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Abstract

This paper studies the implications of changes in the fiscal (spending-tax) policy mix when all categories of spending and taxes are according to their functional breakdown. In so doing, we build a general equilibrium OLG model which naturally incorporates the main functional categories of public spending and taxes as in the euro area. Departing from the crisis year of 2008, the main result is that an increase in public spending on education and health would have outperformed all other changes in fiscal policy that have either happened or have been debated in policy circles.

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1 Introduction

EU policymakers have repeatedly stressed the need to reconsider the composition of public spending and taxes so as to create the right incentives and move the economy to more efficient outcomes in the medium-and long-term (see e.g. European Commission, 2008, part III). In this paper, we revisit this classic problem. We study how permanent changes in the spending-tax policy mix affect private incentives and, in turn, macroeconomic outcomes in the medium- and long-term. The point of departure is the fiscal policy mix in the euro area when the crisis erupted in 2008.

Our work differs from most of the previous literature on the effects of fiscal policy (see section 2 below for related papers), first, because we include all main categories of public spending and taxes in a unified general equilibrium framework and, second, because the breakdown of both public spending and taxes is in terms of their function. A functional decomposition of public spending means that we distinguish among spending on social protection, health, education, defense and public order, etc. Similarly, we distinguish among taxes on capital, labor and consumption. Such a functional decomposition of policy instruments, especially on the expenditure side, allows us to understand better the behavioural channel through which changes in policy affect private incentives and the macro economy.¹

The vehicle of analysis is a general equilibrium OLG model that includes the most important components of public spending and tax revenues in the data. In particular, we include spending on social protection (e.g. pensions), health, general public services (e.g. interest payments), education, economic affairs (e.g. infrastructure) and defense and public-order safety, while, on the taxation side, we have capital, labor and consumption taxes. These are the most important functional categories of spending and taxes as listed in the Eurostat (2012) dataset (see also Table 1 below). Our model accommodates these categories of public spending in a natural way. For instance, in our model, public spending on health increases the probability of reaching the old age, public spending on education contributes to the accumulation of individual human capital of younger generations, public spending on economic affairs improves infrastructure and benefits the productivity of firms, public spending on social protection provides pensions to the old generation via a PAYG system, public spending on defence and public order-safety can protect property rights, etc. To focus on the economy's growth and welfare potential, we abstract from uncertainty so that all dynamics is driven by policy reforms.²

The constructed model is solved numerically using conventional parameter values and fiscal data from the euro area over 2001-2008. Then, the steady state solution of this model, called the status quo, will serve

¹It should be noticed that usually there is an asymmetry between modeling public spending and modeling taxes. While taxes are decomposed by their function (e.g. taxes on labor income, taxes on capital income and taxes on consumption), different categories of public spending are usually modelled according to their economic transaction (e.g. transfer payments, public consumption, public investment) rather than according to their function (e.g. spending on social protection, health, education, defense and public order, etc). But, as is widely recognized, it is not clear what we mean, for instance, by public consumption or transfer payments (see e.g. European Commission, 2008, p. 139). This asymmetry leaves scope for improvement since the behavioural channel - through which changes in the mix of public spending affect private incentives and the macro economy - is not well specified.

²That is, here we do not study counter-cyclical policies aiming at short-term stabilization. Nor we compare alternative public debt consolidation strategies.

as a point of departure to study the implications of various reforms. By reforms, we mean ad hoc permanent changes in the spending-tax policy mix. We study three sets of policy reforms. First, those that resemble policy measures actually taken in the euro area in the aftermath of the 2008 financial crisis (see e.g. Coenen et al., 2013). In particular, we study a rise in public spending on social protection transfers, a rise in public spending on economic affairs (as said, this is mainly public infrastructure spending), a cut in consumption taxes and a cut in labor taxes. Second, reforms that may not have actually happened but have been debated in policy circles. For instance, an horizontal cut in all types of public spending (typically supported by conservative administrations) or a rise in capital taxes (typically supported by socialist administrations). Third, some reforms that have neither happened nor debated but deserve to be studied given the broader set of fiscal instruments available to policymakers. For instance, we experiment with changes in public spending on education, health, or defence and public order-safety. In all cases studied, we start with the assumption that there is one policy change at a time and this change is financed by public debt; this helps us to understand better how the model works. But, at the end, we also study fiscal policy mixes. The comparison of alternative policies will be in terms of discounted lifetime utility and output as well as in terms of steady state results.

Our main results are as follows. An increase in public education spending, financed by a temporary rise in public debt, would have outperformed any other fiscal reforms. The beneficial effects from higher public education spending arise because this type of public spending, not only enhances the productivity of efficient labor time as one would expect, but also because it crowds in private savings. Public spending on health can become equally good as public education spending to the extent that it is necessary for the accumulation of private human capital (unhealthy people cannot be efficient, irrespectively of their education level). It is worth stressing that these results are robust to allowing for private education spending to the extent that we remain within historical ranges for private and public spendings as shares of output.

By contrast, an increase in spending on public pensions, financed by a PAYG system, is found to be particularly bad. Such an increase distorts private incentives, by crowding out private savings more than one for one, and this hurts aggregate output and welfare over time.³ Interestingly, in equilibrium, a rise in spending on public pensions hurts even those that are supposed to be the main beneficiaries, namely the old households. This happens because the indirect harm (from lower savings and less aggregate output) more than outweighs any direct benefits (from the policy attempt to allocate more social resources to pensions). Actually, in most experiments, an increase in public pensions leads to outcomes worse than the status quo (SQ), which, as said above, is defined as the solution with the pre-crisis fiscal policy data.

Combining the above results, our simulations show that a fiscal policy mix, which combines a rise in public education spending with a cut in public spending on social security, would generate substantial benefits not

³This is a well recognized result: social security, via a PAYG system, decreases private saving and hurts capital formation. See e.g. Cooley and Soares (1999), Bouzahzah et al. (2002), Krueger and Kubler (2006) and Soares (2006). Krueger and Kubler (2006) provide a quantitative evaluation of the pluses and minuses of a PAYG social security system, where the tradeoff is between intergenerational risk-sharing and a lower capital stock. Here, as said, we abstract from uncertainty and risk. Instead, we focus on the medium and long term.

only to the aggregate economy but also to all generations even the old. Thus, a growth-enhancing education policy is the means to support the old. 4

Other measures actually taken after the 2008 crisis - like an increase in spending on economic affairs (e.g. public infrastructure) or a reduction in consumption and labor raxes - are better than the SQ but inferior to an increase in public education spending. Finally, debated reforms supported by the political right or the political left - like an horizontal cut in all categories in public spending or a rise in capital taxes respectively - are inferior to an increase in public education spending and, in most cases, they are found to be counter-productive meaning that they are inferior even to the SQ.

The rest of the paper is organized as follows. A brief review of the literature is in section 2. Section 3 reports the public spending data. The model is developed in section 4. Section 5 presents the status quo solution serving as a point of departure. Various reforms are studied in section 6. A sensitivity analysis is in section 7. Section 8 summarizes.

2 Related literature and how we differ

Our work is related to two literatures. The first includes OLG models with fiscal policy. But, to the best of our knowledge, none of the previous papers has studied all main spending policy instruments at the same time. For instance, Arcalean and Schiopu (2010), Abington and Blankenau (2013) and Viaene and Zilcha (2013), among many others, have focused on public education. Cooley and Soares (1999), Krueger and Kubler (2006), Conesa and Garriga (2008), Bassetto (2008), Auerbach and Lee (2011) and Kaganovich and Zilcha (2012), among many others, have focused on social security.⁵ Bhattacharya and Qiao (2007) have focused on health spending. OLG papers with a richer menu of spending policy instruments include Bouzahzah et al. (2002), Soares (2006) and Del Rey and Lopez-Garcia (2013), who have studied both public education and social security, and Dioikitopoulos (2014), who has studied both public health and education.

The second literature includes econometric studies on the growth effects of different types of public spending. See, for instance, Bleaney et al. (1999, 2001), Shelton (2007), Gemmell et al. (2012, 2014) and Acosta-Ormaechea and Morozumi (2015), while a review of previous empirical can be found in European Commission (2008, part III, chapter 3.3).

Our paper belongs to the first literature but it differs from most of the previous papers in that we incorporate all main functional categories of public spending and taxes in a unified neoclassical general equilibrium framework and study the effects of their changes.

⁴For a similar policy recipe, in the context of environmental quality, see Economides and Philippopoulos (2008).

⁵Cooley and Soares (1999) also survey the ealrier literature that dates back to Samuelson (1958) and Diamond (1965).

3 A look at the mix of public spending in the euro area

Table 1 presents the public spending data in the euro zone according to their functional breakdown over 2001-2012. Some data clarifications are helpful for the understanding of the model that follows. Public spending on social protection includes spending on pensions, family support, unemployment benefits, housing, sickness and disability, etc. Public spending on health includes public health, medical products and equipment, hospital services, outpatient services, etc. Public spending on general public services includes public debt payments, administrative spending, foreign economic aid, executive and legislative organs, fiscal affairs and other transfers of a general character between different levels of government. Public spending on education includes spending on pre-primary and primary education, secondary and post secondary non-higher and higher education. Public spending on economic affairs includes public infrastructure spending such as public transport, fuel and energy, mining, manufacturing and construction, communications, licenses and other related support programs. Public spending on public order-safety and defence includes military defence, civil defence, foreign military aid, police and fire protection services, law courts and prisons, etc. Other minor (quantitatively) types of spending are on environmental protection, housing and community amenities and recreation and culture.

In what follows, we will develop a model that gives a natural role to most of these public spending policies.

Table 1: Average public spending 2001-2012 in the Eurozone - COFOG

Functional use	% GDP	% of total public expenditure
Social protection	18.80	39.20
Health	7.00	14.60
General public services	6.40	13.40
Education	5.30	11.00
Economic affairs	4.10	8.50
Public order and safety	1.85	3.90
Defence	1.55	3.20
Environmental protection	0.80	1.70
Housing and community amenities	1.00	1.90
Recreation and culture	1.10	3.30
Total	47.90	

4 Model

Consider a closed economy populated by overlapping generations of three-period-lived households, private firms and the government. Thus, each period corresponds to around 25 calendar years. There are N_t^y young members, N_t^m adult members and N_t^o old members. Thus, the total population is $N_t^y + N_t^m + N_t^o = N_t$ in

each period. For simplicity, we assume that there is no population growth and that the numbers of young, adult and old households remain constant over time and equal to each other. There are also N_t^f private firms owned by the adult households. In order to finance its various categories of public spending, the government levies distorting taxes and issues bonds. Time is discrete and infinite.

4.1 Households

Each household can live for 3 periods, young, adult and old. The household consumes in each period. In addition, when he is young, he spends effort time in education and lives on his parents bequests. When he is adult, he works and can save in the form of capital and government bonds. When he reaches the old age, which happens with probability $0 \le q \le 1$, he uses his own savings and social security (namely, a pension) and dies leaving an optimally chosen bequest. With probability $0 \le 1 - q \le 1$, he dies before reaching the old age leaving an unintended bequest. These bequests (chosen and unintended) are the young people's income.

4.1.1 Household's utility function

The objective of each household born at t is to maximize discounted lifetime utility defined as:

$$u_{t} = \frac{(c_{t}^{y})^{1-\sigma}}{1-\sigma} - \chi_{n} \frac{(e_{t}^{y})^{1+\eta}}{1+\eta} + \chi_{g} \frac{(g_{t}^{u})^{1-\zeta}}{1-\zeta} + \frac{1}{1-\zeta} \left\{ \frac{(c_{t+1}^{m})^{1-\sigma}}{1-\sigma} - \chi_{n} \frac{(l_{t+1}^{m})^{1+\eta}}{1+\eta} + \chi_{g} \frac{(g_{t+1}^{u})^{1-\zeta}}{1-\zeta} \right\} + \beta^{2} q \left\{ \frac{(c_{t+2}^{o})^{1-\sigma}}{1-\sigma} + \chi_{g} \frac{(g_{t+2}^{u})^{1-\zeta}}{1-\zeta} + \beta \chi_{b} \frac{(b_{t+2}^{o})^{1-\xi}}{1-\xi} \right\}$$

$$(1)$$

where c_t^y , c_{t+1}^m , c_{t+2}^o is consumption when young, adult and old respectively, e_t^y is effort time spent in education when young, l_{t+1}^m is effort time spent in work when adult, $g_t^u \equiv \frac{G_t^u}{N_t}$ denotes per capita public spending on "utility-enhancing" public goods and services (see subsection 3.3 below for the definition of utility-enhancing public goods and services) and b_{t+2}^o is a bequest by the old (this is one of household's choice variables). Also, the parameter $0 < \beta < 1$ is the subjective time preference rate and σ , η , ζ , χ_n , χ_g , χ_b , ξ are standard preference parameters. As said, $0 \le q \le 1$ is the probability of reaching the old age; the agent may die before reaching the old age with probability $0 \le 1 - q \le 1$.

4.1.2 Household's budget constraints and bequests

The budget constraints of the household when young, adult and old are respectively:

$$(1 + \tau_t^c) c_t^y + z_t^y = (1 - \tau_t^b) b_{t-1}^y$$
(2)

$$(1 + \tau_{t+1}^c) c_{t+1}^m + k_{t+1}^m + d_{t+1}^m = (1 - \tau_{t+1}^n - \phi_{t+1}) w_{t+1} h_{t+1}^m l_{t+1}^m$$
(3)

$$(1 + \tau_{t+2}^c) c_{t+2}^o + b_{t+2}^o = [1 - \delta^k + (1 - \tau_{t+2}^k) r_{t+2}] k_{t+1}^m + (1 - \tau_{t+2}^k) \pi_{t+2}^o + (1 + \rho_{t+2}) d_{t+1}^m + s_{t+2}^o$$
 (4)

where z_t^y is private spending on education, h_{t+1}^m is the stock of private human capital when adult (see below for its motion), w_{t+1} is the wage earned when adult, k_{t+1}^m is savings in the form of physical capital, d_{t+1}^m is savings in the form of government bonds, r_{t+2} is the return to physical capital, ρ_{t+2} is the return to government bonds, π_{t+2}^o is dividends from firms, s_{t+2}^o is the pension given to each old person from the government, and b_{t-1}^y is an initial endowment which is taken as given by the household (nevertheless, in equilibrium, b_{t-1}^y will be proportional to what is left by the older generations - see below). Finally, $0 < \tau_t^c$, τ_t^n , τ_t^k , $\tau_t^b < 1$ are tax rates on consumption, labor income, capital income and bequests respectively, while $0 < \phi_{t+1} < 1$ is a social security tax (see below for details).

Equation (4) holds with probability $0 \le q \le 1$ only. With probability $0 \le 1 - q \le 1$, the adult dies before reaching the old age. In this case, he leaves an enforced or unintended bequest, denoted as Ω_{t+2}^o . The enforced bequest when the adult dies before reaching the old age is his whole wealth, namely:

$$\Omega_{t+2}^{o} \equiv \left[1 - \delta^{k} + \left(1 - \tau_{t+2}^{k}\right) r_{t+2}\right] k_{t+1}^{m} + \left(1 - \tau_{t+2}^{k}\right) \pi_{t+2}^{o} + \left(1 + \rho_{t+2}\right) d_{t+1}^{m} \tag{5}$$

Therefore, the initial endowment of the young, b_{t-1}^y , is exogenously given to the household when it solves its optimization problem but, in equilibrium, it will be a weighted average of the bequest voluntarily chosen by the old, b_{t+2}^o , and the enforced bequest, Ω_{t+2}^o , in case the adult dies suddenly before reaching the old age, where the weights are respectively the probability of reaching the old age, q, and the probability of suddenly passing away, 1-q. When the universe starts, b_{t-1}^y is given by an initial condition.

4.1.3 Household's human capital

Following the related literature (see e.g. Blankenau and Simpson, 2004, and Blankenau, 2005), the motion of household's human capital is defined as (see also Stokey, 1996, for a similar functional form although in a different context):

$$h_{t+1}^{m} = (1 - \delta^{h}) h_{t}^{y} + B (e_{t}^{y})^{\theta} \left[\gamma (z_{t}^{y})^{\nu} + (1 - \gamma) (\bar{G}_{t}^{e} + \lambda \bar{G}_{t}^{h})^{\nu} \right]^{\frac{1 - \theta}{\nu}}$$
(6)

Thus, individual human capital is augmented by effort time spent in school, e_t^y , by private spending on education, z_t^y , and by government spending per young person on education, $\bar{G}_t^e \equiv \frac{G_t^e}{N_t^y}$, as well as on health, $\bar{G}_t^h \equiv \frac{G_t^h}{N_t^y}$, where the parameter $0 \le \lambda \le 1$ measures how much public spending on health contributes to the quality of human capital (unhealthy people cannot be efficient, irrespectively of their education level).

⁶Thus, individual human capital accumulation can be an increasing function of both private and public spending on education, and this reflects the idea that public spending applies more to primary and secondary education, while private spending applies more to college education and on-the-job training.

Also, B > 0, $0 \le \theta \le 1$, $0 \le \gamma \le 1$ and $0 \le \nu \le 1$ are parameters. The idea behind this functional form is that effort time in school is combined with total (private and public) education spending, with weights θ and $1 - \theta$ respectively, through a Cobb-Douglas technology, to augment private human capital. In turn, private and public education spending, with weights γ and $1 - \gamma$ respectively, are combined into an aggregate through a CES technology with an elasticity of substitution $1/(1 - \nu)$. Finally, B > 0 is a scale parameter.

4.1.4 The probability of reaching the old age

For simplicity, we assume that the probability of an adult reaching the old age, q, depends only on public spending on health and in particular on public spending on health per adult, $\bar{G}_t^h \equiv \frac{G_t^h}{N_t^m}$. 8 This is denoted as $q(\bar{G}_t^h)$, where q(.) is increasing and concave. For convenience, we use the functional form:

$$q\left(\bar{G}_{t}^{h}\right) = \Xi\left(1 + \frac{\bar{G}_{t}^{h}}{1 + \bar{G}_{t}^{h}}\right) \tag{7}$$

where the parameter $\Xi > 0$ is calibrated so as the probability to be within usual ranges (see also e.g. Chakraborty, 2004, and Dioikitopoulos, 2014).

4.1.5 Household's optimality conditions

Households act competitively taking prices, policy instruments and aggregate outcomes as given. They solve their problem at the start of their lives. Thus, they choose c_t^y , c_{t+1}^m , c_{t+2}^o , e_t^y , l_{t+1}^m , k_{t+1}^m , d_{t+1}^m , b_{t+2}^o subject to their budget constraints and the motion of their human capital. The first-order conditions include these constraints and also the optimality conditions for e_t^y , z_t^y , l_{t+1}^m , k_{t+1}^m , d_{t+1}^m and b_{t+2}^o respectively:

$$\chi_{n}\left(e_{t}^{y}\right)^{\eta} = \frac{\beta\left(c_{t+1}^{m}\right)^{-\sigma}\left(1 - \tau_{t+1}^{n} - \phi_{t+1}\right)w_{t+1}l_{t+1}^{m}B\theta\left(e_{t}^{y}\right)^{\theta-1}\left[\gamma\left(z_{t}^{y}\right)^{\nu} + (1 - \gamma)\left(\bar{G}_{t}^{e} + \lambda\bar{G}_{t}^{h}\right)^{\nu}\right]^{\frac{1-\theta}{\nu}}}{\left(1 + \tau_{t+1}^{c}\right)} \tag{8}$$

$$\frac{\left(c_{t}^{y}\right)^{-\sigma}}{\left(1+\tau_{t}^{c}\right)} = \frac{\beta\left(c_{t+1}^{m}\right)^{-\sigma}\left(1-\tau_{t+1}^{n}-\phi_{t+1}\right)w_{t+1}l_{t+1}^{m}B\left(e_{t}^{y}\right)^{\theta}\gamma\left(1-\theta\right)\left[\gamma\left(z_{t}^{y}\right)^{\nu}+\left(1-\gamma\right)\left(\bar{G}_{t}^{e}+\lambda\bar{G}_{t}^{h}\right)^{\nu}\right]^{\frac{1-\theta}{\nu}-1}}{\left(z_{t}^{y}\right)^{1-\nu}\left(1+\tau_{t+1}^{c}\right)} \tag{9}$$

$$\chi_n \left(l_{t+1}^m \right)^{\eta} = \frac{\left(c_{t+1}^m \right)^{-\sigma} \left(1 - \tau_{t+1}^n - \phi_{t+1} \right) w_{t+1} h_{t+1}^m}{\left(1 + \tau_{t+1}^c \right)} \tag{10}$$

⁷Thus, the elasticity parameter, ν , is a measure of sustitutability between the two types of spending, private and public. If $\nu = 1$, the two types become perfect substitutes, while if $\nu = 0$, the function turns to a Cobb-Douglas (see Stokey, 1996).

⁸That is, we assume that the household's probablity of survival does not depend on private decisions, like private spending on health or private income. A generalization could be to assume $q\left(h_{t+1}^{hea}\right) = \Xi\left(1 + \frac{h_{t+1}^{hea}}{1 + h_{t+1}^{hea}}\right)$, where the motion of household's health capital, h_{t+1}^{hea} , is $h_{t+1}^{hea} = \left(1 - \delta^{hea}\right)h_t^{hea} + B^{hea}\left(z_t^{hea}\right)^v\left(\bar{G}_t^{h}\right)^{1-v}$, where z_t^{hea} is private spending on health. If we set $\delta^{hea} = 1$, $B^{hea} = 1$ and v = 0, we go back to the simple specification used here.

⁹Since the household takes policy as given, it also takes q as given. Besides, it does not internalize the link between social security pensions and social security taxes (this is as in e.g. Conesa and Garriga (2008)).

$$\frac{\left(c_{t+1}^{m}\right)^{-\sigma}}{\left(1+\tau_{t+1}^{c}\right)} = \frac{\beta q \left(c_{t+2}^{o}\right)^{-\sigma} \left[\left(1-\delta^{k}\right)+\left(1-\tau_{t+2}^{k}\right) r_{t+2}\right]}{\left(1+\tau_{t+2}^{c}\right)} \tag{11}$$

$$\frac{\left(c_{t+1}^{m}\right)^{-\sigma}}{\left(1+\tau_{t+1}^{c}\right)} = \frac{\beta q \left(c_{t+2}^{o}\right)^{-\sigma} \left(1+\rho_{t+2}\right)}{\left(1+\tau_{t+2}^{c}\right)} \tag{12}$$

$$\frac{\left(c_{t+2}^{o}\right)^{-\sigma}}{\left(1+\tau_{t+2}^{c}\right)} = \beta \chi_b \left(b_{t+2}^{o}\right)^{-\xi} \tag{13}$$

4.2 Private firms

There are $f = 1, 2, ..., N_t^f$ firms. Firms act competitively taking prices, policy instruments and aggregate outcomes as given. Thus, each firm chooses its capital and labor inputs, denoted as K_t^f and L_t^f , to maximize profits:

$$\pi_t^f = Y_t^f - r_t K_t^f - w_t L_t^f \tag{14}$$

where output, Y_t^f , is produced by the function (see also e.g. Lansing, 1998):

$$Y_t^f = A \left(K_t^f \right)^{\alpha_1} \left(L_t^f \right)^{\alpha_2} \left(\frac{K_t^g}{N_t^f} \right)^{1 - \alpha_1 - \alpha_2} \tag{15}$$

where K_t^g is the total stock of public capital (see below for its motion) and A > 0, $0 < \alpha_1$, $\alpha_2 < 1$ are standard technology parameters.

The first-order conditions for the private inputs, K_t^f and L_t^f , are simply:

$$r_t = \frac{\alpha_1 Y_t^f}{K_t^f} \tag{16}$$

$$w_t = \frac{\alpha_2 Y_t^f}{L_t^f} \tag{17}$$

so that each firm's profits are:

$$\pi_t^f = (1 - \alpha_1 - \alpha_2) Y_t^f \tag{18}$$

4.3 Government

We now present government policy. Following most of the related literature, we assume that there is a consolidated general budget and a separate social security budget, where the latter is balanced (see e.g. Soares (2006) and Conesa and Garriga (2008) for similar modeling).

4.3.1 Consolidated government budget constraint

The government uses capital taxes, τ_t^k , labor taxes, τ_t^n , consumption taxes, τ_t^c , and taxes on bequests, τ_t^b , to finance public spending on health, G_t^h , public education spending, G_t^e , public investment spending on infrastructure, G_t^i , public spending on defense and public order-safety, G_t^d , and public spending on the rest of government activities denoted as G_t^c (see below for G_t^c). Note that there is also public spending on pensions but this is financed by social security contributions (see next subsection). Also, D_t is the beginning-of-period total stock of one-period maturity government bonds. We thus have, in aggregate terms, the consolidated government budget constraint:

$$G_t^h + G_t^e + G_t^i + G_t^d + G_t^c + (1 + \rho_t) D_t = D_{t+1} + T_t$$
(19)

where the tax revenues, T_t , are:

$$T_t \equiv \tau_t^c \left(N_t^y c_t^y + N_t^m c_t^m + q N_t^o c_t^o \right) + \tau_t^n N_t^m w_t l_t^m h_t^m + \tau_t^k N_t^o \left(r_t k_t^o + \pi_t^o \right) + \tau_t^b N_t^y b_{t-1}^y$$
(20)

For convenience, concerning the public spending instruments, G_t^h , G_t^e , G_t^i , G_t^d and G_t^c , we will work in terms of their GDP shares defined respectively as $s_t^{g^h} \equiv \frac{G_t^h}{N_t^f Y_t^f}$, $s_t^{g^e} \equiv \frac{G_t^e}{N_t^f Y_t^f}$, $s_t^{g^i} \equiv \frac{G_t^i}{N_t^f Y_t^f}$, $s_t^{g^i} \equiv \frac{G_t^d}{N_t^f Y_t^f}$, and $s_t^{g^c} \equiv \frac{G_t^e}{N_t^f Y_t^f}$.

The meaning of $s_t^{g^h}$, $s_t^{g^e}$, $s_t^{g^i}$ and $s_t^{g^d}$ is straigthforward and their values will be set as in the data. Regarding G_t^c (or its output share, $s_t^{g^c}$), which includes the rest of public spending, it is defined to be as public spending on general public services net of debt payments (spending on debt payments is already captured by $\rho_t D_t$ in our model), plus public spending on housing and community amenities, recreation, culture and environmental protection, plus public spending on social security net of pensions (spending on pensions is already included in our model - see next subsection); these are the rest of government activities in the data (see Table 1 above).

Following usual practice (see e.g. European Central Bank, 2009), we will treat the sum of G_t^c and G_t^d as utility-enhancing public activities, or as "pure public goods", entering directly the household's utility function (1); thus, we assume $G_t^u \equiv G_t^c + G_t^d$ in that equation. We report, however, that when we simply use $G_t^u \equiv G_t^c$ or when we also allow other categories of public spending (like education, etc) to be included into the definition for G_t^u , the main results are not affected, especially the results for output. In addition, in section 7 below, we will allow public spending on defense and public order-safety, G_t^d , to play a more explicit role by also protecting property rights.

Total public capital is augmented via G_t^i and so it evolves over time according to:

$$K_{t+1}^g = (1 - \delta^g) K_t^g + G_t^i \tag{21}$$

4.3.2 Budget constraint of the social security pension system (PAYG)

The government runs a pay-as-you-go (PAYG) social security system. As said above, this operates separately with its own budget. Each old household receives a pension s_t^o when retired (see equation (4)) and these social security benefits are financed by taxing the labor income of the currently working individuals at a rate $0 \le \phi_t \le 1$ (see equation (3)).¹⁰ Thus, aggregating over the relevant households, the budget constraint of the social security pension system is:

$$G_t^s \equiv q N_t^o s_t^o = \phi_t N_t^m w_t l_t^m h_t^m \tag{22}$$

where G_t^s is public spending on pensions and $0 \le \phi_t \le 1$ is the social security tax. For convenience, as we did with the other public spending items, we will work with the output share of public spending on pensions defined as $s_t^{g^s} \equiv \frac{qN_t^os_t^o}{N_t^fY_t^f} = \frac{G_t^s}{N_t^fY_t^f}$.

4.4 Market-clearing conditions

Here, we present the market-clearing conditions in each period.

In the labor market, we have:

$$N_t^f L_t^f = N_t^m l_t^m h_t^m$$

In the capital market, we have:

$$N_t^f K_t^f = N_t^o k_{t-1}^o = N_{t-1}^m k_{t-1}^m$$

In the government bond market, we have:

$$D_{t+1} = N_t^m d_t^m$$

In the dividend market, we have:

$$N_t^f \pi_t^f = N_t^o \pi_t^o$$

In the bequests market we have:

$$N_t^y b_{t-1}^y \equiv q N_t^o b_t^o + (1-q) N_t^m \Omega_t^o$$

¹⁰Here we assume the standard PAYG social security system: all social security benefits of retirees are financed by social security taxes paid by those currently working (see also e.g. Cooley and Soares (1999), Soares (2006), Krueger and Kubler (2006), etc). We could enrich this system by assuming that only a fraction is funded by the PAYG system and the rest is funded by the general government budget. Or by assuming a fully funded system so that social security benefits are financed by social security taxes paid by those employed in the previous period. See e.g. Auerbach and Lee (2011) and Kaganovich and Zilcha (2012), for a comparison of different social security systems.

where the enforced bequest has been defined as:

$$\Omega_{t}^{o} \equiv \left[1 - \delta^{k} + \left(1 - \tau_{t}^{k}\right)r_{t}\right]k_{t-1}^{m} + \left(1 + \rho_{t}\right)d_{t-1}^{m} + \left(1 - \tau_{t}^{k}\right)\pi_{t}^{o}$$

In the PAYG social security system, we have:

$$qN_t^o s_t^o = N_t^m \phi_t w_t l_t^m h_t^m$$

since the social security budget is balanced within the current period.

In the goods market, we have:

$$N_{t}^{y}c_{t}^{y} + N_{t}^{y}z_{t}^{y} + N_{t}^{m}c_{t}^{m} + qN_{t}^{o}c_{t}^{o} + N_{t}^{m}k_{t}^{m} + G_{t}^{c} + G_{t}^{h} + G_{t}^{e} + G_{t}^{i} + G_{t}^{d} = N_{t}^{m}w_{t}l_{t}^{m}h_{t}^{m} + N_{t}^{m}\left(1 - \delta^{k} + r_{t}\right)k_{t-1}$$

where since

$$\begin{split} N_t^f K_t^f &= N_t^o k_{t-1}^o = N_{t-1}^m k_{t-1}^m \\ N_t^f L_t^f &= N_t^m l_t^m h_t^m \\ \\ Y_t^f &= r_t K_t^f + w_t L_t^f \\ \\ N_t^m w_t l_t^m h_t^m + N_t^o r_t k_{t-1}^m = N_t^f Y_t^f \end{split}$$

we get the economy's resource constraint:

$$N_t^y c_t^y + N_t^y z_t^y + N_t^m c_t^m + q N_t^o c_t^o + N_t^m \left[k_t^m - \left(1 - \delta^k \right) k_{t+1}^m \right] + G_t^c + G_t^h + G_t^e + G_t^i + G_t^d = N_t^f Y_t^f$$

or, working with population shares $n_t^y \equiv \frac{N_t^y}{N_t}$, $n_t^m \equiv \frac{N_t^m}{N_t}$, $n_t^o \equiv \frac{N_t^o}{N_t}$, we have:

$$n_t^y c_t^y + n_t^y z_t^y + n_t^m c_t^m + n_t^o c_t^o + n_t^m \left[k_t^m - \left(1 - \delta^k \right) k_{t+1}^m \right] + g_t^c + g_t^h + g_t^e + g_t^i + g_t^d = n_t^f y_t^f$$

where public spending items in small letters denote per capita values and:

$$n_{t}^{f}y_{t}^{f} = A\left(n_{t}^{o}k_{t-1}^{o}\right)^{\alpha_{1}}\left(n_{t}^{m}l_{t}^{m}h_{t}^{m}\right)^{\alpha_{2}}\left(n_{t}^{m}k_{t}^{g}\right)^{1-\alpha_{1}-\alpha_{2}} \text{ and } k_{t}^{g} \equiv \frac{K_{t}^{g}}{N_{t}^{f}}.$$

4.5 Market equilibrium system (for any feasible policy)

We now present the market equilibrium (ME) system which is for any feasible policy. In this equilibrium, households maximize welfare, firms maximize profits, all constraints are satisfied and all markets clear. This system is comprised of the following equations:

Young:

$$\chi_{n}\left(e_{t}^{y}\right)^{\eta} = \frac{\beta\left(c_{t+1}^{m}\right)^{-\sigma}\left(1 - \tau_{t+1}^{n} - \phi_{t+1}\right)w_{t+1}l_{t+1}^{m}B\theta\left(e_{t}^{y}\right)^{\theta - 1}\left[\gamma\left(z_{t}^{y}\right)^{\nu} + (1 - \gamma)\left(\bar{G}_{t}^{e} + \lambda\bar{G}_{t}^{h}\right)^{\nu}\right]^{\frac{1 - \theta}{\nu}}}{\left(1 + \tau_{t+1}^{c}\right)} \tag{23}$$

$$\frac{\left(c_{t}^{y}\right)^{-\sigma}}{\left(1+\tau_{t}^{c}\right)} = \frac{\beta\left(c_{t+1}^{m}\right)^{-\sigma}\left(1-\tau_{t+1}^{n}-\phi_{t+1}\right)w_{t+1}l_{t+1}^{m}B\left(e_{t}^{y}\right)^{\theta}\gamma\left(1-\theta\right)\left[\gamma\left(z_{t}^{y}\right)^{\nu}+\left(1-\gamma\right)\left(\bar{G}_{t}^{e}+\lambda\bar{G}_{t}^{h}\right)^{\nu}\right]^{\frac{1-\theta}{\nu}-1}}{\left(z_{t}^{y}\right)^{1-\nu}\left(1+\tau_{t+1}^{c}\right)} \tag{24}$$

$$(1 + \tau_t^c) c_t^y + z_t^y = (1 - \tau_t^b) b_{t-1}^y$$
(25)

Adult:

$$\chi_n \left(l_t^m \right)^{\eta} = \frac{\left(c_t^m \right)^{-\sigma} \left(1 - \tau_t^n - \phi_t \right) w_t h_t^m}{\left(1 + \tau_t^c \right)} \tag{26}$$

$$\frac{\left(c_{t}^{m}\right)^{-\sigma}}{\left(1+\tau_{t}^{c}\right)} = \frac{\beta q \left(c_{t+1}^{o}\right)^{-\sigma} \left[\left(1-\delta^{k}\right)+\left(1-\tau_{t+1}^{k}\right) r_{t+1}\right]}{\left(1+\tau_{t+1}^{c}\right)} \tag{27}$$

$$\frac{\left(c_{t}^{m}\right)^{-\sigma}}{\left(1+\tau_{t}^{c}\right)} = \frac{\beta q \left(c_{t+1}^{o}\right)^{-\sigma} \left(1+\rho_{t+1}\right)}{\left(1+\tau_{t+1}^{c}\right)} \tag{28}$$

$$(1 + \tau_t^c) c_t^m + k_t^m + d_t^m = (1 - \tau_t^n - \phi_t) w_t h_t^m l_t^m$$
(29)

$$h_{t+1}^{m} = (1 - \delta^{h}) h_{t}^{y} + B(e_{t}^{y})^{\theta} \left[\gamma (z_{t}^{y})^{\nu} + (1 - \gamma) (\bar{G}_{t}^{e} + \lambda \bar{G}_{t}^{h})^{\nu} \right]^{\frac{1 - \theta}{\nu}}$$
(30)

Old:

$$\frac{\left(c_t^o\right)^{-\sigma}}{\left(1+\tau_t^c\right)} = \beta \chi_b \left(b_t^o\right)^{-\xi} \tag{31}$$

$$(1 + \tau_t^c) c_t^o + b_t^o = \left[1 - \delta^k + \left(1 - \tau_t^k\right) r_t\right] k_{t-1}^m + (1 + \rho_t) d_{t-1}^m + s_t^o$$
(32)

Government:

$$\left(s_t^{g^h} + s_t^{g^e} + s_t^{g^i} + s_t^{g^i} + s_t^{g^d} + s_t^{g^c}\right) n_t^f y_t^f + (1+\rho) n_t^m d_{t-1}^m =
= n_t^m d_t^m + \tau_t^c \left(n_t^y c_t^y + n_t^m c_t^m + n_t^o c_t^o\right) + \tau_t^n n_t^m w_t l_t^m h_t^m + \tau_t^k n_t^m r_t k_{t-1}^m + \tau_t^b n_t^y b_{t-1}^y$$
(33)

$$qn_t^o s_t^o = n_t^m \phi_t w_t l_t^m h_t^m \tag{34}$$

$$k_{t+1}^g = (1 - \delta^g) k_t^g + s_t^{g^i} n_t^f y_t^f$$
(35)

$$s_t^{g^s} \equiv \frac{qN_t^o s_t^o}{N_t^f Y_t^f} \tag{36}$$

In the above equations, we use:

$$\begin{split} n_t^f Y_t^f &= A \left(n_t^m k_{t-1}^m \right)^{\alpha_1} \left(n_t^m l_t^m h_t^m \right)^{\alpha_2} \left(n_t^f k_t^g \right)^{1-\alpha_1-\alpha_2} \\ \\ n_t^y b_{t-1}^y &\equiv q n_t^o b_t^o + (1-q) \, n_t^m \Omega_t^o \end{split}$$

$$\begin{split} \Omega_{t}^{o} &\equiv \left[1 - \delta^{k} + \left(1 - \tau_{t}^{k}\right) r_{t}\right] k_{t-1}^{m} + \left(1 - \tau_{t}^{k}\right) \pi_{t}^{o} \\ \pi_{t}^{o} &= \frac{n_{t}^{f}}{n_{t}^{o}} \pi_{t}^{f} = \frac{\left(1 - \alpha_{1} - \alpha_{2}\right) n_{t}^{f} Y_{t}^{f}}{n_{t}^{o}} \\ r_{t} &= \frac{\alpha_{1} Y_{t}^{f}}{K_{t}^{f}} = \frac{\alpha_{1} N_{t}^{f} Y_{t}^{f}}{N_{t}^{f} K_{t}^{f}} = \frac{\alpha_{1} n_{t}^{f} Y_{t}^{f}}{n_{t}^{m} k_{t-1}^{m}} \\ w_{t} &= \frac{\alpha_{2} Y_{t}^{f}}{L_{t}^{f}} = \frac{\alpha_{1} N_{t}^{f} Y_{t}^{f}}{N_{t}^{f} L_{t}^{f}} = \frac{\alpha_{1} n_{t}^{f} Y_{t}^{f}}{n_{t}^{m} l_{t}^{m} h_{t}^{m}} \\ q_{t} &= \Xi \left(1 + \frac{\bar{G}_{t}^{h}}{1 + \bar{G}_{t}^{h}}\right) \end{split}$$

We thus have 14 equations in 14 variables, which are $\left\{z_t^y, c_t^y, c_t^m, c_t^o, e_t^y, l_t^m, k_t^m, \rho_t, h_t^m, s_t^o, \phi_t, b_t^o, k_{t+1}^g\right\}_{t=0}^{\infty}$ and one policy instrument out of $\left\{\tau_t^c, \tau_t^k, \tau_t^n, \tau_t^b, s_t^{g^h}, s_t^{g^e}, s_t^{g^i}, s_t^{g^d}, s_t^{g^c}, s_t^{g^s}, d_t^m\right\}_{t=0}^{\infty}$. In our solutions below, the residual policy instrument will be the end-of-period public debt, d_t^m , except otherwise said.

4.6 Solution methodology

We will work as follows. In the next section (section 5), we solve the above model numerically employing common parameter values and fiscal data from the euro area. The steady state solution of this model will be defined as the "status quo". In turn, section 6 will study the transition dynamics when we depart from this status quo and travel to a new, reformed steady state (reforms are defined in subsection 6.1 below). To compute the transition dynamics, we will take a first-order approximation of the model around the new, reformed steady state, when the initial conditions of the predetermined state variables are those of the status quo solution. We report that non-linear dynamics do not change our main results. Notice that, since the model is deterministic for simplicity, transition dynamics are driven by changes in policy only.

5 Parameters, data and the status-quo solution

This section solves the above model economy numerically using common parameter values and fiscal policy data from the euro area. The values of parameters and policy variables are listed in Tables 2a and 2b respectively, while some details are discussed in subsections 5.1 and 5.2. Then, the resulting steady state solution is in subsection 5.3.

5.1 Parameter values

Parameter values for preference and technology are reported in Table 2a. As said, we use conventional values. We also report that our results are robust to changes in these values (details are in section 6 below).

The time unit is meant to be a a period consisting of 25 years. Regarding preference parameters, we use values used by most of the business cycle literature. The time preference rate, β , is set at 0.985^{25} so as to be consistent with a value for the real interest rate around 5% per year. The weight given to public goods and

services in the household's utility function, χ_g , is set at 0.1. The elasticity of intertemporal substitution, σ , the inverse of Frisch labour elasticity, η , and the elasticity of bequests in the utility are set as in related studies. The preference parameter related to effort time, χ_n , is set at 7; this implies hours of work within usual ranges.

Regarding technology parameters in the production function of goods (see equation (15)), the Cobb-Douglas exponents of physical capital and labor are set at 0.35 and 0.60 respectively, so that the exponent of public capital is 0.05 (the latter is close to public investment as share of output). The scale parameter in the same function, A, is set at 2. As is usual in the OLG literature, we set the values of the depreciation rates of physical, human and public capital equal to 1. For simplicity, we set equal population shares (however, in the data, adults have the highest share in total population). In the human capital production function (see equation (6)), the values of B and θ are close to the values used by the growth literature; these parameter values imply hours of education within usual ranges. Following Stokey (1996), the elasticity parameter, ν , is set at the neutral value of 0.5 (this implies $1/(1-\nu)=2$ so that private and public education spending are good substitutes). The weight given to private vis-a-vis public spending in the same function, γ , is set at 0.25; this implies that, in equilibrium, private education spending as share of GDP is around 0.5%, which is as in the European data (see section 7 below for sensitivity). As regards the parameter λ , measuring the contribution of public spending on health in the creation of private human capital, we start with a modest value of 0.1 (again, see section 7 below for sensitivity).

5.2 Fiscal data

We use spending and tax policy data for the Eurozone. The data are from Eurostat. To solve the model, we use data averages over the pre-crisis period, 2001-2008. Their values are listed in Table 2b. In other words, regarding the exogenously set fiscal policy instruments, we choose to use the means in the data over the pre-crisis period (2001-2008) rather than over the whole euro period (2001-2012). This is because the crisis year, 2008, triggered a number of policy changes or, at least, it led to a discussion of several policy changes. We thus find it more natural to use the 2001-2008 solution as a point of departure to study various reforms. Nevertheless, we report that our qualitative results do not change when we use the means in the data over the whole period (2001-2012) or when we simply use the 2008 values.

Specifically, regarding public spending, we make use of the disaggregation of the international Classification of the Functions of Government (COFOG) in the framework of the European System of National Accounts (ESA95). This is comprised by the functional categories listed in Table 1. Specifically, according to our model above, we use data on education, health, defense and public-order safety, old age pensions (which are a sub-category of social protection expenditure) and economic affairs. These spending items as shares of GDP $(s_t^{g^e}, s_t^{g^h}, s_t^{g^d}, s_t^{g^s}, s_t^{g^i})$ are set at their data averages (see Table 2b). The rest of spending items are included in s^{g^c} , as defined in subsection 4.3.1 above, amounting to around 14% of GDP. Then,

total government spending is around 43% of GDP, which is close to the 2001-2008 data average.

The tax rates on consumption, capital income, bequests and labor income (τ^c , τ^k , τ^b and τ^n) are set respectively at 0.194, 0.295, 0.04 and 0.26, which are the average effective tax rates in the euro zone. Notice that the value of 0.26 for τ^n does not include social security taxes. Social security contributions amount to over one third of total labor tax payments. Hence, along with our value of s^{g^s} , the implied value of the social security tax rate, ϕ , is around 13%. This implies a total labor tax rate of 39%.

Table 2a: Parameter values

Parameters	Value	Description
β	0.985^{25}	time preference rate
σ	2	elasticity of intertemporal substitution
χ_n	7	preference parameter related to effort time
χ_b	1	preference parameter related to bequests
χ_g	0.100	preference parameter related to public spending
ζ	1	elasticity of public consumption in utility
η	1	inverse of Frisch labour supply elasticity
ξ	1	elasticity of bequests in utility
n^y	1/3	share of young in the population
n^m	1/3	share of adults in the population
n^o	1/3	share of old in the population
Ξ	0.600	parameter in the probablity of reaching the old age
Γ	0.800	parameter in the degree of protection of property rights
A	2	scale parameter in production of goods
α_1	0.350	share of capital
α_2	0.600	share of labour
В	15	scale parameter in production of human capital
θ	0.750	productivity of education time
λ	0.100	contribution of public spending on health in human capital
ν	0.500	elasticity parameter in human capital
γ	0.250	weight to private spending in human capital
δ^k	1	physical capital depreciation rate
δ^h	1	human capital depreciation rate
δ^g	1	public capital depreciation rate

Table 2b: Fiscal policy variables (2001-8)

Policy instruments	Value	Description
$ au^c$	0.194	consumption tax rate
$ au^n$	0.260	labour tax rate
$\overline{\phi}$	0.130	social security tax rate
$ au^k$	0.295	capital tax rate
$ au^b$	0.050	bequest tax rate
$\overline{s_t^{g^e}}$	0.052	government spending on education as share of output
$s_t^{g^h}$	0.067	government spending on health as share of output
$\frac{s_t^{g^e}}{s_t^{g^h}}$ $s_t^{g^s}$	0.041	government spending on infrastructure as share of output
	0.081	government spending on pensions as share of output
$s_t^{g^d}$	0.033	government spending on defense & public-order as share of output
$s_t^{g^c}$	0.140	government spending on the rest as share of output

5.3 Status quo solution

This subsection presents the steady state solution of the ME system presented in subsection 4.5 above. In this numerical solution, variables do not change over time and the parameter values and the exogenous policy instruments have been set as in Tables 2a and 2b respectively. This solution is reported in Table 3. As can be seen, the solution is meaningful. Also, most of the implied ratios are not different from their values in the 2001-8 data (e.g. consumption to GDP, public debt to GDP, private education spending to GDP, time allocated to work or school, etc). This solution will serve as a point of departure in what follows.

¹¹Notice that the solution for the probability of an adult reaching the old age (see equation (7) above) is around q = 0.52. We report that, through recalibration of the parameter Ξ in that equation, the solution for q can rise and the main results do not change.

Table 3: Status quo solution

Variable	Solution	Variable	Solution
c^y	0.1121	h^m	2.4793
c^m	0.1228	s^o	0.0874
c^{o}	0.1335	$Y=n^fy^f$	0.1885
e^y	0.3210	b^o	0.1093
l^m	0.4478	b^y	0.1441
k^m	0.0456	r	0.0605
k^g	0.0075	w	0.3059
q	0.5182	$\frac{c}{y}$	0.5400
z^y	0.0028	ho	0.0293
u	-5.9918	$\frac{d^m}{Y}$	0.6035

Notes: (a) u follows from the household's utility function (1). (b) $\frac{c}{y} = \frac{(n^y c^y + n^m c^m + q n^o c^o)}{n^f y^f}$.

6 Fiscal policy reforms

We now study the implications of various policy (spending/tax) reforms. As said above, by reforms we mean permanent changes in the composition of public spending and taxes.

6.1 Discussion of fiscal policy reforms studied

As said in the Introduction, we study three types of policy changes. First, those that are in the spirit of policy changes actually observed in the euro area in the aftermath of the 2008 crisis. Second, policy changes that may not have actually happened but have been debated in policy circles during the same period. Third, some policy changes that have neither happened nor debated but are natural candidates to be studied given the broader set of fiscal instruments in the hands of policymakers.

In all cases studied, we depart from the status quo solution in Table 3 above. Also, in all cases, we assume that any exogenous changes in policy instruments are permanent and are accommodated by adjustments in public debt; in other words, public debt is the residually determined policy variable and this happens both in the transition and in the new reformed steady state (other public financing cases are reported below). To understand better the logic of our results, and following usual practice, we start by considering one reform at a time.

The various reforms studied are listed in Table 4. The first set of reforms (namely, reforms that that are in the spirit of actual policy measures) includes reforms R1-R4. In particular, R1 assumes an one percentage point permanent increase in social protection spending as share of GDP, s^{g^s} ; ¹² R2 assumes an one percentage point permanent increase in public spending on economic affairs, s^{g^i} (which, as said, is basically

¹²By one percentage point (pp), we mean that if it is, say, 18.8% of GDP, it rises to 19.8%.

infrastructure spending); R3 assumes an one percentage point permanent decrease in the consumption tax rate, τ^c ; and R4 assumes an one percentage point permanent decrease in the labor tax rate, τ^n . As said above, these changes, at least qualitatively, are in accordance with fiscal policy measures taken in the euro area after the 2008 crisis (see e.g. Coenen et al. (2013)).

The second set of reforms includes R5-R6. In particular, R5 assumes an horizontal permanent cut in all public spending items by 0.15 percentage points which amounts to a reduction in overall public spending by one percentage point; R6 assumes an one percentage point permanent rise in the capital tax rate, τ^k . We study an horizontal cut in all public spending categories because a smaller public sector is ideologically supported by several right-wing governments in Europe these days¹³ and, in the same spirit, we study a rise in capital taxes because this is ideologically supported by left-wing governments.

The third set includes reforms R7-R9. In particular, R7 assumes an one percentage point permanent increase in public education spending, s^{g^e} ; R8 assumes an one percentage point permanent increase in public spending on health, s^{g^h} ; finally, R9 assumes an one percentage point permanent increase in public spending on defence and public order-safety, s^{g^d} .

We will also report what would have happened if the policy variables had remained at their 2008 values for ever (this SQ serves as a natural benchmark). Notice that, in all cases, for comparability, spending ratios or tax rates are assumed to change by one percentage point (we report that our qualitative results are robust to changes of bigger, or smaller, magnitude) and that it is public debt that adjusts residually in each period.

Table 4: Fiscal (spending-tax) policy reforms studied

Reforms	Description
R1	1 pp increase in spending on pensions, s^{g^s}
R2	1 pp increase in spending on economic affairs, \boldsymbol{s}^{g^i}
R3	1 pp decrease in consumption tax rate, τ^c
R4	1 pp decrease in labor tax rate, τ^n
R5	1 pp decrease in overall public spending
R6	1 pp increase in capital tax rate, τ^k
R7	1 pp increase in spending on education, \boldsymbol{s}^{g^e}
R8	1 pp increase in spending on health, s^{g^h}
R9	1 pp increase in spending on defence and public-order safety, \boldsymbol{s}^{g^d}

6.2 Results for steady state output and welfare

Tables 5 and 6 report the steady state solution of output (y) and utility (u) respectively in each case listed in Table 4 above. As said, the adjusting public finance policy instrument is public debt. Recall that u is as

¹³Our choice of an horizontal cut in all public spending categories can also be motivated by the fact that in several policy reports (see e.g. European Commission, 2014, part IV), it is recommended that debt consolidation is successful when we cut spending on public wages. Here, since we do not have separate data on public wages for each functional category of public spending, we ad hoc cut all public spending categories by 0.15 percentage points.

in equation (1) above.

The results in Tables 5 and 6 indicate that, in the steady state, the best reform (both in terms of steady state output and utility) would be an increase in public education spending, namely R7. At the other end, the worst outcome (again both in terms of steady state output and utility) is given by an increase in spending on public pensions, namely R1; actually, R1 is inferior even to the status quo (SQ). R2, R3 and R4 (namely, other reforms actually implemented) are better than SQ, although worse than R7. Finally, R5 (namely, an horizontal cut in all categories of public spending) and R6 (namely, an increase in the capital tax rate) are worse than SQ in terms of both output and utility. All this is about the steady state equilibrium.

Table 5: Steady state output

SQ	R1	R2	R3	R4	R5	R6	R7	R8	R9
0.1885	0.1778	0.1987	0.1908	0.1935	0.1842	0.1858	0.2028	0.1959	0.1936

Table 6: Steady state utility

\overline{SQ}	R1	R2	R3	R4	R5	R6	R7	R8	R9
-5.9918	-6.0927	-5.9334	-5.9751	-5.9507	-6.0104	-6.0177	-5.8846	-5.9758	-5.9796

6.3 Results for discounted lifetime output and welfare

Tables 7 and 8 report results for discounted lifetime output (defined as $\sum_{t=0}^{\infty} \beta^t y_t$) and discounted lifetime utility (defined as $\sum_{t=0}^{\infty} \beta^t u_t$) respectively for each regime listed in Table 4. Again, as said above, the adjusting public financing instrument is the end-of-period public debt in each period. In turn, the implied utility gains, or losses, for each generation separately (young, adult and old) vis-a-vis the SQ case, measured in terms of consumption equivalence units, are reported in Table 9.

Table 7: Discounted lifetime output

$_{ m SQ}$	R1	R2	R3	R4	R5	R6	R7	R8	R9
0.5992	0.5833	0.6139	0.6022	0.6062	0.5936	0.5954	0.6183	0.6094	0.6062

Table 8: Discounted lifetime utility

\overline{SQ}	R1	R2	R3	R4	R5	R6	R7	R8	R9
-19.0423	-19.2047	-18.9940	-19.0232	-19.9835	-19.0326	-19.0837	-18.8864	-19.0940	-19.0797

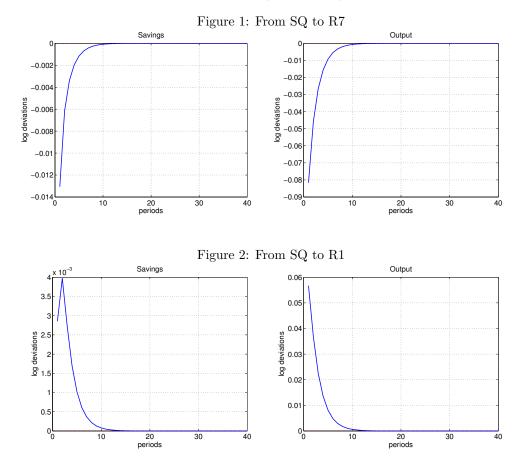
Table 9: Welfare gains/losses (from Table 8)

	R1	R2	R3	R4	R5	R6	R7	R8	R9
ξ^y	-0.0109	-0.0205	-0.0071	-0.0100	0.0181	-0.0019	-0.0065	-0.0307	-0.290
ξ^m	-0.0524	0.0512	0.0213	0.0416	-0.0240	-0.0119	0.0720	0.0361	0.0356
ξ^o	-0.0165	0.0074	-0.0047	0.0043	0.0030	-0.0128	0.0400	-0.0160	-0.0238

Note: A positive number means a welfare gain vis-a-vis SQ.

The main messages from lifetime results are similar to those from steady state results. In other words, both in terms of discounted lifetime output and utility, the best reform would be an increase in public education spending, namely R7. R1 is again the worst; actually, in terms of lifetime utility, R4 (a cut in the labor tax) is the worst but this happens only because leisure falls. R2 and R3 are better than SQ, but worse than R7. R6 (a higher capital tax rate) is worse than SQ both in terms of lifetime output and utility, while R5 (a cut in all categories of public spending) is also worse than SQ although in terms of discounted output only.

The simulated series, and the associated impulse response functions (see Figure 1), reveal that the social benefits from higher public education spending arise, not only because private human capital increases as one would naturally expect, but also because this type of public spending leads to relative high private savings, as measured by $k_{t+1}^m + d_{t+1}^m$ in the adults' budget constraint (see equation (3)). Higher private savings are, in turn, good for aggregate output and hence for social welfare in equilibrium. The opposite happens in the case of public spending on pensions: now an increase in the latter is particularly bad for private savings and eventually for aggregate output (see Figure 2).



Consider finally the results in Table 9. In terms of lifetime welfare gains/losses for each generation separately, an increase in public education spending (R7) produces a welfare gain of 7.2 and 4 percent of consumption for adults and the old respectively (i.e. 7.2 and 4 of a percentage point of consumption). By contrast, an increase in social protection spending (R1) proves to be counter-productive for everybody, even

for the old generation. In other words, even for the old households, the indirect harm from lower savings and aggregate output more than outweighs any direct benefits from the policy attempt to allocate more social resources to their pensions. Notice that, in all cases, the young are affected by less than the adults or the old; this is because the income of the young comes from bequests only.

6.4 Policy mixes

We now study policy mixes. To save on space, and given the above results, we report results only for the case in which we increase public spending on education permanently (as in R7), but now we assume that, in the steady state, this is financed by an adjustment, or fall, in public spending on social protection (which is the opposite of R1), while the steady state public debt-to-GDP ratio is set at its data average value. In the transition, as it has been the case so far, public debt remains the residual public financing instrument. The idea behind this mix is that we increase the most productive type of public spending and this is financed by a cut in the least productive one. Inspection of the new results in Tables 10, 11 and 12 implies that the welfare gains are now much more substantial and this applies to all three generations.

Table 10: Steady state output and utility

	SQ	R7-R1 mix
\overline{y}	0.1885	0.2620
u	-5.9918	-5.4809

Note: s^{g^s} falls from 0.080 to 0.042 at the new steady state.

Table 11: Discounted lifetime output and utility

	$_{ m SQ}$	R7-R1 mix
y	0.5992	0.7160
u	-19.0423	-18.0350

Table 12: Welfare gains/losses (from Table 11)

$=$ ξ^y	0.0851	=
ξ^m	0.3608	
ξ^o	0.1329	

Note: A positive number means a welfare gain vis-a-vis SQ.

7 Sensitivity analysis

We now check the robustness of our results to several changes. We will focus on the importance of private education spending in the creation of new human capital (subsection 7.1), the role of public spending on health again in the creation of new human capital (subsection 7.2) and the possible role of public spending on defense and public order-safety in the protection of property rights (subsection 7.3).

We also report that the main results (to the extent that we still get a solution) are robust to changes in other parameter values or to changes in the exogenous policy instruments of different (smaller or larger) magnitude from what has been assumed so far.

7.1 The role of private spending on education

In the solutions so far, the value of the parameter $0 \le \gamma \le 1$, which is the weight given to private education spending vis-a-vis public education spending in the production function for new human capital (see equation (6)), has been set at 0.25. As said, this parameter value resulted in a solution for private spending on education close to the European data (0.5% of GDP). Our results remain qualitatively unchanged for γ in the range between 0 and around 0.4. Above 0.4, as expected, R7 ceases to be the best policy change. It is important to point out, however, that a value of γ above 0.4 results in solutions for the output share of private education spending outside the range of historicall values. For instance, when we set $\gamma = 0.5$, this share rises to 2.5% of GDP.

We also report that our results do not change when the related elasticity parameter, ν , rises, so that the elasticity of substitution between private and public education spending, $1/(1-\nu)$, also rises. In particular, our results do not change when ν rises from 0.5 (which was its value in the baseline parameterization) to around 0.8.

7.2 The role of public spending on health

We now study higher values of the parameter λ in equation (6). Recall that λ measures the extent to which public spending on health contributes to the creation of new human capital alongside with public spending on education (as argued above, not only education, skills and knowledge, but also health and longevity can be crucial to human capital). Also recall that the results so far have been with $\lambda = 0.1$, which can be thought as a low value. We now go to the other extreme and set, say, $\lambda = 0.9$. The new results are presented in Tables 13, 14 and 15. As expected, now R7 and R8 become almost identical. Namely, an increase in public spending on health becomes as good as an increase in public spending on education to the extent that the former also augments private human capital.

Table 13: Discounted lifetime output

$_{ m SQ}$	R7	R8
0.7293	0.7469	0.7491

Table 14: Discounted lifetime utility

SQ	R7	R8
-17.6238	-17.5604	-17.5749

Table 15: Welfare gains/losses (from Table 14)

	R7	R8
ξ^y	-0.0190	-0.0192
ξ^m	0.0554	0.0555
ξ^o	0.0121	0.0161

Note: A positive number means a welfare gain vis-a-vis SQ.

7.3 The role of public spending on defense and public order-safety

In the analysis so far, we have treated public spending on defense and public-order safety as a utility-enhancing policy activity only. One could claim, however, that this type of policy also provides productivity-enhancing services. For instance, it is widely recognized that public spending on defense and public-order safety (national security, police, courts, prisons, etc) is necessary for the protection of property rights. We therefore augment the model to allow for weak property rights and thus give a richer, and potentially more useful, role to public spending on defense and public order-safety. Thus, now, this type of policy not only provides utility-enhancing services as assumed so far, but it also increases the degree of property right protection.

In the richer model, we assume that each firm can appropriate only a fraction $0 of its output produced, <math>Y_t^f$, because the rest, $(1-p)Y_t^f$, can be taken away by other agents (say, adult households). ¹⁴ Thus, p denotes the degree of protection of property rights in the economy. For simplicity, as we did above with the probability of reaching the old age, we assume that p depends on public expenditure on defense and public order-safety per firm, $\bar{G}_t^d \equiv \frac{G_t^d}{N_t^f}$. This is denoted as $p(\bar{G}_t^d)$, where p(.) is increasing and concave. For convenience, as we did above with the probability of reaching the old age in equation (7), we use the form:

$$p\left(\bar{G}_{t}^{d}\right) = \Gamma\left(1 + \frac{\bar{G}_{t}^{d}}{1 + \bar{G}_{t}^{d}}\right) \tag{37}$$

where the parameter $\Gamma > 0$ is calibrated so as to capture the degree of protection of property rights in the data (see also e.g. Angelopoulos et al., 2009).

Details of the new model are in the Appendix. Here, working as above, we present the final numerical solutions. As can be seen in Tables 16, 17 and 18, the main results do not change. That is, even when we allow public spending on defense and public order-safety to play a new richer role, R9 (namely, an increase in public spending on defense and public order-safety) continues to be inferior to R7 (namely, an increase in public education spending) both in terms of output and utility. It is also important to report that our solution for the degree of property rights, p, is around 0.85, which is close to the international property

¹⁴In a general equilibrium model, nothing is lost. Of course, ill-defined property rights distort individual incentives and this hurts the aggregate economy. This is also the case in our new model. See e.g. Economides et al. (2007) and Angelopoulos et al. (2009) for details.

al. (2009) for details.

15 That is, we assume that the economy's degree of property rights does not depend on private decisions, like expropriation activities, rent seeking effort, etc. For such a generalization, see e.g. Economides et al. (2007) and Angelopoulos et al. (2009).

rights index for the EU (see e.g. the World Bank database).

Table 16: Discounted lifetime output

SQ	R7	R9
0.4107	0.4279	0.4204

Table 17: Discounted lifetime utility

SQ	R7	R9
-21.0862	-20.9774	-21.1561

Table 18: Welfare gains/losses (from Table 17)

	R7	R9
ξ^y	-0.0311	-0.0423
ξ^m	0.0904	0.0504
ξ^o	0.0325	-0.0153

Note: A positive number means a welfare gain vis-a-vis SQ.

8 Concluding remarks

In this paper, we built a general equilibrium OLG model that included the most important functional categories of spending and taxes in the data. We then carried out a number of policy experiments to see what happens when spending and tax instruments change vis-a-vis their data average values.

The main result is that higher public spending on education and health, namely policies that enhance the accumulation of private human capital, would be the best way out of the recent crisis. By contrast, higher public spending on pensions (via a PAYG system) has not been found to be productive.

Regarding social protection spending, however, it is worth keeping in mind that here we assumed that capital markets are complete so that agents can insure themselves via private savings (or private insurance). It would be interesting though to see what happens when capital markets are not complete so that public insurance is not a substitute for private insurance (see e.g. Krueger and Perri (2011)). Also, recall that, in the analysis above, we assumed that differences in age are the only source of heterogeneity across individuals. It would be interesting to add other sources. For instance, we could assume that households are also heterogeneous at birth, namely, there are rich-born households and poor-born households so that there are differences among households within any given cohort. In this case, public education spending may have an even stronger role to play (see e.g. Cunha and Heckman (2009) for this literature). We leave these extensions for future work.

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9 Appendix: Adding weak property rights

In this appendix, we augment the model to allow for weak property rights. We also allow public spending on defense and public order-safety to contribute to the protection of these rights.

As said in the text, each firm can appropriate only a fraction $0 of its output produced, <math>Y_t^f$. The rest, $(1-p)Y_t^f$, can be taken away by adult households. Thus, p is the degree of protection of property rights in the economy. We assume that p depends on public expenditure on defence and public order-safety

per firm, $\bar{G}_t^d \equiv \frac{G_t^d}{N_t^f}$. In particular, the degree of property rights is denoted as $p\left(\bar{G}_t^d\right)$, where $p\left(.\right)$ is increasing and concave. For convenience, we use the functional form:

$$p\left(\bar{G}_t^d\right) = \Gamma\left(1 + \frac{\bar{G}_t^d}{1 + \bar{G}_t^d}\right) \tag{38}$$

where the parameter $\Gamma > 0$ is calibrated so as to capture the degree of protection of property rights in the data.

9.1 Firms

The profit function of each firm is now:

$$\pi_t^f = p_t Y_t^f - r_t K_t^f - w_t L_t^f \tag{39}$$

The variable p_t is taken as given by the firm when it solves its optimization problem. The first-order conditions are now:

$$r_t = p_t \frac{\alpha_1 Y_t^f}{K_t^f} \tag{40}$$

$$w_t = p_t \frac{\alpha_2 Y_t^f}{L_t^f} \tag{41}$$

so that the firm's profits are:

$$\pi_t^f = p_t (1 - \alpha_1 - \alpha_2) Y_t^f \tag{42}$$

9.2 Households

The budget constraint of the adult is now:

$$(1 + \tau_{t+1}^c) c_{t+1}^m + k_{t+1}^m + d_{t+1}^m = (1 - \tau_{t+1}^n - \phi_{t+1}) w_{t+1} h_{t+1}^m l_{t+1}^m + \frac{(1 - p) N_t^f Y_t^f}{N_t^m}$$
 (43)

where $\frac{(1-p)N_t^fY_t^f}{N_t^m}$ is the extra income extracted by each adult in the case of weak property rights. This extra term is taken as given by the household when it solves its optimization problem.

9.3 Market equilibrium system

In the final system, we now use:

$$\pi_{t}^{o} = p_{t} \frac{n_{t}^{f}}{n_{t}^{o}} \pi_{t}^{f} = p_{t} \frac{(1 - \alpha_{1} - \alpha_{2}) n_{t}^{f} Y_{t}^{f}}{n_{t}^{o}}$$

$$r_{t} = p_{t} \frac{\alpha_{1} Y_{t}^{f}}{K_{t}^{f}} = p_{t} \frac{\alpha_{1} N_{t}^{f} Y_{t}^{f}}{N_{t}^{f} K_{t}^{f}} = p_{t} \frac{\alpha_{1} n_{t}^{f} Y_{t}^{f}}{n_{t}^{m} k_{t-1}^{m}}$$

$$w_{t} = p_{t} \frac{\alpha_{2} Y_{t}^{f}}{L_{t}^{f}} = p_{t} \frac{\alpha_{1} N_{t}^{f} Y_{t}^{f}}{N_{t}^{f} L_{t}^{f}} = p_{t} \frac{\alpha_{1} n_{t}^{f} Y_{t}^{f}}{n_{t}^{m} l_{t}^{m} h_{t}^{m}}$$

where, as defined above:

$$p_t = \Gamma \left(1 + \frac{\bar{G}_t^d}{1 + \bar{G}_t^d} \right)$$