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Abstract

The supply of allowances in the European Union Emissions Trading System is determined within a rigid allocation programme. A reform of the EU ETS intends to make allowances allocation exible and contingent on the state of the system. We model the emissions market under adjustable allowance supply in a stochastic partial equilibrium framework and obtain closed form solutions for its dynamics. The model considers a supply control mechanism contingent on the number of allocated and unused allowances, as suggested by the European Commission. We derive analytical dependencies between the allowance allocation adjustment rate and the market equilibrium dynamics, which allows us to represent the quantity thresholds as quantiles for the number of allocated and unused allowances. Finally, we present an analytical tool for the selection of an optimal adjustment rate under both risk-neutrality and risk-aversion. We thereby provide an analytical foundation for the regulator's decision-making in the context of the EU ETS reform and give a novel perspective on the mechanism's overall design.

Keywords: EU ETS reform, policy design, responsiveness, resilience, supply management mechanism, risk-aversion.

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1 Introduction

Cap-and-trade systems are created through the use of transferable emission allowances to control the aggregate amount of pollution emitted by a group of regulated sources. Thus, in contrast to a tax, the system sets a cap on overall emissions without also fixing the allowance price.

Whether or not policymakers should be actively concerned with market price levels (or bounds) has been the focus of numerous debates following the sharp price declines in existing emission trading systems, such as the European Union Emissions Trading System (EU ETS) and the Regional Greenhouse Gas Initiative (see [Borenstein et al., 2015], [Grosjean et al., 2014] and references therein). Ideally, a policymaker would opt for an instrument of central control that has its instructions contingent on the state of the world revealed (e.g. economic shocks or technological advancements). By employing the ideal signal, the ex ante uncertainty would be eliminated ex-post and the optimum solution would be retained ([Weitzman, 1974]). However, designing a contingent instrument as such is very complex in practice and single-order policies are opted for instead ([Hepburn, 2006]).

In general, a policymaker's mandate is primarily dictated by the specific objectives of the system it regulates (e.g. to achieve a certain level of emissions reduction with or without also defending a price interval). Ultimately, however, the policymaker's role is to ensure that its cap-and-trade system enables compliance entities to meet their obligations at minimum costs. Economic theory provides insights into how systems can be designed to facilitate cost minimisation. For example, several studies have explored the effect of banking and borrowing provisions as cost 'smoothing' mechanisms ([Rubin, 1996], [Schennach, 2000], and [Fankhauser and Hepburn, 2010] for a comprehensive overview of the literature). However, these mechanisms alone may not be sufficient when the market is faced with severe uncertainty. In the presence of this market uncertainty, current investment decisions are harder to make and as a consequence, long-term abatement may occur at higher overall costs.

The policymaker's challenge thus quickly amplifies once markets are recognised as inherently unstable. Furthermore, the drivers of market uncertainty are hard to untangle and the impacts of such uncertainties are no easier to control. Notwithstanding, it is possible to design policy mechanisms that can help mitigate these impacts in a way that a non-intervention policy design (such as temporal provisions) alone could not achieve. In the language of [Minsky, 1986], institutions and regulations can be designed to constrain instability.

The aim of an intervention policy design (also referred to as 'contingent' policy), as used in this paper, is to improve the responsiveness of the cap-and-trade system to unforeseen events so that allowance prices or supply are within reasonable bounds. Several studies have explored the choice of the type of indicators for such events, such as activity-based,[1] quantity- or price-based.[2] The implementation of the policy itself can also take different

[1] Using partial equilibrium models, [Jotzo and Pezzey, 2007], [Newell and Pizer, 2008] and [Branger and Quirion, 2014] all discuss whether and how the caps in emissions trading systems should be indexed to the GDP, but reach different conclusions.

[2] In their seminal paper, [Roberts and Spence, 1976] suggest to create a supply curve that is neither fully flat (a pure tax) nor fully vertical (pure cap-and-trade) but dependent on pre-selected price levels. Prices become the indicator used to change allocation of

forms, including: automatic price-based; automatic quantity-based; or discretionary.

Although each of these mechanisms will seek to mitigate the effects of market uncertainty, the persistence of the resulting price impact will depend on the policymaker's mandate or preferences. However, from a policymaker's perspective, the choice between a rule-based (automatic) and a discretionary mechanism is less than straightforward. On the one hand, it is unclear how automatic rules can tackle different sources of uncertainty with varying degrees of market impact. On the other hand, implementing a discretionary intervention policy can render the market inefficient if interventions are too frequent relative to the length of the trading period ([Harstad and Eskeland, 2010]) or can be fraught with many political hurdles that can significantly delay the policy's roll-out.

This paper explores how regulators can improve market efficiency under uncertainty, while reducing the need for discretionary intervention. The analysis is set in the context of the EU ETS, although the results are generally applicable.

As part of the EU ETS reform, the European Commission has indicated a preference for an automatic quantity-based mechanism, the so-called Market Stability Reserve (MSR). The key rationale for using a quantity-based supply management mechanism is to remove the need to specify a price range for triggering allocation adjustments. In the current EU context, the prospect of specifying an acceptable price range for a price-collar mechanism has been faced with significant political challenges. Thus, a quantity-based mechanism provides a practical advantage. Still, how the parameters determining the timing and the intensity of quantity-based adjustments are selected remains an open question both in the political and the academic spheres.

Under the MSR, a rule-based policy is implemented such that auction quantities are adjusted in response to changes in privately-held inventories of unused allowances. According to the EC's proposal, a proportion of allowances that would otherwise be auctioned off to market participants is stored in the reserve if the total number of allowances in circulation (TNA),[3] exceeds an upper trigger level. Allowances are removed from the reserve and placed back into the auction system if the TNA crosses a lower trigger level. Specifically, 12% of the TNA are placed back into the reserve, unless this number is less than 100 million allowances. This corresponds to an implied withholding trigger of 833 million allowances. By contrast, allowances are moved from the reserve back into the auction system if the TNA falls below a lower trigger level. The injection trigger is explicitly set equal to 400 million allowances ([EC, 2014a], [EC, 2014b]).

Under this design, the MSR responds to current market changes by adjusting auction quantities up or down as needed. The resulting increase or decrease in auction quantities is subtracted from or added to the reserve respectively. The indicator used to trigger auction quantity adjustments is the amount of allocated and unused allowances, i.e. the size of the privately-held bank of allowances. Based on the EC's definition of unused

allowances. [Fell and Morgenstern, 2010], [Fell et al., 2012], and [Grüll and Taschini, 2011] demonstrate that such hybrid systems – combination of quantity- and price-based instruments – lower expected control costs.

[3]The TNA is expressed in terms of privately-held inventories of unused allowances in the Model Section.

allowances, this quantity can, at least temporarily, take negative values. De facto, temporary borrowing is possible when full compliance is required by the end of the regulator's planning horizon and the cumulative deficit must only be repaid by then. Under the current EU ETS Directive, borrowing (to some extent) is implicitly possible within a trading phase due to next-year allocation preceding the surrendering of current-year allowances.

In practice, the total number of 'allowances in circulation' is expected to be positive and it will most likely stay slightly above or not far below zero. Because allowances from the reserve are added to the auction amount when the TNA is between 400 million and zero, the EC's absolute injection quantity can be approximated by an injection rate. We will henceforth refer to the trigger levels and adjustment rates as the MSR or policy parameters.

The problem of choosing the policy parameters is characterized by substantial complexity. Recent studies tested a selected combination of parameter choices in a variety of scenarios.[4] The interdependencies between the parametrisation itself and the system dynamics play a crucial role in determining the appropriate design of a contingent policy. The present paper examines the mechanics of the interdependency within an analytical framework and proposes a tool for the selection of an optimal contingent policy.

In particular, we explicitly derive a probabilistic expression for the TNA, from which quantiles for the private bank can be obtained and show how these quantiles depend on the allocation adjustment rate. Finally, we show that when the regulator chooses a level of confidence for the number of unused allowances, the adjustment rate is the key parameter in the design of an MSR.

This paper offers a new approach to the modelling of rule-based supply management mechanisms and contributes to the literature on inter-temporal emissions trading and design of responsive policies. The approach is related to the recent literature that investigates the EC's MSR proposal; however, the methods employed are different. Most notably, the analysis of dependencies between the mechanism parameters and the market equilibrium dynamics proves to be formally similar to that of problems in monetary policies. Therefore, the paper also provides insights for readers who are not directly concerned with supply control on emission allowance markets.

The remainder of the paper proceeds as follows. In Section 2 we provide a theoretical model of emissions control given an adjustable supply of allowances. We first solve the emissions control problem and characterise the equilibrium under risk-neutrality. We then show how a quantity-based supply management programme affects the equilibrium dynamics and derive a probabilistic expression for the quantity indicator as a function of the adjustment rate. Subsequently, we characterise the equilibrium under risk-aversion and illustrate how the adjustment rate affects total compliance costs. In this way, we show how the model presented can be used as

[4] The Deutsches Institut für Wirtschaftsforschung (DIW Berlin) coordinated an international model comparison exercise where different sets of parameters have been tested using economic models and laboratory experiments ([Fell, 2015], [Salant, 2015], [Schopp et al., 2015], [Trotignon et al., 2015], [Holt and Shobe, 2015]).

an analytical tool for selecting an optimal adjustment rate. Section 3 concludes.

2 Model & Results

We present a stochastic partial equilibrium model of a cap-and-trade system in continuous time.[5] Companies in the allowance market choose Markovian abatement and trading strategies to minimise their discounted expected compliance costs under the constraint of perfect compliance at a finite time-horizon. The resulting Nash equilibrium consists of abatement and trading strategies for all companies and a market-clearing price process. Permits demand, i.e. pre-abatement emissions, and allowance allocation are uncertain to the regulated companies. In the spirit of the EC's proposal, allocation therefore serves as a control gateway for the supply management programme.

Allowance Supply and Demand

We introduce the model components by considering the supply and demand of allowances. Let t denote the current time, where $0 \leq t \leq T$ and where T denotes the end of the regulated period. The instantaneous allowance allocation is represented by the process $d\varphi_t$, comprised of the pre-MSR allowance allocation schedule and the MSR quantity adjustment. This constitutes the supply side. The pre-abatement instantaneous emissions are represented by $g_t dt + d\varepsilon_t$, comprised of a deterministic and a noise component respectively, where $d\varepsilon_t$ is a random shock. Finally, let α_t denote the instantaneous abatement. Together, emissions minus abatement constitute the demand side.

Regulated companies are assumed to be atomistic. That is, we let companies $i \in I$ be continuously distributed in a set I, under a probability measure m. In particular, each firm is characterised by her initial endowment of allowances N_0^i, and her emissions process. The aggregate initial endowment of allowances is denoted by $N_0^I = \int_I N_0^i \, dm(i)$. Other individual and aggregate expressions are denoted analogously. Hereafter, we omit the superscript I to simplify the notation. However, it is important to emphasise that the equilibrium results are obtained in terms of individual strategies. Hence, the distribution of companies $i \in I$ has a quantifiable impact.

We now have all the components needed to analytically describe the quantity indicator used to trigger auction adjustments – that is the amount of allocated and unused allowances. The process

$$\mathrm{TNA}_t = N_0 + \int_0^t d\varphi_s - \int_0^t g_s \, ds - \int_0^t d\varepsilon_s + \int_0^t \alpha_s \, ds$$

represents the total number of allowances in circulation (TNA), as per EC's definition. We then define the

[5] We refer to the Appendix for a technical description of the model. Note that the term 'equilibrium' is used in the game-theoretical sense. The equality of marginal abatement costs and the allowance price is obtained as a corollary.

individual allowance net position as

$$X_t^i = N_0^i + \mathbb{E}_t\left[\int_0^T d\varphi_s^i - \int_0^T g_s^i\, ds - \int_0^T d\varepsilon_s^i\right] + \int_0^t \alpha_s^i\, ds\ - \int_0^t \beta_s^i\, ds,$$

where $|\beta_t^i|$ is the number of allowances sold ($\beta_t^i > 0$) or bought ($\beta_t^i < 0$) by company i at time t and where $\mathbb{E}_t = \mathbb{E}[\cdot|\mathcal{F}_t]$ represents the conditional expectation operator.[6]

Under the cap-and-trade system, full compliance is required by the end of the regulated period; that is $\mathbb{E}_t[X_T^i] \geq 0$ for all t. We do not explicitly model the possibility of firms' non-compliance. However, we recognise that regulated firms tend to over-comply with emissions trading regulations ([Requate and Unold, 2003] and [Stranlund et al., 2014] and [Sikorski et al., 2015]). The quantification of such - possibly suboptimal - number of allowances held beyond the compliance phase is not the topic of this paper. Notwithstanding, we acknowledge the existence of such 'over-compliance' demand, which we denote by c^i. We amend the compliance constraint accordingly, by requiring that $\mathbb{E}_t[X_T^i] = c^i$ at all times t.

Uncertainty in the system is modelled by the stochastic demand component, $d\varepsilon_t$. A negative shock decreases demand and, consequently, increases the TNA. For example, a severe negative shock may result in a temporary oversupply of allowances. The duration of the oversupply depends on the future expected net-supply of allowances (in the form of the allocation programme and the MSR quantity adjustments) given by

$$\mathbb{E}_t\left[\int_t^T d\varphi_s^i - \int_t^T g_s^i\, ds - \int_t^T d\varepsilon_s^i\right]$$

and, perhaps more importantly, on how firms choose to temporally adjust abatement and permits trading in order to remain on their cost-minimising strategy path. In what follows, we study how the system responds to shocks and derive expressions for the firms' dynamic behaviour.

Inter-Temporal Decision Problem

When solving their emissions control problem, firms choose how much to trade and how much to abate depending on the cost difference between allowance trading and emissions abatement.

We start by considering the costs associated with trading on the allowance market. We use the bid-ask spread $2\nu > 0$ to account for market trading frictions.[7] Analytically, the amount a firm receives when selling one allowance at a price P is $P - \nu$. We extend this notion to a linear temporary price impact which is given by $P - \nu\beta$. This means that selling ($\beta > 0$) or buying ($\beta < 0$) a number of $|\beta|$ allowances yields an instantaneous

[6] By convention, $\mathcal{F}_t$ represents the information available at time t. We refer to the Appendix for a characterisation of $(\mathcal{F}_t)_{t\geq 0}$.
[7] In practice, the parameter ν can be selected differently but half of the bid-ask spread represents a conventional approximation.

profit of $\beta \cdot (P - \nu\beta)$. We also assume a linear functional form for the marginal abatement cost curve (MACC), $\Pi + 2\varrho\alpha$, where α represents abatement, Π is the MACC intercept, and ϱ is a coefficient that determines the slope of the MACC.[8]

The instantaneous costs of trading and abatement for each company are given by

$$v_t^i = \Pi\alpha_t^i + \varrho(\alpha_t^i)^2 - P_t\beta_t^i + \nu(\beta_t^i)^2.$$

We first consider the case of risk-neutral companies. Under this assumption, let $r > 0$ denote the risk-free interest rate. The solution to the company's inter-temporal decision problem consists of Markovian abatement- and trading strategies, α^i and β^i, respectively, that minimize the company's total cost function

$$J(\alpha, \beta) = \mathbb{E}\left[\int_0^T e^{-rt}\, v_t^i\, dt\right] \tag{1}$$

subject to the compliance constraint $\mathbb{E}_t[X_T^i] = c^i$ for all t. We later consider the model under risk-aversion and derive the expression for the risk-adjusted discount rate ϑ_t.

Equilibrium Strategies & Permits Price

We now characterise the equilibrium under risk-neutrality, associated with the problem in Equation (1). An equilibrium is a set $\{\alpha_t^i, \beta_t^i, P_t; t \in [0,T]\}$ of Markovian strategies α^i, β^i for all firms $i \in I$ and a stochastic price process P that together satisfy the market-clearing condition $\int_I \beta_t^i dm(i) = 0$ for all t. In equilibrium,[9] the abatement and trading strategies are:

$$\alpha_t^i = \frac{P_t - \Pi_t}{2(\nu + \varrho)} - \frac{\nu r\,(X_t^i - c^i)}{(e^{r(T-t)} - 1)(\nu + \varrho)} \quad \text{and} \quad \beta_t^i = \alpha_t^i + \frac{r\,(X_t^i - c^i)}{e^{r(T-t)} - 1},$$

and the price process is

$$P_t = \Pi_t - (X_0 - c)\frac{2re^{rt}\varrho}{e^{rT} - 1} - 2re^{rt}\varrho \int_0^t \frac{d\gamma_s}{e^{rT} - e^{rs}},$$

where

$$d\gamma_s = d\mathbb{E}_s\left[\int_0^T d\varphi_u - \int_0^T d\varepsilon_u\right].$$

The process γ_t reflects changes in the expected future demand and supply affecting companies' abatement- and trading behaviours. For example, an economic shock might have a negative effect on the future expected emissions, as in the case of the recent economic recession. If a contingent supply management system like an

[8] A detailed discussion of the calibration of marginal abatement costs can be found in [Landis, 2015].

[9] The equilibrium price process P does not allow for an individual deviation from the equilibrium strategies to safe costs. Hence, the notion of the equilibrium employed here is that of a Nash equilibrium. We refer to the Appendix for a derivation of the equilibrium.

MSR is in place, the impact of such a shock can be mitigated by adjusting the allowance allocation schedule accordingly.

Note that the individual strategies and the allowance price depend on the pre-MSR allocation programme and the MSR quantity adjustment. Therefore, the solution to the control problem in Equation (1) also includes the market's reaction to a supply management policy. We can thus evaluate the policy impact on the equilibrium allowance price and the abatement distribution.

It can be shown that the aggregate instantaneous abatement α_t is given by

$$\alpha_t = -re^{rt}\frac{X_0(\delta) - c}{e^{rT} - 1} - re^{rt}\int_0^t \frac{d\gamma_s(\delta)}{e^{rT} - e^{rs}}.$$

Only the terms $X_0(\delta)$, determined at time $t = 0$, and the stochastic component $d\gamma_s(\delta)$ depend on the adjustment rate δ. The allowance allocation programme, therefore, does not affect the shape of the expected aggregate abatement curve (as a function of time) when firms are risk-neutral. However, when companies are risk averse, changes in the allocation programme affect their abatement dynamics. We will examine this in more detail below, where we solve the problem under risk-aversion. As for the allocation programme, the standard assumption that the cap should start fairly slack and decline with a fixed rate must be discarded in favour of a more detailed investigation of how market actors actually behave under a cap policy that will gradually unfold over the next 20 or 40 years. We will show how a contingent allocation policy can affect the abatement behaviour, by reducing the impact of variability in net-demand, ultimately improving the system's efficiency.

Supply Management Policy

We now show how a quantity-based supply management programme affects the equilibrium dynamics and derive a probabilistic expression for the quantity indicator as a function of the supply adjustment rate governing the contingent policy.

First, consider the following contingency rule for the supply of allowances: At each time t, if the current TNA is above the level $c \geq 0$, then a fraction $\delta\ dt$ of the difference $|\text{TNA} - c|$ is removed from the auction and added to the reserve. That is, $\delta \cdot |\text{TNA} - c|\ dt$ allowances are removed from the allocation schedule. Conversely, $\delta \cdot |\text{TNA} - c|\ dt$ allowances are added to the allocation schedule if the TNA is lower than c.

Let f_t represent the fixed, ex ante allocation schedule. The dynamics for the TNA is then given by

$$d\text{TNA}_t = f_t dt + \delta(c - \text{TNA}_t)\ dt - g_t dt - d\varepsilon_t + \alpha_t dt.$$

In equilibrium, we then obtain the following expression:

$$\text{TNA}_t = N_0 e^{-\delta t} - \frac{r(e^{rt} - e^{-\delta t})}{(\delta + r)(e^{rT} - 1)}(X_0 - c) - \frac{e^{rt}}{V_t(\delta, r)} \int_0^t e^{-rs}\, V_s(\delta, r)\, d\varepsilon_s + \int_0^t e^{\delta(s-t)}(f_s - g_s + \delta c)\, ds,$$

where $V_t(\delta, r) = (\delta + r)/(e^{(\delta+r)(T-t)} - 1)$. Any random variable ε_t yields a probability distribution of TNA_t, parametrised by the adjustment rate δ. As such, it also yields quantiles for any given confidence level.[10]

For illustration, consider the case of risk-neutral firms and Gaussian distributed $d\varepsilon_t$ with mean zero and deterministic volatility κ_t. We obtain that $\text{TNA}_t \sim \mathcal{N}(a_t, b_t^2)$ where

$$a_t = N_0^I e^{-\delta t} - \frac{r(e^{rt} - e^{-\delta t})}{(\delta + r)(e^{rT} - 1)}(X_0 - c) + \int_0^t e^{\delta(s-t)}(f_s - g_s + \delta c)\, ds$$

is the mean, and

$$b_t^2 = \frac{e^{2rt}}{V_t^2(\delta, r)} \int_0^t e^{-2rs}\, V_s^2(\delta, r)\, \kappa_t^2\, ds$$

is the variance. Now, let λ denote the probability that the TNA stays within the band $[l_t, u_t]$. We can then compute the following

$$\lambda = \Phi\left(d_t^{(1)}\right) - \Phi\left(d_t^{(2)}\right),$$

where $\Phi(\cdot)$ represents the cumulative distribution function of the standard normal distribution and

$$d_t^{(1)} = \frac{u_t - a_t}{b_t} \quad \text{and} \quad d_t^{(2)} = \frac{l_t - a_t}{b_t}.$$

Thus, λ can be expressed as a function $C(\delta) = \lambda$. Any TNA interval can therefore be maintained with a confidence level λ when the adjustment rate δ is set to $C^{-1}(\lambda)$ and vice-versa.

Consider the EC's proposal for the design of an MSR (EC MSR). The proposal specifies a withdrawal rate of 12%, which approximates the adjustment rate δ. The EC MSR also specifies quantity thresholds of $[400, 833]$ million allowances. The analytical dependency between the allowance allocation adjustment rate δ and the market equilibrium dynamics allows us to represent the EC's quanitity thresholds as quantiles for the TNA for a given confidence level. The model is solved in closed form, thus any confidence level for the TNA corresponds to a confidence level for the (uncertain) total compliance costs. The choice of a confidence level for the TNA, however, is less of an operational one, but rather driven by a political choice. In contrast, the adjustment rate δ shall be selected to let compliance entities meet their obligations at minimum costs. Later we show how to select an optimal adjustment rate.

[10]This result is not limited to the case where firms are risk-neutral. However, the distribution of the TNA is subject to a transformation based on companies' risk-aversion. We will examine the necessary change of measure later.

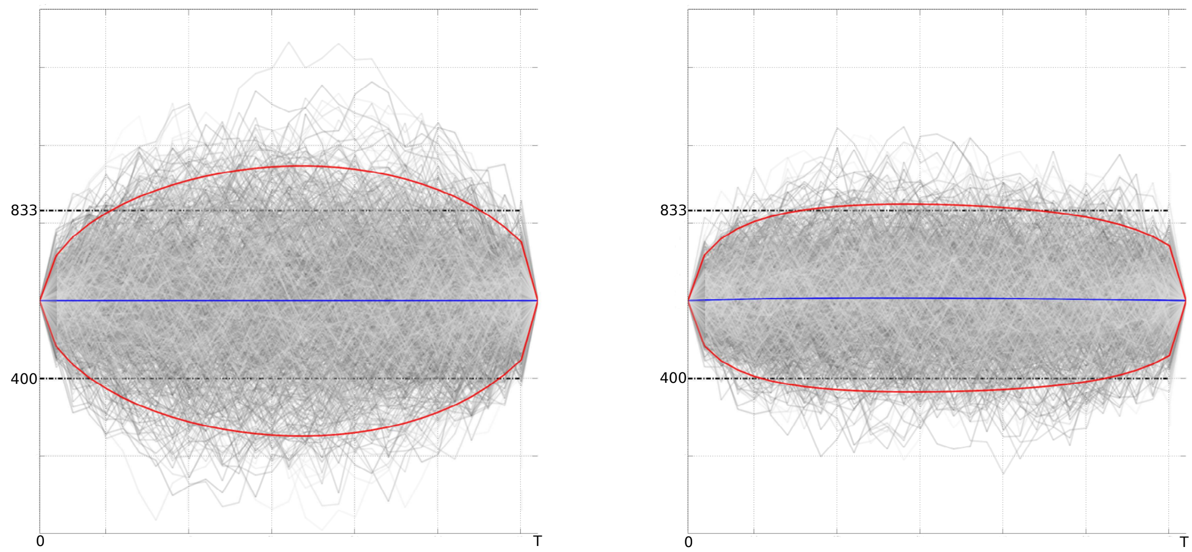

Figure 1: Total number of allowances in circulation (TNA) when the MSR is deactivated, $\delta = 0$ (left diagram) and when the MSR is activated, $\delta = 12\%$ (right diagram). Light grey lines show the TNA paths obtained by simulating 10^3 pre-abatement emissions paths; the two red lines mark the implied TNA quantiles for a 95% confidence level. The parameters used for this example are: $N_0 = c = 600$ million allowances; pre-MSR scheduled allocation and pre-abatement emissions are both equal to 2 billion allowances per year; $\Pi = 5$ Euros; $\varrho = 2 \cdot 10^{-10}$ Euros/tonne^2; $r = 2\%$. Pre-abatement emissions have a volatility of $\kappa = 60$ Million tonnes yearly and companies are assumed to be identical.

Figure 1 illustrates the relation between the allowance adjustment rate δ and the TNA. The figure shows the TNA quantiles for a 95% confidence level when the EC MSR is inactive (left diagram) and when the EC MSR is active with an adjustment rate of 12% (right diagram). In the latter case, market changes force the MSR to adjust the allowance allocation schedule up or down so that the TNA is contained within the allowance quantity corridor of $[400, 833]$ million allowances with a 95% confidence level. The right diagram suggests that this is possible depending on the selected (positive) value of the MSR adjustment rate. By contrast, the left diagram shows that when the MSR adjustment rate is zero (i.e. the MSR is switched off), the chosen quantity corridor cannot be maintained with the desired confidence level. In other words, because the market responds to a supply management policy, the MSR adjustment rate becomes the key policy parameter that allows the regulator to keep the TNA within the intended boundaries.

In the following section we will identify the total compliance costs and quantify the dependence of costs on the adjustment rate δ. We will also derive the expression for the risk-adjusted discount rate as a function of δ. Finally, we show how the key policy parameter, the adjustment rate, can be selected to minimise total expected compliance costs.

Risk-Adjusted Discount Rate & Total Compliance Costs

Below we provide the expression for the risk-adjusted discount rate and show how to identify an optimal supply adjustment rate for a contingent supply mechanism. Finally, we illustrate how the adjustment rate enters the risk-adjusted discount factor, thereby affecting the equilibrium dynamics and the expected total compliance costs.

Let us first recall that P denotes the allowance price and Π represents the MACC intercept. Then, let μ denote the historical rate of return of the difference $\Psi = P - \Pi$ and let k_t denote its time-dependent volatility. Then the risk-adjusted discount factor is given by[11]

$$\vartheta_t = rt + \frac{1}{2}\int_0^t \zeta_s^2 \Psi_s^2 \, ds - \int_0^t \zeta_s \Psi_s \, dW_s,$$

where dW_t is a Gaussian random shock and ζ_t denotes the ratio $(r-\mu)/k_t$. Furthermore, we obtain that, under risk-aversion, the process Ψ_t follows the dynamics

$$d\Psi_t = \left(r + \frac{V_t(\delta, r)}{V_t(0, r)}(\mu - r)\right) \, \Psi_t \, dt - \frac{V_t(\delta, r)}{V_t(0, r)} k_t dW_t. \tag{2}$$

We now observe that, under risk-aversion, the adjustment allocation rate δ becomes a significant determinant of the expected price trajectory - the first term on the right-hand side of Equation (2). To illustrate the impact of the adjustment rate δ on the rate of return and the volatility of the price dynamics, we consider two extreme cases. Recalling that $V_t(\delta, r) = (\delta + r)/(e^{(\delta+r)(T-t)} - 1)$, we note that when the adjustment rate is high, $V_t(\delta, r)$ approaches zero; thus the rate of return approaches the risk-free discount rate, r, and the volatility term approaches zero. This means that the TNA is tight around the level c and the variability of the net-demand is reduced. In contrast, when the adjustment rate δ is very low, the TNA is unconstrained and the risk mitigation provided by the mechanism vanishes. In this case, the risk associated to changes in the net-demand of allowances requires a risk premium, which appears in the first term on the right hand-side of Equation (2).
We note that the aggregate abatement follows the dynamics

$$d\alpha_t = \left(r\alpha_t + \frac{V_t(\delta, r)(r - \mu)}{2\varrho V_t(0, r)}\Psi_t\right) dt + \frac{V_t(\delta, r) k_t}{2\varrho V_t(0, r)} \, dW_t.$$

Earlier we have shown that the expected abatement path does not depend on the specific allocation policy when risk-neutrality is assumed. Conversely, when companies are risk-averse, the expected abatement path reflects the market's reaction to the presence of a supply management policy and its shape (as a function of time) is determined by the supply adjustment rate.

Recall that instantaneous costs are given by $v_t = \int_I \Pi\alpha_t^i + \varrho(\alpha_t^i)^2 - P_t\beta_t^i + \nu(\beta_t^i)^2 \, dm(i)$ and let $w_T = \int_0^T e^{-rt} v_t \, dt$ represent the present value of aggregate total costs.

We note that the equilibrium under risk-neutrality differs from that under risk-aversion. By convention, $\mathbb{E}^{\mathbb{Q}}$

[11] The complete derivation of the risk-adjusted discount factor can be found in the Appendix.

denotes the expectation operator under risk-neutrality and $\mathbb{E}^{\mathbb{P}}$ denotes the expectation operator under risk-aversion. We then have

$$\mathbb{E}^{\mathbb{Q}}\left[\int_0^T e^{-rt}\ v_t\ dt\right] = \mathbb{E}^{\mathbb{P}}\left[\int_0^T e^{-\vartheta_t}\ v_t\ dt\right],$$

where the risk-adjusted discounting term $e^{-\vartheta_t}$ depends on the adjustment rate δ.[12] This dependence is a fundamental aspect to consider when selecting the allowance adjustment rate because companies' dynamic abatement- and trading decisions are influenced by supply and demand risk.

Let v_t^* denote the instantaneous costs under risk-aversion, i.e. companies discount future cashflows using the risk-adjusted interest rate ϑ_t. Without loss of generality, we use the risk-free rate r to evaluate total expected compliance costs:[13]

$$\mathbb{E}^{\mathbb{P}}[w_T^*] = \mathbb{E}^{\mathbb{P}}\left[\int_0^T e^{-rt}\ v_t^*\ dt\right].$$

This can be equivalently decomposed as $\mathbb{E}^{\mathbb{P}}\left[w_T^*\right] = \mathbb{E}^{\mathbb{Q}}[w_T^*] + \mathrm{Cov}^{\mathbb{Q}}\left(e^{\vartheta_T - rT}, w_T^*\right)$ which yields the following expression:

$$\min_{\delta}\ \mathbb{E}^{\mathbb{P}}\left[w_T^*(\delta)\right] = \min_{\delta}\left\{\mathbb{E}^{\mathbb{Q}}[w_T^*(\delta)] + \mathrm{Cov}^{\mathbb{Q}}\left(e^{\vartheta_T(\delta) - rT}, w_T^*(\delta)\right)\right\}. \tag{3}$$

Figure 2 illustrates the total expected compliance costs as a function of the adjustment rate δ. The green line shows the expected total compliance costs under risk-aversion, $\mathbb{E}^{\mathbb{P}}[w_T^*(\delta)]$, and the blue line shows the expected total compliance costs when the covariance between the risk-adjustment and the compliance cost in Equation (3) is ignored. A zero adjustment rate δ leaves the net-demand variability unaffected and implies higher compliance costs both under risk-aversion and risk-neutrality. By increasing δ, the band for the TNA becomes tighter implying a lower net-demand variability. However, net-demand certainty (or lower uncertainty) comes at the cost of less temporal flexibility. Figure 2 illustrates this trade-off. After a certain level for δ, the costs associated to the flexibility loss override the benefits associated to lower net-demand variability. We calculate the optimal values of δ under both risk-neutrality and risk-aversion. In this example, compliance costs are minimised for $\delta = 15\%$ and $\delta = 17\%$, respectively.

Figure 3 illustrates how the adjustment rate δ affects the risk-adjusted interest rate and the total compliance costs. The diagrams show the results of a run of 10^4 simulations using four different adjustment rates, where each blue dot represents the outcome of a model simulation. Start by considering the case where $\delta = 1$, bottom-right diagram. By imposing a very tight band for the TNA, net-demand variability diminishes, the required risk-premium approaches zero, and the average risk-adjusted discount rate converges to the risk-free rate r. When $\delta = 1$, the volatility term in the price dynamics in Equation (2) approaches zero. The allowance price becomes deterministic and increases at the rate r. In practice, this corresponds to a tax regime. Total compliance costs

[12]The equilibrium dynamics under risk-aversion can be obtained by calibrating P_t and hence ϑ_t to historical price data. Technical details are provided in the Appendix. However, the calibration of the model to historical data is not the topic of this paper and is left for future research.

[13]Typically, the regulator might select a different discount factor when evaluating aggregate compliance costs over the period $[0, T]$. Our results hold accordingly for a different discount rate.

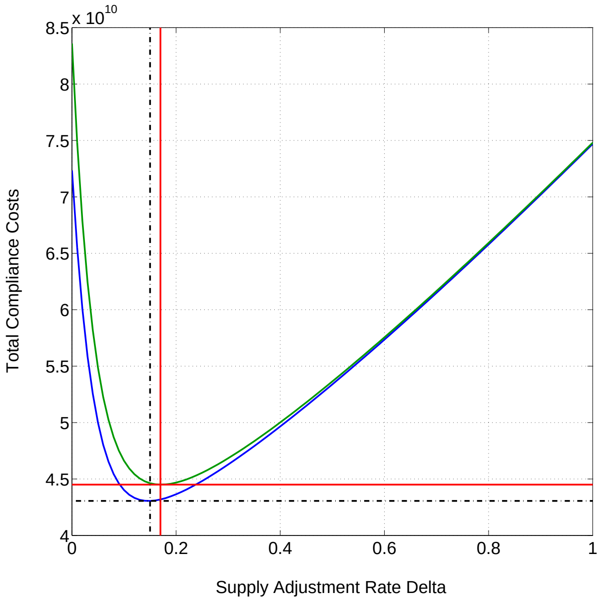

Figure 2: Expected total compliance costs as a function of the adjustment rate δ when $r = 2\%$, $\mu = 2.5\%$, $\varrho = 0.25 \cdot 10^{-9}$ Euros/tonne^2, $\Pi = 0$, $c = 500$ million allowances, a historical price volatility of $k = 0.25$ Euros yearly and expected emissions of $g_t = 4$ billion tonnes yearly. Companies are identical and have an initial supply of 2 billion allowances and a time horizon of $T = 30$ years. The ex ante planned allocation starts at 2 billion allowances and decreases linearly by 2%.

The green line represents the expected total compliance costs $\mathbb{E}^{\mathbb{P}}[w_T^*(\delta)]$ under risk-aversion. Costs are minimised when $\delta = 17\%$ yearly (marked by the vertical red line).

The blue line represents the expected total compliance costs $\mathbb{E}^{\mathbb{Q}}[w_T^*(\delta)]$ under risk-neutrality, obtained when the covariance between risk-adjustment and compliance costs is ignored in the decomposition of Equation (3). Costs are minimised when $\delta = 15\%$ yearly (marked by the dotted line).

are the same for any risk-adjusted interest rate. The reduction in net-demand variability comes, however, at a high cost as shown by the horizontal dotted line. Consider the case $\delta = 0$ (top-left diagram), the band for the TNA is loose, the net-demand variability on allowance prices is unaffected, and there is a positive risk-premium. The average risk-adjusted discount rate is higher than r and risk-aversion has a significant impact on total costs, which can also be observed as a positive correlation in the diagram. This impact is measured by the covariance term in Equation (3), which is exactly the difference between expected compliance costs under risk-neutrality and risk-aversion. Equation (2) then yields the rate of return for permits prices under risk-aversion and a positive risk-premium. Allowance prices volatility is unconstrained. Consequently, total compliance costs are 'uncontrolled' and the average total compliance costs are high. By increasing δ to 15% and 17%, respectively, (top-right and bottom-left diagrams), the TNA is kept in a tighter band and total compliance costs are more concentrated around a level that, on average, is lower than the average aggregated total compliance costs for $\delta = 0$ or $\delta = 1$.

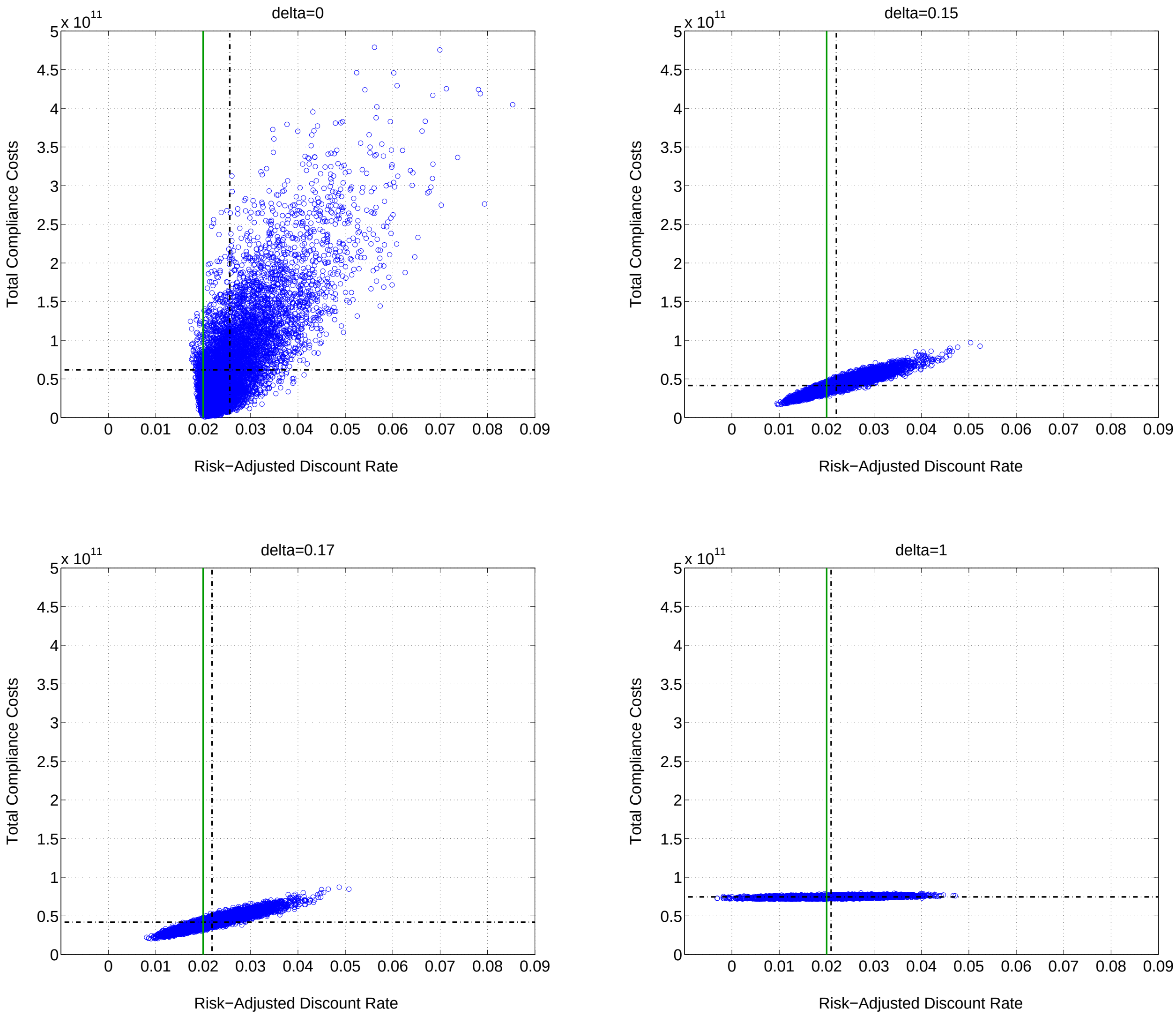

Figure 3: Risk-adjusted discount rates versus total costs under risk-aversion for $r = 2\%$, $\mu = 2.5\%$, $\varrho = 0.25 \cdot 10^{-9}$ Euros/tonne^2, $\Pi = 0$, $c = 500$ million allowances, a historical price volatility of $k = 0.25$ Euros yearly and expected emissions of $g_t = 4$ billion tonnes yearly. Companies are identical and have an initial supply of 2 billion allowances and a time horizon of $T = 30$ years. The ex ante planned allocation starts at 2 billion allowances and decreases linearly by 2%. Each blue dot represents one of 10^4 model simulations. The vertical dotted line marks the average risk-adjusted discount rate. The horizontal dotted line marks the average total compliance cost.

We conclude this section with a general observation implied by Equation (3). It can be shown that the principles of parameter selection discussed above apply analogously for other types of contingent policies. For example, consider a price-based mechanism where the objective of the policy is to maintain the rate of return of the allowance price around a target rate. Let the contingent policy be characterised by a parameter η. It can be shown that the rate of return of the allowances price must equal the risk-adjusted discount rate ϑ_t. We also have, in analogy to Equation (3), that:

$$\min_{\eta} \ \mathbb{E}^{\mathbb{P}}[w_T^*(\eta)] = \min_{\eta} \ \left\{ \mathbb{E}^{\mathbb{Q}}[w_T^*(\eta)] + \mathrm{Cov}^{\mathbb{Q}} \left(e^{\vartheta_T(\eta) - rT}, w_T^*(\eta) \right) \right\}, \qquad (4)$$

where the first term in the covariance depends on the explicit target rate. A natural, albeit debatable, choice of this target rate might be the risk-free rate r. As before, such a decomposition allow us to quantify the impact of the risk-adjusted rate of return $\vartheta_T(\eta)$ on total compliance costs. In practice, enforcing a specific rate of return $\vartheta(\eta)$ for the allowance price is equivalent to the implementation of a tax. For a constant $e^{\vartheta_T(\eta)-rT}$, the covariance term in Equation (4) vanishes and risk-aversion has no effect on total compliance costs. When the price-band is set wider, the permit price reflects economic shocks and total compliance costs are controlled more loosely. As with a quantity-based mechanism, the trade-off between those two effects can be analysed and an optimal adjustment rate can be selected.

3 Conclusions

The supply of permits in the European Union Emissions Trading System (EU ETS) is currently inflexible and is determined within a rigid allocation programme. As such, the system lacks provisions to address imbalances in the demand of allowances resulting from economic shocks. A currently proposed reform of the EU ETS intends to make the allowance allocation adaptive to the system's state. As such, the implementation of a Market Stability Reserve (MSR) could ultimately improve system resilience and market efficiency.

We model an emissions trading system under adjustable supply and obtain closed form solutions for the dynamic market behaviour under uncertainty. In particular, we derive expressions for the individual abatement- and trading strategies of compliance companies, together with a market-clearing price process. We also derive an analytical expression for the total number of allowances in circulation (TNA) in equilibrium, which is the quantity indicator used to adjust allowance supply as per the European Commission's (EC) reform proposal. Our analytical framework comprises the market's reaction to a contingent supply management programme, capturing the feedback between the equilibrium and the MSR through the quantity indicator.

We derive the distribution of the TNA as a function of the adjustment rate (withdrawal and injection). We show this yields an intrinsic relationship between the quantity triggers and the rate of adjustment, which are suggested in the EC proposal. In this way, we show that the adjustment rate becomes the key parameter of the supply management policy given any confidence level for the future state of the system. As such, the model provides us with an explicit representation of dependencies between the adjustment rate parameter and the market's dynamic behaviour.

We derive the expression for the time-dependent risk-adjusted discount rate as a function of the adjustment rate. We then provide an analytical tool to select an optimal adjustment rate which minimises expected compliance costs when regulated companies are risk-averse. We quantify the impact of risk-aversion on compliance costs and provide some general observations on the effect of a contingent supply mechanisms on this dynamics. We conclude with some insights into the relationship between price-based and quantity-based contingent supply mechanisms.

Appendix

In the following sections, we give a technical overview of the model and brief derivations for some of the key results.

The Equilibrium

We model the secondary emissions allowances market in a partial equilibrium framework. Each company $i \in I$ continuously minimises her expected compliance costs at each point in time $t \in [0, T]$, where T is the end of the regulated period. The equilibrium consists of abatement- and trading strategies α_t^i and β_t^i for each company i and a market clearing price process P_t. In equilibrium, no individual deviation from the equilibrium yields expected cost savings for any company. Hence, the notion of an equlibrium employed is that of a Nash equilibrium. We first assume that companies are risk-neutral. Later we will obtain the equilibrium under risk-aversion by deriving the risk-adjusted discount rate along with the necessary change of measure.

We begin by assuming Markovian strategies $\alpha^j = \alpha(Z_t^j)$, $\beta^j = \beta(Z_t^j)$ for every company $j \in I \setminus \{i\}$ except for i. These strategies are given as functions of the companies' individual state processes Z_t^j. Let $\mathcal{F}_t^j = \mathcal{F}_t^j(Z_t^j)$ denote company i's filration, defined as the filtration generated by her state process Z_t^j. The state process will be specified later in the text. The aggregate state process and filratrion is denoted by $Z_t = Z_t^I$ and $\mathcal{F}_t = \mathcal{F}_t^I$, respectively. We then show that it is indeed optimal for company i to replicate the other companies' strategies, as a function of her own state process Z_t^i. For the ease of notation we define

$$h_t = \frac{re^{rt}}{e^{rT} - e^{rt}}.$$

For each company $j \in I \setminus \{i\}$, let her strategies be given by

$$\alpha_t^j = \frac{\Psi_t}{2(\nu + \varrho)} - \frac{\nu}{\nu + \varrho}\, h_t(X_t^j - c^j) \quad \text{and} \quad \beta_t^j = \frac{\Psi_t}{2(\nu + \varrho)} + \frac{\varrho}{\nu + \varrho}\, h_t(X_t^j - c^j),$$

where $\Psi = P_t - \Pi_t$ and Π_t denote company j's MACC-intercept. Given the market clearing condition $\beta^I = 0$, this yields:

$$\Psi_t = -2\varrho h_t(X_t^I - c^I).$$

Since $P_t = \Pi_t + \Psi_t$ and companies are atomistic, this also yields the price process observed by company i. Recall that the net allowances position for company j is given by

$$X_t^j = N_0^j + \gamma_t^j + \int_0^t \alpha_s^j - \beta_s^j \; ds,$$

where

$$\gamma_t^j = \mathbb{E}_t\left[\int_0^T d\varphi_s^j - \int_0^T g_s^j\, ds - \int_0^T d\varepsilon_s^j\right] = \mathbb{E}_t\left[\int_0^T d\varphi_s^j - \int_0^T d\varepsilon_s^j\right]$$

denotes the expectation at time t of the total net allowance supply. Substituting for the strategies α_t^j, β_t^j above, we obtain the following dynamics for the process X_t^j

$$dX_t^j = (\alpha_t^j - \beta_t^j)\, dt + d\gamma_t^j = -\frac{re^{rt}}{e^{rT} - e^{rt}}(X_t^j - c^j)\ + d\gamma_t^j.$$

Solving, we obtain:

$$X_t^j = c^j + (X_0^j - c^j)\, \frac{e^{rT} - e^{rt}}{e^{rT} - 1} + (e^{rT} - e^{rt}) \int_0^t \frac{d\gamma_s^j}{e^{rT} - e^{rs}}.$$

Recall that companies are atomistic. In particular, companies $i \in I$ are assumed to be continuously distributed in the set I. The above representation of X_t^j therefore allows us to consider X_t^I by integrating over I and express Ψ_t as:

$$\Psi_t = -2\varrho h_t(X_t^I - c^I) = -2\varrho\frac{re^{rt}}{e^{rT} - 1}\,(X_0^I - c^I) - 2\varrho re^{rt}\int_0^t \frac{d\gamma_s^I}{e^{rT} - e^{rs}}.$$

In particular we observe that Ψ has dynamics

$$d\Psi_t = r\Psi_t dt - 2\varrho h_t d\gamma_t^I.$$

Let the random shocks $d\varepsilon_t^i$ to emissions (before abatement) be governed by a driftless diffusion $d\varepsilon_t^i = \kappa_t^i\, dW_t^{(1)}$ for some deterministic process κ_t^i, where $dW_t^{(1)}$ denotes a standard Brownian motion. For the moment we assume that γ_t also has the form of a driftless diffusion:

$$d\gamma_t^i = \lambda_t^i dW_t^{(1)},$$

where λ_t^i is deterministic. Later we will introduce a supply management mechanism. We will then prove that in equilibrium, the process γ_t^i has indeed the above form. Note however, that the equilibrium framework under investigation is generally not limited to the specific contingent policy described hereafter.

The process λ_t^i describes how changes to the expected future net-supply of allowances is distributed across the set of companies I. We will abstract from specific assumptions on the form of λ_t^i. However, we note that it is reasonable to assume different λ_t^i for different companies, since pre-abatement emissions levels and allowances demand can vary depending on the type of industry in consideration.

We consider changes in pre-abatement allowances demand and in the (possibly contingent) allowances supply. Their degree of impact can vary for each company. However, all companies are subject to systemic shocks. Hence, we consider the same Brownian motion $W_t^{(1)}$ for each $i \in I$, whereas differences in size, technology etc. are represented by the process λ_t^i.

We consider the problem of optimal pollution control for company i. Let ψ denote an observed value of the difference $P_t - \Pi_t$ and let $P^{t,\psi+\pi}$ denote the price process with time-t value $P_t^{t,\psi+\pi} = \psi + \pi$. Furthermore, let $\Pi^{t,\pi}$ denote the MACC intercept, the present value of which is Gaussian, $d(e^{-rt}\Pi_t) = G_t dW_t^{(2)}$, with $\Pi_t^{t,\pi} = \pi$. At time t, company i has to bear costs v_t^i given by

$$v_t^i = \Pi_t^\pi \ \alpha_t^i + \varrho(\alpha_t^i)^2 - P_t^{\psi+\pi}\beta_t^i + \nu(\beta_t^i)^2.$$

Company i's problem is to find (Markovian) abatement- and trading strategies α^i and β^i respectively, such that the cost function J, given by

$$J(t,x,\psi,\pi) = \mathbb{E}\left[\int_t^T e^{-rs} v_s^i \ ds\right],$$

is minimised by α^i, β^i for all $\pi > 0$, $\psi \in \mathbb{R}$, and such that the compliance constraint $\mathbb{E}_t[X_T^i] = c^i$ is satisfied for all $t \in [0,T]$. Let $w(t,x,\psi,\pi) = \inf_{(\alpha^i,\beta^i)} J(t,x,\psi,\pi)$ denote the value function for company i.
The company observes the state process $Z_t^i = (X_t^i, \Psi_t, \Pi_t)$, where

$$\begin{aligned} dX_t^i = (\alpha_t^i - \beta_t^i)\ dt + d\gamma_t^i &= (\alpha_t^i - \beta_t^i)dt + \lambda_t^i \ dW_t^{(1)}, \\ d\Psi_t = r\Psi_t dt - 2\varrho h_t d\gamma_t^I &= r\Psi_t dt - 2\varrho h_t \lambda_t^I \ dW_t^{(1)}, \\ d\Pi_t &= r\Pi_t dt + e^{rt} G_t \ dW_t^{(2)}. \end{aligned}$$

The Hamilton-Jacobi-Bellman (HJB) equation associated to the minimisation problem above is given by

$$0 = D_t w + r\psi D_\psi w + r\pi D_\pi w + \frac{1}{2}\mathrm{tr}(\sigma\sigma' D_z^2 w) + \inf_{\alpha,\beta}\left\{(\alpha^i - \beta^i)D_x w + e^{-rt}\ v_t^i(\alpha^i,\beta^i)\right\},$$

where σ denotes the matrix:

$$\begin{pmatrix} \lambda_t^i & 0 \\ \lambda_t^I & 0 \\ 0 & e^{-rt}G_t \end{pmatrix}$$

We notice that the minimising strategies α, β in the above equation have to satisfy

$$\alpha^i = -\frac{1}{2\varrho}\left(e^{rt}D_x w + \pi\right) \quad \text{and} \quad \beta^i = \frac{1}{2\nu}\left(e^{rt}D_x w + \psi + \pi\right). \tag{5}$$

Furthermore we notice that the second-order condition is satisfied for all α, β. Therefore, the HJB equation can be rewritten as

$$0 = e^{rt}\left(D_t w + r\psi D_\psi w + r\pi D_\pi w\right) + \frac{e^{rt}}{2}\operatorname{tr}(\sigma\sigma' D_z^2 w) - \frac{1}{4\varrho}\left(e^{rt}D_x w + \pi\right)^2 - \frac{1}{4\nu}\left(e^{rt}D_x w + \psi + \pi\right)^2. \tag{6}$$

In order to enforce the compliance constraint $\mathbb{E}_t[X_T^i] = c^i$ for all t, we impose the singular terminal condition

$$\lim_{t \nearrow T} w(t, x, \psi, \pi) = \begin{cases} 0 & : x = c^i, \\ \infty & : x \neq c^i. \end{cases} \tag{7}$$

The HJB equation (6), together with the terminal conditon (7) is solved by

$$w(t, x, \psi, \pi) = \frac{r\nu\varrho(x - c^i)^2}{(e^{rT} - e^{rt})(\nu + \varrho)} - e^{-rt}\left(\pi + \frac{\varrho\psi}{\nu + \varrho}\right)(x - c^i) - \frac{(e^{r(T-t)} - 1)\psi^2}{4re^{rt}(\nu + \varrho)} + \int_t^T C_s\, ds,$$

where

$$C_s = \frac{(\lambda_s^i)^2 r\nu\varrho}{(e^{rT} - e^{rs})(\nu + \varrho)} - \frac{\varrho}{\nu + \varrho}\lambda_s^i\lambda_s^I e^{-rs} - (\lambda_s^I)^2\frac{e^{r(T-s)} - 1}{4re^{rs}(\nu + \varrho)}.$$

The verification argument for w is straightforward but lengthy. Thus, we omit the full proof. We note that standard arguments of verification confirm α^i, β^i as the company's optimal strategies. Substituting $D_x w$ yields

$$\alpha_t^i = \frac{\Psi_t}{2(\nu + \varrho)} - \frac{\nu}{\nu + \varrho}\, h_t(X_t^i - c^i) \quad \text{and} \quad \beta_t^i = \frac{\Psi_t}{2(\nu + \varrho)} + \frac{\varrho}{\nu + \varrho}\, h_t(X_t^i - c^i).$$

This proves the equilibrium consisting of α^i, β^i to be as above for all $i \in I$ and the market clearing price process to be

$$P_t = \Pi_t - 2\varrho\frac{re^{rt}}{e^{rT} - 1}\left(X_0^I - c^I\right) - 2\varrho re^{rt}\int_0^t \frac{d\gamma_s^I}{e^{rT} - e^{rs}}.$$

Supply Management Policy

We give a brief derivation of the closed-form expression for the total number of allowances in circulation (TNA). In this section, we omit the superscript I for aggregate quantities. Consider the following contingency rule for the supply of allowances: at each time t, if the current TNA is above the level $c \geq 0$, then a fraction δdt of the difference $|\text{TNA} - c|$ is removed from the auction and added to the reserve. That is, $\delta \cdot |\text{TNA} - c| dt$ allowances are removed from the allocation schedule. Conversely, $\delta \cdot |\text{TNA} - c|dt$ allowances are added to the allocation schedule if the TNA is lower than c.

Let f_t represent the fixed pre-MSR allocation schedule and let g_t denote the future expected emissions. We obtain

$$d\text{TNA}_t = f_t dt + \delta(c - \text{TNA}_t)dt - g_t dt - d\varepsilon_t + \alpha_t dt, \tag{8}$$

and notice that Equation (8) yields an expression for TNA_t in terms of the process α_t:

$$\text{TNA}_t = N_0 e^{-\delta t} + \int_0^t e^{\delta(s-t)}(\alpha_s + f_s + \delta c - g_s)\, ds - \int_0^t e^{\delta(s-t)}\, d\varepsilon_s. \tag{9}$$

A straightforward calculation yields that the aggregate abatement α_t at time t is given by

$$\alpha_t = -re^{rt}\frac{X_0 - c}{e^{rT} - 1} - re^{rt}\int_0^t \frac{d\gamma_s}{e^{rT} - e^{rs}},$$

where the dynamics of γ_t are given by

$$d\gamma_t = -d\mathbb{E}_t\left[\int_0^T \delta\text{TNA}_s\, ds + \int_0^T d\varepsilon_s\right] = -\delta d\mathbb{E}_t\left[\int_0^T \text{TNA}_s\, ds\right] - d\varepsilon_t.$$

In order to obtain α_t and TNA_t distributions, we solve the above expression. To simplify notation, we define

$$h_t = \frac{re^{rt}}{e^{rT} - e^{rt}}.$$

This yields

$$d\alpha_t = r\alpha_t dt - h_t d\gamma_t. \tag{10}$$

Since the constraint $\mathbb{E}_t[\text{TNA}_T] = \mathbb{E}_t[X_T] = c$ is satisfied for all t, we have

$$d\mathbb{E}_t\left[\int_0^T e^{\delta s}\alpha_s\, ds\right] = d\mathbb{E}_t\left[\int_0^T e^{\delta s}(g_s - f_s)\, ds - \text{TNA}_0 + \int_0^T e^{\delta s}\, d\varepsilon_s\right] = e^{\delta t} d\varepsilon_t. \tag{11}$$

We use the dynamics of α_t in equation (10) to establish:

$$\int_0^t \alpha_s e^{\delta s}\, ds = \frac{e^{\delta t}}{\delta + r}\left(\alpha_t - \alpha_0 e^{-\delta t} + \int_0^t e^{\delta(s-t)} h_s\, d\gamma_s\right),$$

which, together with equation (11), yields:

$$d\gamma_t = -\frac{V_t(\delta, r)}{h_t}\, d\varepsilon_t, \tag{12}$$

where $V_t(\delta, r) = (\delta + r)/(e^{(\delta+r)(T-t)} - 1)$. From this we finally obtain

$$\text{TNA}_t = N_0 e^{-\delta t} - \frac{r(e^{rt} - e^{-\delta t})}{(\delta + r)(e^{rT} - 1)}(X_0 - c) - \frac{e^{rt}}{V_t(\delta, r)} \int_0^t e^{-rs}\, V_s(\delta, r)\, d\varepsilon_s + \int_0^t e^{\delta(s-t)}(f_s - g_s + \delta c)\, ds. \quad (13)$$

Note that $X_0 = N_0 + \gamma_0$. Using the compliance condition, we can derive the expression for γ_0:

$$\gamma_0 = \frac{(\delta + r)(e^{rT} - 1)}{r(e^{rT} - e^{-\delta T})} \left(N_0 e^{-\delta T} + \int_0^T e^{\delta(s-T)}(f_s - g_s + \delta c)\, ds - c \right) - N_0 + c.$$

Thus, TNA_t is indeed determined in closed-form by Equation (13).

Risk-adjusted Dynamics

We solved the model above under the assumption that all companies are risk-neutral. In particular, companies are assumed to form their expectation under a risk-neutral measure $\mathbb{Q}$. We now evaluate the equilibrium dynamics under the objective measure $\mathbb{P}$ using a risk-adjusted discount rate.

Let μ represent the historical rate of return of $\Psi_t = P_t - \Pi_t$ and let the process k_t represent its historical volatility. That is, the dynamics of Ψ_t under the objective measure $\mathbb{P}$ shall be approximated by

$$d\Psi_t = \mu \Psi_t dt + k_t dW_t^{\mathbb{P}}, \quad (14)$$

where $W_t^{\mathbb{P}}$ is a standard Brownian motion under $\mathbb{P}$. Recall that under $\mathbb{Q}$, the process Ψ_t has dynamics

$$d\Psi_t = r\Psi_t dt - 2\varrho\, h_t d\gamma_t.$$

By substituting for the dynamics of γ_t as given by Equation (12), and recalling that $d\varepsilon_t = \kappa_t dW_t^{(1)}$, we obtain:

$$d\Psi_t = r\Psi_t dt + 2\varrho V_t(\delta, r)\kappa_t dW_t^{\mathbb{Q}},$$

where $W_t^{\mathbb{Q}}$ denotes the $\mathbb{Q}$-Brownian motion $W_t^{(1)}$. When applying the model to a pre-MSR era, the adjustment rate δ vanishes. Therefore we set $\kappa_t = k_t/(2\varrho V_t(0, r))$ and obtain for $\delta = 0$ the dynamics

$$d\Psi_t = r\Psi_t dt + k_t dW_t^{\mathbb{Q}}. \quad (15)$$

Note that for given level of expected emissions g_t, the emissions process given by $g_t dt + d\varepsilon_t = g_t dt + \kappa_t dW_t^{\mathbb{Q}}$ is then fully determined. The volatility of emissions, as assessed by the regulated companies, is therefore implicitly calculated by means of the time series of historical permit prices.

From Equations (14) and (15) we observe that the $\mathbb{Q}$-Brownian motion $W_t^{\mathbb{Q}}$ has to satisfy

$$dW_t^{\mathbb{Q}} = dW_t^{\mathbb{P}} - \frac{r-\mu}{k_t}\,\Psi_t\,dt.$$

By Girsanov's theorem, $\mathbb{Q}$ with $d\mathbb{Q}/d\mathbb{P} = e^{-\vartheta_T + rT}$ satisfies the equality above, where

$$\vartheta_t = rt + \frac{1}{2}\int_0^t \zeta_s^2 \Psi_s^2\,ds - \int_0^t \zeta_s \Psi_s\,dW_s^{\mathbb{P}},$$

and ζ_t denotes the ratio $(r-\mu)/k_t$; since the process $e^{-\vartheta_t + rt}$ is a $\mathbb{P}$-martingale.

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