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# Stranded Assets, the Social Cost of Carbon, and Directed Technical Change: Macroeconomic Dynamics of Optimal Climate Policy

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# Stranded Assets, the Social Cost of Carbon, and Directed Technical Change: Macroeconomic Dynamics of Optimal Climate Policy

## Abstract

The tractable general equilibrium model developed by Golosov et al. (2014), GHKT for short, is modified to allow for stock-dependent fossil fuel extraction costs and partial exhaustion of fossil fuel reserves, a negative impact of global warming on growth, mean reversion in climate damages, steady labour-augmenting technical progress, specific green technical progress driven by learning by doing, population growth, and a direct effect of the stock of atmospheric carbon on instantaneous welfare. We characterize the social optimum and derive simple rule for both the optimal carbon tax and the renewable energy subsidy, and characterize the optimal amount of untapped fossil fuel.

JEL-Codes: H210, Q510, Q540.

Keywords: social cost of carbon, carbon tax, renewable energy subsidy, general equilibrium, Ramsey growth, capital accumulation, stranded assets, simple rules.

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## 1. Introduction

Carbon emissions are at the root of the most important global externality. Damages to production and welfare resulting from global warming are therefore of concern to all citizens of our planet. The best response is to price carbon, either via a carbon tax or an emissions market, at a uniform price throughout the globe. The price must be set to the optimal social cost of carbon (SCC), which corresponds to the present discounted value of marginal damages from global warming. Pricing carbon curbs emissions and global warming in at least three ways.[1] First, it causes substitution away from fossil fuel and towards other factors of production. Second, it brings forward the carbon-free era which relies only on renewable energy and no longer uses fossil fuel. This means that the key source of global warming is stopped earlier. Third, pricing carbon forces makes it more attractive for fossil fuel producers to keep more fossil fuel in the earth. This limits the total amount of global warming and is also a crucial consideration of climate policy.

Technology constrains the options for policymakers fighting climate change. If the direction of technological change builds on existing knowledge, fossil fuel technologies will be favoured in research and development, locking the economy into carbon-emitting energy sources and making it more expensive to shift to carbon-free alternative (Acemoglu et al., 2012). Such a locking-in calls for directed subsidies to renewable energy in order to compensate consumers for the initially higher costs of the renewable infant industry. The first best can be attained with a policy consisting of two instruments, a renewable energy subsidy and a carbon tax or an emissions permit market, which are used to address the two types of market failure separately. The carbon price is set to the optimal SCC in the usual way. The renewable subsidy must be set to the optimal social benefit of learning (SBL) in the renewable energy sector, which corresponds to the present discounted value of marginal reductions in future costs of generation renewable energy due to learning by doing if one extra unit of renewable energy is used today. The renewable energy subsidy boosts green energy production, but also indirectly lowers carbon emissions and the extent of global warming by encouraging the switch away from fossil fuel to renewable energy earlier on.

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[1] Pricing carbon will also encourage carbon capture and sequestration, promote the development of green R&D and curb carbon emissions and global warming.

Although the DICE model, which is used widely in the integrated assessment literature (Nordhaus, 2008; 2014), allows for a negative impact of pricing carbon on fossil fuel use, it does not allow for scarce fossil fuel and for forward-looking expectations and thus does not shed much light on the issue of expectations and timing of optimal energy transitions. Furthermore, integrated assessment studies based on DICE and the tractable general equilibrium model of growth and climate change developed by Golosov et al. (2014), denoted by GHKT from hereon, do not allow for fossil fuel extraction costs that rise as fossil fuel reserves are depleted and more costly fields or deposits have to be explored. These studies thus do not give an answer to the question how the prices of carbon and its energy alternatives and expectations of the future prices affect the optimal amount of abandoned fossil fuel reserves (also known as stranded assets). This is a major shortcoming, since much of the debate among climate scientists is about the crucial importance of limiting cumulative carbon emissions and thereby increasing the size of stranded assets. Clearly, this has a big negative impact on fossil fuel producers.

Our objective is to address these issues in a modified version of the GHKT model and obtain a tractable solution and simple rule for both the optimal SCC and SBL.[2] We extend the GHKT model in five directions. First, we allow for stock-dependent fossil fuel extraction costs and partial exhaustion of fossil fuel reserves. Carbon taxation thus has to be designed to bring forward the carbon-free era but also to leave more fossil fuel abandoned in the crust of the earth. Second, we take account of the empirical findings of Dell et al. (2013) and allow global warming to negatively affect both the level and growth rate of total factor productivity. Empirical evidence suggests that this negative growth effect is substantial for low-income countries. Growth is driven by steady labour-augmenting technical progress. Third, we allow for a direct effect of atmospheric carbon on welfare. Fourth, we allow for population growth and explore its effects on the social cost of carbon under utilitarian welfare. Fifth, we allow for specific green innovations bringing down the cost of renewable production via learning by doing. We thus derive a simple rule for the subsidy for renewable energy given endogenous technical progress in renewable energy production.

[2] We thus analyse a tractable, discrete-time version of the continuous-time model of growth and optimal carbon taxation developed in van der Ploeg and Withagen (2014). This analysis complements the earlier numerical robustness analysis of the GHKT model in the supplementary material of Lint Barrage to Golosov et al. (2014) and in Rezai and van der Ploeg (2015ab).

Section 2 gives our generalization of the GHKT model. Section 3 presents the conditions for the social optimum. Section 4 derives the optimal allocation of labour and production of energy. Section 5 discusses conditions for simultaneous use of fossil fuel and renewable energy. Section 6 discusses the three qualitatively different regimes that can occur for the social optimum. Section 7 presents our optimal policy simulations and discusses the optimal price of carbon and renewable energy subsidy as well as how much assets will be stranded under the various outcomes. Section 8 concludes.

## 2. Generalizing the GHKT model

We adopt the familiar Brock-Mirman (1972) and Golosov et al. (2014) assumptions: logarithmic utility, Cobb-Douglas production function, 100% depreciation of manmade capital in every period, exponential climate damages in production, fossil fuel extraction and production of renewable energy requiring no capital, and a two-box carbon cycle with a part of carbon staying up permanently in the atmosphere and another part that gradually decays and is returned to the surface of the earth and oceans[3]. The amount of energy from one unit of fossil fuel decreases as high-grade reservoirs are depleted. This allows us to analyse the fraction of assets that will be stranded, which is crucial as climate scientists such as Allen et al. (2015) argue that cumulative carbon emissions drive maximum global warming,. The amount of labour in efficiency units required to produce one unit of renewables is given in each period of time, but is subject to dynamically increasing returns to scale as past production spurs green cost-saving innovations. There is a steady rate of labour-augmenting technical progress. In addition we allow carbon emissions resulting from burning fossil fuel to affect either the level or the growth rate of total factor productivity. More generally, we allow for mean reversion around an exogenous growth path for total factor productivity. We suppose that labour supply is exogenous and grows at a constant rate. To keep the exposition simple, we make one simplifying assumptions compared with GHKT by abstracting from coal.

[3] Gerlagh and Liski (2015a) allow for a three-box carbon cycle. Ricke and Caldeira (2014) argue that maximum contribution of a carbon to global warming occurs one decade after its emission.

Subscripts denote periods of time $t$ = 0, 1, 2, … The time impatience factor is constant and is denoted by $0 < \beta < 1$. The consumption level, the level of aggregate output, the aggregate capital stock, and the fossil fuel depletion rate during period $t$ are given by $C_t, Y_t, K_t,$ and $F_t,$ respectively. $A_t, E_t^p, E_t^t, R_t,$ and $S_t$ denote, respectively, the level of total factor productivity, the permanent and transitory stocks of atmospheric carbon (the sum of which is $E_t$), the stock of knowledge about renewable energy production, at the end of period $t$, and the stock of fossil fuel reserves at the start of period $t$. Fossil and renewable energy are perfect substitutes in production, but energy is an imperfect substitute for labour and capital. We use a Cobb-Douglas production function in capital, labour and energy with the constant value shares of capital and energy in output given by $0 < \alpha < 1$ and $0 < \upsilon < 1$, respectively, and $\alpha + \upsilon < 1$. The labour requirement during period $t$ to produce one unit of renewable energy is $r(R_t)$. We suppose that renewable energy is less costly if the stock of knowledge is larger, so that $r'(R_t) > 0$. The decay rate of green knowledge is $0 < \theta < 1$. The energy content of fossil fuel is $s(S_t)$ corresponds to the productivity of fossil fuel extraction and falls as less productive fossil fuel reserves must be accessed, so $s'(S_t) > 0$. $1 - s(S_t)$ represent extraction costs to the fossil fuel sector which are higher if less of the stock of fossil fuel reserves is left. For analytical convenience, we use the exponential functions $\ln\left(r(R_t)\right) = \rho_0 + \rho_1 R_t$ and $\ln\left(s(S_t)\right) = \sigma_0 + \sigma_1 S_t$ where $\rho_0$ and $\sigma_0$ are exogenous positive constants.

The instantaneous welfare loss of global warming is $\psi N_t E_t$, where $\psi \geq 0$. The instantaneous damage of the stock of atmospheric carbon to total factor productivity is denoted by $\chi > 0$. We allow for mean reversion around an exogenous trend growth path for total factor productivity $\overline{A}_t, t = 1, 2, ...$ The mean reversion in the log of total factor productivity is denoted by $0 \leq \delta \leq 1$. Labour supply at time $t$ is $N_t$ and grows at the constant gross rate $N_{t+1}/N_t = \gamma$. The part of labour supply that is allocated to final goods production is $L_t$. The gross steady growth factor of labour-augmenting technical progress is $\omega \geq 1$. Final goods production, resource extraction and renewable energy production all benefit from this, so that the extraction cost of fossil fuel and the renewable energy cost steadily fall at the same rate as the efficiency of labour increases over time. We define the number of (exogenous) efficiency units in the economy as

$M_t \equiv \omega^t N_t$. Labour in efficiency units is allocated either to final goods production $L_t$ or to renewable energy production $L_t^R$, so that $L_t + L_t^R = M_t$. The discount rate corrected for population growth is positive, i.e., $\beta\gamma < 1$.

The fraction of carbon emissions that stays permanently in the atmosphere is $0 < \varphi_L < 1$. Of the part of carbon emissions that stays temporarily in the atmosphere a fraction $0 < \varphi_0 < 1$ is still there at the end of the period (a decade). The decay rate of the stock of atmospheric carbon is $0 < \varepsilon < 1$.

The problem is thus to choose a sequence of consumption levels, fossil fuel depletion rates and renewable energy usages to maximize utilitarian social welfare

$$(1) \qquad \sum_{t=0}^{\infty} \beta^t \left[ N_t \ln(C_t / N_t) - \psi N_t E_t \right]$$

subject to the capital accumulation equation

$$(2) \qquad K_{t+1} = Y_t - C_t, \quad Y_t = A_t K_t^{\alpha} \left[ s(S_t) F_t + r(R_t) L_t^R \right]^{\upsilon} L_t^{1-\alpha-\upsilon}, \quad L_t = M_t - L_t^R,$$

the dynamics of fossil fuel depletion

$$(3) \qquad S_{t+1} = S_t - F_t, \qquad \sum_{t=0}^{\infty} F_t \le S_0,$$

the dynamics of the stock of carbon in the atmosphere

$$(4) \qquad E_t^P = E_{t-1}^P + \varphi_L F_t, \quad E_t^t = \varepsilon E_{t-1}^t + \varphi_0 (1 - \varphi_L) F_t, \quad E_t = E_t^P + E_t^t,$$

the development of total factor productivity

$$(5) \qquad \ln(A_t) = \delta \ln(A_{t-1}) + (1 - \delta) \ln(\overline{A}_t) - \chi E_t,$$

the dynamics of knowledge in the renewable energy sector

$$(6) \qquad R_{t+1} = \theta R_t + L_t^R,$$

and the non-negativity constraints $F_t \ge 0, M \ge L_t^R \ge 0$ and $R_t, S_t \ge 0$ for all $t \ge 0$.

## 3. The social optimum

The social optimum satisfies the properties stated in the following proposition.

**Proposition 1:** The socially optimal saving and consumption functions are

(7) $K_{t+1} = \alpha\beta\gamma Y_t, \qquad C_t = (1-\alpha\beta\gamma)Y_t.$

The demand for fossil fuel and renewable energy follow from

(8) $$\left.\begin{array}{c}\dfrac{\upsilon Y_t}{s(S_t)F_t + r(R_t)L_t^R}s(S_t) \le h_t + \tau_t \\ F_t \ge 0\end{array}\right\}\text{c.s.}, \quad \left.\begin{array}{c}\dfrac{\upsilon Y_t}{s(S_t)F_t + r(R_t)L_t^R}r(R_t) \le w_t - b_t \\ L_t^R \ge 0\end{array}\right\}\text{c.s.},$$

where $h_t, \tau_t, w_t \equiv (1-\alpha-\upsilon)Y_t / L_t$, and $b_t$ are the scarcity rent of fossil fuel, the social cost of carbon (SCC), the social wage (i.e., the marginal product of labour), and the social benefit of learning in renewable energy production (SBL), respectively. The SCC is

(9) $$\tau_t = \varphi Y_t, \qquad \varphi \equiv \left(\frac{\varphi_L}{1-\beta\gamma} + \frac{\varphi_0(1-\varphi_L)}{1-\beta\gamma\varepsilon}\right)\left[(1-\alpha\beta\gamma)\psi + \frac{\chi}{(1-\beta\gamma)(1-\beta\gamma\delta)}\right] \ge 0.$$

The scarcity rents on fossil fuel follow from

(10) $$r_t h_{t-1} = h_t + \upsilon\frac{Y_t}{s(S_t)F_t + r(R_t)L_t^R}s'(S_t)F_t,$$

and the social interest rate is the marginal product of capital, $r_t = \alpha Y_t / K_t$.

The SBL follows from

(11) $$r_t b_{t-1} = \theta b_t + \upsilon\frac{Y_t}{s(S_t)F_t + r(R_t)L_t^R}r'(R_t)L_t^R.$$

**Proof:** see appendix.

The saving and consumption functions (7) follow from the Euler equation and the capital accumulation equation (2). They modify Brock and Mirman (1972) to allow for population growth and indicate that a higher capital share, more patience and higher population growth curb the propensity to consume and boost aggregate investment.

Equations (8) indicate that fossil fuel is not used in the production of final goods if its marginal product is less than its social marginal cost consisting of the scarcity rent plus the SCC. If fossil fuel use is used in production, its marginal product exactly equals its social marginal cost. As fossil fuel reserves are depleted, the marginal product of fossil fuel has to fall. Similarly, if the marginal product of renewable energy falls short of its marginal cost consisting of the wage *minus* the social benefit for learning, it is not used in production. Note that (8) does not preclude simultaneous use of both types of energy.

Equation (9) implies that the optimal SCC is a bigger proportion of aggregate output if the damage parameters $\chi$ and $\psi$ are large, society has more patience (i.e., has a large time impatience factor $\beta$), population growth $\gamma$ is high, and the mean reversion in the process of total factor productivity $\delta$ is big. The GHKT model supposes that climate damages only affect the *level* of total factor productivity so that $\delta = \psi = 0$ and $\tau_t = \frac{\chi}{1-\beta\gamma} Y_t$.[4] If climate damages affect the *rate of growth* of total factor productivity, $\delta = 1$ so that $\tau_t = \frac{\chi}{(1-\beta\gamma)^2} Y_t$. The SCC is thus a bigger proportion of output if global warming damages affect the growth rate rather than the level of total factor productivity. The SCC is also high if a big fraction of carbon emissions stays up permanently in the atmosphere and the residence time of the remaining fraction is small (high $\varphi_L, \varphi_0, \varepsilon$).

Equation (10) is our version of the Hotelling rule. It implies that scarcity rents are the present discounted value of all future reductions in the productivity of fossil fuel extraction resulting from depleting one extra unit of fossil fuel. Once the carbon-free era has commenced and fossil fuel is no longer used, scarcity rents on fossil fuel are zero.

The social benefit of renewable energy use is driven by learning by doing. As more labour is allocated to the renewable energy sector, productivity growth in the sector rises. The SBL in (11) captures the present discounted value of all future increases in productivity from using one more unit of renewable energy today. The SBL benefit equals zero once all learning has occurred in the renewable energy sector.

Since the wage and labour requirements are measured in efficiency units, the saving/consumption decisions (7), energy demands (8), the optimal SCC (9), the Hotelling rule (10), and the subsidy for renewable energy (11) are unaffected by the rate of exogenous technical progress $\omega$.

## 4. Allocation of labour and energy production

[4] The carbon tax rule should react to the unobservable socially optimal level of GDP, not the observable business-as-usual level of GDP. We provide bounds for the socially optimal level of GDP in section 6.

Define the scarcity rent and the renewable energy subsidy as fractions of aggregate output, i.e., $H_t \equiv h_t / Y_t$ and $B_t \equiv b_t / Y_t$. Equations (8) together with the marginal productivity condition for optimal labour use in final goods production then give the optimal allocation of labour to final goods production and renewable energy production.

**Proposition 2:** If only renewable energy is used in final goods production, then $F_t = 0$,

(12Ra) $$b_t = \phi Y_t \quad \text{with} \quad \phi \equiv \frac{\beta\gamma\rho_1}{1-\beta\gamma\theta}, \quad \text{and}$$

(12Rb) $$\frac{\upsilon}{L_t^R} = \frac{1-\alpha-\upsilon}{M_t - L_t^R} - \phi.$$

If only fossil fuel is used in final goods production, then $L_t^R = 0$ and

(12F) $$F_t = \frac{\upsilon}{H_t + \varphi} \quad \text{and} \quad H_{t-1} = \beta\gamma\left(H_t + \upsilon\sigma_1\right).$$

If fossil fuel and renewable energy are used at the same time, we have

(12S) $$\frac{\upsilon}{s(S_t)F_t + r(R_t)L_t^R} = \frac{H_t + \varphi}{s(S_t)} = \frac{1}{r(R_t)}\left(\frac{1-\alpha-\upsilon}{M_t - L_t^R} - B_t\right),$$
$$H_{t-1} = \beta\gamma\left[H_t(1+\sigma_1 F_t) + \varphi\sigma_1 F_t\right] \text{ and } B_{t-1} = \beta\gamma\left[\theta B_t(1-\rho_1 L_t^R) + \rho_1 \frac{1-\alpha-\upsilon}{M_t - L_t^R}\right].$$

**Proof:** see appendix.

Equation (12Ra) indicates that during the carbon-free phase the renewable energy subsidy is a constant fraction of aggregate output. This fraction and thus the subsidy is large if society is relatively patient (high $\beta$), population grows fast (high $\gamma$), learning by doing is substantial (high $\rho_1$), and green knowledge decays slowly (high $\theta$). Equation (12Rb) indicates that during the carbon-free phase the allocation of labour across final goods and renewable energy production is by the technological coefficients of the production function, but also that a higher ratio of renewable energy subsidies (higher $\phi$) tilts the allocation of labour away from final goods towards renewable energy production. Energy use during this phase grows with the population size and technical progress. Employment in renewable energy production, $L_t^R$, also determines fully optimal aggregate production of final goods during the carbon-free phase.

Equations (12F) indicates that if only fossil fuel is used a higher SCC or scarcity rent as fractions of aggregate output (i.e., higher $\varphi$ and $H_t$) depress fossil fuel use. During the fossil fuel era, the scarcity rent (again, as fraction of aggregate output) follows from a linear unstable difference equation independent of the stock of fossil fuel reserves.[5] Generally, the ratio of scarcity rent to aggregate output is high and thus fossil fuel use is low if society is relatively patient, the population grows fast, and fossil fuel extractoin rapidly becomes less productive as more reserves have been used up (high $\beta$, $\gamma$ and $\sigma_1$), and of course if initial fossil fuel reserves are small. The rate of fossil fuel extraction is generally not constant over time. As the end of fossil fuel use draws closer, we note that the scarcity rent as a fraction of output falls and fossil fuel use increases during the phases where only fossil fuel is used. As can be seen from equation (9), the scarcity rent typically has a hump-shaped time profile as aggregate output and marginal extraction cost increase as reserves are depleted but fossil fuel use $F_t$ must eventually fall to ensure zero scarcity rent at the switch point from the carbon to the carbon-free era. If very little fossil fuel is used, the scarcity rent time profile will only have a falling section.

If both energy sources are used together, the social price of fossil energy must equal the social cost of renewable energy: $(h_t + \tau_t)/s(S_t) = (w_t - b_t)/r(R_t)$. Using equations (12S) indicates that total energy use falls with fossil fuel extraction cost and the scarcity rent, but increases with the cost of renewable energy. Solving (12S) for fossil fuel and renewable energy use and for total energy use, we get the comparative statics results:

$$(13)\qquad \begin{aligned} &F_t^S = f(\overset{-}{H_t}, \overset{+}{S_t}, \overset{-}{B_t}, \overset{-}{R_t}), \quad L_t^{RS} = l(\overset{+}{H_t}, \overset{-}{S_t}, \overset{+}{B_t}, \overset{+}{R_t}), \text{ and} \\ &E_t^S = s(S_t)F_t^S + r(R_t)L_t^{RS} = e(\overset{-}{H_t}, \overset{+}{S_t}, \overset{+}{B_t}, \overset{+}{R_t}) \end{aligned}$$

Interestingly, (13) indicates that the energy system is independent of aggregate output and global warming damages. Fossil fuel use increases if the remaining stock of fossil fuel reserves is large and extraction is relatively easy, the scarcity rent is low, and the productivity and subsidy in the renewable sector are low. The use of renewable energy during phases of simultaneous energy use has the exact opposite signs. Total energy use

[5] Although this has a unique saddle-point solution with a constant scarcity rent and thus constant fossil fuel use, this is only relevant for the case of asymptotic depletion of a given stock of fossil fuel reserves. However, in our model we have renewable energy and thus a switch to the carbon-free era in finite time. This implies that scarcity rents are positive until the moment that fossil fuel is phased out.

(the sum of both) decreases in the scarcity rent, increases in the stock of fossil fuel and increases with productivity and subsidy in the renewable sector. Total energy use, however, does not depend on population size or the state of overall technical progress in the economy. A higher $\varphi$, i.e., higher damage coefficients ($\psi$ and $\chi$), more mean reversion or growth effects in TFP damages of global warming (higher $\delta$), less discounting (higher $\beta$) or higher population growth (higher $\gamma$), depresses fossil fuel use by more than it boosts renewable energy. For a given energy mix, a higher social cost of carbon as fraction of GDP implies that more fossil fuel must be left abandoned. A bigger population and labour-saving technical progress induces a shift from the energy mix away from fossil fuel towards renewable energy.

An interesting feature of our model is that the dynamics of the energy sector are decoupled from the dynamics of the overall economy. The evolution of TFP, capital accumulation and environmental degradation has no impact on the depletion of fossil reserves and the progress of renewable technology. The energy sector is governed only by those latter dynamics and the forward-looking dynamics for the scarcity rent $H_t$ and the renewable subsidy $B_t$. These are the non-predetermined state variables whilst $S_t$ and $R_t$ are the predetermined state variables. The solution to this saddle-point system of difference equations pins down the output, investment and consumption decisions, given by equations (2) and (7).

The reader who is more interested in the optimal policy simulations can skip the more technical sections 5 and 6 on when to phase in and out the various types of energy, on whether or not there is simultaneous use of fossil fuel and renewable energy in final goods production and on the determinants of energy transitions, and jump directly to the simulation section 7.

## 5. Conditions for simultaneous use of fossil fuel and renewable energy

To find out whether and if so for how long it is optimal to have simultaneous use of both types of energy, one must first answer the question of when it is optimal to phase in renewable energy and when to phase out fossil fuel.

**Proposition 3:** Consider a phase of the social optimum where initially only fossil fuel is used and $L_0^{RS} \le 0$ (see (11S)) or $S_0 > \frac{1}{\sigma_1}\left[(\rho_0 - \sigma_0) + \rho_1 R_0 + \ln\left(\frac{H_0 + \varphi}{(1-\alpha-\upsilon)/N_0 - B_0}\right)\right]$.

Renewable energy is then phased in as soon as $L_0^{RS}$ becomes positive or as soon as

$$(14) \qquad S_t < \frac{1}{\sigma_1}\left[(\rho_0 - \sigma_0) + \rho_1 R_t + \ln\left(\frac{H_t + \varphi}{(1-\alpha-\upsilon)/M_t - B_t}\right)\right] \equiv S^R$$

Once in a phase with simultaneous use, fossil fuel is phased out if

$$(15) \qquad S_t < \frac{1}{\sigma_1}\left[(\rho_0 - \sigma_0) + \rho_1 R_t + \ln\left(\frac{H_t + \varphi}{(1-\alpha-\upsilon)/(M_t - L_t^R) - B_t}\right)\right] \equiv S_t^F.$$

**Proof:** see appendix.

With negligible or zero renewable energy subsidies and constant population size, it follows that $S_t^R > S_t^F$ reduces to $\upsilon > 0$, which always holds. The stock at which renewable energy is phased in is larger than the stock at which fossil fuel is abandoned even with positive renewable energy subsidies as $B_t$ is falling over time and $H_t$ is positive in inequality (14) and zero in inequality (15). Hence, renewable energy is phased in before fossil fuel is phased out and thus there can be in principle a non-degenerate phase of simultaneous use. Moreover, the right-hand side of (15) rises with time as past periods of renewable energy use increase $R_t$, providing population growth continues, whereas the left-hand sides of (15) falls as extraction depletes reserves. These inequalities must thus eventually start to hold in the order stated as fossil fuel reserves are depleted until the carbon-free phase is reached.[6] It is straightforward to establish that a higher $\varphi, \sigma_0, \rho_0, \rho_1$ and $\upsilon$ boosts both the critical stock of fossil fuel reserves at which renewable energy is phased in and simultaneous use starts, $S_t^R$, and the critical stock of reserves for which fossil fuel is phased out during simultaneous use, i.e., $S_t^F$.[7]

[6] An unanticipated sudden discovery of fossil fuel reserves causes an immediate upward jump in $S_t$ and a downward jump in the scarcity rent. This might imply that one switches back to an earlier phase (e.g., from a carbon-free phase to a phase of simultaneous energy use).

[7] The definition of $S_t^F$ in equation (12S) is implicit. The explicit solution is $S_t^F \equiv \frac{1}{\sigma_1}\left[(\rho_0 - \sigma_0) + \rho_1 R_t + \ln\left(\varphi\left(\phi M_t - (1-\alpha) + \sqrt{(\phi M_t)^2 - 2(1-\alpha-2\upsilon)\phi M_t + (1-\alpha)^2}\right)\right) - \ln(2\upsilon\phi)\right]$.

## 6. Different regimes of energy use

Depending on what the initial stocks of fossil fuel reserves and green knowledge are, we get three potential socially optimal regimes with different phases of energy use.[8]

**Proposition 4:** If $\frac{\mathrm{e}^{\sigma_0+\sigma_1 S_0}}{\mathrm{e}^{\rho_0+\rho_1 R_0}} > \frac{H_0+\varphi}{(1-\alpha-\upsilon)/N_0 - B_0}$, it is optimal to have the carbon-free phase from the outset. If $\frac{H_0+\varphi}{(1-\alpha-\upsilon)/N_0 - B_0} > \frac{\mathrm{e}^{\sigma_0+\sigma_1 S_0}}{\mathrm{e}^{\rho_0+\rho_1 R_0}} > \frac{\varphi}{(1-\alpha-\upsilon)/(N_0-L_0^R)-B_t}$, it is optimal to start with an initial phase where both types of energy are used followed with a final carbon-free phase. If the inequality $\frac{\mathrm{e}^{\sigma_0+\sigma_1 S_0}}{\mathrm{e}^{\rho_0+\rho_1 R_0}} < \frac{\varphi}{(1-\alpha-\upsilon)/(N_0-L_0^R)-B_t}$ holds, it is optimal to have an initial phase with only fossil fuel use followed by an intermediate phase of simultaneous use and then a final carbon-free phase.

**Proof:** see appendix.

If initial fossil fuel reserves are very high, extraction costs are sufficiently low for renewable energy to be uncompetitive. The economy then starts with only fossil fuel. If productivity or subsidies in renewable energy production are high and productivity of fossil fuel extraction is low, it is optimal to start with either a regime of simultaneous use or a carbon-free regime where fossil fuel is never phased in from the outset.

Hence, an upper bound on the stock of fossil fuel reserves to be left abandoned in the crust of the earth is $S_t^F$. More reserves are thus left abandoned and a smaller cumulative carbon budget is adopted if extraction is relatively expensive (low $\sigma_1$) and the cost of renewable energy is low (high $\rho_1$). This is also the case if the SCC as fraction of aggregate output is large, which occurs if damage coefficients from global warming ($\psi$ and $\chi$) are high, mean reversion in climate shocks to total factor productivity ($\delta$) is substantial, the capital share ($\alpha$) is high, population growth ($\gamma$) is high, and society is relatively patient (high $\beta$). Population growth and labour-augmenting technical progress

[8] The social optimum in the continuous-time model of van der Ploeg and Withagen (2014) allows for four qualitatively different regimes depending on the initial capital stock and initial stock of fossil fuel reserves. Here the characterization only depends on $S_0$.

increase the stock of abandoned fossil fuel reserves, which means that there are more stranded assets of fossil fuel producers.

Business as usual corresponds to an outcome with zero carbon taxes (derived by setting $\varphi = 0$ and $b_t = 0$). We then have $S_t^F = S_t^R = 0$ in proposition 4, so that fossil fuel is phased out and renewable energy phased in at the same moment. This implies that under business as usual simultaneous use beyond one period never occurs. Although fossil energy only requires reserves as input, the fossil fuel era can last several periods due to presence of scarcity rents and the incentive to smooth energy consumption over time.

**Proposition 5:** Simultaneous use of fossil fuel and renewable energy in final goods production lasts at most one period if renewable energy subsidies are negligible and

$$(16) \qquad \frac{1-\alpha}{1-\alpha-\upsilon} \leq \gamma\,\omega + \frac{1}{\varphi}\mathrm{e}^{\sigma_1 \upsilon/\varphi}.$$

**Proof:** see appendix.

Proposition 5 holds if policy-makers do not implement a renewable energy subsidy, technological progress in the renewable sector is driven by exogenous forces, or the potential for learning in renewable energy production is small. The proposition illustrates that sustained periods of simultaneous use only occur if the energy share in GDP, $\upsilon$, is large, technological progress in the aggregate economy, $\omega$, is small, population growth, $\gamma$, is small, and the severity of the climate problem, as captured by the carbon tax, $\varphi$, is small. Most empirical analyses of the energy sector and climate change point in the opposite direction, so that phases of simultaneous use of the two types of energy are unlikely to occur.[9]

In the case of a simultaneous phase lasting at most one period and utilizing the fact that towards the end of the carbon era and just before the carbon-free era commences the scarcity rent will have to be zero in line with Heal (1978), we can fully characterize the economy in terms of the switch point $T$.

[9] Condition (16) is satisfied under typical estimates of the income share of capital and energy and of the growth rates of population and productivity. For example, if $\alpha = 0.3$, $\upsilon = 0.07$, $\omega = 1.2$ and $\gamma = 1.1$, then condition (16) boils down to $1.11 < 1.32$.

**Proposition 6:** Let *T* be the period of simultaneous use of fossil fuel and renewable energy or, if there is no periods of simultaneous use, the last period of fossil fuel use. The scarcity rents on fossil fuel are then

$$(17)\qquad h_t = \eta_t Y_t \quad \text{with} \quad \begin{cases} \eta_t \equiv \dfrac{\beta\gamma\upsilon\sigma_1(1-\beta\gamma^{T-t})}{(1-\beta\gamma)} & \text{for } t < T, \\ \eta_t \equiv 0 & \text{for } t \geq T, \end{cases}$$

the renewable energy subsidies are

$$(18)\qquad b_t = \phi Y_t \quad \text{with} \quad \begin{cases} \phi \equiv \dfrac{\beta\gamma\upsilon\rho_1(\beta\gamma)^{T-t}}{(1-\beta\gamma\theta)} & \text{for } t < T, \\ \phi \equiv \dfrac{\beta\gamma\upsilon\rho_1}{(1-\beta\gamma\theta)} & \text{for } t \geq T. \end{cases}$$

Here *T* is the biggest integer that satisfies the cumulative extraction condition

$$(19)\qquad \sum_{t=0}^{T} \frac{\upsilon}{\eta_t + \varphi} \leq S_t^F.$$

**Proof:** see appendix.

If the phase of simultaneous use of fossil and renewable energy is short, the solution of the intertemporal decision problem reduces to solving equation (18) for *T*. The LHS of equation (18) is monotonically increasing in *T* and a solution can be found by testing solutions for *T* counting forward from 0 until the highest *T* satisfying the inequality is determined. It is, however, straightforward to establish lower and upper bounds on the durations of the fossil era and the amount of fossil fuel to be left unexploited in the crust of the earth: the scarcity rent ranges between 0 and $\beta\gamma\upsilon\sigma_1/(1-\beta\gamma)$ from (17) and fossil fuel use between $\upsilon/(\varphi+\beta\gamma\upsilon\sigma_1/(1-\beta\gamma))$ and $\upsilon/\varphi$ from (12F). The integer *T* consequently, ranges between $\varphi S_t^F/\upsilon$ and $(\varphi+\beta\gamma\upsilon\sigma_1/(1-\beta\gamma))S_t^F/\upsilon$.

## 7. Optimal climate policy, cumulative carbon emissions, and stranded assets

In sections 5 and 6 we discussed the nature and sequencing of different energy regimes in the economy. Propositions 5 and 6 provided further characterizations of the economy in cases of at most one period of simultaneous use. Here we briefly present calibrated simulations of the transition from fossil to renewable energy and cumulative carbon emissions and the amount of stranded assets, supplementing the analytical analysis.

Despite the convenient features of the Brock-Mirman (1972) and GHKT assumptions, the energy sector still contains two forward-looking variables: scarcity rents on fossil fuel and the renewable energy subsidies. The first-best solution follows from the usual two-point-boundary-value problem with different phases and endogenous switch times which must be solved with a multiple-shooting algorithm. However, if scarcity rents and renewable energy subsidies are negligible (or policy makers ignore them), forward-looking dynamics are absent and our model can simply be rolled forwards in time. Sufficient conditions for zero scarcity rents are abundant fossil fuel, $S_0 \to \infty$, or fossil fuel extraction costs that do not rise as less accessible reserves have to be explored, $\sigma_1 \to 0$.[10] Likewise a sufficient condition for zero renewable energy subsidies is constant or exogenously falling production costs for renewable energy. Here, however, we compute the fully optimal solution to the two-point-boundary-value problem.

### 7.1. Calibration

Some of the benchmark parameters for the numerical simulations are from GHKT: $\alpha = 0.3, \upsilon = 0.04, \beta = 0.985^{10}, \varphi_L = 0.2, \varphi_0 = 0.393, \varphi = 0.0228$ and $\chi = 2.379 \times 10^{-5}$. This corresponds to a capital share of 30% and an energy share of 4% of value added, and a rate of time impatience of 1.4% per annum. The two-box carbon cycle is calibrated to the following stylized facts: 20% of carbon emissions stay up forever in the atmosphere, of the remainder 60% is absorbed by oceans and the surface of the earth within a year and the rest has a mean life of 300 years, and half of a carbon emissions impulse is removed from the atmosphere after thirty years. Production damages from global warming are 2.379% of global GDP for each trillion ton of carbon in the atmosphere. The benchmark abstracts from utility damages from global warming.

The initial stock of atmospheric carbon is set to 800 GtC. The initial capital stock is 150 \$T. Population is normalized to 1 and initial total factor productivity to 134, which

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[10] If extraction costs are constant, partial exhaustion of reserves must be brought about by specific green technical progress in the production cost of the renewable energy.

ensures that decadal output is calibrated at 700 \$T per decade.[11] In our benchmark results, we set technical progress and population growth to roughly 1% and 0.5% per annum, i.e., $\gamma = 1.1$ and $\omega = 1.05$. Initial fossil fuel reserves are 4000 GtC. This is significantly higher than in GHKT to allow for the conflation of oil and coal into generic fossil fuel resources. We set the utility damage parameter to $\psi = 0.0001$ (whilst GHKT sets this parameter to zero) and limit in our benchmark production damages to the level but not the growth of productivity $\delta = 0$. We set fossil fuel extraction costs such that there are no extraction costs initially, i.e., $\sigma_0 = -\sigma_1 S_0$, and that depletion of half the fossil fuel reserves leads to a doubling of extraction costs, so $\sigma_1 = 0.00138$. The initial stock of renewable technology is set to zero. The learning-by-doing parameters are $\rho_0 = 0$ and $\rho_1 = 0.00138$, where a doubling of the knowledge stock doubles labour productivity in renewable energy production. We suppose that 90% of green knowledge depreciates within one period, so that $\theta = 0.1$.

### 7.2. Simulation of optimal climate policy

Figure 1 plots simulation results for energy use and cumulative carbon emissions under our benchmark calibration (blue bars). Figure 1 also plots the results of two sensitivity exercises: (1) introducing damages from global warming to productivity growth by changing $\delta = 0$ to $\delta = 0.5$ (yellow bars); and (2) raising the rate of time impatience from 1.5% to 3% per annum, i.e. $\beta = 0.97^{10}$ instead of $\beta = 0.985^{10}$ (red bars). Furthermore, figure 1 compares these optimal outcomes with the case of a zero carbon tax ($\tau = 0$). Table 1 presents a summary of the key results for the simulations relating to cumulative carbon emissions, stranded assets, peak global warming and the timing of the transition to the carbon-free era.

For the benchmark calibration, fossil fuel use increases from roughly 10 GtC to 13.5 GtC per annum over the next 30 years. After cumulative use of 345GtC, fossil fuel is abandoned and renewable energy produces all of the economy's energy requirements. There is no simultaneous period of simultaneous use of the two types of energy. Given the carbon budget and a climate sensitivity of 3, global mean temperature peaks at 2°C.

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[11] For purposes of the calibrations, we abstract from $\kappa$ terms in the energy production function. Recalibrating for linear technology gives $\kappa = ½$. The terms can safely be subsumed into the TFP term. Recalibration leaves the productivity parameter of labour in the renewable sector unchanged: $A_3 = 1311$.

**Figure 1: Policy simulations**

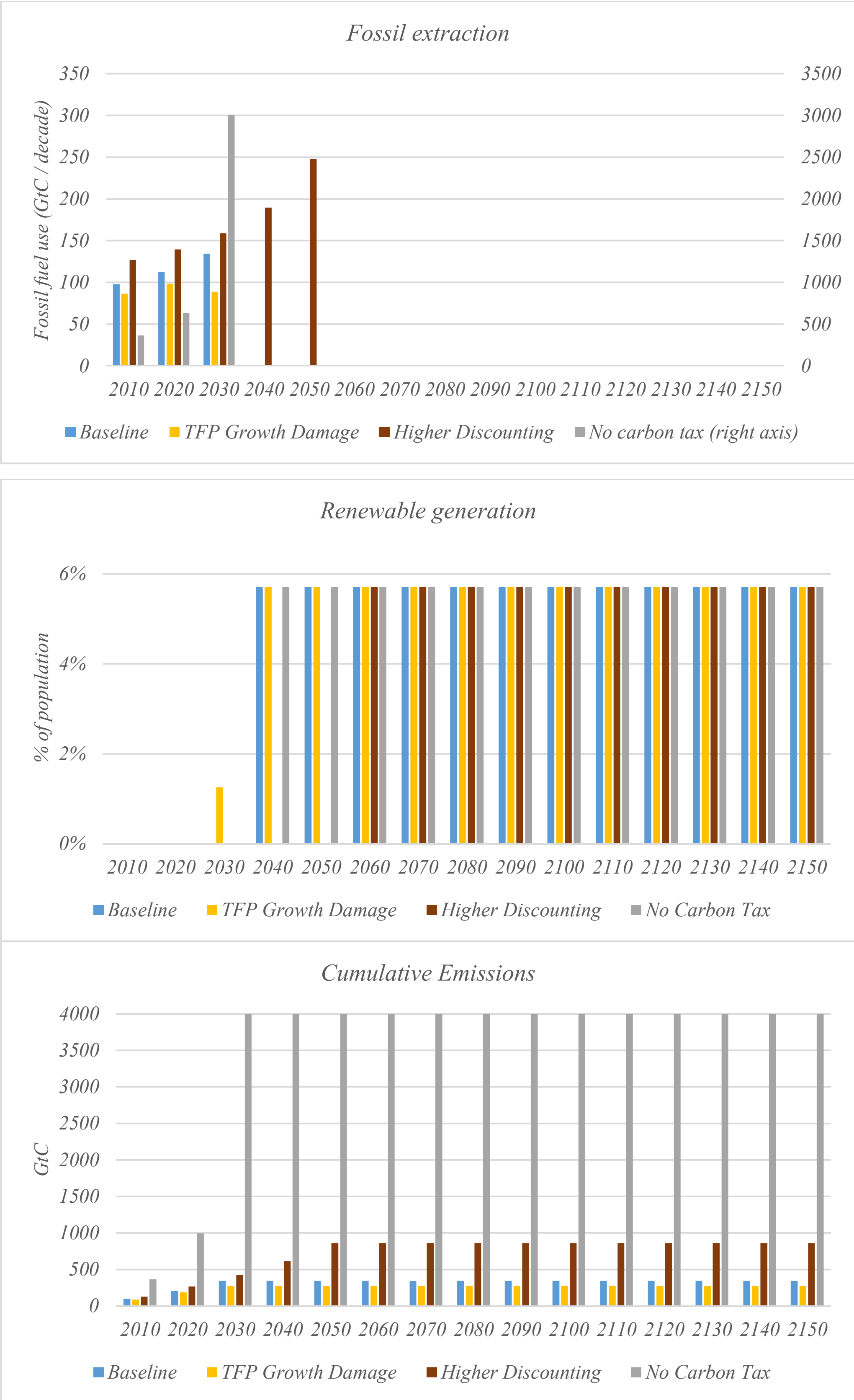
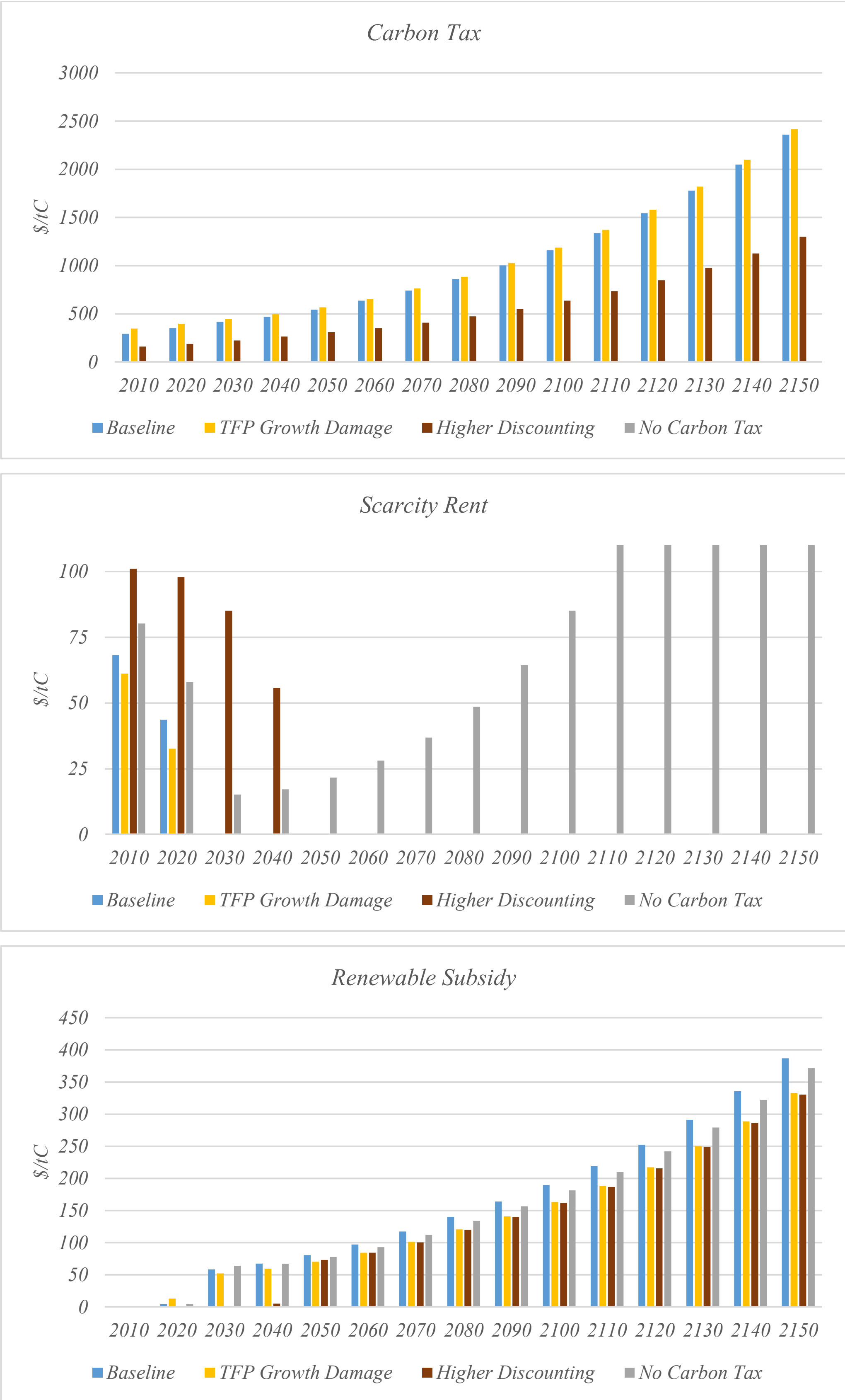


**Figure 1: Policy simulations** (continued)



This rapid decarbonisation of the economy is achieved by implementing a carbon tax of 292 \$/tC in 2010 which rises at the rate of GDP in subsequent periods. This figure is significantly higher than other estimates for the social cost of carbon and is due to the GHKT assumption of fossil fuel not requiring any input and that energy sources are perfect substitutes. The social cost of carbon and the carbon tax increases to compensate for the other missing cost components. If no carbon tax is imposed, the economy continues to operate under business as usual and uses all of the available reserves of 4,000 GtC (note the right hand scale in the first panel of figure 1) which leads to peak warming of 6.1°C. Due to the Cobb-Douglas specification, fossil fuel use is increased radically from 36 GtC in the first year to nearly 300 GtC per annum in 2040.

The first sensitivity run introduces damages to productivity growth ($\delta$ = 0.5). This increases the initial carbon tax to 349 \$/tC and discourages fossil fuel use further. After three instead of four decades, cumulative extraction of carbon has been cut to 274 GtC and renewables are phased in. Peak warming is therefore curbed to below 1.9°C.

The second sensitivity run considers a higher rate of time impatience of 3% per annum. Putting lower weight on the future affects the economy in multiple ways. Beside increasing consumption and lowering investment, the increased impatience increases fossil energy use as the future damages from global warming are discounted more heavily (the carbon tax is cut nearly in half to 160 \$/tC) and the scarcity rent falls. Fossil fuel is also used for longer and the cumulative carbon budget doubles relative to the baseline case to 863 GtC, thus causing peak warming to increase to 2.8°C.

**Table 1: Summary of optimal policy simulations**

| | Carbon Budget | Peak global warming | Switch Time (year) | Initial Carbon Tax |
|---|---|---|---|---|
| Baseline | 345 GtC | 2.0°C | 2040 | 292 \$/tC |
| No Carbon Tax | 4,000 GtC | 6.1°C | 2040 | 0 \$/tC |
| TFP Growth Damage | 274 GtC | 1.9°C | 2040 | 349 \$/tC |
| Higher Discounting | 863 GtC | 2.8°C | 2060 | 160 \$/tC |
| Lower Extraction costs | 717 GtC | 2.6°C | 2070 | 292 \$/tC |
| Stagnant renewable technology | 345 GtC | 2.0°C | 2040 | 292 \$/tC |
| Stagnant population | 629 GtC | 2.5°C | 2060 | 227 \$/tC |
| Stagnant TFP | 345 GtC | 2.0°C | 2040 | 292 \$/tC |

In further sensitivity runs reported in table 1 (but not in figure 1) we have lowered fossil fuel extraction costs by making it less sensitive to depleting reserves (i.e., 50% depletion of fossil fuel reserves leading to only a 25% rather than a 50% loss in fossil fuel extracted). This encourages fossil fuel extraction by lowering the scarcity rent. Fossil fuel is used for longer but its growth rate is flattened. As a result, the cumulative arbon budget doubles compared to the baseline, i.e., 717 GtC instead of 345 GtC. Peak temperature is thus also higher than under the benchmark (2.6°C instead of 2°C). Since fossil fuel use in the first period increases only marginally, the initial carbon tax is virtually identical to the baseline case which is in line with proposition 2.

Stagnant renewable technology due to setting the potential for learning by doing in the production of renewable energy to zero or equivalently a zero renewable energy subsidy delays the start of the carbon-free era and increases the cumulative carbon budget and peak temperature under general circumstances. However, under the assumptions of our model and in line with proposition 6, optimal fossil fuel use and the carbon tax is not affected by more stagnant or more rapid progress in renewable energy technology.

Reducing the exogenous growth rates of the size of the population reduces the willingness of current generations to accept the economic cost of climate policy. The carbon tax falls to 227 $/tC and more fossil fuel is used for longer. The effect is similar to that resulting from more impatience, but less severe. Cumulative carbon emissions increase to 629 GtC and peak warming to 2.5°C. Variations in the growth rate of total factor productivity, $\omega$, has no effect on energy use. Again, this is due to the special assumption of logarithmic utility and is in line with propositions 1-6.

## 8. Conclusion

The GHKT model (Golosov et al., 2014) gives a simple rule for the social cost of carbon and the price of carbon emissions: it should be proportional to aggregate output and thus grow in line with trend growth. The optimal level of GDP, however, remains in the GHKT model subject to the intertemporal, forward-looking depletion decisions of fossil fuel producers who optimally choose to fully deplete their reserves asymptotically. This is unrealistic and ignores that much of the debate on climate policy is about how much carbon one is allowed to emit in total and thus how big stranded fossil fuel assets are. We have therefore extended the GHKT model analytically to allow for stock-dependent fossil fuel extraction costs and partial exhaustion of fossil fuel reserves. To add further realism, we also allow for additive climate damages in social welfare, an effect of global warming on the level or the rate of growth in total factor productivity growth (as in Dell et al. (2012) and studied in Dietz and Stern (2015)), and mean reversion in the process for global warming damages in total factor productivity. Furthermore, we also allow for population growth, labour-augmenting technical progress, and specific green technical progress in the production of renewable energy (as in Acemoglu et al. (2012)). We focus at socially optimal regimes which start with a phase with only fossil fuel use, followed by an intermediate phase where fossil fuel and renewable energy are used alongside each other, and a final carbon-free phase. Under business as usual the intermediate phase does not occur.

Our contributions have been to give a full analytical characterization of our model of the macroeconomics of climate change, the derivation of simple rules for both the optimal carbon tax and the renewable energy subsidy, and an analysis of the effects of these optimal climate policies on the amount of fossil fuel to be locked up, cumulative carbon emissions and the optimal timing of the transition to the carbon-free era. In particular, the social cost of carbon and carbon tax as well as the renewable energy subsidy as fractions of aggregate output are high, cumulative carbon emissions are less, more stranded assets are stranded and peak temperature is less if society is relatively patient or if global warming also has a negative effect on the trend growth rate of the economy. Faster population growth makes climate policy more ambitious. However, in our generalization of the GHKT model changes in the trend growth in labour-augmenting technical progress or in the rate of green technical progress climate policies are neutral.

The GHKT model can be extended analytically in further directions. First, climate risk can be dealt with if the global warming damage coefficient for production damages is unknown and follows an exponential probability distribution function. The certainty-equivalent optimum is then the true stochastic optimum.[12] This justifies using an expected value for the damage coefficient. However, in general, numerical methods are required for calculating optimal climate policies under uncertainty if one departs from

[12] More generally, with relative risk aversion greater (less) the optimal carbon tax is typically higher (lower) and fossil fuel use is lower (bigger) than in the certainty-equivalent outcome.

the GHKT model by having relative risk aversion different from unity.[13] Second, Gerlagh and Liski (2015b) introduce a different type of learning in the GHKT model. If the GHKT model is correct, it is optimal for the carbon tax to grow with GDP over the next hundred years without knowing the exact factor of proportionality. When learning using a hidden-state impact process takes place, it can be shown that, as long as warming is insufficient to generate information about the true social cost of carbon, the carbon tax should rise at a *faster* rate than GDP. Third, Engström and Gars (2015) allow for climate tipping in the GHKT model and argue that the anticipation of such tipping points accelerates fossil fuel extraction and global warming in the short run. Fourth, Iverson (2014) and Gerlagh and Liski (2015a) introduce hyperbolic discounting in the GHKT model. Finally, Li et al. (2015) apply robust stochastic control techniques developed by Hansen and Sargent (2007) to the GHKT model to analyse robust policies for fighting climate change.

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[13] This cannot be done by taking Monte Carlo averages of optimal policies for samples of realizations of stochastic shocks, but needs to be done with dynamic programming methods as in Crost and Traeger (2013) and in Jensen and Traeger (2014). Dietz and Stern (2015) allow for climate risk numerically in quick-and-dirty fashion, but their analysis is numerical and ignores the optimal amount of fossil fuel that should be locked up in the crust of the earth and abstracts from expectations about future developments to affect the optimal timing of the switch to the carbon-free era.

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**APPENDIX**

**Proof of proposition 1:** The Lagrangian for this problem is

$$
\begin{aligned}
&\sum_{t=0}^{\infty}\beta^t N_t\{\ln(C_t/N_t) \\
\text{(A1)}\quad &-\lambda_t\left[K_{t+1}-A_tK_t^{\alpha}(s(S_t)F_t+r(R_t)R_t)^{\upsilon}\left\{\omega^t N_t-L_t^R\right\}^{1-\alpha-\upsilon}+C_t\right] \\
&-\psi(E_t^p+E_t^t)-\mu_t(S_{t+1}-S_t+F_t)-\phi_t\left[\ln(A_t)-\delta\ln(A_{t-1})-(1-\delta)\ln(\bar{A})+\chi E_t\right] \\
&+\left[\eta_t^p(E_{t+1}^p-E_t^p-\varphi_L F_t)+\eta_t^t\left\{E_{t+1}^t-\varepsilon E_t^t-\varphi_0(1-\varphi_L)F_t\right\}\right]-\xi_t\left[R_{t+1}-\theta R_t-L_t^R\right]\},
\end{aligned}
$$

where $\lambda_t$, $\mu_t$, $\xi_t$, and $\phi_t$ denote the shadow values of capital, fossil fuel reserves, and knowledge in the renewable energy sector, and the shadow value of the log of total factor productivity at time $t$ (per private agent), respectively, and the $\eta_t$'s denote the shadow disvalues of the transitory and permanent components of atmospheric carbon.

The first-order conditions for $C_t$ and $K_t$ are $\dfrac{1}{C_t}=\lambda_t$ and $\alpha\gamma\dfrac{Y_t}{K_t}\lambda_t=\dfrac{\lambda_{t-1}}{\beta}$. This gives $\dfrac{C_t}{C_{t-1}}=\alpha\beta\gamma\dfrac{Y_t}{K_t}$ and thus the growth rate of per-capita consumption at time $t$ must equal the product of the discount factor $\beta$ and the gross rate of interest $r_t$:

$$\text{(A2)}\qquad \frac{C_t/N_t}{C_{t-1}/N_{t-1}}=\beta r_t,\qquad r_t\equiv\alpha\frac{Y_t}{K_t}.$$

The Euler equation (A2) and (2) form a difference equation which is saddle-point stable, since $\alpha\beta\gamma<1$. The stable manifold is given by (7).

The first-order optimality condition for the log of total factor productivity gives $\lambda_tN_t\dfrac{Y_t}{A_t}-\dfrac{N_t\phi_t}{A_t}+\rho\beta\dfrac{N_{t+1}\phi_{t+1}}{A_t}=0$ or, using (7), $\dfrac{1}{1-\alpha\beta\gamma}-\phi_t+\beta\gamma\delta\phi_{t+1}=0.$ The only non-explosive solution of this difference equation gives a constant:

$$\text{(A3)}\qquad \phi_t=\frac{1}{(1-\alpha\beta\gamma)(1-\beta\gamma\delta)}.$$

The first-order optimality condition for the carbon stock gives $N_t\eta_t^t-\beta\varepsilon N_{t+1}\eta_{t+1}^t-\psi N_t-\chi\phi_tN_t=0$ and $N_t\eta_t^p-\beta N_{t+1}\eta_{t+1}^p-\psi N_t-\chi\phi_tN_t=0.$ Using (A3) this boils down to

$$\eta_{t+1}^{t} = \frac{1}{\beta\gamma\varepsilon}\eta_{t}^{t} - \frac{1}{\beta\gamma}(\psi + \chi\phi_{t}) = \frac{1}{\beta\gamma\varepsilon}\eta_{t}^{t} - \frac{1}{\beta\gamma}\left[\psi + \frac{\chi}{(1-\alpha\beta\gamma)(1-\beta\gamma\delta)}\right] \quad \text{and}$$

$$\eta_{t+1}^{p} = \frac{1}{\beta\gamma}\eta_{t}^{p} - \frac{1}{\beta\gamma}(\psi + \chi\phi_{t}) = \frac{1}{\beta\gamma}\eta_{t}^{p} - \frac{1}{\beta\gamma}\left[\psi + \frac{\chi}{(1-\alpha\beta\gamma)(1-\beta\gamma\delta)}\right].$$

Since $\beta\gamma < 1$, this difference equation satisfies the saddle-point condition so the only non-explosive solution equation is the following positive constant:

$$\text{(A4)} \quad \begin{aligned} \eta_{t}^{t} &= \frac{1}{1-\beta\gamma\varepsilon}\left[\psi + \frac{\chi}{(1-\alpha\beta\gamma)(1-\beta\gamma\delta)}\right] > 0, \\ \eta_{t}^{p} &= \frac{1}{1-\beta\gamma}\left[\psi + \frac{\chi}{(1-\alpha\beta\gamma)(1-\beta\gamma\delta)}\right] > 0. \end{aligned}$$

Hence, using (7) and $\tau_t \equiv \frac{\varphi_L \eta_t^p + \varphi_0(1-\varphi_L)\eta_t^t}{\lambda_t}$, we get (9). The first-order optimality conditions for fossil fuel and renewables give rise to the Kuhn-Tucker conditions stated in (8), where $w_t \equiv (1-\alpha-\upsilon)Y_t / L_t$, $b_t \equiv \frac{\xi_t}{\lambda_t}$, and $h_t \equiv \mu_t / \lambda_t$.

The first-order optimality condition for reserves is $N_{t-1}\mu_{t-1} = \beta N_t \left( \mu_t + \frac{\upsilon Y_t s'(S_t) F_t \lambda_t}{s(S_t)F_t + r(R_t)L_t^R} \right)$ and recalling that $h_t \equiv \mu_t / \lambda_t$, it follows (using (A2)) that scarcity rents must satisfy (9). The optimality condition for knowledge in the renewable energy sector gives $N_{t-1}\xi_{t-1} = \beta N_t \left( \theta\xi_t + \frac{\upsilon Y_t r'(R_t) L_t^R \lambda_t}{s(S_t)F_t + r(R_t)L_t^R} \right)$ or using (A2), (7), and $b_t \equiv \frac{\xi_t}{\lambda_t}$, we have (11). □

**Proof of proposition 2:** If $F_t = 0$ and $R_t > 0$, the second condition in (8) gives $\upsilon / L_t^R = (1-\alpha-\upsilon)/(M_t - L_t^R) - b_t$ and from (11) $B_{t-1} = \beta\gamma\left(\theta B_t + \upsilon\rho_1\right)$. There is only one stable solution to this difference equation: $B_t = b_t / Y_t = \beta\gamma\upsilon\rho_1 / (1-\beta\gamma\theta)$ and thus (12R). If $F_t > 0$ and $R_t = 0$, we get $\upsilon/(\varphi + H_t) = F_t$ and from (10) $H_{t-1} = \beta\gamma\left(H_t + \upsilon\sigma_1 / (1-\alpha\beta\gamma)\right)$ and thus (12F). If $F_t > 0$ and $R_t > 0$, we have from both parts of (8) that

(A5) $$\frac{\upsilon Y_t}{s(S_t)F_t + r(R_t)L_t^R} = \frac{h_t + \tau_t}{s(S_t)} \text{ and}$$

(A6) $$\frac{\upsilon Y_t}{s(S_t)F_t + r(R_t)L_t^R} = \frac{w_t - b_t}{r(R_t)}$$

The two equations (A5) and (A6) can be solved to give the first part (12S). Substituting the (A5) and (A6) into (10 and (11), respectively, gives the second part of (12S). □

**Proof of proposition 3:** Equations (14) and (15) directly follow from (11S). □

**Proof of proposition 4:** Follows from equations (14) and (15). □

**Proof of proposition 5:** Follows from $S_t^R - S_{t+1}^F < F_t$ and equations (14) and (15) with $L_t^R$, $B_t$, and $H_t$ equal to zero. □

**Proof of proposition 6:** Equations (17) and (18) follow directly from equations (12F) and (12R) and the transversality conditions for $S_t$ and $R_t$. □