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Elisabeth Gugl
Justin Leroux

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Abstract

We assume like Bergstrom (1989) and Dijkstra (2007) that each child's utility is treated as a normal good in the altruistic head's utility function, and show that if utility functions lead to Almost Transferable Utility children can manipulate the tradeoff between their own utility and the parent's utility through their own actions, but they have an incentive to maximize the altruistic head's utility if the altruistic head also considers children's utilities as Hicksian substitutes and hence the rotten kid theorem holds. A special class of such altruistic utility functions that treat utilities of children as normal and Hicksian substitutes are the Generalized Utilitarian Welfare functions.

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Keywords: Rotten Kid Theorem, transferable utility, Hicksian substitutes, fiscal federalism.

Elisabeth Gugl
Department of Economics
University of Victoria
Canada - V8W 2Y2 Victoria BC
egugl@uvic.ca

Justin Leroux
HEC Montréal & CIRPÉE
Canada - Montréal (Québec) H3T 2A7
justin.leroux@hec.ca

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1 Introduction

Becker (1974) introduced the Rotten Kid Theorem (RKT) as a means of reconciling the treatment of a multi-person household as one agent with methodological individualism. He states that if the head of the household "cares sufficiently about all other [household] members to transfer general resources to them, then [an exogenous] redistribution of income [before transfers] among members would not affect the consumption of any member, as long as the head continues to contribute to all" (p. 1076). The reason for this income pooling behavior is that "if a head exists, other members also are motivated to maximize family income and consumption, even if their welfare depends on their own consumption alone" (p. 1080).

Bergstrom (1989) restates the RKT by introducing what he calls "the game rotten kids play." This game consists of two stages. In the first stage, each family member - including the head - chooses an action. The vector of actions taken by the family members results in a specific amount of family income and a specific array of household public goods. In the second stage, the head makes money transfers to the household members using the income generated in the first stage and after observing the actions in the first stage.[1] If the RKT holds, the head successfully uses the money transfers to fully compensate household members who put themselves in an unfavorable position in the first stage through an action that benefits others but not themselves. That is, the money transfers by the head incentivizes good behavior. Bergstrom shows that the theorem holds when the utility functions of the kids lead to transferable utility (TU)[2], given the assumption that the parent treats each kid's utility as a normal good. Dijkstra (2007) generalizes the game rotten kids play by allowing the parent to transfer more than one good to the children.

In this paper we show that there are cardinal utility functions other than those satisfying TU under which the RKT holds, if we assume in addition to normality that the head considers all agents' utilities as Hicksian substitutes. These utility functions must lead to what Gugl and Leroux (2011) define as Almost Transferable Utility (ATU). A special class of such altruistic utility functions that treat utilities of children as normal goods and Hicksian substitutes are the Generalized Utilitarian Welfare functions.[3]

ATU requires the same ordinal properties as TU on the kids' utility functions but allows for cardinal properties like diminishing marginal utility of money,

[1] Note that this game allows for the head to be endowed with a fixed amount of income and kids' actions being restricted to produce public goods. Such a specification of action sets better matches Becker's idea of a head, in which the head controls her own income only.

Another interpretation of this game is that we restrict ourselves to the analysis of subgame perfect equilibria in which the head makes positive money transfers to all kids out of his own income to meet Becker's definition of a head. Given this assumption, it doesn't matter whether the head has control over all family income or just her own portion of it.

[2] Bergstrom (1989) and Dijkstra (2007) refer to "conditional transferable utility" due to the two-stage nature of the game. However, for expositional purposes, we shall simply refer to "transferable utility".

[3] This class has normatively appealing properties. See the conclusion for a brief discussion.

whereas TU implies constant marginal utility of money. For some allocation mechanisms (e.g. the competitive mechanism), this distinction is irrelevant as only ordinal properties of agents' utility functions matter in determining which consumption bundle each person should receive. However, if the distribution of goods is determined by maximizing an altruistic utility function, cardinal properties of people's utility functions make a difference in how much each person receives and in whether agents have an incentive to manipulate their actions. Hence the result in this paper broadens the range of applications for the RKT by substantially increasing the domain of the kids' admissible utility functions while mildly restricting the range of admissible altruistic utility functions.

There has been renewed interest in the conditions under which the RKT holds. Cornes and Silva (1999), Chiappori and Werning (2002) and Kolpin (2006) restrict their analysis of the RKT to a case of one public good and one private good with a linear technology transforming the private good into the public good. They find that there are utility functions other than those leading to TU (and ATU) for which the RKT holds. The reason for this finding is that they severely restrict the production possibility set of the kids. By contrast, we are interested in the conditions under which the RKT holds regardless of the technology, but given a slightly more restrictive assumption on the parent's altruistic utility function (i.e., the Hicksian substitutes assumption). Benjamin (2010) restricts his analysis to two agents, but allows for both players to have altruistic preferences. He emphasizes the role that the assumption that selfish utilities are considered normal goods in each agent's altruistic utility function plays in achieving efficiency (p.29).

The game rotten kids play also plays an important role in understanding the literature on fiscal federalism: The federal government is the altruistic head and the subnational governments are the rotten kids (e.g. Boadway and Tremblay 2006, Hindriks et al. 2008, Koethenbuerger 2007). Our results are of interest because the federal government may not only wish to reduce inequality between regions as reflected in its social welfare function, but may also take into account that more resources devoted to a relatively poorer region has a larger impact on the welfare of this region than the same increase in resources in the richer region, all other things being the same. That is, the federal government makes an interregional welfare comparison and takes a particular cardinal property of region's welfare functions as given (diminishing marginal utility of money). In this case TU no longer holds, but ATU allows us to capture these features.

The remainder of the paper is as follows. In the next section we formally introduce the game rotten kids play. Section 3 introduces Almost Transferability (ATU), a property of the agents' (selfish) utility functions. Section 4 discusses properties of the parent's altruistic utility function. Section 5 contains our main result before concluding.

2 The Game Rotten Kids Play

The players are a set of $n \geq 2$ family members including the parent or "head" of the family. "The game rotten kids play" consists of two stages. In the first stage, agent i chooses an action $a_i \in A_i$; the action set A_i is assumed to be closed and bounded. Let $A = \Pi_{i=1}^{n} A_i$, then vector of chosen actions $a = (a_1, ..., a_n) \in A$ produces wealth for the entire family given by $I(a)$ and an array of public goods $X(a)$. In the second stage, after observing a, the head distributes the family wealth among its members through positive money transfers, $t_i(a)$, so that $\sum_{i=1}^{n} t_i(a) = I(a)$. The preferences of each agent, i, are represented by a continuous utility function, $U_i : A \times \mathbb{R}_+ \to \mathbb{R}$, and denote by $u_i = U_i(X(a), t_i)$ the generic utility level of agent i. We denote by $u = (u_1, u_2, ..., u_n) \in \mathbb{R}^n$ a generic profile of utility levels. For any given a the utility possibility set conditional on a is the set of utility distributions $UP(a)$ that can be achieved. That is,

$$UP(a) = \left\{ u \in \mathbb{R}^n \;\middle|\; \begin{array}{l} u_i = U_i(X(a), t_i) \text{ for all } i = 1, ..., n, \\ (t_1, ..., t_n) \geq 0 \text{ and } \sum_{i=1}^{n} t_i = I(a) \end{array} \right\}$$

We restrict our analysis to utility possibility sets that are convex, closed and bounded. The utility possibility frontier, $\partial UP(a)$, is defined as usual:

$$\partial UP(a) = \{ u \in UP(a) \, | \bar{u} \geq u \implies \bar{u} \notin UP(a) \}$$

Given our assumptions thus far, the utility possibility frontier, $\partial UP(a)$, may not be linear.

In the second stage, the head of household chooses the income distribution $t(a) = (t_1(a), ..., t_n(a))$ by maximizing her altruistic preference, represented by a continuous and strictly increasing welfare function, $W : UP(a) \to \mathbb{R}$, defined over alternative utility distributions.

Definition 1 *We say that kids are* well behaved *if the unique subgame perfect equilibrium of "the game rotten kids play" leads to the household head's preferred [Pareto optimal] utility distribution. (Bergstrom 1989)*

Formally, kids are well-behaved if and only if, for all $a \in A$, all $a_i' \in A_i$, the following holds:

$$\left[u_i^* \left(a_i', a_{-i} \right) - u_i^*(a) \right] \times \left[W\left(u^*\left(a_i', a_{-i} \right) \right) - W\left(u_i^*(a) \right) \right] \geq 0 \text{ for all } i = 1, ..., n,$$

where a_{-i} denotes the action profile of agents other than i and u^* denotes the utility profile resulting from the maximization of the head's altruistic utility function:

$$u^*(a) = \arg\max W(u) \;\text{ s.t. }\; u \in UP(a).$$

3 Almost Transferable Utility

We focus on a property of agents' preferences that is less demanding than transferable utility. We say that the profile of utility functions $U = (U_1, ..., U_n)$ exhibits *Almost Transferable Utility (ATU)* if there exists a profile of positive monotonic and (twice) differentiable transformations $g = (g_1, ..., g_n)$, with $g_i : \mathbb{R} \to \mathbb{R}$, such that:

$$\forall a \in \Pi_{i=1}^{n} A_i, \exists \lambda \in \mathbb{R} \text{ such that } \partial UP\left(a\right) = \{u \in UP(a) | \sum_{i=1}^{n} g_i\left(u_i\right) = \lambda\}. \quad (1)$$

Since λ depends on the action vector a and available income $I\left(a\right)$, we denote it by $\lambda(a)$. We also denote by $\lambda\left(A\right) = \{\lambda \in \mathbb{R} | \exists a \in A \text{ s.t. } \lambda = \lambda(a)\}$.

ATU is a more general class of preference profiles that includes *Transferable Utility (TU)* as the special case where $g_i\left(u_i\right) = u_i$ for all $u_i \in \mathbb{R}$ and all $i = 1, ..., n$. It is well known that in the case of many public goods and one private good the utility functions leading to TU must be of the Generalized Quasi-linear form,[4] i.e.

$$U_i\left(X, t_i\right) = \alpha\left(X\right) t_i + \beta_i\left(X\right) \quad (2)$$

Almost transferable utility requires $U_i\left(X, t_i\right)$ to be concave transformations of (2).

Remark 2 *The fact that $UP(a)$ is a convex set implies that the g_i's must be convex functions, so as to "undo" the curvature of the utility possibility frontier.*

Under ATU, the head of household's maximization program becomes:

$$\max W\left(u\right) \text{ s.t. } \sum_{i=1}^{n} g_i\left(u_i\right) = \lambda(a). \quad (3)$$

When ATU holds and no confusion is possible, we abuse notation slightly and denote by $W\left(\lambda\right)$ the maximum of the above maximization program: $W\left(\lambda\right) \equiv W\left(u^*\right)$. We also denote by $u^*\left(\lambda\right)$ the corresponding utility profile.

4 Normality and ATU-normality

Having discussed properties of the agents' utility functions in the previous section, we now turn to properties of the head's altruistic preferences.

Let U be a profile of utility functions leading to ATU, and let $(g_1, g_2, ..., g_n)$ be the corresponding profile of positive monotonic transformations. Of interest is how the utility profile $u^*\left(\lambda\right)$ is affected by contractions or expansions of the utility possibility set; i.e., by changes in the value of λ.

Definition 3 *The head of household's altruistic preferences are* ATU-normal *if* $\frac{\partial u^*}{\partial \lambda} > 0$ *for all* $\lambda \in \lambda\left(A\right)$ *whenever ATU holds.*

[4] See, e.g., Bergstrom (1989).

This section is devoted to examining how much can be learned from the head's behavior when the utility possibility frontier is linear. This requires additional notation. Specifically, given $M \in \mathbb{R}_+$ and $\beta \in \mathbb{R}^n_+$, we denote by $\hat{u} \in \mathbb{R}^n$ the utility profile resulting from the maximization of the head's altruistic utility function subject to a linear UPF:

$$\hat{u} = \arg\max W(u) \text{ s.t.} \sum_{i=1}^{n} \beta_i u_i = M.$$

Note that the TU setting corresponds to the special case where $\beta_1 = \beta_2 = ... = \beta_n$.

Similarly, denote by $H \in \mathbb{R}$ the result of the minimization problem of the head's linear "expenditure" function subject to a given altruistic utility level, $\bar{W} \in \mathbb{R}$:

$$H = \arg\min \sum_{i=1}^{n} \beta_i u_i \text{ s.t. } W(u) = \bar{W}$$

Like Bergstrom (1989) and Dijkstra (2007) we assume that the head treats the agents' utilities as *normal* goods:

Normality (N) $\partial \hat{u} / \partial M \geq 0$.

In addition we will assume that each child's utility is treated as a *Hicksian substitute*:

Hicksian Substitutes (H) $S_{ij} \equiv \partial H_i / \partial \beta_j > 0$ for all $i \neq j$.

Let $u = (u_1, ..., u_n)$, then the head's problem is given by

$$u^* = \arg \max_{u \in \partial UP(a)} W(u).$$

Lemma 1 *Assumptions (N) and (H) together imply ATU-normality.*

Proof. Follows immediately from Blomquist (1989), Theorem 2, utilizing the fact that the g_i's are convex functions.[5] ■

Remark 4 *It follows from the monotonicity property of W that ATU-normality implies* $\frac{\partial u_i^*}{\partial \lambda} \times \frac{\partial W}{\partial \lambda} \geq 0$ *for all* $i = 1, ..., n$ *and all* $\lambda \in \lambda(A)$ $\{[u_i^*(\lambda') - u_i^*(\lambda)] \times [W(\lambda') - W(\lambda)] \geq 0\}$

These transformations would be irrelevant if the head's altruistic utility function only distributed income differently as ordinal properties of agents' utility functions change. However, as in consumer theory, working with a linear budget constraint (when TU holds) or with a nonlinear budget constraint (when ATU holds but not TU) will have important implications on the demand for a child's utility keeping the ordinal altruistic utility function of the parent the same. Thus cardinal properties of agents' utility functions matter. (See Moulin 1988 for a summary on interpersonal comparison in the context of social welfare functions and Blomquist 1989 on demand for goods.) The same issue of interpersonal welfare comparisons arises in Bergstrom (1989).

[5] See appendix for intuition of Blomquist's theorem.

5 Main Result

Before stating our main theorem, we state a Lemma according to which ATU implies that the utility possibility sets are nested:

Lemma 2 *If U exhibits ATU, then $UP(a) \subseteq UP(a')$ or $UP(a') \subseteq UP(a)$ for all $a, a' \in A$.*

Proof. Let U be such that ATU holds, and consider a change from a to a'. By ATU, $\partial UP(a) = \{u \in UP(a) : \sum g_i(u_i) = \lambda(a)\}$ and $\partial UP(a') = \{u \in UP(a') : \sum g_i(u_i) = \lambda(a')\}$. Hence, either $\lambda(a) = \lambda(a')$, in which case $UP(a) = UP(a')$; or $\lambda(a) > \lambda(a')$ implying $UP(a) \supset UP(a')$; or $\lambda(a) < \lambda(a')$ implying $UP(a) \subset UP(a')$.[6] ■

Corollary 1 *Under ATU, kids are well-behaved if and only if $\frac{\partial u_i^*}{\partial \lambda} \times \frac{\partial W(u^*)}{\partial \lambda} \geq 0$ for all $i = 1, ..., n$ and all $a \in A$.*

We now state our main theorem:

Theorem 5 *Assuming the head's preferences satisfy (N) and (H), kids are well-behaved if and only if the agents' utility profile exhibits ATU.*

Proof. For sufficiency, if U exhibits ATU, Lemma 2 applies. Hence, kids are well-behaved if $\frac{\partial u_i^*}{\partial \lambda} \times \frac{\partial W(u^*)}{\partial \lambda} \geq 0$,for all $i = 1, ..., n$ and all $a \in A$, which is implied by assumptions (N) and (H) through Lemma 1.

For necessity, the proof is split into two parts. First we prove that whenever UPFs cross,[7] there is an incentive for kids not to behave. Second we show that only ATU yields UPFs that do not cross.

Consider an agent i and two action profiles, a and b, such that $a_i \neq b_i$ and $b_j = a_j$ for all $j \neq i$. Assume $\partial UP(a)$ and $\partial UP(b)$ cross at u^a such that $\partial UP(a) \cap \partial UP(b) = u^a$. ■

Moreover, denote

$$u^a = \arg \max_{u \in UP(a)} W(u)$$

and

$$u^b = \arg \max_{u \in UP(b)} W(u).$$

Note that

$$u^b \neq u^a$$

if preferences of the household head are strictly convex; the profile of utility levels that maximizes the head's welfare function given $\partial UP(b)$ must be different from u^a. Since the two $UPFs$ cross at u^a, this implies that neither

$$u^a > u^b$$

[6] This Lemma follows immediately from Lemma 2 in Gugl and Leroux (2011).

[7] This first part is repeating Berstrom (1989)'s argument.

nor

$$u^a < u^b$$

Moreover assume for agent i

$$u_i^a > u_i^b.$$

Since u^a is optimal given $\partial UP\,(a)$ and it is feasible given $\partial UP\,(b)$ we know that

$$W\left(u^b\right) > W\left(u^a\right).$$

However, agent i is better off choosing a_i given $b_j = a_j$ for every agent j other than i. This means b is not a subgame perfect Nash equilibrium even though $W\left(u^b\right)$ is the maximum of the altruistic head's welfare. It follows that if rotten kids must be well behaved for any (well-behaved) welfare function and all technologies, UPFs cannot cross.

For the second part we need to find out the structure of utilities that ensures that UPFs do not cross. In order to do that we use the concept of compensating variation (CV). Again we consider two action profiles, call them a and b. If $\partial UP\,(a) \cap \partial UP\,(b) = \varnothing$, we can assume without loss of generality that for any $u \in \partial UP\,(a)$ there exists a utility profile $u' \in \partial UP\,(b)$ such that

$$u' > u.$$

A switch from u to u' thus increases each agent's utility (and hence the altruistic head's welfare). Note that any utility distribution is reached by transferring money once an action profile is determined. We now ask the question, what transfers lead to the same utility distribution $u' \in \partial UP\,(b)$ if action a is taken instead of action b. Denote by $I\,(a, u, u')$ the amount of income necessary to achieve this goal and by $t_i\,(a, u_i, u_i')$ the transfer to agent i to achieve u_i' with action a. This is the definition of compensating variation. Formally, we write:

$$CV_i\,(u', a) = t_i\,(a, u_i, u_i'),$$

and

$$I\,(a, u, u') = \sum_i CV_i\,(u', a)$$

If utility profiles are such that no UPFs cross, the compensating variation must be the same for all utility distributions $u \in \partial UP\,(a)$. Otherwise, we can always find another action c that yields an income $I\,(c) = I\,(b) - \delta$ where $\delta > 0$ such that there exist $\underline{u}, \overline{u} \in \partial UP\,(c)$ with $I\,(a, u, \overline{u}) > 0$ and $I\,(a, u, \underline{u}) < 0$, implying that $\partial UP\,(a) \cap \partial UP\,(c) \neq \emptyset$; a contradiction.

By construction,

$$\begin{aligned} u_i' &= U_i\,(a, t_i(a) + t_i\,(a, u_i, u_i')) \\ \Longrightarrow & \quad t_i(a) + t_i\,(a, u_i, u_i') = U_i^{-1}\,(b, u_i) \end{aligned}$$

Plugging this into the formula for $I\,(a, u, u')$ we find

$$I\,(a, u, u') = \sum_i U_i^{-1}\,(b, u_i) - I(a)$$

Because $I(a)$ is constant, in order to have $I(a, u, u') = I(a, u, u'')$ for all $u', u'' \in \partial UP(b)$, it must be that there exists some $\lambda \in \mathbb{R}$ such that

$$\sum_i U_i^{-1}(a, u_i') = \lambda$$

for all $u' \in \partial UP(b)$, which is precisely the definition of ATU.

Corollary 2 *Suppose preferences over alternative distributions of agents' utilities take the form of a generalized utilitarian social welfare function*

$$W(u) = \sum_{i=1}^{n} \gamma_i(u_i)$$

where $\gamma_i(\cdot)$ is assumed to be strictly concave and increasing.[8] *Kids are well-behaved if and only if the profile of utility functions U exhibits ATU.*

Proof. It is well known from consumer theory that a separable utility function of the form presented here treats every good as a normal good and as a Hicksian substitute (e.g. Gravelle and Rees 2004, Mas-Collel et al. 1995). Hence assumptions N and H apply. ■

6 Conclusion

Our main result is the following. Under the assumption that the children's utilities are treated by the parent as normals goods—like in Bergstrom (1989) and Dijkstra (2007)—and as Hicksian substitutes—unlike in Bergstrom (1989) and Dijkstra (2007)—, the RKT holds if and only if the household members' utility functions lead to ATU.

Bergstrom (1989)'s proof of the Rotten Kid Theorem (RKT) requires transferable utility, because he assumes that the head treats every child's utility as a normal good in her altruistic utility function: Only if any action by a child, given the actions of all the other children, shifts the utility possibility frontier parallel and it is a simplex, are all the children guaranteed to benefit from taking efficient actions. In comparison to Bergstrom (1989), we can weaken the requirement of TU to ATU by imposing a stronger, yet reasonable condition on the parent's altruistic utility function. In addition to normality (N) we also assume that the head treats each kid's utility as a Hicksian substitute (H).

A particularly appealing class of social welfare functions that satisfies both conditions is the class of Generalized Utilitarian Social Welfare Functions. Separability guarantees that welfare comparisons are independent of non-concerned agents. That is, if we consider actions that redistribute welfare among a subset of agents, we can determine which action is desirable by focussing on the

[8] The result in this paper can be generalized to $\gamma_i(\cdot)$ being a concave but not strictly concave function. In this case a rule to break ties would have to be applied whenever more than one point on the utility possibility frontier maximizes the head's altruistic utility function. Gugl and Leroux (2011) adopt this approach in the context of GUBS.

changes in welfare of this subset of agents only; the fixed welfare level of the non-concerned agents plays no role in the judgement of which action should be taken.

Although utility profiles satisfying ATU share with TU the same ordinal properties of utility functions, ATU allows us to take into account decreasing marginal utility of money. Since we cannot escape some form of interpersonal comparison even if we make no stronger assumption of an altruistic utility function than that it treats kids as normal goods, ruling out the possibility of agents experiencing decreasing marginal utility of money in order for the RKT to hold seems rather restrictive. Of course, a weakening of the assumption of TU comes at a price, but the combination of conditions presented here offers an interesting alternative.

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8 Appendix

In order to keep the exposition succinct, vector notation is used whenever possible. Following Blomquist (1989), we introduce a linearized utility possibility constraint with

$$\beta(\lambda(a))\,u^* \equiv M(\lambda(a)) \tag{4}$$
$$\beta_i(\lambda(a)) \equiv g_i'(u_i^*) \tag{5}$$

such that

$$\begin{array}{c} \max_u W(u_1,...,u_n) \\ s.t. \quad \sum_i \beta_i(\lambda(a))\,u_i = M(\lambda(a)) \end{array} \tag{6}$$

yields $u^L = u^*$.

By our normality assumption, $\partial u^L/\partial M > 0$. By our assumption of Hicksian substitutes,

$$S = \begin{bmatrix} S_{11} & \cdots & S_{1n} \\ \vdots & \ddots & \vdots \\ S_{n1} & \cdots & S_{nn} \end{bmatrix}$$

has negative diagonal elements and positive elements off the diagonal. Given the way we linearized the utility possibility frontier,

$$u^*(\lambda(a)) = u^L(\beta(\lambda(a)), M(\lambda(a))) \tag{7}$$

Taking the derivative with respect to a will change λ. Hence we will from now on suppress a and focus on the impact of λ on each u_i^*. That is, for any $i \in N, j \in N$

$$\frac{\partial u_i^*}{\partial \lambda} = \sum_j \frac{\partial u_i^L}{\partial \beta_j}\frac{\partial \beta_j}{\partial \lambda} + \frac{\partial u_i^L}{\partial M}\frac{\partial M}{\partial \lambda} \tag{8}$$

By (4)

$$\begin{aligned} \frac{\partial M}{\partial \lambda} &= \sum_i \frac{\partial \beta_i}{\partial \lambda}u_i^* + \beta_i\frac{\partial u_i^*}{\partial \lambda} \\ &= 1 + \sum_i \frac{\partial \beta_i}{\partial \lambda}u_i^* \end{aligned} \tag{9}$$

since by Engel aggregation, the sum of the second term in (9) yields 1. By the Slutsky equation,

$$\frac{\partial u_i^L}{\partial \beta_j} = S_{ij} - \frac{\partial u_i^L}{\partial M}u_j^L$$

and hence we cna write (8) as

$$\frac{\partial u_i^*}{\partial \lambda} = \sum_j \left(S_{ij} - \frac{\partial u_i^L}{\partial M} u_j^L \right) \frac{\partial \beta_j}{\partial \lambda} + \frac{\partial u_i^L}{\partial M} \frac{\partial M}{\partial \lambda}$$

Substituting (9) in the equations above yiels

$$\frac{\partial u_i^*}{\partial \lambda} = \sum_j \left(S_{ij} - \frac{\partial u_i^L}{\partial M} u_j^L \right) \frac{\partial \beta_j}{\partial \lambda} + \frac{\partial u_i^L}{\partial M} \left(1 + \sum_i \frac{\partial \beta_i}{\partial \lambda} u_i^* \right)$$

Since $u_i^* = u_i^L$,

$$\frac{\partial u_i^*}{\partial \lambda} = \sum_j S_{ij} \frac{\partial \beta_j}{\partial \lambda} + \frac{\partial u_i^L}{\partial M}$$

By (5)

$$\frac{\partial \beta_j}{\partial \lambda} = g_j'' \frac{\partial u_j^*}{\partial \lambda}$$

Combining all equations we can now relate the non-linear "income effect" with the normality assumption

$$\frac{\partial u_i^*}{\partial \lambda} = \sum_j S_{ij} g_j'' \frac{\partial u_j^*}{\partial \lambda} + \frac{\partial u_i^L}{\partial M}$$

In matrix notation, the vector of welfare distributions changes with a change in action a and hence λ by

$$\frac{\partial u^*}{\partial \lambda} = \left[I - S g'' \right]^{-1} \frac{\partial u^L}{\partial M}.$$

We know the properties of the substitution matrix S. With $g_i'' \geq 0$, $\left[I - S g'' \right]^{-1} > 0$. See Blomquist (1989, p.283-84). Hence $\frac{\partial u^*}{\partial \lambda} > 0$.