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Abstract

This paper studies equilibrium unemployment in a two-region economy with matching frictions, where workers and jobs are free to move and wages are bargained over. Job-seekers choose between searching locally or searching in both regions. Search-matching externalities are amplified by the latter possibility and by the fact that some workers can simultaneously receive a job offer from each region. The rest of the framework builds upon Moretti (2011). Increasing the matching effectiveness out of the region of residence has an ambiguous impact on unemployment rates. While it reduces the probability of remaining unemployed, it also decreases labor demand because of a lower acceptance rate. We characterize the optimal allocation and conclude that the Hosios condition is not sufficient to restore efficiency. A numerical exercise indicates that the loss in net output is non negligible and rising in the matching effectiveness in the other region.

JEL-Code: J610, J640, R130, R230.

Keywords: matching, non-segmented labor markets, spatial equilibrium, regional economics, unemployment differentials.

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1 Introduction

While an abundant literature in urban economics addresses unemployment issues within cities (see Zenou, 2009, for a detailed coverage), less effort has been devoted to analyze the causes of unemployment at the regional level. “Given the large geographical differences in the prevalence of unemployment observed in the real world, understanding spatial equilibrium when the labor market does not instantly clear would appear to be of primary importance.” (Kline and Moretti, 2013, p. 239). The main purpose of this paper is to contribute to this understanding.

The Internet allows both sides of the labor market to find more easily potential partners, even faraway, thanks to job boards and meta-search engines.¹ Moreover, the recruitment process can now also be conducted online through virtual recruiting tools.² Marinescu and Rathelot (2014) provide evidence that the distance between the job-seeker and the job vacancy exceeds 100 km (63 miles) in about 10% of the online applications on CareerBuilder.com. This suggests that a non negligible share of US job-seekers ramp up their job search by expanding it over long distances.³

While most of the literature dealing with regional unemployment assumes that people need to migrate before they can start searching locally, we relax the assumption of segmented regional labor markets. Our main contribution consists in developing a general equilibrium search-matching framework where job-seekers choose whether they search in their region of residence only or all over the country. To the best of our knowledge, this has not been done yet. In this setting, regions are strongly interdependent and several sources of inefficiency explained later are present. A numerical exercise suggests a non-negligible gap between the efficient and the equilibrium allocations.

As a secondary contribution, our analysis also sheds some light on a puzzle. Expectations that the Internet would improve the functioning of the labor market by reducing search-matching frictions were great (see e.g. Autor, 2001). A decade later, the evidence is mixed. Some microeconomic evaluations find that online job search shortens unemployment duration in the US (Kuhn and Mansour, 2014; Choi, 2011). For graduate students in Italy, Bagues and Labini (2009) conclude that the use of an online platform reduces the probability of unemployment and raises geographical mobility. However, via a difference-in-differences approach, Kroft and Pope (2014) find no evidence that the rapid expansion of a major online job board (during the years 2005-2007) has affected city-level unemployment rates in the US. So, the reasons why improvements at the individual level disappear at a more aggregate level need to be understood. This paper proposes an explanation in a spatial economy.

¹ In 2010, 25% of the interviewed Americans who use the Internet declared to do so to find a position (U.S. Census Bureau, 2012, Survey of Income and Program Participation, 2008 Panel). In Europe, in 2005, among the unemployed workers, 25% used the Internet to search for a job. This share has increased to 50% in 2013 (Eurostat, 2014, see http://ec.europa.eu/eurostat/data/database?node_code=isoc_ci_ac_i).

²See e.g. <http://variousthingslive.com/virtual-open-house/> and the links on <http://hiring.monster.com/hr/hr-best-practices/recruiting-hiring-advice/acquiring-job-candidates/virtual-recruitment-strategies.aspx>.

³The evidence is more mixed for the UK (see Manning and Petrongolo, 2011).

We build upon the synthesis of Moretti (2011) who develops a two-region static spatial equilibrium model *à la* Rosen (1979)-Roback (1982). Contrary to these authors, Moretti (2011) assumes that the supply of labor is not perfectly elastic. This property is obtained by assuming that economic agents have heterogeneous idiosyncratic preferences for regions. The aim of Moretti (2011) is to analyze how local shocks propagate in the long run to the rest of the economy, with a focus on the labor market. He discusses the case where agents have different skills, while we keep labor homogeneous. However, regional unemployment disparities are not studied by Moretti. We introduce search-matching frictions and wage bargaining within this framework (Pissarides, 2000) but we abstract from the housing market.

Contrary to most of the search and matching literature, the spatial heterogeneity is explicit in our framework. In each region, imperfect information and lack of coordination among agents create frictions summarized by a constant-returns-to-scale regional-specific matching function in which the number of job-seekers is a weighted sum of the residents and of the non-residents who decide to search all over the country, both numbers being endogenous. We characterize the equilibrium. We show how regional unemployment differentials are affected by the partition of the population between the two regions and between the statuses of national and regional job-seekers. We also conclude that a rise in matching effectiveness out of the region of residence has an ambiguous effect on the unemployment rates. It decreases the probability of remaining unemployed but it also reduces labor demand through a lower acceptance rate of job offers. This ambiguity echoes the main conclusion of Kroft and Pope (2014).

In the standard search-matching literature, frictions generate congestion externalities which are not internalized by decentralized agents unless the Hosios (1990) condition is met. As soon as some workers search all over the country, new sources of inefficiency arise. First, when decentralized agents decide whether to search nationally, they look at their private interest and ignore the consequences of their choices on job creation in all regions. Second, when opening vacancies in a region, firms do not internalize the changes in the matching probability and hence in net output in the rest of the economy. We show that the Hosios condition is never sufficient to decentralize the constrained efficient allocation.

We develop a numerical exercise for a very stylized US economy made of two regions that are initially symmetric and where the Hosios condition prevails. The decentralized economy appears to be far from efficient. For a very wide range of parameters, efficiency requires that nobody searches in the whole country while 10% of the workforce does it in the decentralized economy. Furthermore, the efficient unemployment rate level is lower than the decentralized one. As this exercise assumes symmetric regions, this conclusion is not in contradiction with the recent evidence that geographical mismatch is negligible in the US (see e.g. Sahin *et al.*, 2014, Marinescu and Rathelot, 2014 and Nenov, 2014).

Although a spatial equilibrium model with genuine unemployment has for long been missing, some papers have recently partly filled the gap. Leaving aside the literature where regions are so close that commuting is an alternative to relocation, the literature about regional unemployment differentials can be divided in two groups according to the

type of search: either one needs to move before starting to seek a job in the region of residence or one can search all over the country and then move if needed.

In the first case, some papers extend the island model of Lucas and Prescott (1974) whose economy is populated by a large number of segmented perfectly competitive labor markets where only labor is mobile (workers being allowed to visit only one island per period). Lkhagvasuren (2012) adds search-matching frictions as well as match-location specific productivity shocks in an otherwise standard islands model to reproduce the volatility of unemployment rates in the United States. Focusing also on one (small) region out of many, Wrede (2014) studies the relationships between wages, rents, unemployment and the quality of life in a dynamic framework. He assumes a standard search-matching framework and analyzes how regional amenities affect unemployment and the quality of life. Beaudry *et al.* (2014) introduce search-matching frictions in a spatial equilibrium setting with wage bargaining, free mobility of jobs, a very stylized housing market, and amenities with congestion externalities. In their paper, with some exogenous probability, the jobless population gets the opportunity to move to another city in order to seek jobs, while we let agents choose between two strategies: regional and national search. Furthermore, Beaudry *et al.* (2014) do not look at efficiency while we do. Kline and Moretti (2013) develop a matching model to characterize the optimal (fixed) hiring cost and to look at the rationale for place-based hiring subsidies. Finally, Kline and Moretti (2014) provide a two-region model in which workers have idiosyncratic preferences for a region and are mobile. They use the model for policy purposes, namely to show that place-based policies are not always welfare improving for the whole country. This is due to the fact that taxes generate a deadweight loss.

Second, some recent papers assume that workers can seek a job in the whole country. In a setting with many regions, Amior (2012) studies wages' responses to a housing shock in the presence of skill heterogeneity. He assumes national search in a search-matching framework as well as a random migration cost. Domingues Dos Santos (2011) builds a search-matching dynamic framework with two regions that are each considered as a line. She finds that increasing search effectiveness is beneficial for unemployment rates in both regions. However, she keeps wages exogenous. Using a search-matching dynamic framework with national search and endogenous wages, Antoun (2010) assumes two types of agents who differ in their preference for a region. He finds that a positive productivity shock in one region decreases unemployment locally but raises it in the other region. We extend these models by endogenizing the choice between regional and national search under wage bargaining.⁴ Contrary to these papers, we also develop a normative analysis by looking at efficiency. However, we keep our framework static while they all assume a dynamic setting.

In the new economic geography literature, Epifani and Gancia (2005) analyze the simultaneous emergence of both agglomeration economies and unemployment rate dif-

⁴Molho (2001) develops a partial equilibrium job-search framework with both types of search. Manning and Petrongolo (2011) build a partial equilibrium framework where job-seekers choose their search field. See also Marinescu and Rathelot (2014). We share a common interest with these papers in a general equilibrium model with endogenous wages and vacancies.

ferentials. For this purpose, they build a dynamic two-sector two-region model with transport costs and search-matching frictions. They assume a congestion effect in the utility which could reflect the housing market. They emphasize the role of migration following a productivity shock, which raises the unemployment rate in the short run but decreases it in the long run. Francis (2009) extends this framework to endogenous job destruction.

Galenianos and Kircher (2009) build a directed-search wage-posting model where workers simultaneously apply to a $N > 1$ jobs. They show that multiple applications lead to an inefficient allocation when a vacancy remains unmatched if the job applicant refuses the offer (an assumption revisited by Kircher, 2009). We stick to the random matching assumption with wage bargaining popularized by Pissarides (2000). More importantly, by assumption in our paper, the search for an efficient allocation is constrained by the free choice of agents when they get two job offers. So, the possibility of unmatched vacancies is common to both the decentralized equilibrium and the efficient allocation.

The rest of this paper is organized as follows. Section 2 describes the model and its equilibrium. We first focus on a symmetric equilibrium, to highlight the main mechanisms. We then turn to the case of asymmetric regions. Section 3 studies efficiency. A numerical analysis is conducted in Section 4. Section 5 discusses some extensions to the framework of Section 2. Section 6 concludes.

2 The model

This section develops a model with two distant regions. We first introduce the main assumptions and the matching process. Then, we develop a simple version of the model with symmetric regions and derive the symmetric equilibrium. We allow for differences across regions in a last subsection.

We consider a static model of an economy made of two large regions ($i \in \{1, 2\}$). Each region is a point in space. The distance between the two regions is such that commuting is ruled out, while inter-regional migration is allowed. Topel (1986) and Kennan and Walker (2011) among others have stressed the importance of migration costs. As will soon be clear, idiosyncratic preferences for regions will in our setup play the role of individual-specific relocation costs. The aggregate national labor force is made of an exogenous large number N of homogeneous risk-neutral workers. A worker living in region i supplies one unit of labor if the wage is above the value of time if she stays at home, denoted b_i . Firms are free to locate in the region they prefer. They use labor to produce a unique consumption good which is sold in a competitive market and taken as the *numeraire*.

Workers have idiosyncratic preference for regions. Agent j gets utility c_{ij} from living in region i . As Moretti (2011), we assume that the relative preference for region 1 over region 2, $c_{1j} - c_{2j}$, is uniformly distributed on a given support $[-v; v]$, $v > 0$. The presence of a distribution of relative preferences implies that the elasticity of inter-regional labor mobility is finite. A higher value of v entails less intense responses to regional differences.

The indirect utility V_{ij}^e of an employed individual j living in region i is, as in Moretti (2011), assumed to be additive and defined by:

$$V_{ij}^e = w_{ij} + a_i + c_{ij} \quad (1)$$

where w_{ij} represents the wage earned by agent j in region i and a_i is a measure of exogenous local consumption amenities in region i , such as the climate. These amenities are public goods and are not affected by the number of inhabitants in a region (no rivalry).⁵ Similarly, the indirect utility V_{ij}^u of an unemployed person j residing in region i is:

$$V_{ij}^u = b_i + a_i + c_{ij}. \quad (2)$$

In each regional labor market, we assume a regional-specific random matching process. Adopting a one-job-one-firm setting, firms decide in which region they open at most one vacancy. The cost κ_i of opening a vacancy is constant, exogenous and regional-specific.⁶ Throughout the paper, we assume constant returns in the production of the consumption good.⁷ If the vacancy is filled, a firm produces $y_i > b_i$ units of the consumption good. So, depending on the origin of the worker, a firm makes a profit $J_{ij} = y_i - w_{ij}$ on a filled position.

2.1 The timing of decisions

At the beginning of the unique period, everybody is unemployed, chooses in which region to reside, and decides to search for a job either regionally or nationally (i.e. either one only searches for a job in the region where one lives or one searches in both regions at the same time). The reason why some workers would only search in their region of residence rather than nationally is intuitive. If a worker has a sufficiently strong relative idiosyncratic preference for her region of residence, she will not accept to migrate to take a position. Since, following Decreuse (2008), we assume a small cost of refusing a job offer, this individual will then not take part in the matching process in the other region.

In a second step, firms open vacancies and possibly meet a worker. This process occurs simultaneously in both regions. If a vacancy meets a job-seeker, this worker then accepts or not the job offer. When a match is formed with a job-seeker who does not live in the firm's region, this worker relocates. Allowing unemployed workers to relocate at this stage would complicate the exposition without yielding more insights. After the

⁵Contrary to what is sometimes done in the literature (see e.g. Wrede, 2014 or Brueckner and Neumark, 2014), amenities a_i do not affect the production function either.

⁶Capital is assumed to move freely across regions through vacancy creation. Ignoring credit market imperfections, entrepreneurs have no problem financing their vacancy cost κ_i .

⁷Although very standard in the search-matching literature, this assumption does not account for an empirical regularity according to which firms are more productive in larger cities. The elasticity is quite small, especially when controlling for characteristics such as education, but differences in population sizes can be substantial. In the US however, Beaudry *et al.* (2014) find no significant evidence of agglomeration effects on productivity (over 10-year periods). So, we think that our assumption is not too strong a simplification, at least in the US context.

relocation step, employed workers and firms bargain over wages. Fourth, production takes place and good markets clear.

The moment at which wages are negotiated matters when a relocation of the worker is involved. If this moment occurs before the decision to migrate is taken, through Nash bargaining, the worker will get a partial compensation for the difference in the regional non-wage components of utility $a_i + c_{ij}$. To implement this timing, one has to assume that the employer is aware of the idiosyncratic preferences of the worker for both regions. One can doubt that this information is available.⁸ A survey conducted by CareerBuilder.com at the end 2011 indicates that less than a third of employers are ready to pay for relocation costs of their new employees.⁹ This is casual evidence in favor of the timing indicated above: The bargained wage will then not compensate the worker for the difference in $a_i + c_{ij}$ and hence w_{ij} can be written simply as w_i . We will return to the timing of the wage bargain in Section 3.

Some additional notations have to be introduced. Before the matching process, N_i agents choose to reside in region i (N_i is called the *ex-ante* population in region i , with $N_1 + N_2 = N$). Population in region i is composed of N_i^N agents who search nationally and N_i^R individuals who only search in their region of residence ($N_i = N_i^N + N_i^R$). For agents located in region i , the notation $-i$ will designate the other region.

2.2 The matching process

We allow for distant search, meaning that search in a region can be conducted while living in the other one. The matching effectiveness of those living in the region where vacancies are open is normalized to one. For residents of the other region, this effectiveness takes an exogenous value α with $0 \leq \alpha \leq 1$. The main focus is here on strictly positive values of α .¹⁰ The number of hirings in each region is given by a regional-specific matching function $M_i(\cdot, \cdot)$ with:

$$M_i(\mathcal{V}_i, N_i + \alpha N_{-i}^N) < \min\{\mathcal{V}_i, N_i + \alpha N_{-i}^N\}, \quad i \in \{1, 2\}, \quad (3)$$

where \mathcal{V}_i represents the endogenous number of vacancies opened in region i and $N_i + \alpha N_{-i}^N$ is the endogenous number of job-seekers measured in efficiency units. As Molho (2001) and Manning and Petrongolo (2011) do in a partial equilibrium framework, we endogenize search effort by letting job-seekers choose their search field. Following Pissarides (2000) and a large empirical literature, the matching function has constant returns to scale¹¹ and is increasing and concave in both arguments. Defining tightness in region i as

$$\theta_i = \frac{\mathcal{V}_i}{N_i + \alpha N_{-i}^N},$$

⁸Notice that if the framework was dynamic this timing would raise another issue. Under the standard assumption of automatic renegotiation (Pissarides 2000, p. 15), the wage would be revised after the relocation step and would be chosen exactly as proposed in the timing of events we privilege.

⁹See http://www.careerbuilder.com/share/aboutus/pressreleasesdetail.aspx?id=pr677&sd=1/18/2012&ed=1/18/2099&siteid=cbpr&sc_cmp1=cb_pr677_.

¹⁰Subsection 2.3 of Lutgen and Van der Linden (2013) discusses the case $\alpha = 0$ in detail.

¹¹ Manning and Petrongolo (2011) provide recent evidence at the local level for the UK.

$m_i(\theta_i)$ designates the probability M_i/\mathcal{V}_i that a vacancy in region i meets a worker, with $0 < m_i(\theta_i) < 1$ by the inequality in (3) and $m_i'(\theta_i) < 0$ because of search-matching congestion externalities. So, unfilled jobs find a partner more easily in a region able to attract more job-seekers. The probability that an unemployed worker living in i meets a firm located in region i is $p_i(\theta_i) = \theta_i m_i(\theta_i)$, with $0 < p_i(\theta_i) < 1$. Job-seekers find a job more easily in a thicker local labor market: $[p_i(\theta_i)]' > 0$.¹² The probability that an unemployed worker searching nationally and living in $-i$ meets a firm settled in region i is $\alpha p_i(\theta_i)$. In case of national search, for someone living in i , the probability of getting an offer in i and no offer from the other region is $p_i(\theta_i)(1 - \alpha p_{-i}(\theta_{-i}))$. The probability of the opposite event is $\alpha p_{-i}(\theta_{-i})(1 - p_i(\theta_i))$. The probability of getting an offer from each region is $\alpha p_i(\theta_i)p_{-i}(\theta_{-i})$. In this case, the worker accepts the position that offers the highest indirect utility level. Finally, this worker living in i faces a probability $(1 - p_i(\theta_i))(1 - \alpha p_{-i}(\theta_{-i}))$ of remaining unemployed.

2.3 A model with symmetric regions

Before discussing the general case of asymmetric regions, let us consider the case of the symmetric equilibrium. The main effects of a rise in matching effectiveness out of the region of residence already appear in this framework (in which, when not necessary, we drop the region subscript i).

2.3.1 Wage bargaining

Individual Nash bargaining takes place *ex-post*, once the cost of opening a vacancy is sunk. So, when a vacancy and a job-seeker j have met, the wage solves the following maximization:

$$\max_{w_j} (V_j^e - V_j^u)^\beta (J - V)^{1-\beta}$$

where V is the value of an unfilled vacancy and $\beta \in [0, 1)$ denotes the bargaining power of a worker. The first-order condition can be rewritten as:

$$w_j = w = \beta y + (1 - \beta)b - \beta V. \quad (4)$$

Hence, the wage is independent of the worker's preferences. As $w > b$, under free-entry, workers always take the position.

2.3.2 Acceptation decision

A worker searching locally always accepts a job offer, as $V_j^e > V_j^u$ in a free-entry equilibrium with Nash bargaining. Similarly, a worker searching nationally who only gets a job offer from a firm located in the region where she lives always takes the position. In case this worker only receives a job offer from a firm settled in the other region, she always

¹²As is standard, we assume Inada conditions: $\lim_{\theta \rightarrow 0} m(\theta) = 1$; $\lim_{\theta \rightarrow 0} p(\theta) = 0$; $\lim_{\theta \rightarrow +\infty} m(\theta) = 0$; $\lim_{\theta \rightarrow +\infty} p(\theta) = 1$.

accepts the job, as she decided to search for a job there (as shown in Appendix A). In case a worker gets two job offers, she decides which position to take to maximize her utility in employment (1). When regions are symmetric, the only variables that matter are preferences. One thus gets:

Lemma 1. *Acceptance decisions in case of symmetric regions*

When receiving two job offers, a worker j

- *accepts the position in region 2 if $c_{1j} - c_{2j} < 0$,*
- *takes the job in region 1 otherwise.*

2.3.3 Opening of vacancies

The expected value of a vacant position V is equal to $-\kappa + \pi m(\theta)(y - w)$. π corresponds to the probability that meeting a worker leads to a filled vacancy. Firms open vacancies freely until this value V is nil. Anticipating correctly the outcome of the wage bargain, the free-entry condition becomes:

$$\frac{\kappa}{(1 - \beta)(y - b)} = \pi m(\theta). \quad (5)$$

The probability of filling a vacancy $\pi m(\theta)$ increases with the (*ex-post*) surplus of a match $y - b$ and decreases with the cost of opening a vacancy κ and workers' bargaining power β .

2.3.4 Search and location decisions

Appendix A shows that taking search and location decisions simultaneously or choosing first the location and then the searching area is equivalent (the proof is given for the general case of asymmetric regions). Therefore, to ease the exposition, the presentation below opts for the second timing.

Search decision When deciding whether to search or not in the other region, a worker maximizes her expected utility as regional or national job-seeker. Let p_i be a short notation for $p_i(\theta_i)$. The expected utility if the agent located in i searches nationally is

$$p_i(1 - \alpha p_{-i})V_{ij}^e + \alpha p_{-i}(1 - p_i)V_{-ij}^e + \alpha p_i p_{-i}(\max\{V_{-ij}^e; V_{ij}^e\} - \varepsilon) + (1 - p_i)(1 - \alpha p_{-i})V_{ij}^u, \quad (6)$$

where ε stands for the small cost of refusing a job offer. The expected utility of a job-seeker living in region i and searching for a job in this region only is

$$p_i V_{ij}^e + (1 - p_i) V_{ij}^u. \quad (7)$$

When the small cost of refusing a job offer ε tends to zero, someone searches nationally if her relative preference for region i over region $-i$, $c_{ij} - c_{-ij}$, is above a threshold

z equal to $w - b$. Otherwise, she decides to look for a job in region i only. Perfectly anticipating the wage (4), under free-entry, let $z\beta(y - b)$. Defining

$$z_1 = -z \quad \text{and} \quad z_2 = z, \quad (8)$$

the following lemma applies.

Lemma 2. *When $\alpha > 0$ and the cost of refusing an offer $\varepsilon \rightarrow 0$. Assuming that both z_i 's lie in $(-v, v)$,*

- *If $c_{1j} - c_{2j} < z_1$, agent j searches in region 2 only;*
- *If $z_1 \leq c_{1j} - c_{2j} \leq z_2$, agent j searches nationally;*
- *If $c_{1j} - c_{2j} > z_2$, agent j searches in region 1 only.*

The number of regional job seekers in each region is given by:

$$N^R = \frac{v - z}{2v} N \quad (9)$$

By comparing their expected utility in case of regional and national search, unemployed workers turn out to compare the utility levels when they are actually employed in the other region and when they remain unemployed in their region of residence. These utility levels are not in expected terms and so search decisions are independent of probabilities to get a job offer.¹³ Therefore, the number of workers who search nationally is independent of the matching effectiveness $\alpha > 0$.¹⁴ A higher wage relative to home production yields a higher threshold z , implying that a lower number of workers search for a job regionally.

Location choice As an unemployed worker who decides to look for a job regionally only locates in her region of search, we have to compare the expected utility of an agent j who searches nationally while being located in region 1 or in region 2. These expected utility levels are given by (6), for $i = \{1, 2\}$. As regions are symmetric, the problem simplifies to $c_{1j} \leq c_{2j}$. One gets the following result:

Lemma 3. *Location choice when regions are symmetric*

- *If $c_{1j} - c_{2j} < 0$, then agent j locates in region 2;*
- *Else, worker j settles in region 1.*

The size of the population is therefore equal in both regions.

¹³If ε was non negligible, these probabilities would however play a role in the definition of the z_i 's.

¹⁴This would still be true if ε was non negligible. When α is nil, searching all over the country cannot be ruled out but the probability of finding a job in the other region is zero. So, every job-seeker searches locally.

2.3.5 Acceptance probability, vacancy creation and unemployment rates

A detailed explanation is provided in Appendix C. The number of job-seekers for a firm is $v + \alpha z$ in efficiency units. As we have seen, a worker located in a region always accepts the position there. The workers located in the other region accepts the position if they did not get an offer from a firm located in their region of residence (probability p). Denoting by π the acceptance rate conditional on the meeting, one gets

$$\pi = 1 - \frac{\alpha p z}{v + \alpha z} \quad (10)$$

When workers cannot search out of their region of residence (i.e. $\alpha = 0$), the acceptance rate is 1. When the probability p of meeting a vacancy increases, the acceptance rate shrinks. The same holds for a rise in α . In both cases, the increase leads to a higher number of workers that get two job offers. Because these workers have to refuse one of the two, the acceptance rate goes down.

Combining (5) with (10) leads to the following free-entry condition:

$$\frac{\kappa}{(1 - \beta)(y - b)} = \frac{v + \alpha(1 - p)z}{v + \alpha z} m(\theta) \quad (11)$$

Because α affects negatively the acceptance rate, a rise in the matching effectiveness leads to a lower equilibrium tightness. So does a rise in κ .

The unemployment rate in a region can be written as:

$$u = \frac{(1 - p)(v - \alpha p z)}{v} \quad (12)$$

To construct this rate, one needs to notice that because regions are symmetric, the population leaving *ex-post* in a region is $N/2$. Regarding the number of unemployed, it is equal to $(1 - p)N^R + (1 - p)(1 - \alpha p)N^N$, with

$$N^N = N/2 - N^R = (z/v)(N/2) \quad (13)$$

A higher tightness or a higher matching effectiveness out of the region of residence α yields, other things equal, lower unemployment rates.

2.3.6 Equilibrium and impact of a shock to the matching effectiveness

Definition 1. *An interior equilibrium is a vector $\{w, \theta, u, N^R, N^N\}$ and a scalar z that satisfies equations (4) under free-entry $V = 0$, (8), (9), (11), (12) and (13).*

This equilibrium exists and is unique whenever $v > \beta(y - b)$ (see Appendix E). It is determined recursively. Once w and z are computed, tightness θ is determined using the free-entry condition (11), which allows to set uniquely u .

Proposition 1. *When regions are symmetric, a rise in the matching effectiveness in the other region α has an ambiguous effect on the equilibrium unemployment rates.*

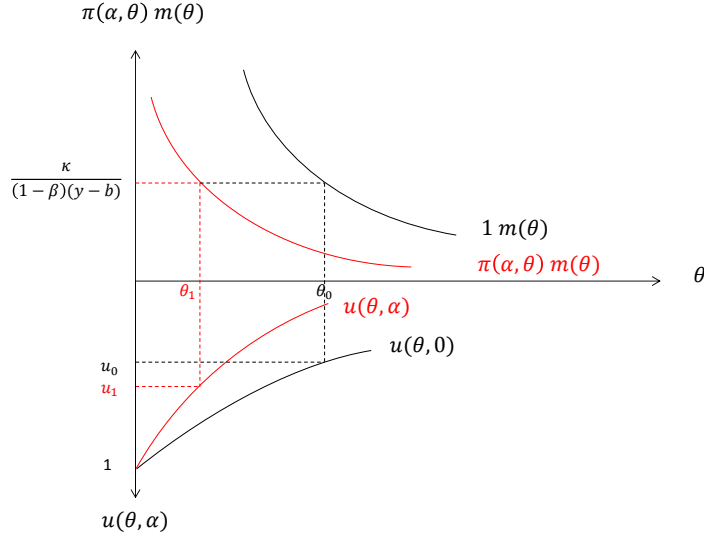


Figure 1: The symmetric equilibrium: $\alpha = 0$ ($\pi = 1$; the black curves) versus $\alpha > 0$ ($\pi < 1$; the red curves).

The consequences of increasing the matching effectiveness can be shown graphically in this simple economy. In the upper part of Figure 1, we draw the left-hand side of (11) when $\alpha = 0$ (in black) and when $0 < \alpha \leq 1$ (in red). A rise in α induces a leftward shift of the curve. The equilibrium level of tightness therefore declines because the acceptance rate π shrinks with α . The lower part of the figure draws (12) and illustrates the favorable partial effect of a rise in α on the unemployment rate conditional on tightness. Depending on the importance of the shifts of the two curves the equilibrium unemployment rate can vary in both directions.

If $\alpha = 0$ (resp., $\alpha > 0$) can be interpreted as a world without (resp., with) the modern communication technologies using the Internet, Figure 1 illustrates that the introduction of these technologies can have ambiguous effects on the regional equilibrium unemployment rates. This is a way of rationalizing the results of Kroft and Pope (2014).

2.4 A model with asymmetric regions

This section highlights the main differences with respect to the symmetric case and shows how the main conclusions from a shock on α are affected. We solve the problem by backwards induction.

The generalized Nash bargaining process is now given by

$$\max_{w_i} (V_{ij}^e - V_{ij}^u)^{\beta_i} (J_i - V_i)^{1-\beta_i}.$$

The first-order condition can be rewritten as:

$$w_i = \beta_i y_i + (1 - \beta_i) b_i - \beta_i V_i. \quad (14)$$

The conclusions drawn in the previous subsection still holds: wages are independent of worker's preferences and are a positive function of the worker's bargaining power and productivity. Wages in region i do not depend on the conditions in the other region.

2.4.1 Acceptance of a job offer

If a worker searching nationally gets two offers, one from each region, she rejects one of them (incurring an arbitrary small cost ε) and accepts the other one. To take this decision, the unemployed worker compares V_{1j}^e with V_{2j}^e . The agent whose relative preference $c_{1j} - c_{2j}$ is above the threshold $w_2 - w_1 + a_2 - a_1$ chooses to work in region 1 rather than in region 2. Let $\Delta = b_2 - b_1 + a_2 - a_1$.

Lemma 4. *When a job-seeker searching nationally has one job offer from each region, there is a threshold*

$$\hat{x} = \Delta + \beta_2(y_2 - b_2) - \beta_1(y_1 - b_1), \quad (15)$$

assumed to be in $(-v, v)$, such that she accepts the job offer in region 2 if $c_{1j} - c_{2j} < \hat{x}$. Otherwise, she accepts the job offer in region 1.

The higher the worker's share of the surplus in region 2 or the higher Δ , the more people will accept the position in region 2 when getting two job offers.

2.4.2 Vacancy creation

Firms open vacancies in region i until the expected gain V_i is nil ($i \in \{1, 2\}$). Combining (14) and the free-entry condition yields

$$\frac{\kappa_i}{(1 - \beta_i)m_i(\theta_i)} = \pi_i(y_i - b_i), \quad \forall i \in \{1, 2\}, \quad (16)$$

where π_i is the conditional acceptance rate in region i . The rate at which vacancies are on average filled, $\pi_i m_i(\theta_i)$, varies as in the previous subsection with the parameters in i .

2.4.3 Search decision and location choice

Search decision An individual j living in region 2 decides to search regionally or nationally by comparing the expected utility in both cases (Equations (6) and (7) for $i = 2$). When the small cost of refusing a job offer ε tends to zero, someone searches nationally if her relative preference for region 1 over region 2, $c_{1j} - c_{2j}$, is above a

threshold $z_1 = b_2 - w_1 + a_2 - a_1$. Otherwise, she decides to look for a job in region 2 only. This is shown in Appendix A (the comparison of cases *e* and *f*).

A similar development is conducted for a worker settled in region 1. A job-seeker living in region 1 whose relative preference for region 1 over region 2 is higher than a threshold $z_2 = w_2 - b_1 + a_2 - a_1$ searches in region 1 only. Below this threshold, the worker looks for a job in the whole country (see the comparison of cases *a* and *c* in Appendix A).

Under perfect anticipation of bargained wages and free-entry, let now the z_i thresholds become:

$$z_1 = \beta_1(b_1 - y_1) + \Delta \quad \text{and} \quad z_2 = \beta_2(y_2 - b_2) + \Delta \quad (17)$$

Then, Lemma 2 applies. The shares of these three groups in the total population are $\frac{v+z_1}{2v}$ for regional job-seekers in region 2, $\frac{z_2-z_1}{2v}$ for national ones and $\frac{v-z_2}{2v}$ for regional job-seekers in 1. Remembering (15), it is easily seen that $z_1 \leq \hat{x} \leq z_2$.

A rise in relative amenity levels in region 2, $a_2 - a_1$, leads to more regional job-seekers in region 2, less regional job-seekers in region 1 but leaves the number of national ones unchanged. A rise in the expected surplus of a match in region 1 does not affect the decision to search or not in region 2 (the threshold z_2 remains constant) but implies less regional job-seekers living in 2. Finally, a rise in α still has no impact on the z thresholds.

Location choice The same comparison as in the symmetric case leads to the following lemma.

Lemma 5. *If relative idiosyncratic preference $c_{1j} - c_{2j} < x$, agent j chooses to reside in region 2, where*

$$x = \Delta + \frac{1 - \alpha}{1 - \alpha p_1 - \alpha p_2 + \alpha p_1 p_2} (p_2 \beta_2 (y_2 - b_2) - p_1 \beta_1 (y_1 - b_1)), \quad (18)$$

with $0 \leq \frac{1 - \alpha}{1 - \alpha p_1 - \alpha p_2 + \alpha p_1 p_2} < 1$ and, by (17), $z_1 \leq x \leq z_2$. Otherwise agent j settles in region 1. The share of the population living *ex-ante* in region 2 (respectively, 1) is then $\frac{v+x}{2v}$ (respectively $\frac{v-x}{2v}$).

Compared to the symmetric case where x was equal to 0, we see that regional differences matter. Notice that when search is equally efficient wherever one looks for a job (i.e. $\alpha = 1$), differences in wages and levels of tightness do not impact the worker's location decision. An increase in relative amenities in region 2, $a_2 - a_1$, as well as a rise (respectively, a drop) in the value of home production in region 2 (resp., 1) induce more workers to locate in 2 *ex-ante*. A rise in α attracts more workers in region 1 if $p_2 \beta_2 (y_2 - b_2) - p_1 \beta_1 (y_1 - b_1) > 0$ and conversely.¹⁵ An increase in tightness in region 1 has several effects if $0 < \alpha < 1$. First, if one lives in region 1, the increase in the probability of being employed in this region equals the decrease in the probability of

¹⁵When this inequality holds, a rise in α augments the chances of benefiting from the better employment prospects in region 2 while staying in the first one.

being unemployed. As the individual stays in the same region, the net gain is proportional to $w_1 - b_1$. Second, if one lives in region 2, the increase in the probability of being employed in region 1 equals the decrease in the probability of staying unemployed in region 2. This effect is proportional to $V_{1j}^e - V_{2j}^u$. Third, the decline in the probability of being employed in 2 is the same wherever one lives. So, this effect cancels out. The first and the second effects push the difference in idiosyncratic preference of the indifferent agent, x , in opposite directions so that the net effect is ambiguous. This conclusion also holds if θ_2 increases. However, the first effect described just above dominates if α is sufficiently small.¹⁶

Proposition 2. *When $0 < \alpha < 1$, a tighter labor market in region i induces more people to reside there under the following sufficient condition:*

$$\alpha < 1 - \frac{(\beta_1(y_1 - b_1) - \beta_2(y_2 - b_2))^2}{(\beta_1(y_1 - b_1) + \beta_2(y_2 - b_2))^2}. \quad (19)$$

When $\alpha = 1$, the levels of tightness do not influence the choice of residence any more.

The proof is provided in Appendix B. Recalling (18), Condition (19) is easy to interpret: The higher the inter-regional difference in workers' shares in the surpluses,¹⁷ the lower α should be in order to get the intuitive relations between the levels of tightness and x .

2.4.4 Summary of the acceptance, search and location decisions

Combining Lemmas 2 and 5, the total labor force is made of four groups with distinct behaviors:

Proposition 3. *Partition of the population*

- If $c_{1j} - c_{2j} < z_1$, agent j locates in region 2 and searches there only;
- If $z_1 \leq c_{1j} - c_{2j} < x$, agent j settles in region 2 and searches in the whole country;
- If $x \leq c_{1j} - c_{2j} \leq z_2$, agent j locates in region 1 and looks for a job nationally;
- If $c_{1j} - c_{2j} > z_2$, agent j settles in region 1 and looks for a job in region 1 only.

Figure 2 illustrates this partition of the total population if $-v < z_1, z_2 < v$.

In general, one cannot rank threshold values x and \hat{x} since x varies with the levels of tightness. When $\alpha = 1$ and regions are asymmetric, the comparison of the thresholds is obvious since $x = \Delta$ then : $\hat{x} \leq x \Leftrightarrow \beta_2(y_2 - b_2) - \beta_1(y_1 - b_1) \leq 0$.

¹⁶We thank one of the anonymous referees who suggested to study this condition.

¹⁷Compared to the sum of these shares in the 2 regions.

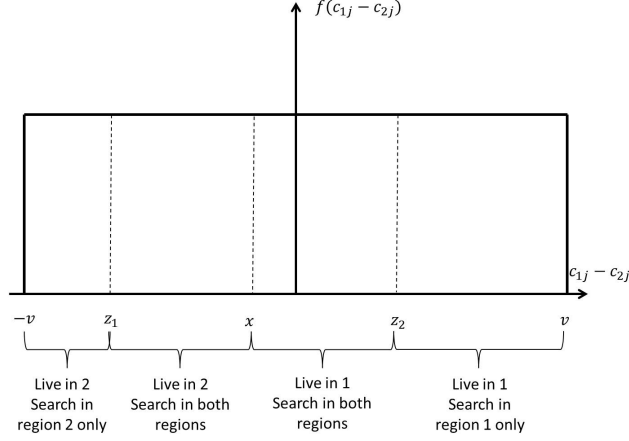


Figure 2: The partition of the population in case of an interior solution.

2.4.5 Acceptance probability and vacancy creation

A detailed explanation is provided in Appendix C. Consider again a vacant position in region 1. The mass of job-seekers searching for a job in 1 is now $[v - x + \alpha(x - z_1)][N/2v]$ in efficiency units. Conditional on meeting one of these unemployed workers, all those whose relative preference $c_{1j} - c_{2j}$ lies above \hat{x} accept for sure an offer from region 1. For those between z_1 and \hat{x} , this is only the case if they get no offer from region 2. So, conditional on a contact between a vacancy in region 1 and a job-seeker, the acceptance probability π_1 is (with a corresponding expression for π_2):

$$\pi_1 = 1 - \frac{\alpha p_2(\hat{x} - z_1)}{v - x + \alpha(x - z_1)} \quad (20)$$

$$\pi_2 = 1 - \frac{\alpha p_1(z_2 - \hat{x})}{v + x + \alpha(z_2 - x)} \quad (21)$$

It is easily checked that the higher \hat{x} , the more workers accept job offers in region 2 and so the lower the acceptance rate in 1. The same is true for x . The higher the number of workers searching in region 2 only (an increasing function of z_1), the higher the acceptance rate in 1. Finally, an increase in the probability of getting a job offer in region 2 decreases the acceptance rate in region 1. Similarly, π_2 increases with \hat{x} and x and decreases with z_2 and p_1 . The impact of the matching effectiveness α should be stressed: A higher α leads to a lower conditional acceptance probability, as the probability of getting two job offers increases. Conversely, $\lim_{\alpha \rightarrow 0} \pi_i = 1$.

Combining (16) with (20)-(21) leads to the following free-entry conditions:

$$\frac{\kappa_1}{(1 - \beta_1)m_1(\theta_1)} = \frac{v - x + \alpha(x - z_1) - \alpha p_2(\theta_2)(\hat{x} - z_1)}{v - x + \alpha(x - z_1)}(y_1 - b_1) \quad (22)$$

$$\frac{\kappa_2}{(1 - \beta_2)m_2(\theta_2)} = \frac{v + x + \alpha(z_2 - x) - \alpha p_1(\theta_1)(z_2 - \hat{x})}{v + x + \alpha(z_2 - x)}(y_2 - b_2) \quad (23)$$

Through the endogenous acceptance rate, vacancy creation in any region is now affected by the level of tightness and the value of parameters in the other region.

2.4.6 Populations and unemployment rates

Since $z_1 \leq x \leq z_2$, and assuming an interior solution, the sizes N_i^N of the workforce searching nationally and N_i^R of those searching in their region of residence only are given by:

$$N_1^R = \frac{(v - z_2)}{2v}N \quad N_1^N = \frac{(z_2 - x)}{2v}N \quad (24)$$

$$N_2^R = \frac{(z_1 + v)}{2v}N \quad N_2^N = \frac{(x - z_1)}{2v}N \quad (25)$$

so that the total *ex-ante* population sizes are $N_1 = N_1^N + N_1^R = \frac{v-x}{2v}N$ and $N_2 = N_2^N + N_2^R = \frac{v+x}{2v}N$, with $N_1 + N_2 = N$.

The unemployment rates which are the ratio of the number of (*ex-post*) unemployed workers over the (*ex-post*) population, are:

$$u_1 = \frac{(1 - p_1)(v - x - \alpha p_2(z_2 - x))}{v - x - \alpha p_2(1 - p_1)(z_2 - x) + \alpha p_1(1 - p_2)(x - z_1) + \alpha p_1 p_2(x - \hat{x})} \quad (26)$$

$$u_2 = \frac{(1 - p_2)(v + x - \alpha p_1(x - z_1))}{v + x + \alpha p_2(1 - p_1)(z_2 - x) - \alpha p_1(1 - p_2)(x - z_1) - \alpha p_1 p_2(x - \hat{x})} \quad (27)$$

The number of (*ex-post*) unemployed workers in, say, region 1 is composed of the agents living *ex-ante* in 1 who did not get a job offer in region 1, $(1 - p_1)\frac{v-x}{2v}N$, to which we subtract the workers who did not get an offer from region 1 but well from region 2 ($\alpha p_2(1 - p_1)\frac{z_2-x}{2v}N$). The denominator corresponds to the population living *ex-post* in the region (up to a $N/2v$ term). *Ex-post*, the number of inhabitants in, say, region 1 is the sum of 4 terms. The first term represents the population living *ex-ante* in region 1. The second term corresponds to the workers who were living *ex-ante* in 1 and who leave region 1 as they only get a position in region 2. The third term is composed of the agents who lived *ex-ante* in region 2 and who move as they only get an offer from region 1. Finally, the fourth term represents the number of workers who get two offers. This term is positive whenever $x > \hat{x}$, meaning that some more workers living in 2 *ex-ante* accept a position in region 1.

The following lemma provides the signs of the partial derivatives of (26) and (27) with respect to the other endogenous variables and α .¹⁸

Lemma 6. *When $\alpha > 0$ and regions are asymmetric, the unemployment rate u_i decreases with the level of tightness θ_i in the same region. A rise in the matching effectiveness α lowers both regional unemployment rates. Furthermore, (i) The sign of the cross-derivatives $\partial u_i / \partial \theta_{-i}$ varies with the sign of $p_2 \beta_2 (y_2 - b_2) - p_1 \beta_1 (y_1 - b_1)$ (more on this below). (ii) A rise in the threshold x (i.e. a bigger workforce N_2 living *ex-ante**

¹⁸The proof, which is available upon request, consists in reorganizing these partial derivatives.

in region 2) decreases the unemployment rate in region 1 while it increases it in region 2. (iii) An increase in the number of regional job-seekers increases the unemployment rate in both regions. (iv) The unemployment rate in region 1 increases with \hat{x} , while the opposite holds for the unemployment rate in region 2.

An increase in region i 's labor market tightness θ_i boosts the probability that a worker living in region i finds a job there and it rises the probability that a worker located in the other region gets a position in region i (which increases the labor force living in region i). Consequently, the unemployment rate in region i goes down. Turning to Property (i) of Lemma 6, augmenting θ_{-i} rises the probability of leaving region i and this reduces both the number of unemployed workers and the size of the labor force in region i . The net effect on the unemployment rate depends on the inter-regional difference in the gains $p_i\beta_i(y_i - b_i)$. More precisely, Appendix D shows that

$$p_2\beta_2(y_2 - b_2) < p_1\beta_1(y_1 - b_1) \Rightarrow \frac{\partial u_1}{\partial \theta_2} < 0, \quad \lim_{\alpha \rightarrow 0^+} \frac{\partial u_2}{\partial \theta_1} > 0 \quad \text{and} \quad \lim_{\alpha \rightarrow 1} \frac{\partial u_2}{\partial \theta_1} < 0, \quad (28)$$

$$p_2\beta_2(y_2 - b_2) > p_1\beta_1(y_1 - b_1) \Rightarrow \frac{\partial u_2}{\partial \theta_1} < 0, \quad \lim_{\alpha \rightarrow 0^+} \frac{\partial u_1}{\partial \theta_2} > 0 \quad \text{and} \quad \lim_{\alpha \rightarrow 1} \frac{\partial u_1}{\partial \theta_2} < 0. \quad (29)$$

Consequently, the cross-partial derivatives are both negative when α is big enough. When $\alpha = 0$, the unemployment rate in a region does not depend on tightness in the other. However, for sufficiently small positive values of α , one cross-derivative $\partial u_i / \partial \theta_{-i}$ is positive.

As x goes up (Property (ii)), the number N_1 of agents living *ex-ante* in region 1 shrinks while N_2 increases. These population sizes are (up to a $N/2v$ term) present in the numerators and the denominators of (26) and (27). In addition, a rise in x affects the numbers of national job-seekers who depletes the regional workforces if they are recruited in the other region. All in all, a rise in x reduces (resp., increases) the unemployment rate in region 1 (resp., 2). A corollary of Property (iii) is that more workers searching all over the country reduces the unemployment rates in both regions. These properties as well as the favorable role of α on the unemployment rates are conditional on the other endogenous variables.

In a standard Mortensen-Pissarides setting (where geographical heterogeneities are concealed in an aggregate matching function), the size of the labor force does not affect the equilibrium unemployment rate (as eventually the number of vacancies rises proportionately, leaving the equilibrium level of tightness unaffected). This equilibrium property is not different here (N plays no role in (26)-(27)). However,

Proposition 4. *If $\alpha > 0$ and regions are asymmetric, the equilibrium unemployment rates are affected by the partition of the population between the two regions and between the two statuses of national versus regional job-seekers.*

2.4.7 Equilibrium

Definition 2. *When $0 < \alpha \leq 1$ and regions are asymmetric, an interior equilibrium is a vector $\{x, \hat{x}, z_1, z_2\}$ assumed to be in $(-v, v)$ and a vector $\{w_i, \theta_i, u_i, N_i^N, N_i^R\}_{i \in \{1,2\}}$,*

solving (14) under free-entry $V_i = 0$, (15), (16), (17), (18), (20), (21), (24), (25), (26) and (27).

As we already know that $z_1 \leq \hat{x}, x \leq z_2$, an equilibrium is interior if $-v < z_1$ and $z_2 < v$. From the definition of Δ and Lemma 2, these conditions become:

Condition 1. *Necessary and sufficient conditions for an interior solution are*

$$v > \beta_1(y_1 - b_1) - (b_2 - b_1 + a_2 - a_1) \quad \text{and} \quad v > \beta_2(y_2 - b_2) + (b_2 - b_1 + a_2 - a_1) \quad (30)$$

The system of equations characterizing an equilibrium is block recursive. The thresholds z_i 's and \hat{x} being explicit functions of the parameters only, the central endogenous variables are $\{\theta_1, \theta_2, x\}$. They are determined by the system (18)-(22)-(23). Once this system is solved, all other endogenous variables get unique values. Under Condition (19), Definition (18) is an implicit relationship which is decreasing in θ_1 and increasing in θ_2 (unless $\alpha = 1$ in which case x is constant). Substituting this relationship into the free-entry conditions (22)-(23) yields:

$$\frac{\kappa_1}{(1 - \beta_1)m_1(\theta_1)} - \pi_1(\theta_1, \theta_2) \underset{\geq 0}{< 0} (y_1 - b_1) = 0 \quad (31)$$

$$\frac{\kappa_2}{(1 - \beta_2)m_2(\theta_2)} - \pi_2(\theta_1, \theta_2) \underset{< 0}{\geq 0} (y_2 - b_2) = 0, \quad (32)$$

where the inequality signs under the θ_i 's designate those of the partial derivatives. The free-entry condition in region 1, (31), can be seen as an implicit reaction function $\theta_1 = \Theta_1(\theta_2)$. Similarly, (32) defines an implicit relationship $\theta_2 = \Theta_2(\theta_1)$. Figure 3 draws these functions for the limit cases, namely $\alpha = 0$ (the two relationships being then orthogonal) and $\alpha = 1$ (the dotted curves),¹⁹ and for an interior value α . By looking at (20) and (21), it is easily seen that the values of $\Theta_i(0)$ are independent of the magnitude of α .

The net effect of a change in a regional gap $y_j - b_j$ on the equilibrium levels of tightness is hard to sign because it affects almost all thresholds present in the acceptance rates $\pi_i, i \in \{1, 2\}$. A rise in α has a clear effect on one of the acceptance probabilities but an ambiguous one on the other. So, the impact on both equilibrium levels of tightness is hard to sign (this effect was negative in the symmetric case). The cost of opening a vacancy is a determinant of unemployment differentials emphasized in the urban search-matching literature (see e.g. Coulson *et al.*, 2001). A rise in the cost of opening a vacancy in, say, region 1 only shifts the $\Theta_1(\theta_2)$ function to the left, leading to a lower equilibrium value of θ_1 and, more interestingly, to a rise in θ_2 (see the red curve and compare equilibrium E and E' in Figure 3). According to Lemma 6, the direct implications are a rise (resp., a decline) in the unemployment rate in region 1 (resp., 2). There are however also

¹⁹If $\alpha = 1$, π_i is a function of tightness in the other region only. $\theta_1 = \Theta_1(\theta_2)$ and $\theta_2 = \Theta_2(\theta_1)$ are negatively sloped. It can easily be checked that $+\infty > \lim_{\theta_2 \rightarrow 0} \Theta_1(\theta_2) > \lim_{\theta_2 \rightarrow +\infty} \Theta_1(\theta_2) > 0$ and $+\infty > \lim_{\theta_1 \rightarrow 0} \Theta_2(\theta_1) > \lim_{\theta_1 \rightarrow +\infty} \Theta_2(\theta_1) > 0$. Furthermore, the $\Theta_i(\theta_{-i})$ functions are convex if the matching function is a Cobb-Douglas. With more general matching functions, the same conclusions hold if the matching rates $m_i(\theta_i)$ are sufficiently convex.

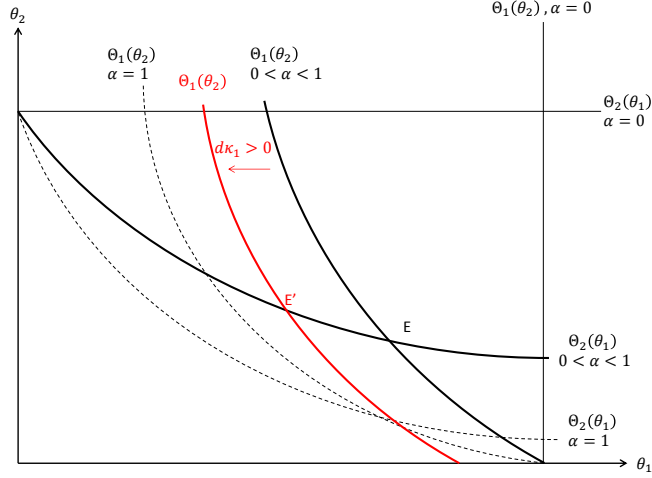


Figure 3: The equilibrium levels of tightness for various values of α .

induced effects in various directions. Given Proposition 2, more people decide to reside in region 2 (x rises). Then, by Result (ii) of Lemma 6, the unemployment rate in region 1 (resp., 2) shrinks (resp., increases). In addition the cross-effects $\partial u_i / \partial \theta_{-i}$ can reinforce or not the direct effects (see (28) and (29)). The net effect on the unemployment rates is therefore ambiguous, contrary to the symmetric case where the net effect was negative.

3 Efficiency

This section studies the efficiency of the *laissez-faire*²⁰ decentralized equilibria introduced in the previous section. We first derive the symmetric optimal allocation and analyze the differences between this allocation and the decentralized equilibrium characterized in Section 2.3. We explain why the Hosios condition is not sufficient to guarantee efficiency of the decentralized equilibrium. We next turn to the analysis of the asymmetric economy and show that additional inefficiencies arise.

In a directed search framework with multiple applications, wage posting with commitment and no recall of applications, Galenianos and Kircher (2009) conclude that the decentralized equilibrium is inefficient because firms cannot influence the probability of retaining applications when they get more than one. By allowing firms to recall all the applicants they receive, Kircher (2009) shows that efficiency is restored. In our framework, firms cannot receive more than one application while some job-seekers can get two job opportunities. Consequently, in the decentralized equilibrium, the probability that a vacancy is rejected is not nil. This feature, which is also present in Galenianos and Kircher (2009), is as such a source of efficiency that could be avoided in a dynamic model in continuous time. We return to this in Section 5. *Here, we show that there are*

²⁰This expression is added since there is no public intervention in Section 2.

other sources of inefficiency that are specific to our regional model.

3.1 The symmetric case

In our static framework, we will first look at the case where the constrained social planner only chooses the levels of tightness. All thresholds are determined as in the decentralized economy. In this environment, the probability of rejecting an offer will be exactly the same in the decentralized and the efficient economies if the corresponding tightness levels are equal too. Next, we will let the planner choose in addition all the threshold values but we will impose that the arrival of two offers cannot be avoided by the planner if it is efficient that some workers search all over the country. If we assume two symmetric regions, the efficient allocation is symmetric as well. We can denote the thresholds $z_2 = -z_1 = z \in [0, v]$ and the probability of being recruited $p_1 = p_2 = p$. The planner's objective function measures net output adjusted to take account of amenities and idiosyncratic preferences for regions:²¹

$$2((y-b)L - \kappa\mathcal{V}) + (b+a)N + \frac{N}{2v} \int_{-v}^0 c_{2j} dj + \frac{N}{2v} \int_0^v c_{1j} dj + \\ - \frac{N}{2v} \alpha p(\theta)(1-p(\theta)) \left[\int_0^z (c_{1j} - c_{2j}) dj \right] + \frac{N}{2v} \alpha p(\theta)(1-p(\theta)) \left[\int_{-z}^0 (c_{1j} - c_{2j}) dj \right]$$

where employment $L = (N/2v) [p(\theta)v + \alpha p(\theta)(1-p(\theta))z]$ and the number of vacancies $\mathcal{V} = \theta(N/2v) [v + \alpha z]$. Hence, up to a constant term, the objective can be rewritten as:

$$\frac{N}{v} [(y-b)p(\theta)(v + \alpha z(1-p(\theta))) - \kappa\theta(v + \alpha z) - \alpha p(\theta)(1-p(\theta))(z^2/2)]. \quad (33)$$

To start with, let z be equal to its decentralized value (8). Then, the optimal tightness verifies:

$$\frac{\kappa}{(1-\eta)m(\theta)} = (y-b) \frac{(v + \alpha z(1-2p(\theta)))}{v + \alpha z} - \frac{\alpha(1-2p(\theta))}{v + \alpha z} \frac{z^2}{2} \quad (34)$$

In a decentralized equilibrium, tightness solves (5), where the acceptance rate π verifies (10) and hence is similar to but different from the term that multiplies $y-b$ in (34). In addition, the last term in the latter expression is not present in (5). So, two sources of inefficiency arise. First, the planner recognizes that an additional vacancy in region i reduces the chances of a match between residents of region i and vacancies in region $-i$, while firms in region i have no reason to share the same concern. By subtracting the nonnegative term $-(y-b)p(\theta)\alpha z/(v + \alpha z)$,²² the planner internalizes an induced effect on expected output in the other region and this pushes optimal tightness downwards. Second, when choosing the number of vacancies in each region, the planner takes into account the impacts of a rise in the number of vacancies on the value of the idiosyncratic

²¹In expressions $\int_{-v}^x c_{2j} dj$ and $\int_x^v c_{1j} dj$, there is a slight abuse of notation since v and x are values for the difference $c_{1j} - c_{2j}$. This notation is equivalent to assuming a bijective relationship between the identifier of workers, j , and their relative preference for region 1, $c_{1j} - c_{2j}$.

²²Merged with the probability of rejecting an offer, it leads to the term $1 - 2p(\theta)$ in (34) while it is only $1 - p(\theta)$ in (11).

preferences when a worker has to migrate to take a job offer. This is represented by the term in z^2 on the RHS (34). A detailed interpretation of this term is given when regions can be asymmetric.

Next, we let the planner also chooses the threshold values. For given numbers of participants to the matching process, the planner divides the population in two equal groups as far as the choice of residence is concerned. Hence, there is no difference with respect to the decentralized economy. In addition, to offer them the highest idiosyncratic level of preference, the planner puts the regional job-seekers at the extremes of the $[-v, v]$ segment and the national job-seekers, if any, in the middle. The national job-seekers located in region 1 are more likely to stay there than national job-seekers located *ex-ante* in region 2 are likely to live in region 1 (if $\alpha < 1$). So, it is efficient to put them to the right of the national job-seekers living in region 2. All this implies that the planner selects unique thresholds z_1, z_2 under the constraints $-v \leq z_1 \leq z_2 \leq v$. Because regions are symmetric, we can assume that $z_2 = z = -z_1$. Therefore, the planner's problem consists now in maximizing Objective (33) with respect to $\{\theta, z\}$. The first-order conditions are given by (34) and:

$$z = y - b - \frac{\kappa}{m(\theta)} \frac{1}{1 - p(\theta)} \quad (35)$$

As a comparison between (35) and the decentralized value $z = \beta(y - b)$ shows, workers do not take their search decision optimally. Two effects are at work. First, because the planner considers the net gain in output, $y - b$, while workers only consider the share $\beta(y - b)$ that accrues to them, the decentralized value of z is lower than the efficient one. As Appendix F.6 explains, this source of inefficiency disappears if the wage bargain occurs before the migration decision, an alternative setting that we henceforth call *ex-ante* bargaining. The second effect is instead present whenever the wages are set. A marginal decrease in z reduces the size of the workforce living in a region and searching also in the other one. This lowers expected net output by an amount $\alpha p(1 - p)(y - b)$ but reduces the cost of vacancy creation by $\alpha \kappa \theta$. After division by $\alpha p(1 - p)$, the loss in expected output becomes $y - b$ while the gain becomes the product of the expected cost of opening a vacancy, $\kappa/m(\theta)$, and of $1/(1 - p)$. This gain is not taken into account by decentralized decisions but well by the planner (see the last term in (35)). So, through this second effect, too many workers search for a job nationally.

3.2 The general case

Let us now consider asymmetric regions. The planner solves²³:

$$\begin{aligned}
\max_{\theta_1, \theta_2, x, \hat{x}, z_1, z_2} & \sum_{i=1}^2 \{ (y_i - b_i) L_i - \kappa_i \mathcal{V}_i + (b_i + a_i) N_i^P \} + \frac{N}{2v} \left[\int_{-v}^x c_{2j} dj \right] + \frac{N}{2v} \left[\int_x^v c_{1j} dj \right] \\
& - \frac{N}{2v} \alpha p_2 (1 - p_1) \left[\int_x^{z_2} (c_{1j} - c_{2j}) dj \right] + \frac{N}{2v} \alpha p_1 (1 - p_2) \left[\int_{z_1}^x (c_{1j} - c_{2j}) dj \right] \\
& + \frac{N}{2v} \alpha p_1 p_2 \left[\int_{\hat{x}}^x (c_{1j} - c_{2j}) dj \right]
\end{aligned} \tag{36}$$

subject to:

$$\begin{aligned}
\mathcal{V}_1 &= \theta_1 \frac{N}{2v} [v - x + \alpha(x - z_1)] & \mathcal{V}_2 &= \theta_2 \frac{N}{2v} [v + x + \alpha(z_2 - x)], \\
L_1 &= \frac{N}{2v} [p_1(v - x) + \alpha p_1(1 - p_2)(x - z_1) + \alpha p_1 p_2(x - \hat{x})], \\
L_2 &= \frac{N}{2v} [p_2(v + x) + \alpha p_2(1 - p_1)(z_2 - x) - \alpha p_1 p_2(x - \hat{x})], \\
N_1^P &= \frac{N}{2v} [v - x - \alpha p_2(1 - p_1)(z_2 - x) + \alpha p_1(1 - p_2)(x - z_1) + \alpha p_1 p_2(x - \hat{x})], \\
N_2^P &= \frac{N}{2v} [v + x + \alpha p_2(1 - p_1)(z_2 - x) - \alpha p_1(1 - p_2)(x - z_1) - \alpha p_1 p_2(x - \hat{x})].
\end{aligned}$$

An efficient allocation is a vector $\{\theta_1, \theta_2, x, \hat{x}, z_1, z_2\}$ solving the following first-order conditions:

$$\begin{aligned}
\frac{\kappa_1}{(1 - \eta_1)m_1(\theta_1)} &= \pi_1(y_1 - b_1) - \frac{\alpha p_2(z_2 - \hat{x})}{v - x + \alpha(x - z_1)}(y_2 - b_2) \\
- \frac{\alpha(1 - p_2)(x - z_1)}{v - x + \alpha(x - z_1)} \left(\Delta - \frac{x + z_1}{2} \right) &- \frac{\alpha p_2(z_2 - x)}{v - x + \alpha(x - z_1)} \left(\Delta - \frac{z_2 + x}{2} \right) \\
- \frac{\alpha p_2(x - \hat{x})}{v - x + \alpha(x - z_1)} \left(\Delta - \frac{x + \hat{x}}{2} \right) &
\end{aligned} \tag{37}$$

$$\begin{aligned}
\frac{\kappa_2}{(1 - \eta_2)m_2(\theta_2)} &= \pi_2(y_2 - b_2) - \frac{\alpha p_1(\hat{x} - z_1)}{v + x + \alpha(z_2 - x)}(y_1 - b_1) \\
+ \frac{\alpha(1 - p_1)(z_2 - x)}{v + x + \alpha(z_2 - x)} \left(\Delta - \frac{z_2 + x}{2} \right) &+ \frac{\alpha p_1(x - z_1)}{v + x + \alpha(z_2 - x)} \left(\Delta - \frac{x + z_1}{2} \right) \\
+ \frac{\alpha p_1(\hat{x} - x)}{v + x + \alpha(z_2 - x)} \left(\Delta - \frac{\hat{x} + x}{2} \right) &
\end{aligned} \tag{38}$$

²³With the same slight abuse of notation as in the previous subsection.

$$\hat{x} = \Delta + y_2 - b_2 - (y_1 - b_1), \quad (39)$$

$$z_1 = b_1 - y_1 + \Delta + \frac{\kappa_1}{m_1(\theta_1)} \frac{1}{1 - p_2}, \quad (40)$$

$$z_2 = y_2 - b_2 + \Delta - \frac{\kappa_2}{m_2(\theta_2)} \frac{1}{1 - p_1} \quad (41)$$

$$x = \Delta + \frac{(1 - \alpha) \left[p_2 \left(y_2 - b_2 - \frac{\kappa_2}{m_2(\theta_2)} \right) - p_1 \left(y_1 - b_1 - \frac{\kappa_1}{m_1(\theta_1)} \right) \right]}{1 - \alpha p_1 - \alpha p_2 + \alpha p_1 p_2}. \quad (42)$$

in which π_1 (resp., π_2) verifies the same expression as in the decentralized economy, namely (20) (resp., (21)).

The same sources of inefficiency as in the previous subsection arise. First, firms do not set the number of vacancies optimally. The planner recognizes that an additional vacancy in region i reduces the chances of a match between residents of region i and vacancies in region $-i$, while firms in region i do not. This planner's concern is captured by subtracting $[\alpha p_2 (z_2 - \hat{x})(y_2 - b_2)]/[v - x + \alpha(x - z_1)]$ from $\pi_1(y_1 - b_1)$ in (37). A corresponding mechanism applies in (38) as well.

Furthermore, when choosing the number of vacancies in each region, the planner takes into account the impacts of a rise in the number of vacancies on the value of leisure, the amenities and the idiosyncratic preference when a worker has to migrate to take a job offer (see the second and third lines of (37) and (38)).²⁴ These compensations are not present in the decentralized equilibrium. Appendix F verifies that *ex-ante* bargaining does not entirely eliminate these sources of inefficiency. For, as Appendix F.6 explains, the decentralized number of vacancies created in a region does not take into account the induced effects (in terms of leisure, amenities and idiosyncratic preferences) on workers who live in this region but are recruited in the other one.

Regarding the search decision, the same sources of inefficiency as in the previous subsection arise, namely that workers do not take the entire surplus into account and do not internalize their impact on the opening of vacancies.

Turning to the choice of residence, a comparison between (42) and (18) indicates one additional source of inefficiency. When fixing x , the planner compares the expected

²⁴Three terms are distinguished in both equations. We now interpret them after multiplication by $(1 - \eta_i)m_i(\theta_i)$. The first part of the second line measures the marginal impact of vacancy creation in region i on workers who seek a job in region i , live in the other region and get a position in region i . This first part is the product of the change in the share of these workers when an additional vacancy is created in i and an expression between parentheses that recognizes that these workers do not enjoy leisure, amenities and their idiosyncratic preference in region $-i$ since they move to region i . The second part of the second line is also the product of two terms. The first one quantifies the change in the share of workers who live in region i , only get an offer in region $-i$ and migrate there when an additional vacancy is created in i . The second term captures that these workers do not enjoy leisure, amenities and their idiosyncratic preference in region i since they move to region $-i$. Finally, the sign of the third line depends on the sign of $x - \hat{x}$. The first term of the line corresponds to the change in the share of workers who get two job offers when an additional vacancy is created in i . These workers are located *ex-ante* in region 2 (resp. 1) whenever $x \geq \hat{x}$ (resp. $x < \hat{x}$). The second term takes into account that these workers do not (resp. do) enjoy leisure, amenities and their idiosyncratic preference in region 2. All these compensations cannot appear if $\alpha = 0$ since workers never migrate to take a position.

increase in net output in each region, $p_i(y_i - b_i - \kappa_i/m_i(\theta_i))$, while job-seekers compare the expected net increase in income $p_i\beta_i(y_i - b_i)$. The Hosios condition does not reconcile the two perspectives since in any case (37) and (38) do not yield an equality between the expected cost of opening a vacancy and $(1 - \eta_i)(y_i - b_i)$ when $\alpha > 0$. This remains true under *ex-ante* bargaining (see Appendix F.6). It should be mentioned that this source of inefficiency in the choice of x would disappear in the limit case where job-seekers are equally effective in the matching process wherever they search ($\alpha = 1$). The decentralized threshold x would in this case be efficient and equal to Δ . In sum,

Proposition 5. *If $\alpha > 0$, even if the Hosios condition is met in both regions, the decentralized laissez-faire equilibrium unemployment rates, regional partition of the workforce and numbers of job-seekers searching in the whole country are inefficient. One cannot rank the optimum and the equilibrium and hence one cannot determine the direction of changes in the workers's bargaining power that could be welfare improving. This proposition is robust to changes in the timing of the wage bargain.*

4 Numerical exercise

In this section, we consider a stylized symmetric economy. We highlight that nobody searches nationally in the efficient allocation. This conclusion resists to many changes in the parameters. We also look at the gap between the efficient and the decentralized allocation when parameter α varies. As a change in α has ambiguous effects on the unemployment rates, we finally show how unemployment rates vary when α rises.

4.1 Parametrization

We consider two symmetric regions in a stylized US economy. We normalize the total size of the population N to 1. We assume Cobb-Douglas matching functions $M_i = \bar{h} \mathcal{V}_i^{0.5} (N_i + \alpha N_{-i}^N)^{0.5}$ with $\bar{h} = 0.7$.²⁵ Following common practice, we assume that the Hosios condition holds (workers' bargaining powers $\beta_i = 0.5$). In the dynamic framework of Pissarides (2009), the expected cost of opening a vacancy amounts to 43% of *monthly* output. Normalizing regional productivity levels y_i to 1 in our static framework can be interpreted as equating monthly output divided by the sum of the discount rate and the separation rate to 1. So, following Pissarides (2009), monthly output equals the sum of the interest rate (0.004) and the separation rate (0.036). Therefore, in our static setting, the expected cost of opening a vacancy, $(\kappa_i/[\pi_i m_i(\theta_i)])$, is $0.43 \times (0.004 + 0.036) = 0.0172$, i.e. 1.72% of intertemporal output. From the free-entry conditions (16), we then derive the value of home production: 0.9656. By (14), the wage equals then 0.98. The difference between the wage and home production may look small. However, as Pissarides (2009) explains, the permanent income of employed workers is only marginally above the permanent income of unemployed workers, even if the difference in a dynamic

²⁵In a static setting, the Cobb-Douglas specification does not guarantee that the hiring rate tends to 1 when θ_i becomes sufficiently big. In the simulations we take care of this difficulty.

framework between current wage and current home production is quite large. Plugging the above-mentioned values for β, y and b into (17) yields $z = 0.0172$. On the basis of the evidence provided by Marinescu and Rathelot (2014) for the US, we assume that 10% of the population is looking for a job nationally. This information is then introduced in the definition of N_i^R (24)-(25) and yields $v = 0.172$. In the absence of evidence about α , we arbitrarily set this parameter to 0.12, which means that workers searching out of their region of residence are 8 times less efficient than in their own region. However, we develop a sensitivity analysis with respect to α . Finally, we calibrate the cost of opening a vacancy κ_i to match an average unemployment rate in the US in the period 2005-2011, namely 7%. So, $\kappa_i = 0.0090$. In the calibrated economy, $p(\theta) = 0.93$, a value well above 0.5.

4.2 The efficient allocation and the efficiency gap

Let us first look at the efficient allocation.²⁶ The central planner chooses higher levels of tightness than in the decentralized equilibrium, so that the efficient unemployment rate is lower than in the decentralized economy (6% versus 7%). As regions are symmetric, this difference is not an indicator of regional mismatch (as measured by e.g. Sahin *et al.*, 2014) but the impact on unemployment of the sources of inefficiency explained after Equality (35). While 10% of the population searches nationally in the decentralized economy, it is optimal that everyone searches regionally only ($z = 0$). A high value of tightness implies that $m(\theta)(1 - p(\theta))$ is low in the RHS of (35) and this explains why the optimal z is zero. Recall that $-\kappa/[m(\theta)(1 - p(\theta))]$ captures the consequences of search decisions on the cost of opening vacancies in the region where the job-seeker does not live.²⁷

The conclusion that workers should only search regionally appears to be very robust to changes in the parameters. Varying the matching effectiveness in the other region, α , from 0.01 to 1 does not modify this conclusion. Let superscript c designate the calibrated values. With $\alpha^c = 0.12$, $z = 0$ is still optimal when we successively consider the following changes in the other parameters: $\kappa \in [0.95 * \kappa^c, 1.85 * \kappa^c]$, $v \in [0.1 * v^c, 30 * v^c]$, $y \in [y^c, 1.0025 * y^c]$ ²⁸ and $\bar{h} \in [0.5 * \bar{h}^c, 1 * \bar{h}^c]$.

Net output levels at the social optimum and at the decentralized equilibrium differ only by the net gain of firms' production minus some idiosyncratic preferences. We compute an "efficiency gap" as this difference in net output divided by the optimal value of net output. Figure 4 draws the evolution of this efficiency gap with α . As already mentioned, the efficiency gap is positive whenever α is positive. The gap is increasing with α and amounts to about 7% when $\alpha \rightarrow 1$.

²⁶To compute the optimal values of $-z_1 = z_2 = z \geq 0$ and of $p_1 = p_2 = p \geq 0$ (from which the corresponding value of θ is deducted), we discretize z in $[0; 0.172]$ and p in $[0, 1]$ (allowing each to take 9000 values), then we evaluate the social objective of the planner for each of the 9000×9000 values. Finally, we select the global optimum.

²⁷The value of $-\kappa/[m(\theta)(1 - p(\theta))]$ is -0.28 at the optimum, which is 8 times bigger than the difference $y - b$. This implies that the optimal z is bounded below by $x = 0$.

²⁸The efficient unemployment rate is already nil when $y = 1.0025$.

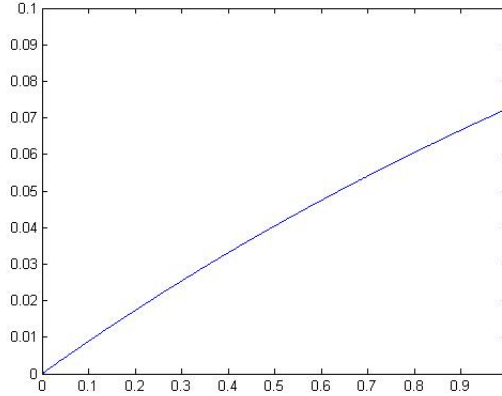


Figure 4: Evolution of the efficiency gap with α

4.3 A shock on α

As explained in Proposition 1, a rise in the matching effectiveness out of the region of residence has ambiguous impacts, even when regions are symmetric. In our stylized economy, increasing α by 25% leads to a limited rise in the unemployment rates (from 7% to 7.2%, see Table 1). This is due to a decrease in the acceptance rate, which induces firms to open less vacancies. As a result, the probability of receiving a job offer in one's region of residence goes down by 0.3%. This effect turns out to outweigh the negative effect of a rise in α on the unemployment rates conditional on tightness levels. Increasing α by 50% pushes the unemployment rates further up (7.5%).

α	π	u	p
0.12	0.989	0.07	0.929
0.15	0.986	0.072	0.927
0.18	0.984	0.075	0.924

Table 1: Impacts of a rise in α in the stylized symmetric economy

5 Extensions

Our main mechanisms should be robust to the introduction of more than two regions. With three regions for instance, the matching functions takes the form: $M_i(V_i, N_i + \alpha N_{-i}^N)$ where now N_{-i}^N stands for the population of the other two regions. Everything else equal, the importance of national job-seekers in the unemployed population (in efficiency units) is growing, which lowers the probability of accepting a job offer.

Lutgen and Van der Linden (2013) extend the theoretical framework by including regional housing markets. As in Moretti (2011), housing demand is totally inelastic (one

unit of dwelling per resident) and housing supply is increasing in the population size. The introduction of an endogenous housing market complicates the model a lot. Many more effects become ambiguous since they hinge upon the slope of the housing supply. For example, more valuable amenities in a region has now an ambiguous impact on the search and location decisions. At given rents, the size of the population increases in the better endowed region. When rents are endogenous, their rise in this region is a disincentive to search and locate there. If the housing supply is very inelastic, one could get that the population shrinks in the better endowed region.

Consider next a dynamic extension with infinitely-lived agents. As in the static framework, for national job-seekers, the partial effect of a rise in α is an increase in the probability of finding a job. In a continuous time setting, the probability of receiving two offers during a very small interval of time tends to zero. A rise in α now negatively affects equilibrium tightness through wages. When α rises, national job-seekers have an improved outside option and hence their bargained wages rise. Then, through the free entry of vacancies, labor demand shrinks. So, although the mechanism at work differs, the ambiguity remains.

6 Conclusion

This paper studies equilibrium unemployment in a two-region static economy where wages are endogenous and homogeneous workers and jobs are free to move. We develop a tractable search-matching equilibrium in which job-seekers can decide to search for a job in another region without first migrating there. Current communication technologies motivate this assumption. Since individuals have idiosyncratic and heterogeneous preferences for regions, part of the population chooses to seek a job all over the country while the rest only searches in the region where they live. These decisions affect the regional unemployment rates. Compared to the case where job-seekers can only search in their region of residence, search-matching externalities are amplified by the opportunity of searching in a region where one does not live and by the fact that some workers can simultaneously receive a job offer from each region. Hence, some vacant positions remain unfilled, which leads to a waste of resources. By this effect, if it is now much easier to search for a job all over the country thanks to the Internet, firms are less inclined to open vacancies. The shrinking labor demand and the higher probability of finding a job conditional on tightness are two opposite forces that hold true in a dynamic framework. They can explain the disagreement between Kroft and Pope (2014) at the city level and Kuhn and Mansour (2014) and Choi (2011) at the individual level.

We characterize the constrained efficient allocation, where the central planner is subject to the same matching frictions as the decentralized agents. In standard non-spatial search-matching models with wage bargaining, the *laissez-faire* decentralized economy is efficient if the Hosios condition is verified. This is also true when search effort is endogenous. In our model where job-seekers decide over their search field, the Hosios condition is not sufficient to guarantee efficiency when workers can search nationally. Workers and firms take decisions without internalizing the effect of their

choices on net output in both regions. Simulations show that the optimal allocation is a corner solution where no one searches all over the country (this result is very robust to changes in parameters) and the efficient regional unemployment rates are one percentage point lower than in the decentralized equilibrium.

This paper does not claim to have evaluated the general equilibrium impact of the Internet on the matching process. It has only focused on the implications of searching before possibly moving to another region under the standard assumptions of constant returns to scale in the matching process and in production. Beaudry *et al.* (2014) find no significant effects of agglomeration forces on productivity in the US. So, we feel confident that the latter assumption is not too strong a simplification. However, the presence of non negligible agglomeration forces would affect our conclusions.

References

- AMIOR, M. (2012). A Labor Market Cushion for Housing Market Shocks, mimeo, University College London, London.
- ANTOUN, R. (2010). Why Almost Similar Regions Have Different Unemployment Rates, mimeo, ERMES, Université Panthéon-Assas, Paris.
- AUTOR, D. H. (2001). Wiring the Labor Market. *Journal of Economic Perspectives*, **15** (1), 25–40.
- BAGUES, M. F. and LABINI, M. S. (2009). Do Online Labor Market Intermediaries Matter? The Impact of AlmaLaurea on the University-to-Work Transition. In *Studies of Labor Market Intermediation*, National Bureau of Economic Research, Inc, pp. 127–154.
- BEAUDRY, P., GREEN, D. A. and SAND, B. (2014). Spatial Equilibrium with Unemployment and Wage Bargaining: Theory and Estimation. *Journal of Urban Economics*, **79**, 2–19.
- BRUECKNER, J. K. and NEUMARK, D. (2014). Beaches, Sunshine, and Public Sector Pay: Theory and Evidence on Amenities and Rent Extraction by Government Workers. *American Economic Journal: Economic Policy*, **6** (2), 198–230.
- CHOI, E. J. (2011). Does the Internet Help the Unemployed Find Jobs?, mimeo, U.S. Bureau of Labor Statistics.
- COULSON, N. E., LAING, D. and WANG, P. (2001). Spatial Mismatch in Search Equilibrium. *Journal of Labor Economics*, **19** (4), pp. 949–972.
- DECREUSE, B. (2008). Choosy Search And The Mismatch Of Talents. *International Economic Review*, **49** (3), 1067–1089.
- DOMINGUES DOS SANTOS, M. (2011). Immigration policy, spatial job search and unemployment, mimeo, ERUDITE, Université Paris-Est, Paris.

- EPIFANI, P. and GANCIA, G. A. (2005). Trade, Migration and Regional Unemployment. *Regional Science and Urban Economics*, **35** (6), 625–644.
- FRANCIS, J. (2009). Agglomeration, job flows and unemployment. *The Annals of Regional Science*, **43** (1), 181–198.
- GALENIANOS, M. and KIRCHER, P. (2009). Directed search with multiple job applications. *Journal of Economic Theory*, **144** (2), 445–471.
- HOSIOS, A. (1990). On the efficiency of matching and related models of search and unemployment. *Review of Economic Studies*, **57**, 279–298.
- KENNAN, J. and WALKER, J. (2011). The Effect of Expected Income on Individual Migration Decisions. *Econometrica*, **79**, 211–251.
- KIRCHER, P. (2009). Efficiency of Simultaneous Search. *Journal of Political Economy*, **117** (5), 861–913.
- KLINE, P. and MORETTI, E. (2013). Place Based Policies with Unemployment. *American Economic Review, Papers and Proceedings*, **103** (3), 238–43.
- and — (2014). People, Places, and Public Policy: Some Simple Welfare Economics of Local Economic Development Programs. *Annual Review of Economics*, **6** (1), 629–662.
- KROFT, K. and POPE, D. G. (2014). Does Online Search Crowd Out Traditional Search and Improve Matching Efficiency? Evidence from Craigslist. *Journal of Labor Economics*, **32** (2), 259 – 303.
- KUHN, P. and MANSOUR, H. (2014). Is Internet Job Search Still Ineffective? *The Economic Journal*, **124** (581), 1213–1233.
- LKHAGVASUREN, D. (2012). Big Locational Unemployment Differences Despite High Labor Mobility. *Journal of Monetary Economics*, **59** (8), 798–814.
- LUCAS, R. E. and PRESCOTT, E. C. (1974). Equilibrium search and unemployment. *Journal of Economic Theory*, **7** (2), 188 – 209.
- LUTGEN, V. and VAN DER LINDEN, B. (2013). *Regional Equilibrium Unemployment Theory at the Age of the Internet*. IZA Discussion Papers 7763, Institute for the Study of Labor (IZA).
- MANNING, A. and PETRONGOLO, B. (2011). *How Local Are Labor Markets? Evidence from a Spatial Job Search Model*. IZA Discussion Papers 6178, Institute for the Study of Labor (IZA).
- MARINESCU, I. and RATHELOT, R. (2014). The Geography of Job Search and Mismatch Unemployment, mimeo.

- MOLHO, I. (2001). Spatial Search, Migration and Regional Unemployment. *Economica*, **68** (270), pp. 269–283.
- MORETTI, E. (2011). Local Labor Markets. In O. Ashenfelter and D. Card (eds.), *Handbook of Labor Economics*, vol. 4, *chapter 14*, Elsevier, pp. 1237–1313.
- NENOV, P. (2014). Regional Reallocation and Housing Markets in a Model of Frictional Migration, mimeo.
- PISSARIDES, C. A. (2000). *Equilibrium Unemployment Theory, 2nd Edition*. The MIT Press.
- (2009). The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer? *Econometrica*, **77** (5), 1339–1369.
- ROBACK, J. (1982). Wages, Rents, and the Quality of Life. *Journal of Political Economy*, **90** (6), 1257–78.
- ROSEN, H. S. (1979). Wage based indexes of urban quality of life. In P. N. Miezowski and M. R. Straszheim (eds.), *Current Issues in Urban Economics*, Johns Hopkins University Press, pp. 74–104.
- SAHIN, A., SONG, J., TOPA, G. and VIOLANTE, G. L. (2014). Mismatch unemployment. *American Economic Review*, **104** (11), 3529–64.
- TOPEL, R. H. (1986). Local Labor Markets. *Journal of Political Economy*, **94** (3), S111–43.
- WREDE, M. (2014). Wages, Rents, Unemployment, and the Quality of Life: A Consistent Theory-Based Measure. *Journal of Regional Science*, forthcoming.
- ZENOU, Y. (2009). *Urban Labor Economics*. Cambridge University Press.

Appendices

A Search and location decisions are taken simultaneously

The following table summarizes the different cases an agent faces when $\alpha > 0$:

	Where to search		
Where to live	Region 1	Region 2	Region 1 and region 2
Region 1	case a	case b	case c
Region 2	case d	case e	case f

We will proceed by first computing the expected utility of an individual in each case. By doing so, we will be able to drop cases b and d. In a second step, we will define 3 thresholds out of the 6 that could be computed from the 4 remaining cases. By ranking these thresholds and comparing the expected utility levels, we will be able to rank the expected utility levels and show that these thresholds are those we get if we assume that agents take the location and search decisions sequentially.

1. Expected utility in each case

Case a: The utility if the individual lives in 1 and searches in 1 only

$$p_1 V_{1j}^e + (1 - p_1) V_{1j}^u$$

Case b: The utility if the individual lives in 1 and searches in 2 only

$$\alpha p_2 V_{2j}^e + (1 - \alpha p_2) V_{1j}^u$$

Case c: The utility if the individual lives in 1 and searches in both regions

$$p_1(1 - \alpha p_2) V_{1j}^e + \alpha p_2(1 - p_1) V_{2j}^e + (1 - p_1)(1 - \alpha p_2) V_{1j}^u + \alpha p_1 p_2 \left[\max \{ V_{1j}^e; V_{2j}^e \} - \varepsilon \right]$$

Case d: The utility if the individual lives in 2 and searches in 1 only

$$\alpha p_1 V_{1j}^e + (1 - \alpha p_1) V_{2j}^u$$

Case e: The utility if the individual lives in 2 and searches in 2 only

$$p_2 V_{2j}^e + (1 - p_2) V_{2j}^u$$

Case f: The utility if the individual lives in 2 and searches in both regions

$$p_2(1 - \alpha p_1) V_{2j}^e + \alpha p_1(1 - p_2) V_{1j}^e + (1 - p_2)(1 - \alpha p_1) V_{2j}^u + \alpha p_1 p_2 \left[\max \{ V_{1j}^e; V_{2j}^e \} - \varepsilon \right]$$

We assume that the cost ε of refusing a job offer tends to zero.

2. Case b is dominated by case c if

$$\begin{aligned} & \alpha p_2 V_{2j}^e + (1 - \alpha p_2) V_{1j}^u \\ & < p_1(1 - \alpha p_2) V_{1j}^e + \alpha p_2(1 - p_1) V_{2j}^e + (1 - p_1)(1 - \alpha p_2) V_{1j}^u + \alpha p_1 p_2 \max \{ V_{1j}^e; V_{2j}^e \} \\ & \Leftrightarrow 0 < (1 - \alpha p_1 p_2) V_{1j}^u + p_1(V_{1j}^e - V_{1j}^u) + \alpha p_1 p_2 \left[\max \{ V_{1j}^e; V_{2j}^e \} - V_{1j}^e - V_{2j}^e + V_{1j}^u \right] \end{aligned}$$

Two sub-cases should be considered:

- If $\max \{ V_{1j}^e; V_{2j}^e \} = V_{2j}^e$, then the comparison becomes:

$$\begin{aligned} 0 & < (1 - \alpha p_1 p_2) V_{1j}^u + p_1(V_{1j}^e - V_{1j}^u) + \alpha p_1 p_2(V_{1j}^u - V_{1j}^e) \\ 0 & < (1 - \alpha p_1 p_2) V_{1j}^u + (1 - \alpha p_2) p_1(V_{1j}^e - V_{1j}^u) \end{aligned}$$

This always holds. Similarly,

- If $\max \{ V_{1j}^e; V_{2j}^e \} = V_{1j}^e$, then the comparison becomes:

$$\begin{aligned} 0 & < (1 - \alpha p_1 p_2) V_{1j}^u + p_1(V_{1j}^e - V_{1j}^u) + \alpha p_1 p_2(V_{1j}^u - V_{2j}^e) \\ 0 & < (1 - \alpha p_1 p_2) V_{1j}^u + (1 - \alpha p_2) p_1(V_{1j}^e - V_{1j}^u) + \alpha p_1 p_2(V_{1j}^e - V_{2j}^e) \end{aligned}$$

This always holds since $V_{1j}^e \geq V_{2j}^e$. As a result, *case b* will never be optimal for agent j .

3. *Case d* is dominated by *case f* if

$$\begin{aligned} & \alpha p_1 V_{1j}^e + (1 - \alpha p_1) V_{2j}^u \\ & < p_2(1 - \alpha p_1) V_{2j}^e + \alpha p_1(1 - p_2) V_{1j}^e + (1 - p_2)(1 - \alpha p_1) V_{2j}^u + \alpha p_1 p_2 \max \{V_{1j}^e; V_{2j}^e\} \\ & \Leftrightarrow 0 < (1 - \alpha p_1 p_2) V_{2j}^u + p_2(V_{2j}^e - V_{2j}^u) + \alpha p_1 p_2 [\max \{V_{1j}^e; V_{2j}^e\} - V_{1j}^e - V_{2j}^e + V_{2j}^u] \end{aligned}$$

Two sub-cases should be considered:

- If $\max \{V_{1j}^e; V_{2j}^e\} = V_{1j}^e$, then the comparison becomes:

$$\begin{aligned} 0 & < (1 - \alpha p_1 p_2) V_{2j}^u + p_2(V_{2j}^e - V_{2j}^u) + \alpha p_1 p_2 (V_{2j}^u - V_{2j}^e) \\ 0 & < (1 - \alpha p_1 p_2) V_{2j}^u + (1 - \alpha p_1) p_2 (V_{2j}^e - V_{2j}^u) \end{aligned}$$

This always holds. Similarly,

- If $\max \{V_{1j}^e; V_{2j}^e\} = V_{2j}^e$, then the comparison becomes:

$$\begin{aligned} 0 & < (1 - \alpha p_1 p_2) V_{2j}^u + p_2(V_{2j}^e - V_{2j}^u) + \alpha p_1 p_2 (V_{2j}^u - V_{1j}^e) \\ 0 & < (1 - \alpha p_1 p_2) V_{2j}^u + (1 - \alpha p_1) p_2 (V_{2j}^e - V_{2j}^u) + \alpha p_1 p_2 (V_{2j}^e - V_{1j}^e) \end{aligned}$$

This always holds since $V_{2j}^e \geq V_{1j}^e$. As a result, *case d* will never be optimal for agent j .

4. Defining the thresholds

With the 4 remaining cases, we define 3 threshold values and show that this is sufficient to get a dominant case for each value of $c_{1j} - c_{2j}$:

Definition of the threshold between *case c* and *case f*

$$\begin{aligned} & p_1(1 - \alpha p_2) V_{1j}^e + \alpha p_2(1 - p_1) V_{2j}^e + (1 - p_1)(1 - \alpha p_2) V_{1j}^u + \alpha p_1 p_2 \max \{V_{1j}^e; V_{2j}^e\} \\ & = p_2(1 - \alpha p_1) V_{2j}^e + \alpha p_1(1 - p_2) V_{1j}^e + (1 - p_2)(1 - \alpha p_1) V_{2j}^u + \alpha p_1 p_2 \max \{V_{1j}^e; V_{2j}^e\} \\ & \Leftrightarrow (1 - \alpha) p_1 V_{1j}^e - (1 - \alpha) p_2 V_{2j}^e + (1 - p_1)(1 - \alpha p_2) V_{1j}^u - (1 - p_2)(1 - \alpha p_1) V_{2j}^u = 0 \\ & \Leftrightarrow (1 - \alpha) p_2 (V_{2j}^e - V_{2j}^u) - (1 - \alpha) p_1 (V_{1j}^e - V_{1j}^u) \\ & = (1 - \alpha p_1 - \alpha p_2 + \alpha p_1 p_2) (V_{1j}^u - V_{2j}^u) = 0 \end{aligned}$$

Using the definitions of utilities, we get equation (18):

$$x = b_2 - b_1 + a_2 - a_1 + (1 - \alpha) \frac{p_2(w_2 - b_2) - p_1(w_1 - b_1)}{1 - \alpha p_1 - \alpha p_2 + \alpha p_1 p_2}$$

Definition of the threshold between case e and case f

$$\begin{aligned}
p_2 V_{2j}^e + (1 - p_2) V_{2j}^u &= p_2 (1 - \alpha p_1) V_{2j}^e + \alpha p_1 (1 - p_2) V_{1j}^e \\
&\quad + (1 - p_2) (1 - \alpha p_1) V_{2j}^u + \alpha p_1 p_2 \max \{ V_{1j}^e; V_{2j}^e \} \\
\Leftrightarrow \alpha p_1 (V_{1j}^e - V_{2j}^u) + \alpha p_1 p_2 [\max \{ V_{1j}^e; V_{2j}^e \} - V_{1j}^e - V_{2j}^e + V_{2j}^u] &= 0
\end{aligned}$$

Two sub-cases should be considered:

- If $\max \{ V_{1j}^e; V_{2j}^e \} = V_{2j}^e$,

$$\begin{aligned}
\alpha p_1 (1 - p_2) (V_{1j}^e - V_{2j}^u) &= 0 \\
\Leftrightarrow V_{1j}^e = V_{2j}^u \text{ as } 0 < p_i < 1 \text{ and } \alpha > 0 \\
\Leftrightarrow z_1 = b_1 - w_1 + b_2 - b_1 + a_2 - a_1
\end{aligned}$$

- If $\max \{ V_{1j}^e; V_{2j}^e \} = V_{1j}^e$,

$$\begin{aligned}
\alpha p_1 (1 - p_2) (V_{1j}^e - V_{2j}^u) + \alpha p_1 p_2 (V_{1j}^e - V_{2j}^e) &= 0 \\
\Leftrightarrow \tilde{z}_1 = b_1 - w_1 + b_2 - b_1 + a_2 - a_1 + p_2 (w_2 - b_2) \text{ as } 0 < p_i < 1 \text{ and } \alpha > 0
\end{aligned}$$

However, the assumption of the sub-cases, $V_{1j}^e \geq V_{2j}^e$, implies that in this case relative preference are such that:

$$c_{1j} - c_{2j} \geq b_1 - w_1 + b_2 - b_1 + a_2 - a_1 + w_2 - b_2 > \tilde{z}_1$$

which leads a contradiction as we assume in the definition of \tilde{z}_1 that $V_{1j}^e \geq V_{2j}^e$. Therefore, the only possible threshold value between case e and case f is z_1 .

Definition of the threshold between case a and case c

$$\begin{aligned}
p_1 V_{1j}^e + (1 - p_1) V_{1j}^u &= p_1 (1 - \alpha p_2) V_{1j}^e + \alpha p_2 (1 - p_1) V_{2j}^e \\
&\quad + (1 - p_1) (1 - \alpha p_2) V_{1j}^u + \alpha p_1 p_2 \max \{ V_{1j}^e; V_{2j}^e \} \\
\Leftrightarrow \alpha p_2 (V_{2j}^e - V_{1j}^u) + \alpha p_1 p_2 [\max \{ V_{1j}^e; V_{2j}^e \} - V_{1j}^e - V_{2j}^e + V_{1j}^u] &= 0
\end{aligned}$$

Two sub-cases should be considered:

- If $\max \{ V_{1j}^e; V_{2j}^e \} = V_{1j}^e$,

$$\begin{aligned}
\alpha p_2 (1 - p_1) (V_{2j}^e - V_{1j}^u) &= 0 \\
\Leftrightarrow V_{2j}^e = V_{1j}^u \text{ as } 0 < p_i < 1 \text{ and } \alpha > 0 \\
\Leftrightarrow z_2 = w_2 - b_2 + b_2 - b_1 + a_2 - a_1
\end{aligned}$$

- If $\max \{V_{1j}^e; V_{2j}^e\} = V_{2j}^e$,

$$\alpha p_2(1 - p_1)(V_{2j}^e - V_{1j}^u) + \alpha p_1 p_2(V_{2j}^e - V_{1j}^e) = 0$$

$$\Leftrightarrow \tilde{z}_2 = w_2 - b_2 + b_2 - b_1 + a_2 - a_1 - p_1(w_1 - b_1) \text{ as } 0 < p_i < 1 \text{ and } \alpha > 0$$

However, the assumption of the sub-case, $V_{2j}^e \geq V_{1j}^e$, implies that in this case relative preference are such that:

$$c_{1j} - c_{2j} \leq w_2 - b_2 + b_2 - b_1 + a_2 - a_1 + b_1 - w_1 < \tilde{z}_1$$

which leads to a contradiction as we assume in the definition of \tilde{z}_2 that $V_{2j}^e \geq V_{1j}^e$. Therefore, the only possible threshold value between *case a* and *case c* is z_2 .

5. Ranking the thresholds

It is easily seen that $z_1 \leq x \leq z_2$.

6. Dominant strategies

	z_1	x	z_2
f dominates c	f dominates c	c dominates f	c dominates f
e dominates f	f dominates e	f dominates e	f dominates e
c dominates a	c dominates a	c dominates a	a dominates c
$e > f > c > a$	$f > c > a$ and $f > e$	$c > f > e$ and $c > a$	$a > c > f > e$
e gives the highest utility	f gives the highest utility	c gives the highest utility	a gives the highest utility

Figure 5: Dominant strategies

- the individual whose relative preference is x is indifferent between living in 1 and searching in both regions (*case c*) and living in 2 and searching in both regions (*case f*);
- the individual whose relative preference is z_1 is indifferent between living in 2 and searching in 2 only (*case e*) and living in 2 and searching in both regions (*case f*);
- the individual whose relative preference is z_2 is indifferent between living in 1 and searching in 1 only (*case a*) and living in 1 and searching in both (*case c*);

We conclude from Figure 5 that the three threshold values we chose at first are sufficient to get a dominant strategy for each value of the relative preference $c_{1j} - c_{2j}$. These values are equivalent to those obtained when location and search decisions are taken sequentially.

B Proof of Proposition 2

When $\alpha = 1$, Proposition 2 is obvious.

When $0 < \alpha < 1$, we first provide conditions guaranteeing that $\partial x/\partial p_2 > 0$ and $\partial x/\partial p_1 < 0$. These conditions depend on endogenous variables. We finally derive a unique sufficient condition on α .

$$\frac{\partial x}{\partial p_2} = \frac{1 - \alpha}{(1 - \alpha p_1 - \alpha p_2 + \alpha p_1 p_2)^2} ((1 - \alpha p_1)\beta_2(y_2 - b_2) - \alpha p_1(1 - p_1)\beta_1(y_1 - b_1))$$

which has a positive sign if:

$$\alpha < \frac{\beta_2(y_2 - b_2)}{p_1(\beta_2(y_2 - b_2) + (1 - p_1)\beta_1(y_1 - b_1))} \quad (43)$$

To obtain a sufficient condition depending on parameters only, we should now minimize the RHS of (43) by maximizing the denominator with respect to $p_1 \in (0, 1)$. The unique maximum is

$$p_1^* = \min\left(\frac{\beta_1(y_1 - b_1) + \beta_2(y_2 - b_2)}{2\beta_1(y_1 - b_1)}, 1\right)$$

If $\beta_2(y_2 - b_2) < \beta_1(y_1 - b_1)$, $p_1^* < 1$ and a sufficient condition for (43) is

$$\alpha < \frac{4\beta_1(y_1 - b_1)\beta_2(y_2 - b_2)}{(\beta_1(y_1 - b_1) + \beta_2(y_2 - b_2))^2} \quad (44)$$

Otherwise, a sufficient condition is $\alpha < 1$.

Similarly $\partial x/\partial p_1 < 0$ if

$$\alpha < \frac{\beta_1(y_1 - b_1)}{p_2(\beta_1(y_1 - b_1) + (1 - p_2)\beta_2(y_2 - b_2))} \quad (45)$$

Minimizing this ratio with respect to $p_2 \in (0, 1)$ yields

$$p_2^* = \min\left(\frac{\beta_1(y_1 - b_1) + \beta_2(y_2 - b_2)}{2\beta_2(y_2 - b_2)}, 1\right)$$

If $\beta_2(y_2 - b_2) > \beta_1(y_1 - b_1)$, $p_2^* < 1$ and a sufficient condition for (45) is (44). Otherwise, a sufficient condition is $\alpha < 1$. To guarantee that both $\partial x/\partial p_2 > 0$ and $\partial x/\partial p_1 < 0$, we conclude that (44), i.e. (19) in Proposition 2, is the unique sufficient condition, its RHS being in any case lower than one.

C Conditional acceptance rates

In this appendix, we show formulas (20) and (21). In the first part, we focus on the conditional acceptance rate when the vacancy is located in region 1. We then turn to the opposite case.

C.1 Conditional acceptance rate in region 1

A vacant position located in region 1 faces $\frac{v-x+\alpha(x-z_1)}{2v}N$ possible workers in efficiency units. These workers always accept a job offer from the firm if their relative preference for region 1 over region 2, $c_{1j} - c_{2j}$, is higher than \hat{x} . If their relative preference is below \hat{x} , job-seekers only accept the offer if they have not received one from a firm located in region 2.

We thus compute the conditional acceptance rate as :

$$\frac{2v}{v-x+\alpha(x-z_1)} \{P(c_{1j} - c_{2j} \geq \hat{x})1 + P(c_{1j} - c_{2j} < \hat{x}) P(\text{no offer from 2})\}$$

There are two sub-cases: Whether x is lower or greater than \hat{x} .

Whenever $x < \hat{x}$,

Relative preference	Proba to have this preference	Proba to accept a position in 1
$v > c_{1j} - c_{2j} > \hat{x}$	$\frac{v-\hat{x}}{2v}$	1
$\hat{x} > c_{1j} - c_{2j} > x$	$\frac{\hat{x}-x}{2v}$	$1 - \alpha p_2$
$x > c_{1j} - c_{2j} > z_1$	$\frac{\alpha(x-z_1)}{2v}$	$1 - p_2$

The conditional acceptance rate is thus:

$$\frac{2v}{v-x+\alpha(x-z_1)} \left\{ \frac{v-\hat{x}}{2v} + \frac{\hat{x}-x}{2v}(1 - \alpha p_2) + \alpha \frac{x-z_1}{2v}(1 - p_2) \right\}$$

which leads to equation (20).

Whenever $x > \hat{x}$,

Relative preference	Proba to have this preference	Proba to accept a position in 1
$v > c_{1j} - c_{2j} > x$	$\frac{v-x}{2v}$	1
$x > c_{1j} - c_{2j} > \hat{x}$	$\frac{\alpha(x-\hat{x})}{2v}$	1
$\hat{x} > c_{1j} - c_{2j} > z_1$	$\frac{\alpha(\hat{x}-z_1)}{2v}$	$1 - p_2$

The condition acceptance rate is thus:

$$\frac{2v}{v-x+\alpha(x-z_1)} \left\{ \frac{v-x}{2v} + \frac{\alpha(x-\hat{x})}{2v}1 + \alpha \frac{\hat{x}-z_1}{2v}(1 - p_2) \right\}$$

which leads to equation (20) as well.

C.2 Conditional acceptance rate in region 2

A vacant position located in region 2 faces $\frac{v+x+\alpha(z_2-x)}{2v}N$ possible workers in efficiency units. These workers always accept a job offer from the firm if their relative preference for region 1 over region 2, $c_{1j} - c_{2j}$, is lower than \hat{x} . If their relative preference is higher

\hat{x} , job-seekers only accept the offer if they have not received one from a firm located in region 1.

We thus compute the conditional acceptance rate as :

$$\frac{2v}{v+x+\alpha(z_2-x)} \{P(c_{1j}-c_{2j} < \hat{x})1 + P(c_{1j}-c_{2j} \geq \hat{x}) P(\text{no offer from 1})\}$$

Here again there are two sub-cases: Whether x is lower or greater than \hat{x} .

Whenever $x < \hat{x}$,

Relative preference	Proba to have this preference	Proba to accept a position in 2
$-v < c_{1j} - c_{2j} < x$	$\frac{v+x}{2v}$	1
$x < c_{1j} - c_{2j} < \hat{x}$	$\frac{\alpha(\hat{x}-x)}{2v}$	1
$\hat{x} < c_{1j} - c_{2j} < z_2$	$\frac{\alpha(z_2-\hat{x})}{2v}$	$1 - p_1$

The condition acceptance rate is thus:

$$\frac{2v}{v+x+\alpha(z_2-x)} \left\{ \frac{v+x}{2v} + \frac{\alpha(\hat{x}-x)}{2v} 1 + \alpha \frac{z_2-\hat{x}}{2v} (1-p_1) \right\}$$

which leads to equation (21).

Whenever $x > \hat{x}$,

Relative preference	Proba to have this preference	Proba to accept a position in 2
$-v < c_{1j} - c_{2j} < \hat{x}$	$\frac{v+\hat{x}}{2v}$	1
$\hat{x} < c_{1j} - c_{2j} < x$	$\frac{x-\hat{x}}{2v}$	$1 - \alpha p_1$
$x < c_{1j} - c_{2j} < z_2$	$\frac{\alpha(z_2-x)}{2v}$	$1 - p_1$

The condition acceptance rate is thus:

$$\frac{2v}{v+x+\alpha(z_2-x)} \left\{ \frac{v+\hat{x}}{2v} + \frac{x-\hat{x}}{2v} (1-\alpha p_1) + \alpha \frac{z_2-x}{2v} (1-p_1) \right\}$$

which leads again to equation (21).

D Cross partial derivative of unemployment rates

The cross-partial derivatives of the unemployment rates with respect to tightness in the other region are respectively:

$$\text{sign} \left\{ \frac{\partial u_1}{\partial \theta_2} \right\} = \text{sign} \{ \alpha p_1 [(v-x)(x-\Delta) - \alpha(z_2-x)(x-z_1)] \} \quad (46)$$

$$\text{sign} \left\{ \frac{\partial u_2}{\partial \theta_1} \right\} = \text{sign} \{ \alpha p_2 [(v+x)(\Delta-x) - \alpha(z_2-x)(x-z_1)] \} \quad (47)$$

We directly notice that one of these two derivatives is always negative:

$$\begin{aligned} x - \Delta < 0 \quad \text{i.e.} \quad p_2\beta_2(y_2 - b_2) < p_1\beta_1(y_1 - b_1) &\Rightarrow \frac{\partial u_2}{\partial \theta_1} \begin{matrix} \geq \\ \leq \end{matrix} 0 \text{ and } \frac{\partial u_1}{\partial \theta_2} < 0, \\ x - \Delta > 0 \quad \text{i.e.} \quad p_2\beta_2(y_2 - b_2) > p_1\beta_1(y_1 - b_1) &\Rightarrow \frac{\partial u_1}{\partial \theta_2} \begin{matrix} \geq \\ \leq \end{matrix} 0 \text{ and } \frac{\partial u_2}{\partial \theta_1} < 0. \end{aligned}$$

To limit the extent of these ambiguities, we first compute the limits when α tends to 0 and to 1. We then check how the functions could behave between these two limits. From (46), (47) and (18), the following limit values are obvious:

$$\begin{aligned} \lim_{\alpha \rightarrow 0} \frac{\partial u_1}{\partial \theta_2} &= \lim_{\alpha \rightarrow 0} \frac{\partial u_2}{\partial \theta_1} = 0; \\ \lim_{\alpha \rightarrow 1} \text{sign} \left\{ \frac{\partial u_1}{\partial \theta_2} \right\} &= \lim_{\alpha \rightarrow 1} \text{sign} \left\{ \frac{\partial u_2}{\partial \theta_1} \right\} = \text{sign} \{ -(z_2 - x)(x - z_1) \} < 0. \end{aligned}$$

It can furthermore be shown²⁹ that:

$$\begin{aligned} \lim_{\alpha \rightarrow 0^+} \text{sign} \left\{ \frac{\partial u_1}{\partial \theta_2} \right\} &= \text{sign} \{ (x - \Delta)(v - x) \}, \\ \lim_{\alpha \rightarrow 0^+} \text{sign} \left\{ \frac{\partial u_2}{\partial \theta_1} \right\} &= \text{sign} \{ (\Delta - x)(v + x) \}. \end{aligned}$$

Between the two extremes, we cannot go further in the analysis. Because of the limit when $\alpha \rightarrow 1$, we however can conclude that:

- When $x - \Delta > 0$, $\frac{\partial u_2}{\partial \theta_1} < 0 \forall \alpha > 0$ and $\frac{\partial u_1}{\partial \theta_2} < 0$ if α is large enough;
- When $x - \Delta < 0$, $\frac{\partial u_1}{\partial \theta_2} < 0 \forall \alpha > 0$ and $\frac{\partial u_2}{\partial \theta_1} < 0$ if α is large enough;
- When $x - \Delta = 0$, both partial derivatives are always negative.

²⁹A proof is available upon request. It can actually be shown that the partial derivatives (46) and (47) have the same sign as a polynomial of degree four in α , whose constant term is respectively $(v - x)(x - \Delta)$ and $(v + x)(\Delta - x)$.

E Existence of a symmetric equilibrium

When both regions have the same exogenous characteristics, a free-entry symmetric equilibrium is characterized by the following conditions:

$$\begin{aligned}\hat{x} &= x = 0 \\ z_2 &= \beta(y - b) = -z_1 \\ \pi m(\theta) &= \frac{\kappa}{(1 - \beta)(y - b)}\end{aligned}\tag{48}$$

$$\pi = 1 - \frac{\alpha p(\theta) z_2}{v + \alpha z_2}\tag{49}$$

$$u = \frac{(1 - p(\theta))(v - \alpha p(\theta) z_2)}{v}$$

$$N_1^P = N_2^P = 1/2$$

and (24) and (25). This system of equations can be solved recursively. First, the z_1 and z_2 thresholds are functions of parameters only. Second, combining equations (48) and (49) leads to the following implicit relationship in equilibrium tightness:

$$\frac{v + \alpha(1 - p(\theta))\beta(y - b)}{v + \alpha\beta(y - b)} = \frac{1}{m(\theta)} \frac{\kappa}{(1 - \beta)(y - b)}$$

The left-hand side is a negative function of tightness, while the right-hand side depends positively on tightness. So, there is at most one equilibrium. To show the existence of the equilibrium, consider the limit of each side of the last equality when θ tends to 0:

$$\begin{aligned}\lim_{\theta \rightarrow 0} \frac{v + \alpha(1 - p(\theta))\beta(y - b)}{v + \alpha\beta(y - b)} &= 1 \\ \lim_{\theta \rightarrow 0} \frac{1}{m(\theta)} \frac{\kappa}{(1 - \beta)(y - b)} &= 0, \text{ by the Inada conditions.}\end{aligned}$$

So, a unique symmetric equilibrium tightness exists. The other endogenous variables are then determined uniquely as well.

F *Ex-ante* bargaining

The aim of this appendix is to check whether the *ex-ante* bargaining process leads to an efficient allocation under the Hosios condition. In the five following subsections, we derive and briefly explain the key equations that are modified compared to the model described in the paper. In the last subsection, we compare the decentralized equilibrium free-entry conditions with the efficient allocation, and show that the Hosios condition is never sufficient to restore efficiency.

When bargaining *ex-ante*, the timing is defined as follows³⁰:

³⁰An alternative timing would be that workers who get two job offers bargain with the firms before refusing one of the job offers. This would lead to Bertrand competition between the two firms for this worker. It has however been checked that this lead to further sources of inefficiency.

1. Workers decide where to locate and where to search.
2. Firms open vacancies and the matching process takes place.
3. In case a worker gets two offers, she chooses one of them.
4. Workers bargain with the firm.
5. Workers relocate if the accepted position is in the other region.
6. Production takes place and both the housing and the good markets clear.

F.1 Wage bargaining

Since there are, for each worker, two potential fall-back positions when bargaining (being unemployed in region 1 or in region 2), there will be four wages in the economy, which may depend on the worker's relative idiosyncratic preference. Let w_{ikj} denote the wage of individual j living in region k *ex-ante* and who works for a firm located in i and V_{ikj}^e the utility she gets in this case. The wage w_{ikj} verifies:

$$\max_{w_{ikj}} (V_{ikj}^e - V_{kj}^u)^{\beta_i} (J_{ij})^{1-\beta_i}$$

where

$$V_{ikj}^e - V_{kj}^u = w_{ikj} - b_k + a_i - a_k + c_{ij} - c_{kj}$$

This leads to the following wages:

$$w_{ikj} = \beta_i y_i + (1 - \beta_i) b_i - (1 - \beta_i) (b_i - b_k + a_i - a_k + (c_{ij} - c_{kj})) \quad (50)$$

or

$$\begin{aligned} w_{11j} &= w_{11} = \beta_1 y_1 + (1 - \beta_1) b_1 \\ w_{12j} &= \beta_1 y_1 + (1 - \beta_1) b_1 + (1 - \beta_1) (\Delta - (c_{1j} - c_{2j})) \end{aligned} \quad (51)$$

$$\begin{aligned} w_{22j} &= w_{22} = \beta_2 y_2 + (1 - \beta_2) b_2 \\ w_{21j} &= \beta_2 y_2 + (1 - \beta_2) b_2 - (1 - \beta_2) (\Delta - (c_{1j} - c_{2j})) \end{aligned} \quad (52)$$

It is worth mentioning that *ex-ante* bargaining allows workers coming from the other region to be compensated for the difference in leisure, amenities and idiosyncratic preferences (see the term $\Delta - (c_{1j} - c_{2j})$ in (51) and (52)).

F.2 Acceptance decisions

Acceptance decisions are conditional on the region of residence, as wages in the other region depend on it. Workers located in region 1 decide whether they prefer working in region 1 by comparing V_{11j}^e and V_{21j}^e . This leads to the following threshold:

$$\hat{x}_2 = \Delta + y_2 - b_2 - \frac{\beta_1}{\beta_2} (y_1 - b_1)$$

Considering now workers located in region 2 *ex-ante*, one gets:

$$\hat{x}_1 = \Delta + \frac{\beta_2}{\beta_1}(y_2 - b_2) - (y_1 - b_1)$$

When regions are asymmetric, these two thresholds are equal if and only if $\beta_1 = \beta_2$. As we aim at checking whether an *ex-ante* bargaining process leads to an efficient allocation under Hosios, we make the assumption that $\beta_1 = \beta_2$, so that we get a unique threshold \hat{x} . This assumption is made as the central planner would always choose a unique \hat{x} threshold. For, assume that $\beta_1 \neq \beta_2$, so that there exist two threshold values \hat{x}_1 and \hat{x}_2 . The only possibility that these two thresholds are simultaneously meaningful is when we have $\hat{x}_1 < x < \hat{x}_2$.³¹ This case is represented by Figure 6 and is never optimal. For, take a first worker whose relative preference for region 1 over region 2 lies between \hat{x}_1 and x . This worker would choose to work in region 1. Take then a second worker whose relative preference lies between x and \hat{x}_2 . She chooses to worker in region 2. As assumed in the paper, workers are homogenous in productivity in a given region. Thus, interchanging these 2 workers (the first worker would then work in region 2 and the second one in region 1) does not modify the total levels of production nor the ex-post population sizes. However, this will change the levels of workers' idiosyncratic preferences. In this regard, interchanging the first worker and the second worker would lead to a higher level of preferences (as workers having a relative preference between x and \hat{x}_2 value more region 1 relative to region 2 than workers with a relative preference between \hat{x}_1 and x), without modifying the level of production nor the population sizes. The central planner thus prefer this situation than the initial one. This induces that having two distinct thresholds \hat{x} is never optimal. Therefore, for the rest of this appendix, we assume that $\beta_1 = \beta_2 = \beta$, so that we do not face a multi-thresholds equilibrium for the acceptance decision.

F.3 Opening of vacancies

We assume free-entry of firms. Firms open vacancies until the expected profit is nil: $\pi_i m_i(\theta_i)(y_i - w_i^e) - \kappa_i = 0$, where w_i^e is the wage a firm located in region i is expected to pay. This can be rewritten as:

$$\frac{\kappa_i}{m_i(\theta_i)} = \pi_i(y_i - w_i^e)$$

F.4 Location and search decisions

The location and search decisions are taken simultaneously. With the assumption on the β 's, it is easily shown that taking the decision simultaneously of choosing first where to locate and then where to search is equivalent (one needs to proceed as in Appendix A). To ease the exposition, we focus on the second procedure.

³¹This situation assumes that the threshold x is unique. This unicity is verified with the assumption we make regarding the β 's.

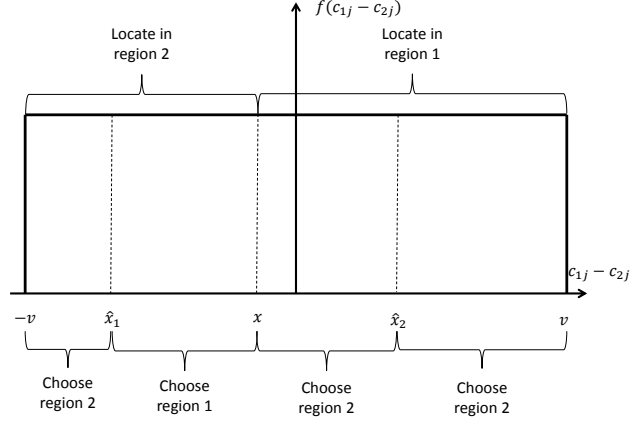


Figure 6: Case of two different \hat{x} thresholds

F.4.1 Search decision

A worker located in region 2 searches regionally when:

$$0 > -\alpha p_1 p_2 V_{22j}^e + \alpha p_1 p_2 [\max \{V_{22j}^e; V_{12j}^e\} - \epsilon] + \alpha p_1 (1 - p_2)(V_{12j}^e - V_{2j}^u)$$

Assuming that ϵ tends to 0, one gets the following threshold value whenever $V_{22j}^e \geq V_{12j}^e$:

$$z_1 = \Delta - (y_1 - b_1) \quad (53)$$

When $V_{22j}^e < V_{12j}^e$, one gets:

$$\tilde{z}_1 = \Delta - (y_1 - b_1) + p_2(y_2 - b_2)$$

However, $V_{22j}^e < V_{12j}^e$ implies that:

$$c_{1j} - c_{2j} = \Delta - (y_1 - b_1) + y_2 - b_2 > \tilde{z}_1$$

which leads to a contradiction, as we assume in the definition of \tilde{z}_1 that $V_{22j}^e < V_{12j}^e$. We thus get a unique threshold value, z_1 , for the search decision of agents located in region 2.

Similarly, we get the following threshold for workers located in region 1:

$$z_2 = \Delta + y_2 - b_2 \quad (54)$$

F.4.2 Location choice

The marginal worker that decides where to locate is a national job-seeker. One thus needs to compare the expected utility of a national job-seeker in both regions. One gets as threshold value:

$$x = \Delta_2 + \frac{(1 - \alpha)(p_2 \beta (y_2 - b_2) - p_1 \beta (y_1 - b_1))}{1 - \alpha \beta p_1 - \alpha \beta p_2 + \alpha \beta p_1 p_2} \quad (55)$$

F.5 Population, acceptance rates and expected wages

Populations can be described as in the paper, using the new threshold definitions. Acceptance rates are given, as in the *ex-post* bargaining case, by equations (20)-(21). Knowing how workers locate and search, one can compute the expected wage of a firm. In region 1, the wage bill can be rewritten as:

$$\begin{aligned} \frac{N}{2v}(\beta y_1 + (1 - \beta)b_1) [p_1(v - x) + \alpha p_1(1 - p_2)(x - z_1) + \alpha p_1 p_2(x - \hat{x})] \\ + \frac{N}{2v}(1 - \beta)\alpha p_1(1 - p_2) \int_{z_1}^x (\Delta - (c_{1j} - c_{2j}))dj \\ + \frac{\max\{x - \hat{x}; 0\}}{x - \hat{x}} \frac{N}{2v}(1 - \beta)\alpha p_1 p_2 \int_{\hat{x}}^x (\Delta - (c_{1j} - c_{2j}))dj \end{aligned}$$

As the number of employed workers in region 1 is given by

$$L_1 = \frac{N}{2v} [p_1(v - x) + \alpha p_1(1 - p_2)(x - z_1) + \alpha p_1 p_2(x - \hat{x})],$$

we can write the expected wage in region 1 as:

$$\begin{aligned} w_1^e = \beta y_1 + (1 - \beta)b_1 + \frac{(1 - \beta)(1 - p_2)\alpha(x - z_1)(\Delta - \frac{x+z_1}{2})}{v - x + \alpha(x - z_1) - \alpha p_2(\hat{x} - x)} \\ + \frac{(1 - \beta)\alpha p_2 \max\{x - \hat{x}; 0\} (\Delta - \frac{x+\hat{x}}{2})}{v - x + \alpha(x - z_1) - \alpha p_2(\hat{x} - z_1)} \end{aligned}$$

So, plugging this equation in the free-entry condition yields:

$$\begin{aligned} \frac{\kappa_1}{(1 - \beta)m_1(\theta_1)} = \pi_1(y_1 - b_1) - \frac{\alpha(1 - p_2)(x - z_1)}{v - x + \alpha(x - z_1)} \left(\Delta - \frac{x + z_1}{2} \right) \\ - \frac{\alpha p_2 \max\{x - \hat{x}; 0\}}{v - x + \alpha(x - z_1)} \left(\Delta - \frac{x + \hat{x}}{2} \right) \end{aligned} \quad (56)$$

Similarly, one gets for the expected wage in region 2:

$$\begin{aligned} w_2^e = \beta y_2 + (1 - \beta)b_2 - \frac{(1 - \beta)(1 - p_1)\alpha(z_2 - x)(\Delta - \frac{x+z_2}{2})}{v + x + \alpha(z_2 - x) + \alpha p_1(\hat{x} - x)} \\ - \frac{(1 - \beta)\alpha p_1 \max\{\hat{x} - x; 0\} (\Delta - \frac{x+\hat{x}}{2})}{v - x + \alpha(z_2 - x) - \alpha p_1(z_2 - \hat{x})} \end{aligned}$$

so that the free-entry condition in region 2 writes:

$$\begin{aligned} \frac{\kappa_2}{(1 - \beta)m_2(\theta_2)} = \pi_2(y_2 - b_2) + \frac{\alpha(1 - p_1)(z_2 - x)}{v + x + \alpha(z_2 - x)} \left(\Delta - \frac{z_2 + x}{2} \right) \\ + \frac{\alpha p_1(\max\{\hat{x} - x; 0\})}{v + x + \alpha(z_2 - x)} \left(\Delta - \frac{\hat{x} + x}{2} \right) \end{aligned} \quad (57)$$

F.6 Efficiency

Comparing (56)-(57) with (37)-(38), one directly sees that the *ex-ante* bargaining helps restoring efficiency, but is not sufficient.

First, regarding the search thresholds definitions (equations (40)-(41) and (53)-(54)), we notice that it partially corrects the search decisions, by taking the whole surplus formed rather than a share β . Workers however still do not take into account the impact of their search decision on the opening of vacancies in the other region.

Second, even if thresholds were set optimally, firms do not open the optimal number of vacancies. This is due to the fact that workers get compensated for leisure, amenities, rents and idiosyncratic preferences by a firm if they are currently working in it, while the central planner compensate all workers. For example, for a firm settled in region 1, the third term on the RHS of (37) is equivalent to the second term on the RHS of (56). Furthermore, if $x > \hat{x}$, then the fifth term in (37) corresponds to the third term of (56). As workers, when getting two job offers and being located in region 2, prefer to work in region 1, firms located in region 1 compensate them for not living in region 2 anymore. However, when deciding how many vacancies they open in region 1, firms do not take into account the implications of their decisions on the workers who live in region 1 but accept a job offer in region 2. Such an event is of course influenced among other things by the number of vacancies created in region 1.

Furthermore, *ex-ante* bargaining does not allow to internalize the loss in output in the other region (in (56)-(57) there is no expression corresponding to the second term on the RHS of (37)-(38) because when they bargain over wages agents have no reason to internalize the induced effect of the wage *via* vacancy creation on the net output created in the other region).