CESifo Working Papers

Firm-Productivity and Export under Non-Constant Marginal Costs

Arijit Mukherjee

CESifo
Center for Economic Studies & Ifo Institute

Firm-Productivity and Export
under Non-Constant Marginal Costs

Abstract

Recent theoretical research shows that exporters are more productive than nonexporters. We show that this result holds almost trivially for the case of constant marginal cost of production, as mainly assumed in the literature, but it may not hold true if the marginal cost is not constant. Our result provides a simple explanation for recent empirical evidence showing exporters are less productive than non-exporters.

JEL-Codes: F100, F120.

Keywords: export, non-constant marginal cost of production, productivity.

Arijit Mukherjee
Nottingham University Business School
Jubilee Campus, Wollaton Road
United Kingdom – Nottingham, NG8 1BB
arijit.mukherjee@nottingham.ac.uk

February 2016

Firm-productivity and export under non-constant marginal costs

1. Introduction

Melitz (2003) analyses the relationship between firm-productivity and export decision. Using constant marginal costs of production, he shows that exporters are more productive than non-exporters. While this finding got support from many empirical papers (Greenaway and Kneller, 2007 and Bernard et al., 2012), several recent empirical papers suggest the opposite relation, implying that exporters are lower productive than non-exporters (Hallak and Sivadasan, 2013, Wagner, 2013 and 2014).

Given this background, the purpose of this paper is two-fold. First, we provide a simple proof for Melitz's result and show that his result holds almost trivially for the case of constant marginal cost of production, as mainly assumed in the literature. Second, we show that his result may not hold under non-constant marginal costs of production.[1]

If the marginal cost of production is constant, 'higher productive firms export' follows immediately from a firm's standard investment decision, where investment occurs if the firm's operating profit is higher than the fixed cost. Since a lower marginal cost of production, which can be the outcome of a higher productivity of the firm, increases operating profit of the firm, a relatively higher productive firm will have higher incentive for investment.

The situation is not so straightforward if the marginal cost is not constant. Unlike constant marginal cost, the profits from domestic sell are affected by the export decision if

---

[1] See, e.g., Ahn and McQuoid (2012), Vannoorenberghe (2012) and Senalp (2015) for some recent works on international trade with non-constant marginal costs. However, the focus of those papers is different from ours.

the marginal cost is not constant. We first show the condition required for Melitz's result under non-constant marginal costs and then provide an example to show that more productive firms can be non-exporters. The difference between the results under non-constant marginal cost of production and the constant marginal cost of production is attributed to the fact that export affects the concerned firm's profit from domestic sells under the former but not under the latter.

The remainder of the paper is organised as follows. Section 2 describes the model and derives the results. Section 3 concludes.

2. The model and the results

2.1. Constant marginal cost of production

The purpose of this subsection is to provide a simple proof for Melitz's result showing that exporters are more productive than non-exporters.

Assume that there is a firm, called firm 1, which sells its product in its domestic country and is deciding whether or not to export to a foreign country. If firm 1 exports to the foreign country, it needs to incur a fixed cost, F. For simplicity, we assume away any variable cost of exporting.

Assume that the total cost of firm 1 is $C = C(L,Q)$, where L is the inverse of labour-productivity, Q is the total amount of output, $\frac{\partial C}{\partial L} > 0$, $\frac{\partial C}{\partial Q} > 0$ and $\frac{\partial^2 C}{\partial Q^2} = 0$. Hence, we consider here that the marginal cost of production is constant, as assumed in Melitz (2003). Further, as L increases, i.e., productivity falls, it increases cost. We have $Q = q_d$ when firm 1 sells only in the domestic country and $Q = (q_d + q_f)$ when firm 1 sells in the domestic country and also exports to the foreign country.

2

Assume that the demand functions in the domestic country and in the foreign country are $p_d(q_d)$ and $p_f(q_f)$ respectively.

If firm 1 sells only in the domestic country, it determines output to maximise $[p_d(q_d)q_d - C(L, q_d)]$. Denote firm 1's equilibrium profit in this situation by $\pi_1^x(L)$.

If firm 1 sells to the domestic and the foreign countries (i.e., selling in the domestic country and also exporting), it determines the domestic output, q_d, and the amount of export, q_f, to maximise $[p_d(q_d)q_d + p_f(q_f)q_f - C(L, q_d + q_f) - F]$. Since firm 1's equilibrium domestic output is unaffected by the amount of export, firm 1's equilibrium profit is given by $\pi_1^x(L) + \pi_1^f(L) - F$, where $\pi_1^x(L)$ is firm 1's profit from domestic sell and $\pi_1^f(L) - F$ is firm 1's net profit from export.

Firm 1 sells in the domestic market and also exports to the foreign market if $\pi_1^x(L) + \pi_1^f(L) - F > \pi_1^x(L)$ or $\pi_1^f(L) > F$. Using the envelope theorem, it is easy to show that if firm 1's productivity increases, i.e., L decreases, it increases $\pi_1^f(L)$. Hence, firm 1 exports if its productivity is higher than a threshold level, since that creates an operating profit that is higher than the fixed cost. In other words, a relatively higher productive firm sells in the domestic market and also exports while a low productive firm sells in the domestic market only, as shown by Melitz (2003).

The following proposition summarises the above discussion.

Proposition 1: *If the total cost of firm 1 is $C = C(L, Q)$ with $\dfrac{\partial C}{\partial L} > 0$, $\dfrac{\partial C}{\partial Q} > 0$ and $\dfrac{\partial^2 C}{\partial Q^2} = 0$, firm 1 exports if $\pi_1^f(L) > F$, and since a higher productivity of firm 1 increases $\pi_1^f(L)$, a relatively higher productivity of firm 1 encourages it to export.*

2.2. Non-constant marginal cost of production

Now consider a situation where firm 1 faces a non-constant marginal cost of production. Assume that the total cost of firm 1 is $C = C(L,Q)$ with $\dfrac{\partial C}{\partial L} > 0$, $\dfrac{\partial C}{\partial Q} > 0$ and $\dfrac{\partial^2 C}{\partial Q^2} \neq 0$.

If firm 1 sells only in the domestic country, it determines output to maximise $[p_d(q_d)q_d - C(L,q_d)]$ and its equilibrium profit is $\pi_1^x(L)$.

If firm 1 sells in the domestic country and also exports to the foreign country, it determines the domestic output, q_d, and the amount of export, q_f, to maximise $[p_d(q_d)q_d + p_f(q_f)q_f - C(L,q_d + q_f) - F]$. In this situation, denote firm 1's equilibrium profit by $\pi_{df}^* = \widetilde{\pi}_d^* + \widetilde{\pi}_f^*$, where $\widetilde{\pi}_d^*$ and $\widetilde{\pi}_f^*$ are firm 1's profit from the domestic market and export respectively. Since we are considering non-constant marginal cost of production in this subsection, firm 1's profit from domestic market is affected by the amount of export. Hence, unlike constant marginal cost of production, firm 1 sells in the domestic market and exports to the foreign market if $\widetilde{\pi}_d^* + \widetilde{\pi}_f^* - F > \pi_d^*$ or $(\widetilde{\pi}_d^* - \pi_d^*) + \widetilde{\pi}_f^* \equiv G > F$. Using the envelope theorem, we get that if firm 1's productivity increases, i.e., L decreases, it increases (decreases) G, i.e., $\dfrac{\partial G}{\partial L} < (>)0$, if $\left.\dfrac{\partial C}{\partial L}\right|_{Q=q_d^*} < (>) \left.\dfrac{\partial C}{\partial L}\right|_{Q=(q_d^*+q_f^*)}$. Hence, if the marginal cost is not constant, the result of Melitz (2003), i.e., exporters are more productive than non-exporters, does not hold if $\left.\dfrac{\partial C}{\partial L}\right|_{Q=q_d^*} > \left.\dfrac{\partial C}{\partial L}\right|_{Q=(q_d^*+q_f^*)}$.

The above result suggests that if the marginal cost is not constant, a higher productivity increases (decreases) the incentive for export if the direct cost reduction due to a higher productivity is less (more) at the 'equilibrium domestic sell only' compared to the total equilibrium outputs under 'domestic sell and export'. If the direct effect of a higher

productivity creates lower (higher) cost reduction at the 'equilibrium domestic sell only' compared to the 'equilibrium outputs under domestic sell and export', it creates lower (higher) gain from 'domestic sell only' compared to 'domestic sell and export', thus increasing (decreasing) the incentive for export.

The following proposition is immediate from the above discussion.

Proposition 2: *If the total cost of firm 1 is* $C = C(L,Q)$ *with* $\frac{\partial C}{\partial L} > 0$, $\frac{\partial C}{\partial Q} > 0$ *and* $\frac{\partial^2 C}{\partial Q^2} \neq 0$, *a higher productivity of firm 1 increases (decreases) its incentive for export for*

$$\left.\frac{\partial C}{\partial L}\right|_{Q=q_d^*} < (>) \left.\frac{\partial C}{\partial L}\right|_{Q=(q_d^*+q_f^*)}.$$

2.2.1. An example

Now consider an example with a specific cost function. Assume that the cost function is $C(L,Q) = 10L^2Q - L^3Q^2$. This cost function has the following property:

$\frac{\partial C}{\partial Q} = 2L^2(5 - LQ) > 0$ for $LQ < 5$, $\frac{\partial^2 C}{\partial Q^2} = -2L^3 < 0$ and $\frac{\partial C}{\partial L} > 0$ for $3L < 20$. Hence, we consider the parameter values such that *LQ < 5* and *3L < 20*. We consider in the following analysis that $0 < L < .5$ and the domestic and foreign demand functions are respectively $P_d = 10 - q_d$ and $P_f = 10 - q_f$.

Standard calculation gives the following equilibrium values. If firm 1 sells only in the domestic country, its equilibrium output and profit are respectively $q_d^* = \frac{10(1 - L^2)}{2(1 - L^3)}$ and $\pi_d^* = \frac{(10 - 10L^2)^2}{4(1 - L^3)}$. If firm 1 sells in the domestic country and also exports, its equilibrium

outputs and profit are respectively $q_d^* = q_f^* = \dfrac{5(1-L^2)}{(1-2L^3)}$ and $\pi_{df}^* = \dfrac{50(1-L^2)^2}{(1-2L^3)} - F$. It is easy to

check that the equilibrium values satisfy the constraint $LQ < 5$ and $3L < 20$ for $0 < L < .5$.

Comparing π_d^* and π_{df}^*, we get that firm 1 exports if

$$F < G \equiv \frac{50(1-L^2)^2}{(1-2L^3)} - \frac{(10-10L^2)^2}{4(1-L^3)} = \frac{25(1-L)(1+L)^2}{(1+L+L^2)(1-2L^3)}.$$
We get that

$$\frac{\partial G}{\partial L} = \frac{25L(1+L)[L+5L^2-4(1-L)(1-2L^3-L^4)]}{(1+L+L^2)^2(1-2L^3)^2}.$$
Hence, $\dfrac{\partial G}{\partial L} > (<)0$ for

$[L+5L^2-4(1-L)(1-2L^3-L^4)] > (<)0$. We get that $[L+5L^2-4(1-L)(1-2L^3-L^4)] < 0$ at

$L = 0$ but $[L+5L^2-4(1-L)(1-2L^3-L^4)] > 0$ at $L = .5$, implying that $\dfrac{\partial G}{\partial L} > 0$ for L close to

.5.

Plotting $\dfrac{\partial G}{\partial L}$ with respect to $L \in [0,.5]$, we get the following diagram (**Figure 1**),

which shows that a higher productivity of firm 1 decreases the incentive for export (i.e.,

$\dfrac{\partial G}{\partial L} > 0$) for L close to .5 but it increases the incentive for export (i.e., $\dfrac{\partial G}{\partial L} < 0$) otherwise.
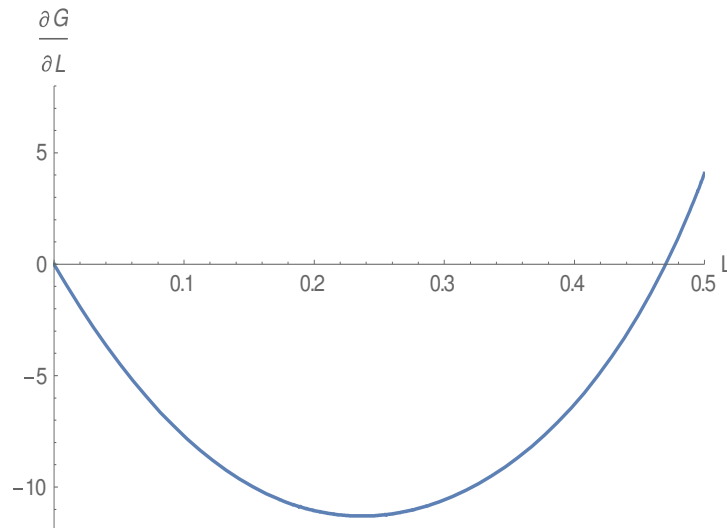

Figure 1: The relationship between $\dfrac{\partial G}{\partial L}$ and L

One can find a similar result by comparing $\left.\dfrac{\partial C}{\partial L}\right|_{Q=q_d^*}$ and $\left.\dfrac{\partial C}{\partial L}\right|_{Q=(q_d^*+q_f^*)}$. We have

$$\frac{\partial C}{\partial L} = 2L^2\left(5-Lq\right).$$
Hence,
$$\left.\frac{\partial C}{\partial L}\right|_{Q=q_d^*} = \frac{25L\left(1+L\right)\left(4+L+L^2\right)}{\left(1+L+L^2\right)^2}$$
and

$$\left.\frac{\partial C}{\partial L}\right|_{Q=(q_d^*+q_f^*)} = \frac{100L\left(1-L^2\right)\left(2-3L-L^3\right)}{\left(1-2L^3\right)^2}.$$
Denoting
$$\left(\left.\frac{\partial C}{\partial L}\right|_{Q=q_d^*}-\left.\frac{\partial C}{\partial L}\right|_{Q=(q_d^*+q_f^*)}\right) = \frac{\Delta C}{\Delta L}$$
and

plotting $\dfrac{\Delta C}{\Delta L}$ with respect to $L \in [0,.5]$, we get the following diagram (**Figure 2**), which is
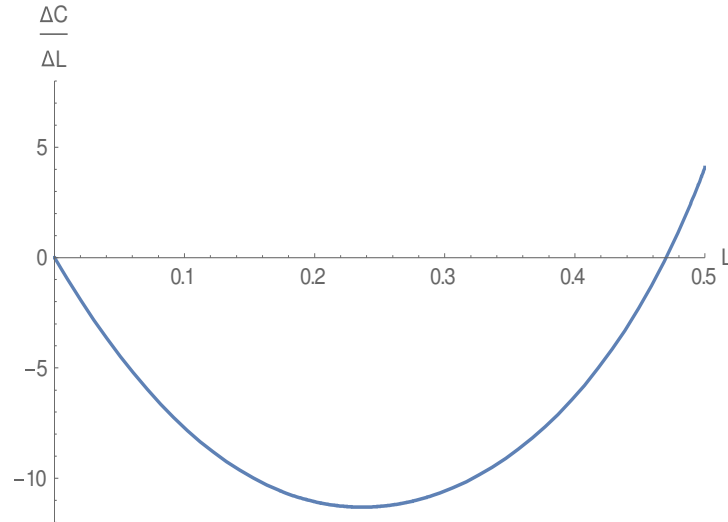
similar to Figure 1.

Figure 2: The relationship between $\dfrac{\Delta C}{\Delta L}$ and L

3. Conclusion

We show that the result showing exporters are more productive than non-exporters holds almost trivially for the case of constant marginal cost of production, as mainly assumed in the literature. However, this result may not hold if the marginal cost is not constant. Our result provides a simple explanation for recent empirical works showing exporters are less productive than non-exporters.

References

Ahn, J. and A.F. McQuoid, 2012, 'Capacity constrained exporters: micro evidence and macro implications', *Mimeo*, Florida International University, USA.

Bernard, A.B., J.B. Jensen, S.J. Redding and P.K. Schott, 2012, 'The empirics of firm heterogeneity and international trade', *Annual Review of Economics*, 4: 283-313.

Greenaway, D. and R. Kneller, 2007, 'Firm heterogeneity, exporting and foreign direct investment', *Economic Journal*, 117: F134-F161.

Hallak, J.C. and J. Sivadasan, 2013, 'Firms' exporting behavior under quality constraints', *Journal of International Economics*, 91: 53-67.

Melitz, M.J., 2003, 'The impact of trade on intra-industry reallocations and aggregate industry productivity', *Econometrica*, 17: 1692-1725.

Senalp, U., 2015, 'Essays on firm heterogeneity and international trade', *PhD thesis*, Loughborough University, UK.

Vannoorenberghe, G., 2012, 'Firm-level volatility and exports', Journal of International Economics, 86: 57–67.

Wagner, J., 2013, 'Are low-productive exporters marginal exporters? evidence from Germany', *Economics Bulletin*, 33: 467-481.

Wagner, J., 2014, 'Low-productive exporters are high-quality exporters: evidence from Germany', *Working Paper Series in Economics, No. 292*, University of Lüneburg.