Horizontal Mergers and Product Quality

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Abstract

We study the effects of horizontal mergers when firms compete on quality and price. Two key factors are identified: (i) the magnitude of variable quality costs, and (ii) the relative magnitudes of cross-quality and cross-price effects on demand. The merging firms will increase (reduce) both quality and price if the degree of competition is sufficiently stronger (weaker) on price than on quality. If variable quality costs are sufficiently small, non-merging firms will respond to a merger by either reducing or increasing both price and quality. Welfare implications are not clear-cut and mergers might improve welfare through endogenous fixed-cost savings.

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1 Introduction

It is by now widely recognised that horizontal mergers may affect consumer welfare not only through price changes but also through changes in key non-price characteristics such as product quality. In the recent empirical merger literature, several studies show, based on merger simulations, that quality effects of mergers can be hugely important. For example, Fan (2013) develops a structural model of newspaper markets and show that ignoring adjustments to product characteristics as a result of a merger substantially affects the simulated merger effects. Similar conclusions are reached by Israel et al. (2013) and Tenn et al. (2010) based on merger simulations in the airline and ice cream industries, respectively.[1] In recent antitrust practice, the quality dimension is also frequently mentioned. For example, a feared reduction of service quality was one of the elements determining the European Commission's decision to reject the proposed takeover of Aer Lingus by Ryanair in 2007.[2]

However, the effects of mergers on quality remain an under-researched issue, where the potential theoretical mechanisms are less well understood, which poses a considerable challenge to competition policy practitioners who aim to deal with quality effects in a comprehensive way. This knowledge gap is aptly summarised by the OECD Competition Committee (based on a 2013 roundtable debate) as follows:

> "While the importance of quality is undisputed and issues about quality are mentioned pervasively in competition agency guidelines and court decisions, there is no widely-agreed framework for analysing it which often renders its treatment superficial [.......] the role of quality effects in merger controls, and in particular, trading off between quality and price effects, remains to be one of the most vexatious – and still unresolved – issues." (OECD, 2013, p. 1)

In the present paper we offer a contribution towards filling this knowledge gap by pre-

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[1] Tenn et al. (2010) consider promotional effort rather than quality, but these two non-price dimensions have of course many similarities.

[2] Commission decision of 27 June 2007 in Case No COMP/M.4439 – Ryanair / Aer Lingus I.

senting what is, to our knowledge, the first comprehensive and general theoretical analysis of mergers when firms compete on both price and quality. We perform our analysis within a very general framework in which firms produce differentiated products and where demand responds negatively (positively) to own price (quality) and positively (negatively) to competing firms' prices (qualities). We also allow for both fixed and variable costs associated with increasing the quality of a product.

Our analysis reveals that the price and quality effects of a merger are far from clear-cut and that a number of different equilibrium configurations are possible regarding the responses of merging and non-merging firms. However, our analysis also shows that these effects are to a large extent determined by two different factors: (i) the magnitude of variable quality costs, which determines the nature of strategic interaction along the quality and price dimensions, and (ii) the relative magnitude of cross-quality and cross-price effects on demand, which determines the relative intensity of competition along the quality and price dimensions.

Whether the merged firm will increase or reduce qualities and prices depend on the relative strength of two counteracting incentives, which contributes to the general ambiguity of the merger effects. These incentives will be carefully explained later on; here we will just summarise a few of our main findings. If firms compete sufficiently strongly on quality relative to price, the merged firm will increase both price and quality, and, *vice versa*, if competition is sufficiently much stronger on prices than on qualities, the merged firm will reduce both price and quality. However, for intermediate cases, it is also possible that the merged firm will increase the price and reduce the quality.

The response from non-merging firms depends on the nature of strategic interaction. If variable quality costs are sufficiently small, qualities are net strategic substitutes and prices are net strategic complements.[3] In this case, we show that the non-merging firms' quality and price responses always go in the same direction. If the merged firm's incentives to reduce quality are sufficiently strong, the non-merging firms will respond by *increasing* both quality

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[3]The concept of net strategic substitutability/complementarity is explained in Section 2.

and price. On the other hand, if the merged firm's have sufficiently strong incentives to increase prices, the non-merging firms will respond by *reducing* both quality and price. The former case arises if competition is sufficiently strong along the quality dimension, whereas the latter case requires that competition is sufficiently strong along the price dimension.

Because of the general ambiguity of the merger effects, we also present a parametric example based on a three-firm Salop (1979) model, in order to illustrate some of the main mechanisms of the general model. Among other things, this parametric model reveals that when competition takes place also along a quality dimension, this opens up for the possibility that mergers might improve social welfare through endogenous fixed-cost savings. This and other potential welfare implications are discussed in a separate section of the paper.

Finally, we also extend the main analysis to allow for the possibility that, after the merger, the merger participants might withdraw one of their products (or, depending on the interpretation of the model, close down production at one of their plants). In this case, we show that a merger will lead to higher quality and price for all firms in the industry if two conditions are met: (i) the merger increases the demand-responsiveness to price to a sufficient degree, and (ii) variable quality costs are sufficiently small.

Besides extending the standard horizontal merger literature[4] by including a quality dimension, our paper is also related to the vast literature on competition and quality, dating back at least to Swan (1970), who compared the incentives of a monopolist and a competitive firm with respect to a particular quality dimension, namely product durability. Much of the subsequent literature consists of papers that apply a vertical differentiation framework, often with firms that offer a range of products with different qualities.[5] Models of price-quality competition in a horizontal differentiation framework are fewer and include, i.e., Economides

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[4]See the seminal contributions by Salant et al. (1983), Perry and Porter (1985), and Farrell and Shapiro (1990) for horizontal mergers in a Cournot oligopoly. Our study is more related to the seminal paper by Deneckere and Davidson (1985) who were the first to analyse horizontal mergers when firms produce differentiated products and compete in prices, showing that the merger effects are very different than in a Cournot setting.

[5]Some early key contributions to this strand of the literature include Gabszewicz and Thisse (1979, 1980) and Shaked and Sutton (1982, 1983) for the case of single-product firms, and Mussa and Rosen (1979), Gal-Or (1983) and Champsaur and Rochet (1989) for the case of multi-product firms.

(1993), Ma and Burgess (1993), Gravelle (1999) and Brekke et al. (2010).[6]

Although the effect of competition on quality provision is addressed in some of the above mentioned papers, there is no explicit merger analysis. In fact, theoretical studies that explicitly analyse the effects of a horizontal merger on the price and quality offered by merging and non-merging firms are almost non-existent.[7] A rare recent exception is Pinto and Sibley (2014), who add quality as a demand-shifter to a standard Marshallian-type demand system with horizontal product differentiation and perform a merger analysis based on numerical simulations. There are several differences between their paper and ours. Importantly, while their analysis is based on a parametric model with explicit functional forms applied to the demand and cost structure, our analysis is based on a much more general framework.[8] Furthermore, besides applying a more general model, we also consider the case of product withdrawal following a merger (what we refer to as 'closure'), an aspect that is missing from their analysis.

Despite the obvious importance of the topic, the empirical literature on the effects of horizontal mergers on quality is also relatively scarce. Besides the recent papers mentioned earlier, there are also a few studies addressing the quality effects of mergers in hospital markets, where quality is clearly a key issue.[9] These studies tend to find large price effects and weak (though mostly negative) effects on quality (see Gaynor and Town, 2012).[10]

Our paper is also somewhat related to the literature on horizontal mergers and product

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[6]There are also a few empirical papers studying the effect of more competition on quality; for example Mazzeo (2003), who finds a positive relationship between competition and quality in the US airline industry; and Matsa (2011) who studies the effect of competition on supermarkets' incentive to provide quality, and finds that competition from Wal-Mart decreases inventory shortfalls by up to 24 percent.

[7]Willig (2011) includes product quality in an analysis of unilateral competitive effects of horizontal mergers ('upward pricing pressure'), but there is no equilibrium analysis with strategic interaction between merging and non-merging firms.

[8]This implies, among other things, that we are able to capture some effects that are missing from their study. For example, while Pinto and Sibley assume constant marginal production costs, we allow for marginal production costs to increase with quality. Holding prices fixed, the latter assumption implies that qualities are strategic complements whereas the former assumption implies strategic independence.

[9]There also exists a couple of theoretical studies on hospital mergers: Calem, Dor and Rizzo (1999) and Brekke (2004). Among several differences with the present study, an important limitation of these papers is that the analysis is cast in a duopoly setting, implying that a merger leads to a monopolisation of the hospital market. In contrast, an important feature of our merger analysis is how non-merging firms respond to the merger.

[10]See also Ho and Hamilton (2000), Capps (2005) and Romano and Balan (2011).

choice, which acknowledges that a merger might lead the merging (and possibly non-merging) firms to reposition their products or to change their product line. Theoretical contributions in this strand of the literature include Lommerud and Sørgard (1997), Posada and Straume (2004) and Gandhi et al. (2008), whereas key empirical contributions include Berry and Waldfogel (2001) and Sweeting (2010).

The rest of the paper is organised as follows. In Section 2 we present the general model and explore the nature of strategic interaction in the model. We also present a parametric example based on the Salop model in this section. Section 3 contains the main part of the analysis, where we explore the effects of a merger on the quality and price choices of merging and non-merging firms. In Section 4 we discuss how adding a quality dimension to firms' strategic interaction might affect the welfare implications of a merger. In Section 5 we extend the analysis to consider the case of closure, where the merging firms withdraw one of their products (or close down one of their plants) after the merger. Finally, in Section 6 we summarise our findings and offer some concluding reflections.

2   Model

Consider a market with n single-product firms, each producing a differentiated product. Demand for good i is given by $D_i\left(q_1, .., q_n, p_1, .., p_n, n\right)$, where q_i and p_i are the quality and price, respectively, of good i, with $\frac{\partial D_i}{\partial p_i} < 0$, $\frac{\partial D_i}{\partial p_j} > 0$, $\frac{\partial D_i}{\partial q_i} > 0$, $\frac{\partial D_i}{\partial q_j} < 0$, $\frac{\partial^2 D_i}{\partial q_i^2} \lesseqgtr 0$ and $\frac{\partial^2 D_i}{\partial p_i^2} \lesseqgtr 0$. We also assume that an increase in n reduces the demand for each good and makes demand more responsive to quality and price: $\frac{\partial D_i}{\partial n} < 0$, $\frac{\partial^2 D_i}{\partial q_i \partial n} > 0$ and $\frac{\partial^2 D_i}{\partial p_i \partial n} < 0$. We assume that the demand system is symmetric and that demand for each good is separable in all qualities and prices.

The cost function of Firm i is assumed to be given by

$$C_i\left(q_i, D_i\right) = c\left(q_i\right) D_i + K\left(q_i\right), \tag{1}$$

where $\frac{\partial c}{\partial q_i} > 0$, $\frac{\partial^2 c}{\partial q_i^2} \lessgtr 0$, $\frac{\partial K}{\partial q_i} > 0$ and $\frac{\partial^2 K}{\partial q_i^2} > 0$. Thus, we assume constant marginal production costs for a given quality level, but there are both variable and fixed costs associated with an increase in the quality of the good produced.[11] For example, higher quality implies higher variable production costs if more expensive inputs are required to produce a higher-quality product.

Firms are assumed to be profit-maximisers with price and quality as their strategic choice variables. With the above demand and cost functions, the profit of Firm i is given by

$$\pi_i\left(q_i, p_i\right) = \left[p_i - c\left(q_i\right)\right] D_i - K\left(q_i\right). \tag{2}$$

We assume the firms play a non-cooperative game where price and quality are chosen simultaneously.

2.1 Parametric example

Throughout the general analysis, in order to illustrate some of the main mechanisms of the model, we will present results from a parametric example based on spatial competition. Suppose that $n = 3$ with the firms being equidistantly located on a circle with circumference equal to 1. A total mass of 1 consumers, each with unit demand, are uniformly distributed on the same circle. The net utility of a consumer located at z and buying the good from Firm i, located at x_i, is given by

$$u_{z,x_i} = v + bq_i - p_i - t\left(z - x_i\right)^2, \tag{3}$$

where v, b and t are all strictly positive. The corresponding demand for the good offered by Firm i is

$$D_i\left(p_i, p_{i-1}, p_{i+1}, q_i, q_{i-1}, q_{i+1}\right) = \frac{1}{3} + \frac{3\left(b\left(2q_i - q_{i+1} - q_{i-1}\right) - 2p_i + p_{i+1} + p_{i-1}\right)}{2t}. \tag{4}$$

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[11] The function $K\left(\cdot\right)$ might also capture fixed costs that are not related to quality, in which case $K\left(0\right) > 0$.

The cost function is parameterised by setting $c(q_i) = cq_i$ and $K(q_i) = \frac{k}{2}q_i^2 + F$, where c, k and F are positive parameters. We also impose the restriction $b > c$ in order to ensure interior solutions with strictly positive equilibrium quality levels in the pre- and post-merger games.[12]

2.2 Strategic relationship between qualities and prices

Before analysing the effects of a merger on prices and qualities, it is highly instructive to investigate the strategic relationship between the firms' choice variables, which to a large extent determines the nature of the different mechanisms at play. Consider, for simplicity, the case of $n = 2$, in which the definition of strategic substitutability/complementarity is straightforward. The symmetric Nash equilibrium is then implicitly characterised by the following pair of first-order conditions:

$$\frac{\partial \pi_i}{\partial q_i} = (p_i - c(q_i))\frac{\partial D_i}{\partial q_i} - D_i\frac{\partial c}{\partial q_i} - \frac{\partial K}{\partial q_i} = 0, \tag{5}$$

$$\frac{\partial \pi_i}{\partial p_i} = D_i + (p_i - c(q_i))\frac{\partial D_i}{\partial p_i} = 0, \tag{6}$$

$i = 1, 2$. From this system of equations we can derive two different sets of best-response functions: (i) $q_i(p_i, q_j, p_j)$ and $p_i(q_i, q_j, p_j)$, which are the best-quality-response and best-price-response functions, respectively, when keeping all other variables constant; and (ii) $q_i(q_j, p_j)$ and $p_i(q_j, p_j)$, which are the best-quality-response and best-price-response functions when the firm optimally adjust its own price and quality, respectively. The first set of best-response functions determine whether qualities (prices) are *gross* strategic substitutes or complements, whereas the second set of best-response functions determine *net* strategic substitutability or complementarity.

Consider first the strategic relationship between qualities for given price levels, and between

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prices for given quality levels. Regarding qualities, the strategic relationship is given by

$$sign\left(\frac{\partial q_i\,(p_i, q_j, p_j)}{\partial q_j}\right) = sign\left(\frac{\partial^2 \pi_i}{\partial q_i \partial q_j} = -\frac{\partial D_i}{\partial q_j}\frac{\partial c}{\partial q_i}\right) > 0. \tag{7}$$

Thus, as long as there is a positive relationship between quality and marginal production costs, qualities are *gross strategic complements*. The intuition is the following. If one firm increases its quality, the competing firm loses demand, which in turn reduces its marginal cost of quality provision. This firm will therefore respond by increasing its quality. Notice that this strategic relationship requires the presence of variable quality costs.

Regarding prices, the strategic relationship is given by

$$sign\left(\frac{\partial p_i\,(q_i, q_j, p_j)}{\partial p_j}\right) = sign\left(\frac{\partial^2 \pi_i}{\partial p_i \partial p_j} = \frac{\partial D_i}{\partial p_j}\right) > 0. \tag{8}$$

Thus, prices are *gross strategic complements*, which is a standard result from the oligopoly literature. All else equal, a unilateral price increase by one firm leads to higher demand for the competing firm, which optimally responds by increasing its price.

However, if a firm changes its price (quality), it will also have an incentive to optimally adjust its quality (price). The strategic relationship between price and quality within a firm is given by

$$sign\left(\frac{\partial^2 \pi_i}{\partial q_i \partial p_i} = \frac{\partial^2 \pi_i}{\partial p_i \partial q_i} = \frac{\partial D_i}{\partial q_i} - \frac{\partial D_i}{\partial p_i}\frac{\partial c_i}{\partial q_i}\right) > 0. \tag{9}$$

Thus, *price and quality are strategic complements within firms*. Abstracting from any strategic responses by the competing firm, a price increase has two effects on incentives for quality provision. It increases the firm's profit margin and also reduces the marginal cost of quality provision through lower demand. Both effects contribute to a higher optimal level of quality. *Vice versa*, a higher quality level leads to higher demand and also increases marginal production costs, and both effects contribute to a higher optimal price.

By internalising the above price-quality relationship, we can derive the conditions for qualities (prices) to be net strategic substitutes or complements. By differentiating (5)-(6) with

respect to q_i, p_i and q_j, and applying Cramer's Rule, we have

$$sign\left(\frac{\partial q_i\left(q_j, p_j\right)}{\partial q_j}\right) = sign\left(\left(\frac{\partial D_i}{\partial q_i} + \frac{\partial c}{\partial q_i}\left(\frac{\partial D_i}{\partial p_i} + (p_i - c)\frac{\partial^2 D_i}{\partial p_i^2}\right)\right)\frac{\partial D_i}{\partial q_j}\right). \qquad (10)$$

Thus, whether qualities are net strategic substitutes or complements depends on the relative strength of two opposing forces. On the one hand, qualities are gross strategic complements (second term in (10)). On the other hand, if competing Firm j increases quality, then Firm i will have lower demand and its profits are therefore maximised, all else equal, at a lower price. Since price and quality are within-firm strategic complements, the quality level will also be adjusted downwards (first term in (10)). If latter effect dominates the former, qualities are net strategic substitutes.

Similarly, the net strategic relationship between prices are given by

$$sign\left(\frac{\partial p_i\left(q_j, p_j\right)}{\partial p_j}\right) = sign\left(\left(-\frac{\partial^2 \pi_i}{\partial q_i^2} - \frac{\partial c}{\partial q_i}\left(\frac{\partial D_i}{\partial q_i} - \frac{\partial D_i}{\partial p_i}\frac{\partial c}{\partial q_i}\right)\right)\frac{\partial D_i}{\partial p_j}\right). \qquad (11)$$

Again, the sign of this expression is determined by the relative strength of two counteracting forces. On the one hand, prices are gross strategic complements (first term in (11)). On the other hand, a price increase by competing Firm j leads to higher demand for Firm i. As a result, Firm i's marginal costs of quality provision will increase and it will optimally respond by reducing its quality. Since price and quality are within-firm strategic complements, the price level will also be adjusted downwards (second term in (11)). If the former effect dominates the latter, prices are net strategic complements.

In our parametric example, it is easily shown that qualities are always net strategic substitutes, whereas prices are net strategic complements. More generally, the above analysis can be summarised as follows:

Lemma 1 *If variable quality costs are sufficiently small, qualities are net strategic substitutes whereas prices are net strategic complements.*

3 Quality and price effects of a merger

Consider a merger between two of the n firms in the industry. In a differentiated products model, given that the merger does not affect the number of goods produced, the post-merger game is an asymmetric game between one multi-product firm (the merged firm) and $n - 2$ single-product firms.[13] Thus, a merger is a discrete change of market structure that, in a general (non-parameterised) model, makes it hard to use standard comparative statics tools to assess the effects of the merger. One way to overcome this problem is to consider a 'marginal merger'. Suppose that the objective functions of the merger candidates (denoted i and j) are $\Pi_i := \pi_i + \alpha \pi_j$ and $\Pi_j := \pi_j + \alpha \pi_i$, respectively, where $\alpha \in (0, 1)$. The pre- and post-merger games appear then as the special cases of $\alpha = 0$ and $\alpha = 1$, respectively. Given the assumption that the equilibrium outcomes (prices and qualities) are monotonic in α, we can qualitatively assess the effects of a merger on these variables by considering a marginal increase in α.[14,15]

Let $N = \{1, .., n\}$ be the set of pre-merger firms/products in the industry, let $M = \{i, j\}$ be the set of merger participants, and let $O = N \backslash M$ be the set of outside (non-merging) firms. For the merging Firm i, which merges with Firm j, the first-order conditions for optimal quality and price are given by

$$\frac{\partial \Pi_i}{\partial q_i} = (p_i - c(q_i)) \frac{\partial D_i}{\partial q_i} - D_i \frac{\partial c}{\partial q_i} - \frac{\partial K}{\partial q_i} + \alpha (p_j - c(q_j)) \frac{\partial D_j}{\partial q_i} = 0, \tag{12}$$

$$\frac{\partial \Pi_i}{\partial p_i} = D_i + (p_i - c(q_i)) \frac{\partial D_i}{\partial p_i} + \alpha (p_j - c(q_j)) \frac{\partial D_j}{\partial p_i} = 0. \tag{13}$$

Because of symmetry, $p_j = p_i$ and $q_j = q_i$ in equilibrium, which implies that (12)-(13) can be re-written as

$$(p_i - c(q_i)) \left(\frac{\partial D_i}{\partial q_i} + \alpha \frac{\partial D_j}{\partial q_i} \right) - D_i \frac{\partial c}{\partial q_i} - \frac{\partial K}{\partial q_i} = 0, \tag{14}$$

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[13]See Section 5 for an analysis of the case of merger with 'closure', where the merged firm stops producing one of the goods after the merger.

[14]We are truly grateful to an anonymous referee for suggesting this ingenious approach to the problem.

[15]This approach is somewhat similar to the concept of an 'infinitesmal merger' proposed by Farrell and Shapiro (1990), where such a merger is defined as a small change in the output of the merger participants (the insiders).

$$D_i + (p_i - c(q_i)) \left(\frac{\partial D_i}{\partial p_i} + \alpha \frac{\partial D_j}{\partial p_i} \right) = 0. \tag{15}$$

For the non-merging Firm k, the first-order conditions are

$$\frac{\partial \pi_k}{\partial q_k} = (p_k - c(q_k)) \frac{\partial D_k}{\partial q_k} - D_k \frac{\partial c}{\partial q_k} - \frac{\partial K}{\partial q_k} = 0, \tag{16}$$

$$\frac{\partial \pi_k}{\partial p_k} = D_k + (p_k - c(q_k)) \frac{\partial D_k}{\partial p_k} = 0, \quad k \in O. \tag{17}$$

The Nash equilibrium is thus implicitly given by a system of four equations, (14)-(17), where all demand functions and their first-order derivatives are evaluated at the quality-price vector $(q_i, q_i, q_k, ..., q_k, p_i, p_i, p_k, ...p_k)$. By differentiating the system (14)-(17) with respect to (q_i, p_i, q_k, p_k) and α, and applying Cramer's Rule, we can derive the equilibrium effects of the merger on the qualities and prices of all firms in the industry.

3.1 The quality of the merging firms

Qualitatively, the effect of a merger on the merging firms' quality provision is given by[16]

$$sign(\frac{\partial q_i}{\partial \alpha}) = sign \left(\begin{array}{c} \left[-\frac{\partial D_j}{\partial q_i} \frac{\partial^2 \pi_i}{\partial p_i^2} + \frac{\partial D_j}{\partial p_i} \left(\frac{\partial (D_i - D_j)}{\partial q_i} - \frac{\partial c}{\partial q_i} \frac{\partial (D_i + D_j)}{\partial p_i} \right) \right] \Phi \\ +2(n-2)\frac{\partial D_k}{\partial p_i} \left(\frac{\partial D_j}{\partial q_i} + \frac{\partial c}{\partial q_i} \frac{\partial D_j}{\partial p_i} \right) \left(\frac{\partial D_i}{\partial p_k} \Omega_p - \frac{\partial D_i}{\partial q_k} \Omega_q \right) \end{array} \right), \tag{18}$$

where $\Phi > 0$ is a function of the equilibrium variables (see Appendix A for an explicit definition)[17], and

$$\Omega_p := \frac{\partial^2 \pi_k}{\partial q_k^2} + \frac{\partial c}{\partial q_k} \left(\frac{\partial D_k}{\partial q_k} - \frac{\partial c}{\partial q_k} \frac{\partial D_k}{\partial p_k} \right), \tag{19}$$

$$\Omega_q := \frac{\partial D_k}{\partial q_k} + \frac{\partial c}{\partial q_k} \left(\frac{\partial^2 \pi_k}{\partial p_k^2} - \frac{\partial D_k}{\partial p_k} \right). \tag{20}$$

We can classify the various sub-effects into two categories: (i) first-order effects through the merged firm's quality and price setting, and (ii) second-order (feedback) effects through the strategic responses of non-merging firms. The two sets of effects are grouped together

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[16]See Appendix A for full details of the derivations of all expressions in this section.

[17]The positive sign of Φ follows from the assumption of a negative definite Jacobian matrix.

in the two terms on the right-hand side of (18), where the first (second) term represents the first-order (second-order) effects.

There are two different first-order effects, which have opposite sign. First, the merger allows the merger participants to internalise a negative quality competition externality, which implies a lower quality level. This effect is captured by the first term in the square brackets of (18). Second, the merger also allows the merged firm to internalise a negative price competition externality, which implies a higher price. Since price and quality are within-firm strategic complements, this contributes to a higher quality level as a result of the merger. This effect is captured by the second term in the square brackets.

Thus, the sum of the above described first-order effects is *a priori* indeterminate and depends on how strong competition is on quality versus price; more specifically, the overall effect depends on the cross-quality versus cross-price effects on demand. Suppose that the cross-quality effects on demand are large relative to the cross-price effects (i.e., $|\partial D_j/\partial q_i|$ is large relative to $\partial D_j/\partial p_i$), which implies that firms compete harder on quality than on price. In this case, the first of the two above described effects is large relative to the second, implying a drop in quality provision by the merger participants. *Vice versa*, if the degree of price competition is sufficiently strong relative to the degree of quality competition, a merger might lead to higher quality by the merging firms.

The first-order effects of a merger are complemented by feedback effects through the non-merged firms' price and quality responses. These effects are given by the second term in (18), with an *a priori* indeterminate sign. Whether the feedback effects counteract or reinforce the first-order effects depends on the relative strength of quality and price competition, and on the strategic nature of competition along these two dimensions. By comparing (19)-(20) with (10)-(11), we see that the signs of Ω_p and Ω_q are exactly determined by whether prices and qualities, respectively, are net strategic substitutes or complements. More specifically, $\Omega_p > (<) 0$ if prices are net strategic substitutes (complements), whereas $\Omega_q > (<) 0$ if qualities are net strategic substitutes (complements), as these concepts are defined in Section 2.2.

Consider the case of small variable quality costs, implying that $(\partial D_j/\partial q_i)+(\partial c/\partial q_i)\,(\partial D_j/\partial p_i)<$ 0 and that prices are net strategic complements whereas qualities are net strategic substitutes (i.e., $\Omega_p<0$ and $\Omega_q>0$). In this case, the sign of the feedback effects is determined by the relative magnitude of the terms $|(\partial D_i/\partial p_k)\,\Omega_p|$ and $|(\partial D_i/\partial q_k)\,\Omega_q|$. If the degree of quality competition is sufficiently strong relative to the degree of price competition, the second term dominates and the feedback effects are negative. On the other hand, if the firms compete sufficiently much harder on price than on quality, the first term dominates and the feedback effects are positive. In both cases, though, the feedback effects tend to reinforce the first-order effects, since the first-order quality effects of a merger is negative (positive) when competition is sufficiently much stronger along the quality (price) dimension. More generally, though, whether the feedback effects reinforce or counteract the first-order effects, we would expect that the feedback effects do not dominate, implying that the sign of the merger effect on quality is determined by the previously described first-order effects.

The ambiguity of the general model is resolved in our parametric example. In the linear Salop model, the effect of a merger on the merged firm's quality provision is found always to be negative. More generally, we can summarise the above analysis as follows:

Proposition 1 *Suppose that the effect of a merger on the merged firm's quality provision is determined by the sign of the first-order effects. A merger will then lead to lower (higher) quality by the merged firm if the cross-quality effect on demand is sufficiently large (small) relative to the cross-price effect.*

3.2   The price of the merging firms

The effect of the merger on the prices charged by the merging firms is given, in qualitative terms, by

$$
sign(\frac{\partial p_i}{\partial \alpha}) = sign\left(
\begin{array}{c}
\left[-\frac{\partial D_j}{\partial p_i}\frac{\partial^2 \pi_i}{\partial q_i^2} + \frac{\partial D_j}{\partial q_i}\left(\frac{\partial(D_i+D_j)}{\partial q_i} - \frac{\partial c}{\partial q_i}\frac{\partial(D_i-D_j)}{\partial p_i}\right)\right]\Phi \\
-2(n-2)\frac{\partial D_k}{\partial q_i}\left(\frac{\partial D_j}{\partial q_i} + \frac{\partial c}{\partial q_i}\frac{\partial D_j}{\partial p_i}\right)\left(\frac{\partial D_i}{\partial p_k}\Omega_p - \frac{\partial D_i}{\partial q_k}\Omega_q\right)
\end{array}
\right). \tag{21}
$$

As for the quality effect of the merger, the price effect is a sum of two sets of sub-effects: first-order effects (first term) and feedback effects (second term). Also equivalently to the quality effect, the first-order price effects consist of two counteracting mechanisms. On the one hand, the internalisation of the price competition externality leads to higher prices. This is captured by the first term in the square brackets of (21). On the other hand, the internalisation of the quality competition externality leads to lower quality and therefore to lower prices, because of within-firm strategic complementarity between price and quality. This effect is captured by the second term in the square brackets. Once more, the relative magnitude of these effects depend on the relative strength of price competition versus quality competition. If the cross-price effects on demand are sufficiently large relative to the cross-quality effects, implying that the firms compete more strongly on price than on quality, the former effect dominates and a merger leads (by the first-order effects) to higher prices. However, if the degree of quality competition is sufficiently strong relative to the degree of price competition, the latter effect will dominate and a merger might, perhaps paradoxically, lead to lower prices.

The non-merging firms' price and quality responses create feedback effects on the merged firm's optimal price setting. These feedback effects are given by the second term in (21), with a generally indeterminate sign. However, notice by comparison of the second terms in (18) and (21) that the direction of these feedback effects are equivalent for price and quality. Thus, as for the case of quality, the feedback effects tend to reinforce the first-order price effects when variable quality costs are sufficiently small, and when the degree of competition is sufficiently much stronger in either the price or the quality dimension.

The above analysis illustrates how introducing a quality dimension to firms' strategic interaction can dramatically alter the price effects of a merger. In the absence of quality competition, a merger without cost synergies would always lead to higher prices. However, when firms also compete on quality, the price effect of a merger becomes generally indeterminate, and depends on the relative strength of price and quality competition, as explained above. This indeterminacy is also captured in our parametric example, where it can be shown (see

Appendix B) that the price set by the merging firms increases (decreases) if the demand responsiveness to quality is sufficiently low (high). More generally, the price effects of a merger can be summarised as follows:

Proposition 2 *Suppose that the effect of a merger on the merged firm's price decisions is determined by the sign of the first-order effects. A merger will then lead to higher (lower) prices by the merged firm if the cross-price effect on demand is sufficiently large (small) relative to the cross-quality effect.*

3.3   Quality and price responses by non-merging firms

The quality and price responses of the non-merging firms are given by, respectively,

$$sign(\frac{\partial q_k}{\partial \alpha}) = sign\left(\Omega_q \left[\frac{\partial D_k}{\partial q_i}\Psi_q + \frac{\partial D_k}{\partial p_i}\Psi_p\right]\right) \tag{22}$$

and

$$sign(\frac{\partial p_k}{\partial \alpha}) = sign\left(-\Omega_p \left[\frac{\partial D_k}{\partial q_i}\Psi_q + \frac{\partial D_k}{\partial p_i}\Psi_p\right]\right), \tag{23}$$

where

$$\Psi_p := -\frac{\partial D_j}{\partial p_i}\frac{\partial^2 \pi_i}{\partial q_i^2} + \frac{\partial D_j}{\partial q_i}\left(\frac{\partial(D_i + D_j)}{\partial q_i} - \frac{\partial c}{\partial q_i}\frac{\partial(D_i - D_j)}{\partial p_i}\right), \tag{24}$$

$$\Psi_q := -\frac{\partial D_j}{\partial q_i}\frac{\partial^2 \pi_i}{\partial p_i^2} + \frac{\partial D_j}{\partial p_i}\left(\frac{\partial(D_i - D_j)}{\partial q_i} - \frac{\partial c}{\partial q_i}\frac{\partial(D_i + D_j)}{\partial p_i}\right). \tag{25}$$

Notice here that $\Psi_p > (<)\,0$ and $\Psi_q > (<)\,0$ if the first-order effects of the merger on the merged firm's price and quality are positive (negative).

It follows from (22)-(23) that the quality and price responses of non-merging firms depend on two different factors: (i) the size of variable quality costs, which determines whether qualities and prices are net strategic substitutes or complements (i.e., the signs of Ω_q and Ω_p), and (ii) the relative magnitude of cross-quality and cross-price effects on demand, which determines the direction of the first-order effects on the merged firm's quality and price (i.e., the signs of Ψ_q and Ψ_p).

Notice that whether the non-merging firms' quality and price responses go in the same direction or not depends only on the net strategic substitutability/complementarity of qualities and prices. If qualities are net strategic substitutes ($\Omega_q > 0$) and prices are net strategic complements ($\Omega_p < 0$), or *vice versa* ($\Omega_q < 0$ and $\Omega_p > 0$), then quality and price responses from the non-merging firms always go in the same direction. Otherwise, if qualities and prices are both net strategic substitutes or net strategic complements, quality and price responses go in opposite directions.

Consider the case of $\Omega_q > 0$ and $\Omega_p < 0$, which requires that variable quality costs are sufficiently small (see Lemma 1). In this case, the non-merging firms will either increase or reduce both quality and price, depending on the sign of the expression in square brackets in (22) and (23), which in turn depends on the quality and price adjustments of the merging firms. If the merged firm reduces its quality ($\Psi_q < 0$) and increases its price ($\Psi_p > 0$), the non-merging firms have an unambiguous incentive to increases both their qualities (since qualities are net strategic substitutes) and their prices (since prices are net strategic complements).

However, if the merged firm reduces both quality and price, the second term in the square brackets is negative and the effect on the non-merging firms' price and quality responses is ambiguous. The same ambiguity appears in the case where the merged firm increases both quality and price. This ambiguity is resolved, though, if the degree of competition is sufficiently much stronger along one or the other of the two dimensions (quality and price). Suppose that cross-quality effects on demand are much stronger than cross-price effects, such that $|(\partial D_k/\partial q_i)\,\Psi_q| > |(\partial D_k/\partial p_i)\,\Psi_p|$ and $\Psi_q < 0$. In this case, where the firms compete harder on quality than on price, the merged firm will reduce its quality (and price) and the non-merging firms respond by increasing their qualities and prices. On the other hand, suppose that cross-price effects on demand are much stronger than cross-quality effects, such that $|(\partial D_k/\partial q_i)\,\Psi_q| < |(\partial D_k/\partial p_i)\,\Psi_p|$ and $\Psi_p > 0$. In this case, where firms compete harder on price than on quality, the merged firm will increase its price (and quality) but the non-merging firms still respond by increasing their qualities and prices.

Our analysis throughout this section suggests that there is a large number of possibilities regarding the equilibrium quality and price responses by merging and non-merging firms. Our parametric example captures two of these possibilities: (i) If competition along the quality dimension is sufficiently strong, the merged firm reduces both quality and price, whereas the non-merged firm increases both quality and price; (ii) if demand is less quality-responsive, the merged firm reduces quality but increases the price, whereas the non-merged firm still increases both quality and price. The former case is characterised by $\Psi_p < 0$ and latter by $\Psi_p > 0$. In both cases, $\Omega_q > 0$, $\Omega_p < 0$, $\Psi_q < 0$ and $(\partial D_k/\partial q_i)\,\Psi_q + (\partial D_k/\partial p_i)\,\Psi_p > 0$.[18] One interesting result from the Salop model is that, although the merged firm always reduces quality, the *average quality* in the market goes up. This illustrates a more general point. If qualities are net strategic substitutes, a reduction in the merged firm's quality provision does not necessarily imply that average quality provision in the market is reduced.

Based on our general analysis, the quality and price responses of non-merging firms can be summarised as follows:

Proposition 3 *(i) If prices are net strategic complements (substitutes) and qualities are net strategic substitutes (complements), then non-merging firms' quality and price responses always go in the same direction. Otherwise, the quality and price responses of non-merging firms go in opposite directions. (ii) If variable quality costs are sufficiently small, non-merging firms will respond to the merger by increasing both quality and price if the degree of competition is sufficiently much stronger on either price or quality.*

4 Welfare

What are the potential welfare effects of a merger when firms compete on both quality and price? A merger will generally change all the prices and qualities, and, as we know from the analysis of the previous section, the direction of price and quality changes for merging and

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[18]Recall that, in the Salop model, qualities are net strategic substitutes whereas prices are net strategic complements.

non-merging firms are generally ambiguous. Because of these ambiguities, a general charac-
terisation of the welfare effects of a merger is hard to produce. However, the main purpose of
this welfare section is to point out how and why competition along two different dimensions
(price and quality) might alter the welfare implications of a merger in a fundamental way,
compared with the standard case of pure price competition, and give rise to the possibility of
welfare-improving mergers.

In the absence of quality competition, a merger without any cost synergies will generally
reduce welfare because it leads to higher prices in the industry. However, this conclusion is no
longer so clear-cut when firms also compete on quality, for several reasons. As the analysis in
the previous section has shown, a merger might well lead to lower prices or higher qualities,
at least for some firms, which makes the welfare assessment more ambiguous. For example,
if qualities are net strategic substitutes and prices are net strategic complements, which will
be the case if variable quality costs are sufficiently small, one implication of the first part
of Proposition 3 is that a merger will always lead to either higher quality or lower prices by
non-merging firms.

Furthermore, the presence of fixed quality costs opens up for the possibility that mergers
might improve welfare by generating *endogenous fixed-cost synergies*. Suppose that qualities
are net strategic substitutes. A merger will then typically make the quality provision more
asymmetric between merging and non-merging firms, which potentially implies a reduction
in total fixed costs. To see why more asymmetric quality provision can lead to endogenous
fixed-cost savings, consider the following simple example. Suppose that $n = 2$. Suppose also
that $q_i < q_j$ and $p_i = p_j$, implying $D_i < D_j$. Consider now a marginal change in q_i. The
necessary change in q_j that keeps average quality (denoted by $\bar{q}$) unchanged is given by

$$dq_j = -\frac{\partial \bar{q}/\partial q_i}{\partial \bar{q}/\partial q_j} dq_i. \tag{26}$$

The corresponding change in total fixed costs is

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$$\frac{\partial K}{\partial q_i}dq_i + \frac{\partial K}{\partial q_j}dq_j = \left(\frac{\partial K}{\partial q_i} - \frac{\partial K}{\partial q_j}\frac{\partial \overline{q}/\partial q_i}{\partial \overline{q}/\partial q_i}\right)dq_i > 0 \quad if \quad \frac{\partial \overline{q}/\partial q_j}{\partial \overline{q}/\partial q_i} > \frac{\partial K/\partial q_j}{\partial K/\partial q_i}. \tag{27}$$

Since K is strictly convex, $\partial K/\partial q_i < \partial K/\partial q_j$ if $q_i < q_j$. Thus, a necessary condition for more asymmetric quality provision (i.e., $dq_i < 0$ such that $d\overline{q} = 0$) to reduce total fixed quality costs is $\partial \overline{q}/\partial q_j > \partial \overline{q}/\partial q_i$, which holds if equilibrium market shares are convex, linear or 'not too concave' in qualities. In this case, a merger that makes quality provision more asymmetric might lead to endogenous fixed-cost savings which can potentially be large enough to make the merger welfare improving.

An example of such a welfare improving merger appears in the linear Salop model for a parameter set which is characterised by a relatively low degree of fixed-cost convexity, a relatively high degree of demand-responsiveness to quality, and a relatively low degree of product differentiation.[19]

5   Extension: Mergers with closure

In this section we extend the analysis by considering the case where the merged firm decides to reduce its product line by offering only one product after the merger, or, if the model is given a spatial interpretation, the merged firm closes down one of its two plants and allocates all production to the remaining plant. Such closure would be profitable if there are sufficiently large product-specific (or plant-specific) fixed costs. Thus, the analysis in this extension applies to cases where the realisation of fixed-cost savings is an important motivation for the merger.

When the merged firm is a single-product (or single-plant) firm, the pre- and post-merger equilibria are symmetric and a merger is equivalent to reducing the number of firms/products, which implies that the quality and price effects of a merger is the same for merging and

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[19]See Appendix B for further details.

non-merging firms. The symmetric (pre-merger) equilibrium is characterised by

$$\frac{\partial \pi_i}{\partial q_i} = (p_i - c(q_i)) \frac{\partial D_i}{\partial q_i} - D_i \frac{\partial c}{\partial q_i} - \frac{\partial K}{\partial q_i} = 0 \tag{28}$$

and

$$\frac{\partial \pi_i}{\partial p_i} = D_i + (p_i - c(q_i)) \frac{\partial D_i}{\partial p_i} = 0, \quad i \in N, \tag{29}$$

where all demand functions and their first-order derivatives are evaluated at the quality-price vector $(q_i, ..., q_i, p_i, ..., p_i)$.

By differentiating (28)-(29) with respect to p_i, q_i and n, and applying Cramer's Rule, the effect of a merger on equilibrium quality is given by[20]

$$sign\left(-\frac{\partial q_i}{\partial n}\right) = sign \left[\begin{array}{c} -(p_i - c(q_i)) \left(\frac{\partial D_i}{\partial q_i} - \frac{\partial c}{\partial q_i} \left(\frac{\partial D_i}{\partial p_i} + (n-1) \frac{\partial D_j}{\partial p_i} \right) \right) \frac{\partial^2 D_i}{\partial p_i \partial n} \\ + (p_i - c(q_i)) \left(\frac{\partial^2 \pi_i}{\partial p_i^2} + (n-1) \frac{\partial D_j}{\partial p_i} \right) \frac{\partial^2 D_i}{\partial q_i \partial n} \\ - \left(\frac{\partial D_i}{\partial q_i} + \frac{\partial c}{\partial q_i} \left(\frac{\partial D_i}{\partial p_i} + (p_i - c(q_i)) \frac{\partial^2 D_i}{\partial p_i^2} \right) \right) \frac{\partial D_i}{\partial n} \end{array} \right]. \tag{30}$$

The quality effect of a merger is given by the sum of three sub-effects, represented by the three terms on the right-hand side of (30). First, a merger reduces the demand responsiveness to price, which leads to a higher price and therefore higher quality. Second, a merger reduces the demand responsiveness to quality, which leads directly to a lower quality provision.[21] The third effect is a priori ambiguous. A merger increases demand (per firm), which increases the marginal cost of quality and therefore leads to lower quality. On the other hand, the demand increase also leads to a higher price and therefore higher quality (since price and quality are within-firm strategic complements). The third effect is positive if variable quality costs are sufficiently small, or if demand responds sufficiently stronger to quality than to price.

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[20]See Appendix A for the full derivation of all results in this section.

[21]Notice that a negative definite Jacobian matrix requires

$$\frac{\partial^2 \pi_i}{\partial p_i^2} + (n-1) \frac{\partial D_j}{\partial p_i} < 0.$$

Similarly, the effect of a merger on equilibrium price is given by

$$
sign\left(-\frac{\partial p_i}{\partial n}\right) = sign \begin{bmatrix} (p_i - c(q_i))\left(\frac{\partial^2 \pi_i}{\partial q_i^2} - (n-1)\frac{\partial c}{\partial q_i}\frac{\partial D_j}{\partial q_i}\right)\frac{\partial^2 D_i}{\partial p_i \partial n} \\ -(p_i - c(q_i))\left(\frac{\partial D_i}{\partial q_i} + (n-1)\frac{\partial D_j}{\partial q_i} - \frac{\partial c}{\partial q_i}\frac{\partial D_i}{\partial p_i}\right)\frac{\partial^2 D_i}{\partial q_i \partial n} \\ \left(\frac{\partial^2 \pi_i}{\partial q_i^2} + \frac{\partial c}{\partial q_i}\left(\frac{\partial D_i}{\partial q_i} - \frac{\partial c(q_i)}{\partial q_i}\frac{\partial D_i}{\partial p_i}\right)\right)\frac{\partial D_i}{\partial n} \end{bmatrix} \tag{31}
$$

Once more, the total effect is a sum of three sub-effects, given by the three terms in (31). First, a merger reduces the demand responsiveness to price, which leads directly to a higher price.[22] Second, a merger also reduces the demand responsiveness to quality, which leads to lower quality and therefore to a lower price. The third term has an ambiguous sign. A merger increases demand (per firm), which makes demand less price-elastic and leads to a higher price. However, higher demand also leads to lower quality because of increased marginal cost of quality, which in turn leads to a lower price. The third term is positive if variable quality costs are sufficiently small.

In our parametric example, the third term in both (30) and (31) are positive. Furthermore, in each expression the two positive terms dominate the one negative term, implying that a merger increases both the quality and the price.[23] In the more general model, the above analysis can be summarised as follows:

Proposition 4 *A merger with closure leads to higher quality and price for all firms if (i) the merger leads to a sufficiently large reduction in the demand responsiveness to price, and if (ii) variable quality costs are sufficiently small.*

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[22]Notice that a negative definite Jacobian matrix requires

$$
\frac{\partial^2 \pi_i}{\partial q_i^2} - (n-1)\frac{\partial c}{\partial q_i}\frac{\partial D_j}{\partial q_i} < 0.
$$

[23]This result is also found by Economides (1993) who report an inverse relationship between firm density and equilibrium quality in a Salop model with quality and price competition. However, Brekke et al. (2010) show that this result could be reversed if utility is non-linear in income.

6 Concluding remarks

In this paper we have analysed the effects of horizontal mergers when firms compete along two different dimensions: price and quality. We have shown that the effects of a merger are quite involved because of the strategic relationship between quality and price, and we report some perhaps surprising results. First, whether a merger will induce the merger participants to increase or reduce prices and qualities are far from clear-cut. If firms compete sufficiently strongly on quality, a merger might lead to lower prices, and if firms compete sufficiently strongly on price, a merger might lead to higher quality. The non-merging firms' price and quality responses are also far from obvious. We have shown that, if variable quality costs are sufficiently small, a merger will induce non-merging firms either to increase both price and quality, or to reduce both price and quality, depending on the relative strength of price and quality competition.

The general pattern of our results suggest that welfare implications of mergers are much less clear-cut when firms compete along two different dimensions, compared with the standard case of price competition. As we have discussed in Section 4, there are two reasons for this. First, a merger will most likely result in higher quality or lower price for some firms in the industry, which makes the welfare assessment of a merger *a priori* more ambiguous. Second, the presence of fixed quality costs implies that a merger might improve welfare through endogenous fixed-cost savings, something that we are able to explicitly confirm in a parametric example.

Although we have conducted our analysis within a very general framework, we have nevertheless been forced to make a couple of simplifications in order to make the analysis feasible. We have assumed that demand is separable in all qualities and prices, and we have assumed constant marginal production costs for given quality levels. Although these simplifications somewhat reduce the generality of the analysis, we still believe that we have been able to capture the most important mechanisms that determine the strategic choices in markets where firms compete on both price and quality.

Appendix A

In this appendix we present the details of the comparative statics results; first from the main model presented in Sections 2-3 and then from the model extension presented in Section 5.

A.1. Comparative statics in the main model

The Nash equilibrium is implicitly given by a system of four equations, given by (14)-(17) in Section 3, which are here redefined as

$$F_1 \;:\; = (p_i - c(q_i))\left(\frac{\partial D_i}{\partial q_i} + \alpha\frac{\partial D_j}{\partial q_i}\right) - D_i\frac{\partial c}{\partial q_i} - \frac{\partial K}{\partial q_i} = 0, \tag{A1}$$

$$F_2 \;:\; = D_i + (p_i - c(q_i))\left(\frac{\partial D_i}{\partial p_i} + \alpha\frac{\partial D_j}{\partial p_i}\right) = 0, \tag{A2}$$

$$F_3 \;:\; = (p_k - c(q_k))\frac{\partial D_k}{\partial q_k} - D_k\frac{\partial c}{\partial q_k} - \frac{\partial K}{\partial q_k} = 0, \tag{A3}$$

$$F_4 \;:\; = D_k + (p_k \; - c(q_k))\frac{\partial D_k}{\partial p_k} = 0. \tag{A4}$$

where the demand functions are given by

$$D_i\left(p_i, p_i, p_k, ..., p_k, q_i, q_i, q_k, ..., q_k, n\right)$$

$$D_j\left(p_i, p_i, p_k, ..., p_k, q_i, q_i, q_k, ..., q_k, n\right)$$

$$D_k\left(p_i, p_i, p_k, ..., p_k, q_i, q_i, q_k, ..., q_k, n\right), \quad k \in O.$$

By differentiating (A1)-(A4) with respect to q_i, p_i, q_k, p_k and α, we can write the system on matrix form as

$$\begin{bmatrix} \frac{\partial F_1}{\partial q_i} & \frac{\partial F_1}{\partial p_i} & \frac{\partial F_1}{\partial q_k} & \frac{\partial F_1}{\partial p_k} \\[4pt] \frac{\partial F_2}{\partial q_i} & \frac{\partial F_2}{\partial p_i} & \frac{\partial F_2}{\partial q_k} & \frac{\partial F_2}{\partial p_k} \\[4pt] \frac{\partial F_3}{\partial q_i} & \frac{\partial F_3}{\partial p_i} & \frac{\partial F_3}{\partial q_k} & \frac{\partial F_3}{\partial p_k} \\[4pt] \frac{\partial F_4}{\partial q_i} & \frac{\partial F_4}{\partial p_i} & \frac{\partial F_4}{\partial q_k} & \frac{\partial F_4}{\partial p_k} \end{bmatrix} \begin{bmatrix} dq_i \\[4pt] dp_i \\[4pt] dq_k \\[4pt] dp_k \end{bmatrix} + \begin{bmatrix} \frac{\partial F_1}{\partial \alpha} \\[4pt] \frac{\partial F_2}{\partial \alpha} \\[4pt] \frac{\partial F_3}{\partial \alpha} \\[4pt] \frac{\partial F_4}{\partial \alpha} \end{bmatrix} d\alpha = 0, \tag{A5}$$

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where

$$\frac{\partial F_1}{\partial q_i} = -\frac{\partial^2 K_i}{\partial q_i^2} - \frac{\partial^2 c}{\partial q_i^2}D_i - \frac{\partial c}{\partial q_i}\left(2\frac{\partial D_i}{\partial q_i} + (1+\alpha)\frac{\partial D_j}{\partial q_i}\right) + (p_i - c(q_i))\frac{\partial^2 D_i}{\partial q_i^2},$$

$$\frac{\partial F_1}{\partial p_i} = \left(\frac{\partial D_i}{\partial q_i} + \alpha\frac{\partial D_j}{\partial q_i}\right) - \frac{\partial c}{\partial q_i}\left(\frac{\partial D_i}{\partial p_i} + \frac{\partial D_j}{\partial p_i}\right), \quad \frac{\partial F_1}{\partial q_k} = -\frac{\partial c}{\partial q_i}\frac{\partial D_i}{\partial q_k}(n-2),$$

$$\frac{\partial F_1}{\partial p_k} = -\frac{\partial c}{\partial q_i}\frac{\partial D_i}{\partial p_k}(n-2), \quad \frac{\partial F_2}{\partial q_i} = \frac{\partial D_i}{\partial q_i} + \frac{\partial D_j}{\partial q_i} - \frac{\partial c}{\partial q_i}\left(\frac{\partial D_i}{\partial p_i} + \alpha\frac{\partial D_j}{\partial p_i}\right),$$

$$\frac{\partial F_2}{\partial p_i} = 2\frac{\partial D_i}{\partial p_i} + (1+\alpha)\frac{\partial D_j}{\partial p_i} + (p_i - c(q_i))\frac{\partial^2 D_i}{\partial p_i^2}, \quad \frac{\partial F_2}{\partial q_k} = \frac{\partial D_i}{\partial q_k}(n-2),$$

$$\frac{\partial F_2}{\partial p_k} = \frac{\partial D_i}{\partial p_k}(n-2), \quad \frac{\partial F_3}{\partial q_i} = -2\frac{\partial c}{\partial q_k}\frac{\partial D_k}{\partial q_i}, \quad \frac{\partial F_3}{\partial p_i} = -2\frac{\partial c}{\partial q_k}\frac{\partial D_k}{\partial p_i}, \quad \text{(A6)}$$

$$\frac{\partial F_3}{\partial q_k} = -\frac{\partial^2 K}{\partial q_k^2} - \frac{\partial^2 c}{\partial q_k^2}D_k + (p_k - c(q_k))\frac{\partial^2 D_k}{\partial q_k^2} - \frac{\partial c}{\partial q_k}\left(2\frac{\partial D_k}{\partial q_k} + (n-3)\left.\frac{\partial D_k}{\partial q_l}\right|_{q_l=q_k}\right),$$

$$\frac{\partial F_3}{\partial p_k} = \frac{\partial D_k}{\partial q_k} - \frac{\partial c}{\partial q_k}\left(\frac{\partial D_k}{\partial p_k} + (n-3)\left|\frac{\partial D_k}{\partial p_l}\right|_{p_l=p_k}\right),$$

$$\frac{\partial F_4}{\partial q_i} = 2\frac{\partial D_k}{\partial q_i}, \quad \frac{\partial F_4}{\partial p_i} = 2\frac{\partial D_k}{\partial p_i}, \quad \frac{\partial F_4}{\partial q_k} = \frac{\partial D_k}{\partial q_k} + (n-3)\left|\frac{\partial D_k}{\partial q_l}\right|_{q_l=q_k} - \frac{\partial c}{\partial q_k}\frac{\partial D_k}{\partial p_k},$$

$$\frac{\partial F_4}{\partial p_k} = 2\frac{\partial D_k}{\partial p_k} + (n-3)\left|\frac{\partial D_k}{\partial p_l}\right|_{p_l=p_k} + (p_k - c(q_k))\frac{\partial^2 D_k}{\partial p_k^2},$$

$$\frac{\partial F_1}{\partial \alpha} = (p_i - c(q_i))\frac{\partial D_j}{\partial q_i}, \quad \frac{\partial F_2}{\partial \alpha} = (p_i - c(q_i))\frac{\partial D_j}{\partial p_i}, \quad \frac{\partial F_3}{\partial \alpha} = \frac{\partial F_4}{\partial \alpha} = 0,$$

where we have exploited the fact that, by symmetry, $\left.\left|\frac{\partial D_i}{\partial q_j}\right|\right|_{q_j=q_i} = \frac{\partial D_j}{\partial q_i}$, $\left.\left|\frac{\partial D_i}{\partial p_j}\right|\right|_{p_j=p_i} = \frac{\partial D_j}{\partial p_i}$, $\left.\left|\frac{\partial D_k}{\partial q_j}\right|\right|_{q_j=q_i} = \frac{\partial D_k}{\partial q_i}$ and $\left.\left|\frac{\partial D_k}{\partial q_j}\right|\right|_{q_j=q_i} = \frac{\partial D_k}{\partial q_i}$.

A.1.1. The effect of a merger on the merged firm's quality

Assuming that the Jacobian matrix is negative definite, the sign of $\partial q_i/\partial \alpha$ is given by the sign of

$$\begin{vmatrix} -\frac{\partial F_1}{\partial \alpha} & \frac{\partial F_1}{\partial p_i} & \frac{\partial F_1}{\partial q_k} & \frac{\partial F_1}{\partial p_k} \\[2mm] -\frac{\partial F_2}{\partial \alpha} & \frac{\partial F_2}{\partial p_i} & \frac{\partial F_2}{\partial q_k} & \frac{\partial F_2}{\partial p_k} \\[2mm] 0 & \frac{\partial F_3}{\partial p_i} & \frac{\partial F_3}{\partial q_k} & \frac{\partial F_3}{\partial p_k} \\[2mm] 0 & \frac{\partial F_4}{\partial p_i} & \frac{\partial F_4}{\partial q_k} & \frac{\partial F_4}{\partial p_k} \end{vmatrix}. \quad \text{(A7)}$$

This determinant can be written as

$$
\begin{aligned}
-&\left(\frac{\partial F_2}{\partial p_i}\frac{\partial F_1}{\partial \alpha} - \frac{\partial F_1}{\partial p_i}\frac{\partial F_2}{\partial \alpha}\right)\left(\frac{\partial F_3}{\partial q_k}\frac{\partial F_4}{\partial p_k} - \frac{\partial F_3}{\partial p_k}\frac{\partial F_4}{\partial q_k}\right) \\
-&\left(\frac{\partial F_1}{\partial \alpha}\frac{\partial F_2}{\partial p_k} - \frac{\partial F_2}{\partial \alpha}\frac{\partial F_1}{\partial p_k}\right)\left(\frac{\partial F_3}{\partial p_i}\frac{\partial F_4}{\partial q_k} - \frac{\partial F_3}{\partial q_k}\frac{\partial F_4}{\partial p_i}\right) \\
-&\left(\frac{\partial F_1}{\partial q_k}\frac{\partial F_2}{\partial \alpha} - \frac{\partial F_2}{\partial q_k}\frac{\partial F_1}{\partial \alpha}\right)\left(\frac{\partial F_3}{\partial p_i}\frac{\partial F_4}{\partial p_k} - \frac{\partial F_3}{\partial p_k}\frac{\partial F_4}{\partial p_i}\right),
\end{aligned}
\tag{A8}
$$

or, when substituting from (A6),

$$
\begin{aligned}
&(p_i - c(q_i))\left[-\frac{\partial D_j}{\partial q_i}\frac{\partial^2 \pi_i}{\partial p_i^2} + \frac{\partial D_j}{\partial p_i}\left(\frac{\partial (D_i - D_j)}{\partial q_i} - \frac{\partial c}{\partial q_i}\frac{\partial (D_i + D_j)}{\partial p_i}\right)\right]\Phi \\
&+ (p_i - c(q_i))\, 2(n-2)\frac{\partial D_i}{\partial p_k}\frac{\partial D_k}{\partial p_i}\left(\frac{\partial D_j}{\partial q_i} + \frac{\partial c}{\partial q_i}\frac{\partial D_j}{\partial p_i}\right)\Omega_p \\
&- (p_i - c(q_i))\, 2(n-2)\frac{\partial D_i}{\partial q_k}\frac{\partial D_k}{\partial p_i}\left(\frac{\partial D_j}{\partial q_i} + \frac{\partial c}{\partial q_i}\frac{\partial D_j}{\partial p_i}\right)\Omega_q,
\end{aligned}
\tag{A9}
$$

where

$$
\frac{\partial^2 \pi_i}{\partial p_i^2} = 2\frac{\partial D_i}{\partial p_i} + (p_i - c(q_i))\frac{\partial^2 D_i}{\partial p_i^2} < 0,
\tag{A10}
$$

$$
\Phi := \frac{\partial F_3}{\partial q_k}\frac{\partial F_4}{\partial p_k} - \frac{\partial F_3}{\partial p_k}\frac{\partial F_4}{\partial q_k} > 0,
\tag{A11}
$$

$$
\Omega_p := \frac{\partial^2 \pi_k}{\partial q_k^2} + \frac{\partial c}{\partial q_k}\left(\frac{\partial D_k}{\partial q_k} - \frac{\partial c}{\partial q_k}\frac{\partial D_k}{\partial p_k}\right),
\tag{A12}
$$

$$
\Omega_q := \frac{\partial D_k}{\partial q_k} + \frac{\partial c}{\partial q_k}\left(\frac{\partial^2 \pi_k}{\partial p_k^2} - \frac{\partial D_k}{\partial p_k}\right).
\tag{A13}
$$

Notice that $\Phi > 0$ by the assumption of a negative definite Jacobian matrix. Since $(p_i - c(q_i)) > 0$ in equilibrium, we can factor this out of (A9) and arrive at:

$$
sign\left(\frac{\partial q_i}{\partial \alpha}\right) = sign\left(\begin{array}{c}\left[-\frac{\partial D_j}{\partial q_i}\frac{\partial^2 \pi}{\partial p_i^2} + \frac{\partial D_j}{\partial p_i}\left(\frac{\partial (D_i - D_j)}{\partial q_i} - \frac{\partial c}{\partial q_i}\frac{\partial (D_i + D_j)}{\partial p_i}\right)\right]\Phi \\ +2(n-2)\frac{\partial D_k}{\partial p_i}\left(\frac{\partial D_j}{\partial q_i} + \frac{\partial c}{\partial q_i}\frac{\partial D_j}{\partial p_i}\right)\left(\frac{\partial D_i}{\partial p_k}\Omega_p - \frac{\partial D_i}{\partial q_k}\Omega_q\right)\end{array}\right).
\tag{A14}
$$

A.1.2. The effect of a merger on the merged firm's price

The sign of $\partial p_i / \partial \alpha$ is given by the sign of

$$
\begin{vmatrix}
\frac{\partial F_1}{\partial q_i} & -\frac{\partial F_1}{\partial \alpha} & \frac{\partial F_1}{\partial q_k} & \frac{\partial F_1}{\partial p_k} \\[6pt]
\frac{\partial F_2}{\partial q_i} & -\frac{\partial F_2}{\partial \alpha} & \frac{\partial F_2}{\partial q_k} & \frac{\partial F_2}{\partial p_k} \\[6pt]
\frac{\partial F_3}{\partial q_i} & 0 & \frac{\partial F_3}{\partial q_k} & \frac{\partial F_3}{\partial p_k} \\[6pt]
\frac{\partial F_4}{\partial q_i} & 0 & \frac{\partial F_4}{\partial q_k} & \frac{\partial F_4}{\partial p_k}
\end{vmatrix}.
\tag{A15}
$$

This determinant can be written as

$$
\begin{aligned}
& -\left(\frac{\partial F_1}{\partial q_i}\frac{\partial F_2}{\partial \alpha} - \frac{\partial F_2}{\partial q_i}\frac{\partial F_1}{\partial \alpha}\right)\left(\frac{\partial F_3}{\partial q_k}\frac{\partial F_4}{\partial p_k} - \frac{\partial F_3}{\partial p_k}\frac{\partial F_4}{\partial q_k}\right) \\
& -\left(\frac{\partial F_2}{\partial \alpha}\frac{\partial F_1}{\partial p_k} - \frac{\partial F_1}{\partial \alpha}\frac{\partial F_2}{\partial p_k}\right)\left(\frac{\partial F_3}{\partial q_i}\frac{\partial F_4}{\partial q_k} - \frac{\partial F_3}{\partial q_k}\frac{\partial F_4}{\partial q_i}\right) \\
& -\left(\frac{\partial F_2}{\partial \alpha}\frac{\partial F_1}{\partial q_k} - \frac{\partial F_2}{\partial q_k}\frac{\partial F_1}{\partial \alpha}\right)\left(\frac{\partial F_3}{\partial p_k}\frac{\partial F_4}{\partial q_i} - \frac{\partial F_3}{\partial q_i}\frac{\partial F_4}{\partial p_k}\right),
\end{aligned}
\tag{A16}
$$

or, when substituting from (A6),

$$
\begin{aligned}
& -(p_i - c(q_i))\left[\frac{\partial D_j}{\partial p_i}\frac{\partial^2 \pi_i}{\partial q_i^2} - \frac{\partial D_j}{\partial q_i}\left(\frac{\partial (D_i + D_j)}{\partial q_i} - \frac{\partial c}{\partial q_i}\frac{\partial (D_i - D_j)}{\partial p_i}\right)\right]\Phi \\
& -(p_i - c(q_i))\,2(n-2)\frac{\partial D_i}{\partial p_k}\frac{\partial D_k}{\partial q_i}\left(\frac{\partial D_j}{\partial p_i}\frac{\partial c}{\partial q_i} + \frac{\partial D_j}{\partial q_i}\right)\Omega_p \\
& +(p_i - c(q_i))\,2(n-2)\frac{\partial D_i}{\partial q_k}\frac{\partial D_k}{\partial q_i}\left(\frac{\partial D_j}{\partial p_i}\frac{\partial c}{\partial q_i} + \frac{\partial D_j}{\partial q_i}\right)\Omega_q,
\end{aligned}
\tag{A17}
$$

where

$$
\frac{\partial^2 \pi_i}{\partial q_i^2} = -\frac{\partial^2 K_i}{\partial q_i^2} - \frac{\partial^2 c}{\partial q_i^2}D_i - 2\frac{\partial c}{\partial q_i}\frac{\partial D_i}{\partial q_i} + (p_i - c(q_i))\frac{\partial^2 D_i}{\partial q_i^2} < 0.
\tag{A18}
$$

After factoring out $(p_i - c(q_i)) > 0$ from (A17) we arrive at

$$
sign(\frac{\partial p_i}{\partial \alpha}) = sign\left(
\begin{array}{c}
\left[-\frac{\partial D_j}{\partial p_i}\frac{\partial^2 \pi_i}{\partial q_i^2} + \frac{\partial D_j}{\partial q_i}\left(\frac{\partial (D_i + D_j)}{\partial q_i} - \frac{\partial c}{\partial q_i}\frac{\partial (D_i - D_j)}{\partial p_i}\right)\right]\Phi \\[8pt]
-2(n-2)\frac{\partial D_k}{\partial q_i}\left(\frac{\partial D_j}{\partial q_i} + \frac{\partial c}{\partial q_i}\frac{\partial D_j}{\partial p_i}\right)\left(\frac{\partial D_i}{\partial p_k}\Omega_p - \frac{\partial D_i}{\partial q_k}\Omega_q\right)
\end{array}
\right).
\tag{A19}
$$

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A.1.3. The effect of a merger on the non-merging firms' qualities

The sign of $\partial q_k/\partial\alpha$ is given by the sign of

$$
\begin{vmatrix}
\frac{\partial F_1}{\partial q_i} & \frac{\partial F_1}{\partial p_i} & -\frac{\partial F_1}{\partial\alpha} & \frac{\partial F_1}{\partial p_k} \\[1.2ex]
\frac{\partial F_2}{\partial q_i} & \frac{\partial F_2}{\partial p_i} & -\frac{\partial F_2}{\partial\alpha} & \frac{\partial F_2}{\partial p_k} \\[1.2ex]
\frac{\partial F_3}{\partial q_i} & \frac{\partial F_3}{\partial p_i} & 0 & \frac{\partial F_3}{\partial p_k} \\[1.2ex]
\frac{\partial F_4}{\partial q_i} & \frac{\partial F_4}{\partial p_i} & 0 & \frac{\partial F_4}{\partial p_k}
\end{vmatrix} . \tag{A20}
$$

This determinant can be written as

$$
\begin{aligned}
&-\left(\frac{\partial F_1}{\partial p_i}\frac{\partial F_2}{\partial\alpha}-\frac{\partial F_2}{\partial p_i}\frac{\partial F_1}{\partial\alpha}\right)\left(\frac{\partial F_4}{\partial p_k}\frac{\partial F_3}{\partial q_i}-\frac{\partial F_3}{\partial p_k}\frac{\partial F_4}{\partial q_i}\right) \\
&-\left(\frac{\partial F_1}{\partial q_i}\frac{\partial F_2}{\partial\alpha}-\frac{\partial F_2}{\partial q_i}\frac{\partial F_1}{\partial\alpha}\right)\left(\frac{\partial F_3}{\partial p_k}\frac{\partial F_4}{\partial p_i}-\frac{\partial F_3}{\partial p_i}\frac{\partial F_4}{\partial p_k}\right) \\
&-\left(\frac{\partial F_1}{\partial\alpha}\frac{\partial F_2}{\partial p_k}-\frac{\partial F_2}{\partial\alpha}\frac{\partial F_1}{\partial p_k}\right)\left(\frac{\partial F_3}{\partial q_i}\frac{\partial F_4}{\partial p_i}-\frac{\partial F_3}{\partial p_i}\frac{\partial F_4}{\partial q_i}\right) .
\end{aligned} \tag{A21}
$$

Notice, however, that

$$
\frac{\partial F_3}{\partial q_i}\frac{\partial F_4}{\partial p_i}-\frac{\partial F_3}{\partial p_i}\frac{\partial F_4}{\partial q_i}=-2\frac{\partial c}{\partial q_k}\frac{\partial D_k}{\partial q_i}2\frac{\partial D_k}{\partial p_i}+2\frac{\partial c}{\partial q_k}\frac{\partial D_k}{\partial p_i}2\frac{\partial D_k}{\partial q_i}=0,
$$

which eliminates the third term in (A21). The remaining two terms can, after substituting from (A6), be written as

$$
\begin{aligned}
&(p_i-c(q_i))\left(-\frac{\partial D_j}{\partial q_i}\frac{\partial^2\pi_i}{\partial p_i^2}+\frac{\partial D_j}{\partial p_i}\left(\frac{\partial(D_i-D_j)}{\partial q_i}-\frac{\partial c}{\partial q_i}\frac{\partial(D_i+D_j)}{\partial p_i}\right)\right)2\frac{\partial D_k}{\partial q_i}\Omega_q \\
&+(p_i-c(q_i))\left(-\frac{\partial D_j}{\partial p_i}\frac{\partial^2\pi_i}{\partial q_i^2}+\frac{\partial D_j}{\partial q_i}\left(\frac{\partial(D_i+D_j)}{\partial q_i}-\frac{\partial c}{\partial q_i}\frac{\partial(D_i-D_j)}{\partial p_i}\right)\right)2\frac{\partial D_k}{\partial p_i}\Omega_q
\end{aligned} \tag{A22}
$$

Factoring out $2\,(p_i-c(q_i))>0$ from (A22), we get

$$
sign\left(\frac{\partial q_k}{\partial\alpha}\right)=sign\left(\Omega_q\left(\frac{\partial D_k}{\partial q_i}\Psi_q+\frac{\partial D_k}{\partial p_i}\Psi_p\right)\right), \tag{A23}
$$

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where

$$\Psi_q := -\frac{\partial D_j}{\partial q_i}\frac{\partial^2 \pi_i}{\partial p_i^2} + \frac{\partial D_j}{\partial p_i}\left(\frac{\partial (D_i - D_j)}{\partial q_i} - \frac{\partial c}{\partial q_i}\frac{\partial (D_i + D_j)}{\partial p_i}\right) \tag{A24}$$

and

$$\Psi_p := -\frac{\partial D_j}{\partial p_i}\frac{\partial^2 \pi_i}{\partial q_i^2} + \frac{\partial D_j}{\partial q_i}\left(\frac{\partial (D_i + D_j)}{\partial q_i} - \frac{\partial c}{\partial q_i}\frac{\partial (D_i - D_j)}{\partial p_i}\right). \tag{A25}$$

A.1.4. The effect of a merger on the non-merging firms' prices

The sign of $\partial p_k/\partial \alpha$ is given by the sign of

$$\begin{vmatrix} \frac{\partial F_1}{\partial q_i} & \frac{\partial F_1}{\partial p_i} & \frac{\partial F_1}{\partial q_k} & -\frac{\partial F_1}{\partial \alpha} \\[2mm] \frac{\partial F_2}{\partial q_i} & \frac{\partial F_2}{\partial p_i} & \frac{\partial F_2}{\partial q_k} & -\frac{\partial F_2}{\partial \alpha} \\[2mm] \frac{\partial F_3}{\partial q_i} & \frac{\partial F_3}{\partial p_i} & \frac{\partial F_3}{\partial q_k} & 0 \\[2mm] \frac{\partial F_4}{\partial q_i} & \frac{\partial F_4}{\partial p_i} & \frac{\partial F_4}{\partial q_k} & 0 \end{vmatrix}. \tag{A26}$$

This determinant can be written as

$$\begin{aligned} &-\left(\frac{\partial F_1}{\partial q_i}\frac{\partial F_2}{\partial \alpha} - \frac{\partial F_2}{\partial q_i}\frac{\partial F_1}{\partial \alpha}\right)\left(\frac{\partial F_3}{\partial p_i}\frac{\partial F_4}{\partial q_k} - \frac{\partial F_3}{\partial q_k}\frac{\partial F_4}{\partial p_i}\right) \\ &-\left(\frac{\partial F_1}{\partial p_i}\frac{\partial F_2}{\partial \alpha} - \frac{\partial F_2}{\partial p_i}\frac{\partial F_1}{\partial \alpha}\right)\left(\frac{\partial F_3}{\partial q_k}\frac{\partial F_4}{\partial q_i} - \frac{\partial F_3}{\partial q_i}\frac{\partial F_4}{\partial q_k}\right) \\ &-\left(\frac{\partial F_1}{\partial q_k}\frac{\partial F_2}{\partial \alpha} - \frac{\partial F_2}{\partial q_k}\frac{\partial F_1}{\partial \alpha}\right)\left(\frac{\partial F_3}{\partial q_i}\frac{\partial F_4}{\partial p_i} - \frac{\partial F_3}{\partial p_i}\frac{\partial F_4}{\partial q_i}\right), \end{aligned} \tag{A27}$$

but we already know that

$$\frac{\partial F_3}{\partial q_i}\frac{\partial F_4}{\partial p_i} - \frac{\partial F_3}{\partial p_i}\frac{\partial F_4}{\partial q_i} = 0,$$

so the third term in (A27) vanishes. After substituting from (A6), the remaining two terms can be written as

$$\begin{aligned} &-(p_i - c(q_i))\left(-\frac{\partial^2 \pi_i}{\partial q_i^2}\frac{\partial D_j}{\partial p_i} + \frac{\partial D_j}{\partial q_i}\left(\frac{\partial (D_i + D_j)}{\partial q_i} - \frac{\partial c}{\partial q_i}\frac{\partial (D_i - D_j)}{\partial p_i}\right)\right)2\frac{\partial D_k}{\partial p_i}\Omega_p \\ &-(p_i - c(q_i))\left(-\frac{\partial^2 \pi_i}{\partial p_i^2}\frac{\partial D_j}{\partial q_i} + \frac{\partial D_j}{\partial p_i}\left(\frac{\partial (D_i - D_j)}{\partial q_i} - \frac{\partial c}{\partial q_i}\frac{\partial (D_i + D_j)}{\partial p_i}\right)\right)2\frac{\partial D_k}{\partial q_i}\Omega_p.\tag{A28} \end{aligned}$$

After factoring out $2\left(p_i - c(q_i)\right) > 0$, we get

$$sign\left(\frac{\partial p_k}{\partial \alpha}\right) = sign\left(-\Omega_p\left[\frac{\partial D_k}{\partial p_i}\Psi_p + \frac{\partial D_k}{\partial q_i}\Psi_q\right]\right). \tag{A29}$$

A.2. Comparative statics in the model where a merger leads to closure

The symmetric Nash equilibrium is implicitly given by (28)-(29), which can be redefined as

$$G_1 \quad : \quad = (p_i - c(q_i))\frac{\partial D_i}{\partial q_i} - D_i\frac{\partial c}{\partial q_i} - \frac{\partial K}{\partial q_i} = 0, \tag{A30}$$

$$G_2 \quad : \quad = D_i + (p_i - c(q_i))\frac{\partial D_i}{\partial p_i} = 0, \quad i \in N, \tag{A31}$$

where the demand functions are given by $D_i\left(p_i, ..., p_i, q_i, ..., q_i, n\right)$. By differentiating (A30)-(A31) with respect to q_i, p_i and n, we can write the system on matrix form as

$$\begin{bmatrix} \frac{\partial G_1}{\partial q_i} & \frac{\partial G_1}{\partial p_i} \\ \frac{\partial G_2}{\partial q_i} & \frac{\partial G_2}{\partial p_i} \end{bmatrix}\begin{bmatrix} dq_i \\ dp_i \end{bmatrix} + \begin{bmatrix} \frac{\partial G_1}{\partial n} \\ \frac{\partial G_2}{\partial n} \end{bmatrix} dn = 0, \tag{A32}$$

where

$$\begin{aligned}
\frac{\partial G_1}{\partial q_i} &= -\frac{\partial^2 K_i}{\partial q_i^2} - \frac{\partial^2 c}{\partial q_i^2}D_i - \frac{\partial c}{\partial q_i}\left(2\frac{\partial D_i}{\partial q_i} + (n-1)\frac{\partial D_j}{\partial q_i}\right) + (p_i - c(q_i))\frac{\partial^2 D_i}{\partial q_i^2}, \\
\frac{\partial G_1}{\partial p_i} &= \frac{\partial D_i}{\partial q_i} - \frac{\partial c}{\partial q_i}\left(\frac{\partial D_i}{\partial p_i} + (n-1)\frac{\partial D_j}{\partial p_i}\right), \quad \frac{\partial G_2}{\partial q_i} = \frac{\partial D_i}{\partial q_i} + (n-1)\frac{\partial D_j}{\partial q_i} - \frac{\partial c}{\partial q_i}\frac{\partial D_i}{\partial p_i}, \\
\frac{\partial G_2}{\partial p_i} &= 2\frac{\partial D_i}{\partial p_i} + (n-1)\frac{\partial D_j}{\partial p_i} + (p_i - c(q_i))\frac{\partial^2 D_i}{\partial p_i^2}, \tag{A33} \\
\frac{\partial G_1}{\partial n} &= (p_i - c(q_i))\frac{\partial^2 D_i}{\partial q_i \partial n} - \frac{\partial c(q_i)}{\partial q_i}\frac{\partial D_i}{\partial n}, \quad \frac{\partial G_2}{\partial n} = \frac{\partial D_i}{\partial n} + (p_i - c(q_i))\frac{\partial^2 D_i}{\partial p_i \partial n}.
\end{aligned}$$

Assuming that the Jacobian matrix is negative definite, the sign of $\partial q_i/\partial n$ is given by the

sign of

$$
\begin{vmatrix} -\frac{\partial G_1}{\partial n} & \frac{\partial G_1}{\partial p_i} \\ -\frac{\partial G_2}{\partial n} & \frac{\partial G_2}{\partial p_i} \end{vmatrix},
\tag{A34}
$$

which, after substituting from (A33), can be written as

$$
\begin{aligned}
&(p_i - c(q_i)) \left(\frac{\partial D_i}{\partial q_i} - \frac{\partial c}{\partial q_i} \left(\frac{\partial D_i}{\partial p_i} + (n-1) \frac{\partial D_j}{\partial p_i} \right) \right) \frac{\partial^2 D_i}{\partial p_i \partial n} \\
&- (p_i - c(q_i)) \left(\frac{\partial^2 \pi_i}{\partial p_i^2} + (n-1) \frac{\partial D_j}{\partial p_i} \right) \frac{\partial^2 D_i}{\partial q_i \partial n} \\
&+ \left(\frac{\partial D_i}{\partial q_i} + \frac{\partial c(q_i)}{\partial q_i} \left(\frac{\partial D_i}{\partial p_i} + (p_i - c(q_i)) \frac{\partial^2 D_i}{\partial p_i^2} \right) \right) \frac{\partial D_i}{\partial n}.
\end{aligned}
\tag{A35}
$$

Similarly, the sign of $\partial p_i / \partial n$ is given by the sign of

$$
\begin{vmatrix} \frac{\partial G_1}{\partial q_i} & -\frac{\partial G_1}{\partial n} \\ \frac{\partial G_2}{\partial q_i} & -\frac{\partial G_2}{\partial n} \end{vmatrix},
\tag{A36}
$$

which, after substituting from (A33), can be written as

$$
\begin{aligned}
&- (p_i - c(q_i)) \left(\frac{\partial^2 \pi_i}{\partial q_i^2} - (n-1) \frac{\partial c}{\partial q_i} \frac{\partial D_j}{\partial q_i} \right) \frac{\partial^2 D_i}{\partial p_i \partial n} \\
&+ (p_i - c(q_i)) \left(\frac{\partial D_i}{\partial q_i} + (n-1) \frac{\partial D_j}{\partial q_i} - \frac{\partial c}{\partial q_i} \frac{\partial D_i}{\partial p_i} \right) \frac{\partial^2 D_i}{\partial q_i \partial n} \\
&- \left(\frac{\partial^2 \pi_i}{\partial q_i^2} + \frac{\partial c(q_i)}{\partial q_i} \left(\frac{\partial D_i}{\partial q_i} - \frac{\partial c}{\partial q_i} \frac{\partial D_i}{\partial p_i} \right) \right) \frac{\partial D_i}{\partial n}.
\end{aligned}
\tag{A37}
$$

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Appendix B

In this appendix we give the explicit results of the Salop model presented in Section 2.1. Notice that some of the signs of the effects are determined by invoking the condition $t > \frac{9(b-c)^2}{4k}$, which guarantees that only profitable mergers are considered.

B.1. Quality and price effects of a merger

In the symmetric pre-merger Nash equilibrium, qualities and prices are given by

$$q_i^* = \frac{b-c}{3k}, \quad p_i^* = \frac{t}{9} + \frac{c(b-c)}{3k}. \tag{B1}$$

Suppose that two of the firms merge. Qualities and prices in the post-merger Nash equilibrium (where we use subscript m to denote the merging firms and subscript o to denote the non-merging firm) are given by

$$q_m^* = \frac{(b-c)\left(5kt - 9(b-c)^2\right)}{9k\left(2kt - 3(b-c)^2\right)}, \quad p_m^* = \frac{\left(5kt - 9(b-c)^2\right)\left(2kt + 3c(b-c)\right)}{27k\left(2kt - 3(b-c)^2\right)}, \tag{B2}$$

$$q_o^* = \frac{(b-c)\left(8kt - 9(b-c)^2\right)}{9k\left(2kt - 3(b-c)^2\right)}, \quad p_o^* = \frac{\left(kt + 3c(b-c)\right)\left(8kt - 9(b-c)^2\right)}{27k\left(2kt - 3(b-c)^2\right)}. \tag{B3}$$

The effects of the merger on qualities and prices are given by

$$q_m^* - q_i^* = -\frac{(b-c)t}{9\left(2kt - 3(b-c)^2\right)} < 0, \tag{B4}$$

$$p_m^* - p_i^* = t\left(\frac{4kt - 3(3b-2c)(b-c)}{27\left(2kt - 3(b-c)^2\right)}\right) > (<)\, 0 \quad if \quad t > (<)\frac{3(3b-2c)(b-c)}{4k}, \tag{B5}$$

$$q_o^* - q_i^* = \frac{2t(b-c)}{9\left(2kt - 3(b-c)^2\right)} > 0. \tag{B6}$$

$$p_o^* - p_i^* = \frac{2t\left(kt + 3c(b-c)\right)}{27\left(2kt - 3(b-c)^2\right)} > 0. \tag{B7}$$

The effects of the merger on *average* quality and price are given by

$$\bar{q} - q_i^* = \frac{2kt^2 (b - c)}{27 \left(2kt - 3 (b - c)^2 \right)^2} > 0, \tag{B8}$$

$$\bar{p} - p_i^* = \frac{2t \left(81 (b - c)^4 + kt (28kt - 9 (b - c) (10b - 11c)) \right)}{243 \left(2kt - 3 (b - c)^2 \right)^2} > 0, \tag{B9}$$

where

$$
\begin{aligned}
\bar{q} \quad : \quad &= 2 D_m \left(q_m^*, p_m^*, q_o^*, p_o^* \right) q_m^* + D_o \left(q_m^*, p_m^*, q_o^*, p_o^* \right) q_o^* \\
&= \frac{\left(81 (b - c)^4 + 2kt \left(19kt - 54 (b - c)^2 \right) \right) (b - c)}{27k \left(2kt - 3 (b - c)^2 \right)^2}
\end{aligned} \tag{B10}
$$

and

$$
\begin{aligned}
\bar{p} \quad : \quad &= 2 D_m \left(q_m^*, p_m^*, q_o^*, p_o^* \right) p_m^* + D_o \left(q_m^*, p_m^*, q_o^*, p_o^* \right) p_o^* \\
&= \frac{729c (b - c)^5 + kt \left(81 (5b - 17c) (b - c)^3 + 2kt (82kt - 9 (28b - 47c) (b - c)) \right)}{243k \left(2kt - 3 (b - c)^2 \right)^2}
\end{aligned} \tag{B11}
$$

B.2. Welfare effects of a merger

Social welfare, measured as the sum of producers' and consumers' surplus, is given by

$$W = \sum_{i=1}^{3} \left(\int_0^{\widehat{x}_i^{i+1}} (v + bq_i - ts) \, ds + \int_0^{\widehat{x}_i^{i-1}} (v + bq_i - ts) \, ds - cq_i D_i - \frac{k}{2} q_i^2 \right) - 3F, \tag{B12}$$

where

$$\widehat{x}_i^{i+1} = \frac{1}{6} + \frac{3 \left(b (q_i - q_{i+1}) - (p_i - p_{i+1}) \right)}{2t} \tag{B13}$$

is the location (measured *clockwise* from Firm i) of the consumer who is indifferent between Firm i and Firm $i+1$, and

$$\widehat{x}_i^{\,i-1} = \frac{1}{6} + \frac{3\left(b\left(q_i - q_{i-1}\right) - \left(p_i - p_{i-1}\right)\right)}{2t} \tag{B14}$$

is the location (measured *counter-clockwise* from Firm i) of the consumer who is indifferent between Firm i and Firm $i-1$.[24]

Inserting the equilibrium values of quality and price in the welfare function, a comparison of the pre- and post-merger equilibrium welfare levels yields:

$$W\left(q_m^*, p_m^*, q_o^*, p_o^*\right) - W\left(q_i^*, p_i^*\right) = -\frac{\left(2kt - 9\left(b-c\right)^2\right)kt^2}{243\left(2kt - 3\left(b-c\right)^2\right)^2} < (>)\,0 \quad if \quad t > (<) \frac{9\left(b-c\right)^2}{2k}. \tag{B15}$$

B.3. Quality and price effects of a merger with closure

If a merger leads to closure of one of the merging firms' plants/products, the post-merger equilibrium is symmetric in terms of qualities and prices, and these are given by

$$q_m^* = q_o^* = \frac{b-c}{2k}, \quad p_m^* = p_o^* = \frac{2t}{9} + \frac{c\left(b-c\right)}{2k}. \tag{B16}$$

A comparison of (B1) and (B16) yields

$$q_m^* - q_i^* = q_o^* - q_i^* = \frac{b-c}{6k} > 0 \tag{B17}$$

and

$$p_m^* - p_i^* = p_o^* - p_i^* = \frac{t}{9} + \frac{c\left(b-c\right)}{6k} > 0. \tag{B18}$$

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[24]This implies a slight abuse of notation, since $i - 1 = 3$ if $i = 1$ and $i + 1 = 1$ if $i = 3$.

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