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College Expansion and Curriculum Choice

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CESIFO WORKING PAPER NO. 5299
CATEGORY 5: ECONOMICS OF EDUCATION
APRIL 2015

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ISSN 2364-1428

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Abstract

This paper analyzes the impact of college enrollment expansion on student academic achievements and labor market outcomes in the context of competition among colleges. When public policies promote “access” to college education, colleges adjust their curricula: Less selective public colleges adopt a less demanding curriculum in order to accommodate the influx of less able students. As we argue in the paper, this adjustment benefits low-ability college students at the expense of those of medium ability. At the same time, this reduces the competitive pressure faced by elite colleges, as less selective colleges become a less appealing alternative for the medium ability students. The selective, elite colleges therefore adopt a more demanding curriculum to better serve their most able students, again at the expense of medium ability students. The model offers an explanation to two sets of empirical phenomena: (i) the observed U-shaped earnings growth profile among college-educated workers in the U.S. and (ii) the diverging selectivity trends of American colleges.

JEL-Code: I210, I230, I240, J240, H440.

Keywords: curriculum, postsecondary education, enrollment expansion, income distribution.

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March 2015

We thank Rachana Bhatt, William Blacknell, Kristin Butcher, Stephanie Cellini, Susan Dynarski, David Frisvold, Art Goldsmith, Barry Hirsch, Tilman Klumpp, Tim Sass, Diane Whitmore Schanzenbach, and Itzak Zilcha for helpful comments. All remaining errors are our own.

1 Introduction

During the last several decades, the landscape of postsecondary education in the United States has changed significantly. College education, once a gateway to the elite, has become increasingly accessible to the general public. As shown in Table 1, between 1959 and 2008, enrollment in postsecondary education has increased from 3.64 million to 19.10 million, or 525%. This growth was mainly driven by enrollment in public colleges, which has increased from 2.18 million to 13.97 million (641%). During the same period, enrollment in not-for-profit private colleges has increased from 1.46 million to 3.66 million (251%).¹

Table 1: Enrollment in postsecondary degree-granting institutions (in thousands)

Year	Total	Public	Private		
			All	Not-for-profit	For-profit
1959	3,640	2,181	1,459	n/a	n/a
1969	8,005	5,897	2,108	2,088	20
1979	11,570	9,037	2,533	2,461	71
1989	13,539	10,578	2,961	2,731	229
1999	14,791	11,309	3,482	3,052	430
2008	19,103	13,972	5,131	3,662	1,469

Furthermore, the steady increase in college enrollment far outpaced the growth of population. As shown in Table 2, after controlling for population size, the enrollment rates within given age cohorts have shown similar patterns of multi-fold increases.² Part of the increase in the enrollment rate reflects better “access” to higher education, driven by public policies such as the G.I. bill and the Higher Education Act of 1965.

Another point of departure for this paper is that over the same period the earnings of those who attended college evolved unevenly for different parts of the distribution. We document this by using March Current Population Survey (IPUMS-CPS) data to examine the earnings growth profile for college-educated workers. As detailed in Section 2, we use the repeated cross-sectional data to calculate the growth factors of earnings for

¹Source: National Center for Education Statistics, Digest of Education Statistics 2009, Tables 003 and 189. (<http://nces.ed.gov/programs/digest/d09>. Accessed April 13th, 2011.)

²Source: National Center for Education Statistics, Digest of Education Statistics 2009, Table 007.

Table 2: Enrollment in postsecondary education by age group (in %)

Year	18–19 years old			20–24 years old			25–29 years old	30–34 years old
	Total	In basic edu.	In higher edu.	All	20–21	22–24		
1959	36.8	n/a	n/a	12.7	n/a	n/a	4.9*	2.4*
1969	50.2	n/a	n/a	23.0	34.1	15.4	7.9	4.8
1979	45.0	10.3	34.6	21.7	30.2	15.8	9.6	6.4
1989	56.0	14.4	41.6	27.0	38.5	19.9	9.3	5.7
1999	60.6	16.5	44.1	32.8	45.3	24.5	11.1	6.2
2008	66.0	17.4	48.6	36.9	50.1	28.2	13.2	7.3

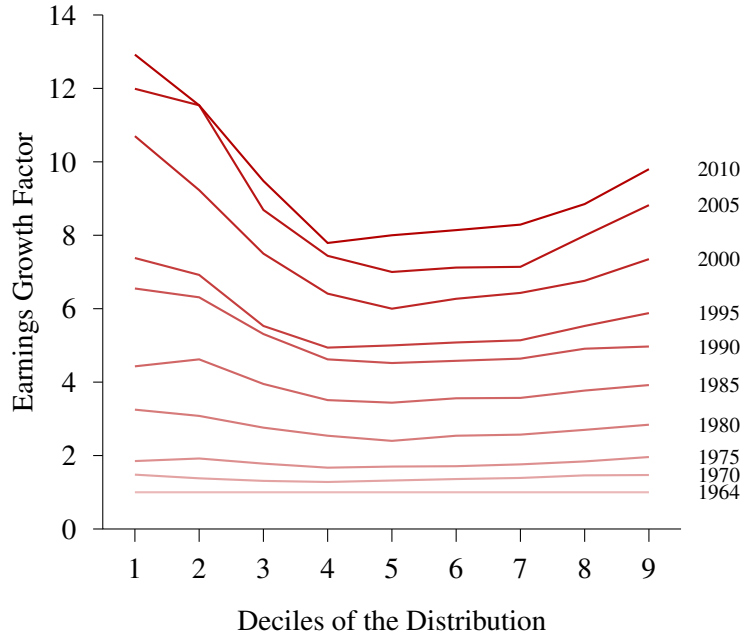
*Data for 1959 unavailable; reported for 1960.

each decile of the annual earnings distributions relative to the corresponding decile in a benchmark year. As can be seen in Figure 1, growth factors in the top and bottom deciles are markedly higher than those in the middle of the distribution. In other words, there is a “sagging middle” in the earnings growth distribution among the college-educated workers. As a stylized pattern, this “sagging middle” phenomenon also shows up if we narrow the sample of college-educated workers to specific age cohorts, or use alternative base years to calculate the earnings growth profile.

While the fact of stagnating mid-section, relative to the ends, has been observed in the dynamics of the overall wage distribution, and theories (most notably, the “routinization” hypothesis: see, e.g., Autor et al., 2008, Acemoglu and Autor, 2011) have been advanced to explain it, the “sagging middle” phenomenon in the dynamics of wage distribution within the group of college-educated workers has not been, to our knowledge, specifically addressed, let alone explained by the existing literature. This paper develops a model where colleges respond to the demand for expanded access by strategically adjusting their curricula, whose impact on students’ college outcomes offers an explanation to the observed U-shaped earnings growth profile among college-educated workers.

In our model, education technology of a college, which converts a student’s ability and learning effort into human capital, is characterized by a discretionary level of curriculum. It is given by curricular standard, a threshold for student abilities (which can be alternatively interpreted as levels of pre-college human capital attainment) min-

Figure 1: Earnings growth profile for college-educated workers, 1964 benchmark



initially required for students to derive any educational benefit in college, such that students with abilities below or slightly above the threshold will have none or very little benefit, respectively. We further posit that the choice of curricular standard also affects, positively, the marginal educational gains, or the “progress rate” term of the education technology, which represents the marginal product of effort and ability for qualified students in terms of their human capital outcomes. Thus, each curriculum represents a distinct education production technology, with two interrelated parameters capturing the curriculum-specific threshold and the progress rate. Lower ability students are better served by a less challenging curriculum (i.e., the one with lower threshold requirement and slower progress rate). Higher ability students are better served by a more challenging curriculum (with higher threshold requirement and faster progress rate). Thus, each curriculum has a comparative advantage among certain segments of student population, and different curricula across colleges can be viewed as horizontally differentiated product offerings.

In our model, higher education is provided by two types of colleges. One prioritizes quality of the graduating students, specifically their human capital outcomes, and is not directly subject to growing pressures to expand access to higher education. It can be thought of as an elite college. The other college is less selective, and while it also cares

about human capital of its graduates, has their quantity as an additional distinct priority. A straightforward interpretation of the latter that we follow for simplicity is that it is a result of political pressure to ensure greater access to higher education, which can be expressed directly or through financial incentives by a government.³

As a result, in our model, the two college types split the population of students such that the less selective public college serves the lower segment of the ability distribution of college-bound population and is compelled to respond to the public policy to let more students in, while the elite college attracts relatively higher ability students. We show that the public college will respond to policies aimed at increased access to higher education by adopting a less demanding curriculum, i.e., one with a lower threshold requirement which also implies a lower marginal return to effort and ability to accommodate the influx of lower-ability students. For example, the downward adjustment in curriculum could manifest itself in more remedial course work offered, fewer challenging topics, and a slower pace of learning in general. It benefits lower ability students, who would otherwise struggle to keep up, at the expense of medium ability students who are ready to learn but are not sufficiently challenged. As the less selective public college adjusts its curriculum downward, we show that the elite college finds it optimal to respond by elevating its curriculum due to the reduced pressure to compete with the public college for medium ability students: “watered down” curriculum of the public college makes it a less appealing alternative for those students, which implies that the elite college can “beef up” its curriculum to better serve its higher ability students without risking losing some of its less able students to the competitor. Thus this response benefits high-ability students, again at the expense of their medium ability peers. Although perfect sorting of students by ability across colleges with respect to their selectivity rankings does not obtain in reality since location and cost factors vary among students of equal ability, a substantial rise in such sorting among American colleges has been observed over the last four decades (Hoxby, 2009) whereby “selective” colleges have been shown to have

³This interpretation would well fit the case of less selective public colleges most subject to such government policies. However, there are of course other realistic interpretations of this assumption, equally suitable for motivating our model. A bias of less selective colleges, public or private, in favor of quantity of their students has much to do with the colleges’ increased reliance on tuition revenues (which are not, however, explicitly featured in our model). Indeed, the public policies to expand access to higher education are often expressed in the U.S. through tuition subsidies, either through direct appropriation for public colleges, or through financial aid to students. On the other hand, the business model of elite private as well as some top public colleges (whose lucid formalization is offered by Hoxby 2012) is based in part on operating a private endowment which allows the college to balance its budgets intertemporally while banking on future contributions by graduates commensurate with their career earnings, whose expected levels can be deemed proportionate to the attained human capital.

become more selective, and vice versa. Our model’s prediction of increasing college selectivity is consistent with and offers an explanation for this phenomenon.

The endogenous curriculum adjustments at both types of colleges and their resulting enrollment expansion have a non-uniform impact on students’ human capital outcomes according to our model: It benefits the lower and higher segments of the ability distribution of college students, but not the ones in the middle. Translating this effect into labor market outcomes for college-educated workers, this model offers an explanation for the observed U-shaped earnings growth profile during the period of enrollment expansion. This explanation complements those offered in the existing literature based on the demand side of the labor market, particularly in response to technological changes.

The remainder of the paper is organized as follows. Section 2 presents more detailed evidence of the “sagging middle” phenomenon in the earnings growth profile for college-educated workers. Section 3 relates this paper to the existing literature on education and labor market outcomes. Section 4 develops a theoretical model of college education technologies characterized by college-specific curricula, and derives students’ optimal college choices given the curricula of the colleges. Section 5 endogenizes the colleges’ curriculum choice strategies and defines their Nash equilibrium. Section 6 contains our main comparative statics results: they characterize how equilibrium college curricula and the economy’s human capital distribution respond to a policy of increased college access. Section 7 concludes. All proofs are in the Appendix.

2 U.S. Earnings Growth Profile, 1964–2010

2.1 Data

Figure 1 presented in the previous section is based on the March Current Population Survey (IPUMS-CPS), a household survey conducted jointly by the U.S. Census Bureau and the Bureau of Labor Statistics and covering the period from 1962 to 2010. We use the information on respondents’ age, highest educational level attained, and annual wage income for the previous year. The education variable is missing for the year 1963, so we use data from 1964 onward. Table 1 in the Introduction shows that this period featured drastic enrollment expansion in U.S. higher educations, both in absolute terms and as a fraction of the college-age population.

With respect to the educational outcome, we categorize as “college-educated” all

workers with at least some college education, ranging from workers with one or two years of college to those with graduate or professional degrees. In contrast, non-college-educated workers are defined as those with at most a high school diploma. With respect to the labor market outcome, we focus on workers’ earnings—namely, their annual wage incomes—instead of their weekly or hourly wage rates. Besides the wage rate, a worker’s earnings may also depend on factors such as the nature of the job (part-time or full-time), unemployment risks (likelihood and duration), compensation schemes (wage versus bonus), etc. These factors are known to be related to a worker’s educational attainment and human capital level, so the earnings variable provides a broader measurement of a worker’s labor market outcome. Our sample includes all workers whose annual earnings are not missing or equal zero.

We are interested in documenting the differences in the dynamics at the high and the low end of the earnings distribution of college-educated workers as compared to that in its middle section. To do so, we use the earnings distribution in the year 1964 as a benchmark. For each n -th, $n \in \{1, 2, \dots, 9\}$, decile of the distribution, we find the nominal earnings level E_{1964}^n . Similarly, we find the nominal earnings for all deciles of the earnings distribution E_t^n in year $t > 1964$. We then calculate the growth factor at each decile relative to the benchmark in 1964, $g_t^n = \frac{E_t^n}{E_{1964}^n}$. Note that we use the nominal earnings as recorded in the data; accounting for inflation would have proportionate effects at all the deciles and therefore would not change the shape of the curves depicting relative changes in the distribution of earnings.

2.2 Empirical patterns

Although our focus is on the relative differences in the earnings growth factors across deciles within the group of college-educated workers, we start with a look at the entire population to replicate what has been documented in the literature. Figure 2A shows the earnings growth profile for all workers, regardless of age or education attainment. Similarly to what has been documented in the literature (e.g., Acemoglu and Autor 2011), Figure 2A exhibits the “polarization” phenomenon, i.e., the fact that earnings at both ends of the distributions grow faster than those in the middle, producing U-shaped graphs of the earnings growth profiles.⁴

⁴See Figures 1 and 11 in Autor et al. (2008) and Figure 9 in Acemoglu and Autor (2011). Unlike these authors, who do employ similar percentile-wise growth measures but only look at full-time, full-year workers, we include all workers with positive earnings. This may explain why we obtain a relatively steeper declining segment of the “U-shape” and a somewhat earlier onset of this phenomenon: early

Figure 2: Earnings growth profile, all age groups, 1964 benchmark

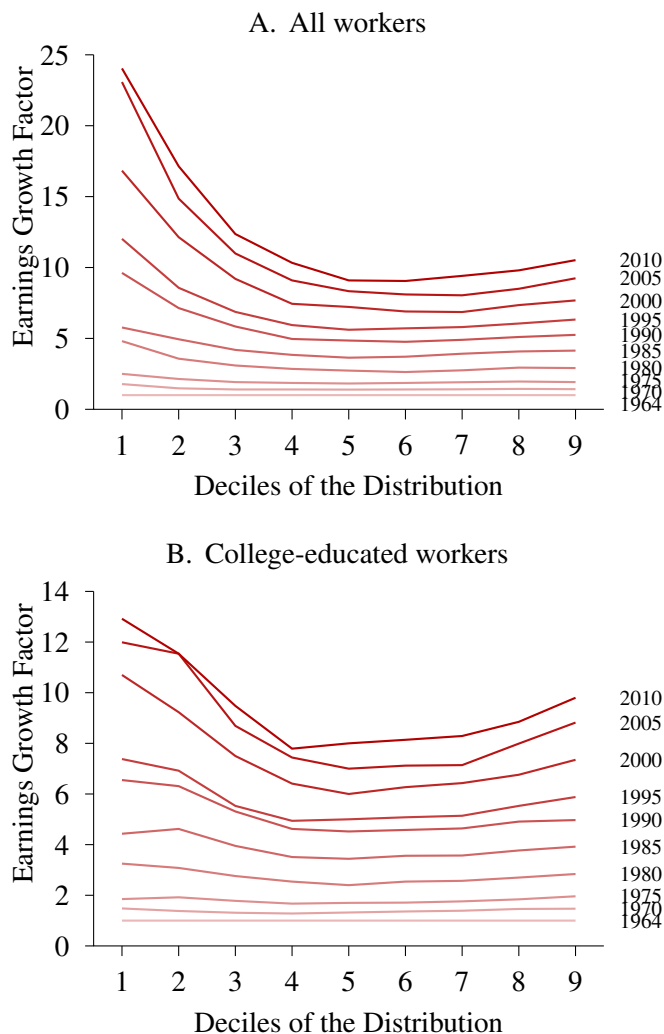


Figure 2B, which replicates the graph given by Figure 1 in the Introduction, shows the earnings growth profile we obtain applying the same methodology to the group of college-educated workers only.⁵ Remarkably, the sagging middle phenomenon observed in Figure 2A for the entire working population persists after excluding workers with at most high school education.⁶

1980's as opposed to mid- to late 1980's observed in the aforementioned literature which focuses on wage rate dynamics of full-time workers.

⁵As a reflection of the dramatic rise in postsecondary enrollment over the period 1964 to 2010, the number of workers with at least some college education in IPUMS-CPS survey samples that we use here increased from 9,143 out of a total 54,539 (16.8%) in 1964, to 82,369 of 159,609 (51.6%) in 2010.

⁶One must of course recognize compositional changes in the distributions compared across the years.

Next, we restrict our sample to specific age-cohorts of the college-educated workers. Figures 3A and 3B represent, respectively, the cohorts of 30 to 34 year olds and 35 to 39 year olds. The U-shaped earning growth profile persists in both age cohorts, which shows that the sagging middle phenomenon is not an artifact of major changes in the age composition of the workers.

Lastly, we use 1980 instead of 1964 as the benchmark to examine the robustness of the sagging middle phenomenon in the later part of the period. Figure 4 reproduces the earnings growth profiles for the two aforementioned age cohorts of college-educated workers relative to the 1980 benchmark. Again, the sagging middle phenomenon persists.

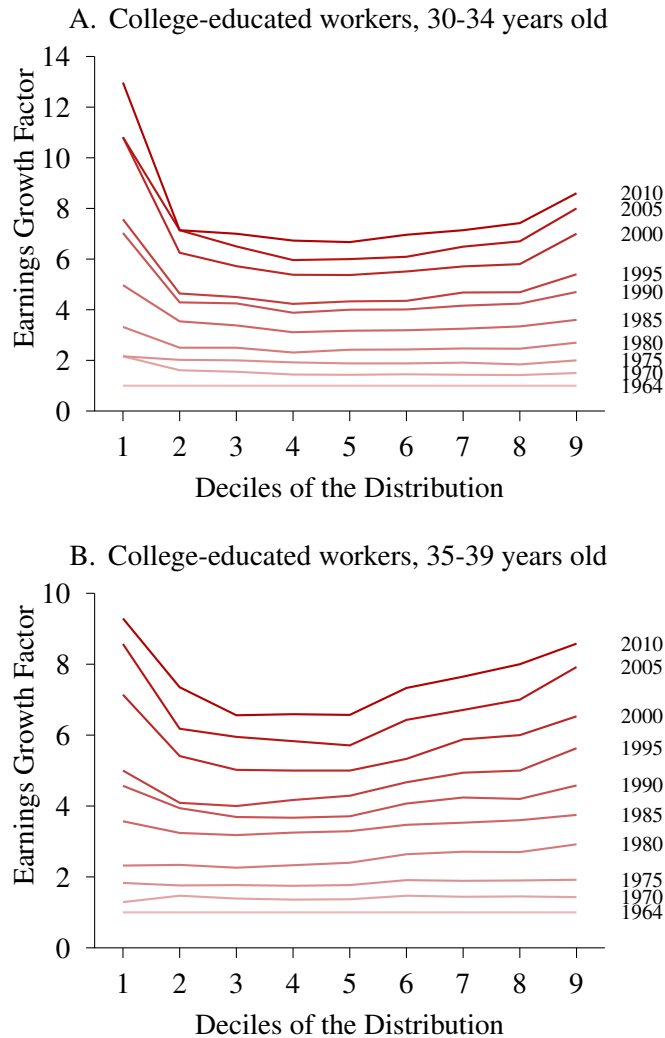
The goal of our paper is to develop a theoretical model that offers an explanation of this observed U-shape in the earnings dynamics among the college-educated workers based on the supply side of the labor market, specifically, the one based on the evolution of the distribution of such workers' human capital attainment.

3 Literature Review

An extensive literature links students' academic achievements to their labor market outcomes. The main focus of this literature is on the college premium, i.e., the wage differential between the groups of college educated workers (with an adjustment for workers with "some college" education) and those with at most high school education. Changes in the college premium over the recent decades have been linked, in the literature focusing on the demand side of labor market, to skill-biased technological improvements. Among others, Katz and Murphy (1992), Autor et al. (1998), and Autor et al. (2008) show that such technological changes account for several salient changes in the U.S. wage distribution over time. Davis (1992), Katz et al. (1995), Murphy et al. (1998), Card and Lemieux (2001), and Atkinson (2008) demonstrate that this explanation is consistent with cross-country differences among developed economies. An extensive

While these issues are present in comparing the distributions of all workers, they are even more pronounced when it comes to the distributions of those with (at least some) college education, as this group has been expanding as a share of the population during the period under consideration. Thus n -th percentiles in the earnings measured in different years are likely to represent different percentiles in the ability distribution at large. However, our results in obtaining the U-shaped curves in the earnings growth profiles robustly persist as one looks at more recent evolutions involving smaller compositional changes. Furthermore, accounting for the compositional change would likely only further steepen the declining portion of the U-curve, which is of particular interest for our purposes.

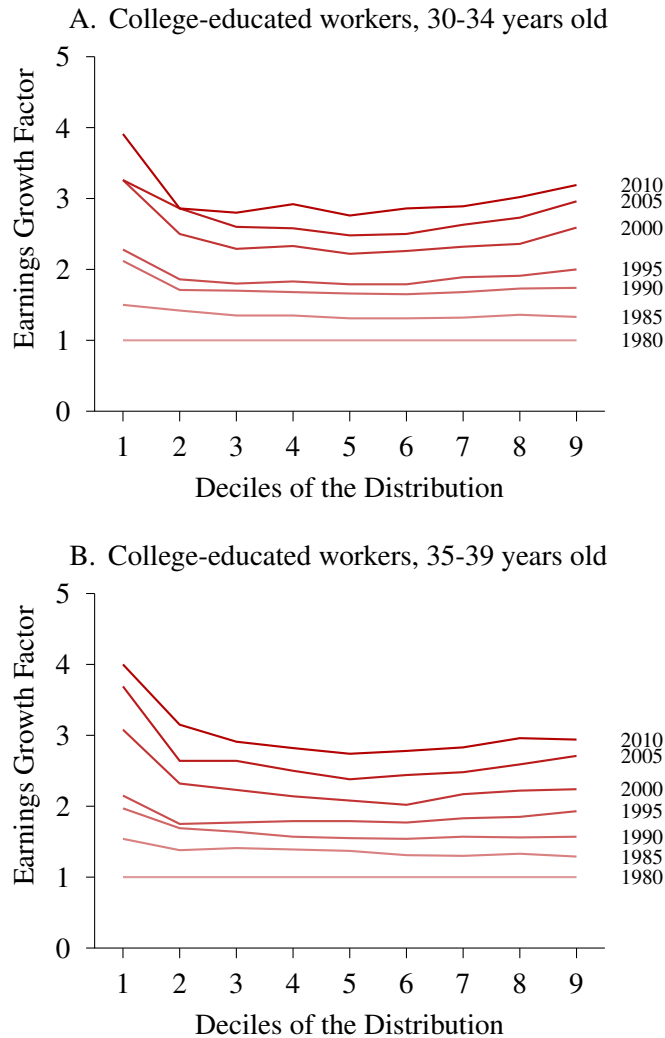
Figure 3: Earnings growth profile by age cohort, 1964 benchmark



survey of this literature is provided by Acemoglu and Autor (2011).

This paper is motivated, in part, by the observed pattern of earnings growth profile within the group of college-educated workers. There is a substantial body of literature analyzing the evolution and recent growth of variance of earnings within this group attributable to its observed or unobserved heterogeneity. Some results point to growth, over recent decades, of this within-group residual inequality due to the variation in learning ability in particular (see, e.g., Taber, 2001, and Lochner and Shin, 2014). Some theoretical models (Galor and Moav, 2000, Gould et al., 2002) helped explain these results within the directed technological change paradigm by arguing that the change

Figure 4: Earnings growth profile by age cohort, 1980 benchmark



is biased toward innate ability, including the ability to adjust to change.⁷ According to this “ability-bias” literature, however, the magnitude of wage growth should exhibit monotone rise along the ability distribution. One might expect, therefore, that it will be the highest in the right tail of the wage distribution of college graduates, and the lowest in its left tail.⁸ However, the data we presented in Section 2 shows a non-monotone,

⁷Laitner (2000) analyzes a model where individual return to investment in education is enhanced by a higher individual ability as well as exogenous unbiased technological change. He notes, however, that the overall variance of income inequality within the higher education group is lowered due to composition effect, as this group expands being joined by less able agents.

⁸Gilpin and Kaganovich (2012) obtain the same kind of effect, which they call “rising talent premium” within the college educated group, as purely a phenomenon of non-linear returns to ability in college

U-shaped pattern of growth across deciles of the earnings distribution of college educated workers. We argue that this pattern cannot be easily reconciled with existing labor demand-side theories of skill premium changes. Our model thus contributes to the literature by illuminating differences in wage growth within the group of college educated workers based on the changes in the distribution of human capital attainments while in college.

Thus this paper's focus is on the heterogeneous human capital gains in college. It builds on a growing literature that emphasizes the hierarchical structure of the education process one of whose important new insights is that the benefits from investing in superior quality of education at a given stage may critically depend, and even be contingent upon sufficient preparation at its prior stage. Driskill and Horowitz (2002), Su (2004, 2006), Blankenau (2005), Blankenau, Cassou, and Ingram (2007), Cunha and Heckman (2007), and Gilpin and Kaganovich (2012) model education as a sequence of stages, where human capital output from lower stages acts as an input in the education technology at higher stages. In particular, the models of Su (2004, 2006) and Gilpin and Kaganovich (2012) feature a curricular threshold at the higher education stage, which sets the minimum pre-college preparation level necessary for making educational gains in college.⁹ This paper takes student outcomes at the basic education stage as given, and focuses instead on curricular choices at different colleges as discussed in the Introduction.

This paper also contributes to the literature on inter-school competition. Rothschild and White (1995) (see also a review by Winston, 1999), Epple and Romano (1998), and Epple et al. (2006) model segmentation of the higher education market based on students' ability to study and to pay. This literature assumes that all schools use the same curriculum. That is, schools may differ in the levels of their educational inputs, including the peer factors, but not in their education production technologies. When peer effects are present, students benefit from attending school with high-ability peers,

education, which stems from the presence of a curricular threshold in the education technology, similar to that used in the present paper. The disproportional benefit derived from college by more able students keeps rising further, if the quality of the overall pre-college preparation grows over time. While this non-linearity of returns to ability is a general consequence of a pre-requisite curricular threshold feature of the education technology, the main distinction of the present paper is the availability of alternative competing curricula featuring different levels of preparation thresholds.

⁹In these papers, the curriculum threshold is intrinsic to the education production technology and not a policy variable. For modeling of academic standards as a policy choice, see Costrell (1994, 1997) and Betts (1998). Eisenkopf and Wohlschlegel (2012) follow this line of work with a duopoly model where two colleges choose their distinct standards while competing for students of heterogeneous ability.

and hence are willing to pay higher tuition fees for such a benefit. Heterogeneity in both student ability and their family income then generates a stratification of school qualities in equilibrium and the sorting of students across schools according to learning ability and the ability to pay.¹⁰ De Fraja and Iossa (2002) add a geographic dimension to intercollegiate duopoly competition where students incur mobility as well as tuition costs. They demonstrate the emergence of two types of equilibria where colleges are either stratified in quality or offer identical quality and serve as unique regional providers, with the level of mobility costs determining which of the outcomes will obtain. In their model, school quality is determined by its admission standard. Furthermore, any student admitted to a school will benefit from a higher standard, as long as he continues to pass it. Thus a common feature of the above literature is that the educational attainment of an individual student, if admitted, increases in the school's quality characteristics.

In our paper, each school adopts an optimal curriculum, its defining quality characteristics, which is thus endogenous and school-specific. Furthermore, our model differs from the above literature in the following important respect: not all students would gain the most human capital from the school, even if admitted, with the highest curricular standard. Instead, students self-segregate by ability based on the best match between it and the curriculum, rather than due to peer effects. Indeed, here, if a lower ability student were to attend a school with predominantly high-ability peers, then instead of benefiting from a peer-group effect, he would find the school's curriculum geared toward them too challenging for him in terms of maximizing his academic achievement. It also follows that not all students choosing to attend a given school will benefit from its curricular standard being raised: some will, in fact, suffer from it. This is an essential, and arguably realistic feature of our model.

4 The Model

In this section, we describe a simple economy consisting of a continuum of students and two colleges, each equipped with an education production technology characterized by a pair of college-specific parameters which we call a *curriculum*. We analyze students'

¹⁰An interesting extension to the line of work on quality differentiation among colleges is offered by Brezis and Hellier (2013) who analyze the long-term intergenerational implications, in terms of social mobility, of the higher education market segmentation into elite and non-elite institutions. Our paper can offer additional insights for this research as it demonstrates that a policy of expanded access to lower ranked schools enhances polarization, as this evinces a strategy of increased selectivity among the higher ranked schools.

decisions about choosing a college, if any, as well as about their effort level when in college, given curricula of the colleges. Section 5 will characterize each college’s choice of a curriculum as an equilibrium outcome in the competition between the colleges.

4.1 Education technology

A *curriculum* of a college’s education technology is defined by two parameters: A curricular standard c , which sets the threshold of prerequisite level of preparation to the course of study in this college, and the progress rate A , which determines students’ learning gains while in college. Thus, under curriculum (A, c) , a student’s (value-added) human capital g is produced according to

$$g(q, e) = \begin{cases} 0 & \text{if } q \leq c, \\ A(q - c)e & \text{if } q > c, \end{cases} \quad (1)$$

where $q \geq 0$ denotes the students pre-college ability, and $e \geq 0$ is his learning effort. Student’s ability and effort are his two inputs in his human capital production. A student will benefit from learning under curriculum (A, c) if and only if his pre-college ability level exceeds the curricular threshold c .¹¹

The curricular threshold c represents the prerequisite knowledge and skills required to study at a college under this curriculum. For example, if a course in intermediate microeconomics requires algebra as prerequisite, a student not possessing such background will not benefit from learning in this course for lack of required skills, even if he attends classes regularly. On the other hand, if a part of the course is devoted to studying the necessary math, we interpret this as lowering the curricular threshold. The same student will derive benefit from the course albeit to a lesser extent than a student with superior prior preparation.¹² The progress rate A represents the rate at which students

¹¹A student’s pre-college ability can be interpreted as his human capital level reached prior to college. This in turn can be modeled as the output of the basic education stage, where inputs may include the student’s innate ability, learning effort, family inputs, as well as school inputs such as funding, teacher quality, class size. More importantly, the production technology at the basic education stage may also be subject to different curricular choices. In this paper, we abstract from intertemporal decisions across different education stages, and treat a student’s pre-college ability as exogenous.

¹²Note that the presence of a threshold c implies that there are increasing marginal returns to a student’s pre-college ability level: For any $q' > q > c$, we have $(q' - c)/(q - c) > q'/q > 1$. In other words, high-ability students benefit disproportionately more from a challenging curriculum, compared to low-ability students, i.e., they enjoy a “talent premium” as also discussed by Gilpin and Kaganovich (2012) whose model features a similar education production function.

can expand their knowledge in the course of study.

It is clear that if there was a curriculum (A, c) with a very large value for A and a very small value for c , it would give large benefits to students of almost any level of preparation. In a world of trade-offs, such a technology is likely not available. Realistically, curricular choices involve a tradeoff such that, larger values for A (greater learning progress) are associated with larger values for c (higher level of prior preparation). That is, the higher a curriculum's progress rate, the fewer students are potentially able to benefit from this curriculum. We will capture this trade-off through the following simplifying assumption:

Assumption 1. Curriculum (A, c) is feasible if $A = \tilde{A}c$, where $\tilde{A} > 0$ is a given constant.

As described further in this section, students choose a college and the learning effort to exert there while taking curricula of the colleges as given. In Section 5, we will model the colleges' choices of their curricula as endogenous outcomes of inter-collegiate competition for students.

4.2 Colleges

There are two colleges, denoted $s \in \{1, 2\}$. The curriculum of college s is (A_s, c_s) . For now, we assume that these curricula are fixed, and ordered as follows:

$$c_1 < c_2.$$

which, according to Assumption 1, implies $A_1 < A_2$.

That is, college 1 has a lower threshold and slower progress rate, and can be thought of as a less selective college. College 2 has a higher threshold and faster progress rate, and can be thought of as a more selective, elite college.¹³

Of course, many factors can affect the progress rate at any given college. For example, more experienced teachers can better motivate students and allow them to learn faster than other teachers. Similarly, a small class size may allow the instructor to provide more individual feedback to students. Colleges may differ in these aspects,

¹³The limitation of the number of colleges to two, while allowing us to address the distinct roles and interactions of less selective and elite institutions, the key matter of interest in this paper, allows us to keep the model analytically tractable. An emerging literature on strategic interaction between colleges in a duopoly context includes, in addition to the already mentioned, papers by Del Rey (2001) and Kemnitz (2007) and focuses, unlike this paper, on questions of university funding.

especially if they receive different levels of funding, which can affect class sizes and instructor quality. If this is the case, then students in one college will learn better and faster than students in the other, even if both have the same curriculum. We, however, do not explicitly include financial aspect of education production, including tuition and other sources of college funding in the model, noting only that these variables tend to correlate with curricular standards of colleges. We assume that all differences between colleges are captured by the differences in the parameters of their curricula.

4.3 Students

There is a continuum of students of measure 1. Pre-college ability level of student i is denoted by q_i . Students are heterogeneous in their pre-college ability levels; specifically, we assume that the q_i are uniformly distributed on $[0, \bar{q}]$ with density $1/\bar{q}$. Students know their own ability and observe the curriculum offered at each college.

A student faces two choices. First, he must decide whether to go to college at all, and if so, to which college. Second, if he decides to attend college s , he must also decide how much learning effort to invest. We assume that there is no capacity constraint in either of the two colleges, and that attending a college is free.¹⁴

A student's objective is to maximize his (value-added) human capital less the disutility of effort, which we assume to be quadratic. If student i chooses to attend college s and exerts effort e_i , then his net benefit is given by

$$\begin{aligned}
 U_i(s, e_i) &= g_s(q_i, e_i) - \frac{\theta}{2} \cdot e_i^2 \\
 &= \begin{cases} -\theta/2 \cdot e_i^2 & \text{if } q_i \leq c_s, \\ A_s(q_i - c_s)e_i - \theta/2 \cdot e_i^2 & \text{if } q_i > c_s, \end{cases} \quad (2)
 \end{aligned}$$

with $\theta > 0$.

Conditional on his choice to attend college s , a student's effort decision is easy to characterize, as it is obviously complementary to the adequacy of his pre-college preparation, i.e., the difference $q_i - c_s$. The optimal effort of student i in college s is easily

¹⁴Introducing (potentially different) tuition payments at the two colleges will not change our main results qualitatively and will only affect the identities of two marginal students. The first marginal student is the one who is indifferent between attending college 1 and college 2 (i.e., the student whose net benefits are equal at both colleges). The second marginal student is indifferent between attending college 1 and staying out of college. If tuition levels are given and fixed, one can show that the model will yield qualitatively similar results obtained here for the free tuition model.

computed given our assumption of a quadratic disutility from effort. It is given by

$$e_i^*(s) = \begin{cases} 0 & \text{if } q_i \leq c_s, \\ \frac{1}{\theta} A_s (q_i - c_s) & \text{if } q_i > c_s. \end{cases} \quad (3)$$

Substituting the effort decision (3) back into the human capital production function (1), we obtain a student's (value-added) human capital after attending college s :

$$g_i^*(s) = \begin{cases} 0 & \text{if } q_i \leq c_s, \\ \frac{1}{\theta} [A_s (q_i - c_s)]^2 & \text{if } q_i > c_s. \end{cases} \quad (4)$$

4.4 Choosing a college

Taking the colleges' curricula as given, a student will choose to attend the college (if at all) which better serves his needs. Using (3)–(4) in the objective function (2), we find that a student's benefit from attending college s net of effort costs is given by

$$U_i^*(s) = U_i(s, e_i^*(s)) = \begin{cases} 0 & \text{if } q_i \leq c_s, \\ \frac{1}{2\theta} [A_s (q_i - c_s)]^2 & \text{if } q_i > c_s. \end{cases} \quad (5)$$

Student i will choose college 2 over college 1 if $U_i^*(2) \geq U_i^*(1)$, unless the choice to stay out of college is superior, which will be the case if $U_i^*(s) \leq 0$ for $s = 1, 2$.

We denote student i 's enrollment decision by $s_i^* \in \{0, 1, 2\}$, where $s_i^* = 0$ means that the student does not attend college. The following result characterizes this choice:

Lemma 1. *Given the curricula (A_1, c_1) and (A_2, c_2) offered at colleges 1 and 2, student i 's optimal enrollment decision is the following:*

$$s_i^* = \begin{cases} 2 & \text{if } q_i \geq c_1 + c_2, \\ 1 & \text{if } q_i \in (c_1, c_1 + c_2), \\ 0 & \text{if } q_i \leq c_1. \end{cases} \quad (6)$$

Thus, higher ability students attend the elite college 2, medium ability students attend the non-elite college 1, and the lowest ability students do not seek higher education, where the cutoffs between these nominal ability categories are determined by the choices of curricular standards by the colleges.

We conclude this section with offering a flavor of the results to come. Note that, because there is a finite number of colleges (two, in our model), curricula cannot be individually designed to best serve each student's needs. Instead, each college enrolls students of different pre-college ability levels pooled together to be educated using the same curriculum. For all but a measure zero of students, this will not be an ideal learning technology. To examine a student's preference ranking over the entire set of feasible curricula, let us call the term $A(q_i - c)$ in (1) the *learning effectiveness* for a student with pre-college ability level q_i under curriculum (A, c) . Because the cost of effort is independent of the curriculum, a student's preference over curricula is ranked by his (student-specific) learning effectiveness. In other words, a higher learning effectiveness directly translates into higher learning effort (3), higher human capital (4), and higher utility level (5). Furthermore, if curriculum (A, c) is feasible, as per Assumption 1, its learning effectiveness for student i can be written as

$$A(q_i - c) = \tilde{A}c(q_i - c). \quad (7)$$

Therefore, the individually optimal curriculum for student i is the one which maximizes expression (7):

$$(\hat{A}_i, \hat{c}_i) = \left(\frac{\tilde{A}q_i}{2}, \frac{q_i}{2} \right).$$

Thus, high-ability students prefer curricula with higher thresholds (curricular standards) and, accordingly, faster progress rates, while low-ability students prefer curricula with lower thresholds and slower progress rates.

Now consider a student whose pre-college ability is such that

$$c_1 < \hat{c}_i = \frac{q_i}{2} < c_2.$$

Relative to this student's individually optimal curriculum, college 1 is too easy and college 2 is too demanding. Of course, if either c_1 or c_2 is not far from \hat{c}_i , student i will be able to study in an "almost ideal" learning environment, and the fact that no college offers exactly i 's ideal curriculum (\hat{A}_i, \hat{c}_i) will not affect this student's learning outcomes much. If, however, c_1 is located substantially below \hat{c}_i , and c_2 is substantially above \hat{c}_i , then student i will find himself "stuck in the middle", i.e., placed in a suboptimal learning environment regardless of which college he chooses.

5 Equilibrium Curricula

In the previous section, the curricula of the two colleges were taken as given and used to derive students' enrollment choices and human capital outcomes. In this section, we focus on the choices of curricula by the colleges. We model the colleges' curricular choices as a Nash equilibrium outcome of a game played between the two schools. To this end, we first need to introduce objective functions of the two colleges.

5.1 Objectives of the colleges

As discussed before, we assume that the elite college 2 is free of any outside pressure or incentives to increase the number of its students *per se*, and is exclusively motivated to maximize the aggregate human capital output of its students.

As long as its curriculum is more challenging than that of college 1, students with ability above $c_1 + c_2$ will enroll in the elite college (see Lemma 1). Therefore the human capital of college 2's student body is given by,

$$H_2 \equiv \int_{c_1+c_2}^{\bar{q}} g_i^*(2) \frac{1}{\bar{q}} dq_i = \int_{c_1+c_2}^{\bar{q}} \frac{1}{\theta} [A_2(q_i - c_2)]^2 \frac{1}{\bar{q}} dq_i, \quad (8)$$

where $A_2 = \tilde{A}_{c_2}$. We assume that college 2 chooses curriculum (A_2, c_2) to maximize (8) while taking college 1's threshold c_1 as given.

We argue that the objective to maximize (8) is a meaningful proxy for the goals of an elite college. Although we do not explicitly model the financial side of a college operation, the argument is based on the fact that the aggregate human capital of a cohort of college graduates H_2 correlates with their aggregate lifetime income which, for a typical selective American college, serves as a basis for expected future alumni contributions. If college 2 were able to charge a full tuition payment from each student, commensurate with this expected lifetime return, the aggregate tuition revenue would be an increasing function of H_2 . If college 2 is unable to charge thus differentiated tuition, or charges none at all as assumed in this model, the college will be arguably motivated by the future contributions of its alumni, which would tend to be proportionate to their human capital value added while in college.¹⁵

¹⁵This understanding, already mentioned in the Introduction, is well aligned with Hoxby's (2012) detailed analysis of the business model of elite private colleges and arguably to some extent applies to more selective public colleges as well.

Unlike college 2, college 1 is less selective and shares the policy maker's concern for "access" to higher education. So, besides the aggregate human capital output of its student body, this less selective college is also compelled to give college accessibility a priority. This assumption reflects, without being explicitly modeled, the realities of a combination of direct pressures and financial incentives from state legislatures as well as the greater budgetary reliance of less selective, public as well as private, colleges on tuition revenues. We therefore assume, that in addition to a concern for the aggregate value added human capital of its graduates (i.e., their aggregate quality), the objective function of college 1 includes an additional term reflecting the quantity of its students.

As is intuitively clear and will be demonstrated below, this objective will compel the public college to choose a curriculum which is less demanding than that of the elite college. Therefore, according to Lemma 1, it implies that the aggregate human capital of students enrolled in the public college is given by

$$H_1 \equiv \int_{c_1}^{c_1+c_2} g_i^*(2) \frac{1}{\bar{q}} dq_i = \int_{c_1}^{c_1+c_2} \frac{1}{\theta} [A_1(q_i - c_1)]^2 \frac{1}{\bar{q}} dq_i,$$

where $A_1 = \tilde{A}c_1$.

The fraction of individuals *not* enrolled in higher education is c_1/\bar{q} . We assume that the motive to lower this fraction, reflecting the goal to expand access to college education, is represented as a distinct component of college 1's objective function. Specifically, for the sake of tractability, we posit the loss function to be $(c_1/\bar{q})^5$.¹⁶ The objective of college 1 is then to maximize

$$H_1 - \gamma \cdot \left(\frac{c_1}{\bar{q}} \right)^5, \quad (9)$$

where $\gamma > 0$ is the weight placed on college accessibility. The less selective college maximizes (9) by choice of a feasible curriculum (A_1, c_1) . In making this decision, it takes college 2's threshold c_2 as given.

The parameter γ is essential to our analysis as a policy parameter capturing the weight policy makers and public universities place on the access to higher education *per se*, and are therefore willing to pursue enrollment expansion even at the expense of the aggregate quality of education. The less selective public college's main tool to

¹⁶This particular form helps in obtaining tractable solutions because, as one can see, the aggregate human capital value H_1 is a 5th-degree polynomial of variables c_1 and c_2 . However, our main results remain qualitatively robust as long as the loss function is sufficiently convex, namely, $(c_1/\bar{q})^n$ for $n \geq 3$.

pursue the policy of increasing access is lowering the curricular threshold c_1 . This gives a larger fraction of the population access to college education. But this comes with two side effects. First, and directly, it implies that the curriculum's other parameter, progress rate A_1 , is lowered as well, which may sacrifice the college's human capital goal. Second, and indirectly, this will trigger a curricular change at college 2 and will thus have an effect on human capital attainment by its students.

5.2 Nash equilibrium

We now examine equilibrium curricular choices by the colleges, given their objectives described above. The public college chooses a feasible curriculum (A_1, c_1) to maximize (9), taking (A_2, c_2) as given. Likewise, the elite college chooses a feasible (A_2, c_2) to maximize (8), taking (A_1, c_1) as given. Therefore curricular choices constitute a (pure strategy) Nash equilibrium in a game played by the two colleges.

Since for each college s , A_s depends on c_s via Assumption 1 (i.e., $A_s = \tilde{A}_{c_s}$), it is sufficient to treat each college's problem as choosing the optimal threshold $c_s \in [0, \bar{q}]$. The first-order condition with respect to c_2 for the elite college 2 is

$$\frac{\tilde{A}^2}{\theta \bar{q}} c_2 \left[-\frac{2}{3} c_1^3 - \frac{5}{3} c_2^3 + 4c_2^2 \bar{q} - 3c_2 \bar{q}^2 + \frac{2}{3} \bar{q}^3 \right] = 0,$$

which can be reduced to

$$(2\bar{q} - 5c_2)(\bar{q} - c_2)^2 = 2c_1^3. \quad (10)$$

Denote by $c_2(c_1)$ the value for c_2 solving (10), given c_1 . Since $c_1 \in (0, \bar{q})$, the solution must satisfy $c_2(c_1) \in (0, 2\bar{q}/5)$. Within this range, $c_2(\cdot)$ will be a differentiable function with $c_2'(c_1) < 0$. In other words, if college 1 adopts a less challenging curriculum, college 2's best response is to adopt a more challenging one.

Similarly, the first order condition for the less selective college 1 is

$$\frac{2\tilde{A}^2}{3\theta \bar{q}} c_1 c_2^3 - \frac{5\gamma}{\bar{q}^5} c_1^4 = 0.$$

This condition can be solved explicitly for the best response function of college 1:

$$c_1(c_2) = \left(\frac{2\tilde{A}^2 \bar{q}^4}{15\gamma\theta} \right)^{1/3} c_2. \quad (11)$$

Note that $c_1'(c_2) > 0$. In other words, if college 2 adopts a more challenging curriculum, college 1's best response is to adopt a more challenging curriculum as well. For both colleges, it is straightforward to verify that the second-order conditions of maximum do hold.

Figure 5 illustrates the best response curve for the elite college $c_2(c_1)$ and that for the less selective college $c_1(c_2)$. An *equilibrium* is then a pair of curricular thresholds (c_1^*, c_2^*) which are mutual best responses—that is, $c_1^* = c_1(c_2^*)$ and $c_2^* = c_2(c_1^*)$. The equilibrium is stable, if a small perturbation of the equilibrium results in best response dynamics converging back to it. This will be the case when $c_1'(c_2^*)c_2'(c_1^*) > -1$. The following result provides a sufficient condition for existence and uniqueness of a stable equilibrium in curriculum choices.

Lemma 2. *When γ is sufficiently large, namely, $\gamma > \frac{2\tilde{A}^2\bar{q}^4}{15\theta}$, a unique, stable Nash equilibrium in curricular choices exists.*

Lemma 2 relates the less selective school's concern for college accessibility γ to the education technology parameter \tilde{A} , the pre-college human capital endowment parameter \bar{q} , and the preference parameter θ . The Lemma guarantees equilibrium existence, if the public college cares sufficiently about accessibility of higher education.

6 The Effects of Increased Access

Our main parameter of interest is γ , reflecting the weight placed by the policy maker on access to higher education. We now explore how changes in this parameter impact the equilibrium outcomes of our model, when colleges choose their curricula optimally, and present our main results.

Our first result describes the changes in equilibrium curricula and enrollments, as γ changes.

Proposition 3. *In the Nash equilibrium, the following holds:*

$$\frac{dc_1^*}{d\gamma} < 0, \quad \frac{dc_2^*}{d\gamma} > 0, \quad \frac{d(c_1^* + c_2^*)}{d\gamma} < 0.$$

That is, when policy makers place more weight on access to college education, the less selective college adopts an even less demanding curriculum and the elite college makes its curriculum even more demanding. Furthermore, enrollment in both colleges increases.

When policy makers attach a higher weight to access to college education, the less selective college lowers its curriculum threshold further to pursue enrollment expansion. Remarkably, the elite college finds it optimal to elevate its curricular threshold, i.e., to make its curriculum even more demanding. The reason for this is that the easing of the less selective college's curriculum lessens the competitive pressure on the elite college to appeal to the medium ability students. In other words, the downward adjustment of curriculum at college 1 makes college 1 a less appealing option for students of intermediate ability, who most prefer a curriculum of intermediate difficulty. But this implies that college 2 can afford to raise its curricular threshold without risking to lose its medium ability students. Note that, by increasing its threshold c_2 , the elite college reduces the human capital gain of its medium ability students for whom this college remains a superior choice anyway. However, this loss is more than offset by the increase in human capital of the elite college's high-ability students, due to the increased marginal returns to ability (see Footnote 12). The pursuit of enrollment expansion therefore causes the less selective college to make its curriculum less demanding, which then compels the elite college to further elevates its curriculum.

Figure 5 illustrates these adjustments. College 2's best response curve (implicitly defined by (10)) is downward sloping and college 1's best response curve (given by (11)) is upward sloping. As γ increases, college 1 best response curve shifts downward (the dotted straight line in the graph is the best response when $\gamma = 0$). College 2 best response curve, on the other hand, is independent of γ and stays fixed. Thus, as γ increases, the equilibrium slides down along college 2's best response curve. This implies that c_1^* decreases and c_2^* increases, as stated in Proposition 3.

Proposition 2 also states that $c_1^* + c_2^*$ decreases, as γ increases. Recall from Lemma 1 that, given a pair of curricula with $c_2 > c_1$, students with ability between c_1 and $c_1 + c_2$ enroll in college 1 and students with ability between $c_1 + c_2$ and \bar{q} enroll in college 2. Because the ability distribution is uniform, it follows that student enrollment in both the less selective *and* the elite college increases when γ increases. This happens despite the fact that the elite college becomes harder to get into (i.e., c_2^* increases). The reason is that students of relatively high ability who were enrolled in the less selective college will switch to the elite college when the former's curriculum becomes less demanding.

Let us now examine how increased access to higher education affects the welfare and human capital outcomes of students. Consider an increase in γ , and let $(c_1^{\text{old}}, c_2^{\text{old}})$ and $(c_1^{\text{new}}, c_2^{\text{new}})$ denote the equilibrium curricula before and after the change in γ . By

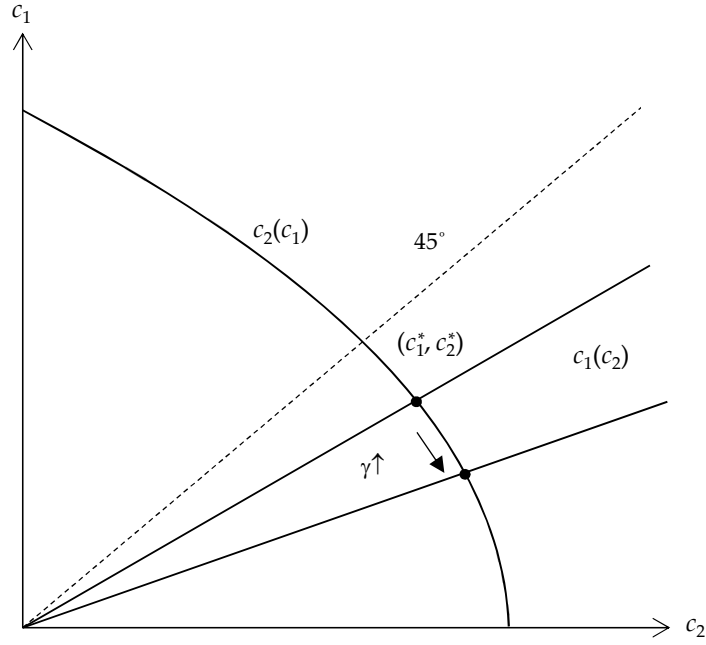


Figure 5: Best responses and Nash equilibrium

Proposition 3,

$$c_1^{\text{new}} < c_1^{\text{old}} < c_2^{\text{old}} < c_2^{\text{new}}.$$

That is, the “wedge” between less selective and elite curricular thresholds widens. Our next result describes how students’ welfare and human capital are affected by this.

Proposition 4. *Suppose γ increases, and curricular thresholds adjust from $c_1^{\text{old}}, c_2^{\text{old}}$ to $c_1^{\text{new}}, c_2^{\text{new}}$ respectively. Then the distribution of payoffs and the human capital distribution are affected as follows:*

- (a) *Students with ability levels $q_i \in (c_1^{\text{new}}, c_1^{\text{old}} + c_1^{\text{new}})$ and $q_i \in (c_2^{\text{old}} + c_2^{\text{new}}, \bar{q}]$ are made better off, and accumulate more human capital.*
- (b) *Students with ability levels $q_i \in (c_1^{\text{old}} + c_1^{\text{new}}, c_2^{\text{old}} + c_2^{\text{new}})$ are made worse off, and accumulate less human capital.*
- (c) *Students with ability levels $q_i \in [0, c_1^{\text{new}}]$ do not attend college before or after the change in γ . They are equally well off, and accumulate the same amounts of human capital, before and after the change.*

Thus, the changes in equilibrium college curricula affect students differently, depending on their initial ability. Proposition 4 characterizes the distributional impacts of curricular adjustments. If γ increases, medium ability students lose out, while high ability and low ability college enrollees are made better off. The intuition for this differential impact was already discussed in Section 4.4. As the gap between the less selective college's curriculum (A_1, c_1) and the elite curriculum (A_2, c_2) widens, the elite curriculum moves closer to the ideal curriculum for high-ability students. Similarly, the less selective college's curriculum moves closer to the ideal curriculum for less able students. Both curricula, however, move away from the ideal curriculum for medium ability students.

Thus, with endogenous curricular choices, a shift in the policy maker's preference toward greater access has a non-monotone impact on students belonging to different parts of the ability distribution. It is important to emphasize that the presence of effective strategic interaction between colleges is necessary for this non-monotonicity. If the elite college did not have an incentive to adjust its curriculum in response to the public college's move, enrollment expansion at the less selective college would increase the human capital of low ability students and negatively affect medium ability students (some of whom would switch to the elite college as a result) without affecting the high-ability students who are already enrolled in the elite college. It is the elite college's strategic "beefing up" of its curriculum—in response to the less selective college's "watering down" of its curriculum—that helps the most able students. But now, the elite curriculum adjustment hurts the medium ability students for the second time, making the elite education a less adequate match for them as well.

7 Concluding Remarks

This paper is a first step toward understanding endogenous curricular choices of colleges and their effects on the distribution of human capital attainment by heterogeneous students. Our model predicts (consistent with some evidence of diverging selectivity of American colleges) a downward adjustment in the curricula of already less selective colleges, and an upward adjustment of the curricula in the more selective ones, when policy makers place increased weight on access to college education relative to the concerns for the aggregate quality of human capital being produced. The distributional impact of these changes is non-monotone: While low and high-ability students gain in terms

of human capital acquisition and welfare, medium ability students lose out. The model thus offers a novel explanation for the observed U-shape earning growth profile among college-educated workers in the macro data and the diverging selectivity patterns among the American colleges, while suggesting a connection between the two phenomena. To be sure, the model we use is highly stylized. Among its limitations are the restriction to the case of two colleges and the exclusion of an explicit account for the financial side of college economics, both necessitated by keeping the focus on the key insights while preserving analytical tractability.

The “two-college” assumption might be less restrictive in the world where students’ mobility cost is high in relation to tuition. In this case, it is not unrealistic to assume that in light of high fixed cost of college entry, local choices of more and less selective college options are highly limited. Relative costs of mobility are, however, decreasing (Hoxby, 2009, refers to this fact as an explanation for increasing selectivity of the elite colleges), which makes the question of a potential extension of our model to the multi-college case important. One might conjecture, in particular, a horizontally differentiated set of colleges which differ in the degrees to which a separate concern for the quantity of students, apart from their aggregate quality, guides their operation. Indeed, there is a range of American colleges whose funding models combine, with differing weights, reliance on current tuition revenues and future alumni contributions. In this horizontally differentiated multi-college situation, specific segments of student population can be better served in terms of a closer match between curricular challenge and student preparation. While an extension of the issues raised in this paper to such situation would not be analytically tractable (particularly given that a pure-strategy Nash equilibrium might not exist), we conjecture that our stylized insights would be sustained under additional assumptions to ensure the existence of the equilibrium. Namely if under policy pressures to increase access to higher education (for example, through increased subsidies to the least selective schools) college-bound population expands, strategic interaction between the colleges is likely to result in the divergence of selectivity patterns, with the less selective schools responding by lowering their curricula further, while a subset of selective ones raising their respective curricular standards.

An extension of our analysis to incorporate education expenditure in the education production technology could help capture school quality aspects beside their curricula, such as teacher quality, class size, and classroom equipment, and so forth. While different students prefer different curricula, all students can benefit from higher levels of

the aforementioned characteristics of quality. Allowing for interactions across different school quality characteristics would possibly permit richer results. For example, under what conditions do changes in curricular thresholds and changes in other aspects of school quality go hand-in-hand? If it is possible for a school to reduce its curricular threshold and learning speed, while improving the other quality variables, the tradeoff between access and human capital production may shift in ways that alter the distributional impact of enrollment expansion. These questions are especially important when examining policies which allocate public funds to achieve accessibility objectives.

Finally, a framework in which schools choose their curricula could also be useful for reexamining the role of peer effects. In the existing literature on peer effects (e.g., Epple and Romano (1998)), students directly benefit from interacting with better qualified peers. When school curricula are endogenous, as proposed here, a new dimension of peer interactions emerges: If a school's curriculum is geared toward the majority of its student body, a less prepared student who attends a school with significantly better qualified peers may indirectly suffer from by being exposed to too challenging a learning environment. The interaction between these two effects suggests another non-monotone relationship: While having slightly better peers has a positive effect on human capital, facing a more challenging curriculum geared toward these better peers can have a detrimental impact on one's learning outcomes. These effects are especially relevant when examining the efficiency and equity of outcomes under different education systems, such as *mixing* (i.e., pooling students of different ability in one class) or *tracking* (i.e., separating students by ability).

Appendix

Proof of Lemma 1. Note that staying out of college is equivalent to learning under curriculum with $c = 0$, $A = 0$. Because the cost of effort is independent of the curriculum under which a student learns, a student of ability q prefers curriculum (A, c) to curriculum (A', c') whenever $A(q - c) > A'(q - c')$. Now suppose $0 < c_1 < c_2$. A student with ability q prefers staying out of college to attending college 1 if $q < c_1$. He prefers college 1 over college 2 if $A_1(q - c_1) > A_2(q - c_2)$. By Assumption 1, $A_1 = \tilde{A}c_1$ and $A_2 = \tilde{A}c_2$; a student of ability q student therefore prefers college 1 over college 2 if $\tilde{A}c_1(q - c_1) > \tilde{A}c_2(q - c_2)$, or $q < c_1 + c_2$. This implies the Lemma's result. \square

Proof of Lemma 2. An equilibrium is a point (c_1^*, c_2^*) such that $0 \leq c_1^* < c_2^* \leq \bar{q}$ and $c_1^* = c_1(c_2^*)$, $c_2^* = c_2(c_1^*)$. Since $c_1' > 0$ and $c_2' < 0$, then if such a point exists it must be unique. To establish existence, first observe from (10) that $c_2(0) = \frac{2}{5}\bar{q}$. Next, from (11) we see that college 1's best response curve is linear in c_2 and starts at the origin. When $\gamma > \frac{2\tilde{A}^2\bar{q}^4}{15\theta}$, $c_1(c_2) < c_2$, and therefore a unique equilibrium exists with $c_1^* < c_2^*$.

Next, we establish stability of the equilibrium. We need to show that $c_1'(c_2^*)c_2'(c_1^*) > -1$. When $\gamma > \frac{2\tilde{A}^2\bar{q}^4}{15\theta}$, $c_1'(c_2) < 1$. Therefore, a sufficient condition for stability is that $c_2'(c_1^*) > -1$. Implicitly differentiating (10) with respect to c_1 we obtain

$$c_2'(c_1) = \frac{-6c_1^2}{9q^2 - 24qc_2 + 15c_2^2},$$

which is negative. Thus, $c_2'(c_1) > -1$ if and only if $6c_1^2 < 9\bar{q}^2 - 24\bar{q}c_2 + 15c_2^2$. Since $c_1^* < c_2^*$, it is sufficient to show that

$$6(c_2^*)^2 < 9\bar{q}^2 - 24\bar{q}c_2^* + 15(c_2^*)^2,$$

which is indeed satisfied given that $c_2^* < \frac{2}{5}\bar{q}$. \square

Proof of Proposition 3. The inequalities $dc_1^*/d\gamma < 0$ and $dc_2^*/d\gamma > 0$ follow immediately from the fact that $c_2(c_1)$ decreases in c_1 and is independent of γ , while $c_1(c_2)$ increases in c_2 and decreases in γ (see Figure 5). Recall from the proof of Lemma 2 that $c_2'(c_1^*) > -1/2$. Thus, $dc_1^*/d\gamma + dc_2^*/d\gamma = (1 + c_2'(c_1^*))dc_1^*/d\gamma < 0$. \square

Proof of Proposition 4. Recall that by Proposition 3, when γ increases,

$$c_1^{\text{new}} < c_1^{\text{old}} < c_2^{\text{old}} < c_2^{\text{new}}, \quad c_1^{\text{new}} + c_2^{\text{new}} < c_1^{\text{old}} + c_2^{\text{old}}.$$

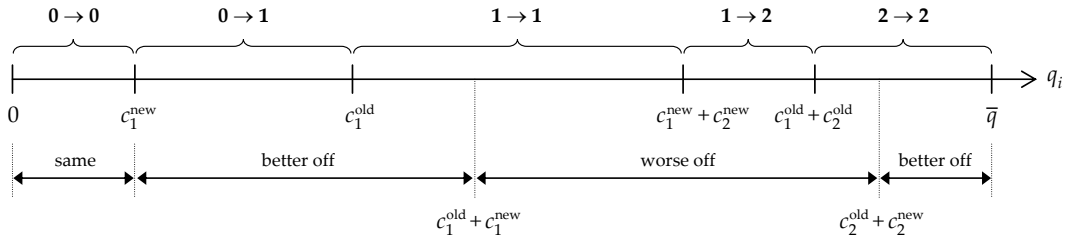
According to Lemma 1 the population can be partitioned into the following five subgroups:

1. Students with $q_i \in [0, c_1^{\text{new}}]$ do not attend college after the policy change.
2. Students with $q_i \in (c_1^{\text{new}}, c_1^{\text{old}}]$ do not attend college before the policy change, but attend college 1 after the change.
3. Students with $q_i \in (c_1^{\text{old}}, c_1^{\text{new}} + c_2^{\text{new}}]$ attend college 1 before and after the change.
4. Students with $q_i \in (c_1^{\text{new}} + c_2^{\text{new}}, c_1^{\text{old}} + c_2^{\text{old}}]$ attend college 1 before the policy change, but switch to college 2 after the change.
5. Students with $q_i \in (c_2^{\text{old}} + c_2^{\text{old}}, \bar{q}]$ attend college 2 before and after the change.

By (4)–(5), a student’s human capital and net-of-effort payoff both increase in his learning effectiveness $A(q_i - c)$ of the curriculum of the college he attends. Furthermore, $A = \tilde{A}c$ by Assumption 1. This implies that a student is better off and prefers curriculum c over $c' > c$, iff

$$\tilde{A}c(q_i - c) > \tilde{A}c'(q_i - c') \iff q_i < c + c'.$$

The following graph illustrates the students’ college choices, human capital output, and net-of-effort payoff before and after the policy change. (Students with ability levels at the boundaries of these intervals are indifferent, hence could be moved to an adjacent group without affecting the result.)



Students in the first subgroup are obviously unaffected by the policy change. Students in the second subgroup gain in higher human capital and payoff levels after the policy change.

A student in the third group (who attends college 1 before and after the policy change) prefers c_1^{new} to c_1^{old} , if $q_i < c_1^{\text{old}} + c_1^{\text{new}}$; otherwise he prefers c_1^{old} to c_1^{new} .

All students in the fourth group (who attend college 1 before, and college 2 after the change) prefer c_1^{old} to c_2^{new} because for these students $q_i < c_1^{\text{old}} + c_2^{\text{old}} < c_1^{\text{old}} + c_2^{\text{new}}$.

Any student in the fifth group (who attends college 2 before, and after the change) prefers c_2^{old} to c_2^{new} if $q_i < c_2^{\text{old}} + c_2^{\text{new}}$; otherwise he prefers c_2^{new} to c_2^{old} .

Putting these observations together, after the policy change, students with $q_i \in (c_1^{\text{new}}, c_1^{\text{old}} + c_1^{\text{new}})$ or $q_i \in (c_2^{\text{old}} + c_2^{\text{new}}, \bar{q}]$ are better off and acquire more human capital, but students with $q_i \in (c_1^{\text{old}} + c_1^{\text{new}}, c_2^{\text{old}} + c_2^{\text{new}})$ are worse off and acquire less human capital.

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