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Abstract

We investigate the effect of a ban on third-degree price discrimination on the sustainability of collusion. We build a model with two firms that may be able to discriminate between two consumer groups. Two cases are analyzed: (i) Best-response symmetries so that profits in the static Nash equilibrium are higher if price discrimination is allowed. (ii) Best-response asymmetries so that profits in the static Nash equilibrium are lower if price discrimination is allowed. In both cases, firms' discount factor has to be higher in order to sustain collusion in grim-trigger strategies under price discrimination than under uniform pricing.

JEL-Codes: D430, K210, L130, L410.

Keywords: collusion, duopoly, grim-trigger strategies, third-degree price discrimination.

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1. Introduction

A classic topic in the economic literature on antitrust are the effects of price discrimination – or the ban of it – on consumer surplus and total welfare. The static effects of permitting third-degree price discrimination are by now well understood. Little is known, however, regarding the effects of a ban on price discrimination on dynamic competition. Does a legal ban on price discrimination facilitate a collusive outcome instead of promoting competition? Whether tacit collusion can be sustained in equilibrium depends, on the one hand, on the gains from collusion, and, on the other hand, on the temptation of a firm to deviate unilaterally from the collusive agreement. Permitting price discrimination affects both the gains from collusion and the temptation to deviate. Under price discrimination, a deviation from the collusive agreement can be targeted to specific consumer groups, enhancing the one-period profit from a deviation. Due to this effect, collusion is harder to sustain if price discrimination is permitted. On the other hand, price discrimination may enhance the gains from collusion, in particular when price discrimination enhances competition.[1] Thus, there can be opposing effects at play regarding whether a ban on price discrimination facilitates collusion or supports competition.

Our main finding is that permitting price discrimination enhances the temptation to deviate significantly and thus hampers the formation of cartels; i.e., the set of discount factors for which collusion can be sustained is larger under uniform pricing than under price discrimination. This result holds true for both cases: (i) best-response symmetries – static Nash equilibrium profits are higher if price discrimination is allowed and (ii) best-response asymmetries – static Nash equilibrium profits are lower if price discrimination is allowed.[2]

To the best of our knowledge, this is the first paper that investigates the effect of a ban on third-degree price discrimination on the "likelihood" of collusion to occur. The analysis of third-degree price discrimination is extended to competitive markets – i.e., imperfectly competitive markets where firms produce differentiated products, by Borenstein (1985), Holmes (1989), and Jacques-Francois Thisse (1988). One important finding of this literature is that – with best-response

[1]For instance, Gehrig and Stenbacka (2005, p.152) conjecture that "[a]s price discrimination leads to more intense competition than uniform prices, the industry can sustain implicit collusion under circumstances with lower discount factors.[...] discriminatory pricing schemes promote the stability of collusion..."

[2]The phrases best-response symmetry and best-response asymmetry go back to Corts (1998). A review of this literature is provided by Stole (2007) and also by Armstrong (2006).

asymmetries – firms are better off when committing to uniform pricing but price discrimination emerges as the unique equilibrium outcome. In other words, firms face a *prisoners' dilemma* situation.

The sustainability of collusion when firms are able to price discriminate is analyzed by Liu and Serfes (2007) and Colombo (2010b). Liu and Serfes (2007) analyze a model where two horizontally differentiated firms can acquire information about consumers' preferences. They investigate when firms decide to collude on a uniform price and when do they decide to collude on discriminatory prices. The result depends on how costly it is to acquire information. Colombo (2010b) investigates how the degree of differentiation affects the sustainability of collusion on a uniform price and on discriminatory prices.[3] The crucial difference of these papers to our approach is that, if a firm deviates from the collusive agreement, this firm acquires information and engages in price discrimination. In other words, even if the firms agree to charge a uniform price the optimal deviation exhibits price discrimination. In our model price discrimination may be banned by law. If this is the case and the firms collude, they collude on a uniform price. Crucially, if a firm now deviates, it is still restricted by law to charge a uniform price. Moreover, these papers only investigate situations with best-response asymmetries, while we analyze collusion also under best-response symmetries.

2. The Model

We consider an industry with two symmetric firms, A and B, producing differentiated goods. Each firm produces at constant marginal cost $c \geq 0$ and without fixed costs.

There is a continuum of consumers with measure normalized to one. A consumer is interested in purchasing at most one unit. We use a simple discrete choice model with perfectly negatively correlated brand preferences – similar to Hotelling (1929)'s model. The utility of a consumer with type (θ, ρ) is

$$u = \begin{cases} v - \rho\theta - p_A & \text{if purchasing from firm } A \\ v + \rho\theta - p_B & \text{if purchasing from firm } B \;, \\ 0 & \text{if not purchasing a good} \end{cases} \tag{1}$$

[3] Similar questions are also addressed by Colombo (2009, 2010a). The former analyzes varies degrees of differentiation and the latter various forms of information (shared or unshared information between firms).

where p_i is the price charged by firm $i \in \{A, B\}$ from this consumer. We assume that θ is uniformly distributed around mean zero; i.e., $\theta \sim U[-\bar{\theta}, \bar{\theta}]$ with $\bar{\theta} > 0$. Moreover, $\rho \in \{\rho_L, \rho_H\}$ with $0 < \rho_L < \rho_H$. A fraction α of the consumers has type ρ_L and the remaining $1 - \alpha$ consumers have type ρ_H. The ρ-types are distributed independently from the θ-types. Consumers of type ρ_L react more strongly to price differences than consumers of type ρ_H and are in that sense more price sensitive; types ρ_H could be students and types ρ_L could be non-students. Consumers with a high (low) θ have strong brand preferences. Those with high (low) θs strongly prefer brand B (A) to brand A (B). These brand preferences may be correlated with observable characteristics like place of residence, age, gender, or revealed by the customer's purchase history.

Time is discrete and denoted by $t = 0, 1, \ldots, \infty$. At the beginning of each period, each firm chooses a price for its own product without knowing the other firm's price choice. Consumers are interested in purchasing one unit every period and are unable to store the good. Consumers first observe the prices and decide thereafter whether to buy and if they buy from which firm. At the end of each period, all choices are publicly observed. Firms discount future profits at the constant rate $\delta \in (0, 1)$.

The main research question is how does a ban on price discrimination affect the sustainability of collusion. The infinitely repeated price game has a continuum of equilibria and thus we have to rely on the comparison of equilibria in certain strategies in order to answer this question. We focus on the sustainability of collusion as a subgame perfect equilibrium in grim-trigger strategies. In the punishment phase – after a deviation by one of the two firms –, the strategies prescribe that the prices in any period are equal to the static Nash equilibrium prices. In the cooperation phase, the strategies prescribe to charge the price (or prices) that maximizes joint profits. As it is well-known, the grim-trigger strategy is an equilibrium only if the firms are sufficiently patient, i.e., if δ is sufficiently high.

In the following, we will analyze three scenarios.

(i) Uniform pricing (U): Both firms are restricted to charge the same price from all consumers.

(ii) Price discrimination with best-response symmetries (DS): Firms can discriminate between consumers with ρ_L and ρ_H; i.e., each firm $i = A, B$ can charge two prices, p_i^L and p_i^H, with the former being charged from consumers with type ρ_L and the latter from consumers with type ρ_H.

(iii) Price discrimination with best-response asymmetries (DA): Firms can discriminate between consumers with $\theta \in [-\bar{\theta}, 0)$ and consumers with $\theta \in [0, \bar{\theta}]$. Each firm $i = A, B$ charges two prices, p_i^- and p_i^+, with the former being charged from consumers with type $\theta < 0$ and the latter from consumers with type $\theta \geq 0$.

For all scenarios $j \in \{U, DS, DA\}$, we calculate the one period static Nash equilibrium profit of a firm, π_j^N, the one period (maximal) profit of a firm under collusion, π_j^C, and the one period profit a firm makes when deviating optimally from the collusive agreement, π_j^D. It is well-known that collusion is sustainable in scenario j – in grim-trigger strategies – if and only if $\sum_{t=0}^{\infty} \delta^t \pi_j^C \geq \pi_j^D + \sum_{t=1}^{\infty} \delta^t \pi_j^N$, which is equivalent to

$$\delta \geq \frac{\pi_j^D - \pi_j^C}{\pi_j^D - \pi_j^N} =: \bar{\delta}_j. \tag{2}$$

In order to answer our main research question, we compare $\bar{\delta}_U$ to $\bar{\delta}_{DS}$ and $\bar{\delta}_U$ with $\bar{\delta}_{DA}$.

We will focus on equilibria in which the market is fully covered. Moreover, we restrict attention to situations where a firm that deviates does not serve the whole market; i.e., both firms have a positive demand even if one firm deviates from the collusive agreement. The former provides a lower bound on $v - c$, while the latter provides an upper bound. The precise formal assumptions that guarantee this are provided below.

3. The Analysis

3.1. Uniform Pricing

In this subsection, we assume that price discrimination is banned and thus each firm $i = A, B$ sets a single price p_i. There are two marginal consumers, denoted by $\hat{\theta}_k$ for $k = L, H$, who are indifferent between purchasing from firm A and firm B. Formally, the marginal consumers are given by $\hat{\theta}_k = (p_B - p_A)/2\rho_k$. Consumers with a lower θ-type than the marginal consumer purchase from firm A, while those with a higher type purchase from firm B. The demand functions of firm A and B – if the market is fully covered – are

$$D_A(p_A, p_B) = \alpha \left[\frac{p_B - p_A}{2\rho_L} + \bar{\theta}\right] \frac{1}{2\bar{\theta}} + (1 - \alpha) \left[\frac{p_B - p_A}{2\rho_H} + \bar{\theta}\right] \frac{1}{2\bar{\theta}} \tag{3}$$

$$\text{and} \quad D_B(p_B, p_A) = \alpha \left[\bar{\theta} - \frac{p_B - p_A}{2\rho_L}\right] \frac{1}{2\bar{\theta}} + (1 - \alpha) \left[\bar{\theta} - \frac{p_B - p_A}{2\rho_H}\right] \frac{1}{2\bar{\theta}}, \tag{4}$$

respectively. Firm i's profit function is $\pi_i(p_i, p_j) = (p_i - c)D_i(p_i, p_j)$ with $i, j \in \{A, B\}$ and $i \neq j$.

It is readily established that in the static Nash equilibrium each firm charges $p_U^N = c + 2R\bar{\delta}$ and makes a profit of

$$\pi_U^N = R\bar{\theta}, \tag{5}$$

where

$$R := \frac{\rho_L \ \rho_H}{(1-\alpha)\rho_L + \alpha\rho_H}. \tag{6}$$

Now suppose that the two firms form a cartel and collude, i.e. they maximize joint profits. If it is optimal to serve all consumers, then the highest price that can be charged is $p_U^C = v$. The corresponding per firm profit is

$$\pi_U^C = \frac{1}{2}(v-c). \tag{7}$$

Finally, we derive the optimal static deviation from the collusive agreement. If, say firm A, charges $p_A = v$, the best-response of firm B is to charge $p_U^D = (v+c)/2 + R\bar{\theta}$. The profit of firm B from this deviation is

$$\pi_U^D = \frac{1}{4R\bar{\theta}}\left[\frac{1}{2}(v-c) + R\bar{\theta}\right]^2. \tag{8}$$

Now we can state our first finding.

Lemma 1 (Uniform Pricing)**.** *Suppose that*

$$\frac{2\rho_H}{(1-\alpha)\rho_L + \alpha\rho_H}\rho_L\bar{\theta} < v - c < \frac{(4-4\alpha)\rho_L + (2+4\alpha)\rho_H}{(1-\alpha)\rho_L + \alpha\rho_H}\rho_L\bar{\theta}. \tag{9}$$

Then, under uniform pricing, collusion can be sustained in grim-trigger strategies if and only if $\delta \geq \bar{\delta}_U$, with

$$\bar{\delta}_U = \frac{[(v-c)/2 - R\bar{\theta}]^2}{[(v-c)/2 + R\bar{\theta}]^2 - 4R^2\bar{\theta}^2}. \tag{10}$$

The lower bound on $v-c$ provided by (9) ensures that the static Nash equilibrium is interior; i.e., the market is fully covered even if a firm slightly deviates from the Nash equilibrium price.[4] The upper bound on $v-c$ ensures that a firm that unilaterally deviates from the collusive agreement does not want to serve the whole market; i.e., completely undercutting the rival is not optimal.[5] The critical discount factor $\bar{\delta}_U$ then follows almost immediately from inserting the three static profits derived above into (2).

[4]For lower values on $v-c$ there is a "kinked equilibrium" in which the marginal consumer is indifferent between purchasing from A, purchasing from B, and not purchasing the good.

[5]A formal analysis of these issues is presented in Appendix A.

3.2. Price Discrimination with Best-Response Symmetries

Now, we assume that both firms are able and allowed to discriminate between consumers that react strongly to price differences (types ρ_L) and consumers that weakly react to price differences (types ρ_H). Each firm i sets two prices, p_i^L and p_i^H. With firm i's production technology exhibiting constant returns to scale, it solves two independent maximization problems. The profit of firm $i = A, B$ from consumers with type ρ_k, with $k = L, H$, is

$$\pi_i^k = \omega_k (p_i^k - c) \left[\bar{\theta} - \frac{p_i^k - p_j^k}{2\rho_k} \right] \frac{1}{2\bar{\theta}}, \tag{11}$$

where $\omega_L = \alpha$ and $\omega_H = 1 - \alpha$.

In the static Nash equilibrium, each firm charges the same price from consumers with type ρ_k, which is $p_{DS}^{N,k} = c + 2\rho_k \bar{\theta}$. The corresponding equilibrium profit of a firm from both markets is

$$\pi_{DS}^N = [\alpha \rho_L + (1-\alpha)\rho_H]\bar{\theta}. \tag{12}$$

Suppose the firms collude, i.e. they maximize joint profit. If it is optimal to serve all consumers, the optimal price is $p_{DS}^C = v$ for both consumer types. The corresponding per firm profit is

$$\pi_{DS}^C = \frac{1}{2}(v - c). \tag{13}$$

Suppose now that one firm deviates from the collusive agreement while the other one sticks to it, charging the cartel price $p_{DS}^C = v$. The deviating firm's best response is to charge $p_{DS}^{D,k} = \frac{1}{2}(v + c) + \rho_k \bar{\theta}$ from consumer type ρ_k. The corresponding profit is

$$\pi_{DS}^D = \sum_{k \in \{L,H\}} \frac{\omega_k}{4\rho_k \bar{\theta}} \left[\frac{1}{2}(v - c) + \rho_k \bar{\theta} \right]^2. \tag{14}$$

Lemma 2 (Price Discrimination with Best-Response Symmetries)**.** *Suppose that*

$$2\rho_H \bar{\theta} < v - c < 6\rho_L \bar{\theta}. \tag{15}$$

Then, under price discrimination with best-response symmetries, collusion can be sustained in grim-trigger strategies if and only if $\delta \geq \bar{\delta}_{DS}$*, with*

$$\bar{\delta}_{DS} = \frac{\sum_{k \in \{L,H\}} \frac{\omega_k}{\rho_k} \left[\frac{1}{2}(v - c) - \rho_k \bar{\theta} \right]^2}{\sum_{k \in \{L,H\}} \frac{\omega_k}{\rho_k} \left[\frac{1}{4}(v - c)^2 + (v - c)\rho_k \bar{\theta} - 3\rho_k^2 \bar{\theta}^2 \right]}. \tag{16}$$

Condition (15) ensures that, in the static Nash equilibrium, all consumers are served and that a firm unilaterally deviating from the collusive agreement does not serve all consumers in a market. As before, the critical discount factor is derived by inserting the static profits into (2).

Now we are prepared to answer our main research question for the case of best-response symmetries.

Proposition 1 (Best-Response Symmetries)**.** *Suppose (15) holds and suppose that firms are able to discriminate between consumers with type* ρ_L *and* ρ_H*. Then, banning price discrimination facilitates collusion; i.e.,* $\bar{\delta}_U < \bar{\delta}_{DS}$*.*

According to Proposition 1, it is more likely – in the sense of set inclusion – that a collusive outcome is obtained under uniform pricing than under price discrimination. This result is intuitive. The critical discount factor (2) is increasing in the deviation profit π_j^D and in the static Nash profit π_j^N. It is decreasing in the collusive profit π_j^C. The profit from collusion is independent of whether firms can price discriminate or not. Price discrimination allows for a targeted deviation from the collusive agreement and thus the profit from a unilateral deviation is higher if price discrimination is permitted, $\pi_{DS}^D > \pi_U^D$. Moreover, with best-response symmetries, the static Nash equilibrium profit is higher under price discrimination than under uniform pricing, $\pi_{DS}^N > \pi_U^N$. Hence, the set of discount factors so that collusion can be sustained is larger under uniform pricing.

3.3. Price Discrimination with Best-Response Asymmetries

Suppose now that firms can discriminate between consumers with $\theta \in [-\bar{\theta}, 0)$, the '$(-)$-market', and consumers with $\theta \in [0, \bar{\theta}]$, the '$(+)$-market'. Firms cannot discriminate with respect to type ρ. For simplicity, we assume $\rho_L = \rho_H = \rho$. Let p_i^z with $z \in \{-, +\}$ denote the price that firm $i = A, B$ charges from consumers in the (z)-market. In both markets, there exists a marginal consumer $\hat{\theta}^z = (p_B^z - p_A^z)/2\rho$, who is indifferent between purchasing from firm A and firm B. At prices p_i^+ and p_i^-, demand functions for firms A and B in the two markets are

$$D_A^-(p_A^-, p_B^-) = \left(\bar{\theta} + \frac{p_B^- - p_A^-}{2\rho}\right)\frac{1}{2\bar{\theta}} \qquad D_A^+(p_A^+, p_B^+) = \left(\frac{p_B^+ - p_A^+}{2\rho}\right)\frac{1}{2\bar{\theta}} \tag{17}$$

$$D_B^-(p_A^-, p_B^-) = \left(-\frac{p_B^- - p_A^-}{2\rho}\right)\frac{1}{2\bar{\theta}} \qquad D_B^+(p_A^+, p_B^+) = \left(\bar{\theta} - \frac{p_B^+ - p_A^+}{2\rho}\right)\frac{1}{2\bar{\theta}}. \tag{18}$$

Firm i's profit in market z is $\pi_i^z(p_i^z; p_j^z) = (p_i^z - c)D_i^z(p_i^z, p_j^z)$.

In the static Nash equilibrium, each firm charges $p^{N-}_{DA,A} = p^{N+}_{DA,B} = c + \frac{4}{3}\rho\bar{\theta}$ in its 'strong' and $p^{N+}_{DA,A} = p^{N-}_{DA,B} = c + \frac{2}{3}\rho\bar{\theta}$ in its 'weak' market. The profits made in the strong and the weak market are $4\rho\bar{\theta}/9$ and $\rho\bar{\theta}/9$, respectively. Thus, each firm makes an overall profit of

$$\pi^{N}_{DA,i} = \pi^{N-}_{DA,i} + \pi^{N+}_{DA,i} = \frac{5}{9}\rho\bar{\theta}. \tag{19}$$

Suppose now that the firms form a cartel. Again, we assume that it is optimal to serve all consumers and thus a cartel price of $p^{C}_{DA} = v$ for both markets is optimal. Each firm earns an overall profit of $\pi^{C}_{DA} = \frac{1}{2}(v - c)$.

Both firms may have an incentive to deviate unilaterally from the collusive agreement. Note that, when both firms charge the cartel price from all consumers, firm A serves the $(-)$-market and firm B the $(+)$-market. Price discrimination now allows a firm to increase its profit by setting a lower price to consumers in its 'weak' market, while still charging the cartel price and making the cartel profit in its own, 'strong', market. The deviating firm's best response to the other firm setting $p^{C}_{DA} = v$ is $p^{D}_{DA} = \frac{1}{2}(v+c)$. Charging p^{D}_{DA} in the weak market and $p^{C}_{DA} = v$ in the strong market, a firm can make a profit of

$$\pi^{D}_{DA} = \frac{1}{2}\left[(v - c) + \frac{1}{8\rho\bar{\theta}}(v - c)^2\right]. \tag{20}$$

Lemma 3 (Price Discrimination with Best-Response Asymmetries)**.** *Suppose that*

$$\rho\bar{\theta} < v - c < 4\rho\bar{\theta} \tag{21}$$

Then, under price discrimination with best-response asymmetries, collusion can be sustained in grim-trigger strategies if and only if $\delta \geq \bar{\delta}_{DA}$, with

$$\bar{\delta}_{DA} = \frac{(v - c)^2}{8\rho\bar{\theta}(v - c) + (v - c)^2 - \frac{80}{9}\rho^2\bar{\theta}^2}. \tag{22}$$

As in the previous lemmas, the condition (21) ensures that all consumers are served in the static Nash equilibrium and that a unilaterally deviating firm does not serve all consumers.

From Lemmas 1 and 3 the next result – the effect of a ban on price discrimination on the sustainability of collusion – is readily obtained.

Proposition 2 (Best-Response Asymmetries)**.** *Assume that $2\rho\bar{\theta} < v - c < 4\rho\bar{\theta}$ and suppose that firms are able to discriminate between consumers with types $\theta \in [-\bar{\theta}, 0)$ and $\theta \in [0, \bar{\theta}]$. Then, banning price discrimination facilitates collusion; i.e., $\bar{\delta}_U < \bar{\delta}_{DA}$.*

According to Proposition 2, with best-response asymmetries a ban on price discrimination makes a collusive outcome "more likely" – as in the case of best-response symmetries. Now, however, there are opposing effects at play. As with best-response symmetries, profits from a unilateral deviation are higher if price discrimination is permitted which makes it more difficult for firms to sustain collusion under price discrimination. In contrast to the case of best-response symmetries, static Nash equilibrium profits are lower under price discrimination. Thus, now the gains from forming a cartel are higher if price discrimination is permitted. In our model, the former effect always outweighs the latter and thus permitting price discrimination makes it harder for firms to form a cartel.

4. Conclusion

In this paper we have shown that a ban on price discrimination facilitates collusion. We used a stylized model of horizontally differentiated firms to obtain the result. The model is particularly restrictive or peculiar with regard to two features. First, the profits made by a firm in a given period when engaging in collusion do not depend on whether price discrimination is permitted or not. Second, we focus on moderate degrees of competition. If the two goods are strong substitutes, a firm that deviates serves the whole market. To investigate how a ban on price discrimination affects the "likelihood" of a collusive outcome for more general demand structures and various degrees of competition is an open question for future research.

A. Proofs of Propositions and Lemmas

Proof of Lemma 1. First, we consider the static Nash equilibrium. The profit function of firm B is (symmetric for firm A)

$$\pi_B = (p_B - c)\left\{\alpha\left[\bar{\theta} - \frac{p_B - p_A}{2\rho_L}\right] + (1-\alpha)\left[\bar{\theta} - \frac{p_B - p_A}{2\rho_H}\right]\right\}\frac{1}{2\bar{\theta}}. \tag{A.1}$$

The first-order condition of profit maximization is

$$\alpha\left[\bar{\theta} - \frac{p_B - p_A}{2\rho_L}\right] + (1-\alpha)\left[\bar{\theta} - \frac{p_B - p_A}{2\rho_H}\right] - (p_B - c)\left[\frac{\alpha}{2\rho_L} + \frac{1-\alpha}{2\rho_H}\right] = 0. \tag{A.2}$$

In the symmetric equilibrium each firm sets the price $p_U^N = c + 2R\bar{\theta}$. It can readily be established that no asymmetric equilibrium exists. Inserting the equilibrium price in the profit function yields the Nash equilibrium profit $\pi_U^N = R\bar{\theta}$.

The price p_U^N indeed constitutes an equilibrium of the static game only if all consumers purchase either from firm A or firm B. Type $\theta = 0$ prefers to buy one unit (either from A or from B) instead of not buying a good if $v - p_U^N > 0$. This condition is equivalent to

$$v - c > \frac{2\rho_L \rho_H \bar{\theta}}{(1-\alpha)\rho_L + \alpha\rho_H}, \tag{A.3}$$

which holds by assumption (by (9)).

If the firms collude and it is optimal to serve all consumers, then the optimal price is $p_U^C = v$ leading to a firm profit of $\pi_U^C = (v-c)/2$. It might, however, be optimal not to serve all consumers; i.e., to charge a price $p > v$ so that some types with θ close to zero purchase from neither firm. For prices $p > v$ each firm is a (local) monopolist and thus a consumer of type $\theta = (p_B - v)/\rho_k$ is indifferent between purchasing from B and not purchasing the good. The profit of firm B is given by

$$\pi_B = (p_B - c)\left\{\alpha\left[\bar{\theta} - \frac{p_B - v}{\rho_L}\right] + (1-\alpha)\left[\bar{\theta} - \frac{p_B - v}{\rho_H}\right]\right\}\frac{1}{2\bar{\theta}}. \tag{A.4}$$

From the first-order condition the optimal price is readily obtained,

$$p^* = \frac{1}{2}\left(v + c + \frac{\rho_L \rho_H}{(1-\alpha)\rho_L + \alpha\rho_H}\bar{\theta}\right). \tag{A.5}$$

This price – and thus not serving all consumers – is optimal only if it is higher than the willingness to pay v. Note that $p^* > v$ is equivalent to

$$v - c < \frac{\rho_L \rho_H \bar{\theta}}{(1-\alpha)\rho_L + \alpha\rho_H}, \tag{A.6}$$

which is never satisfied under the imposed assumption (by (9)).

Next, we consider the optimal deviation from the collusive agreement. From the first-order condition (A.2) we obtain that the best response to $p_A = v$ is

$$p_U^D = \frac{1}{2}(v-c) + \frac{\rho_L \rho_H \bar{\theta}}{(1-\alpha)\rho_L + \alpha\rho_H}. \tag{A.7}$$

This price is derived under the presumption that the marginal consumers are interior. This is indeed the case if and only if for all $k \in \{L, H\}$ it holds that $-\bar{\theta} < \hat{\theta}_k$. This condition can be written as

$$2\rho_k \bar{\theta} > \frac{1}{2}(v-c) - \frac{\rho_L \rho_H}{(1-\alpha)\rho_L + \alpha\rho_H}\bar{\theta} \quad \forall k \in \{L, H\}. \tag{A.8}$$

The above condition is hardest to satisfy for $k = L$. Setting $\rho_k = \rho_L$ in (A.7) and rearranging yields

$$v - c < \frac{(4-4\alpha)\rho_L + (2+4\alpha)\rho_H}{(1-\alpha)\rho_L + \alpha\rho_H}\rho_L \bar{\theta}. \tag{A.9}$$

Finally, we derive the critical discount factor. The critical discount factor is defined by $\bar{\delta}_U = (\pi_U^D - \pi_U^C)/(\pi_U^D - \pi_U^N)$. Hence,

$$\begin{aligned}\bar{\delta}_U &= \frac{\frac{1}{4R\bar{\theta}}\left[\frac{1}{2}(v-c)+R\bar{\theta}\right]^2 - \frac{1}{2}(v-c)}{\frac{1}{4R\bar{\theta}}\left[\frac{1}{2}(v-c)+R\bar{\theta}\right]^2 - R\bar{\theta}} \\ &= \frac{\left[(v-c)/2 - R\bar{\theta}\right]^2}{\left[(v-c)/2 + R\bar{\theta}\right]^2 - 4R^2\bar{\theta}^2},\end{aligned} \tag{A.10}$$

which concludes the proof. □

Proof of Lemma 2. The overall profit of firm $i = A, B$ is given by the sum of profits from consumer groups L and H; i.e. $\pi_{DS,i} = \sum_{k\in\{L,H\}} \pi_i^k$. Profits can be maximized independently for each consumer group. Note that the maximization problem for one consumer group k is equivalent to the maximization problem with uniform pricing and $\rho_L = \rho_H = \rho_k$.

First, we consider the static Nash equilibrium. Note that with $\rho_L = \rho_H = \rho_K$, the expression defined as R in (6) simplifies to ρ_k. Thus, in the static Nash equilibrium, each firm charges the price $p_{DS}^{N,k} = c + 2\rho_k\bar{\theta}$ and makes a profit of $\pi_{DS}^k = \rho_k\bar{\theta}$ from selling to consumer group k. Overall profit is given by the sum of profits from both groups, i.e. $\pi_{DS}^N = \alpha\rho_L\bar{\theta} + (1-\alpha)\rho_H\bar{\theta}$.

The price $p_{DS}^{N,k} = c + 2\rho_k\bar{\theta}$ constitutes an equilibrium of the static game only if all consumers purchase either from firm A or from firm B. Type $\theta = 0$ and $\rho = \rho_H$, who has the lowest utility, still prefers to buy one unit of the good if $v - p_{DS}^{N,H} > 0$. This is equivalent to $v - c > 2\rho_H\bar{\theta}$, which holds by (15).

If the firms collude and if it is optimal to serve all consumers, the profit maximizing price is $p_{DS}^C = v$ for both consumer groups. Both firms earn an overall profit of $\pi_{DS}^C = \frac{1}{2}(v-c)$. It could, however, be optimal not to serve all consumers in a group; i.e., setting a price higher than v at which some consumers do not purchase the good. Suppose both firms agree on prices $p_i^k > v$. Firm B's profit from type ρ_k consumers now is given by $\pi_B^k = (p_B^k - c)\left[\bar{\theta} - (p_B^k - v)/\rho_k\right]/(2\bar{\theta})$. The optimal price is $p_B^{k*} = (v + c + \rho_k\bar{\theta})/2$. It is optimal not to serve all consumers only if $p_B^{k*} > v$ which is equivalent to $\rho_k\bar{\theta} > v - c$. Under assumption (15), this condition is not satisfied.

Consider next firm B unilaterally deviating from the collusive agreement. For each consumer group k, firm B maximizes the profit function π_B^k given that A charges the cartel price $p_{DS}^C = v$. The best response to the rival firm charging the cartel price is $p_{DS}^{D,k} = \frac{1}{2}(v+c) + \rho_k\bar{\theta}$. With $p_{DS}^{D,k}$, we have interior marginal consumers only if for all $k \in L, H$ it holds that $\hat{\theta}_k > -\bar{\theta}$. After inserting prices, this condition can be written as $v - c < 6\rho_k\bar{\theta}$, which is harder to satisfy for

$k = L$. Thus, the deviation price we computed is indeed optimal if and only if $v - c < 6\rho_L\bar{\theta}$, which is satisfied under assumption (15).

Now, we can compute the critical discount factor $\bar{\delta}_{DS} = (\pi^D_{DS} - \pi^C_{DS})/(\pi^D_{DS} - \pi^N_{DS})$:

$$\bar{\delta}_{DS} = \frac{\sum_{k\in\{L,H\}} \frac{\omega_k}{4\rho_k\bar{\theta}} \left[\frac{1}{2}(v-c) + \rho_k\bar{\theta}\right]^2 - \frac{1}{2}(v-c)}{\sum_{k\in\{L,H\}} \frac{\omega_k}{4\rho_k\bar{\theta}} \left[\frac{1}{2}(v-c) + \rho_k\bar{\theta}\right]^2 - \omega_k\rho_k\bar{\theta}}$$

$$= \frac{\sum_{k\in\{L,H\}} \frac{\omega_k}{\rho_k} \left[\frac{1}{2}(v-c) - \rho_k\bar{\theta}\right]^2}{\sum_{k\in\{L,H\}} \frac{\omega_k}{\rho_k} \left[\frac{1}{4}(v-c)^2 + (v-c)\rho_k\bar{\theta} - 3\rho_k^2\bar{\theta}^2\right]} \tag{A.11}$$

□

Proof of Proposition 1. The critical discount factors, $\bar{\delta}_U$ and $\bar{\delta}_{DS}$, are given by (10) and (16), respectively, only if the constraints (9) and (15) are jointly satisfied. It is easy to verify that (15) sets both the more restrictive lower and upper bound on $v - c$; i.e. whenever (15) is satisfied, (9) is fulfilled, too. To prove that $\bar{\delta}_U < \bar{\delta}_{DS}$, we show that the fraction defining $\bar{\delta}_{DS}$ has a larger numerator (N) and smaller denominator (D) than the fraction defining $\bar{\delta}_U$.

We start with the numerator. $N_U - N_{DS} > 0$ is equivalent to

$$\frac{1}{R}\left[\frac{1}{2}(v-c) + R\bar{\theta}\right]^2 - \alpha\left[\frac{1}{\rho_L}\left(\frac{1}{2}(v-c) + \rho_L\bar{\theta}\right)^2\right] - (1-\alpha)\left[\frac{1}{\rho_H}\left(\frac{1}{2}(v-c) + \rho_H\bar{\theta}\right)^2\right] > 0. \tag{A.12}$$

Condition (A.12) can be simplified to $(\rho_H - \rho_L)^2 > 0$, which is always satisfied.

Next, consider the denominator. $D_{NS} - D_U > 0$ is equivalent to

$$\frac{1}{4\bar{\theta}R}\left[\frac{1}{2}(v-c) + R\bar{\theta}\right]^2 - R\bar{\theta} - \frac{\alpha}{4\bar{\theta}\rho_L}\left[\frac{1}{2}(v-c) + \rho_L\bar{\theta}\right]^2 - \frac{1-\alpha}{4\bar{\theta}\rho_H}\left[\frac{1}{2}(v-c) + \rho_H\bar{\theta}\right]^2 + \alpha\rho_L\bar{\theta} + (1-\alpha)\rho_H\bar{\theta} > 0. \tag{A.13}$$

Condition (A.13) can be simplified to $(\rho_H - \rho_L)^2 > 0$, which is always satisfied. □

Proof of Lemma 3. First we consider the static Nash equilibrium. Firms maximize their profit in the (–)-and in the (+)-market independently. The profit functions of firms A and B in the (–)-market are

$$\pi^-_{DA,A} = (p^-_A - c)\left(\bar{\theta} + \frac{p^-_B - p^-_A}{2\rho}\right)\frac{1}{2\bar{\theta}} \tag{A.14}$$

$$\pi^-_{DA,B} = (p^-_B - c)\left(-\frac{p^-_B - p^-_A}{2\rho}\right)\frac{1}{2\bar{\theta}}. \tag{A.15}$$

From the two first-order conditions, we get the equilibrium prices $p_{DA,A}^{N-} = c + \frac{4}{3}\rho\bar{\theta}$, $p_{DA,B}^{N-} = c + \frac{2}{3}\rho\bar{\theta}$ and profits $\pi_{DA,A}^{N-} = \frac{4}{9}\rho\bar{\theta}$, $\pi_{DA,B}^{N-} = \frac{1}{9}\rho\bar{\theta}$. The (+)-market is symmetric to the (−)-market, only firms' roles are reversed. Thus, the equilibrium prices and profits are $p_{DA,A}^{N+} = c + \frac{2}{3}\rho\bar{\theta}$, $p_{DA,B}^{N+} = c + \frac{4}{3}\rho\bar{\theta}$, $\pi_{DA,A}^{N+} = \frac{1}{9}\rho\bar{\theta}$, and $\pi_{DA,B}^{N+} = \frac{4}{9}\rho\bar{\theta}$.

Firm i's overall profit is given by

$$\pi_{DA,i}^{N} = \pi_{DA,i}^{N-} + \pi_{DA,i}^{N+} = \frac{5}{9}\rho\bar{\theta}. \tag{A.16}$$

The prices $p_{DA,i}^{N,z}$ constitute Nash equilibria only if all consumers purchase the good. Note that there are two marginal consumers, who are indifferent between buying from A or B: $\hat{\theta}^{-} = -\frac{1}{3}\bar{\theta}$ and $\hat{\theta}^{+} = \frac{1}{3}\bar{\theta}$. The marginal consumers obtain the lowest utility from purchasing a good. Thus, if the utility from purchasing either good of the marginal consumer is positive, then all consumer types obtain a strictly positive utility from buying. The utility of the marginal consumers is positive if $v - c > \rho\bar{\theta}$, which is fulfilled under assumption (21).

If the firms form a cartel, the profit maximizing price is $p_{DA}^{C} = v$ for both markets. At this price, firm A serves the (−)-market and B the (+)-market. Each firm earns an overall profit of $\pi_{DA}^{C} = \frac{1}{2}(v - c)$. To check that setting p_{DA}^{C} and serving all consumers is indeed optimal, suppose, for example, that firm B sets a price $p_{B}^{+} > v$ in the (+)-market (analogous for A and the (−)-market). The profit is given by $\pi_{B}^{+} = (p_{B}^{+} - c)\left[\bar{\theta} - \frac{p_{B}^{+} - v}{\rho}\right]\frac{1}{2\bar{\theta}}$ (see proof of Lemma 2). The optimal price is $p_{B}^{*} = \frac{1}{2}(v + c + \rho\bar{\theta})$. It is optimal not to serve all consumers only if $p_{B}^{*} > v$ which is equivalent to $v - c < \rho\bar{\theta}$. However, this condition is not satisfied under assumption (21).

Consider next one firm deviating from the collusive agreement. Due to price discrimination, a firm can gain demand and thereby increase its profit by setting a lower price in the market served by the competitor while still charging the cartel price p_{DA}^{C} and making the cartel profit $\pi_{DA}^{C} = \frac{1}{2}(v - c)$ in its strong market. The profit from deviation is

$$\frac{1}{2}(v - c) + \left(p^{D} - c\right)\left(\frac{v - p^{D}}{2\rho}\right)\frac{1}{2\bar{\theta}}. \tag{A.17}$$

From the first-order condition, we get the optimal deviation price $p_{DA}^{D} = \frac{1}{2}(v + c)$. Inserting this price in the profit function yields the deviating firm's overall profit:

$$\pi_{DA}^{D} = \frac{1}{2}\left[(v - c) + \frac{1}{8\rho\bar{\theta}}(v - c)^{2}\right]. \tag{A.18}$$

With the deviation price, we have interior marginal consumers only if $\hat{\theta} > -\bar{\theta}$.

After inserting prices, this condition simplifies to $v - c < 4\rho\bar{\theta}$, which is satisfied by assumption (21).

Finally, we can compute the critical discount factor $\bar{\delta}_{DA} = (\pi^D_{DA} - \pi^C_{DA})/(\pi^D_{DA} - \pi^N_{DA})$:

$$\begin{aligned}\bar{\delta}_{DA} =& \frac{\frac{1}{2}(v-c) + \frac{1}{16\rho\bar{\theta}}(v-c)^2 - \frac{1}{2}(v-c)}{\frac{1}{2}(v-c) + \frac{1}{16\rho\bar{\theta}}(v-c)^2 - \frac{5}{9}\rho\bar{\theta}} \\ =& \frac{(v-c)^2}{8\rho\bar{\theta}(v-c) + (v-c)^2 - \frac{80}{9}\rho^2\bar{\theta}^2}. \end{aligned} \tag{A.19}$$

□

Proof of Proposition 2. To compare the critical discount factors, the assumptions on $\bar{\delta}_U$ from (9) as well as those on $\bar{\delta}_{DA}$ from (21) have to be fulfilled. For $\rho_L = \rho_H = \rho$, (9) simplifies to $2\rho\bar{\theta} < v - c < 6\rho\bar{\theta}$, which implies that (9) sets the more restrictive lower bound, and (21) the more restrictive upper bound on $v-c$. Thus, $v - c$ is restricted to $2\rho\bar{\theta} < v - c < 4\rho\bar{\theta}$.

We want to show that $\bar{\delta}_U < \bar{\delta}_{DA}$, which is equivalent to

$$\frac{\left[\frac{1}{2}(v-c) - \rho\bar{\theta}\right]^2}{\frac{1}{4}(v-c)^2 + \rho\bar{\theta}(v-c) - 3\rho^2\bar{\theta}^2} < \frac{(v-c)^2}{8\rho\bar{\theta}(v-c) + (v-c)^2 - \frac{80}{9}\rho^2\bar{\theta}^2}, \tag{A.20}$$

where we simplified $\bar{\delta}_U$ setting $\rho_L = \rho_H = \rho$. To simplify our expressions, we define $v - c =: x$ and $\rho\bar{\theta} =: y$. Substituting $v - c$ and $\rho\bar{\theta}$ with x and y and rearranging (A.20) yields

$$x^2 - \frac{19}{7}xy + \frac{10}{7}y^2 > 0. \tag{A.21}$$

Note that the left hand side of (A.21) is a function of x with a U-shaped graph. This function is equal to zero for $x_1 = \frac{5}{7}\rho\bar{\theta}$ and $x_2 = 2\rho\bar{\theta}$; it takes on negative values in the interval (x_1, x_2) and is positive otherwise. Hence, under the considered restrictions on $v - c$ it always holds that $\bar{\delta}_U < \bar{\delta}_{DA}$.[6] □

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[6]If the restrictions on $v - c$ are relaxed – in particular the lower bound, then this does not imply a reversed ordering of the critical discount factors because then $\bar{\delta}_U$ is no longer given by (10).

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