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Abstract

Using a household production model of educational choices, we characterise a free market situation in which some agents (high wagers) educate their children full-time and spend a sizable amount of resources on them, while others (low wagers) educate them only partially. The free-market equilibrium is iniquitous, both because the households have different resources and because the children have different access to education. Public policy is thus called for, for vertical as well as horizontal equity purposes. Conventional wisdom has it that both objectives could be achieved using price control instruments, i.e. income taxes and price subsidies. We find instead that income taxes reduces equality of opportunity and that price subsidies cannot remedy this. Quantity controls become necessary: a compulsory education package, financed by a redistributive tax system, achieves both types of equity. Redistributive taxation and compulsory education are therefore best seen as complementary policies.

JEL-Code: H420, H520.

Keywords: education, in-kind transfers, redistributive taxation.

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I Introduction

In what sense might parents constrain rather then favour the development of their children? Mostly by underinvesting in their education, a phenomenon which is by now accepted as a stylized fact in the literature. There are two competing explanations for this.

- First, there is the standard beckerian view (e.g. Becker et al. 1990) according to which parents see education as a consumption good whose enjoyment may be limited by liquidity constraints: parents are altruistic towards their children, and would like to spend as much as possible in their education, but they might be unable to afford the level of outlay which would be optimal given the potential abilities of the children. The obvious remedy for this is a redistributive policy that transfers more resources towards the needy. A more market-oriented solution is difficult to find, as there is no credit market for the investment in education due to the lack of collateral (future income is normally unacceptable).

- An alternative view sees education as an investment also from the standpoint of the parents and not only of the children: this perspective is related to the "exchange model" of the family pioneered e.g. by Cigno (1993). Selfish family members engage in transfers regulated by self-enforcing rules specifying rewards for obedience and punishments for deviations. The resulting system may be inefficient for several reasons, the most relevant being that parents, when investing in their children's education, foresee that they will be able to reap only a fraction of the return, and tend therefore to underinvest. Redistribution is clearly ineffective, whereas the subsidization of educational expenditure, by lowering the cost of investment, might work (Anderberg and Balestrino, 2003).

The results from the empirical literature are hardly decisive. It is true that the testable implications of the altruistic model are usually not verified (e.g. Altonji et al. 1992, 1997), whereas those of the exchange model are more consistently found to be holding (e.g. Cigno et al. 1998, 2006). It has however been argued that the test usually employed for the altruistic model is unnecessarily restrictive, and that at this stage of our general knowledge there is no definitive case in favour of one or the other approach (McGarry 2000).

A point which we might want to stress is however that neither view recommends an education policy that includes, among other things, compulsory schooling. This is in stark contrast with what actually happens in virtually all the developed countries, and has been happening for the past 150-plus years. It is an historical fact that education policy was conceived in terms of free and mandatory public schooling (financed by public funds) when it was introduced during the XIX century in the West (Germany, France and later UK and US); and free and mandatory schooling is still at the basis of our educational systems today.[1] Compulsory schooling is, instead, still at stake in many less developed countries where universal primary education is far from having been achieved, especially for girls.

Economists are always suspicious towards policy interventions that seem to thwart individual freedom or consumer sovereignity. It has however been recognised, at least since the contributions of Neary and Roberts (1980) and Guesnerie and Roberts (1984), that in a second-best world quantitative restrictions may be welfare-improving inasmuch as they enhance the efficiency and the redistributive impact of the tax system.[2] While these arguments certainly pave the way for our present line of research, they are too vague for our purposes. They refer to generic commodities, and not specifically to education, a service that can of course be bought on the market as many others but has its own pecularities. Two aspects, normally recognised in the literature on education, but not in that on rationing, are, in our opinion, worth emphasising:

1. unlike most commodities, education is purchased not by those who consume it, but by a third party (at least for primary and secondary education, the parents bear the costs of education, while the benefit will be reaped, in time, by the children);

2. the enjoyment of its fruits, no matter whether they are seen in terms of investment or consumption value, require out-of-pocket expenses *and* a large amount of time, i.e. ample opportunity costs (education is a long process: it goes on for years).[3]

In order to account for these peculiarities, we employ a household economics approach. We recognise that there are two actors involved in the purchase and consumption of education, the parents and the child (point 1 above) and we model time allocation in a detailed way, trying to

[1] For example, see Go (2013) for a paper presenting a political economy explanation for the American achievement of universal free public schooling according to a historical perspective.

[2] The surveys by Balestrino (1999, 2000) illustrate the state of the art in this stream of work at the end of the '90s. For a more recent outlook, see Blomquist et al. (2010).

[3] According to Cipolla (1969) opportunity costs appear to be the single most important factor behind the development of education systems. Thus, as long as the kid's time was valuable for help in the farm or for employment in the Dickensian factories of the early industrial revolution, large-scale education programs were not undertaken in the Western world, but became important starting from the second half of the XIX century, when productivity began to be high enough to make the kids' work dispensable.

account for its key role in the educational process (point 2). From a normative standpoint, we develop an argument showing i) that education policy is socially desirable, and ii) that it must preferably include a period of compulsory schooling rather than follow another intervention design.

The plan of the paper is the following. Section II discusses how policy objectives should be formulated. Section III presents the model of educational choice and the *laissez-faire* outcome. Section IV analyses different public policies. Finally, section V contains some concluding remarks.

II Horizontal and vertical equity in educational policy

A distinction is often made between vertical and horizontal equity. The former refers to the equalisation of resources among agents, and is normally pursued *via* progressive tax systems; the latter implies an equal treatment of equals, and requires that policies do not discriminate motivelessly. In our setting, then, vertical equity would be pursued by redistributing income among the parents, while the most important achievement in terms of horizontal equity would be that all the children are given equal access to education – what one might call "equality of opportunity". In this sense, an equitable distribution of education among the children could be judged fair.

Typically, normative economic analyses present the government's objective in terms of vertical equity and efficiency objectives. The social planner is assumed to maximise a Paretian, quasi-concave social welfare function that will pick up a point that lies on the second-best Pareto-frontier and satisfies the vertical equity requirements implied by the convexity of the social indifference curves.

The policy approaches to education that we mentioned above (the beckerian view and the exchange view) fall, broadly speaking, within this framework: the optimal education policy is defined together with the optimal intragenerational redistribution policy as the result of social welfare maximisation. In the beckerian view, the two policies coincide, as the optimal income tax also remedies the inequality in education achievements; in the exchange view, a specific educational subsidy is needed in addition to the income tax. It is interesting to notice, however, *that the achievement of vertical equity among families does not come at the expense of equality of opportunity for the children*: income is redistributed among the parents or educational expenditure is subsidised and, at the same time, the children are all given access to education.

This outcome is in fact somewhat surprising, as it is often found that horizontal and vertical equity requirements conflict with each other (see e.g. Auerbach and Hassett 2002); however, it is predicated on models of educational choice that, in our view, do not take into full account the implications of the two peculiar characteristics of education that we highlighted before, namely the role of the parents and the relevance of opportunity costs.

We shall see that in our model, where these two characteristics are thoroughly explored, neither the policy recommandation of the beckerian model nor that of the exchange model work as far as equality of opportunity is concerned: a different policy instrument, to wit compulsory education, is needed. This, despite the assumption of altruism within the family that we maintain throughout.

One way of viewing this is to say that, in our model, equality of opportunity for the children must be pursued with a specific quantity control, rather than with the usual armoury of price controls. In this sense, our model may be considered as an attempt to apply the now sizable literature on equality of opportunities to education.[4] Indeed, as stated by Brunori *et al.* (2012, p. 765), 'equality of educational opportunity is a widely agreed principle, almost universally considered to be a funding principle of education policy.' There are of course different interpretations of the principle in the literature; in any case, even the more restrictive interpretations do not deny the importance of a minimum education level (Friedman and Friedman 1962, for example, state that 'both the imposition of a minimum required level of schooling and the financing of this schooling by the state can be justified by the "neighborhood effects" of schooling.')

The approach by Roemer (1998) is particularly close to our set-up. Indeed, Roemer points out that equality of educational opportunity can be obtained by equalizing or compensating all those individual's circumstances affecting the individual's final outcome for which she cannot be held responsible, and letting, instead, unaltered the effects of choices for which individuals are to be held responsible. Of course, this requires that it must be possible to distinguish the share of inequalities due to unequal circumstances (opportunities) from the one due to individual choices. Now, the first characteristic of (primary and perhaps secondary) education we mentioned above is that it is not chosen by its users, who cannot therefore taken to be responsible for the consequences of the choice. Then, a compensation principle can be applied

[4] This literature started with Rawls (1971), and nowadays equality of opportunity can be considered 'the prevailing conception of social justice in contemporary western societies' (Ferreira and Peragine 2014, p. 2). Beyond the case of education, such approach has been applied to different areas of public policy such as health, anti-poverty schemes, income taxation and redistribution.

to neutralize inequality in education achievements.

Our position seems to capture closely what has historically happened with the introduction of compulsory education in the Western countries (see Section I above) and also the way in which current policy proposals for the developing countries are formulated, namely directly in terms of percentages of children who have access to primary education. For example, at the World Education Forum in Dakar, 2000, Goal 2 was stated as: "Ensuring that by 2015 all children, particularly girls, children in difficult circumstances and those belonging to ethnic minorities, have access to, and complete, free and compulsory primary education of good quality". No mention, in this statement, is made of the parents or of the tax system: it is said that 100% of the children must receive *compulsory* education, which means that this specific policy instrument is called for, and no other. In other words, policy makers in the past, as well as today, appear to have pursued and to pursue horizontal equity objectives alongside the more commonly assumed vertical equity/efficiency objectives, and appear to have believed and to believe that the former requires a dedicated instrument. In this paper, we will see how these prescriptions follow naturally from our model of educational choice.

III A model of educational choice

We consider a finite-horizon model.[5] The economy is made of two-persons households: one parent and one child. We posit that, in order to earn an income, each parent supplies a certain amount of labour l to the market at a wage rate denoted by w; w varies continuously on $[0, \overline{w}]$ according to a density function $F(w)$, and the agents have unit mass.

Income can be spent on the parent's own consumption C, the child's consumption c, and the child's education e. The latter also requires time d (of the child): in fact, we attribute extreme importance to the fact that education is a very time-intensive activity. Total time endowment is normalised to unity for both agent types. The time of the parent that is not employed on the market, denoted by H, together with the time of the child that is not employed for educational purposes, denoted by h, is used to produce a non-marketable household public good y, nonrivalrous and nonexcludable within the family; for simplicity, no other input is required. A perfect substitute for the households public good, z, is available on the market at

[5] The model could be recast in an OLG framework. All the results reached in the simpler case treated here would however carry over, and we would have to add many unnecessary details, with the consequent risk of making the model lose its focus.

the price p.

We assume that the parent is altruistically linked to the child. For simplicity, altruism is taken to be full, i.e. the parent weighs the kid's utility as her own. This is the simplest setting in which the model can be developed, and also one in which the cards are not too obviously staked in favour of policy intervention, which indirectly reinforces our arguments. The arguments could however be adapted for a model based on the exchange view of the family.

The *laissez-faire* outcome

Let us begin by considering what would happen in a free market, in which there is no government intervention. For simplicity, we set c fixed at some conventional level normalized to zero. We also normalize to zero the subsistence level of consumption of the parent. The household production function (linearly homogeneous, strictly concave and increasing) is

$$y = y\left(H, h\right). \tag{1}$$

We assume that the child cannot produce without a contribution from her parent, who must at least coordinate and supervise home production. Formally, H is an essential input, so that $y\left(0, h\right) = 0$.[6] Also, we introduce

$$x = x\left(e, d\right), \tag{2}$$

again a linearly homogeneous, strictly concave and increasing function, representing the value of education for the child in consumption terms (the child's gross income). We assume that e and d are technogical complements (the more time you spend on education, the more effective is the money you spend on it and *viceversa*), and that both time and money are essential to production,

$$\partial^2 x/\partial e \partial d = \partial^2 x/\partial d \partial e > 0;\; x\left(0, d\right) = x\left(e, 0\right) = 0. \tag{3}$$

Assuming additive separability for the utility functions, we write the parent's preferences as

$$\left[U\left(C\right) + F\left(y\left(H, h\right) + z\right)\right] + \left[u\left(x\left(e, d\right)\right) + f\left(y\left(H, h\right) + z\right)\right], \tag{4}$$

where $U\left(\cdot\right)$ and $F\left(\cdot\right)$ as well as $u\left(\cdot\right)$ and $f\left(\cdot\right)$ are strictly concave sub-utility functions, $U\left(\cdot\right)$ and $u\left(\cdot\right)$ refer to private consumption for the parent and the kid, respectively, while $F\left(\cdot\right)$ and $f\left(\cdot\right)$

[6] This seems a reasonable assumption. Our qualitative results would carry over also in its absence but the exposition would be less straightforward.

refer to the household public good still for the parent and the kid, respectively. Normalising the price of the consumption good to unity, the budget constraint for the agents is

$$C + pz + e = wl. \tag{5}$$

The time constraints for the parent and the kid, respectively, are

$$H + l = 1 \text{ and } h + d = 1. \tag{6}$$

Using these elements, we write the parent's problem as one of choosing $C,\ H,\ z,\ e$ and h so as to

$$\text{Max } \left[U(C) + F(y(H,h) + z)\right] + \left[u(x(e, 1-h)) + f(y(H,h) + z)\right]$$
$$\text{s.t. } C + pz + e - w(1-H) = 0;$$
$$H \geq 0;\ z \geq 0;\ e \geq 0;\ h \geq 0.$$

The first order conditions (FOCs) are as follows:

$$U' = \lambda; \tag{7}$$
$$\left(F' + f'\right) y_H \leq \lambda w, \text{ plus complementary slackness;} \tag{8}$$
$$F' + f' \leq \lambda p, \text{ plus complementary slackness;} \tag{9}$$
$$u' x_e \leq \lambda \text{ plus complementary slackness;} \tag{10}$$
$$\left(F' + f'\right) y_h \leq u' x_d \text{ plus complementary slackness,} \tag{11}$$

where the subscripts denote partial derivatives, and λ is the marginal utility of income.

To begin with, let us investigate the question whether the household public good is purchased on the market or produced internally. Recall that y and z are perfectly substitutable: the parent will clearly use the least expensive. Or, to put it in a different language, she will act according to where her comparative advantage lies, in home production or in market work.

If the parent choses home-production, and assuming an interior solution in which both time-inputs, H and h, are used,[7] it must be the case that

$$F' + f' = \frac{U'}{y_H} w \text{ and } F' + f' = \frac{u'}{y_h} x_d, \tag{12}$$

[7] We could also consider a corner solution where $H = 1$ and the parent devotes no time to labour on the market. This would imply no money to spend in education which in turn implies a level of $h = 1$ (or $d = 0$) because expenditure is an essential input in education. Such a situation may occur for very low levels of the wage rate but it is of no particular interest for our argument.

where we used (7), (8) and (11). In turn, this implies that the input mix in home-production is defined according to the following arbitrage equation:

$$\frac{U'}{y_H}w = \frac{u'}{y_h}x_d. \tag{13}$$

That is, the opportunity cost of the parent's home-time equals that of the child's home-time.[8]

If the parent instead chooses to purchase z, (9) would become, using again (7):

$$F' + f' = U'p. \tag{14}$$

Then, comparing the rewritten FOCs (12) and (14), we can see that the household public good will be produced at home if $w/\left.y_H\right|_{H=0} < p$ (i.e. if the opportunity cost of home-production, measured by the ratio of the productivities on the market and at home, is less than the market price of the substitute) and purchased on the market otherwise.

It is important to establish how w/y_H varies with w. We assume the following:

$$\frac{\partial\left(w/y_H\right)}{\partial w} > 0, \tag{15}$$

which corresponds to the idea that y_H, at most, grows moderately (less then proportionally) as w increases.[9] As w and w/y_H increase, there will be some wage rate w^* for which

$$\frac{w^*}{y_H} = p. \tag{16}$$

Then, all households with $w < w^*$ produce y at home, while those with $w \geq w^*$ purchase z on the market.[10]

We will thus have:

1. a group of low-wage households, $w \in [0, w^*)$, where the kid's time is split between school and home-production ($h > 0$ and therefore $d < 1$), the parent works partly at home and partly in the market ($H > 0$ and $l > 0$, with income wl), and the household public good is not purchased on the market ($z = 0$);

[8] To simplify the exposition, we do not consider the case where (8) is binding and (11) is slack, i.e., the marginal utility of education for the child at $d = 1$ is higher than the marginal utility of income for the parent. Given that $x(.)$ represents the present value of the return to education this case seems less plausible; however the analysis could easily be extended to include it.

[9] For a similar assumption, see Becker and Murphy (2007).

[10] Actually, the choices of the households at the threshold wages are indeterminate, but we can assign these households to one group or the other without loss of generality. It is also possible, of course, that w/y_H never reaches p, so that all households produce y at home, and nobody sends the kids to school full-time. This could be treated as a special case of our more general hypothesis.

2. a group of high-wage households, $w \in [w^*, w^+]$, where the kids go to school full-time ($h = 0$ and $d = 1$), the parents work full-time ($H = 0$ and $l = 1$, with income w), and the household public good is purchased on the market ($z > 0$).

We can now describe the main comparative statics results.

Comparative statics

The details of the comparative statics analysis are reported in Appendix A: here, we give the main results and a few words of interpretation. As shown in the appendix, the results are predicated on mild restrictions and, of course, on the separability assumption. For the high wagers, we saw that $H = h = 0$, that is $l = d = 1$. Substituting the budget constraint (5) into the utility function (4), the optimisation problem becomes

$$\max_{z,e} \left[U\left(w - pz - e\right) + F\left(z\right)\right] + \left[u\left(x\left(e, 1\right)\right) + f\left(z\right)\right], \tag{17}$$

and the consumption mix is determined by the arbitrage condition

$$\frac{F' + f'}{p} = u' x_e. \tag{18}$$

The value for e that emerges is then combined with $d = 1$ to give the equilibrium value for x. The comparative statics results are:

$$\frac{\partial z}{\partial w} > 0; \; \frac{\partial e}{\partial w} > 0. \tag{19}$$

Summing up, all the kids from the high-wage families are educated full-time; furthermore, the richer is the parent, the more she'll spend on education. As a consequence, x increases in the wage rate. Income is trivially increasing in the wage rate.

As for the low wagers, we know that $z = 0$. So, the problem is

$$\max_{H,e,h} \left[U\left(w - e - wH\right) + F\left(y\left(H, h\right)\right)\right] + \left[u\left(x\left(e, 1 - h\right)\right) + f\left(y\left(H, h\right)\right)\right]. \tag{20}$$

The consumption mix is determined by

$$U' = u' x_e, \tag{21}$$

while the input mix (in the production of y) is determined by (13) above. The emerging values for e and d determine x. The comparative statics results are

$$\frac{\partial H}{\partial w} < 0; \; \frac{\partial e}{\partial w} > 0. \tag{22}$$

while the sign of $\partial h/\partial w$ is ambigous. The low wagers' labour supply is increasing in the wage rate, and therefore so is their income. Expenditure in education increases in the wage rate and this of course increases the value of education. However, we cannot be sure that a higher wage rate implies an increase in d (and a consequent decrease in h). It might be the case that, the higher is the wage rate, the more the kids will be working at home: in other words, some of the low wagers might replace their own home time with that of the kids as working outside home become more advantageous. If this is so, we cannot even say whether the total returns to education x increase, because less school-time makes expenditure less effective (d and e are complements).[11]

The characteristics of the market equilibrium

From the analysis above, it is clear that the economy presents an "educational divide". First of all, educational expenditure increases with the wage rate, so there is of course a source of difference there. But, mostly, the difference originates from the fact that those households whose wage is in the upper range give their children a full-time education, while the households with lower wage rates send their kids to school only for a part of their time, and employ the remaining time for the production of a household public good. Further, since time and expenditure are, plausibly, seen as complementary inputs in our model, it follows that the money that the high wagers spend is more effective than that spent by the low wagers, thereby further accentuating the divide.

The divide has nothing to do with altruism, which is just as strong for the low as for the high wagers. Indeed, the main force behind the separation between those who enjoy a full-time education and those who don't is the logic of *comparative advantages* for the production of the household public good. If such an advantage lies in home-production, i.e. w/y_H is lower than p, then the child will not achieve a full-time education.

If, as we argued in Section II above, it is plausible to assume that the government has (also)

[11] If one should make an assumption, it would probable be safe to postulate that increasing expenditure will entail little compensation for a reduced time at school, which is probably mostly irreplaceable due to the teacher's inputs, the peer effects, etc.. An alternative approach is that proposed by Glewwe (2002) for less developed countries where years of schooling and school quality are considered as alternative inputs in the production of the child cognitive skills. In that set-up, when the learning efficiency of the child increases or the cost of education decreases, parents prefer to increase the quality of the children education rather than the quantity because the latter has a greater opportunity cost in terms of less time for the child market work.

a horizontal equity objective such as granting equal opportunities to all the children, then, the educational divide is of course a matter of concern. In this case, it would clearly become desirable to devise some kind of policy package so as to allow also the low wagers to send their children to school full-time.

IV Alternative policy frameworks

We now assess the performance of the policy packages that economists usually recommend. Since we are in an altruistic setting, we start with a redistributive income tax, that we take for simplicity to be linear, with tax rate $\tau > 0$ and lump-sum subsidy $T > 0$. In a beckerian model, using such a policy instrument would lead to a social equilibrium in which both vertical and horizontal equity objectives are attained (see Sections 1 and 2 above).

Our own results, however, stands in stark contrast, as we show that *the income tax is not at all effective as far as horizontal equity is concerned.* Things do not change if we add an educational subsidy, $\sigma > 0$; in fact, we argue that no price instrument can guarantee equality of opportunity in our setting. Put another way, in our framework, the presence of redistributive taxation, which is necessary to achieve some degree of vertical equity, damages horizontal equity. Therefore, we consider the merits of an alternative policy in which we replace the educational subsidy σ with a compulsory education package consisting in a given amount of per-child expenditure and a given number of years of mandatory schooling, and perform a full analysis of this case.

Redistributive taxation (with an educational subsidy)

With an income tax in place, the budget constraint becomes:

$$C + pz + e + w(1-\tau)H = w(1-\tau) + T. \tag{23}$$

We assume that also the future income of the children is taxed at the same rate as that of the parents so that children earn $(1-\tau)x(\cdot)$ after tax. The utility function therefore is:

$$[U(C) + F(y(H,h) + z)] + [u((1-\tau)x(e, 1-h)) + f(y(H,h) + z)]. \tag{24}$$

Let us check the effects on the model. The FOCs become

$$U' = \lambda; \tag{25}$$

$$\left(F' + f'\right) y_H \leq \lambda \left(1 - \tau\right) w, \text{ plus complementary slackness;} \tag{26}$$

$$F' + f' \leq \lambda p, \text{ plus complementary slackness;} \tag{27}$$

$$u' \left(1 - \tau\right) x_e \leq \lambda \text{ plus complementary slackness;} \tag{28}$$

$$\left(F' + f'\right) y_h \leq u' \left(1 - \tau\right) x_d \text{ plus complementary slackness.} \tag{29}$$

Suppose now that a social welfare maximisation exercise had been performed, and that the desired level of redistribution has been actuated via the optimally set tax rate and poll subsidy, say $\widetilde{\tau} > 0$ and $\widetilde{T} > 0$. From the FOCs above, we can then identify a cut-off wage rate $w^* \left(\widetilde{\tau}, \widetilde{T}\right)$ implicitly defined by

$$w^* = \frac{p}{1 - \widetilde{\tau}} y_H. \tag{30}$$

In principle $\widetilde{\tau}$ and $\widetilde{T}$ may act on H and thus on y_H. However, we assumed that the agents at $w = w^*$ purchase the household public good on the market and therefore for them $H = 0$ always: hence, y_H is virtually unaffected by changes in the policy tools (ignoring second-order effects). There only remains a direct effect for $\widetilde{\tau}$:

$$\frac{\partial w^*}{\partial \widetilde{\tau}} = \frac{p}{\left(1 - \widetilde{\tau}\right)^2} y_H > 0. \tag{31}$$

Relative to the free-market situation, then, in an equilibrium with policy there will be *less* agents who educate their kids full time! This is because the marginal tax rate $\widetilde{\tau}$ alters the comparative advantage situation: working outside home become less advantageous, and more agents choose to produce the household public good domestically. The achievement of a redistributive objective in vertical equity terms *via* the income tax would therefore limit equality of opportunity.[12]

Adding an educational subsidy σ will not alter the general outcome. The budget constraint becomes

$$C + pz + \left(1 - \sigma\right) e + w \left(1 - \tau\right) H = w \left(1 - \tau\right) + T, \tag{32}$$

[12] Incidentally notice that also a tax on z, which we do not consider in the model to keep things simple, would have a negative effect on the threshold wage rate - much in the same way as an increase in the income tax rate. Since the tax would have mostly a redistributive purpose (hitting a good that is only consumed by the high-wagers) the logic is the same: vertical equity conflicts with horizontal equity.

while the utility function is the same as above. The FOCs are also the same except for the one w.r.t e, which becomes $u'(1-\tau)x_e \leq (1-\sigma)\lambda$ (plus complementary slackness); and, once the social welfare maximisation has been carried out, the threshold is as in (30). The same reasoning as above then applies: the income tax reduces equality of opportunity, and the price subsidy is ineffective to remedy this. We would of course expect the subsidy to increase educational expenditure: but it is doubtful whether this might compensate the kids who have been obliged to leave full-time schooling for the reduced educational time (we have already made this point above – see fn. 11). Indeed, it is easy to see that, if the low wagers choose not to educate their kids full-time, no price instrument can alter this choice. So, for example, if, plausibly, $\partial h/\partial w < 0$ and the low wagers in the proximity of w^* have $d \simeq 1$, then all the others will have $d < 1$, no matter what the tax arrangements are.

In our setup, therefore, the standard beckerian prescription of using redistributive policy to make even the less well-off prone to educate their children is ineffective, despite the assumption of altruism within the family, as far as horizontal equity objectives are concerned. Rather, redistributive taxation has a strong negative effect, since the presence of a marginal tax rate forces some individuals not to educate their children full-time. This cannot be remedied by the education subsidy, which only works on the intensive margin.[13]

Redistributive taxation with a compulsory education package

Given the weak performance of the standard set of policy tools, it makes sense to investigate whether compulsory education can be more effective. Consider then, along with the linear income tax with tax rate $\tau > 0$ and lump-sum subsidy $\widehat{T} > 0$, a compulsory education package (E, D) where E is per-child expenditure and D is the number of years of mandatory schooling. The parents can top up both rations, adding expenditure and school time beyond the mandatory level: then, e and d are now the amounts of expenditure and school-time that the parent can employ for topping up the compulsory levels, respectively.

We characterise the policy problem as one of choosing the policy tools that maximise a social welfare function, thereby satisfying a vertical equity requirement, subject to a constraint

[13] This rather sharp result would need to be qualified if we were to assume that the agents at $w = w^*$ produce the household public good at home. It would still be true that the presence of a marginal tax rate induces an increase in the number of kids who do not go to school full-time, but the price subsidy and/or the poll-subsidy might partially compensate the effect.

that imposes a horizontal equity requirement. The constraint is simply that D be set at unity, thus forcing $h = d = 0$: this way, all kids will go to school full-time (setting a lower level would be self-defeating, as the high wagers would adjust in order to have $D + d = 1$, while the low wagers would continue to have $D + d < 1$). This corresponds to a current practice in virtually all Western countries today.[14]

We further investigate whether it is optimal to set also a fixed level of expenditure E along with the tax instruments; this means in principle that the total expenditure $E + e$ might differ for kids from different families, although we will focus below on a case in which it is the same for all.

By substituting the constraint that $D = 1$ into the utility function, the latter becomes

$$[U(C) + F(y(H,0) + z)] + [u((1-\tau)x(E+e,1)) + f(y(H,1) + z)], \tag{33}$$

while the budget constraint is

$$C + pz + e + (1-\tau)wH = (1-\tau)w + \widehat{T}. \tag{34}$$

Notice that it is possible to write the budget constraint also as

$$C + pz + (e+E) + (1-\tau)wH = (1-\tau)w + T, \tag{35}$$

that is, as if the agent were paying the educational expenditure herself. In fact, as formally shown in Appendix C, T now also covers E.

The problem of the agent is then to maximise (33) by choice of C, H, z, and e s.t. (35) and the non-negativity constraints. The FOCs are as follows:

$$U' = \lambda; \tag{36}$$

$$\left(F' + f'\right) y_H \leq \lambda (1-\tau) w, \text{ plus complementary slackness;} \tag{37}$$

$$\left(F' + f'\right) \leq \lambda p, \text{ plus complementary slackness;} \tag{38}$$

$$u' (1-\tau) x_e \leq \lambda \text{ plus complementary slackness.} \tag{39}$$

[14] A parallel can be drawn with health economics, where some theories of justice analyse the interpersonal distribution of health by establishing different types of condition upon outcomes that need to be satisfied before solving any social planner maximisation problem. For example, a common condition is that a minimum decent level of health has to be fulfilled for some specified groups, and no trade-off is allowed between such a goal and any other (Williams and Cookson 2000).

The choice between home production and market purchase depends on the measure of comparative advantage, just as before. The threshold wage rate is as in (30), and the distinction between high- and low wagers works in the same way, with the added twist that, for the high wagers, the presence of $D = 1$ is of no consequence because this is what the parents would have chosen anyway.

The indirect utility for both types of parent can be written as a function of the policy instruments, $V = V(\tau, T, E)$, and the derivatives w.r.t. the policy tools are

$$\frac{\partial V}{\partial T} = \lambda > 0;\ \frac{\partial V}{\partial \tau} = -\lambda w (1 - H) - u' x < 0; \tag{40}$$

$$\frac{\partial V}{\partial E} = u' (1 - \tau) x_e - \lambda < 0 \text{ if } e = 0; \tag{41}$$

Notice that $H > 0$ for low wagers and $H = 0$ for high wagers in the expression for $\partial V/\partial \tau$. The sign of $\partial V/\partial E$ depends on whether E exceeds the quantity that the agent would have chosen in the free market or not: $\partial V/\partial E$ is negative if it does, equal to 0 otherwise.

The next step requires us to check the comparative statics in this new setting with the policy instruments. This task is made extremely cumbersome by the fact that there are many possibilities concerning the extent to which the compulsory educational package actually constrains the family choices. We noticed that $D = 1$ is always inframarginal for the high wagers, while for the low wagers the constraint will definitely bite. Instead E may or may not bite for both types. Here we focus on the case that seems more interesting,[15] i.e. the one in which E constrains the choices of all households. Therefore, $e = 0$ for all agents. This is the scenario in which the quantity constraints interfere the most with the free choices of the agents: can in this extreme case those constraints be welfare-improving?

Let us start from the comparative statics for both the high and the low wagers (calculations are found in Appendix D). Notice that the high wagers only choose z. We find that

$$\partial z/\partial w > 0;\ \partial z/\partial \tau < 0;\ \partial z/\partial T > 0;\ \partial z/\partial E < 0. \tag{42}$$

As for the low wagers, they only choose H, and we find that

$$\partial H/\partial w < 0;\ \partial H/\partial \tau > 0;\ \partial H/\partial T > 0;\ \partial H/\partial E < 0. \tag{43}$$

Parents with higher wage rates devote less time to home production, because for them market work is more attractive. Also, the parents will increase the time devoted to home production

[15] As far as the effects of the education policies are concerned, the conclusions in the other cases are analogous to those discussed here. The results for the other cases are available from the authors.

following an increase in the tax rate, τ, because this makes market work less attractive, and also following an increase in the lump-sum subsidy, T, because this increases the sum that can be used to buy goods on the market. Conversely, an increase in the compulsory educational expenditure, E, forces parents to reduce the time devoted to home production.

Optimal second-best policy

Consider now a second-best policy problem with compulsory education. We have the following:

$$\max_{\tau,T,E} \int_0^{\overline{w}} \beta(w) V(\tau, T, E; w) F(w) \, \mathrm{d}w$$
$$\text{s.t. } \tau \int_0^{\overline{w}} x(E) F(w) \, \mathrm{d}w + \tau \int_0^{\overline{w}} wF(w) \, \mathrm{d}w - \tau \int_0^{w^*(\tau)} [wH(w) F(w)] \, \mathrm{d}w - T = R,$$

where R is a fixed revenue requirement. The FOCs are

$$\int_0^{\overline{w}} \beta(w) \frac{\partial V}{\partial \tau} F(w) \, \mathrm{d}w + \mu \left\{ \int_0^{\overline{w}} x(w) F(w) \, \mathrm{d}w + \int_0^{\overline{w}} wF(w) \, \mathrm{d}w - \right.$$
$$\left. - \int_0^{w^*(\tau)} wH(w) F(w) \, \mathrm{d}w - \tau \int_0^{w^*(\tau)} w \frac{\partial H(w)}{\partial \tau} F(w) \, \mathrm{d}w - \tau w^* H(w^*) F(w^*) \frac{\partial w^*}{\partial \tau} \right\} = 0; \quad (44)$$

$$\int_0^{\overline{w}} \beta(w) \frac{\partial V}{\partial T} F(w) \, \mathrm{d}w - \mu \left\{ \tau \int_0^{w^*(\tau)} w \frac{\partial H(w)}{\partial T} F(w) \, \mathrm{d}w + 1 \right\} = 0; \quad (45)$$

$$\int_0^{\overline{w}} \beta(w) \frac{\partial V}{\partial E} F(w) \, \mathrm{d}w + \mu\tau \left\{ \int_0^{\overline{w}} x_e F(w) \, \mathrm{d}w - \int_0^{w^*(\tau)} w \frac{\partial H(w)}{\partial E} F(w) \, \mathrm{d}w \right\} = 0; \quad (46)$$

where the derivatives with respect to the indirect utility functions are given by (40) and (41). We can then state the main results concerning the policy rules.

First, consider the second FOC:

$$\frac{\int_0^{\overline{w}} \beta(w) (\partial V / \partial T) F(w) \, \mathrm{d}w}{\mu} - \tau \int_0^{w^*(\tau)} w \frac{\partial H(w)}{\partial T} F(w) \, \mathrm{d}w = 1; \quad (47)$$

the net social marginal utility of income, inclusive of its effect on revenue and weighted by μ, equals unity. This is a standard result that characterises the optimal T.

The third FOC can be rearranged as follows:

$$\tau \left\{ \int_0^{\overline{w}} x_e F(w) \, \mathrm{d}w - \int_0^{w^*(\tau)} w \frac{\partial H(w)}{\partial E} \right\} F(w) \, \mathrm{d}w = - \frac{\int_0^{\overline{w}} \beta(w) (\partial V / \partial E) F(w) \, \mathrm{d}w}{\mu}, \quad (48)$$

that is the marginal benefit in terms of increased revenue must equal the marginal cost in terms of forcing the agents out of the chosen consumption bundle – recall that $\partial V / \partial E < 0$ when the ration bites. Increased revenue depends on the fact that setting E higher leads to a larger future income of the children x, as well as to less home-production time, or equivalently more time devoted to market work, $(\partial H / \partial E < 0)$ and therefore more taxable income from the parents.

This analysis presupposes that $\tau > 0$. In order to check whether this is the case, we can rearrange the first FOC. To this end, define

$$\Psi \equiv \int_0^{\overline{w}} x(w) F(w) \mathrm{d}w + \int_0^{\overline{w}} wF(w) \mathrm{d}w - \int_0^{w^*(\tau)} wH(w) F(w) \mathrm{d}w. \tag{49}$$

We can then write

$$\tau = \frac{\Psi - \int_0^{\overline{w}} \beta(w) (\partial V/\partial \tau) F(w) \mathrm{d}w/\mu}{\int_0^{w^*(\tau)} w\frac{\partial H(w)}{\partial \tau} F(w) \mathrm{d}w + w^* H(w^*) F(w^*) \frac{py}{(1-\tau)^2}}, \tag{50}$$

where we used (31).

From the fact that $\partial H/\partial \tau > 0$ we deduce that the denominator in (50) is positive. This term represents the total revenue loss associated with a marginal increase in τ: the reduction in labour supply implies a reduction of tax base, and the fact that w^* varies inversely with τ implies a further reduction because more agents start employing home-production to get the household public good, and therefore work less. The larger is this term, the smaller will be τ.

Instead, the first term at the numerator, Ψ, is, as we just said, the marginal revenue gain from the tax. It is positive because

$$\int_0^{\overline{w}} x(w) F(w) \mathrm{d}w + \int_0^{\overline{w}} wF(w) \mathrm{d}w > \int_0^{w^*(\tau)} wH(w) F(w) \mathrm{d}w. \tag{51}$$

Further, the second term at the numerator of (50), also positive, is the marginal welfare loss. Therefore, if the revenue gain exceeds the welfare loss, the numerator is positive as well. In that case, we have $\tau > 0$; and the larger is the difference between the two terms above the line, the larger is the tax rate.

To sum up, the fact that $\tau > 0$ implies that, at the optimum, we have $E > 0$: if some form of redistributive taxation is in place, it is optimal to force the agents to spend on education more than they would have done in a free-market. The desirability of the quantity controls is justified by the fact that they imply a gain in revenue terms. If this gain is large enough to compensate the costs in terms of displaced consumption, then some form of quantitative restriction is welfare-improving.[16]

[16] It may be worth noting that, had we chosen D optimally rather than have it fixed at the outset, we would have reached a similar conclusion to that for E, that is, we would have found that it is socially desirable, on vertical equity grounds, to establish mandatory schooling. Of course, there would not have been, at this level of generality, any guarantee that the optimal level of D were exactly unity, so in this sense we would not have necessarily achieved a full equality of opportunity.

A remarkable feature of this result is that redistributive taxation and education policy in the form of compulsory education appear to be strongly intertwined. Neither works without the other, if we assume that both horizontal and vertical equity are policy concerns. Redistributive taxation satisfies the vertical equity requirements but damages the future earnings of (some of) the children, as we know from the analysis of the impact of the tax rate on the threshold wage rate; horizontal equity thus requires a specific education policy, in the form of mandatory schooling. At the same time, the full package of compulsory education, in which agents are forced to provide resources to finance the expenditure on education for their children, cannot be optimal without redistributive taxation.

V Concluding remarks

We began by asking whether there is a reason why education policy should involve a mandatory and (virtually) free-of-charge schooling period, as it commonly does in the Western countries. Economists should be particularly interested in obtaining an answer, as quantity controls are traditionally considered outperformed by price controls in standard economic theory. Having established that education policy must, for some reason, be implemented, many would argue that it should take the form of a price subsidy (making education less costly should make agents more prone to purchase it for their children) or simply be embedded in tax policy (redistributing resources in favour of the poor should automatically help them to send their children to school).

Now, it is well-known, at least since Guesnerie and Roberts (1984), that the superiority of price controls is only valid in first-best, and that quantity controls can be welfare-improving in a variety of second-best contexts: the last 20 years have seen a vast research effort on this that traces its roots to the contributions of Blomquist and Christiansen (1995) and Boadway and Marchand (1995) and continues to this day (e.g. Blomquist et al. 2010). Our work follows this stream of the literature, and aims to fill a blank space, because none of those works has dealt specifically with education as we believe it should be characterised, namely as i) an extremely expensive and time-consuming process that ii) involves a decision-maker (the parent) who is not the direct beneficiary (the child).

Using a model that accounts for both these features, we have first depicted a free market situation in which some agents ("high-wagers") educate their children full-time and spend a sizable amount of resources on them, while others ("low-wagers") educate them only partially (and in principle might even not educate them at all).[17] This outcome is generated by the presence of an alternative usage of the children's time: rather than be sent to school, they can be employed in producing a household public good (alternatively, we might have assumed that the children can be sent to work in the market: for a model with child labour, see Cigno 2013). The high-wagers can afford to replace this home-produced good with a marketed substitute; the low-wagers' comparative advantage, instead, lies in home-production. The free-market equilibrium is iniquitous where parents are concerned, due to their having different exogenous skills and thus different incomes (a vertical equity problem), and also where children are concerned, even if the parents are fully altruistic, because the kids receive different educations depending on whether they are born in a high-wage or a low-wage family. Further, the differences in the education they receive today imply that there will be a disparity in earning abilities tomorrow due to choices made by the parents, not by themselves (a horizontal equity problem).[18]

Public policy is thus called for, both for vertical and horizontal equity reasons. In this framework, we argued that it is indeed socially optimal to introduce a compulsory education package, using a standard redistributive tax system to finance it. Mandatory schooling fully compensates the kids for the disadvantages at which their parents' choices might have put them. Adding a mandatory expenditure requirement forces the parents away from their equilibrium choices, which is of course costly but entails also an advantage in terms of increased revenue that can be used for redistributive purposes. Indeed, it may finance the poll subsidy, that goes to the family as a whole, and the educational expenditure for the children, including those from the less well-off families.

From the point of view of horizontal equity, a compulsory education policy is shown to be superior to the use of price subsidies, that only work on the intensive margin, i.e. boost education expenditure for those who would have educated their children full-time anyway in

[17] This result can be linked to the recent empirical literature that try to analyse how exogenous changes in parents' education due to variations in compulsory schooling laws may affect the intergenerational transmission of education. For example, Piopiunik (2014) provides evidence that individuals with more schooling (and thus on avergae higher wages) value their kids' education more highly.

[18] Another possible reason for public intervention, which we do not explore but just mention briefly here, is the fact that the children, despite all having the same ability, are educated at different level: this might indeed have efficiency implications. The *laissez-faire* equilibrium is clearly efficient from the point of view of the parents (or of the families as a whole), but if we look at it from the point of view of the children (e.g. if the social welfare function were given by the sum of children's sub-utility functions) this is no longer the case.

a free market, but are unable to induce those who didn't educate their kids full-time to start doing so. And we also argued that redistributive taxation alone is in fact counter-productive, as it forces more agents than in *laissez-faire* to avoid educating their children full-time, because it tips the comparative advantage balance in favour of making child household work more desirable. This suggests that redistributive taxation and compulsory education are best seen as complementary policies if we assume that the government pursues both vertical and horizontal equity objectives.

Appendix A - Comparative statics in *laissez-faire*

Recall that we are using a separable utility function throughout. This implies that strict concavity is enough to guarantee the signs. With a general utility function, the same results would have to be obtained by adding a few assumptions and restrictions on the sign and the magnitude of the cross-derivatives of the utility function.

High-wagers

In the case of high-wagers, we can write the maximisation problem as

$$\max_{z,e} \ (U(w - pz - e) + F(z)) + (u(x(e,1)) + f(z)). \tag{A1}$$

The FOCs are

$$-U'p + F' + f' = 0 \ \ (z); \tag{A2}$$

$$-U' + u'x_e = 0 \ \ (e). \tag{A3}$$

By totally differentiating, we have:

$$-U''p\mathrm{d}w + \left[U''p^2 + F'' + f''\right]\mathrm{d}z + U''p\mathrm{d}e = 0; \tag{A4}$$

$$-U''\mathrm{d}w + U''p\mathrm{d}z + \left[U'' + \left(u''(x_e)^2 + u'x_{ee}\right)\right]\mathrm{d}e = 0. \tag{A5}$$

Therefore:

$$\begin{bmatrix} U''p^2 + F'' + f'' & U''p \\ U''p & U'' + \left(u''(x_e)^2 + u'x_{ee}\right) \end{bmatrix} \begin{bmatrix} \mathrm{d}z/\mathrm{d}w \\ \mathrm{d}e/\mathrm{d}w \end{bmatrix} = \begin{bmatrix} U''p \\ U'' \end{bmatrix} \tag{A6}$$

Then, the signs are as follows

$$sgn \ \mathrm{d}z/\mathrm{d}w = sgn \ \left[U''p\left(U'' + \left(u''(x_e)^2 + u'x_{ee}\right)\right)\right] - \left(U''\right)^2 p > 0; \tag{A7}$$

$$sgn \ \mathrm{d}e/\mathrm{d}w = sgn \ \left[\left(U''p^2 + F'' + f''\right)U'' - \left(U''p\right)^2\right] > 0. \tag{A8}$$

Low wagers

In the case of low wagers, we can write the maximisation problem as

$$\max_{H,e,d} U(w-e-wH)+F(y(H,1-d))+u(x(e,d))+f(y(H,1-d)). \tag{A9}$$

The FOCs for an interior solution are:

$$-U'w+F'y_H+f'y_H=0\ \ (H); \tag{A10}$$

$$-U'+u'x_e=0\ \ (e); \tag{A11}$$

$$-F'y_h-f'y_h+u'x_d=0\ \ (d). \tag{A12}$$

Totally differentiating, we have:

$$\begin{aligned}&-\left[U''(1-H)w+U'\right]\mathrm{d}w+\left[U''w^2+\left(F''+f''\right)(y_H)^2+\left(F'+f'\right)y_{HH}\right]\mathrm{d}H+\left[U''w\right]\mathrm{d}e-\\&-\left[\left(F''+f''\right)y_Hy_h+\left(F'+f'\right)y_{Hh}\right]\mathrm{d}d=0;\end{aligned} \tag{A13}$$

$$\begin{aligned}&-\left[U''(1-H)\right]\mathrm{d}w+\left[U''w\right]\mathrm{d}H+\left[U''+\left(u''(x_e)^2+u'x_{ee}\right)\right]\mathrm{d}e+\\&+\left[u''x_ex_d+u'x_{ed}\right]\mathrm{d}d=0;\end{aligned} \tag{A14}$$

$$\begin{aligned}&0\mathrm{d}w-\left[\left(F''+f''\right)y_hy_H+\left(F'+f'\right)y_{hH}\right]\mathrm{d}H+\left[\left(u''x_dx_e+u'x_{de}\right)\right]\mathrm{d}e+\\&+\left[\left(F''+f''\right)(y_h)^2+\left(F'+f'\right)y_{hh}+\left(u''(x_d)^2+u'x_{dd}\right)\right]\mathrm{d}d=0.\end{aligned} \tag{A15}$$

Now, let

$$\begin{bmatrix}a_{11}\\a_{21}\\a_{31}\end{bmatrix}=\begin{bmatrix}U''w^2+(F''+f'')(y_H)^2+(F'+f')y_{HH}<0\\U''w<0\\-[(F''+f'')y_hy_H+(F'+f')y_{hH}]>0\end{bmatrix}; \tag{A16}$$

$$\begin{bmatrix}a_{12}\\a_{22}\\a_{32}\end{bmatrix}=\begin{bmatrix}U''w<0\\U''+u''(x_e)^2+u'x_{ee}<0\\u''x_ex_d+u'x_{ed}\end{bmatrix}; \tag{A17}$$

$$\begin{bmatrix}a_{13}\\a_{23}\\a_{33}\end{bmatrix}=\begin{bmatrix}-[(F''+f'')y_Hy_h+(F'+f')y_{Hh}]>0\\u''x_dx_e+u'x_{de}\\(F''+f'')(y_h)^2+(F'+f')y_{hh}+u''(x_d)^2+u'x_{dd}<0\end{bmatrix}, \tag{A18}$$

where the sign of a_{32} is not determined. Then

$$\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}\begin{bmatrix}\mathrm{d}H/\mathrm{d}w\\\mathrm{d}e/\mathrm{d}w\\\mathrm{d}d/\mathrm{d}w\end{bmatrix}=\begin{bmatrix}[U''(1-H)w+U']\\U''(1-H)\\0\end{bmatrix}. \tag{A19}$$

Further assuming that

$$-\frac{U''}{U'} < \frac{1}{(1-H)\,w}, \tag{A20}$$

the comparative statics signs are as follows. Take $\mathrm{d}H/\mathrm{d}w$ first:

$$\begin{aligned} sgn\ \mathrm{d}H/\mathrm{d}w &= -sgn\left\{\left[\underbrace{U''(1-H)\,w+U'}_{+}\right]\left(\underbrace{a_{22}a_{33}-a_{32}a_{23}}_{+}\right)\right. \\ &\left. +\underbrace{U''(1-H)}_{-}\underbrace{\left(\underset{?}{a_{23}}\underset{+}{a_{31}}-\underset{-}{a_{21}}\underset{-}{a_{33}}\right)}_{?}\right\}. \end{aligned} \tag{A21}$$

The term $(a_{22}a_{33}-a_{32}a_{23})$ is positive because of the maximization condition. The sign of the term $(a_{23}a_{31}-a_{21}a_{33})$ is ambigous. It is negative if $a_{23} \leq 0$. If instead $a_{23} > 0$, for $(a_{22}a_{33}-a_{32}a_{23})$ to be non positive it must be

$$-\frac{u''x_ex_d+u'x_{ed}}{U''w} \leq \frac{(F''+f'')\,(y_h)^2+(F'+f')\,y_{hh}+u''\,(x_d)^2+u'x_{dd}}{(F''+f'')\,y_hy_H+(F'+f')\,y_{hH}}. \tag{A22}$$

It can be shown that a sufficient, but far from necessary, condition for the inequality to be satisfied is that h and H are sufficiently substitutable for each other. We will here consider cases where (A22) is satisfied, ensuring that

$$\frac{\mathrm{d}H}{\mathrm{d}w} < 0 \text{ or } \frac{\mathrm{d}l}{\mathrm{d}w} > 0. \tag{A23}$$

Consider then $\mathrm{d}e/\mathrm{d}w$:

$$sgn\ \mathrm{d}e/\mathrm{d}w =$$

$$-sgn\left\{\underbrace{U''(1-H)}_{-}\underbrace{\left(\underset{-}{a_{11}}\underset{-}{a_{33}}-\underset{+}{a_{31}}\underset{+}{a_{13}}\right)}_{+}+\underbrace{\left[U''(1-H)+U'\right]}_{+}\underbrace{\left(\underset{?}{a_{32}}\underset{+}{a_{13}}-\underset{-}{a_{12}}\,\underset{-}{a_{33}}\right)}_{?}\right\} \tag{A24}$$

We know that the term $a_{11}a_{33}-a_{31}a_{13}$ is positive because of the maximization conditions, and, as we have discussed above, we are considering the case where $\left(\underset{+}{a_{32}}\underset{+}{a_{13}}-\underset{-}{a_{12}}\,\underset{-}{a_{33}}\right) < 0$, then

$$\mathrm{d}e/\mathrm{d}w > 0.$$

Finally, let us consider $\mathrm{d}d/\mathrm{d}w$.

$$sgn\ \mathrm{d}d/\mathrm{d}w =$$

$$= -sgn\left\{\underbrace{\left[U''(1-H)+U'\right]}_{+}\underbrace{\left(\underset{-}{a_{12}}\underset{?}{a_{23}}-\underset{+}{a_{13}}\underset{-}{a_{22}}\right)}_{?}+\underbrace{U''(1-H)}_{-}\underbrace{\left(\underset{-}{a_{21}}\underset{+}{a_{13}}-\underset{-}{a_{11}}\ \underset{?}{a_{23}}\right)}_{?}\right\} \tag{A25}$$

Therefore the sign of $\mathrm{d}d/\mathrm{d}w$ and consequently of $\mathrm{d}h/\mathrm{d}w$ is ambiguous.

VI Appendix B - Equivalence of budget constraints

To see that budget constraint (34) is equivalent to budget constraint (35), let us first write the government budget if E is paid for by the government itself and then if E is paid by the parent:

$$\tau\int_0^{\overline{w}} w\,(1-H(w))\,F(w)\,\mathrm{d}w+\tau\int_0^{\overline{w}} x\,(E+e,D+d)\,F(w)\,\mathrm{d}w-E=\widetilde{T} \tag{B1}$$

$$\tau\int_0^{\overline{w}} w\,(1-H(w))\,F(w)\,\mathrm{d}w+\tau\int_0^{\overline{w}} x\,(E+e,D+d)\,F(w)\,\mathrm{d}w=T. \tag{B2}$$

To check the equivalence, integrate (34) to yield

$$\int_0^{\overline{w}}(C+pz+e)\,F(w)\,\mathrm{d}w-(1-\tau)\int_0^{\overline{w}} w\,(1-H)\,F(w)\,\mathrm{d}w=\widetilde{T}, \tag{B3}$$

and substitute the revenue constraint (B1); then integrate (35) to yield

$$\int_0^{\overline{w}}(C+pz+e)\,F(w)\,\mathrm{d}w+E-(1-\tau)\int_0^{\overline{w}} w\,(1-H)\,F(w)\,\mathrm{d}w=T, \tag{B4}$$

and substitute (B2) (recall that the agents have unit mass). It is immediate to see that the resource constraints computed using the two procedures coincide:

$$\int(C+pz+e)\,F(w)\,\mathrm{d}w+E+\tau\int_0^{\overline{w}} x\,(E+e,D+d)\,F(w)\,\mathrm{d}w=$$
$$=\int_0^{\overline{w}} w\,(1-H)\,F(w)\,\mathrm{d}w. \tag{B5}$$

Appendix C - Comparative statics under compulsory education

High wagers

Having no need to employ their time in home production, the high wagers set $H=h=0$: all the parent's time goes into working and all the kid's time goes into education. The constraint that $D = 1$ is of no consequence because that is what the parents would have chosen anyway. On the contrary, E is an actual constraint ($e = 0$). Then, high-wagers choose z to maximise

$$\left[U\left((1-\tau)\,w + T - pz - E\right) + F\left(z\right)\right] + \left[u\left((1-\tau)\,x\left(E,1\right)\right) + f\left(z\right)\right]. \tag{C1}$$

The FOC is:

$$-U'p + F' + f' = 0, \tag{C2}$$

and it follows that

$$\frac{\partial z}{\partial w} = -\frac{-U''p\left(1-\tau\right)}{U''p^2 + F'' + f''} > 0;\ \frac{\partial z}{\partial \tau} = -\frac{U''wp}{U''p^2 + F'' + f''} < 0; \tag{C3}$$

$$\frac{\partial z}{\partial T} = -\frac{-U''p}{U''p^2 + \alpha F'' + f''} > 0;\ \frac{\partial z}{\partial E} = -\frac{U''p}{U''p^2 + F'' + f''} < 0. \tag{C4}$$

Low wagers

Low-wagers are constrained by both E and D ($d = 0$, $e = 0$). Consequently, they only choose H to maximise

$$\left[U\left((1-\tau)\,w + T - E - (1-\tau)\,wH\right) + F\left(y\left(H,0\right)\right)\right] + \left[u\left((1-\tau)\,x\left(E,1\right)\right) + f\left(y(H,0\right)\right]. \tag{C5}$$

The FOC is

$$-\left(1-\tau\right)U'w + \left(F' + f'\right)y_H = 0. \tag{C6}$$

Hence, since

$$-\frac{U''}{U'} < \frac{1}{\left(1-\tau\right)w(1-H)}, \tag{C7}$$

from (A20), we have that

$$\begin{aligned}\frac{\partial H}{\partial w} &= -\frac{-\left(1-\tau\right)U' - \left(1-\tau\right)^2 w\left(1-H\right)U''}{U''\left(1-\tau\right)^2 w^2 + \left(F'+f'\right)y_{HH} + \left(F''+f''\right)\left(y_H\right)^2} = \\ &-\frac{-\left(1-\tau\right)\left(U' + \left(1-\tau\right)w\left(1-H\right)U''\right)}{U''\left(1-\tau\right)^2 w^2 + \left(F'+f'\right)y_{HH} + \left(F''+f''\right)\left(y_H\right)^2} < 0;\end{aligned} \tag{C8}$$

$$\frac{\partial H}{\partial \tau} = -\frac{w\left[U' + \left(1-\tau\right)w\left(1-H\right)U''\right]}{U''\left(1-\tau\right)^2 w^2 + \left(F'+f'\right)y_{HH} + \left(F''+f''\right)\left(y_H\right)^2} > 0; \tag{C9}$$

$$\frac{\partial H}{\partial T} = -\frac{-\left(1-\tau\right)wU''}{U''\left(1-\tau\right)^2 w^2 + \left(F'+f'\right)y_{HH} + \left(F''+f''\right)\left(y_H\right)^2} > 0; \tag{C10}$$

$$\frac{\partial H}{\partial E} = -\frac{\alpha\left(1-\tau\right)wU''}{U''\left(1-\tau\right)^2 w^2 + \left(F'+f'\right)y_{HH} + \left(F''+f''\right)\left(y_H\right)^2} < 0. \tag{C11}$$

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