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Workforce Location and Equilibrium Unemployment in a Duocentric Economy with Matching Frictions

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Abstract

This article examines unemployment disparities and efficiency in a densely populated economy with two job centers and workers distributed between them. We introduce commuting costs and search-matching frictions to deal with the spatial mismatch between workers and firms. In a decentralized economy job-seekers do not internalize a composition externality they impose on all the unemployed. With symmetric job centers, a change in the distribution of the workforce can lead to asymmetric equilibrium outcomes. We calibrate the model for Los Angeles and Chicago Metropolitan Statistical Areas. Simulations suggest that changes in the workforce distribution have non-negligible effects on unemployment rates, wages, and net output, but cannot be the unique explanation of a substantial mismatch problem.

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Keywords: spatial mismatch, commuting, urban unemployment, externality.

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I Introduction

Does the spatial structure of a city affect labor markets outcomes? How do job-seekers organize their search activity along the spatial dimension? What are the implications of this activity on equilibrium unemployment rates and efficiency? These questions have already been studied in monocentric cities (see Zenou, 2009, for an overview) or in the case of uniformly distributed agents and jobs around the circle (see Marimon and Zilibotti, 1999; Hamilton et al., 2000; Decreuse, 2008). Cities with similar population sizes are often believed to be comparable. Fig. 1 illustrates that this is however not true for major cities.[1] Paris and Shanghai have on average 7.6 millions of people, yet the population density in Shanghai is 3.4 times higher than in Paris. London and Moscow seem to have a more uniform population distribution compared to Jakarta, Berlin or New York which are more populated in the center. In the U.S. Los Angeles Metropolitan Statistical Area (MSA) has twice the population density of Chicago MSA. Moreover for the U.S. the monocentric view seems outdated: "America changed from a nation of distinct cities separated by farmland, to a place where employment and population density is far more continuous" according to Glaeser (2007).

The aim of this paper is to better understand disparities in unemployment rates in metropolitan areas, in particular in the U.S. We consider a densely populated city with two business districts and a possibly non-uniform distribution of workers located along a line connecting them. Each business district is a distinct labor market characterized by search-matching frictions. An endogenous number of firms choose to set up in either of these centers. To capture that residential changes are typically much less frequent than transitions on the labor market, we set up a two-stage model as in e.g. Zenou (2009c). Initially, individuals freely choose once and for all where to reside. Next, the unemployed use their time endowment to look for vacant jobs in the business districts. Firms freely decide where they open a vacancy. Employed workers commute to the job center where they have been recruited until the match is exogenously destroyed.

In equilibrium, unemployed workers specialize their search in only one job center. The closer a job-seeker resides to a job center, the lower are the commuting costs, so the higher is the total surplus created if a firm located in this job center matches with this job-seeker. As we assume individual Nash bargaining over the wage, commuting costs are shared between the employer and the employee. Moreover, the vacancies open in a job center are generic in the sense of being accessible to any job-seeker wherever she lives. The expected profit made in a job center is higher

[1] Fig. 1 presents a three dimensional perspective where the boundaries of a city are the result of overlaying population density and built-up areas. For instance, London is limited to its 52 boroughs, Shanghai to "the city proper" and Paris to the municipal area and "la petite couronne." Jakarta is represented by the Jabotabek area which is Jakarta municipality plus Tangerang, Bekasi, and Bogor. Moscow is limited to the area within its municipal boundary.

when those seeking a job there are concentrated in the neighborhood of this center. When an additional individual joins the queue of job-seekers in a center, she ignores the consequences of this decision on expected profits and hence on vacancy creation. This generates an externality. If agents chose where they search in an efficient way (i.e. so as to maximize net output), the so-called Hosios condition would be sufficient to internalize standard search-matching externalities. This condition which is familiar in the search-matching literature expresses that agents' shares of the total surplus created by a match equal respectively the elasticities of the matching function with respect to the stocks of buyers (vacant jobs) and sellers (job-seekers) in the labor market. However, as the decisions of where to search a job is generically inefficient, the decentralized economy is typically not efficient even if the Hosios condition is met.

Numerical analyses provide orders of magnitude of the impacts of changes in the shape of the workforce distribution on unemployment rates and on efficiency. A first exercise considers a uniformly distributed workforce of mass lower than one and a complementary mass of workers located in the central business district (CBD). Letting this mass rise lowers the unemployment rate everywhere. Yet, the decentralized economy is almost efficient. Next, we consider Los Angeles and Chicago MSAs. We calibrate the model in both MSAs with census data for the year 2000. Then, we develop several counterfactual exercises either interchanging the two workforce distributions or replacing the actual ones by some standard parametric distributions. The counterfactual assumptions we consider can cause changes in unemployment rates up to about half a percentage point and in net output up to 5% when the workforce is more concentrated far from the job centers. These are non-negligible effects.

Because of our focus on duocentric cities, this paper is mainly related to Coulson et al. (2001).[2] They show that, in the presence of heterogeneous commuting abilities, the equilibrium unemployment rate is higher in the CBD where vacancy costs are by assumption higher. This outcome holds despite a commuting flow of CBD residents to the job center in the suburbs (the SBD), called "reverse commuting." Although we are also interested in spatial unemployment disparities, we depart from their approach in the following ways. First, while in Coulson et al. (2001) workers are located only in the job centers, we spatially distribute the workforce between the job centers (with total commuting costs increasing with distances). This assumption seems to us more in accordance with the stylized fact described in Glaeser (2007)'s quote above. Second, while Coulson et al. (2001) base all their analysis on the assumption of heterogeneous firms' entry costs, we are agnostic about that. Instead we look at how the shape of the distribution

[2] In addition to the already-mentioned monocentric city and circular models, Rupert and Wasmer (2012) have recently developed a new framework under the isotropy assumption according to which space looks the same wherever an agent is located.

of the workforce affects various spatial outcomes. Furthermore, simulations provide an order of magnitude of these effects. Third, we develop a formal welfare analysis while Coulson et al. (2001) only sketch it in their proposition 6. In our framework, after their initial choice of residence, workers face high relocation costs and do not change residency, as assumed by Raphael and Riker (1999), Brueckner and Zenou (2003), Zenou (2006, 2009a,c, 2013) among others.

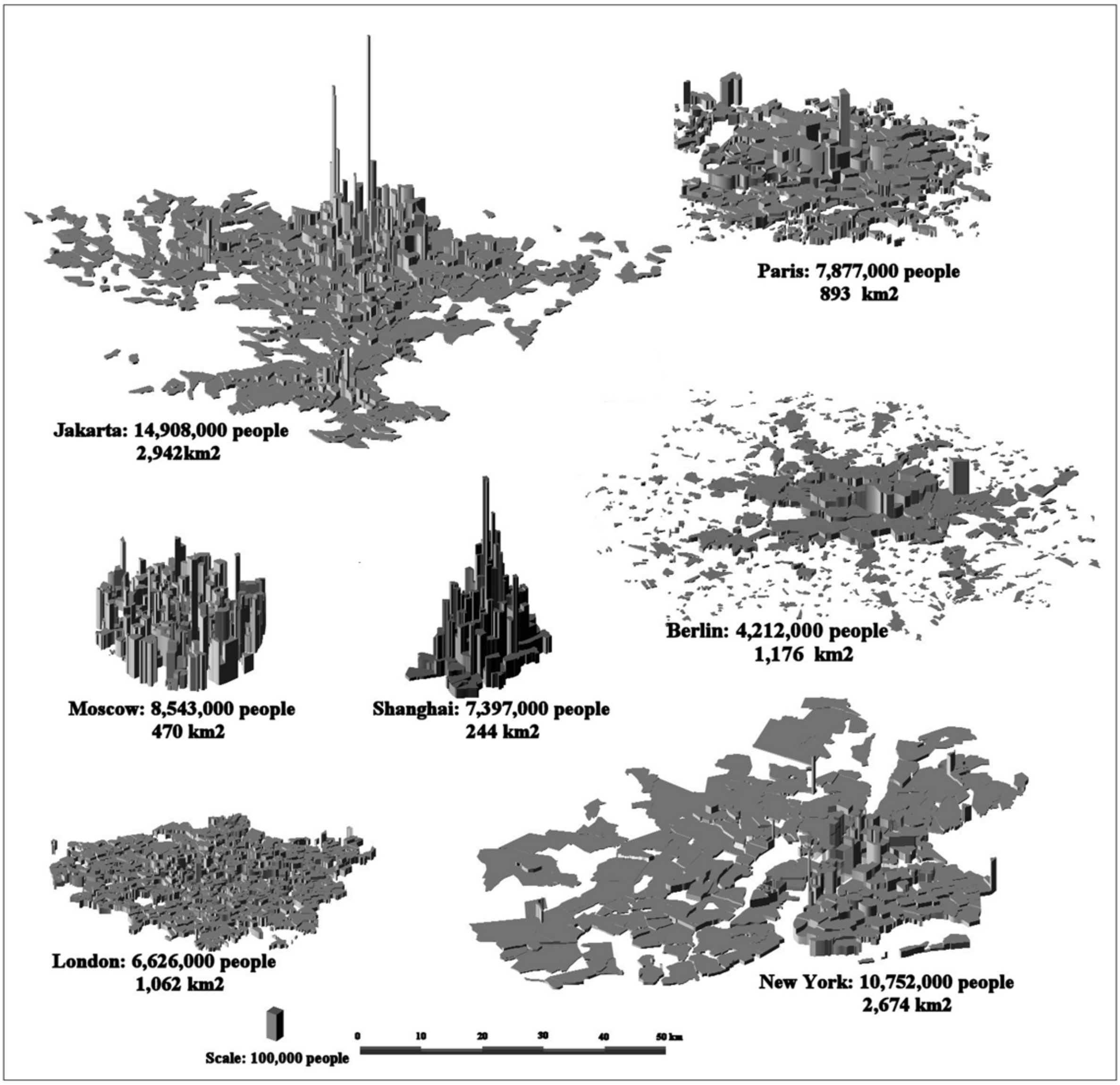


Fig. 1. Spatial distribution of population in seven major metropolises, represented at the same scale
Source: Bertaud (2008)

The next sections present the model and the welfare analysis. Section IV discusses two numerical analyses. Section V concludes.

II The model

We consider two job centers (indexed by $j = \{A, B\}$) in a densely populated area where people are distributed along a straight line joining the two centers. The workforce is normalized to unity and homogeneous in every sense, but their location, denoted x. The workforce is distributed according to a continuous density function $f : x \in [0,1] \mapsto f(x) \in \mathbb{R}_*^+$, with CDF $F(\cdot)$.

We build a model in steady state with three goods: a consumption good which serves as the numéraire, labor, and housing. There are three types of agents: workers, firms, and absentee landlords. We consider a two-stage model: first, workers decide once and for all the location $x \in [0,1]$ where they live and pay rents $R(x)$; second, the transactions occur in the labor market in a continuous-time matching model. This two steps structure captures the idea that transactions on the housing market are typically much less frequent than transitions between employment and unemployment, especially when the labor market is very flexible as in the US. This structure seems to us more realistic than the polar one where individuals would relocate at no cost after each transition on the labor market.[3] Risk-neutral and infinitely-lived agents discount the future at a common rate r. Each job center presents a distinct labor market where an endogenous number of firms choose to set up. Firms open job center specific vacancies. Workers supply inelastically one unit of labor and demand one unit of housing in a single location. Firms produce under perfect competition and constant returns to scale the consumption good using only labor. Individuals are either unemployed or employed. Job-seekers spend their time endowment looking for a job in A and in B. If employed, they can be occupied in either of the job centers and commute at a unit cost $\tau > 0$.[4]

The matching process is represented by a standard differentiable matching function $M_j(V_j, U_j)$ specific to each job center $j \in \{A, B\}$. This function yields the flow of matches per unit of time in j, M_j, as a function of the stocks of vacant jobs in job center j, V_j, and the stock of unemployed, U_j, in market j.[5] As standard in the literature (Petrongolo and Pissarides, 2001), we assume M_j to be increasing and concave in both of its arguments, exhibiting Constant Returns to Scale. The rate at which a vacancy is filled in j is:

$$\frac{M_j}{V_j} = M_j\left(1, \frac{U_j}{V_j}\right) = M_j\left(1, \frac{1}{\theta_j}\right) = \mu_j(\theta_j), \text{ with } \mu_j'(\theta_j) < 0$$

where $\theta_j \equiv \frac{V_j}{U_j}$ is named the labor market tightness in j. A tighter labor market makes it more difficult to recruit workers due to a congestion effect. Similarly, the rate at which a job-seeker

[3] Our approach follows e.g. Zenou (2009c).

[4] More precisely, τ denotes the pecuniary and time cost per unit of distance commuted to the job center.

[5] It should be noticed that by assumption firms in any center j do not open vacancies that are only accessible to job-seekers in a specific location x. This realistic assumption plays a major role in the model.

finds a vacancy in j is:

$$\frac{M_j}{U_j} = \theta_j \mu_j(\theta_j) = \psi_j(\theta_j), \text{ with } \psi_j'(\theta_j) > 0.$$

A tighter labor market increases the rate at which job-seekers find a job (the so-called thick market externality). As is standard, we assume the following Inada conditions:

$$\lim_{\theta_j \to 0} \psi_j(\theta_j) = \lim_{\theta_j \to +\infty} \mu_j(\theta_j) = 0 \quad \text{and} \quad \lim_{\theta_j \to +\infty} \psi_j(\theta_j) = \lim_{\theta_j \to 0} \mu_j(\theta_j) = +\infty.$$

When a match is formed in job center j, y_j units of output are produced.[6] In the presence of search-matching frictions, when a vacancy and a job-seeker have matched, a surplus is created. For, if they separate, each partner has to start again a new search process. Let $\Upsilon(x)$ denote the present-discounted value of the expected utility of an unemployed worker located in x. $W_j(x)$ has the same meaning for a worker employed in job center j. Let $\Pi_j(x)$ be the present-discounted profit made on a job in j filled with a worker located in x. These functions verify the Bellman equations introduced below. Under free entry of vacancies, the present-discounted expected profit made on a vacant position is nil. So, the (total) surplus of a match in j with a worker located in x, denoted $S_j(x)$, is defined by:

$$S_j(x) = \Pi_j(x) + W_j(x) - \Upsilon(x).$$

II.1 Wage formation

A contact between a worker and a vacancy leads to a contractual relationship whenever the surplus $S_j(x)$ is positive. Workers have then no incentive to quit. Jobs are destroyed at an exogenous separation rate δ_j. In that case, both parties search for a new suitable partner. Under free entry, $\Pi_j(x)$ verifies the following Bellman equation:

$$r\,\Pi_j(x) = y_j - w_j(x) - \delta_j\,\Pi_j(x) \tag{1}$$

where $w_j(x)$ is the wage of a worker living in x and occupied in j.[7] Let $z_j(x)$ denote the worker's commuting distance to her workplace:

$$z_j(x) = \begin{cases} x & \text{for } j = A \\ 1 - x & \text{for } j = B. \end{cases}$$

[6] We do not assume that commuting affects the productivity of workers. The average commuting time in the U.S. is rather short according to Gobillon et al. (2007) and the latest American Community Survey by the U.S. Census. If productivity was negatively affected by the time devoted to commuting (as suggested e.g. by van Ommeren and i Puigarnau, 2011, for Germany), the model could be adapted by introducing a weakly decreasing relationship between y_j and the commuting distance x. The model developed below should then be adapted (in particular Assumptions 1 and 2). However, the qualitative conclusions would remain unaffected.

[7] Zenou (2009b) (p. 24) summarizes the empirical evidence confirming that either firms reimburse their workers' commuting costs or provide transport-related fringe benefits. Mulalic et al. (2014) exploit a Danish quasi-natural experiment and conclude that a rise in commuting distance has a long-run effect on compensations.

The inter-temporal value of having a job $W_j(x)$ solves:

$$r\,W_j(x) = w_j(x) - \tau\, z_j(x) - R(x) - \delta_j[W_j(x) - \Upsilon(x)] \tag{2}$$

Using Eqs. (1) and (2), the surplus writes:

$$(r + \delta_j)\; S_j\,(x) = y_j - \tau\, z_j\,(x) - r\,U\,(x) \tag{3}$$

where we define:

$$r\,U(x) = r\,\Upsilon(x) + R(x). \tag{4}$$

Wages $w_j(x)$ are set through Nash bargaining. During the negotiation, agents take tightness and $\Upsilon(x)$ as given. Wages are determined by:

$$w_j\,(x) = \arg\max_{w}\,[W_j\,(x) - \Upsilon\,(x)]^{\beta_j}\;[\Pi_j\,(x)]^{1-\beta_j}$$

where $\beta_j \in (0,1)$ denotes the exogenous worker's bargaining power. Then, the first-order condition of this problem implies that the surplus accruing to the worker (resp., the employer) verifies:

$$W_j\,(x) - \Upsilon\,(x) = \beta_j\, S_j(x) \qquad \text{resp.,} \qquad \Pi_j(x) = (1 - \beta_j)\; S_j(x). \tag{5}$$

The wage equation is solved by plugging Eqs. (1) and (3) into Eq. (5):

$$w_j\,(x) = \beta_j y_j + (1 - \beta_j)\;[\tau\, z_j\,(x) + r\,U(x)]\,. \tag{6}$$

II.2 The search decision

The unemployed only commute from time to time for an interview. Hence, they incur commuting costs that we neglect.[8] Let Σ_j the expected returns to search in location j for a worker located in x be defined by:

$$\Sigma_j = \psi_j(\theta_j)\,[W_j(x) - \Upsilon(x)] = \beta_j\,\psi_j(\theta_j)\,\frac{y_j - \tau\, z_j(x) - r\,U(x)}{r + \delta_j} \tag{7}$$

where the second equality follows from Eqs. (3) and (5). The inter-temporal value in unemployment $\Upsilon(x)$ solves the following Bellman equation:

$$\begin{aligned} r\,\Upsilon(x) &= b - R(x) \\ &\quad + \max\Big\{0, \max_{\varepsilon(x)\in[0,1]}\Big[\varepsilon(x)\,\Sigma_A(\theta_A, x), (1 - \varepsilon(x))\,\Sigma_B(\theta_B, x)\Big]\Big\} \\ \Leftrightarrow\; r\,U(x) &= b \\ &\quad + \max\Big\{0, \psi_A(\theta_A)\,(W_A(x) - \Upsilon(x))\,, \psi_B(\theta_B)\,(W_B(x) - \Upsilon(x))\Big\} \end{aligned} \tag{8}$$

[8] Their main search activity is made from where they live or close to it via the reading of newspapers, surfing on the web, visiting the nearest one-stop career center, sending out resumes, contacting friends and relatives, and the like (for descriptive evidence, see Kuhn and Mansour, 2011, for the U.S. and Longhi and Taylor, 2011, for Great Britain). The assumption of absence of commuting cost can easily be relaxed to the case where the unemployed commute a non negligible amount of time, but anyway less than employed individuals (see e.g. Zenou, 2009).

where b is the instantaneous value in unemployment. Eq. (8) tells that an unemployed has first to decide whether she searches for a job or not. If she does, as in Coulson et al. (2001), she optimizes the use of her unit time endowment to search for work in A and B. The rate at which a job offer is found in A (respectively, in B) is $\varepsilon(x)\,\psi_A(\theta_A)$ (respectively, $(1-\varepsilon(x))\,\psi_B(\theta_B)$). These rates are multiplied by the job center-specific gain of becoming employed, $W_j(x) - \Upsilon(x)$.

As the marginal returns to search in one location $\psi_j(\theta_j)\,(W_j(x) - \Upsilon(x))$ do not depend on the share $\varepsilon(x)$ of time spent searching in job center j, either the unemployed living in x search only in job center A (i.e. $\varepsilon(x) = 1$), or they search only in job center B (i.e. $\varepsilon(x) = 0$), or they do not search at all.

After substitution of $rU(x)$ into (7), Σ_j is in general a function of θ_A, θ_B and x such that:

Lemma 1. *For any location $x \in [0,1]$, $\Sigma_A(\theta_A, \theta_B, x)$ and $\Sigma_B(\theta_A, \theta_B, x)$ do not depend on rents $R(x)$, verify*

$$\frac{\partial \Sigma_A}{\partial x}\,(\theta_A, \theta_B, x) \leqslant 0 \leqslant \frac{\partial \Sigma_B}{\partial x}\,(\theta_A, \theta_B, x)\,, \tag{9}$$

with strict inequalities if the unemployed seek jobs. Moreover,

$$\begin{aligned} &\frac{\partial \Sigma_A}{\partial \theta_A}(\theta_A, \theta_B, x) > 0 = \frac{\partial \Sigma_A}{\partial \theta_B}(\theta_A, \theta_B, x) \\ \textit{if} \qquad &\Sigma_A(\theta_A, \theta_B, x) > \max[0, \Sigma_B(\theta_A, \theta_B, x)] \end{aligned}$$

and

$$\begin{aligned} &\frac{\partial \Sigma_B}{\partial \theta_B}(\theta_A, \theta_B, x) > 0 = \frac{\partial \Sigma_B}{\partial \theta_A}(\theta_A, \theta_B, x) \\ \textit{if} \qquad &\Sigma_B(\theta_A, \theta_B, x) > \max[0, \Sigma_A(\theta_A, \theta_B, x)] \end{aligned}$$

Proof. Three cases need to be distinguished.

Case *(i)*: If $\Sigma_A(\theta_A, \theta_B, x) > \Sigma_B(\theta_A, \theta_B, x)$ and $\Sigma_A(\theta_A, \theta_B, x) > 0$.

As $\varepsilon(x) = 1$, combining Eqs. (7) and (8) yields:

$$r\,U(x) = \frac{\beta_A\,\psi_A(\theta_A)(y_A - \tau\,x) + (r + \delta_A)b}{r + \delta_A + \beta_A\,\psi_A(\theta_A)} \tag{10}$$

which is decreasing in x conditional on tightness and independent on rents $R(x)$. Now, considering Eq. (7) in A leads to

$$\Sigma_A\,(\theta_A, \theta_B, x) = \beta_A\,\psi_A(\theta_A)\,S_A(x) = \beta_A\,\psi_A(\theta_A)\frac{y_A - \tau\,x - b}{r + \delta_A + \beta_A\,\psi_A(\theta_A)} \tag{11}$$

which is decreasing in x (conditional on tightness) because the surplus of a match $S_A(x)$ is decreasing. It is also independent on rents $R(x)$. Then, substituting $r\,U(x)$ into Eq. (7)

evaluated in $j = B$ yields:

$$\Sigma_B(\theta_A, \theta_B, x) = \frac{\beta_B\ \psi_B(\theta_B)}{(r+\delta_B)(r+\delta_A+\beta_A\psi_A(\theta_A))}\{(y_B - \tau(1-x) - b)(r+\delta_A) \\ + \beta_A\psi_A(\theta_A)(y_B - y_A - \tau(1-2x))\}.$$

Therefore, in Case *(i)*, we have the following property:

$$\frac{\partial \Sigma_A}{\partial x} < 0 < \frac{\partial \Sigma_B}{\partial x} \qquad \text{and} \qquad \frac{\partial \Sigma_A}{\partial \theta_A} > 0 = \frac{\partial \Sigma_A}{\partial \theta_B}.$$

Case *(ii)* If $\Sigma_B(\theta_A, \theta_B, x) > \Sigma_A(\theta_A, \theta_B, x)$ and $\Sigma_B(\theta_A, \theta_B, x) > 0$.

As $\varepsilon(x) = 0$, combining Eqs. (7) and (8) yields:

$$r\,U(x) = \frac{\beta_B\,\psi_B(\theta_B)\ (y_B - \tau(1-x)) + (r+\delta_B)\,b}{r+\delta_B+\beta_B\,\psi_B(\theta_B)} \tag{12}$$

which is increasing in x conditional on tightness and independent on rents $R(x)$. Now, considering Eq. (7) in B leads to

$$\Sigma_B(\theta_A, \theta_B, x) = \beta_B\,\psi_B(\theta_B)\,S_B(x) = \beta_B\,\psi_B(\theta_B)\frac{y_B - \tau(1-x) - b}{r+\delta_B+\beta_B\,\psi_B(\theta_B)} \tag{13}$$

which is increasing in x (conditional on tightness) because the surplus $S_B(x)$ is increasing. It is also independent on rents $R(x)$. Then, considering Eq. (7) also in A yields:

$$\Sigma_A(\theta_A, \theta_B, x) = \frac{\beta_A\psi_A(\theta_A)}{(r+\delta_A)(r+\delta_B+\beta_B\psi_B(\theta_B))}\{(y_A - \tau x - b)(r+\delta_B) \\ + \beta_B\psi_B(\theta_B)(y_A - y_B + \tau(1-2x))\}.$$

Therefore, in Case *(ii)*, we have the following property:

$$\frac{\partial \Sigma_A}{\partial x} < 0 < \frac{\partial \Sigma_B}{\partial x} \qquad \text{and} \qquad \frac{\partial \Sigma_B}{\partial \theta_B} > 0 = \frac{\partial \Sigma_B}{\partial \theta_A}.$$

Case *(iii)* If $\Sigma_A(\theta_A, \theta_B, x) < 0$ and $\Sigma_B(\theta_A, \theta_B, x) < 0$.

Then, the unemployed workers do not search and $r\,U(x) = b$, so (9) again applies. □

According to Lemma 1, the expected surpluses of searching a job do not depend on rents $R(x)$. This is because employed and unemployed workers need both to pay rents where they live.

Lemma 1 shows that conditional on the level of tightness in each center the expected return to search in A (respectively, in B) shrinks (respectively, grows) as the distance to A rises (and hence the one to B shrinks). These effects are entirely driven by the evolution of the surpluses, which in turn vary as commuting costs do. Moreover, the relationships $x \mapsto \Sigma_j(\theta_A, \theta_B, x)$, for $j \in \{A, B\}$, are differentiable except at the threshold $\tilde{x}$ such that $\Sigma_A(\theta_A, \theta_B, \tilde{x}) = \Sigma_B(\theta_A, \theta_B, \tilde{x})$, if any. To guarantee that the surplus is positive for all x and j, we henceforth rule out Case *(iii)* by fixing an upper-bound on τ:[9]

Assumption 1. $\tau < \min\{y_A, y_B\} - b$.

According to Assumption 1, all jobless individuals are searching for a job. Lemma 1 then implies the existence of a threshold location denoted $\tilde{x}$ that separates the pool of unemployed searching a job in A to those searching a job in B.

Lemma 2. *For any θ_A and θ_B, either*

- *there exists a unique $\tilde{x} \in [0,1]$ such that*

$$\Sigma_A\left(\theta_A, \theta_B, \tilde{x}\right) = \Sigma_B\left(\theta_A, \theta_B, \tilde{x}\right) \tag{14}$$

- *or $\Sigma_A\left(\theta_A, \theta_B, 1\right) > \Sigma_B\left(\theta_A, \theta_B, 1\right)$ in which case all unemployed workers search in A*

- *or $\Sigma_A\left(\theta_A, \theta_B, 0\right) < \Sigma_B\left(\theta_A, \theta_B, 0\right)$ in which case all unemployed workers search in B.*

Henceforth, we concentrate on the case where $\tilde{x} \in [0,1]$ verifies Eq. (14). Assumption 2 below will guarantee such a configuration.

II.3 The labor demand

We adopt a one-job-one-firm setting. Each vacancy can be either filled of vacant. Opening a vacant job costs k_j per unit of time. Under free-entry, firms open vacancies in j until the expected cost of hiring a worker equals the expected profit made on a filled position. In job centers A and B, this condition is respectively:[10]

$$\begin{aligned} k_A &= \mu_A(\theta_A) \int_0^{\tilde{x}} \max\{\Pi_A(x), 0\} \frac{f(x)}{F(\tilde{x})} dx, \\ k_B &= \mu_B(\theta_B) \int_{\tilde{x}}^1 \max\{\Pi_B(x), 0\} \frac{f(x)}{1 - F(\tilde{x})} dx. \end{aligned} \tag{15}$$

[9] Relaxing this assumption would somewhat complicate the model as there would be a reservation distance above which surplus in A would become negative and hence matches in A would not be formed and there would be a reservation distance below which matches in B would not be formed for the same reason. This would not add much insight into our analysis.

[10] We here consider that the spatial distribution of applicants coincides with the one of the labor force, i.e. is given by the density $f(x)$. This condition obviously holds at the steady state. It also holds along transitional dynamics where the unemployment rate is initially uniform and then the threshold $\tilde{x}$ does not change. To see this, let $u_t(x)$ denote the unemployment rate among individuals living in x at time t and $\dot{u}_t(x)$ its time derivative. The low of motion of this unemployment rate is given by:

$$\dot{u}_t(x) = \begin{cases} \delta_A - \left(\delta_A + \psi_A(\theta_{A,t})\right) u_t(x) & \text{if} \quad x \leqslant \tilde{x} \\ \delta_B - \left(\delta_B + \psi_B(\theta_{B,t})\right) u_t(x) & \text{if} \quad x > \tilde{x} \end{cases}$$

Therefore, if the unemployment rates are initially uniform and the threshold $\tilde{x}$ does not change, then all the unemployment rates $u_t(x)$ for location below the threshold remain identical. The same holds above the threshold. Consequently, the spatial distribution of job applicants remains identical to the one of the labor force.

The flow cost k_j of opening a vacancy (on the left) equals the rate μ_j at which a vacancy meets an applicant times the expected profit made on an applicant. Given the above-explained search decisions, the latter is a conditional expectation, respectively $\mathbb{E}_x\left\{\max\left\{\Pi_A(x),0\right\} \mid x \leqslant \tilde{x}\right\}$ and $\mathbb{E}_x\left\{\max\left\{\Pi_B(x),0\right\} \mid x \geqslant \tilde{x}\right\}$. By Assumption 1 and the surplus sharing rule (5), $\Pi_j(x)$ is always positive ($j \in \{A, B\}$). Let

$$\Gamma_A(\tilde{x}) = \int_0^{\tilde{x}} x \, \frac{f(x)}{F(\tilde{x})}\, dx \qquad \text{and} \qquad \Gamma_B(\tilde{x}) = \int_{\tilde{x}}^1 (1-x)\, \frac{f(x)}{1-F(\tilde{x})}\, dx \tag{16}$$

denote the conditional expected commuting distance respectively to job centers A and B. These functions verify:

$$0 \leqslant \Gamma_A(\tilde{x}) \leqslant \tilde{x} \;\text{ and }\; \Gamma_A'(\tilde{x}) = \frac{f(\tilde{x})}{F(\tilde{x})}\left(\tilde{x} - \Gamma_A(\tilde{x})\right) \geqslant 0 \tag{17}$$

$$0 \leqslant \Gamma_B(\tilde{x}) \leqslant 1-\tilde{x} \;\text{ and }\; \Gamma_B'(\tilde{x}) = -\frac{f(\tilde{x})\left(1-\tilde{x}-\Gamma_B(\tilde{x})\right)}{1-F(\tilde{x})} \leqslant 0. \tag{18}$$

Using Eqs. (1), (5) (11) and (13), the free entry conditions (15) become:

$$\frac{r+\delta_j}{1-\beta_j}\frac{\theta_j}{\psi_j(\theta_j)} + \frac{\beta_j}{1-\beta_j}\theta_j = \frac{y_j - \tau\,\Gamma_j(\tilde{x}) - b}{k_j}, \quad j \in \{A, B\}. \tag{19}$$

Lemma 3. *For any value of the threshold $\tilde{x}$, under Assumption 1, the equilibrium value of tightness is unique. In $j = A$ (respectively, $j = B$), tightness decreases (respectively, increases) with the threshold $\tilde{x}$.*

Proof. Under the matching functions' Inada conditions, the left-hand side (LHS) of Eq. (19) is increasing in θ_j from 0 to $+\infty$. By Assumption 1, the RHS is positive for all values of $\tilde{x}$. Hence, for any $\tilde{x} \in [0, 1]$ and any $j \in \{A, B\}$, Eq. (19) implicitly defines a unique level of tightness $\theta_j = \Theta_j(\tilde{x})$ with $\Theta_A'(\tilde{x}) < 0$ and $\Theta_B'(\tilde{x}) > 0$. □

A rise in $\tilde{x}$ raises the conditional commuting distance to job center A and hence reduces the expected surplus of a match. This induces a decline in tightness in A. A similar phenomenon applies in B with the opposite implication on tightness. A composition effect is at work. When a firm decides whether to open a vacancy, it compares the expected cost to the expected profit made when the position is filled. Since vacant jobs are specific to the job center, but accessible to individuals located anywhere, and as workers' commuting costs are partly reimbursed by the employer, this expected profit shrinks when job-seekers living further away enter the queue of unemployed in the job center.[11] In other contexts, similar composition effects have been emphasized by e.g. Decreuse (2008) and Albrecht et al. (2010).

[11] If, as in Gautier and Zenou (2010), the location of each worker is not observed or not verifiable, two cases can occur. Either, the average commuting cost is common knowledge and firms compensate their employee for this average. Then, the same qualitative effect on the expected profit would be observed. Or, there is no compensation at all for commuting costs. Then, this effect disappears. However, as explained in Footnote 7, there is empirical evidence that such compensations are present in reality.

II.4 The labor market equilibrium

The steady-state equilibrium can be defined recursively. First, we need to characterize the 3-tuple $\left\{\tilde{\theta}_A, \tilde{\theta}_B, \tilde{x}\right\}$. Second, the size of the population in unemployment and the number of vacancies are then determined.

Definition 1. *A steady-state equilibrium is 3-tuple $\left\{\tilde{\theta}_A, \tilde{\theta}_B, \tilde{x}\right\}$ that verifies the free-entry condition (19) on each labor market and search indifference condition (14).*

Let us define

$$\mathscr{S}_j(\tilde{x}) = \Sigma_j\left(\Theta_A(\tilde{x}), \Theta_B(\tilde{x}), \tilde{x}\right),$$

that is the expected return to search in j at any value of the threshold $\tilde{x}$, in or out of equilibrium, once the effect of commuting distance on tightness, $\Theta_j(\tilde{x})$ implicitly defined by (19), is taken into account. Figure 2 illustrates Lemma 4 and Proposition 1 introduced below.

Lemma 4. *The expected return to search in market A, $\mathscr{S}_A(\tilde{x})$, decreases with the value of the threshold $\tilde{x}$. The opposite is true in B.*

Proof. Lemma 1 shows that Σ_A (respectively, Σ_B) is not affected by θ_B (resp., θ_A) in the interval of the $[0,1]$ segment where job-seekers search in A (resp., B). From Lemmas 1 and 3,

$$\frac{\partial \mathscr{S}_j(\tilde{x})}{\partial \tilde{x}} = \frac{\partial \Sigma_j}{\partial \theta_j}\frac{\partial \Theta_j(\tilde{x})}{\partial \tilde{x}} + \frac{\partial \Sigma_j}{\partial \tilde{x}} < 0 \text{ if } j = A, \quad > 0 \text{ if } j = B.$$

□

Taking the effect of the threshold on the level of tightness under free entry, the equilibrium condition (14) becomes:

$$\mathscr{S}_A(\tilde{x}) = \mathscr{S}_B(\tilde{x}) \tag{20}$$

which admits at most one solution. Hence, if an equilibrium exists, it is unique. To ensure existence, one needs:

$$\mathscr{S}_A(0) > \mathscr{S}_B(0) \qquad \text{and} \qquad \mathscr{S}_A(1) < \mathscr{S}_B(1). \tag{21}$$

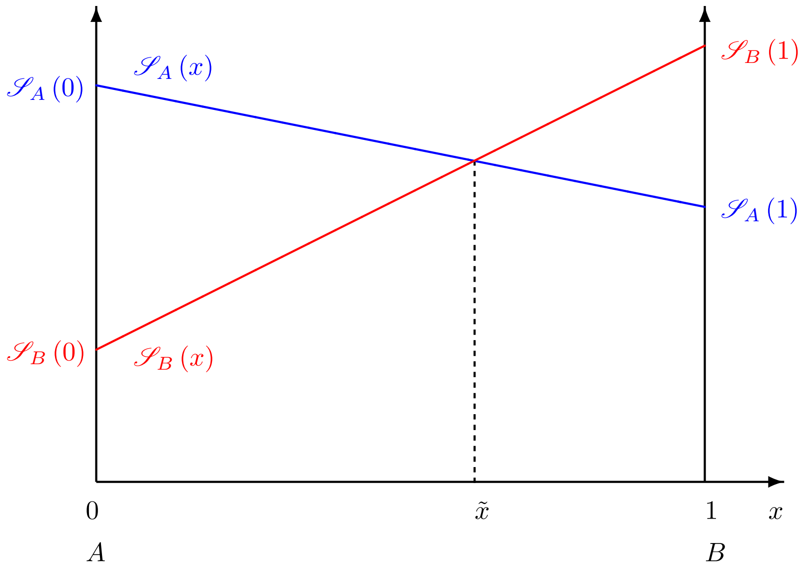


Fig. 2. The equilibrium

Let us define $\underline{\psi}_A = \psi_A(\Theta_A(1))$ the exit rate to employment in job center A when everybody on the segment $[0,1]$ seeks a job there since $\tilde{x} = 1$. At the other extreme, we denote $\overline{\psi}_A = \psi_A(\Theta_A(0))$. Obviously, we have $0 < \underline{\psi}_A < \overline{\psi}_A$. Similarly in B, let $\underline{\psi}_B = \psi_B(\Theta_B(0))$, $\overline{\psi}_B = \psi_B(\Theta_B(1))$, with $0 < \underline{\psi}_B < \overline{\psi}_B$. From the definition of $\mathscr{S}_j$ and (7), Inequalities (21) are equivalent to the following assumption:

Assumption 2. *Parameter τ is such that:*

$$\tau > \max\left\{y_A - b - \frac{\overline{\nu}_B}{\underline{\nu}_A}(y_B - b), y_B - b - \frac{\overline{\nu}_A}{\underline{\nu}_B}(y_A - b)\right\}$$

where we define

$$\overline{\nu}_A = \frac{\beta_A \overline{\psi}_A}{r + \delta_A + \beta_A \overline{\psi}_A}, \quad \overline{\nu}_B = \frac{\beta_B \overline{\psi}_B}{r + \delta_B + \beta_B \overline{\psi}_B},$$
$$\underline{\nu}_A = \frac{\beta_A \underline{\psi}_A}{r + \delta_A + \beta_A \underline{\psi}_A}, \quad \underline{\nu}_B = \frac{\beta_B \underline{\psi}_B}{r + \delta_B + \beta_B \underline{\psi}_B}.$$

Assumption 2 puts a lower bound on the commuting cost τ. Intuitively, if τ is too low and $\mathscr{S}_A(1) > \mathscr{S}_B(1)$ (resp. $\mathscr{S}_B(0) > \mathscr{S}_A(0)$), even workers located very close to B (resp. A) are better off searching a job in A (B). Search costs should then be high enough to prevent that a single labor market exists. It can be checked that Assumptions 1 and 2 can be incompatible if the two marginal products y_A and y_B are too different.[12] A direct consequence of the two previous lemmas is:

[12] For instance, one can imagine that y_A is so low compared to y_B, so that: $0 < y_A - b < y_B - b - \frac{\overline{\nu}_A}{\underline{\nu}_B}(y_A - b) < y_B - b$. In this example, if the marginal commuting cost τ verifies Assumption 1 (i.e. is lower than $y_A - b$), it can obviously not be compatible with Assumption 2 at the same time. In this example, $\mathscr{S}_A(0) < \mathscr{S}_B(0)$, which amounts to saying that job center A ceases to exist due to a lack of productivity compared to B. Henceforth, we neglect such uninteresting cases where the two-center model collapses to a one-business-district setting.

Proposition 1. *An equilibrium exists, is unique, and interior under Assumptions 1 and 2.*

Knowing the steady-state equilibrium $\left\{\tilde{\theta}_A, \tilde{\theta}_B, \tilde{x}\right\}$, the wage in any location x on the left of $\tilde{x}$ is obtained by plugging $\tilde{\theta}_A$ in Eq. (6) (resp. $\tilde{\theta}_B$ on the right of $\tilde{x}$.) The equilibrium population sizes in each state and the number of vacancies are also easily computed. Let

$$G_j(x) = \begin{cases} F(x) & \text{for } j = A \\ 1 - F(x) & \text{for } j = B. \end{cases} \tag{22}$$

In job center j, the steady-state equilibrium numbers of unemployed $\tilde{U}_j$, of employed $\tilde{L}_j$, and of vacancies are given by:

$$\tilde{U}_j = \frac{\delta_j G_j(\tilde{x})}{\delta_j + \psi_j(\tilde{\theta}_j)}, \quad \tilde{L}_j = G_j(\tilde{x}) - \tilde{U}_j \qquad \text{and} \qquad \tilde{V}_j = \tilde{\theta}_j \tilde{U}_j \tag{23}$$

The equilibrium density of workers employed in job center A, (respectively, unemployed and searching a job in job center B) in any location x is:

$$\begin{aligned} L_A(x) =& \frac{\psi_A\left(\tilde{\theta}_A\right) f(x)}{\delta_A + \psi_A\left(\tilde{\theta}_A\right)}, \left(\text{resp., } U_A(x) = \frac{\delta_A f(x)}{\delta_A + \psi_A(\tilde{\theta}_A)}\right) \text{ for } x \leqslant \tilde{x} \\ L_A(x) =& \, 0, \qquad\qquad\qquad (\text{resp., } U_A(x) = 0) \qquad\qquad \text{for } x > \tilde{x} \end{aligned} \tag{24}$$

$$\begin{aligned} L_B(\tilde{x}) =& \frac{\psi_B\left(\tilde{\theta}_B\right) f(x)}{\delta_B + \psi_B\left(\tilde{\theta}_B\right)}, \left(\text{resp., } U_B(\tilde{x}) = \frac{\delta_B f(x)}{\delta_B + \psi_B(\tilde{\theta}_B)}\right) \text{ for } x > \tilde{x} \\ L_B(\tilde{x}) =& \, 0, \qquad\qquad\qquad (\text{resp., } U_B(\tilde{x}) = 0) \qquad\qquad \text{for } x \leqslant \tilde{x} \end{aligned} \tag{25}$$

In segment $(0, \tilde{x})$ (respectively, $(\tilde{x}, 1)$), the equilibrium unemployment rate is constant:

$$\frac{\delta_A}{\delta_A + \psi_A(\tilde{\theta}_A)}, \qquad \left(\text{resp., } \frac{\delta_B}{\delta_B + \psi_B(\tilde{\theta}_B)}\right). \tag{26}$$

II.5 Comparative static analysis

Conditional on $\tilde{x}$, the comparative statics of the model is fully standard (see e.g. Pissarides, 2000). From the free-entry conditions (19), for a given value of the threshold, firms post less vacancies and hence equilibrium tightness falls in any job center after a marginal rise in the cost of opening a vacancy, the job destruction rate or the workers' bargaining power in this center. The same holds if the instantaneous value in unemployment or the discount rate rises. Moreover, a rise in the marginal product y_j increases θ_j.

Turning to the comparative statics on the threshold $\tilde{x}$, Appendix A shows that the *partial* effect of a rise in k_j, δ_j, b or r or a decline in y_j on the expected return of search in j, $\mathscr{S}_j(x)$, reinforces the above-mentioned effect through equilibrium tightness (see the summary in Table 6 of this appendix). This is however not true for the bargaining power β_j, for reasons explained later on. Fig. 3 illustrates the total effect of parameters' changes on the $\Sigma_j(\Theta_A(x), \Theta_B(x), x)$ (i.e. $\mathscr{S}_j(x)$) schedules for any value of the threshold x when this effect has a clear sign.

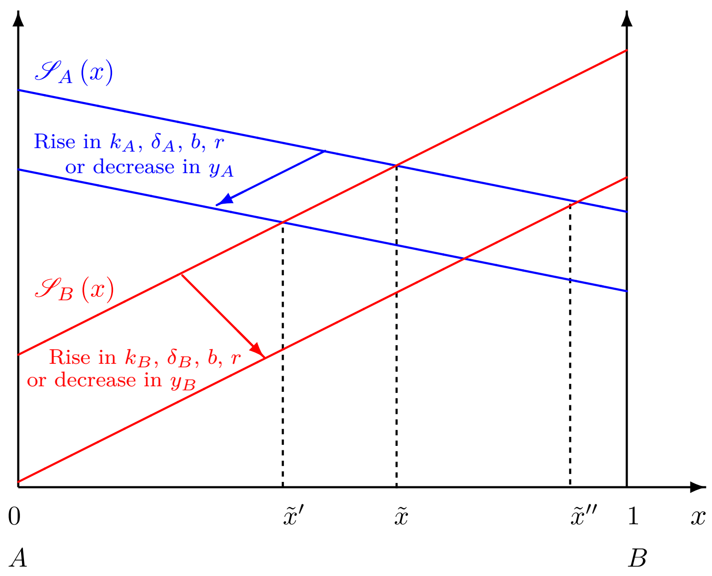


Fig. 3. Comparative statics

In Fig. 3, a rise in k_A or in δ_A (or a decline in y_A) shifts the whole curve $\mathscr{S}_A(x)$ downwards without affecting the same curve in B, so the equilibrium threshold declines (see $\tilde{x}'$). The equilibrium value of θ_B then declines because $\Theta_B'(\tilde{x}) > 0$ and the $\Theta_B(x)$ schedule is not *directly* affected by changes in any of the parameters k_A, δ_A or y_A. On the contrary, the total effect on equilibrium tightness in A is ambiguous for the direct effect of a rise in k_A or in δ_A (or a decline in y_A) on tightness and the effect through the threshold $\tilde{x}$ go in opposite directions (see Appendix I.3 for more explanations). The case where k_B or δ_B rises (or y_B declines) is symmetric. It induces a rise in the equilibrium threshold value (see $\tilde{x}''$ on Fig. 3). The equilibrium value of θ_A then declines because $\Theta_A'(\tilde{x}) < 0$ and the $\Theta_A(x)$ schedule is not *directly* affected by changes in any of the parameters k_B, δ_B or y_B. Furthermore, opposite forces lead as above to an ambiguous impact on equilibrium tightness in B.

We obtain ambiguous marginal impacts of the instantaneous value in unemployment and of the interest rate on the equilibrium threshold value since both $\mathscr{S}_j(x)$ curves shift in the same direction (namely downwards in Fig. 3). Then, the total effects on equilibrium tightness levels are obviously ambiguous as well. The impacts of a change in the bargaining power depend on the magnitude of this power. Rising the bargaining power in center j has a positive partial effect on $\Sigma_j(\theta_A, \theta_B, x)$ but, as we have seen at the beginning of this subsection, it also has a negative direct effect on tightness θ_j for any x and hence a negative effect on the expectation Σ_j. Appendix A shows that a rise in any bargaining power β_j does not affect the $\mathscr{S}_j(x)$ curve when the Hosios condition (i.e. $\beta_j = \eta_j \equiv -\theta_j \mu_j'(\theta_j)/\mu_j(\theta_j)$[13]) is verified. In this particular case, the positive partial effect and the negative induced effect through tightness cancel out. Therefore, the threshold $\tilde{x}$ remains unaffected.[14] Under the Hosios condition, a rise in workers' bargaining power in center j has therefore only a direct negative effect on equilibrium tightness in the same center along (19) (and no effect in the other one). If now the workers' bargaining powers are both "too high" (i.e. $\beta_j > \eta_j, j \in \{A, B\}$), a rise in β_A lowers $\tilde{x}$ and a rise in β_B increases $\tilde{x}$. This occurs because the negative direct effect of a rise in β_j on tightness outweighs the positive *partial* effect on the expected return of search Σ_j. So, the latter curve shifts downwards. The opposite effects are observed when the workers' bargaining powers are both "too low." The total impact of rise in the workers' bargaining power can sometimes be signed if the Hosios condition does not apply. See Table 1.

Table 1: Comparative statics with respect to workers' bargaining powers β_j

	$\eta_A \geqslant \beta_A$	$\eta_A < \beta_A$		$\eta_B \geqslant \beta_B$	$\eta_B < \beta_B$
$d\tilde{x}/d\beta_A$	$\geqslant 0$	< 0	$d\tilde{x}/d\beta_B$	$\leqslant 0$	> 0
$d\theta_A/d\beta_A$	< 0	?	$d\theta_A/d\beta_B$	$\geqslant 0$	< 0
$d\theta_B/d\beta_A$	$\geqslant 0$	< 0	$d\theta_B/d\beta_B$	< 0	?

II.6 The housing market

We finally consider the stage 1 of our model where rents and workers' locations are decided once and for all on a perfectly competitive housing market. A unit mass of dwellings that are identical in all aspects, but their locations is available. The housing supply is inelastically determined by some spatial and institutional constraints on the building of new dwellings. Each individual occupying a single dwelling, the endogenous density function $f(x)$ describing the distribution of workers along the space line $[0, 1]$ is identical to the exogenous distribution of dwellings in the economy. Initially, all workers are unemployed and bid for renting the dwellings. They also have outside opportunities which yield an expected lifetime utility $\overline{\Upsilon}$. Workers being perfectly mobile at the initial housing stage, they must be indifferent across the different

[13] η_j is the elasticity of the matching function with respect to the stock of unemployment.

[14] Pissarides (2000) shows a related result in a setting without explicit spatial heterogeneity, but endogenous participation decisions. The participation rate reaches a maximum when the Hosios condition is met. Then, a marginal rise in the workers' bargaining power does not modify participation decisions.

locations. Therefore, the no-arbitrage condition on the housing market leads to:[15]

$$\Upsilon(x) = \overline{\Upsilon} \qquad \Leftrightarrow \qquad R(x) = r\left(U(x) - \overline{\Upsilon}\right)$$

Using Eqs. (10) and (12), the rents are:

$$R(x) = \begin{cases} \dfrac{\beta_A\,\psi_A(\theta_A)(y_A - \tau\,x) + (r+\delta_A)b}{r + \delta_A + \beta_A\,\psi_A(\theta_A)} - r\,\overline{\Upsilon} & \text{if} \quad x \leqslant \tilde{x} \\ \dfrac{\beta_B\,\psi_B(\theta_B)(y_B - \tau(1-x)) + (r+\delta_B)b}{r + \delta_B + \beta_B\,\psi_B(\theta_B)} - r\,\overline{\Upsilon} & \text{if} \quad x \geqslant \tilde{x} \end{cases} \tag{27}$$

As rents cannot be negative, $\overline{\Upsilon}$ cannot be too high. Rents perfectly compensate the unemployed workers for leaving further away from the job center they might be working in.

They decrease with distance at pace $\tau\,\beta_A\,\psi_A(\theta_A)/(r + \delta_A + \beta_A\,\psi_A(\theta_A))$ until the threshold $\tilde{x}$ and then increase at pace $\tau\,\beta_B\,\psi_B(\theta_B)/(r + \delta_B + \beta_B\,\psi_B(\theta_B))$ above $\tilde{x}$.

III The social optimum and its decentralization

Conditional on the available matching technology, the social planner has three instruments, namely the two levels of tightness and the *threshold* value denoted x which directs the job-search process towards job center A or B. We add a superscript * to designate the efficient value of these instruments. As is often the case in this literature, we limit the analysis to the case where $r \mapsto 0$.[16] Then, the social welfare function maximizes aggregate net output in steady state (ignoring the transitional dynamics). Aggregate net output Ω is defined as output produced net of commuting costs plus the value of time in unemployment minus the cost of creating vacant jobs (being transfers among agents, wages, and rents do not appear in Ω). Using (16) and (23), Ω can be written as a function of the two levels of tightness and of the threshold x:

$$\begin{aligned} \Omega\left(\theta_A, \theta_B, x\right) &= \sum_{j\in\{A,B\}} \left(y_j - \tau\Gamma_j(x)\right)\frac{\psi_j(\theta_j)G_j(x)}{\delta_j + \psi_j(\theta_j)} + b\frac{\delta_j G_j(x)}{\delta_j + \psi_j(\theta_j)} - k_j\,V_j \\ &= b + \sum_{j\in\{A,B\}} G_j\left(x\right)\,\mathcal{W}_j\left(\theta_j, x\right) \end{aligned} \tag{28}$$

[15] Let us verify that there exists an equilibrium transitional dynamics from the initial stage where all individuals are unemployed along which the threshold $\tilde{x}$ and tightness values θ_j jump instantaneously to their steady-state values. If this was the case, the expected surpluses $\Sigma_j(\theta_A, \theta_B, x)$ would also jump instantaneously to their steady-state values. Therefore, the threshold $\tilde{x}$ would do the same according to the indifference condition (14). Because the threshold $\tilde{x}$ does not change and the unemployment rate is initially uniform, so do tightness levels through the free-entry conditions, according to Footnote 10. Hence, our initial presumption would be verified. We can then also conclude that the lifetime values of being unemployed $\Upsilon(x)$ are constant, and so do the rents.

[16] As explained e.g. by Cahuc and Zylberberg (2004) in the case of the basic matching model, the social planner problem can be studied in two ways. First, in the more general approach, the planner solves a dynamic optimal control problem subject to the law of motion of the unemployment rate. The second approach sets aside the problem of dynamic optimization by looking directly at the maximization of net output in a steady state subject to the equation characterizing the steady-state unemployment rate. Both approaches lead to the same equation characterizing optimal tightness in steady state when $r \mapsto 0$. In our setting, optimal control techniques cannot be applied since we would have a continuum of law of motions (namely, one in each location x). So, unless the threshold is set once for all and the unemployment rates are initially uniform (see Footnote 10), we would have a continuum of state variables to manage, namely, one unemployment rate per location. We therefore adopt the second approach and assume $r \mapsto 0$.

where

$$\mathcal{W}_j\left(\theta_j, x\right)=\frac{\psi_j\left(\theta_j\right)}{\delta_j+\psi_j\left(\theta_j\right)}\left(y_j-\tau\, \Gamma_j\left(x\right)-b\right)-\frac{\theta_j\, \delta_j}{\delta_j+\psi_j\left(\theta_j\right)} k_j, \tag{29}$$

designates the per capita surplus created in job center j. We proceed in three steps. First, for any level of the threshold x, we determine the optimal values of tightness, $\theta_A^*(x)$ and $\theta_B^*(x)$, in each job center. Second, we show that the Hosios condition is necessary to decentralize the optimal allocation. Third, we select the optimal threshold x^* that maximizes $b+\sum_{j\in\{A,B\}} G_j\left(x\right) \mathcal{W}_j\left(\theta_j^*(x), x\right)$ and conclude that the equilibrium threshold $\tilde{x}$ chosen in the decentralized economy is typically inefficient. The first-order condition with respect to θ_j is:

$$\frac{\delta_j}{1-\eta_j(\theta_j^*(x))}\,\frac{\theta_j^*(x)}{\psi_j(\theta_j^*(x))}+\frac{\eta_j(\theta_j^*(x))}{1-\eta_j(\theta_j^*(x))}\theta_j^*(x)=\frac{y_j-\tau\,\Gamma_j\left(x\right)-b}{k_j} \tag{30}$$

where $\eta_j=\eta_j(\theta_j)=-\frac{\theta_j\mu_j'(\theta_j)}{\mu_j(\theta_j)}\in(0,1)$. In the standard matching literature, if the worker's bargaining power β_j happens to be equal to elasticity η_j, the surplus sharing rule (5) internalizes search-matching externalities. This equality is the already-mentioned Hosios condition (see Hosios, 1990, and Pissarides, 1990). Put differently, the decentralized economy is efficient (i.e. it maximizes net output) despite the presence of these externalities. In our setting, when the threshold x is taken as given, the average commuting cost $\Gamma_j(x)$ in each job center is fixed. The problem of determining the optimal level of tightness in each market takes the same form as in the basic matching model, with an additional cost that is exogenous. Therefore, as in the basic matching model, the Hosios condition ensures that the decentralized equilibrium generates the social optimal allocation. The next lemma - our second step - shows that the Hosios condition is necessary to decentralize the optimal allocation $\{\theta_A^*, \theta_B^*, x^*\}$.

Lemma 5. *The Hosios condition* $\beta_j=\eta_j(\theta_j^*(x^*))$, $\forall j\in\{A,B\}$, *is necessary to decentralize the optimal allocation, i.e. to guarantee that* $\tilde{\theta}_j=\theta_j^*$ *and* $\tilde{x}=x^*$.

Proof. Let $r\mapsto 0$. If the economy decentralizes the optimal allocation, then $\tilde{x}=x^*$. The RHS of Eqs. (19) and (30) being independent of tightness, they are identical. The unique decentralized level of tightness $\tilde{\theta}_j$ and the unique optimal one θ_j^* can only be equal if the LHS of Eqs. (19) and (30) are identical. This can only hold if the Hosios condition $\beta_j=\eta_j(\theta_j^*(x^*))$, $\forall j\in\{A,B\}$, is verified. □

The Hosios condition is only necessary and sufficient if the decentralized value of the threshold $\tilde{x}$, which solves Eq. (20), equals the efficient one x^*. Otherwise, the RHSs of (30) and of (19) are different. Consequently, we can get $\tilde{\theta}_j\neq\theta_j^*$ even though the Hosios condition is met.

Third, we turn to the optimality condition with respect to x. To this end, we get using (30):

$$k_j = (y_j - \tau\,\Gamma_j\,(x) - b)\,\frac{(1-\eta_j(\theta_j^*(x)))\;\mu_j\left(\theta_j^*(x)\right)}{\delta_j + \eta_j(\theta_j^*(x))\;\psi_j\left(\theta_j^*(x)\right)}$$

so, from Eq. (29)

$$\mathcal{W}_j\left(\theta_j^*(x), x\right) = (y_j - \tau\,\Gamma_j\,(x) - b)\,\frac{\eta_j(\theta_j^*(x))\;\psi_j\left(\theta_j^*(x)\right)}{\delta_j + \eta_j(\theta_j^*(x))\;\psi_j\left(\theta_j^*(x)\right)}. \tag{31}$$

Remembering (8) and (11), this way of expressing $\mathcal{W}_j$ makes clear that under the Hosios condition, $\beta_j = \eta_j\left(\theta_j^*(x)\right)$, the product $b + \mathcal{W}_j\left(\theta_j^*(x), x\right)$ is the expected discounted utility of an unemployed searching in j. The optimal threshold value x therefore maximizes the expected utility of an unemployed,[17] namely:

$$\begin{aligned}\Omega\left(\theta_A^*(x), \theta_B^*(x), x\right) = b &+ \int_0^x (y_A - \tau\,\zeta - b)\,\frac{\eta_A\left(\theta_A^*(x)\right)\;\psi_A\left(\theta_A^*(x)\right)}{\delta_A + \eta_A\left(\theta_A^*(x)\right)\;\psi_A\left(\theta_A^*(x)\right)}\;f(\zeta)\;d\zeta \\ &+ \int_x^1 (y_B - \tau\,(1-\zeta) - b)\,\frac{\eta_B\left(\theta_B^*(x)\right)\;\psi_B\left(\theta_B^*(x)\right)}{\delta_B + \eta_B\left(\theta_B^*(x)\right)\;\psi_B\left(\theta_B^*(x)\right)} f(\zeta) d\zeta.\end{aligned}$$

Let us define

$$\mathscr{E}_j(\theta) = \frac{\delta_j}{\left(\delta_j + \eta_j(\theta)\;\psi_j(\theta)\right)^2}\;\frac{\partial(\eta(\theta)\;\psi(\theta))}{\partial\theta}\;\left.\frac{\partial\theta_j}{\partial\Gamma_j}\right|_{\text{Eq. (30)}} < 0 \tag{32}$$

the marginal change of $\frac{\eta_j\left(\theta_j^*(x)\right)\;\psi_j\left(\theta_j^*(x)\right)}{\delta_j+\eta_j\left(\theta_j^*(x)\right)\;\psi_j\left(\theta_j^*(x)\right)}$ when Γ_j increases by one unit along Eq. (30). This term is negative as a rise in average transportation cost Γ_j decreases tightness. Appendix B shows that under the Hosios condition $\beta_j = \eta_j\left(\theta_j(x^*)\right)$, the optimal threshold x^* verifies:

$$\Sigma_A(\theta_A^*(x^*), \theta_B^*(x^*), x^*) + I_A(x^*) = \Sigma_B(\theta_A^*(x^*), \theta_B^*(x^*), x^*) + I_B(x^*) \tag{33}$$

where $I_j(x) = (z_j(x) - \Gamma_j(x))\,(y_j - \tau\Gamma_j(x) - b)\,\mathscr{E}_j\left(\theta_j^*(x)\right),\; j \in \{A, B\}$. Under the Hosios condition, $\Sigma_j\left(\theta_A^*(x), \theta_B^*(x), x\right)$ is nothing else than the expected return to search in location j in the decentralized economy $\Sigma_j(\Theta_A(x),\,\Theta_B(x), x)$ evaluated at the threshold x, where the Σ_j functions were defined by respectively (11) and (13), and $\Theta_j(x)$ yields tightness under free entry for any threshold x (see the proof of Lemma 3).

Each expression $I_j(x^*)$, henceforth I_j for short, has no reason to be nil unless $\eta_j \mapsto 0$,[18] which is a degenerate case where the number of vacancies has no influence on the number of hirings. If I_A and I_B happen to be equal, the Hosios condition is sufficient to guarantee the equality between the decentralized and the optimal triple $(\theta_A^*, \theta_B^*, x^*)$. This would be the case if the two job centers were identical and the distribution of the population was uniform on $[0, 1]$. In general, I_A and I_B have however no reason to be equal and hence the Hosios condition is not sufficient to guarantee that the steady-state equilibrium is efficient.

[17]As $r \mapsto 0$, this is also the expected utility of a member of the labor force.

[18]When $\eta \mapsto 0$ the term $\frac{\partial(\eta(\theta)\psi(\theta))}{\partial\theta} \mapsto 0$ (see (32)), hence $\mathscr{E}_j(\theta) \mapsto 0$.

The negative effects I_j omitted by decentralized agents have a clear interpretation. In the decentralized economy, the threshold location verifies Eq. (20), which expresses that the private gains $\mathscr{S}_j(x) = \Sigma_j(\Theta_A(x), \Theta_B(x), x)$, of searching in A and in B are equal in equilibrium. This indifference condition (20) overlooks that a change in the threshold affects the conditional expected commuting distance, Γ_j, of all workers and thereby the expected surplus accruing to employers. This in turn modifies the number of vacancies created in both job centers under free entry and eventually the levels of tightness. Finally, this change in both levels of tightness has an impact on the expected utility of all job-seekers. This externality is different from the standard search-matching externalities that are internalized under the Hosios condition. At the root of this additional externality, one finds the composition effect introduced in Subsection II.3. Since vacant jobs are specific to the job center, but accessible to individuals located anywhere and as workers' commuting costs are shared through the wage bargain, the expected profit of opening a vacancy shrinks when job-seekers further away enter the queue of unemployed seeking an occupation in the job center. We henceforth talk about the composition externality. This externality is made of two opposite effects $I_A < 0$ and $I_B < 0$. If $I_A < I_B$, Eq. (33) implies that at a social optimum, the return to search of the pivotal job-seeker (i.e. someone located in x^*) is higher in job center A than in B. Therefore, the social optimum needs to instruct some job seekers to search in market B rather than in A. In other words, in the decentralized economy too many job-seekers are searching for a job in business district A.

With a Cobb-Douglas matching function on each labor market, a popular functional form used in the numerical analysis below, η_j becomes a parameter. Then, as explained in Appendix B,

$$I_j(x) = -\tau \ (z_j(x) - \Gamma_j(x)) \left[\frac{\psi_j(\theta_j^*(x))}{\delta_j + \psi_j(\theta_j^*(x))} - \frac{\eta_j \ \psi_j(\theta_j^*(x))}{\delta_j + \eta_j \ \psi_j(\theta_j^*(x))} \right] < 0. \tag{34}$$

I_j is the opposite of the product of the positive difference between the marginal commuting cost $\tau \, z_j(x)$ and the (conditional) average one $\tau \, \Gamma_j(x)$ and a second positive difference, $\left[\frac{\psi_j(\theta_j^*(x))}{\delta_j + \psi_j(\theta_j^*(x))} - \frac{\eta_j \ \psi_j(\theta_j^*(x))}{\delta_j + \eta_j \ \psi_j(\theta_j^*(x))} \right]$, which depends on tightness $\theta_j^*(x)$ only. The latter difference is decreasing in tightness if and only if $\sqrt{\eta_j}\psi_j(\theta_j^*(x)) > \delta_j$,[19] which corresponds to an unemployment rate lower than $\frac{\sqrt{\eta_j}}{1+\sqrt{\eta_j}}$. For $\eta_j = 0.1$, this ratio corresponds to an unemployment rate of 24%, while for $\eta_j = 0.9$, it corresponds to an unemployment rate of 48%. Therefore, as most empirical analyses find a value of $\eta \in [0.4, 0.7]$, we can take for granted that the latter difference is decreasing in tightness. If $x - \Gamma_A(x)$ is increasing in x - which is not guaranteed - what we here learn is that the LHS of (33) is decreasing in x. Then if $1 - x - \Gamma_B(x)$ is decreasing in x,

[19] The derivative of $\frac{\psi}{\delta+\psi} - \frac{\eta \ \psi}{\delta+\eta \ \psi}$ with respect to ψ is $\frac{\delta}{(\delta+\psi)^2} - \frac{\eta \ \delta}{(\delta+\eta \ \psi)^2}$. This term is negative whenever $(\delta + \eta \ \psi)^2 < \eta \ (\delta + \psi)^2$ or $\sqrt{\eta}\psi > \delta$.

the RHS of (33) is increasing in x so that if (33) has a solution x^* it is unique.

This section can be summarized as follows:

Proposition 2. *The Hosios condition is necessary, but generically not sufficient to guarantee that the decentralized equilibrium is efficient. The decentralized threshold $\tilde{x}$ can be lower or above the efficient one x^* depending on the relative strength of the composition externalities in the two job centers.*

As explained above, a counter-example where the Hosios condition is sufficient is the case where the two centers are symmetric and the population is uniformly distributed.

IV Numerical analyses

In this section, we provide numerical simulations to quantify the effects found in the theoretical section. In particular, we illustrate how the shape of the distribution of the workforce influences the decentralized allocation and we look at the gap between the efficient and the decentralized allocations. In addition, we provide some evidence on the impacts of policies that change the commuting cost and the cost of opening vacancies. In the first part, we look at a uniform distribution combined with a parametrized mass of workers located in job center A. Next, we look at two U.S. Metropolitan Statistical Areas (MSA), Los Angeles and Chicago, and after calibrating the model for two major job centers of these MSAs we develop a counterfactual analysis with respect to the distribution of the workforce.

We assume Cobb Douglas matching functions, $M_j = m_j V_j^{1-\eta_j} U_j^{\eta_j}$, $j \in \{A, B\}$. The job finding rate and the rate of filling a vacancy respectively are:

$$\psi_j(\theta_j) = m_j \theta_j^{1-\eta_j} \quad (35)$$

$$\mu_j(\theta_j) = m_j \theta_j^{-\eta_j} \quad (36)$$

IV.1 A numerical illustration

In the first exercise we take symmetric job centers and we set the discount rate to zero. In both business districts, productivity y_j is normalized to 1, the value of leisure is normalized to $b = 0.4$ (Shimer, 2005), the quarterly separation rate in 2000[20] is $\delta_A = \delta_B = 0.03$ (Shimer, 2012) and we set $m_A = m_B = 1$. The vacancy costs k_j are set to match unemployment rates of 6%. We take the transportation cost $\tau = 0.4$ (Zenou, 2009b, p. 40). The Hosios condition is assumed by imposing $\beta_j = \eta_j = 0.5$. The labor force is distributed according to a uniform distribution whose total mass is $1 - \alpha_A$ and a mass point of $\alpha_A \in [0, 1)$ is located at $x = 0$. So, the CDF

[20] This data was constructed by Robert Shimer. For additional details, see Shimer (2007) and his webpage http://sites.google.com/site/robertshimer/research/flows.

is $F(x) = \alpha_A + (1-\alpha_A)x$, implying that as α_A tends to 1, the population becomes more and more concentrated at $x = 0$. The model is calibrated for $\alpha_A = 0$. The average commuting costs towards job centers A and B for $x \in (0,1)$ respectively are,

$$\Gamma_A(x) = \frac{(1-\alpha_A)\frac{x^2}{2}}{\alpha_A + (1-\alpha_A)x}, \quad \text{with } \frac{\partial \Gamma_A(x)}{\partial \alpha_A} < 0, \quad \text{and} \quad \Gamma_B(x) = \frac{1-x}{2}.$$

The simulation results in Fig. 4 depict how the allocation at the decentralized equilibrium (blue solid curves) and the optimal one (red dashed curves) are modified when the mass in job center A, α_A, exogenously increases from 0 to 1. The two allocations coincide when $\alpha_A = 0$ as the two job centers are symmetric and the workforce is uniformly distributed. Then, we know from the previous section that the Hosios condition guarantees efficiency. When the mass point α_A increases, vacant jobs in A have a larger probability of meeting job-seekers close to them. So, for any threshold value $\tilde{x}$ the conditional expected commuting distance $\Gamma_A(\tilde{x})$ decreases. Therefore, the schedule $\Theta_A(\tilde{x})$ shifts upwards when α_A rises and so does the expected returns to search in A, $\mathscr{S}_A(\tilde{x})$, in Fig. 3. On the contrary, the schedule $\Theta_B(\tilde{x})$ and hence $\mathscr{S}_B(\tilde{x})$ are unaffected by α_A. From (20) and Fig. 3, $\tilde{x}$ has to rise with α_A. The bottom left panel of Fig. 4 shows that this effect is however very small.

In the same panel, the optimal threshold x^* slightly *declines* with α_A. To see why, we need to understand how changes in α_A modify the I_j terms (34) in both job centers. Notice first that because the schedule $\Gamma_A(x)$ shifts downwards when α_A increases, the difference between the marginal and the average commuting distance to job center A, $x - \Gamma_A(x)$, increases whatever the value of x, while the corresponding term in B is not a function of α_A. Furthermore, the last term defining I_j shifts upwards for any x when α_A increases (because the schedule $\Theta_A(x)$ shifts upwards) while, again the corresponding term in B remains unaffected. For these two reasons, as α_A rises, the schedule $I_A(x)$ shifts downwards while nothing changes in B. So, compared to the decentralized equilibrium, the efficient threshold will be lower (more job-seekers should search in B instead of A). This does not however explain why the efficient x^* shrinks with α_A. Two opposite movements affect the left-hand side of (33): the one we have just explained and the upward shift of the schedule $\mathscr{S}_A$ discussed earlier. From the profile of x^* in the bottom left panel of Fig. 4, we deduct that the former slightly outweighs the latter. However, the difference between x^* and the the decentralized $\tilde{x}$ is not sizable.

As far as the decentralized equilibrium value of $\tilde{\theta}_A$ is concerned, two forces are here also at work. First, when α_A rises, for the reason explained earlier, the $\Theta_A(\tilde{x})$ schedule shifts upwards, inducing a rise in $\tilde{\theta}_A$ for any threshold $\tilde{x}$. However, the decentralized $\tilde{x}$ increases a bit. This in turn induces a decline in the decentralized value $\tilde{\theta}_A$ since $\Theta'_A(\tilde{x}) < 0$. Fig. 4 shows that the former effect dominates and triggers a sizable increase in $\tilde{\theta}_A$. In comparison, the gap between

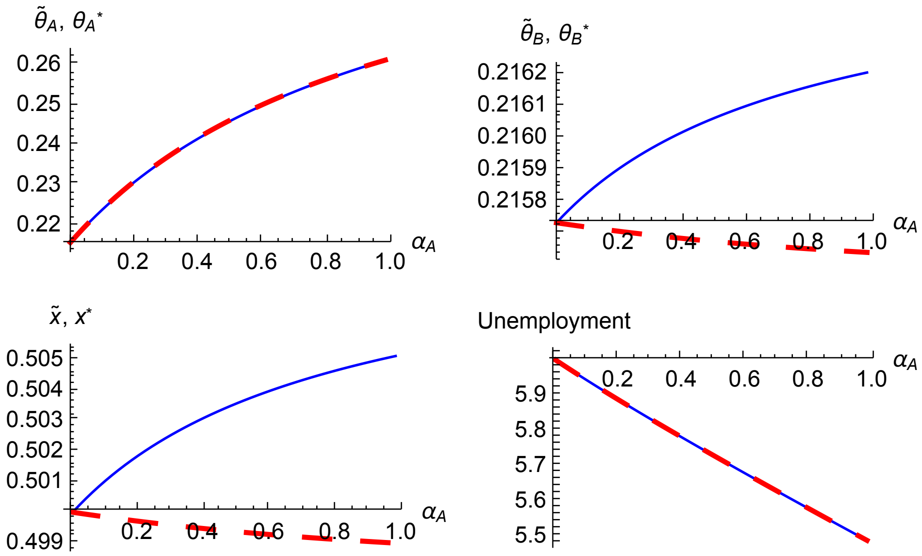


Fig. 4. Decentralized versus Optimal allocation

$\tilde{\theta}_A$ and θ_A^* looks negligible.

In job center B, only one mechanism is at work since the conditional expected commuting distance to B, $\Gamma_B(\tilde{x})$, is not affected by α_A. As $\tilde{x}$ rises with α_A, $\Gamma_B(\tilde{x})$ declines and hence the decentralized value $\tilde{\theta}_B$ increases as well, but the impact is very small. For a symmetric reason, the efficient value θ_B^* somewhat declines because x^* slightly declines with α_A. Both the decentralized and the efficient levels of the aggregate unemployment decrease with α_A, from 6% when $\alpha_A = 0$ to 5.5% when $\alpha_A = 1$. In sum, concentrating the population in job center A has some sizable effects on the decentralized and the efficient allocations, but the gap between these two is tiny.

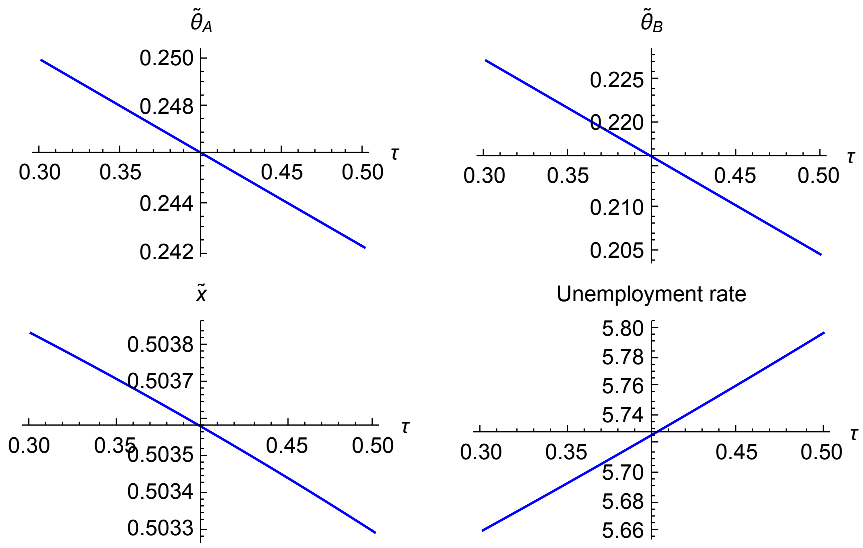


Fig. 5. Decentralized allocation when τ rises ($\alpha_A = 0.5$)

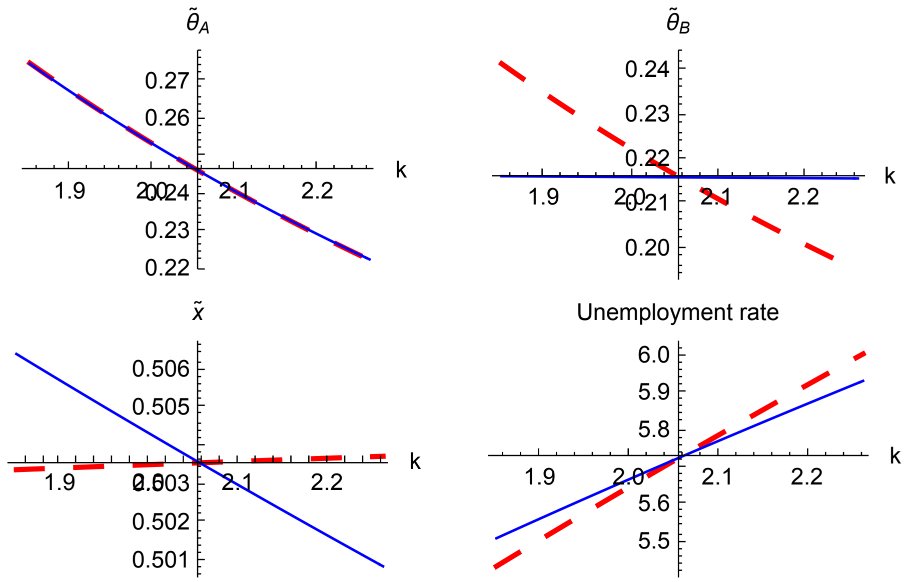


Fig. 6. Decentralized allocation when only k_A rises (the continuous lines) and when both k_j grow in exactly the same way (the interrupted lines); $\alpha_A = 0.5$

We now consider the impacts on the decentralized equilibrium of policies financed by a tax on rents (which has no allocative consequence in our framework). We set α_A to 0.5, but keep the other parameters at their calibrated values. Fig. 5 quantifies the effects when the unit commuting cost τ varies between 0.3 and 0.5. As the calibrated value is 0.4, we illustrate what happens when this cost is either subsidized or taxed. When τ rises, tightness declines in both job centers, but less so in A because of the mass α_A of workers who do not have to commute to A. The impact on the threshold $\tilde{x}$ is negligible and the aggregate unemployment rate rises slightly only.[21] Finally, we look at the effects of changes of the costs of opening a vacancy around their calibrated values (2.06). Fig. 6 displays the outcomes when the cost of opening a vacancy varies only in job center A (the continuous lines) and when both costs grow in exactly the same way (the dashed lines). Tightness $\tilde{\theta}_A$ declines almost identically in the two scenarios. Conversely, tightness $\tilde{\theta}_B$ is only affected when the vacancy cost k_B is also rising. The threshold $\tilde{x}$ is almost unaffected when both k_j's are growing and slightly declining when only k_A rises. In both cases the increase in the aggregate unemployment rate is non negligible.

In all these experiments, the two job centers were symmetric. Consider as an example the following asymmetry: α_A is set to 0.5, a change that favors job center A, and the marginal product of labor in B rises (say by 25%), which favors this center instead. The most striking implication is that the threshold $\tilde{x}$ shrinks to 0.18 while the aggregate unemployment rate declines to 5.3%. Then, rising both k_j's generates a pattern of adjustment that is qualitatively

[21]The latter conclusion is sensitive to the magnitude of α_A. When this parameter is set to 0, a rise of the commuting cost by 25% increases the aggregate unemployment rate by 0.2 percentage points.

the same as in the previous figure. Hence, we do not report those simulations.

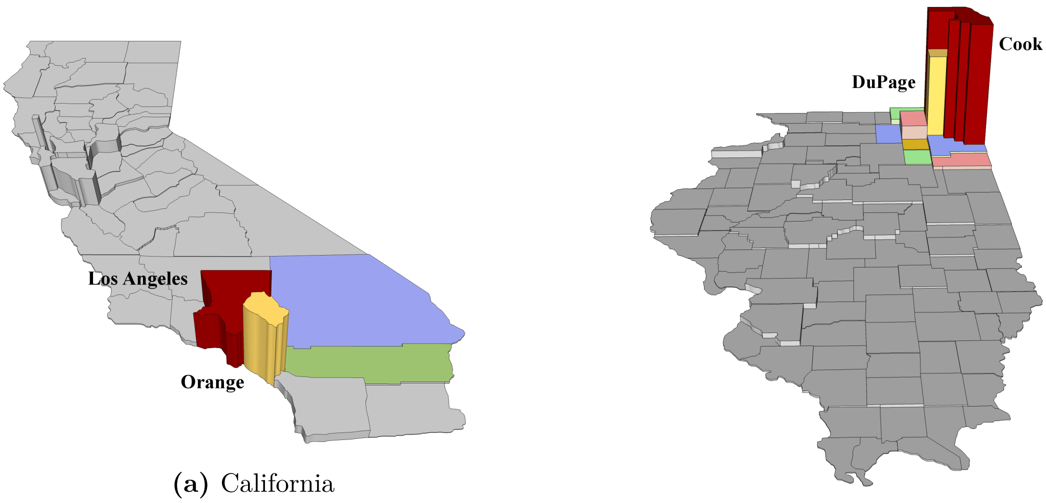
IV.2 Los Angeles and Chicago MSAs

This subsection focuses on the consequences of changing the shape of the distributions of the workforce. Here, the two job centers will be asymmetric.

IV.2.a The data

In the second part of the numerical exercise we calibrate our model with data on MSAs in the U.S. The leading MSAs, ranked by population, are also those with the highest mean travel time to work i.e. New York (34 minutes), Los Angeles (29 minutes) and Chicago (31 minutes) (see McGuckin and Srinivasan, 2003; Rapino et al., 2011). We calibrate the model using data from the U.S. 2000 census.

We have access to data of the total workforce, the number of employed, unemployed, and commuters at the zip code and county levels on the basis of the location of residence. Average wages and the number of employees are available only at the county level on the basis of the location of the job. We collect information for Los Angeles and Chicago MSAs,[22] which we respectively take as representative of "new" and "old" cities.[23]



(a) California

(b) Illinois

Fig. 7. Paid employees by county, 2000
(Thousands per sq mi)
Source: U.S. Census Bureau, Department of commerce.

We assume there is one job center per county and determine its size by the number of paid

[22]New York MSA's configuration is out of the scope of this model since it presents four important job centers on a row: New York, Queens, Nassau and Suffolk counties. San Francisco MSA is a multicentric MSA and hence out of the scope of our model.

[23]Old cities used to be the ten most populated in 1900, i.e. New York, Chicago, Philadelphia, Detroit, Boston, and San Francisco. In contrast, new cities like Los Angeles, Atlanta, Houston, Dallas, Miami, and Nassau-Suffolk had much smaller populations during that century.

employees per square mile.[24] Fig. 7 depicts the states of California and Illinois by county and each county's size is measured by the height on the map. According to the U.S. 2000 Census definition, Los Angeles MSA sprawls over the counties of Los Angeles, Orange, Riverside and San Bernardino, that are colored on Fig. 7a. Two job centers exceed the others in size: Los Angeles county (in red) and Orange county (in yellow) respectively account for 63% and 23% of the total paid employees in Los Angeles MSA. Chicago MSA is made of a number of counties that are colored on Fig. 7b.[25] The largest job centers in Chicago MSA, see Fig. 7b, are in Cook county (in red) with a share of 60% and DuPage county (in yellow) with 14% of paid employees.

Interestingly for our study the highway Route 5 links Los Angeles and Orange counties, see Fig. 8. We only take into account the active population with residence along this highway. In most cases Route 5 passes through a zip code, while in others it is at the border of two zip code areas, in which case we average their populations. Job center A or CBD is assumed to be located in Los Angeles city center and job center B or SBD in Santa Ana city center. They are separated by 33.9 miles. In Chicago MSA, we only consider the active population with residence along Routes 290 and 88, which connect Cook and DuPage counties, see Fig. 9. The CBD is assumed to be located in Chicago city center and the SBD in Naperville city center. They are separated by 33.6 miles. The labor force in each zip code area located between the specified job centers forms

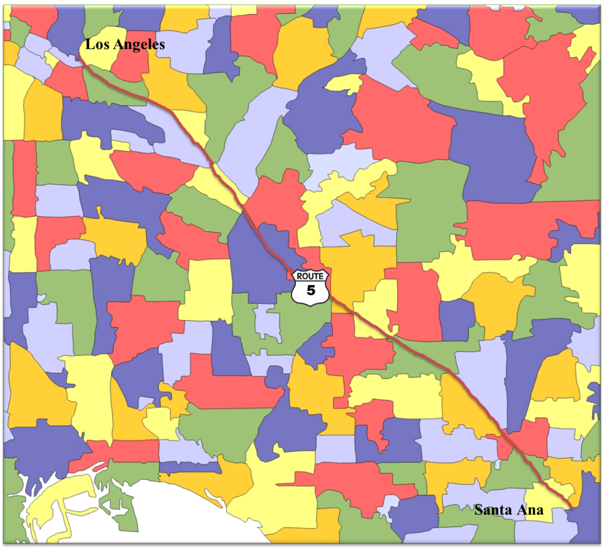


Fig. 8. California, job centers connected through Route 5
Source: http://www.zipmap.net/California.htm

a discrete workforce distribution, which we transform into a continuous density $f(x)$ and CDF

[24]Data available only at the county level. Source: U.S. Census Bureau, Department of Commerce, 2000 and the National Association of Counties.

[25]Chicago MSA spreads over the states of Illinois, Indiana (Lake and Porter counties) and Wisconsin (Kenosha county). The counties included from the state of Illinois are: Cook, DeKalb, DuPage, Grundy, Kane, Kendall, Lake, McHenry, Will and Kankakee. Metropolitan areas defined by the Office of Management and Budget, June 30th, 1999. Source: Population division, U.S. Census Bureau. Released online on July 1999.

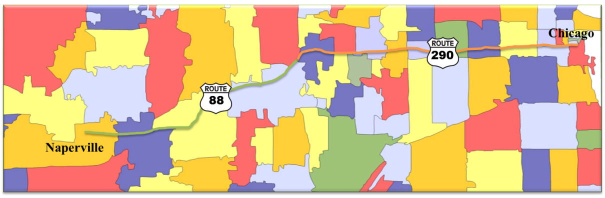
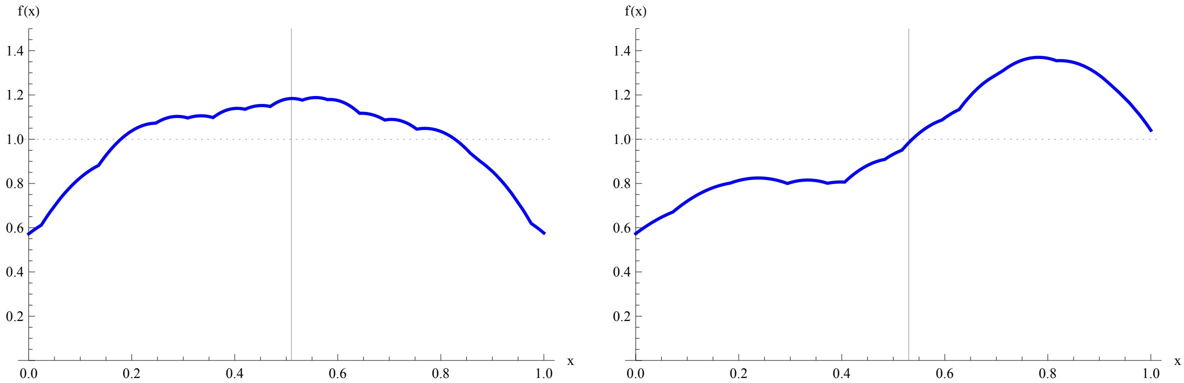


Fig. 9. Chicago, job centers connected through Route 88 and 290
Source: http://www.zipmap.net/Illinois.htm



(a) Los Angeles MSA **(b)** Chicago MSA

Fig. 10. Kernel distributions of the labor force

$F(x)$ on the segment $[0,1]$. For this purpose, for both MSAs, we estimate non parametrically the density using a Kernel with bandwidth with smoothing factor 5 which yields a bandwidth of 0.27 and 0.31 for Los Angeles and Chicago MSA, respectively.[26] The densities are displayed in Fig. 10. We denote x_0 as the geographical boundary between the two counties. Now that we have the continuous distribution F of the workforce with residence along the selected routes, we can fix x_0 such that $F(x_0)$ matches the observed share of the workforce living in the county on the left ("LC" for short). Then of course, $1-F(x_0)$ matches the share in the county to the right ("RC" for short). In Los Angeles MSA, Los Angeles (resp. Orange) county spreads over the segment $[0,0.51]$ (resp. $(0.51,1]$). In Chicago MSA, Cook (resp. DuPage) county spreads over the segment $[0,0.53]$ (resp. $(0.53,1]$). In Los Angeles MSA, see Fig. 10a, the population density is at its highest level on the border between counties, whereas at the extremes, i.e. $x=0$ and $x=1$, we observe the lowest densities. In Chicago MSA, however, Fig. 10b, the distributed is skewed to the right. We also observe an inverted-U-shape density within DuPage county. To summarize these differences by two numbers, $F(x_0)=0.5$ in Los Angeles MSA while it is close to 0.4 in the other one.

[26]We use a quadratic Kernel (Epanechnikov) $k_2(x) = \frac{3}{4}\left(1-x^2\right)$. We calculate the bandwidth using the Silverman rule-of-thumb: $h = 5\cdot\hat{\sigma}^2\cdot C_\nu(k)\cdot N^{-1/(2\nu+1)}$ where the bandwidth h, equals the product of the smoothing factor 5, times the sample standard deviation, $\hat{\sigma}^2$, a constant, $C_\nu(k)=2.34$, the sample N, and the order of the Kernel $\nu=2$.

IV.2.b The calibration

The parameters of the model are: y_j, δ_j, η_j, β_j, m_j for $j = \{A, B\}$, and r, b, and τ, the unknowns being $\tilde{\theta}_j$ and $\tilde{x}$. The reference year is 2000. We match the *means* of the unemployment rates in the selected zip codes along the indicated connecting routes respectively within the left and the right counties. Since average wages are only available for the year 2000 at the county level, we also match the average wages respectively in the left and the right counties.[27] We use a quarter as the unit of time. The real interest rate in the U.S. in 2000 was 4%,[28] thus we take $r = 0.98\%$. Following Petrongolo and Pissarides (2001) we choose an elasticity of the matching function $\eta_A = \eta_B = 0.5$, and as common practice we assume the Hosios condition, $\beta_A = \beta_B = 0.5$. Due to the small gap between unemployment rates in Los Angeles MSA we assume the scale factor of the matching function to be equal across job centers (see Table 2). Indeed, along Route 5, the average unemployment rate of the zip codes areas that belong to Los Angeles (resp. Orange) county is 8% (resp. 7%). For Chicago MSA, where the unemployment rate is 6 percentage points higher in Los Angeles County than in DuPage county, we assume $m_A < m_B$ (see Table 3). The unemployment insurance (UI) replacement rate in the states of California and Illinois is around 0.5 (Taylor, 2011).

As we observe different unemployment rates across counties, it is natural to think that different separation rates δ_j's might be part of the explanation. We do not have this information at the county level for the year 2000. However, we have found more recent data about the number of initial claims for UI at the county level (this includes new, additional, and transitional claims).[29] We select the year 2007[30] and compute the ratio between these initial claims in 2007 and employment in the same year for the counties under scrutiny. This proxy for separation rates is not exactly what we need to calibrate separation rates. So, we do not use their levels in each MSA. We only use the ratio between these proxies for the *county to the right* (RC) and the *county to the left* (LC). We set the separation rate to 0.036 (Pissarides, 2009) for the LCs (Los Angeles and Cook counties) and this ratio is only used to scale the separation rate in the RCs

[27] Wage estimates are calculated from data collected from employers in all industry divisions by Occupational Employer Statistics (OES), Bureau of Labor Statistics. The information. Source for California: Employment Development Department. Source for Illinois: The Workforce Information center.

[28] Source: Federal Reserve, Daily Treasury Real Long-Term Rates in 2000 (average).

[29] This data is provided by the State of California Employment Development Department and the Illinois Department of Employment Security. A *"new claim"* is the first claim for a benefit year period (e.g. for the regular UI program it is 52 weeks). An individual would only have one new claim during a benefit year period. An *"additional claim"* is when another claim is filed during the same benefit year and there is intervening work between the first claim and the second claim. An individual can have multiple additional claims during the same benefit year if she meets the eligibility requirements. A *"transitional claim"* is when a claimant is still collecting benefits at the end of their benefit year period and had sufficient wage earnings during that year to start up a new claim once the first benefit year period ends.

[30] This is the first year where the relevant information is available. Moreover the years 2000 and 2007 share similar economic conditions, both being characterized by an unemployment rate reaching a local minimum.

(Orange and DuPage counties). In both MSAs, this leads to $\delta_B < \delta_A$ (see Table 2 and Table 3).

Due to the importance of commuting by car,[31] to calibrate τ we first use the mileage reimbursement rate for privately owned automobile (POA). This information is provided by the U.S. General Service administration (GSA) and for the year 2000 it was calculated to be 0.325 USD/mile.[32] Second, τ takes into account the opportunity cost of time spent commuting. We find that the commuting times during peak hours between the two job centers are 60 and 53 minutes, respectively in Los Angeles and Chicago MSA. Since in Los Angeles (Chicago) MSA the average hourly wage is 16 USD (21 USD), the opportunity cost component of the commuting costs equals 27 USD (29 USD respectively). For the final commuting cost parameter we add the mileage reimbursement and the opportunity cost, double it to take into account a round trip and multiply it by 66 working days in a quarter.

Recall that for each MSA we aim to match in 2000 the average unemployment rates of the zip codes areas that belong to a county to the left and to the right, denoted $\bar{u}_{LC}$ and $\bar{u}_{RC}$, and the corresponding average wages, denoted $\bar{w}_{LC}$ and $\bar{w}_{RC}$.

Hence, for a given value of the threshold $\tilde{x}$, we have four unknowns: y_A, y_B, $\tilde{\theta}_A$ and $\tilde{\theta}_B$. We define a system of four equations according to the relative position of the boundary of the two counties (x_0) and the threshold ($\tilde{x}$). We equate the observed share[33] of employed workers in LC and respectively RC and the formulas coming from the model:

$$e_{LC} = \frac{\psi_A}{\delta_A + \psi_A} F(\underline{x}) + \frac{\psi_B}{\delta_B + \psi_B} \left(F(x_0) - F(\underline{x})\right) \tag{37}$$

$$e_{RC} = \frac{\psi_A}{\delta_A + \psi_A} \left(F(\overline{x}) - F(x_0)\right) + \frac{\psi_B}{\delta_B + \psi_B} \left(1 - F(\overline{x})\right) \tag{38}$$

where ψ_j stands for $\psi_j(\theta_j)$, $\underline{x} = \min\{x_0, \tilde{x}\}$ and $\overline{x} = \max\{x_0, \tilde{x}\}$. The system of four equations is then:

$$\bar{w}_{LC} = \int_0^{\underline{x}} w_A(x) \frac{\psi_A}{\delta_A + \psi_A} \frac{f(x)}{e_{LC}} dx + \int_{\underline{x}}^{x_0} w_B(x) \frac{\psi_B}{\delta_B + \psi_B} \frac{f(x)}{e_{LC}} dx \tag{39}$$

$$\bar{w}_{RC} = \int_{x_0}^{\overline{x}} w_A(x) \frac{\psi_A}{\delta_A + \psi_A} \frac{f(x)}{e_{RC}} dx + \int_{\overline{x}}^{1} w_B(x) \frac{\psi_B}{\delta_B + \psi_B} \frac{f(x)}{e_{RC}} dx \tag{40}$$

$$\bar{u}_{LC} = \frac{\psi_A}{\delta_A + \psi_A} \frac{F(\underline{x})}{F(x_0)} + \frac{\psi_B}{\delta_B + \psi_B} \frac{\left(F(x_0) - F(\underline{x})\right)}{F(x_0)} \tag{41}$$

$$\bar{u}_{RC} = \frac{\psi_A}{\delta_A + \psi_A} \frac{\left(F(\overline{x}) - F(x_0)\right)}{1 - F(x_0)} + \frac{\psi_B}{\delta_B + \psi_B} \frac{\left(1 - F(\overline{x})\right)}{1 - F(x_0)} \tag{42}$$

where $\frac{\psi_A}{\delta_A+\psi_A} \frac{f(x)}{e_{LC}}$ and $\frac{\psi_B}{\delta_B+\psi_B} \frac{f(x)}{e_{RC}}$ are the conditional employed population density in LC and

[31] In 2000, in Los Angeles MSA around 86% of the labor force resident in the zip code zones around Route 5 commuted by car. In Chicago MSA, for the same year, 80% of the labor force resident in zip code zones around Route 88 and 290 were car-commuters.

[32] It includes (i) gasoline and oil (excluding taxes), (ii) depreciation of original vehicle cost, (iii) maintenance, accessories, parts, and tires, (iv) insurance and (v) state and Federal taxes.

[33] Since in the model the total labor force is normalized to one, we match the share and not the level of employment.

RC, respectively. Then, Eq. (6) is used to express $w_j(x)$ in terms of unknowns and parameters (see Appendix C for more details).

We set the initial condition $\tilde{x} = x_0$[34] and then solve Eqs. (39) to (42). Next, we compute the two expected returns to search Σ_j respectively locations A and B, and according to the sign and the magnitude of the difference between the two Σ_j's, a new value of $\tilde{x}$ is computed and the system Eqs. (39) to (42) is solved again. This iterative procedure is applied until equality (14) is verified. Finally, we compute the cost of opening a vacancy, k_j, using the free entry condition (19).

From the calibration in Table 2, the threshold $\tilde{x}$ separating job seekers in two groups is very close to the boundary, x_0, between the two counties. We do not have data about commuters for the zip codes we consider. Still, information *at the county level* is worth to look at. According to the U.S. Census Bureau, 97% of work-commuters in Los Angeles and Orange counties travel within and between these counties.[35] In line with the calibration property $\tilde{x} \approx x_0$, the commuters' flow within each of these two counties is much higher than between them. The share of inner-county commuters in Los Angeles and Orange counties in 2000 was 96% and 85%, respectively. The higher average unemployment rate in Los Angeles county compared to Orange county is the consequence of a bigger separation rate, despite a slightly higher productivity level and lower unit cost of opening vacancies in job center A. All in all, tightness is higher in job center A than in the job center B.

In the calibration for Chicago MSA, Table 3, the threshold $\tilde{x}$ separating job seekers in two groups is again close to the boundary, x_0, between the two counties. According to the U.S. Census Bureau in 2000, 94% of work-commuters in Cook and DuPage counties travel within and between these counties. A 93% of work-commuters in Cook county are inner-county commuters. This is higher than in DuPage county where 65% of commuters travel to work within the county and the rest go to work in Cook county. The average unemployment rate is considerably higher in Cook county. This is first due to a higher separation rate. Next, a lower scale factor of the matching function in job center A (Cook county) and a much higher vacancy cost lead to lower tightness and lower probability of being recruited in job center A (Cook county) despite a higher productivity than in job center B (DuPage county).

[34]We have however checked that the calibration is robust to changes in this initial condition.

[35]Data of commuting patterns in the state of California, at the county level (U.S. Census Bureau).

Table 2: Calibration of Los Angeles MSA, quarterly data

	Value	Description	Source/Target
1. Parameters			
1.1. From the literature, data and assumptions			
r	0.98	Interest rate (%)	Federal Reserve
$\eta_A = \eta_B$	0.5	Matching fn. Elasticity	Petrongolo and Pissarides (2001)
$\beta_A = \beta_B$	0.5	Workers' bargaining power	Hosios condition $\eta_j = \beta_j$
b	$4,452$	Unemployment Insurance	Bureau of Labor Statistics
δ_A	0.036	Separation rate LC	Pissarides (2009)
δ_B	0.023	Separation rate RC	Data
$m_A = m_B$	0.5	Matching fn. scale factor of LC and RC	Unemployment rates
τ	$3,542$	Commuting cost (USD per unit of scaled distance)	U.S. General Service Administration and hourly wage
x_0	0.51	Boundary between Los Angeles - Orange counties	Data
$F(x_0)$	0.51	CDF for Los Angeles county	Data
1.2. Computed by the model (USD/quarter)			
y_A	$9,322$	Labor productivity LC	Eqs. (39) - (42)
y_B	$9,190$	Labor productivity RC	Eqs. (39) - (42)
k_A	$4,825$	Vacancy cost LC	Eqs. (39) - (42)
k_B	$9,340$	Vacancy cost RC	Eqs. (39) - (42)
2. Outcomes			
2.1. Matched labor market outcomes			
$\bar{w}_{LC}$	$8,964$	Average wages Los Angeles county (USD/quarter)	State of California EDD [b]
$\bar{w}_{RC}$	$8,843$	Average wages Orange county (USD/quarter)	State of California EDD [b]
$\bar{u}_{LC}$	8.18	Average unempl. rate Los Angeles county (%)	U.S. 2000 census
$\bar{u}_{RC}$	7.31	Average unempl. rate Orange county (%)	U.S. 2000 census
2.2. Endogenous variables computed by the model [a]			
$\tilde{x}$	0.52	Location of the marginal worker	Eq. (14)
$\tilde{\theta}_A$	0.65	Market tightness LC	Eq. (19)
$\tilde{\theta}_B$	0.33	Market tightness RC	Eq. (19)
$\psi_A\left(\tilde{\theta}_A\right)$	0.40	Exit rate of unempl. LC	Eq. (35)
$\psi_B\left(\tilde{\theta}_B\right)$	0.29	Exit rate of unempl. RC	Eq. (35)
$\mu_A\left(\tilde{\theta}_A\right)$	0.62	Vacancy filling rate LC	Eq. (36)
$\mu_B\left(\tilde{\theta}_B\right)$	0.87	Vacancy filling rate RC	Eq. (36)
$k_A/\mu_A\left(\tilde{\theta}_A\right)$	$7,801$	Exp. cost of opening a vacancy in LC	
$k_B/\mu_B\left(\tilde{\theta}_B\right)$	$10,770$	Exp. cost of opening a vacancy in RC	

[a] Different initial values for $\tilde{x}$ do not affect the calibrated results.
[b] Employment Development Department. www.ca.gov

Table 3: Calibration of Chicago MSA, quarterly data

	Value	**Description**	**Source/Target**
1. Parameters			
1.1. From the literature, data and assumptions			
r	0.98	Interest rate (%)	Federal Reserve
$\eta_A = \eta_B$	0.5	Matching fn. Elasticity	Petrongolo and Pissarides (2001)
$\beta_A = \beta_B$	0.5	Workers' bargaining power	Hosios condition $\eta_j = \beta_j$
b	$5,438$	Unemployment Insurance	Bureau of Labor Statistics
δ_A	0.036	Separation rate LC	Pissarides (2009)
δ_B	0.024	Separation rate RC	Data
m_A	0.6	Matching fn. scale factor of	Unemployment rate Cook county
m_B	0.8	LC and RC	Unemployment rate DuPage county
τ	$3,890$	Commuting cost (USD per unit of scaled distance)	U.S. General Service Administration and hourly wage
x_0	0.53	Boundary between Cook - DuPage counties	Data
$F(x_0)$	0.42	CDF for Cook county	Data
1.2. Computed by the model (USD/quarter)			
y_A	$11,686$	Labor productivity LC	Eqs. (39) - (42)
y_B	$10,841$	Labor productivity RC	Eqs. (39) - (42)
k_A	$11,056$	Vacancy cost LC	Eqs. (39) - (42)
k_B	$4,671$	Vacancy cost RC	Eqs. (39) - (42)
2. Outcomes			
2.1. Matched labor market outcomes			
$\bar{w}_{LC}$	$11,169$	Average wage Cook county (USD/quarter)	IDES [b]
$\bar{w}_{RC}$	$10,663$	Average wage DuPage county (USD/quarter)	IDES [b]
$\bar{u}_{LC}$	8.96	Unemployment rate Cook county (%)	U.S. 2000 census
$\bar{u}_{RC}$	3.19	Unemployment rate DuPage county (%)	U.S. 2000 census
2.2. Endogenous variables computed by the model [a]			
$\tilde{x}$	0.54	Location of the marginal worker	Eq. (14)
$\tilde{\theta}_A$	0.37	Market tightness LC	Eq. (19)
$\tilde{\theta}_B$	0.89	Market tightness RC	Eq. (19)
$\psi_A\left(\tilde{\theta}_A\right)$	0.37	Exit rate of unempl. LC	Eq. (35)
$\psi_B\left(\tilde{\theta}_B\right)$	0.75	Exit rate of unempl. RC	Eq. (35)
$\mu_A\left(\tilde{\theta}_A\right)$	0.98	Vacancy filling rate LC	Eq. (36)
$\mu_B\left(\tilde{\theta}_B\right)$	0.85	Vacancy filling rate RC	Eq. (36)
$k_A/\mu_A\left(\tilde{\theta}_A\right)$	$11,233$	Exp. cost of opening a vacancy in LC	
$k_B/\mu_B\left(\tilde{\theta}_B\right)$	$5,506$	Exp. cost of opening a vacancy in RC	

[a] Different initial values for $\tilde{x}$ do not affect the calibrated values.
[b] Illinois Department of Employment Security. www.illinois.gov

IV.2.c The counterfactual simulations

In Table 4 (resp., 5) we take the calibrated parameters of Tables 2 (resp., 3) and simulate the implications of substituting counterfactual distributions of the workforce. In Table 4 (resp., 5), column (1) reproduces the key endogenous indicators of Table 2 (resp., 3). In column (2), we swap the active population distribution between MSAs. Next, we look at the consequences of a uniform distribution in column (3). Finally, in columns (4) to (6) we do the same for three truncated normal distributions on a support $[0,1]$: A symmetric density ($\mathcal{N}(0.5, 0.15)$), a normal distribution positively skewed because of the truncation ($\mathcal{N}(0.25, 0.5)$), and a negatively skewed ($\mathcal{N}(0.75, 0.5)$). The latter density functions are illustrated in Fig. 11. As in the initial stage of the model, rents equalize the lifetime discounted utility in unemployment wherever one lives and since each individual occupies a dwelling whose size is normalized to unity, the driver of the above-mentioned changes in the distribution of the workforce is a modification in the (here exogenous) supply of housing.

Table 4: Simulations for Los Angeles MSA, quarterly data

		Counterfactual population distributions				
	Los Angeles MSA [a]	**Chicago MSA**	**Uniform**	**Normal $\mathcal{N}(mean, st.dev.)$ truncated at $(0,1)$** [b]		
	(From Table 2)		$unif(0,1)$	$\mathcal{N}(0.5, 0.15)$	$\mathcal{N}(0.25, 0.5)$	$\mathcal{N}(0.75, 0.5)$
	(1)	(2)	(3)	(4)	(5)	(6)
x_0	0.51	0.51	0.51	0.51	0.51	0.51
$F(x_0)$	0.51	0.40	0.51	0.53	0.63	0.40
$\bar{w}_{LC}$	8,964	8,964	8,961	8,988	8,962	8,969
$\bar{w}_{RC}$	8,843	8,837	8,839	8,865	8,847	8,838
$\bar{u}_{LC}$	8.18	8.15	8.10	8.65	8.10	8.26
$\bar{u}_{RC}$	7.31	7.21	7.23	7.72	7.36	7.23
$e_{RC} + e_{LC}$	92.25	92.41	92.32	91.78	92.18	92.36
$\tilde{x}$	0.518	0.518	0.518	0.582	0.520	0.517
$\tau \Gamma_A (\tilde{x})$	1,002	967	918	1,388	918	1,062
$\tau \Gamma_B (\tilde{x})$	933	833	853	1,307	983	858
$\tilde{\theta}_A$	0.65	0.66	0.67	0.58	0.67	0.64
$\tilde{\theta}_B$	0.33	0.34	0.34	0.30	0.33	0.34
$\psi_A \left(\tilde{\theta}_A\right)$	0.40	0.41	0.41	0.38	0.41	0.40
$\psi_B \left(\tilde{\theta}_B\right)$	0.29	0.29	0.29	0.27	0.29	0.29
$\mu_A \left(\tilde{\theta}_A\right)$	0.62	0.62	0.61	0.66	0.61	0.62
$\mu_B \left(\tilde{\theta}_B\right)$	0.87	0.85	0.86	0.92	0.87	0.86
$k_A/\mu_A \left(\tilde{\theta}_A\right)$	7,801	7,840	7,894	7,361	7,895	7,727
$k_B/\mu_B \left(\tilde{\theta}_B\right)$	10,770	10,925	10,894	10,173	10,692	10,877
$\Omega \left(\tilde{\theta}_A, \tilde{\theta}_B, \tilde{x}\right)$	7,749	7,818	7,825	7,398	7,781	7,769

[a] Different initial values for $\tilde{x}$ do not affect the final results of the calibration.
[b] A normal distribution that is truncated at 0 on the left and at 1 on the right is defined in density form as $f(x) = \frac{\phi(x) I(x)}{\Phi(1) - \Phi(0)}$ where ϕ (resp. Φ designates the Normal density (resp. cumulative density) function and $I_{(0,1)}(x) = 1$ if $0 \leqslant x \leqslant 1$, $I_{(0,1)}(x) = 0$ otherwise.

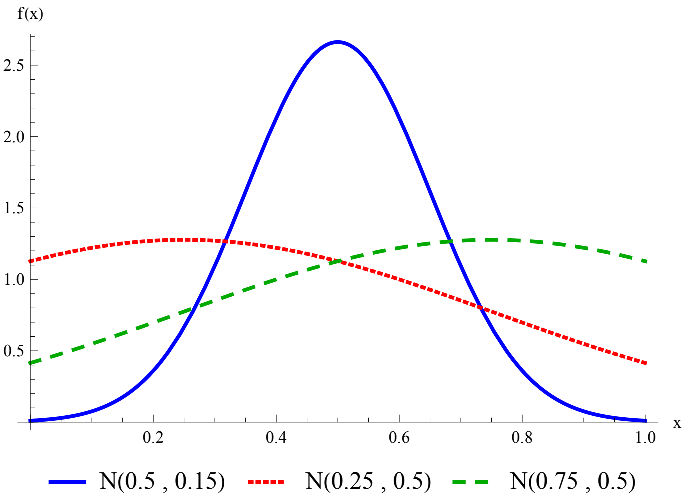


Fig. 11. Distributions for simulations Columns (4) to (6)

Substituting the counterfactual distribution of the workforce in Chicago into Los Angeles MSA consists of concentrating more population around job center B. Comparing columns (1) and (2) of Table 4 we observe that the threshold location $\tilde{x}$ does not change at the two-digit level. Tightness turns out to be higher in both job centers and net output is 1% higher when we take the workforce distribution of Chicago MSA. This is due to a lower average commuting cost towards both job centers. Therefore, firms get a higher surplus from a match and are induced to post more vacancies. This explains 0.16 percentage points rise in the MSA's employment rate and a decline in both average unemployment rates, especially $\bar{u}_{RC}$ in Orange county.

Since the actual distribution of Los Angeles MSA's workforce is not too far from uniform (see Fig. 10a), it is natural to consider this assumption in column (3) of Table 4. The same mechanisms as in column (2) are at work. Eventually, net output is 1% higher than in column (1). The most dramatic change appears in column (4) where we assume that the labor force is concentrated around the boundary between the two counties ($\mathcal{N}(0.5, 0.15)$). The substantial increase in $\tau\,\Gamma_j(\tilde{x})$ in both counties eventually leads to a drop in tightness levels by more than 10% and of net output by 4.5%. Lower tightness levels cause a decline in the total employment rate and a rise of both unemployment rates by nearly half a percentage point. Finally, the distribution $\mathcal{N}(0.75, 0.5)$ is interesting because it looks similar to the actual distribution in Chicago MSA. However, even if $F(x_0)$ is the same in both cases, the workforce near the CBD is less important (e.g. $F[0.25] = 0.18$ in column (2) versus 0.15 in column (6)). In addition in the RC (Orange county), the inversed-U shape profile of the workforce in the RC is more pronounced in column (2) than in column (6). These differences are sufficient to induce that, compared to the actual distribution in column (1), $\tau\,\Gamma_A(\tilde{x})$ rises in column (6) while it decreases in column (2). In column (6), $\tau\,\Gamma_B(\tilde{x})$ is lower than in column (1), but the decline is less pronounced than in column (2). These differences lead to lower tightness in job center A under the assumption $\mathcal{N}(0.75, 0.5)$ while it rises with the counterfactual distribution of Chicago MSA. Additionally, the average unemployment rate in the left county, $\bar{u}_{LC}$, is higher than in column (1) while it was lower with the Chicago distribution. So, limited differences in the distribution of the workforce turn out to have opposite effects on the average unemployment rates in the LC.

Comparing columns (1) and (2) of Table 5 we observe that differences of the outcomes of the model are small at the two-digit level (see $\tilde{x}$ and $\tilde{\theta}_A$). Substituting Los Angeles MSA workforce distribution implies however a drop in average wages (4.3% for the LC and 1.6% for the RC) and a rise in average unemployment rates (+0.15 percentage points in the LC and +0.08 in the RC). Moreover, the aggregate employment rate falls by 0.8 percentage points and net output is reduced by 0.6%. The more concentrated distribution around the boundary between counties explains these differences. Hence, higher average commuting costs towards both job centers diminish welfare. Comparing a uniform distribution with Chicago MSA's actual distribution of the active population, see Fig. 10b, one can expect a stronger effect in job center A since the workforce density of DuPage county is strictly below one. With equal density all along the MSA the average commuting costs to job center A (B) fall (rise) by 5.6% (1.6%). However, total employment rate and net output drop by 0.7 percentage points and 0.3%, respectively. We observe larger differences particularly in unemployment rates once we introduce a normal distribution positively or negatively skewed. The normal distribution of column (4) is specially worth considering since commuting costs to both job centers rise substantially. Consequently, net output shrinks by 4% and the average unemployment rate in the LC (resp., RC) is 0.6 (resp. 0.3) percentage points higher than in column (1). Finally, in column (6) net output drops by 0.6%. It should be noted that the changes in the distributions of the population introduced in Table 5 do not deeply change the strong unemployment rate gap between the two counties.

Table 5: Simulations for Chicago MSA, quarterly data

		Counterfactual population distributions				
	Chicago MSA[a]	**Los Angeles MSA**	**Uniform**	**Normal $\mathcal{N}(mean, st.dev.)$ truncated at $(0,1)$**[b]		
	(From Table 3)		$unif(0,1)$	$\mathcal{N}(0.5, 0.15)$	$\mathcal{N}(0.25, 0.5)$	$\mathcal{N}(0.75, 0.5)$
	(1)	(2)	(3)	(4)	(5)	(6)
x_0	0.53	0.53	0.53	0.53	0.53	0.53
$F(x_0)$	0.42	0.53	0.53	0.58	0.65	0.42
$\bar{w}_{LC}$	11,169	11,184	11,177	11,213	11,176	11,189
$\bar{w}_{RC}$	10,663	10,670	10,666	10,687	10,676	10,664
$\bar{u}_{LC}$	8.96	9.11	9.01	9.57	9.00	9.19
$\bar{u}_{RC}$	3.19	3.27	3.22	3.53	3.34	3.20
$e_{RC} + e_{LC}$	94.38	93.62	93.70	92.95	92.99	94.27
$\tilde{x}$	0.540	0.540	0.541	0.537	0.542	0.539
$\tau\,\Gamma_A(\tilde{x})$	1,114	1,146	1,052	1,569	1,046	1,220
$\tau\,\Gamma_B(\tilde{x})$	879	979	893	1,384	1,028	902
$\tilde{\theta}_A$	0.37	0.37	0.38	0.34	0.38	0.36
$\tilde{\theta}_B$	0.89	0.87	0.89	0.79	0.86	0.88
$\psi_A(\tilde{\theta}_A)$	0.37	0.36	0.37	0.35	0.37	0.36
$\psi_B(\tilde{\theta}_B)$	0.75	0.75	0.75	0.71	0.74	0.75
$\mu_A(\tilde{\theta}_A)$	0.98	0.99	0.98	1.04	0.98	1.00
$\mu_B(\tilde{\theta}_B)$	0.85	0.86	0.85	0.90	0.86	0.85
$k_A/\mu_A(\tilde{\theta}_A)$	11,233	11,193	11,308	10,667	11,315	11,103
$k_B/\mu_B(\tilde{\theta}_B)$	5,506	5,442	5,497	5,176	5,411	5,491
$\Omega(\tilde{\theta}_A, \tilde{\theta}_B, \tilde{x})$	9,714	9,660	9,744	9,278	9,718	9,660

[a] Different initial values for $\tilde{x}$ do not affect the final results of the calibration.
[b] A normal distribution that is truncated at 0 on the left and at 1 on the right is defined in density form as $f(x) = \frac{\phi(x)I(x)}{\Phi(1)-\Phi(0)}$ where ϕ (resp. Φ designates the Normal density (resp. cumulative density) function and $I_{(0,1)}(x) = 1$ if $0 \leqslant x \leqslant 1$, $I_{(0,1)}(x) = 0$ otherwise.

V Conclusions

Because of job decentralization in many Western cities, this research about the causes of unemployment fits in the literature about polycentric cities. Since the introduction of the spatial mismatch hypothesis in the U.S., the duocentric case is a frequently used framework that we also adopt. While the spatial mismatch hypothesis often focuses on the causes of the black and white divide, our article analyzes how the shape of distribution of the workforce living between the two job centers influences a number of equilibrium outcomes among which the unemployment rates. Apart from the location of residence, the workforce is here homogeneous.

We build a two-stage framework. First, perfectly mobile workers decide once and for all their location of residence and location-specific rents verify an indifference condition. Second, transactions occur on the labor market. During this second stage, jobs are perfectly mobile, job-seekers decide where to search and employed workers commute to the job center where they matched with a vacant position. This two-step structure seems to us less extreme than the assumption that individuals reconsider their location of residence at each transition on the labor market. We provide a sharp dynamic theoretical framework with a unique interior equilibrium under a small number of assumptions about the commuting cost. The equilibrium allocation is found by first solving the job-search problem and then characterizing the firms' labor demand choices. The search-matching frictions, the distribution of commuting distances and the wage bargain shape the payoffs associated with these decisions. The comparative static analysis leads to very intuitive intermediate results but, given the number of endogenous variables, the net effects are often ambiguous.

This paper emphasizes that commuters create an externality on job creation. Because wages compensate partly for commuting costs, the expected value of opening a vacancy in a job center shrinks when the pool of job-seekers spreads over longer distances. When jobless people decide where to seek jobs, they do not internalize that their decision affects job creation and hence has an impact on all the unemployed. Because of this composition externality, we show that the regional unemployment rates are typically inefficient even under the Hosios condition.

We conduct two numerical analyses. First, we assume symmetric job centers and impose that the distribution of the workforce is a mixture of a uniform distribution and a mass of workers located in one of the two job centers. We let this mass grow from 0 to 1 in order to see the consequences of a progressive disappearance of the spatial dimension (the limit case being a standard matching framework where jobs and workers are concentrated in a point in space). Simulations show first that concentrating the population in a unique job center has sizable effects on the decentralized and the efficient allocations (e.g. the aggregate unemployment rate shrinks from 6 to 5.5% when the mass varies varies from 0 to 1). So, remarkably, with symmetric job centers, different outcomes emerge from changing only the distribution of the workforce. Second, the quantitative gap between these two allocations is tiny when the Hosios condition applies. As a complement, we look at the impacts on the decentralized equilibrium of policies financed by a tax on rents (which has no allocative consequence in our framework). A striking conclusion is that subsidizing or taxing the commuting cost does not affect much the decentralized allocation. For instance, a rise of the commuting cost by 25% does nearly not affect job-search decisions and increases the aggregate unemployment rate by at most 0.2 percentage points (the calibrated value being 6%).

Second, we calibrate the model for Los Angeles and Chicago MSAs in 2000. We simulate the impacts of substituting counterfactual distributions of the workforce. It turns out that the location of the population has non-negligible effects on unemployment rates, wages, and net output. Within the range of distributions we have considered for Los Angeles, changes in unemployment rates (respectively, in net output) can reach half a percentage point (resp., 5%). In Chicago, where unemployment rates were more heterogeneous, the order of magnitude is similar. The range of population distributions considered in this article does however not deeply affect the spatial disparities in unemployment rates.

With the assumed commuting costs, the bottom line of these numerical exercises is that a reshaping of the distribution of the workforce has non-negligible effects on the equilibrium allocations, but cannot be *the unique* explanation of a substantial spatial mismatch problem. Moreover, under the Hosios condition, the composition externality is not a sizable source of inefficiency.

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A Comparative statics

We define $\psi'_j(\theta_j) = (1-\eta_j(\theta_j))\,\psi_j(\theta_j)/\theta_j$, where η_j is the elasticity of the matching rate $\mu_j(\theta_j)$ (i.e. $\psi_j(\theta_j)/\theta_j$) with respect to tightness.

I.1 Equilibrium tightness for a given level of the threshold $\tilde{x}$

For $j \in \{A, B\}$ we can rewrite Eq. (19) as $\mathcal{F}_j(\theta_j, \tilde{x}) = 0$ where we define

$$\mathcal{F}_j(\theta_j, \tilde{x}) = \frac{\theta_j k_j}{\psi_j(\theta_j)} - (1-\beta_j)\frac{y_j - \tau\,\Gamma_j(\tilde{x}) - b}{r + \delta_j + \beta_j\,\psi_j(\theta_j)}$$

Let ζ_j denote any of the parameters in $\{k_j, \delta_j, y_j, \beta_j, b, r\}$. Using the implicit function theorem,

$$\frac{\partial\theta_j}{\partial\zeta_j} = -\frac{\partial\mathcal{F}_j/\partial\zeta_j}{\partial\mathcal{F}_j/\partial\theta_j} \tag{A.1}$$

where $\partial\theta_j/\partial\zeta_j$ can also be written $\partial\Theta_j(\tilde{x})/\partial\zeta_j$ (see the proof of Lemma 3) and in which

$$\frac{\partial\mathcal{F}_j}{\partial\theta_j} = \frac{k_j}{\psi_j(\theta_j)}\frac{\eta_j(\theta_j)(r+\delta_j) + \beta_j\psi_j(\theta_j)}{r + \delta_j + \beta_j\psi_j(\theta_j)} > 0.$$

The sign of $\partial\theta_j/\partial\zeta_j$ is therefore given by the one of $\partial\mathcal{F}_j/\partial\zeta_j$. So, along $\mathcal{F}_j(\theta_j, \tilde{x}) = 0$,

- $\frac{\partial\theta_j}{\partial k_j} < 0$, because $\frac{\partial\mathcal{F}_j}{\partial k_j} = \frac{\theta_j}{\psi_j(\theta_j)} > 0$.
- $\frac{\partial\theta_j}{\partial\delta_j} = \frac{\partial\theta_j}{\partial r} < 0$, because $\frac{\partial\mathcal{F}_j}{\partial\delta_j} = \frac{1}{r+\delta_j+\beta_j\psi_j(\theta_j)}\frac{k_j\theta_j}{\psi_j(\theta_j)} > 0$.
- $\frac{\partial\theta_j}{\partial y_j} > 0 > \frac{\partial\theta_j}{\partial b}$, because $\frac{\partial\mathcal{F}_j}{\partial y_j} = -\frac{\partial\mathcal{F}_j}{\partial b} = -\frac{1-\beta_j}{r+\delta_j+\beta_j\psi_j(\theta_j)} < 0$.
- $\frac{\partial\theta_j}{\partial\beta_j} < 0$, because $\frac{\partial\mathcal{F}_j}{\partial\beta_j} = \frac{r+\delta_j+\psi_j(\theta_j)}{(1-\beta_j)(r+\delta_j+\beta_j\psi_j(\theta_j))}\frac{k_j\theta_j}{\psi_j(\theta_j)} > 0$.

I.2 Equilibrium threshold

By totally differentiating Eq. (20), on gets:

$$\frac{d\tilde{x}}{d\zeta_j} = \frac{\frac{d\mathscr{S}_B(\tilde{x})}{d\zeta_j} - \frac{d\mathscr{S}_A(\tilde{x})}{\partial\zeta_j}}{\frac{d\mathscr{S}_A(\tilde{x})}{d\tilde{x}} - \frac{d\mathscr{S}_B(\tilde{x})}{d\tilde{x}}} \tag{A.2}$$

where ζ_j denotes any of the parameters in job center j. By Lemma 4 the denominator of Eq. (A.2) is negative, and

$$\frac{d\mathscr{S}_j}{d\zeta_{j'}} = \frac{\partial\Sigma_j}{\partial\zeta_{j'}} + \frac{\partial\Sigma_j}{\partial\theta_j}\frac{\partial\Theta_j}{\partial\zeta_{j'}} \tag{A.3}$$

is nil when $j \neq j'$ except for $\zeta_{j'} \in \{r, b\}$. By Lemma 1, we know that $\partial\Sigma_j/\partial\theta_j > 0$. So, in order to study the sign of the numerator of Eq. (A.2) we need to sign the second term on the right-hand side of (A.3):

- $\frac{\partial\Sigma_j}{\partial k_j} = 0.$

- $\frac{\partial \Sigma_j}{\partial \delta_j} = \frac{\partial \Sigma_j}{\partial r} = -\Sigma_j \, \frac{1}{r+\delta_j+\beta_j\,\psi_j(\theta_j)} < 0.$
- $\frac{\partial \Sigma_j}{\partial y_j} = -\frac{\partial \Sigma_j}{\partial b} = \frac{\beta_j\,\psi_j(\theta_j)}{r+\delta_j+\beta_j\,\psi_j(\theta_j)} > 0.$
- $\frac{\partial \Sigma_j}{\partial \beta_j} = \Sigma_j \, \frac{r+\delta_j}{\beta_j\,[r+\delta_j+\beta_j\,\psi_j(\theta_j)]} > 0.$

The comparative static analysis is summarized in Table 6.

Table 6: Comparative statics

$\zeta_{j'}$	$\frac{\partial \mathscr{S}_B}{\partial \zeta_{j'}} =$	$\frac{\partial \Sigma_B}{\partial \theta_B}$	$\frac{\partial \Theta_B}{\partial \zeta_{j'}}$	$+\frac{\partial \Sigma_B}{\partial \zeta_{j'}}$	$\frac{\partial \mathscr{S}_A}{\partial \zeta_{j'}} =$	$\frac{\partial \Sigma_A}{\partial \theta_A}$	$\frac{\partial \Theta_A}{\partial \zeta_{j'}}$	$+\frac{\partial \Sigma_A}{\partial \zeta_{j'}}$	$\frac{\partial \mathscr{S}_B}{\partial \zeta_j} - \frac{\partial \mathscr{S}_A}{\partial \zeta_j}$	$\frac{d\tilde{x}}{d\zeta_j}$
k_A	0	+	0	0	−	+	−	0	+	−
δ_A	0	+	0	0	−	+	−	−	+	−
y_A	0	+	0	0	+	+	+	+	−	+
β_A	0	+	0	0	?†	+	−	+	?	?
b	−	+	−	−	−	+	−	−	?	?
r	−	+	−	−	−	+	−	−	?	?
k_B	−	+	−	0	0	+	0	0	−	+
δ_B	−	+	−	−	0	+	0	0	−	+
y_B	+	+	+	+	0	+	0	0	+	−
β_B	?†	+	−	+	0	+	0	0	?	?

† It can be checked that $\frac{\partial \mathscr{S}_j}{\partial \beta_j} \gtreqless 0 \Leftrightarrow \eta_j \gtreqless \beta_j$.

Given that $\frac{\partial \mathscr{S}_j}{\partial \beta_j} \gtreqless 0 \Leftrightarrow \eta_j \gtreqless \beta_j$, the signs of the partial derivatives with respect to the workers' bargaining power verify:

$$\frac{d\tilde{x}}{d\beta_A} \gtreqless 0 \Leftrightarrow \eta_A \gtreqless \beta_A \qquad \text{and} \qquad \frac{d\tilde{x}}{d\beta_B} \gtreqless 0 \Leftrightarrow \eta_B \lesseqgtr \beta_B.$$

So, an ambiguity remains for $d\tilde{x}/db$ and $d\tilde{x}/dr$ only.

I.3 Equilibrium tightness

The total effect of a marginal change in parameter ζ_j on equilibrium tightness in j is

$$\frac{d\theta_j}{d\zeta_j} = \frac{\partial \theta_j}{\partial \zeta_j} + \frac{\partial \Theta_j}{\partial \tilde{x}}\,\frac{d\tilde{x}}{d\zeta_j}, \qquad j \in \{A, B\} \tag{A.4}$$

where on the right-hand side, the first term is given by (A.1), the second one equals $-\left(\partial \mathcal{F}_j/\partial \tilde{x}\right) / \left(\partial \mathcal{F}_j/\partial \theta_j\right)$ (negative for $j = A$ and positive for $j = B$), while the third one is given by by (A.2). This total effect (A.4) has an ambiguous sign for all parameters.

B Efficient value of the threshold x^*

The first-order condition of the maximization of $\Omega\left(\theta_A^*(x), \theta_B^*(x), x\right)$ with respect to the threshold x writes:

$$\begin{aligned}
&(y_A - \tau x - b)\,\frac{\eta_A\left(\theta_A^*(x)\right)\ \psi_A\left(\theta_A^*(x)\right)}{\delta_A + \eta_A\left(\theta_A^*(x)\right)\ \psi_A\left(\theta_A^*(x)\right)} + \frac{F(x)}{f(x)}\ \Gamma_A'(x)\ \mathscr{I}_A\left(x, \theta_A^*(x)\right) \\
=\ &(y_B - \tau(1-x) - b)\,\frac{\eta_B\left(\theta_B^*(x)\right)\ \psi_B\left(\theta_B^*(x)\right)}{\delta_B + \eta_B\left(\theta_B^*(x)\right)\ \psi_B\left(\theta_B^*(x)\right)} - \frac{1-F(x)}{f(x)}\ \Gamma_B'(x)\ \mathscr{I}_B\left(x, \theta_B^*(x)\right)
\end{aligned} \tag{B.1}$$

in which $\mathscr{I}_j\left(x, \theta_j^*(x)\right) = (y_j - \tau\Gamma_j(x) - b)\,\mathscr{E}_j\left(\theta_j^*(x)\right)$. Under the Hosios condition and $r \mapsto 0$, the first terms on both sides of Eq. (B.1) equal $\Sigma_j\left(\theta_A^*(x), \theta_B^*(x), x\right)$ with $j = A$ on the LHS and $j = B$ on the RHS (See respectively (11) and (13)). Eq. (B.1) also depends on how sensitive the conditional expected commuting distance $\Gamma_j(x)$ is to the value of the threshold x and on the mass of individuals $F(x)$ (respectively, $1 - F(x)$). Remembering the value of the derivatives $\Gamma_j'(x)$ in (17) and (18), the first-order condition (B.1) can be rewritten as:

$$\begin{aligned}
\Sigma_A\left(\theta_A^*(x), \theta_B^*(x), x\right) + (x - \Gamma_A(x))\,\mathscr{I}_A\left(x, \theta_A^*(x)\right) =&\,\Sigma_B\left(\theta_A^*(x), \theta_B^*(x), x\right) \\
&+ (1 - x - \Gamma_B(x))\,\mathscr{I}_B\left(x, \theta_B^*(x)\right)
\end{aligned} \tag{B.2}$$

Expression (B.2) can be reformulated as (33).

With a Cobb-Douglas matching function on each labor market, differentiating Eq. (30) gives:

$$\left.\frac{\partial\theta_j}{\partial\Gamma_j}\right|_{\text{Eq. (30)}} = -\frac{\tau}{k_j}\ \frac{1-\eta_j}{\eta_j}\ \frac{\psi_j(\theta_j)}{\delta_j + \psi_j(\theta_j)}$$

Using Eq. (30):

$$(y_j - \tau\Gamma_j(x) - b))\left.\frac{\partial\theta_j}{\partial\Gamma_j}\right|_{\text{Eq. (30)}} = -\tau\ \frac{\theta_j}{\eta_j}\ \frac{\delta_j + \eta_j\ \psi_j(\theta_j)}{\delta_j + \psi_j(\theta_j)}$$

Hence, by (32), $\mathscr{I}_j(x, \theta_j^*(x))$ can be rewritten as:

$$-\tau\frac{(1-\eta_j)\delta_j\psi_j(\theta_j^*(x))}{\left(\delta_j + \eta_j\psi_j(\theta_j^*(x))\right)\left(\delta_j + \psi_j(\theta_j^*(x))\right)} = -\tau\left[\frac{\psi_j(\theta_j^*(x))}{\delta_j + \psi_j(\theta_j^*(x))} - \frac{\eta_j\psi_j(\theta_j^*(x))}{\delta_j + \eta_j\psi_j(\theta_j^*(x))}\right].$$

This leads to (34).

C Computation of the wage equations for the program

Using the wage Eq. (6), Eqs. (8), (37) and (38) respectively become:

$$
\begin{aligned}
\bar{w}_{LC} &= \int_0^{\underline{\mathsf{x}}} w_A(x)\,\frac{\psi_A}{\delta_A+\psi_A}\,\frac{f(x)}{e_{LC}}dx + \int_{\underline{\mathsf{x}}}^{x_0} w_B(x)\,\frac{\psi_B}{\delta_B+\psi_B}\,\frac{f(x)}{e_{LC}}dx \\
&= \frac{1}{e_{LC}}\left\{\frac{\psi_A}{\delta_A+\psi_A}\int_0^{\underline{\mathsf{x}}} w_A(x)f(x)dx + \frac{\psi_B}{\delta_B+\psi_B}\int_{\underline{\mathsf{x}}}^{x_0} w_B(x)f(x)dx\right\} \\
&= \frac{1}{e_{LC}}\left\{\frac{\psi_A}{\delta_A+\psi_A}\left[\left(\beta_A y_A + (1-\beta_A)\left(b+\frac{\beta_A\psi_A(y_A-b)}{r+\delta_A+\beta_A\psi_A}\right)\right)F(\underline{\mathsf{x}}) + \frac{(1-\beta_A)(r+\delta_A)}{r+\delta_A+\beta_A\psi_A}\,\tau\int_0^{\underline{\mathsf{x}}} xf(x)dx\right]\right. \\
&\left. +\frac{\psi_B}{\delta_B+\psi_B}\left[\left(\beta_B y_B + (1-\beta_B)\left(b+\frac{\beta_B\psi_B(y_B-b)}{r+\delta_B+\beta_B\psi_B}\right)\right)(F(x_0)-F(\underline{\mathsf{x}})) + \frac{(1-\beta_B)(r+\delta_B)}{r+\delta_B+\beta_B\psi_B}\,\tau\int_{\underline{\mathsf{x}}}^{x_0}(1-x)f(x)dx\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
\bar{w}_{RC} &= \int_{x_0}^{\overline{\mathsf{x}}} w_A(x)\,\frac{\psi_A}{\delta_A+\psi_A}\,\frac{f(x)}{e_{RC}}dx + \int_{\overline{\mathsf{x}}}^{1} w_B(x)\,\frac{\psi_B}{\delta_B+\psi_B}\,\frac{f(x)}{e_{RC}}dx \\
&= \frac{1}{e_{RC}}\left\{\frac{\psi_A}{\delta_A+\psi_A}\int_{x_0}^{\overline{\mathsf{x}}} w_A(x)f(x)dx + \frac{\psi_B}{\delta_B+\psi_B}\int_{\overline{\mathsf{x}}}^{1} w_B(x)f(x)dx\right\} \\
&= \frac{1}{e_{RC}}\left\{\frac{\psi_A}{\delta_A+\psi_A}\left[\left(\beta_A y_A + (1-\beta_A)\left(b+\frac{\beta_A\psi_A(y_A-b)}{r+\delta_A+\beta_A\psi_A}\right)\right)(F(\overline{\mathsf{x}})-F(x_0)) + \frac{(1-\beta_A)(r+\delta_A)}{r+\delta_A+\beta_A\psi_A}\,\tau\int_{x_0}^{\overline{\mathsf{x}}} xf(x)dx\right]\right. \\
&\left. +\frac{\psi_B}{\delta_B+\psi_B}\left[\left(\beta_B y_B + (1-\beta_B)\left(b+\frac{\beta_B\psi_B(y_B-b)}{r+\delta_B+\beta_B\psi_B}\right)\right)(1-F(\overline{\mathsf{x}})) + \frac{(1-\beta_B)(r+\delta_B)}{r+\delta_B+\beta_B\psi_B}\,\tau\int_{\overline{\mathsf{x}}}^{1}(1-x)f(x)dx\right]\right\}
\end{aligned}
$$