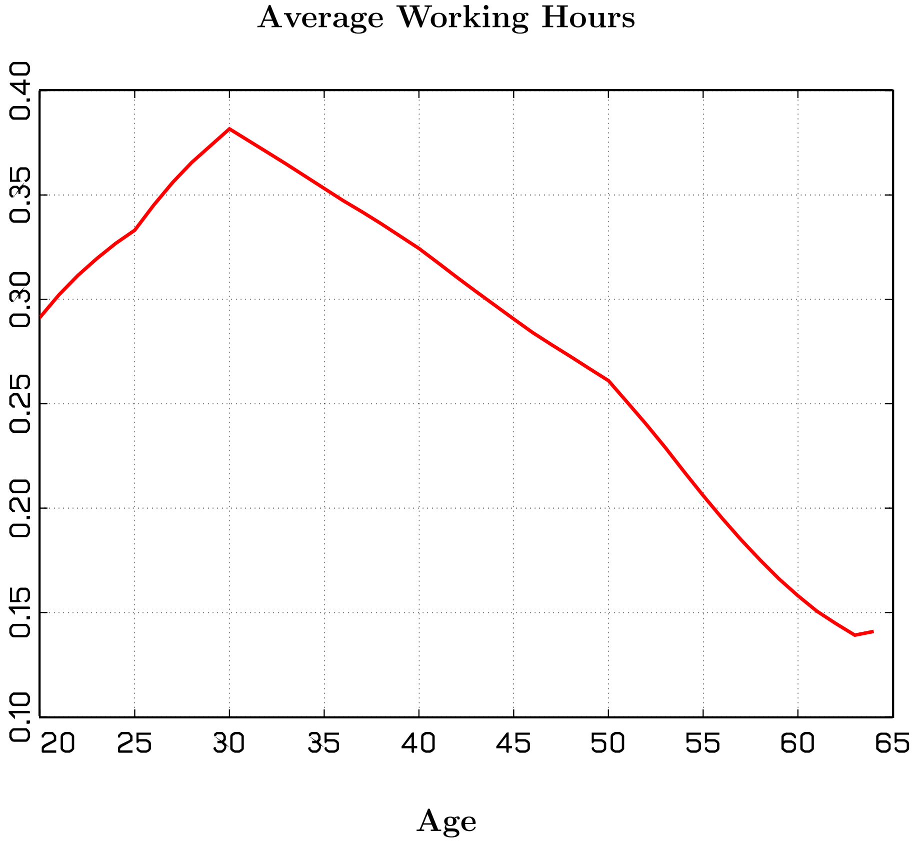


measures fall short of values observed empirically. For example, Budría Rodriguez, Díaz-Giménez, Quadrini, and Ríos-Rull (2002) report Gini coefficients of (gross) income and wealth equal to 0.553 and 0.803. Our model values fall short of the empirical ones for mainly two reasons: 1) We do not consider self-employed workers and entrepreneurs. Quadrini (2000) presents empirical evidence that the concentration of income and wealth is higher among entrepreneurs and that the introduction of an endogenous entrepreneurial choice in a dynamic general equilibrium model helps to reconcile the inequality in the model with that of the US economy. 2) We omit bequests.[18]

heterogeneity was Huggett (1996).

[18]Among others, Heer (2001) considers the effect of endogenous bequests in a life-cycle model.

Figure A.1.2: Labor-supply-age profile



A.2 Computation

The main computational problem is the numerical solution of the intertemporal household decision problem. We use value function iteration as described in Chapter 9.3 of Heer and Maußner (2009).[19] There are various numerical methods that are able to compute this two-dimensional optimization problem. We have chosen to transform the problem into two nested one-dimensional optimization problems and apply Golden Section Search in each step. Our reason for this approach is that the Golden Section Search is a very robust method that can easily handle non-negativity constraints such as $l \geq 0$ or $k \geq 0$. In the outer iteration for the worker, the maximum is searched over $\tilde{x}_{t+1}(s+1)$. Given the definition of pension contributions, (2.13), we can derive l_t from

[19]The computer programs are available from the author upon request.

this expression. For given $\tilde{x}_t$ and l_t, we compute the optimal next-period capital stock k_{t+1} that maximizes the right-hand side of the Bellman equation (A.2.1):

$$\tilde{V}_t(\tilde{k}_t(s), \tilde{x}_t(s), s, \epsilon, \eta) = \begin{cases} \max\limits_{\tilde{k}_{t+1}(s+1), \tilde{c}_t(s+1), l_t(s)} \Big[u\left(\tilde{c}_t(s), l_t(s)\right) + \\ \quad (1+g)^{\gamma(1-\sigma)} \beta E_t \tilde{V}_{t+1}(\tilde{k}_{t+1}(s+1), \tilde{x}_{t+1}(s+1), s+1, \epsilon, \eta') \Big], \\ \quad s = 1, \ldots, T \\ \\ \max\limits_{\tilde{k}_{t+1}(s+1), \tilde{c}_t(s+1)} \Big[u\left(\tilde{c}_t(s), 1\right) + \\ \quad (1+g)^{\gamma(1-\sigma)} \beta E_t \tilde{V}_{t+1}(\tilde{k}_{t+1}(s+1), \tilde{x}_{t+1}(s+1), s+1, \epsilon, \eta) \Big], \\ \quad s = T+1, \ldots, T + T^{R-1}, \end{cases} \tag{A.2.1}$$

subject to (2.3), (2.13), and (2.14). We continue for different values of $\tilde{x}_{t+1}(s+1)$ until we have found the maximum. Over the capital stock and the accumulated pensions contributions, $\tilde{k}$ and $\tilde{x}$, we use a grid of 200 and 20 equidistant points. Between gridpoints, we interpolate linearly. We also tested the sensitivity of our results with respect to a higher number of grid points and cubic spline instead of linear interpolation. However, results did not change significantly.

A.3 Optimal Policy in the Year 2050

The equilibrium values for the stationary state and the population in the year 2050 are described in Table A.3.1. For the computaton of the stationary equilibrium, we have assumed that absolute government expenditures $\tilde{G}$ (relative to the technology level A_t) remain at the constant level of 2013.

Table A.3.1: Allocation Effects of Social Security in 2050 for $\rho_b = 0\%$

τ_b	0.0%	0.5%	1.0%	1.5%	2.0%
$\tilde{Y}$	0.526	0.520	0.514	0.511	0.508
$\tilde{K}$	1.833	1.787	1.745	1.706	1.672
$\tilde{L}$	0.261	0.259	0.258	0.257	0.256
$\bar{l}$	0.353	0.351	0.349	0.348	0.346
replacement ratio	0%	1.6%	3.1%	4.4%	5.4%
Δ	0%	0.37%	0.60%	**0.64%**	0.44%

Notes: The welfare change Δ is computed relative to the case without social security ($\tau_b = 0\%$).

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