Student Loans and the Allocation of Graduate Jobs

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Abstract

Higher education is not just a signal of innate ability. At least a certain level of educational achievement (degree level, degree mark) is strictly required to perform a graduate job. School leavers fall into two categories, the rich and the poor. Ability is distributed in the same way in both groups. Graduate jobs are differentiated by quality. The output of each graduate job-worker match depends on the worker's ability and educational achievement as well as on the quality of the job. Individual wealth and ability are private information. Educational achievement and realized productivity are common knowledge. Graduates and graduate jobs are matched by tournament. In laissez faire, only the rich can buy enough education and enter the tournament. The poor are confined to the non-graduate labour market. This is doubly inefficient because some of the rich buy too much education, and some of the graduates have lower ability than some of the non-graduates. Student loans allow the more able among the poor to buy a higher education, discourage the less able among the rich from so doing, and bring individual education investments closer to their efficient levels. Unless the loan is large enough to allow a poor school leaver to invest as much, and thus get as good a job, as a rich one of the same ability, however, jobs of the same quality are assigned to graduates with the same education but different ability. Competition among employers will then result in poor graduates being paid a higher salary than rich graduates doing the same job.

JEL-Code: C780, D820, I220, J240.

Keywords: higher education, matching tournaments, credit rationing, separating equilibria.

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1 Introduction

The present paper examines the effects of student loans in a situation where non-graduate jobs are allocated by a conventional market and graduate jobs are allocated by a matching tournament.[1] A tournament is a contest where heterogeneous participants compete for one or more prizes. In a matching tournament, there are two categories of participants (men and women, employers and employees, schools and students), and each member of each category seeks to form the match most advantageous to itself with a member of the other category (in other words, the "prize" is a match). An early example of matching tournament is provided by Becker (1973), where the participants are young men and women intent on marriage. Exploiting the result in Koopmans and Beckmann (1957) that an efficient location pattern associates the most productive economic activity with the most advantageous site, the second most productive activity with the second most advantageous site, etc., Becker shows that the most attractive man will marry the most desirable woman, the second most attractive man will marry the second most desirable woman, etc. ("positive assortative matching"). Gale and Shapley (1962) show that an efficient allocation can be reached by a ritualized search or "courting" routine where (i) each man proposes to his favourite woman; (ii) each proposed-to woman keeps the best suitor waiting and rejects all others; (iii) each rejected man proposes to his next favourite woman; (iv) steps (ii) and (iii) are repeated until either there are no rejected men or every rejected man exhausts the list of women. Such an allocation will be reached also if this male-chauvinist routine is replaced by a liberated one, where the persons making marriage proposals are the women.[2] There are obvious parallels between these procedures and the exchanges of CVs and job offers that occur between graduates and potential employers.

The early matching literature abstracts from informational problems. Following Spence (1973), however, such problems have gradually gained centre-stage. The marriage problem is revisited by Peters and Siow (2002) under the assumption that the quality of a match depends not only on the parties' innate personal characteristics, but also on the investments parents may have made to enhance their children's attractiveness. The authors argue that the outcome will be a Pareto optimum. Hoppe et al. (2009), in a context where signals are wasteful, show, however, that the costs of signaling may counterbalance the gains from assortative matching with respect to random matching. In a recent paper, Bhaskar and Hopkins (2013) show that there may exist inefficient equilibria, and that uniqueness of the optimal equilibrium can be restored by introducing stochastic returns. Similar issues and results arise in other contexts. The allocation of workers differentiated by innate ability and educational investment to jobs differentiated by quality is studied by Hopkins (2012) under

[1] Some evidence of this is reported in, among others, Bratti et al. (2004) and Castagnetti and Rosti (2009).

[2] Cigno (1991, Ch. 1) shows that there may be more than one efficient allocation and that, if this is the case, the male-chauvinist and the liberated courting procedures will seek out different allocations.

the assumption that there is a continuum of both workers and jobs. Assuming that ability is unobservable, but reduces the cost of education, education is a signal of ability. The tournament then ranks workers on the basis of their educational level, and matches them with jobs in such a way that the candidate with the highest educational level will get the highest quality job, the one with the second-highest educational level will get the second-highest quality job, and so on. Fernandez and Galí (1999) compare the performance of tournaments and conventional markets in the allocation of students differentiated by wealth as well as ability to schools of different quality. They find that, if at least some of the students are effectively credit constrained, tournaments dominate conventional markets in terms of matching efficiency and possibly also of aggregate consumption.

Like Hopkins (2012), we are concerned with job matching and associated educational investments. Unlike those authors, however, we distinguish between graduate jobs, which require a university degree and are assigned by tournament, and non-graduate jobs, which do not require a university degree and are allocated by a conventional labour market. As in Fernandez and Galí (1999), credit is rationed, and potential university students differ not only in their innate ability, but also in their initial wealth. In contrast with those authors, however, we are interested in how the government can improve the matching, and bring individual educational investments closer to their efficient levels conditional on the matching, rather than in whether markets or tournaments produce the better result. Furthermore and more crucially, we assume that the number of graduate jobs is given, so that any policy facilitating access to higher education for the poor will restrict the number of graduate jobs available for the rich, and thus affect the educational investment behaviour not only of the poor but also of the rich. In Fernandez and Galí, where the number of school places is infinitely expandable, any such policy would only affect the behaviour of the poor.

Without policy intervention, some workers would be excluded from higher education and thus from graduate jobs not because they are insufficiently talented, but because they are insufficiently wealthy. That is undesirable not only on equity (equality of opportunity), but also on efficiency grounds. Assuming that initial wealth is uncorrelated with native talent, some graduate jobs will in fact be occupied by untalented but wealthy workers, and some non-graduate jobs by talented but impecunious ones. Furthermore, the less wealthy among those who invest in a higher education will invest less, and the more wealthy among them will invest more, than would be efficient. We show that the government can improve job matching and bring individual investments closer to their efficient levels by borrowing wholesale on the international money market and lending to individual students.[3] By relaxing the credit constraints faced by the poor, this policy would in fact replace the less talented among the rich

[3] In a context where graduate jobs are allocated by conventional markets, and equity is an issue, Cigno and Luporini (2009) show that a scholarship scheme financed by a graduate tax dominates student loans (even income-contingent ones) because it does not allow the talented rich to opt out of the scheme (and thus refuse to subisidize the poor) like the latter.

with the more talented among the poor in the performance of graduate jobs It would also reduce underinvestment by the poor and, perhaps surprisingly, overinvestment by the rich. If student loans are so generous that nobody's investment decisions are credit constrained, graduate jobs of the same quality will go to graduates of the same learning ability and educational level. Otherwise, graduate jobs of the same quality will go to graduates with the same level of education but possibly different level of ability. Given that personal ability can be inferred ex post, when the productivity of the job-worker match is observed, competition among employers to secure the best graduates will then result in an ex-post wage improvement for those (as it happens the poor) who turn out to have higher productivity than others (as it happens the rich) with the same education level. These findings constitute a novel contribution to the theoretical job-worker matching literature, and bring the predictions of this theory closer to our perception of reality.

2 Framework

Our agents are school leavers. There is a continuum of them differentiated by native ability, z, and wealth, y. Wealth takes only two values, $y \in \{0, \overline{y}\}$ where $\overline{y} > 0$. Ability is distributed over "poor" ($y = 0$) and "rich" ($y = \overline{y}$) agents with the same distribution function $G(z)$ and density function $g(z)$, such that $g(z|0) = g(z|\overline{y})$ $\nabla z \in [0, \overline{z}]$. The Lebesgue measure of the rich is a proper fraction α of that of the total agent population, which we normalize to unity.[4] An agent can go into the labour market straight after leaving school, or after a period in higher education. There is also a continuum of graduate jobs differentiated by quality, $s \in [0, \overline{s}]$, with distribution function $H(s)$. We can think of s as an index of technological sophistication or entrepreneurial ability. The Lebesgue measure of graduate jobs is a fraction $\beta \leq \alpha$ of that of rich agents. Therefore, not all agents (possibly not even all the rich ones) can get a graduate job. Those who do not will take a non-graduate job, and earn a fixed wage w_0. As our focus is on the allocation of graduate jobs, we assume that there are enough graduate and non-graduate jobs to occupy all school leavers, but nothing of substance changes if we allow for unemployment.

Let x denote the educational level achieved by an agent who attended university. We can think of this as either a degree level (e.g., BA, MA, Ph.D.) or a degree mark. The output produced by a graduate with learning ability z and education x, employed in a job of quality s, is $\pi(z, s, x)$, with $\pi_s > 0$, $\pi_x > 0$, $\pi_z > 0$, $\pi_{xx} < 0$, $\pi_{zz} = 0$, $\pi_{sx} = 0$, $\pi_{zs} > 0$, and $\pi_{zx} = 0$. The first three assumptions say that productivity is increasing in job quality, higher education, and worker's ability. The fourth and fifth say that the marginal productivity of x is decreasing, but that of z is constant. The latter is only a simplifying assumption. We will show later (see Section 3.2.1) that nothing of substance changes if $\pi_{zz} < 0$. The sixth assumption is required for integrabil-

[4] If the number of agents were finite, we would be saying that the number of rich agents may be different from the number of poor agents.

ity. The seventh says that graduate job quality and graduate worker's ability are complements in production. Together with the eighth, this assumption is required for stability of the matching equilibrium. Where π_{zx} is concerned, stability requires only nonnegativity, but we set it equal to zero to simplify the algebra. The cost of acquiring x units of education for an agent of ability z is $c(z,x)$, with $c(0,.) = 0$, $c_x > 0$, $c_z < 0$, $c_{xx} = 0$, $c_{zz} \leq 0$ and $c_{zx} < 0$. The assumption that c_{xx} is zero simplifies the analysis.[5] We further assume that $\pi_x(0,0,0) > c_x(0,0)$. Therefore, z has a dual role. First, it reduces the cost of x. Second, it directly increases output. As at least a certain educational achievement is necessary to carry out a graduate job, the function $\pi(.)$ is defined only for $x \geq x_0$, where $x_0 > 0$ is the minimum level of education required for such a job (say, a BA with a low graduation mark). Without loss of generality, we set x_0 equal to the efficient level of education for an agent of ability $z = 0$ employed in a graduate job of quality $s = 0$,

$$x_0 = \arg\max \pi(0,0,x) - c(0,x).$$

Assuming for simplicity that the interest rate is zero (but nothing of substance changes if the interest rate is positive, so long as it is lower than the return to educational investment for at least the more talented agents), the utility of an agent endowed with wealth y and ability z who buys x units of university education is

$$u = y + w - c(z,x), \tag{1}$$

where w is the worker's wage, obviously no higher than π and no lower than w_0. In general, w will depend on (z,s,x), but the functional form will differ according to the type of equilibrium. The utility of an agent who does not invest in higher education and thus goes into the non-graduate labour market straight after leaving school is $y + w_0$.

As in all the relevant literature, we assume that employers cannot offer their employees a full contingent contract (i.e., cannot make w contingent on z as well as x). In contrast with much of the theoretical literature, however, we realistically assume that employers may offer a contract specifying an initial wage and a fixed bonus conditional on productivity reaching at least a certain predetermined level. In the light of this, w is to be interpreted as the sum of an initial wage and a possible bonus.

3 First best

In first best (FB), s, x, y and z are common knowledge. The policy maker then prescribes educational investments to agents and assigns graduate jobs to graduates so as to maximize the social surplus

$$\int_z \int_s [\pi(z,s,x) - c(z,x)]\, ds dz$$

[5] If c_{xx} were positive, we would need a further assumption (see Proof of Proposition 4).

subject to the resource constraint

$$\int_{z} \left[\alpha\overline{y} - 2c\left(z,x\right)\right] g(z)dz \geq 0.$$

We assume that the resource constraint is never binding. In other words, there are enough initial resources to finance the efficient level of education.

Koopmans and Beckmann (1957) demonstrate that this maximization yields assortative matching in (z, s). Given that β is less than 1 (i.e., there are fewer graduate jobs than agents), there will be a threshold value of z, $\widetilde{z} > 0$, defined by

$$G(\widetilde{z}) = 1 - \beta, \tag{2}$$

such that all agents with $z \geq \widetilde{z}$ will attend university independently of their y. This subpopulation of agents is distributed with distribution function $\frac{G(z)-(1-\beta)}{\beta}$, and density function $\frac{g(z)}{\beta}$. Positive assortative matching then means that a worker of ability $z_i \in [\widetilde{z}, \overline{z}]$ is matched with a job of quality $s_i \in [0, \overline{s}]$, such that

$$\frac{G\left(z_i\right) - (1-\beta)}{\beta} = \phi\left(\frac{G\left(z_i\right) - (1-\beta)}{\beta}\right) = H\left(s_i\right), \tag{3}$$

where ϕ: $[0, 1] \rightarrow [0, 1]$ is the matching function.[6] This defines the function

$$s_{FB}\left(z\right) = H^{-1}\left(\frac{G(z) - (1-\beta)}{\beta}\right),$$

which associates a job of quality s to an agent of quality z.

The first-best level of university education for a school leaver of ability $z \geq \widetilde{z}$ matched with a job of quality $s_{FB}\left(z\right)$, will be

$$x_{FB}(z) = \arg\max\left[\pi\left(z, s_{FB}\left(z\right), x\right) - c\left(z,x\right)\right],$$

and will thus satisfy

$$\pi_x\left(z, s_{FB}\left(z\right), x\right) - c_x\left(z,x\right) = 0. \tag{4}$$

Given that $\widetilde{z} > 0$, it follows from the assumptions on $c(z,x)$ and $\pi(z,s,x)$ that $x_{FB}(z) > x_0 \; \forall \; z \geq \widetilde{z}$. For future reference, we define $\widetilde{x}$ as the first-best value of education for the least able agents employed in graduate jobs, so that $\widetilde{x} \equiv x_{FB}(\widetilde{z})$.

Given that, in FB, the distribution of the surplus is independent of resource allocation, we say nothing on the matter. Our interest here is just to characterize an efficient allocation.

[6] This function is measure-preserving and one-to-one on $\phi([0, 1])$. See Hopkins (2012) for details.

4 Laissez faire

In laissez faire (LF), s and x are common knowledge, and π is observable ex post, but y and z are private information. As lenders do not observe z, we assume like Fernandez and Galí (1999) that agents cannot borrow. Employers do not observe z either, but can infer it ex post in a separating equilibrium. Following Hopkins (2012), we represent the equilibrium process as a two-stage game. At the first (non-cooperative) stage, the agents choose whether and how much to invest in education subject to the liquidity constraint

$$y - c(z, x) \geq 0. \tag{5}$$

At the second (cooperative) stage, graduate jobs are allocated by a matching tournament, and the product of each match is shared between the parties in such a way that the matching scheme will be stable.

As the minimum educational investment required to carry out a graduate job is positive, and given that $c(., x)$ is positive for any positive x, (5) is binding for all the poor, who will consequently invest $x = 0$ and be excluded from the tournament. By contrast, (5) may be slack for some of the rich. Without loss of generality, we assume that there are as many graduate jobs as there are rich agents ($\alpha = \beta$), so that all the rich can participate in the tournament if it is to their advantage. In equilibrium all the rich will actually participate, because the lowest graduate salary is equal to the non graduate salary plus the cost for the lowest ability agent of acquiring the minimum level of education required to participate, $c(x_0, 0)$. Consequently, the support of the ability distribution of graduate workers is wider than in FB, where it includes only agents with $z \geq \widetilde{z}$. Our line of reasoning is a development of Hopkins (2012), which in turn draws on Mailath (1987). Hopkins implicitly assumes that all agents are "rich" according to our definition, and thus that none of them is liquidity constrained.[7] Given this assumption, he establishes necessary conditions for the stability of a positively assorted matching equilibrium under complete information (assuming that it exists) and then shows that the same conditions hold under incomplete information and finally goes on to demonstrate that a separating incomplete-information equilibrium exists.[8] Our LF differs from Hopkins in that the equilibrium concerns only a subset of the population, namely the rich. In this section, therefore, we limit ourselves to summarizing the main result of Hopkins (2012) and pointing out the minimal differences introduced by our approach. In subsequent sections, we will see that the equilibrium can be substantially modified by government intervention.

We start by assuming that, in the LF equilibrium, rich agents adopt a symmetric, differentiable and strictly increasing investment strategy $x_{LF}(z)$. Later

[7] Hopkins considers both the transferable utility case, where wages are bargained between employers and employees, and the nontransferable utility one, where wages are sticky (Clark (2006) establishes conditions for the existence of a unique stable matching in this case). As the second of these assumptions seems more appropriate for non-graduate wages than for graduate ones, we have assumed transferable utility for the graduate labour market, and non-transferable utility for non-graduate one.

[8] There also exists a pooling equilibrium in which wages reflect the average productivity.

we will show that this is actually the case. Let $F(x)$ be the distribution function of x induced by the distribution of z, $G(z)$, and by the investment strategy $x_{LF}(z)$. The rank position, $F(x(z_i))$, of an agent of ability $z_i \in [0, \overline{z}]$ buying $x_{LF}(z_i)$ will then be equal to this agent's rank $G(z_i)$ in the ability distribution. Positive assortative matching, whereby a worker buying $x_i = x_{LF}(z_i)$ is matched with a job of quality $s_i \in [0, \overline{s}]$, is such that

$$F(x_i) = G(z_i) = \phi(G(z_i)) = H(s_i). \tag{6}$$

This condition differs from (3) in that the matching is now based on the education level rather than directly on ability, but it still yields a relationship between job quality and agent's ability,

$$s_{LF}(z) = H^{-1}(G(z)).$$

The first derivative of this function is

$$s'_{LF}(z) = \frac{g(z)}{h(s_{LF}(z))}.$$

Notice that, in contrast with FB, some graduate jobs are now filled by (rich) graduates of ability $z < \widetilde{z}$.

Stage-2 stability conditions determine the wage schedule. For the equilibrium to be stable, the sum of the profit of a firm of quality $s(z)$ matched with a worker of ability z and of the wage of a worker of ability $z + \varepsilon$ matched with a firm of quality $s(z + \varepsilon)$, with ε arbitrarily small, must be no lower than the output that the first of these two firms would produce if it were matched with the second of these two workers,

$$\pi(z, s(z), x) - w(z, s(z), x) + w(z + \varepsilon, s(z + \varepsilon), x) \geq \pi(z + \varepsilon, s(z), x). \tag{7}$$

Moreover, the x chosen by a worker of ability z must satisfy

$$w(z, s(z), x + \varepsilon) + \pi(z, s(z), x) - w(z, s(z), x) \geq \pi(z, s(z), x + \varepsilon), \tag{8}$$

Taking the limit of (7) and (8) for ε going to zero, we find

$$w_z(z, s(z), x) + w_s(z, s(z), x)\, s'(z) = \pi_z(z, s(z), x) \tag{9}$$

and

$$w_x(z, s(z), x) = \pi_x(z, s(z), x). \tag{10}$$

Let $\underline{w}_{LF}$ denote the lowest graduate wage, yet to be determined. Applying Proposition 2 of Hopkins (2012) to the present context, it can be shown that the only stable matching is the positive assortative one with the wage schedule

$$w_{LF}(z, s_{LF}(z), x) = \int_0^z \pi_z(r, s_{LF}(r), x_0)\, dr + \int_{x_0}^x \pi_x(z, s_{LF}(z), t)\, dt + \underline{w}_{LF} \tag{11}$$

derived from the stability conditions (9)-(10). In stating these conditions, we took z and thus $s(z)$ as observable. Having assumed $\pi_{sx} = 0$, however, the implied wage schedule does not depend on the functional form of $x_{LF}(.)$, and will thus be the same under incomplete information.

Let us now move to stage 1. In a separating equilibrium, it must be unprofitable for an agent of ability z to choose the education level x appropriate for an agent of ability $z' \neq z$. Exploiting this incentive-compatibility condition and (9)-(10), Hopkins derives the following differential equation

$$x'_{LF}(z) = \frac{\pi_z\left(z, s_{LF}\left(z\right), x\right)}{c_x\left(z, x\right) - \pi_x\left(z, s\left(z\right), x\right)} \tag{12}$$

which has a unique solution. To establish the boundary condition recall that the efficient level of education for an agent of learning ability $z = 0$, employed in a graduate job of quality $s = 0$, is equal to x_0. Workers of ability $z = 0$ assigned to jobs of quality $s\left(0\right) = 0$ have nothing to signal and will thus choose $x(0) = x_0$, so that $c_x\left(x, 0\right) = \pi_x\left(0, s\left(0\right), x\right)$. By contrast, participating agents with $z > 0$ will want to signal their ability and thus invest more than would be efficient given the job allocation. For these agents, the choice of x will be such that $c_x\left(z, x\right)$ is greater than $\pi_x\left(z, s\left(z\right), x\right)$. The investment function is thus

$$x_{LF}(z) = \int_{\tilde{z}}^{z} x'_{LF}(z)dz + \; x_0. \tag{13}$$

Together with the assortative matching scheme (6) and wage schedule (11), (13) constitutes a symmetric equilibrium of the matching tournament (equivalent to Proposition 3 of Hopkins (2012)).

We are now equipped to determine $\underline{w}_{LF}$. In Hopkins (2012), where all jobs are assigned by tournament and $x_0 = 0$, this minimum wage is set arbitrarily. Here, by contrast, $\underline{w}_{LF}$ must satisfy

$$\underline{w}_{LF} \geq w_0 + c\left(0, x_0\right).$$

Competition among graduates will ensure that this constraint is satisfied as an equation (i.e., that the lowest paid graduate will be indifferent between investing in education and getting a graduate job, or going straight into the non-graduate labour market).

The equilibrium defined by (6), (11) and (13) is inefficient for two reasons. First, because all graduate jobs other than those of quality $s_{LF}(\overline{z})$ are occupied by graduates of lower ability than in FB. Second, because x is inefficiently high for all $z > 0$. The former derives from the fact that the agents excluded from the tournament are the poor rather than the less able. This source of inefficiency is absent in Hopkins (2012), where all agents are rich. The latter reflects the fact that, as graduate workers are ranked according to their educational level, all rich agents other than the marginal ones (those who are indifferent between going to university or straight into the labour market) have an incentive to invest more in order to make a better match. Moreover, all rich agents other than those with the highest ability level have a better match in laissez faire than in first best.

Proposition 1. For all $z \in [\widetilde{z}, \overline{z})$, $s_{LF}(z) > s_{FB}(z)$.

Proof. See Appendix.

5 Student loans

Recall that y is private information. If the government (unlike individual agents) can borrow against its future tax revenue, it can raise efficiency (and, incidentally, equity) by lending to students at stage 1 of the game, and, assuming that the repayment is enforceable,[9] recovering the credit at stage 2. It may be that the government can borrow unlimited amounts from the market, and that the maximum it will lend to each potential student is then determined by ability-to-repay considerations only. But, it may also be that overall public debt management considerations dictate a lower ceiling.

Having set the interest rate equal to zero, the utility function remains (1), because the loan and the loan repayment cancel out, but the liquidity constraint facing potential students is now

$$y + b - c(z, x) \geq 0, \tag{14}$$

where b is the maximum each of them is allowed to borrow from the government. Let b_0 denote the value of b that makes (14) slack for all $z \geq \widetilde{z}$. Let $\widetilde{b}$ denote that which allows poor students of ability $\widetilde{z}$ to buy the efficient amount of education $\widetilde{x}$. We show below that an equilibrium with student loans exists and leads to the same job allocation (but not to to the same investments in education) as in FB if $b \geq b_0$. For $b_0 \geq b \geq \widetilde{b}$ a student-loans equilibrium will in general exist for a sufficiently high level of b. Such an equilibrium will allow poor and rich agents with $z \geq \widetilde{z}$ to participate in the tournament, and will be only partially separating. Graduate jobs of the same quality will be assigned to graduates with the same educational level, but different abilities (a lower one for the rich, and a higher one for the poor). Equilibrium beliefs will reflect true ability values. Out-of-equilibrium beliefs will satisfy the Divinity Criterion (Banks and Sobel, 1987).[10] A partially separating equilibrium does not exist for $0 \leq b \leq \widetilde{b}$. An equilibrium will exist if the value of b is so low that not even the most able of the poor agents are able to buy the minimum level of education, x_0, required for a graduate job, but the equilibrium will then be the same as in LF. Let

[9] But see Cigno and Luporini (2009) for a discussion of the enforceability issue.

[10] In the absence of this refinement, there may exist other student-loans partially separating equilibria where some of the rich of ability $z < \widetilde{z}$ go to university while some of the poor with $z > \widetilde{z}$ go straight to the labour market. There also exists a pooling equilibrium where all agents of ability $z \geq \widetilde{z}$ choose $\widetilde{x}$, and firms hold to their priors.

$\widetilde{\widetilde{b}} = c(\overline{z}, x_0)$. If $\widetilde{b} > b > \widetilde{\widetilde{b}}$, the most able of the poor agents would find it profitable to take the student loan and invest in education, but there will be no equilibrium.

5.1 Unconstrained equilibrium: $b \geq b_0$

If b is sufficiently large to relax the liquidity constraint for the poor with $z \geq \widetilde{z}$, all these agents will invest in education (though not, as we will see, at the efficient level) and participate in the matching tournament. Given the policy, the support of the ability distribution of participating agents is $[\widetilde{z}, \overline{z}]$ as in FB, and thus narrower than in LF. Again as in FB, the distribution function is $\frac{G(z)-(1-\beta)}{\beta}$. As we did for LF, we start by assuming that job allocation is positively assorted, and then argue that this is the only stable matching.

For an agent of ability z_i matched with a job of quality s_i, x_i satisfies

$$F\left(x_i\right) = \frac{G(z_i) - (1-\beta)}{\beta} = \phi\left(\frac{G(z_i) - (1-\beta)}{\beta}\right) = H(s_i),$$

where i represents both the agent's education ranking and the job's quality ranking. An agent of ability z is then assigned to a job of quality

$$s_{USL}\left(z\right) = H^{-1}\left(\frac{G(z) - (1-\beta)}{\beta}\right),$$

where USL stands for unconstrained student loans, even though the matching is now made on the basis of the choice of education level. Consequently, for all $z \in [\widetilde{z}, \overline{z})$,

$$s_{USL}\left(z\right) = s_{FB}\left(z\right) < s_{LF}\left(z\right).$$

As in the LF case, we should now show that the only stable allocation is positively assortative and implies a particular wage schedule. As the argument is again analogous to the one in Hopkins (2012), we use stability conditions analogous to (9)-(10) to derive the wage schedule

$$w_{USL}\left(z, s_{USL}\left(z\right), x\right) = \int_{\widetilde{z}}^{z} \pi_z\left(r, s_{USL}\left(r\right), x_0\right) dr + \int_{\widetilde{x}}^{x} \pi_x\left(z, s_{USL}\left(z\right), t\right) dt + \underline{w}_{USL}.$$

At $z = \widetilde{z}$, $w = \underline{w}_{USL}$, where

$$\underline{w}_{USL} = w_0 + c\left(\widetilde{z}, \widetilde{x}\right).$$

These agents have no interest in buying more than the efficient amount of x, $\widetilde{x}$, because they have nothing to signal. Those with $z > \widetilde{z}$, by contrast, have an interest in signaling that their z is higher than the minimum, and will thus adopt an investment strategy different from FB. The educational investment of these agents, derived from stability conditions analogous to (9)-(10) and from the incentive-compatibility condition that it must be unprofitable for an agent

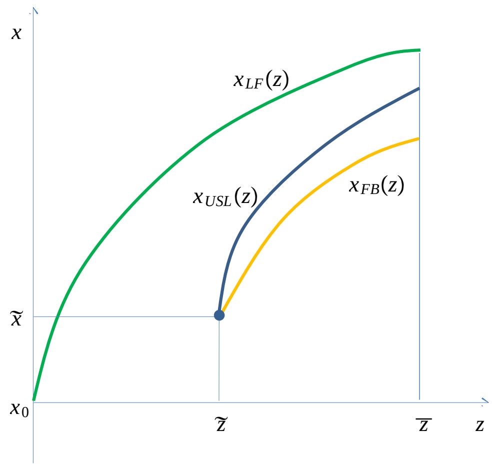

Figure 1: USL equilibrium vs. Laissez Faire and First Best

of ability z to choose the education level x appropriate for an agent of ability $z' \neq z$, satisfies

$$x'_{USL}(z) = \frac{\pi_z\left(z, s_{USL}\left(z\right), x\right)}{c_x\left(z, x\right) - \pi_x\left(z, s_{USL}\left(z\right), x\right)}. \tag{15}$$

Integrating this equation from $\widetilde{x}$, we find the USL equilibrium investment strategy $x_{USL}(z)$.

Figure 1 shows the graphs of the $x_{FB}(z)$, $x_{USL}(z)$ and $x_{LF}(z)$ curves.

Proposition 2. *The $x_{USL}(z)$ curve lies above the $x_{FB}(z)$ curve everywhere except at $z = \widetilde{z}$, where $x_{USL}(\widetilde{z}) = x_{FB}(\widetilde{z})$, and below the $x_{LF}(z)$ curve everywhere except at $z = \overline{z}$, where $x_{USL}(\overline{z})$ could equal $x_{LF}(\overline{z})$.*

Proof. See Appendix.

Corollary 1. *USL is less inefficient than LF.*

For $z > \widetilde{z}$, $x_{USL}(z)$ is higher than $x_{FB}\left(z\right)$. In comparison with FB, there is thus overinvestment as and for the same reason as in LF. However, $x_{USL}\left(z\right)$ is lower than $x_{LF}(z)$. As the $x_{USL}(z)$ curve starts from a higher z than the $x_{LF}(z)$ curve, poor agents with ability $z < \widetilde{z}$ invest the same amount $(x = 0)$ in both regimes, but all rich agents invest less, and poor agents of ability $z \geq \widetilde{z}$ more, in USL than in LF. The finding that the rich invest less in USL than in LF may seem surprising because a rich agent of any given ability faces more competition from agents of the same or higher ability. The explanation is that overinvestment is driven by the desire to separate from agents of lower ability, and that there are fewer of the latter in USL than in LF. Given that those who would have overinvested in LF (the rich) now invest less, and those who would have underinvested (the high-ability poor) now invest more, student loans with $b \geq b_0$ will then raise efficiency not only because they improve job matching, but also because they bring individual investments closer to their FB levels.

5.2 Constrained equilibrium: $b < b_0$

Our LF and USL equilibria differ from the matching equilibrium in Hopkins (2012) in that they apply only to a segment of the population (the rich in LF, the more able in USL) rather than to the entire population as in that article. In the case where $b < b_0$, an adapted version of the Hopkins approach can be applied to each of the two wealth categories. We assume (that the parameters are such) that the cost of the FB educational investment, $c(z, x_{FB}(z))$, is increasing in z, but nothing of substance would happen if that were not the case.[11] Further down we will show (Proposition 4 and 6) that, for an interval of z included in or coinciding with $(\widetilde{z}, \overline{z})$, agents with the same z will buy different amounts of x, higher if the agent is rich than if the agent is poor.

We know that graduates and jobs are matched on the basis of their observable characteristics (x for the former, s for the latter). As we did with regard to LF and USL, we start by assuming positive assortative matching with respect to x, find necessary conditions for stability, and then demonstrate that the equilibrium is in fact positively assorted. In the LF and USL equilibria, however, there was only one investment function, and consequently only one matching function. Here, by contrast, there are two investment functions, $x_R(.)$ for the rich and $x_P(.)$ for the poor. Consequently, there will also be two matching functions, $s_R(.)$ and $s_P(.)$, and positive assortative matching with respect to z will occur within each wealth category rather than across the entire population.

Assuming that jobs with the same s will be assigned at random among graduates with the same x, those among these graduates who are liquidity constrained will have a higher z and thus produce a larger π than those who are not. If employers could observe wealth, they would infer that, for any given x, the graduates with the lower y have a higher z and employ them in preference to graduates with the higher y. But we are assuming that y is private information. Could it be argued that it is in the poor's interest to disclose (e.g., write in their CV) that they received a loan, and thus reveal that they are more able than others with the same x? The answer is no, because the rich would counter the poor's strategy by taking a loan too,[12] and thus pretend to be poor. Ex

[11] All that could happen, if $c(z, x_{FB}(z))$ were not increasing in z, is that a CSL equilibrium where not all the agents of ability $\widetilde{z} < z < \overline{z}$ are constrained arises even for $b = \widetilde{b}$. If that were the case, there would be no CSL equilibrium where all the agents in that ability range are constrained.

[12] If the interest rate charged by the government were equal to the return on investments other than education (not necessarily zero as assumed here), the rich would in fact be indifferent between financing their education out of their own resources and accepting a government loan. Were it lower, the rich would accept the government loan anyway.

post, when π is observed, the employer will infer z. Competition to retain the employees with the higher z will then force employers to pay a bonus to their employees with the higher z. There will thus be two wage schedules, $w_R(.)$ for the rich and $w_P(.)$ for the poor.

Let CSL denote a student loan equilibrium where at least some of the participating agents are liquidity constrained. In what follows, we assume that such an equilibrium exists and establish some of its necessary characteristics. In the next subsection, we examine the existence issue. Remember that, for $b \geq b_0$, (14) is slack for all agents of ability $z \geq \widetilde{z}$, and that, for $b = \widetilde{b}$, poor agents with $z = \widetilde{z}$ can buy their efficient level of education $\widetilde{x}$.

Proposition 3. *If a CSL equilibrium exists for $b < b_0$, it will be such that the least (most) able rich participating in the tournament have the same ability level, $z = \widetilde{z}$ ($z = \overline{z}$) and buy the same amount of education as the least (most) able of the participating poor. The least able agents buy the efficient amount $\widetilde{x}$.*

Proof. See Appendix.

Corollary 2. *For a CSL equilibrium to exist b must be no lower than $\widetilde{b}$.*

In order to have a CSL equilibrium, it is thus necessary that at least the poor with the lowest ($z = \widetilde{z}$) and the highest ($z = \overline{z}$) ability level are not liquidity constrained in their choice of x, implying that b must be at least equal to $\widetilde{b}$. The intuition is that those of ability $z = \widetilde{z}$ may not be liquidity constrained at the given b because they invest little, but those of slightly higher ability will be because they would like to buy more x than they can with that b. Conversely, those with $z = \overline{z}$ may not be liquidity constrained at the given b because they are clever, but those with slightly lower ability may because they are not as clever.

5.2.1 All poor agents with $\widetilde{z} < z < \overline{z}$ are liquidity constrained

Consider first the case where b is such, that all the poor other than those at the two extremes ($z = \widetilde{z}$ and $z = \overline{z}$) are liquidity constrained (the existence and uniqueness of such a level of b will be the subject of Proposition 4 below). Let $\widetilde{b} < \bar{b} < b_0$ denote the value of b that has this property. For $b = \bar{b}$, a poor of ability $\overline{z}$ will buy the same amount of x, let us call it $\overline{x}$, as an equally talented rich, and a poor of ability $\widetilde{z}$ will buy the efficient amount of x, $\widetilde{x}$, like an equally talented rich. For $\widetilde{z} < z < \overline{z}$, there will be two different levels of x, lower for the rich than for the poor. In what follows, we start by assuming the existence of the two matching functions, $s_P(.|b)$ for the poor and $s_R(.|b)$ for the rich, both increasing in z, such that $s_P(z|b) < s_R(z|b)$, and derive the form of the wage schedules that ensure the stability of such matchings. We then prove the existence of an equilibrium where the matching functions are $s_P(.|b)$ and $s_R(.|b)$.

The CSL equilibrium associated with $b = \overline{b}$ must satisfy stability conditions analogous to (7) and (8). The only difference is that there is now a pair of these conditions for the rich, and another for the poor. The pair applicable to the rich determines the wage schedule

$$w_R\left(z, s_R\left(z|\overline{b}\right), x\right) = \int_{\widetilde{z}}^{z} \pi_z\left(r, s_R\left(r|\overline{b}\right), x\right) dr + \int_{\widetilde{x}}^{x} \pi_x\left(z, s_R\left(z|\overline{b}\right), t\right) dt + \underline{w}_{CSL}. \tag{16}$$

The pair applicable to the poor determines

$$w_P\left(z, s_P\left(z|\overline{b}\right), x\right) = \int_{\widetilde{z}}^{z} \pi_z\left(r, s_P\left(r|\overline{b}\right), x\right) dr + \int_{\widetilde{x}}^{x} \pi_x\left(z, s_P\left(z|\overline{b}\right), t\right) dt + \underline{w}_{CSL}. \tag{17}$$

In each case, $\underline{w}_{CSL} = \underline{w}_{USL} = w_0 + c\left(\widetilde{x}, \widetilde{z}\right)$, where $\widetilde{x} > x_0$ is the efficient education level bought by all agents (rich or poor) of ability $\widetilde{z}$.

Take two agents, one rich and one poor, both with the same (s, x), such that $s_P\left(z'|\overline{b}\right) = s_R\left(z|\overline{b}\right)$ for $z', z \in \left(\widetilde{z}, \overline{z}\right)$. Given that

$$\int_{\widetilde{x}}^{x} \pi_x\left(z, s_P\left(z|\overline{b}\right), t\right) dt = \int_{\widetilde{x}}^{x} \pi_x\left(z, s_R\left(z|\overline{b}\right), t\right) dt$$

for the assumption that $\pi_{zx} = \pi_{sx} = 0$, it follows that

$$\begin{aligned} w_P\left(z', s_P\left(z'|\overline{b}\right), x\right) - w_R\left(z, s_R\left(z|\overline{b}\right), x\right) &= \\ \int_{\widetilde{z}}^{z'} \pi_z\left(r, s_P\left(r|\overline{b}\right), x\right) dr - \int_{\widetilde{z}}^{z} \pi_z\left(r, s_R\left(r|\overline{b}\right), x\right) dr,\ z' &> z, \end{aligned} \tag{18}$$

where w_P is the wage of the poor, and w_R that of the rich. As the first of the two integrals on the RHS of this equation is calculated over the same interval of s as, but over a wider interval of z than, the second integral, and given that $\pi_{zz} = 0$, the difference between the two will be positive even though π_{zs} is positive. In words, an employer hiring a worker with education x in a job of quality s will promise to pay this worker a bonus equal to $w_P\left(z', s_P\left(z'|\overline{b}\right), x\right) - w_R\left(z, s_R\left(z|\overline{b}\right), x\right)$ if the productivity turns out to be $\pi\left(z', s_P\left(z'|\overline{b}\right), x\right)$ rather than $\pi\left(z, s_R\left(z|\overline{b}\right), x\right)$. If he did not do that, the worker would in fact be offered such a bonus by another employer.[13] For $z = \overline{z}$, the interval of z over which the integral is calculated will be the same for both the rich and the poor, and the bonus due to the latter will consequently be zero.

[13] This may be true even if π_{zz} were negative rather than zero as we are assuming, so long as π_z did not fall too fast as z increases. What would happen it that were not true? Would the bonus have to be paid to the rich? The answer is no, because the poor would then have an incentive to destroy output. If π_z falls very fast, the wage schedule of the rich will then apply also to the poor, and the analysis that follows remains the same.

Let $x_R(z|\bar{b})$ and $x_P(z|\bar{b})$ denote the investment functions, yet to be determined, of respectively the rich and the poor. Assume that $x_R(.|\bar{b})$ and $x_P(.|\bar{b})$ are increasing functions. The matching condition is now

$$F(x_i) = \alpha F_R(x_i) + (1-\alpha)F_P(x_i) = H(s_i), \tag{19}$$

where $F_R(x)$ is the distribution of x induced by $x_R(z|\bar{b})$, and $F_P(x)$ the one induced by $x_P(z|\bar{b})$.

All agents of ability $\widetilde{z}$ now invest $\widetilde{x}$ as in USL, because they have nothing to signal. Above that ability level, however, investment behaviour depends on whether the agent is rich or poor. If a CSL equilibrium exists, it satisfies the incentive-compatibility condition that it must be unprofitable for a rich agent of ability z to choose the x appropriate for a rich agent of ability $z' \neq z$. Given conditions analogous to (9)-(10), the investment strategy of the rich will then satisfy

$$x_R'(z|\bar{b}) = \frac{\pi_z\left(z, s_R\left(z|\bar{b}\right), x\right)}{c_x\left(x,z\right) - \pi_x\left(z, s_R\left(z|\bar{b}\right), x\right)}, \tag{20}$$

so that, if a solution to (20) exists with initial condition $x_R(\widetilde{z}|\bar{b}) = \widetilde{x}$,

$$x_R(z|\bar{b}) = \int_{\widetilde{z}}^{z} x_R'(z|\bar{b})dz + \widetilde{x}. \tag{21}$$

We do not have an incentive-compatibility condition for the participating poor, because these agents borrow all that the government is willing to lend them, and their choice of x is thus determined by

$$c\left(x,z\right) = b. \tag{22}$$

By the implicit function theorem, therefore,

$$x_P'\left(z|b\right) = -\frac{c_z}{c_x}, \tag{23}$$

and the investment function of the poor is

$$x_P\left(z|\bar{b}\right) = \int_{\widetilde{z}}^{z} x_P'(z)dz + \widetilde{x}. \tag{24}$$

We now turn to the issue of the existence of a CSL equilibrium where all poor agents with $\widetilde{z} < z < \overline{z}$ are liquidity constrained, and establish its characteristics. Recall that a poor of ability $\overline{z}$ will buy the same amount of x as an equally talented rich,

$$x_R(\overline{z}|\bar{b}) = x_P\left(\overline{z}|\bar{b}\right).$$

Note also that poor agents are not liquidity constrained if their z is equal to $\widetilde{z}$, but will be if z is even only slightly larger than $\widetilde{z}$ because, in view of (20),

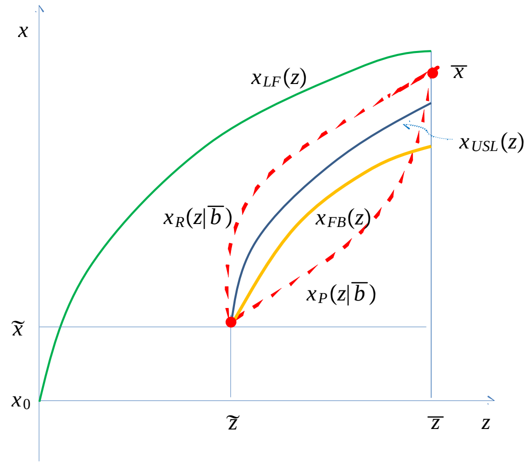

Figure 2: CSL equilibrium with $b = \overline{b}$

$x_R'(z|\overline{b})$ tends to infinity in a neighborhood of $\widetilde{z}$, and the level of x chosen by the rich thus increases very rapidly as z does.

Proposition 4. *There is a value of* b, $\widetilde{b} < \overline{b} < b_0$, *such that there exists a CSL equilibrium where rich and poor agents of ability* $\overline{z}$ *buy the same level of education* $\overline{x}$, *while rich and poor agents of ability* $\widetilde{z}$ *buy the FB level of education* $\widetilde{x}$. *For* $\widetilde{z} < z$ *the rich buy more education than in FB. For* $\widetilde{z} < z < \overline{z}$, *the rich buy more education than the poor.*

Proof. See Appendix.

The equilibrium in question is illustrated in Figure 2. Everywhere except at $z = \widetilde{z}$ and $z = \overline{z}$, the $x_R(z|\overline{b})$ curve lies above the $x_{FB}(z)$ and $x_P(z|\overline{b})$ curves. Using an argument analogous to that used in relation to Proposition 2 for the USL case, it can be easily shown that the following also is true.

Proposition 5. *The* $x_R(z|\overline{b})$ *curve lies above the* $x_{USL}(z)$ *curve everywhere except at* $z = \widetilde{z}$, *and everywhere below the* $x_{LF}(z)$ *curve. Up to a certain* z *lower than* $\overline{z}$, *the* $x_P(z|\overline{b})$ *curve lies below the* $x_{USL}(z)$ *curve and, up to an even lower* z, *also below the* $x_{FB}(z)$ *curve. Above those two critical levels of* z, *the* $x_P(z|\overline{b})$ *curve lies above the* $x_{USL}(z)$ *and* $x_{FB}(z)$ *curves.*

In CSL, therefore, the rich overinvest more than in USL, but still less than in LF. The poor of ability $z = \widetilde{z}$ buy the efficient x. Those of ability higher than $\widetilde{z}$, but lower than a certain $z < \overline{z}$, underinvest. Those of even higher ability overinvest.

5.2.2 Not all poor agents with $\widetilde{z} < z < \overline{z}$ are liquidity constrained

Consider next the case where $\overline{b} < b < b_0$. The demonstration that an equilibrium exists is analogous to that of the previous case. At this higher level of b, however, the poor are not liquidity-constrained not only for $z = \widetilde{z}$ and $z = \overline{z}$, but also for a range of values of z just below $\overline{z}$. Let $\underline{z}(b)$ denote the lowest value of z for which this is true given b. The poor with $z \geq \underline{z}(b)$ will then buy the same amount of x as the rich of the same ability, but those with $\widetilde{z} < z < \underline{z}(b)$ will buy less. Therefore, there will again be two investment functions, one for those who are not liquidity constrained (which now include the cleverer poor, those with $z \geq \underline{z}(b)$, as well as all the rich of ability $z \geq \widetilde{z}$) and one for those who are so constrained (the poor with $\widetilde{z} < z < \underline{z}(b)$). For $z = \underline{z}(b)$, the two functions have the same value. For $\widetilde{z} < z < \underline{z}(b)$, the amount invested by those who are not liquidity constrained (i.e., in this case, the rich) is higher than that invested by those who so are constrained.

Now let $\underline{x}(b)$ denote the amount of education bought by agents of ability $\underline{z}(b)$. For $x < \underline{x}(b)$, jobs of the same quality s will again be assigned at random to agents with the same educational achievement x, but different ability z. There will consequently be two job allocation functions, $s_U(.)$ for those who are not liquidity constrained (i.e., for all the rich with $z \geq \widetilde{z}$, and for the poor with $z > \underline{z}(b)$), and $s_P(.)$ for those who are constrained (i.e., for the poor with $\widetilde{z} < z < \underline{z}(b)$). The wage schedule, derived from stability conditions analogous to (7) and (8), is

$$w_U\left(z, s_U\left(z|b\right), x\right) = \int_{\widetilde{z}}^{z} \pi_z\left(t, s_U\left(z|b\right), \widetilde{x}\right) dt + \int_{\widetilde{x}}^{x} \pi_x\left(z, s_U\left(z|b\right), t\right) dt + \underline{w}_{CSL}$$

for those who are not liquidity constrained, and

$$w_P\left(z, s_P\left(z|b\right), x\right) = \int_{\widetilde{z}}^{z} \pi_z\left(t, s_P\left(z|b\right), \widetilde{x}\right) dt + \int_{\widetilde{x}}^{x} \pi_x\left(z, s_P\left(z|b\right), t\right) dt + \underline{w}_{CSL}, \quad \widetilde{z} < z < \underline{z}(b),$$

for those who are. The latter includes a bonus calculated as in (18).[14]

The functions that allocate jobs to agents, $s_U(z|b)$ and $s_P\left(z|b\right)$, are derived in Proposition 6 below, together with the equilibrium strategies of the two categories, $x_U(z|b)$, and $x_P(z|b)$. Up to $\underline{x}(b)$, the matching condition is

$$F(x_i) = \alpha F_R(x_i) + (1-\alpha)F_P(x_i) = H(s_i).$$

Above $\underline{x}(b)$, the matching condition becomes

$$F(x_i) = \frac{G(z_i) - (1-\beta)}{\beta} = H(s_i),$$

and we have then the same job allocation as in FB. The educational investment of those who are not liquidity constrained for $\widetilde{z} < z < \underline{z}(b)$ (i.e., the rich) is

[14] But, of course, the size of the bonus will be different.

still governed by (21) as in the case where b is equal to $\overline{b}$. Now, however, b is greater than $\overline{b}$, and the s associated with each z is lower than in the previous case, because the poor can buy more x. In other words, some bright poor agents displace some of the rich. The amount invested by an unconstrained agent of ability $z > \underline{z}(b)$ is

$$x_U(z|b) = \int_{\underline{z}(b).}^{\overline{z}} x'_U(z|b)dz + \underline{x}(b),$$

where $x'_U(z|b)$ has the same form as (20). The investment strategy of the liquidity-constrained poor will still satisfy (23), and will thus be given by

$$x_P(z|b) = \int_{\widetilde{z}}^{z} x'_P(z)dz + \widetilde{x}.$$

All poor agents participating in tournament, other than those with either $z = \widetilde{z}$ or $z = \overline{z}$, are now matched with higher quality jobs than in the case where b is equal to $\overline{b}$.

Proposition 6. *In a CSL equilibrium with $\overline{b} < b < b_0$, the poor are not liquidity constrained not only for $z = \widetilde{z}$ and $z = \overline{z}$, but also for $z \geq \underline{z}(b)$. These agents buy the same amount of x as the rich of the same ability. The poor with $\widetilde{z} < z < \underline{z}(b)$ buy less x than the rich of the same ability.*

Proof. See Appendix.

The equilibrium in question is illustrated in Figure 3. For the poor with $\widetilde{z} < z < \underline{z}(b)$, the investment strategy is represented by the $x_P(z|b)$ curve. Notice that the relatively less talented among these agents invest less, and the relatively more talented more, than in USL. The investment strategy of the rich with $z > \widetilde{z}$ is represented by the $x_U(z|b)$ curve. The extremely talented poor, namely those with $z \geq \underline{z}(b)$ who, at this level of b, are not liquidity constrained in their investment decisions, behave like the equally talented rich. The upper part of the $x_U(z|b)$ curve (less steep than the rest) represents, therefore, the investment strategies of both wealth categories. As b rises, the $x_P(z|b)$ curve shifts upwards. At the same time, the $x_U(z|b)$ curve gets closer to the USL curve, and the segment of the $x_U(z|b)$ curve common to rich and poor agents gets longer. For b sufficiently large ($b \geq b_0$), nobody would be rationed, and the $x_U(z|b)$ curve would coincide with the USL curve, along which graduates are matched with graduate jobs as in FB, but there would still be some overinvestment.

6 Conclusion

In our model, higher education increases productivity. Furthermore, at least a certain level of educational achievement (degree level, degree mark) is strictly

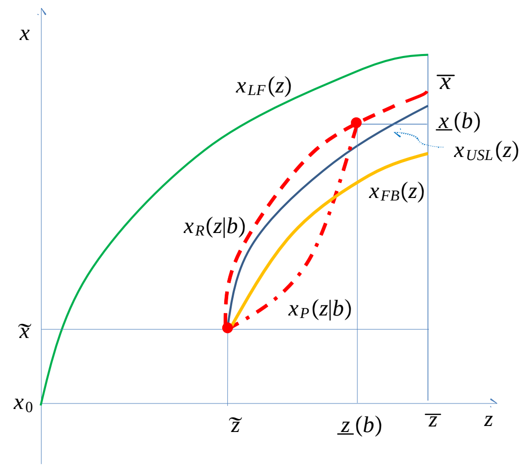

Figure 3: CSL equilibrium with $b > \overline{b}$

required to perform a graduate job. Therefore, educational investment is not just a costly signal. Graduate jobs are differentiated by quality. Graduates and graduate jobs are matched by tournament. Non-graduate jobs are allocated by a conventional labour market. School leavers fall into two wealth categories, the rich and the poor. Native ability is distributed in the same way in both groups. Individual wealth and ability are private information. By contrast, job quality and individual educational achievement are common knowledge. The output of each graduate job-worker match depends on the job's quality as well as on the worker's ability and educational achievement. In the absence of policy, the poor are excluded from higher education and do not participate in the tournament. The government can change that by borrowing wholesale on the international money market and lending to individual students. We have shown that, if the maximum a student can borrow from the government is sufficiently large, the policy will have the effect of replacing some of the less able rich with some of the more able poor in the performance of graduate jobs, and that the properties of the resulting equilibrium will depend on how high this maximum is. The different possibilities are synthesized in Table 1, where b denotes the maximum that a student is allowed to borrow from the government, FB the first-best equilibrium and LF the laissez faire. USL denotes an equilibrium with student loans where poor agents are not liquidity constrained (rich ones never are), and CSL one where at least some of them are.

$0 \leq b \leq \widetilde{\widetilde{b}}$	LF separating equilibrium
$\widetilde{\widetilde{b}} < b < \widetilde{b}$	No separating equilibrium
$\widetilde{b} \leq \bar{b} = b < b_0$	Partially separating CSL equilibrium including rich and poor students with $z \geq \widetilde{z}$; all poor students with $\widetilde{z} < z < \overline{z}$ are liquidity constrained
$\widetilde{b} \leq \bar{b} < b < b_0$	Partially separating CSL equilibrium including rich and poor students with $z \geq \widetilde{z}$; poor students with $\widetilde{z} < \underline{z} < z < \overline{z}$ are not liquidity constrained
$b \geq b_0$	USL separating equilibrium with same matching pattern as FB, but overinvestment.

Table 1. Taxonomy

The LF allocation occurs if b is either zero, or so low that none of the poor goes to university. This allocation is doubly inefficient. First, because some graduate jobs are performed by relatively low-ability agents even though relatively higher-ability ones are available. Second, because (as in all signalling models) all participating agents other than those with the lowest ability buy too much education. A CSL equilibrium arises if the amount the government can lend to each student is at least equal to $\widetilde{b}$, but smaller than b_0. This equilibrium is less inefficient than LF because it replaces some of the less able rich with some of the more able poor, and brings investment levels closer to FB not only by allowing the more able poor to invest more than zero, but also by discouraging the rich from investing too much. It also has an interesting feature. Contrary to what we are used to see in tournament models, graduate jobs of the same quality are assigned to graduates with the same educational level, but different ability levels. As poor school leavers cannot spend more than b for their education, poor graduates will in fact enter the tournament with less education than rich ones of the same ability. Consequently, poor graduates will end up doing the same jobs as relatively less able rich graduates. Given, however, that ability can be inferred ex post when the productivity of the match is observed, competition among employers to secure the best workers will result in an ex-post wage improvement (a "productivity bonus") for poor graduates. This feature disappears if b is large enough to support a USL equilibrium, where jobs are allocated as in FB. Compared with CSL, USL educational investments will be even closer to their FB levels, but there will still be some overinvestment. The finding that, in CSL as in USL, the rich invest less than in LF may seem surprising because, in the presence of a student loan scheme, a rich agent of any given ability faces more competition from agents of the same or higher ability and might thus be expected to invest more rather than less than in the absence of such a scheme. The explanation is that overinvestment is driven by the desire to separate from agents of lower ability, and that there are fewer of the latter in either CSL or USL than in LF.

Educational overinvestment occurs in a model like ours where education enhances productivity as in others where it does not, simply because ability is private information, and educational achievement is a signal of ability. Without wealth inequalities or given a perfect credit market, this informational asymmetry would not prevent positive assortative matching of graduate jobs differentiated by quality with graduates differentiated by learning ability. Given wealth inequalities and an imperfect credit market, however, the best jobs will not go to the best graduates because education is a distorted signal. The reason, first pointed out by Hoff and Lyon (1995) in a context where jobs are allocated by a conventional labour market, is that employers cannot tell, ex ante, whether a worker is willing to stake money on education because he knows he is clever, or because he is rich. All means of directly ascertaining a school leaver's native ability – from cognitive tests, to the gathering of "soft information" as advocated by Gary-Bobo and Trannoy (2008) – are thus beneficial.[15] As pointed out by the same authors, however, such means reduce overinvestment by the rich, but do not reduce underinvestment by the poor. By contrast, as we have seen, student loans reduce both.

7 Appendix

7.1 Proof of Proposition 1

The support of the ability distribution is narrower in FB than in LF because, unlike the latter, the former includes only agents with $z \geq \widetilde{z}$. Therefore, as the ability distribution is the same for the rich and for the poor,

$$\frac{G(z)-(1-\beta)}{\beta} < G(z) \ \forall \ z \in [\widetilde{z}, \overline{z}).$$

Given assortative matching in both FB and LF, and given that $H^{-1}(.)$ is monotonically increasing, it then follows that

$$s_{FB}(z) = H^{-1}\left(\frac{G(z)-(1-\beta)}{\beta}\right) < H^{-1}(G(z)) = s_{LF}(z).$$

7.2 Proof of Proposition 2

We know that, for $z < \widetilde{z}$, $x_{LF}(z) > x_{USL}(z) = x_{FB}(z) = 0$. We also know that $x_{USL}(\widetilde{z}) = x_{FB}(\widetilde{z}) = \widetilde{x}$. We must demonstrate that $x_{LF}(\widetilde{z}) > \widetilde{x}$, $x_{LF}(z) > x_{USL}(z)$ for $z \in (\widetilde{z}, \overline{z})$, and $x_{USL}(z) > x_{FB}(z)$ for $z \in (\widetilde{z}, \overline{z}]$. Concerning $x_{LF}(\widetilde{z}) > \widetilde{x}$, notice that, in USL, graduates of ability $\widetilde{z}$ are matched with jobs of quality $s = 0$, while in LF they are matched with jobs of quality

[15] Cigno and Luporini (2009) argue that school records, cognitive tests, etc. should be used not only to select students with the highest learning ability, but also to ascertain their aptitude for different kind of studies (e.g., arts *vs.* science), and that government help should be made conditional on the student choosing the appropriate course of studies.

$s_{LF}(\widetilde{z}) > 0$. Then, $x_{USL}(\widetilde{z}) < x_{LF}(\widetilde{z})$ because $x_{USL}(\widetilde{z}) = \widetilde{x}$ is found maximizing $\pi(\widetilde{z}, x, 0) - c(\widetilde{z}, x)$, while $x_{LF}(\widetilde{z})$ is calculated integrating (12) from x_0 and is consequently higher than $argmax\,(\pi(\widetilde{z}, s_{LF}(\widetilde{z}), x) - c(\widetilde{z}, x))$, which is in turn higher than $x_{USL}(\widetilde{z})$ for the assumption that $\pi_{sz} > 0$. Hence, at $z = \widetilde{z}$, the $x_{USL}(z)$ curve lies below the $x_{LF}(z)$ curve.

To demonstrate that $x_{LF}(z) > x_{USL}(z)$ for $z \in (\widetilde{z}, \overline{z})$, take any z in that interval. Considering that the slope of the $x_{USL}(z)$ curve is given by (15), while that of the $x_{LF}(z)$ curve is given by (12), the two curves cannot cross. Notice that the numerator of (15) is lower than the numerator of (12) because $\pi_{sz} > 0$ and $\pi_{zx} = 0$ by assumption, and $s_{LF}(z) > s_{USL}(z)$ for any $z \in (\widetilde{z}, \overline{z})$. Notice also that the denominators of (15) and (12) are increasing in x. Consequently, if there existed values of z such that $x_{USL}(z) > x_{LF}(z)$, the slope of the $x_{USL}(z)$ curve would be lower than that of the $x_{LF}(z)$ curve. Considering that $x_{USL}(\widetilde{z}) < x_{LF}(\widetilde{z})$, for the two curves to cross at a value $z' \in (\widetilde{z}, \overline{z})$, it would then have to be true that $x_{USL}(z)$ is steeper than $x_{LF}(z)$ in some interval belonging to $(\widetilde{z}, z')$. But, for any $z = z' + \delta$, with δ arbitrarily small, the slope of $x_{USL}(z)$ should then be lower than that of $x_{LF}(z)$, thus contradicting $x_{USL}(z) > x_{LF}(z)$. Neither can the two curves coincide from point $z' \in (\widetilde{z}, \overline{z})$ upwards. Given that $\pi_{sx} = 0$, this would in fact imply that (15) and (12) have the same denominator. The numerators should then be the same too. But this is impossible because $\pi_{sz} > 0$ implies that, for any given x, the numerator of (15) is lower than the numerator of (12). This however does not exclude $x_{USL}(\overline{z}) = x_{LF}(\overline{z})$.

The demonstration that $x_{USL}(z) > x_{FB}(z)$ $z \in (\widetilde{z}, \overline{z})$ is in Proposition 3 of Hopkins (2012) . This demonstration refers to what we call LF, but it applies equally to our USL.

7.3 Proof of Proposition 3

Assuming that x_R and x_P are strictly increasing in z (the demonstration is in the proof of propositions 4 and 6), we can prove Proposition 3 in a series of steps.

Step 1. For values of z such that the poor are liquidity constrained, the amount of x bought by a rich of ability z is higher than the amount bought by a poor of the same ability. Therefore, the $x_R(z)$ curve lies above the $x_P(z)$ curve for values of z such that the poor are liquidity constrained. The two curves coincide for values of z such that the poor are unconstrained. This in turn implies that the minimum ability level for which an agent invests in education cannot be higher for the rich than for the poor.

Step 2. There cannot exist a CSL equilibrium where some rich agents of ability $z \leq \overline{z}$ buy more x than the poor of ability $z = \overline{z}$. If such an equilibrium existed, there would in fact be a level of z, z^m, and a corresponding level of x, x^m, such that the rich of ability $z \geq z^m$ for whom it is optimal to buy $x \geq x^m$ separate themselves from the poor, by buying more x than the poor of ability $z = \overline{z}$. That, however, cannot be an equilibrium because the employer hiring a graduate of education level x^m would be better-off hiring a worker of education level $x^m - \delta$ with δ arbitrarily small. By so doing, he would in fact have a positive probability of hiring a poor of ability $\overline{z} > z^m$. Therefore, if an equilibrium exists, all agents of ability $z = \overline{z}$ buy the same amount of education, independently of their wealth.

Step 3. There cannot exist a CSL equilibrium where the lowest x bought by the rich, x^n, is higher than the the lowest x bought by the poor, x^q. Given that $x_P(z)$ is strictly increasing and recalling Step 1, if $x^n$is higher than x^q, the ability of a poor choosing x^n would be strictly higher than that of a rich choosing the same level of x. Then, for an argument analogous to the one used in Step 2, a firm hiring a graduate of education level x^n would be better-off hiring a worker of education level $x^n - \delta$ because, if it did that, it would hire a poor of ability level higher than the average ability of the agents choosing x^n.

Step 4. There cannot exist a CSL equilibrium where the lowest x bought by the rich, x^n, is lower than the lowest x bought by the poor, x^q. Suppose that such an equilibrium exists. Let z_P^q be the ability level of the poor, and z_R^q that of the rich, buying x^q in this equilibrium. We know from Step 1 that $z_R^q \leq z_P^q$, and that there will thus be rich agents of ability lower than z_R^q buying positive amounts of x. Consider a level of x, $x' < x^q$. If it is profitable for a rich of ability $z' < z^r$ to buy x', it will be even more profitable to buy that amount of x for a poor of ability level z'', $z' < z'' < z_P^q$, such that $c(x^q - \delta, z'') = b$ with δ arbitrarily small (so that z'' can thus afford x'). Hence, the equilibrium in question cannot exist.

Steps 1 to 4 tell us that, if an equilibrium exists for $b < b_0$, it will be such that the least able rich participating in the tournament buy the same amount of education as the least able of the participating poor and that all agents of ability $z = \overline{z}$ buy the same amount of education independently of their wealth.

Step 5. There cannot exist an equilibrium satisfying the Divinity Criterion (Banks and Sobel, 1987) such that the common lowest level of x, say $\widehat{x}$, is chosen by rich and poor with different ability levels. If such an equilibrium existed, $\widehat{x}$ would in fact be chosen by rich of ability z' and poor of ability z'', $z'' > z'$. Then, $\underline{w}_{CSL}$ would have to satisfy $\underline{w}_{CSL} = w_0 + c(z', \widehat{x})$, where $c(z', \widehat{x}) > c(z'', \widehat{x})$. Since $c_{xz} < 0$, however, there is a level of x, $\widehat{x} - \delta$, such that i) $c(z'' - \varepsilon, \widehat{x} - \delta) = b$, ii) $\underline{w}_{CSL} - c(z' - \varepsilon, \widehat{x} - \delta) < w_0$, and iii) $\underline{w}_{CSL} - c(z'' - \varepsilon, \widehat{x} - \delta) > w_0$ for ε arbitrarily small. If the Divinity Criterion is to be satisfied, firms observing $x = \widehat{x} - \delta$ cannot attribute a positive belief either to $z \leq z' - \varepsilon$ or to $z \geq z'' - \varepsilon$, because a poor agent of ability $z = z'' - \varepsilon$ is the type that can mostly profit from the choice of $\widehat{x} - \delta$ as he is the only type that would strictly improve upon his equilibrium utility from any of the contracts offered by the firms. But if belief $z = z'' - \varepsilon$ is attached to $x = \widehat{x} - \delta$, then poor agents with $z'' - \varepsilon$ would have an incentive to actually deviate to $\widehat{x} - \delta$. In fact, the offer of $\underline{w}_{CSL}$ from a firm of quality $s = 0$ dominates their equilibrium payoff because of iii), while such offer is clearly profitable for a firm of quality $s = 0$ holding belief $z'' - \varepsilon$. Hence, the equilibrium considered cannot satisfy the Divinity Criterion.

Therefore, if an equilibrium exists for $b < b_0$, it will be such that the least able rich participating in the tournament not only buy the same amount of education but also have the same ability level, as the least able of the participating poor.

Given that the measure of graduate jobs is the same in CSL as in FB and USL, the common minimum ability level will then be $\widetilde{z}$.

7.4 Proof of Proposition 4

We first show that it is possible to construct two matching functions, $s_P(z|b)$ for the poor and $s_R(z|b)$ for the rich, such that there is assortative matching over z within wealth categories, and over x for the population as a whole. Recall that, in contrast with Hopkins (2012), where no agent is liquidity constrained, condition (21) concerning x applies only to the rich, because the poor are constrained by (22).

Consider a value of b such that $\widetilde{b} \leq b < b_0$. Denote by $x_P(z|b)$ the function that solves (22). By the implicit function theorem we know that such a function exists. Given our assumptions on $c(z,x)$, $x_P(z|b)$ is continuous, convex and strictly increasing in z, with derivative $x'_P(z|b) = -\frac{c_z}{c_x}$. If we relax the assumptions on $c(z,x)$ by allowing c_{xx} to be positive, convexity requires $\frac{c_{xz}}{c_z} > \frac{c_{xx}}{2c_x}$.Clearly, $x_P(z|b)$ and $x'_P(z|b)$ are increasing in b.

Given our assumptions on $\pi(z,s,x)$ and $c(z,x)$, and assuming that $s_R(z|b)$ is increasing (positive assortative matching),

$$x'_R(z|b) = \frac{\pi_z\left(z, s_R\left(z|b\right), x\right)}{c_x\left(z,x\right) - \pi_x\left(z, s_R\left(z|b\right), x\right)}, \tag{25}$$

is monotonically increasing in z. Consequently, if a solution

$$x_R(z|b) = \int_{\widetilde{z}}^{z} x'_R(t|b)dt + \widetilde{x}, \tag{26}$$

exists, it is monotonically increasing in z. Then $x_i^{-1}(x|b)$, $i = P, R$, is defined and decreasing in b. The distribution of x conditional on b thus satisfies

$$F(x|b) = \alpha F_R(x|b) + (1-\alpha)F_P(x|b) = \alpha G\left[x_R^{-1}(x|b)\right] + (1-\alpha)G\left[x_P^{-1}(x|b)\right] \tag{27}$$

over $[\widetilde{x}, \overline{x}_R]$, where $\overline{x}_R$ denotes the level of education chosen by a rich of ability $\overline{z}$. Assortative matching implies

$$F\left(x_i|b\right) = \phi\left(\alpha G\left[x_R^{-1}(x_i|b)\right] + (1-\alpha)G\left[x_P^{-1}(x_i|b)\right]\right) = H\left(s_i\right), \tag{28}$$

so that an agent buying x_i is matched with a job of quality s_i.

The function $x_P(z|b)$ has already been defined and does not depend on $s_P(z)$. We want to demonstrate that there is only one $s_R(z)$ function simultaneously satisfying (26) and (28), and that $s'_R(z) > 0$ (positive assortative matching). Take an ability level, $z \geq \widetilde{z}$, such that the rich of that ability participate in the tournament. Suppose that (26) and (28) are simultaneously satisfied at this level of z. Consider a value $z + \varepsilon$ with $\varepsilon > 0$ arbitrarily small.

As $G\left[x_P^{-1}(x_i|b)\right]$ is given, there is only one value of $s_R(z+\varepsilon)$ that simultaneously satisfies (26) and (28). Given that, by construction, (26) and (28) are satisfied at $z=\widetilde{z}$ where $s_R\left(\widetilde{z}|b\right)=0$, and given also that $x_R(z|b)$ is continuous, if we follow this procedure starting from $z=\widetilde{z}$, we identify the unique function $s_R\left(z|b\right)$ satisfying (26) and (28) at all $z\geq\widetilde{z}$. Hence, $s_R\left(z|b\right)$ is uniquely defined by (26) and (28), and increasing in z. The properties of $s_P(z|b)$, in particular that this function is increasing in z, can then be derived from (28).

The fact that $s_R\left(z|b\right)$ is increasing implies that the function $x_R(z|b)$ is strictly concave in z. Given $\pi_{zs}<0$, $x_R'(z|b)$ is increasing in $s_R\left(z|b\right)$ for any given z. Consequently, $x_R(z|b)$ is increasing in $s_R\left(z|b\right)$ for that and any lower z, because z is the upper limit of the integral in (26).

So far, we have assumed that (26) has a unique solution. To prove it, we show that Theorem 1 and 2 in Mailath (1987) apply to the present case. For that to be true, the function $V(z,\widehat{z},x)\equiv w(\widehat{z},s(\widehat{z}),x)-c(z,x)$ must satisfy Mailath's regularity conditions (1) $V(z,\widehat{z},x)$ is C^2 on the set of possible messages, (2) $V_2>0$, (3) $V_{13}>0$, (4) $V_3=0$ has a unique solution and (5) (boundedness) $V_{33}(z,s(z),x)<0$. Condition (1) is obviously satisfied. Condition (2) also is satisfied because, as we have shown, $V_2=w_z+w_s s'=\pi_z$, and π_z is positive by assumption. Condition (3) is satisfied because $V_{13}=-c_{zx}$, and c_{zx} is negative by assumption. Condition (4) is satisfied because $V_3=\pi_x-c_x$, and π_{xx} is negative and c_{xx} zero by assumption. Condition (5), finally, is satisfied because $V_{33}(z,s(z),x)=\pi_{xx}$. Additionally, the investment function $x_R(z|b)$ must be incentive-compatibile, and satisfy the initial condition $x_R(\widetilde{z}|b)=\widetilde{x}$. In our case, (25) does satisfy the incentive-compatibility condition for the rich, and the initial condition is the one indicated. As argued in Hopkins (2012), Mailath's Theorem 1 implies that $x_R(z|b)>x_{FB}(z)$ for all $z>\widetilde{z}$. Therefore, as stated in our Proposition 4, all agents other than those with the minimum ability level required to participate in the tournament overinvest.

Having demonstrated the existence of a unique $x_P(z|b)$ and a unique $x_R(z|b)$ curve, we now want to show that the two lie as in Figure 2, and thus that a CSL equilibrium exists. Note first that, $x_P(\widetilde{z}|b)=x_R(\widetilde{z}|b)$ and that, in view of (25), $x_R'(z|b)$ tends to infinity in a neighborhood of $\widetilde{x}$. Given that $x_P(z|.)$ is convex and $x_R(z|.)$ concave, the $x_R(z|b)$ curve will then lie above the $x_P(z|b)$ curve in that neighborhood. Suppose that $b=\widetilde{b}$. Given that $c(z,x_{FB}(z))$ is increasing in z,

$$\overline{x}_R\equiv x_R(\overline{z}|\widetilde{b})>x_P(\overline{z}|\widetilde{b}),$$

where $x_P(\overline{z}|\widetilde{b})$ solves $c(\overline{z},x)=\widetilde{b}$. Were it true that $\overline{x}_R\leq x_P(\overline{z}|\widetilde{b})$, $c(\overline{z},x_P(\overline{z}|\widetilde{b}))$ would in fact be higher than $\widetilde{b}$, because $\overline{x}_R>x_{FB}(\overline{z})$, and thus the liquidity constraint would be violated. As the equilibrium requires $\overline{x}_R\equiv x_R(\overline{z}|b)=x_P(\overline{z}|b)$, there is then no equilibrium for $b=\widetilde{b}$, but we can still construct the $x_R(z|\widetilde{b})$ and $x_P(z|\widetilde{b})$ curves following the procedure outlined above. Knowing that the former lies above the latter in a neighborhood of $\widetilde{z}$, that $x_R(\overline{z}|\widetilde{b})>x_P(\overline{z}|\widetilde{b})$, that $x_R(z|b)$ is concave and that $x_P(z|b)$ is convex, the $x_R(z|\widetilde{b})$ curve will then lie above the $x_P(z|\widetilde{b})$ curve at all $z>\widetilde{z}$.

Raising b will relax the liquidity constraints of the poor. Consequently $x_P(z|.)$ will rise and, given (28), $x_R(z|.)$ will fall (because the rich other than those of ability $z = \widetilde{z}$ will be hired for lower quality jobs than they otherwise would, i.e., $s_P(z)$ will rise and $s_R(z)$ will fall). For b sufficiently high, some of the poor will cease to be liquidity constrained. Given that $x_P(z|b)$ is convex and $x_R(z|b)$ is concave, as b rises, the two curves will come closer together and eventually coincide at the point where $z = \overline{z}$. At that level of b, call it $\overline{b}$, the poor of ability $z = \overline{z}$ are not liquidity constrained, and the two investment curves lie as in Figure 2. Therefore, a CSL equilibrium exists for $b = \overline{b}$.

7.5 Proof of Proposition 6

Consider the last part of the proof of Proposition 4. If we let b rise above $\overline{b}$, the $x_P(z|b)$ curve shifts upwards the $x_R(z|b)$ curve (now called $x_U(z|b)$) shifts downward and the two curves cross at a value of $z < \overline{z}$. Some poor agents of ability $z < \overline{z}$ will then cease to be liquidity constrained, and buy the same amount of x as the rich of the same ability. But, as the $x_U(z|b)$ curve shifts downward, the educational investment of the rich does not remain the same. Recalling that $\underline{z}(b)$ denotes the lowest level of z such that the poor are unconstrained, the rich of ability $z \geq \underline{z}(b)$ in fact face additional competition from the poor, and get lower quality jobs. In the new equilibrium, the educational investment of (rich and poor) unconstrained agents, $x_U(z|b)$, is then lower than $x_R(z|\overline{b})$. For $z \geq \underline{z}(b)$, the job allocation function $s_U(z|b)$ is such that

$$G(z) = F(x) = H(s),$$

and thus the same as in FB. The associated investment function is $x_U(z|b) = \int_{\underline{z}(b)}^{z} x'_U(t)dt + \underline{x}$, where $x'_U(z) = \frac{\pi_z(z,s_U(z|b),x)}{c_x(z,x)-\pi_x(z,s_U(z|b),x)}$ for rich and poor alike, and $\underline{x}$ is the common value of x at the point where the curve representing the investment behaviour of poor agents, $x_P(z|b)$, crosses the one representing the investment behaviour of unconstrained agents, $x_U(z|b)$. Proofs analogous to those developed for the LF and USL equilibria then apply.

For $\widetilde{z} < z < \underline{z}(b)$, there are two job allocation functions, $s_P(z|b)$ for the poor and $s_U(z|b)$ for the rich and two investment functions $x_P(z|b)$ and $x_U(z|b)$, constructed following the same procedure as in the proof of Proposition 4.

REFERENCES

Banks, J. and J. Sobel (1987), Equilibrium Selection in Signaling Games, *Econometrica* 55, 647-661.

Bhaskar, V. and E. Hopkins (2013), Marriage as a Rat Race: Noisy Pre-Marital Investments with Assortative Matching, mimeo.

Becker, G. S. (1973), A Theory of Marriage: Part I, *Journal of Political Economy* 81, 813-846

Bratti, M., A. McKnight, R. Naylor and J. Smith (2004), Higher Education Outcomes, Graduate Employment and University Performance Indicators, *Journal of the Royal Statistical Society Series A* 167, 475-496

Castagnetti, C. and L. Rosti (2009), Effort Allocation in Tournaments: The Effect of Gender on Academic Performance in Italian Universities, *Economics of Education Review* 28, 357–369

Cigno, A. (1991), *Economics of the Family.* Oxford and New York: Clarendon Press and Oxford University Press.

——— and A. Luporini (2009), Scholarships or Student Loans? Subsidizing Higher Education in the Presence of Moral Hazard, *Journal of Public Economic Theory* 11, 55-87

Clark, S. (2006), The Uniqueness of Stable Matching, *Contributions to Theoretical Economics* 6.

Fernandez, R. and J. Galí (1999), To Each According to ... ? Markets, Tournaments, and the Matching Problem with Borrowing Constraints, *Review of Economic Studies* 66, 799-824.

Gale, D. and L. Shapley (1962), College Admissions and the Stability of Marriage, *American Mathematical Monthly* 69, 9-15.

Gary-Bobo, R. J. and A. Trannoy (2008), Efficient Tuition Fees and Examinations, *Journal of the European Economic Association* 6, 1211-1243

Hoff, K. and A. B. Lyon (1995), Non-Leaky Buckets: Optimal Redistributive Taxation and Agency Costs, *Journal of Public Economics* 58, 365-390

Hopkins, E. (2012), Job Market Signaling of Relative Position, or Becker Married to Spence, *Journal of the European Economic Association* 10, 290-322

Hoppe, H. C., B. Moldovanu and A. Sela (2009), The Theory of Assortative Matching Based on Costly Signals, *Review of Economic Studies* 76, 253–281

Koopmans, T. C. and M. Beckmann (1957), Assignments Problems and the Location of Economic Activities, *Econometrica* 25, 53-76

Mailath G. J. (1987), Incentive Compatibility in Signaling Games with a Continuum of Types, *Econometrica* 55, 1349-1365.

Peters M. and A. Siow (2002), Competing Premarital Investments, *Journal of Political Economy* 110, 592-608.