The international platform of Ludwig-Maximilians University's Center for Economic Studies and the Ifo Institute

Working Papers

www.cesifo.org/wp

Altruistic Observational Learning

Christoph March
Anthony Ziegelmeyer

CESIFO WORKING PAPER NO. 5792
CATEGORY 13: BEHAVIOURAL ECONOMICS
MARCH 2016

ISSN 2364-1428

CESifo
Center for Economic Studies & Ifo Institute

CESifo *Working Paper No. 5792*

Altruistic Observational Learning

Abstract

We report two information cascade game experiments that directly test the impact of altruism on observational learning. Participants interact in two parallel sequences, the *observed* and the *unobserved* sequence. Only the actions of the *observed* entail informational benefits to subsequent participants. We find that *observed* contradict their private information significantly less often than *unobserved* when the monetary incentives to herd are moderately weak. Long laboratory cascades accumulate substantial public information which increases the earnings of participants. In Experiment 2, participants have better opportunities to learn about the strategies played by *observed* which amplifies the impact of altruism on observational learning.

JEL-Codes: D640, D820, D830.

Keywords: social learning, informational herding, altruistic behavior.

*Christoph March**
TUM School of Management
Technical University of Munich
Arcisstraße 21
Germany – 80333 Munich
christoph.march@wi.tum.de

Anthony Ziegelmeyer
Queen's Management School
Queen's University Belfast
185 Stranmillis Road
United Kingdom – Belfast BT9 5EE
email@anthonyziegelmeyer.com

*corresponding author

For valuable comments and discussions we are grateful to Thomas Daske, Sebastian Krügel, Michael Kurschilgen, Birendra Rai, Marco Sahm, and Georg Weizsäcker. Helpful comments were also received from seminar audiences at Berlin, Belfast, Munich, Nottingham, and Strasbourg, as well as from conference participants at the 2015 EEA annual meeting and the 2015 Thurgau Experimental Economics Meeting. We thank Jörg Cyriax, Tobias Gschnaidtner, Florian Inderst, Leonard Przybilla, Alexander Schlimm, and Laura von Lekow for excellent research assistance. Finally, the first author gratefully acknowledges financial support from the European Research Council and hospitality of Paris School of Economics during his time as a postdoctoral researcher.

1 Introduction

In myriad settings where individuals with limited payoff-relevant information take actions observable by others, there is convincing empirical support for observational learning: the behavior of individuals is influenced by others' actions partly *because* of the information contained therein. For instance, learning from observing peers' purchasing decisions matters for experience goods like movies, music or restaurant meals (Cai, Chen, and Fang, 2009; Moretti, 2011; Salganik, Dodds, and Watts, 2006), and observational learning effects have also been identified in financial, kidney and microloan markets as well as in sequential elections (Cipriani and Guarino, 2014; Knight and Schiff, 2010; Zhang, 2010; Zhang and Liu, 2012). The empirical confirmation of observational learning is hardly surprising since each individual is likely to benefit from combining her private information with the information revealed by others' actions. At the societal level, however, learning from others is potentially detrimental given that the more an individual relies on public information to guide her decision the less informative her decision.

This self-defeating property of observational learning is the central message of the full-rationality literature which concludes that learning from others results in serious failures to achieve efficient economic outcomes (Chamley, 2004). Information cascade models even make the compelling prediction that when a sequence of players each in turn take one of several actions, with each player observing all of her predecessors' actions, an information cascade eventually occurs in which players rationally take uninformative imitative actions (Banerjee, 1992; Bikhchandani, Hirshleifer, and Welch, 1992).[1] If few actions can be taken, information cascades are born quickly, and since public information stops accumulating once a cascade starts, players benefit little from observing others' actions. The empirical relevance of such spectacular failures of information aggregation and inefficient economic outcomes ultimately depends on the relative weight that individuals put on their private information when combining it with the information contained in observed actions. The present paper investigates, both theoretically and experimentally, whether altruism increases the response to private information and has the potential to improve economic welfare in observational learning.[2]

Starting with Anderson and Holt (1997), economists have mainly utilized laboratory experiments to test the predictions of information cascade models.[3] The bulk of the experimental evidence is summarized in Weizsäcker (2010) which compiles a meta-dataset from 13 information cascade experiments and introduces a reduced-form approach to measure the success of observational learning by

[1]The self-defeating property holds unless players' actions are always sufficient statistics for their information in which case observational learning is efficient (Lee, 1993). In settings where economic outcomes are inefficient, information cascades need not arise as observational learning is asymptotically complete with private information of unbounded strength (Smith and Sørensen, 2000). Acemoglu, Munther, Lobel, and Ozdaglar (2011) characterize necessary and sufficient conditions in general observation structures for learning to be asymptotically complete.

[2]In the observational learning settings we consider, economic welfare or efficiency simply means the sum of monetary payoffs and we posit that an altruistic individual is willing to decrease her own monetary payoff in order to increase the monetary payoff of another individual.

[3]More easily than naturally-occurring data, laboratory data allow economists to empirically identify the existence and isolate the impact of observational learning since the information structure is under the experimenter's control. For example, Stone and Zafar (2014) investigate the optimality of observational learning behavior using rankings of top American football teams submitted by sports journalists. They conclude that their findings corroborate the laboratory findings but they acknowledge that many other explanations cannot be ruled out given the limitations of their data.

controlling for the empirical profitability of actions (see also Ziegelmeyer, March, and Krügel, 2013). Reassuringly, the meta-study confirms that participants learn from others' actions and it also finds that in situations where it is empirically optimal for participants to follow their private information they most often do so. In the complementary set of situations, however, participants are reluctant to contradict their private information unless the monetary incentives to follow others are strong enough. Observational learning behavior differs from its rational counterpart in that private information is overweighted relative to public information and participants take more uninformative imitative actions after longer laboratory cascades (Kübler and Weizsäcker, 2005). These two behavioral regularities have been either explained as resulting from a commonly known non-Bayesian updating rule where participants exaggerate the precision of their private information (Goeree, Palfrey, Rogers, and McKelvey, 2007) or as resulting from participants wrongly assigning a low precision to others' actions and applying short chains of reasoning (Kübler and Weizsäcker, 2004).[4] The experimental evidence presented in this paper shows that the explanatory power of cognitive biases is limited and that the future informational gains of actions further drive the overemphasis on private information and its attenuation in long laboratory cascades.

We report two information cascade game experiments that directly test the impact of altruism on observational learning. Our first experiment consists of nine sessions where participants, in two parallel sequences, guess which of two options has been randomly selected at the beginning of each repetition of the cascade game. There are seven and eight participants in the *observed* and *unobserved* sequence, respectively. In the first part of the session, each participant obtains a single draw from an urn which has a two-thirds chance of indicating the selected option (i.e. signal quality is 2/3). Then, one participant from each of the two parallel sequences is randomly assigned to one of the first seven guessing periods and the remaining *unobserved* is assigned to the last period. Once both guesses have been submitted in a given period, the guess of the *observed* is made public knowledge. Guesses of *unobserved* always remain private. In each of the three repetitions of the cascade game, a correct guess yields 1 Euro and an incorrect guess yields zero. In the second part of the session, we collect many more guesses by doubling the number of repetitions of the cascade game and by relying on the strategy method at the history level (this design feature is borrowed from Cipriani and Guarino, 2009). In each repetition, all fifteen participants guess one of the two options in the first period. The guess of one *observed* is then randomly selected to be made public at the beginning of the next period and this participant stops guessing. This process continues until the last period where each of the *unobserved* submits a guess. For each participant, only one randomly selected guess is paid in each repetition. The last two parts of each session are identical to part 2 except that *unobserved* receive private signals of quality 18/21 and 12/21 in part 3 and 4, respectively. To cleanly identify the impact of altruism on observational learning behavior, *unobserved* guesses in the last two parts are excluded from the analysis.

In our experimental setting all participants face the same cognitive challenge when learning from others. However, *observed* have an incentive to overweight their private information relative to public information as long as they recognize the value of signaling information to subsequent participants in any of the two sequences. To generate testable hypotheses about behavioral differences between

[4]A recurrent justification for non-Bayesian updating is overconfidence (Bernardo and Welch, 2001; Kariv, 2005; Nöth and Weber, 2003). Recently, a growing literature has examined the nature of observational learning outcomes when players make boundedly rational inferences (Bohren, 2015; Eyster and Rabin, 2010; Guarino and Jehiel, 2013).

observed and *unobserved*, we study simple models of observational learning with altruistic players. First, we show that, in the 2-by-2-by-2 setting of Bikhchandani, Hirshleifer, and Welch (1992) with sufficiently many players, sequential equilibria exist where altruism generates additional informative guesses. Consequently, the onset of information cascades is delayed and the expected correctness of subsequent players' guesses is increased. Second, we apply the logit quantal response equilibrium (LQRE) approach to the analysis of altruistic observational learning. We show that a LQRE exists where even a small degree of altruism increases the informativeness of guesses when the monetary incentives to follow others are moderately weak which in turn increases the probability to contradict private information after long herds. With the help of numerical computations, we then derive full and precise predictions in our laboratory cascade game regarding the influence of altruism on the equilibrium choice probabilities after any history of previous guesses.

The results of our first experiment confirm the main implications of altruistic observational learning. First, by using the approach introduced by Weizsäcker (2010), we find that *observed* contradict their private information significantly less often than *unobserved* in situations where the monetary incentives to follow others are moderately weak. Once the incentives to follow others are strong enough *observed* contradict their private information to the same extent as *unobserved*. Second, we show that the more informative observational learning is the more public information is accumulated. Finally, we observe that altruistic behavior often enhances the monetary payoffs of participants as its associated benefits more than compensate for its associated costs. *Unobserved*, on the other hand, act as if they slightly overweight their private information relative to the public information only when the monetary incentives to follow others are the weakest.

In Experiment 1, participants at the end of the cascade game are informed about the selected option and their earnings, and they are reminded of the entire sequence of *observed* guesses which were made public. But, in line with previous cascade experiments, the draws of other *observed* are not disclosed. Our second experiment is identical to the first one except that the feedback screens also disclose the private signals of the *observed* guesses which were made public. In Experiment 2, participants are therefore offered better opportunities to learn about the strategies played by *observed* in the cascade game. We find that reducing the level of behavioral uncertainty amplifies the impact of altruism on observational learning. The response to private information in the *observed* sequence is stronger and more public information is aggregated in the second than in the first experiment. In the *unobserved* sequence, participants are still slightly reluctant to contradict their private information when the monetary incentives to follow others are the weakest. This finding suggests that in the absence of future informational benefits of actions the overemphasis on private information is more driven by judgment biases rather than by inferential biases.

Related Literature. The theoretical section of the present paper relates to the few models of observational learning where players take into account the future informational gains of their actions.

Smith, Sørensen, and Tian (2014) study the altruistic observational learning model with a general distribution of private information and an infinite number of players, and their analysis focuses on the constrained efficient equilibrium.[5] They show that with private information of bounded strength higher degrees of altruism lead to smaller cascade sets which entails that uninformative actions are

[5] Vives (1997) also studies an altruistic observational learning model but in a market setting with Gaussian information. He shows that socially optimal learning accumulates more public information than individually optimal learning.

taken less readily. We prove related results in a simple model of observational learning with a finite sequence of altruistic players who receive symmetric binary private signals. In particular, Proposition 2 shows that equilibria exist in which the onset of information cascades is delayed and the likelihood of a herd on the less profitable action is reduced.

Ali and Kartik (2012) consider a simple setting of observational learning with *collective* preferences: a player's payoff depends on a binary state of nature and on the profile of actions of any subset of all players, players may differ in how they care about the choices of others, and each player weakly prefers others to take the most profitable action.[6] They show that an equilibrium exists in which players behave as in the unique equilibrium of the standard model where payoffs are independent. By focusing on the specific case of homogeneous altruism in a similar observational learning setting, we provide detailed results about the impact of forward-looking incentives on players' response to private information. In particular, we establish that the standard LQRE outcome differs from any of the LQRE outcomes with altruistic players meaning that players' relative emphasis on private information is systematically influenced by altruism in the presence of commonly known payoff-responsive errors.

Altruistic behavior is an intuitive explanation for the fact that observational learning is more informative in the laboratory than predicted by rational herding. Indeed, participants are likely to understand that a stronger response to private information is an altruistic act as participants have collectively (almost) full information. In addition, the monetary cost for a participant who ignores a short laboratory cascade is rather trivial while the monetary benefits for her successors are potentially substantial, and there is ample evidence of altruistic behavior in laboratory games where own costs are low while others' benefits are large (Andreoni, Harbaugh, and Vesterlund, 2008). Still, the experimental literature on observational learning has either ignored the altruistic explanation or favored an explanation in terms of judgment or inferential biases.[7]

Most cascade experiments have implemented short sequences of participants which undermine the influence of efficiency concerns in observational learning (two-thirds of the observations in the meta-dataset of Weizsäcker, 2010, come from sequences with at most six participants). But even the longer cascade experiments did not invoke altruism as a possible explanation for participants' relative overemphasis on private information with the exception of Goeree, Palfrey, Rogers, and McKelvey (2007). These authors acknowledge that the reluctance to contradict private information after short laboratory cascades might be a manifestation of altruistic behavior rather than a base rate fallacy (BRF). They however reject the altruistic interpretation since they find no significant difference in the structural estimates of the BRF parameter when comparing earlier and later periods of the cascade game or when comparing sequences of 20 and 40 participants. The validity of this conclusion is questionable since the specification of the structural model has a critical impact on inferences about

[6]Thanks to the richness of the class of preferences considered by Ali and Kartik (2012), their observational learning model encompasses many applications. For example, each player could be altruistic toward a different set of other players i.e. the structure of altruism could be captured by a network. Another leading application is sequential voting with rich motivations for voters such as expressive-voting preferences or preferences about margins of victory.

[7]Alevy, Haigh, and List (2007) is a notable exception. They find that financial market professionals respond more strongly to private information *and* make better use of available public information than do students. Consequently, the professionals are involved in weakly fewer overall laboratory cascades and significantly fewer laboratory cascades on the wrong action. The authors conclude that "...data reveal that the decisions of market professionals are consistent with behaviors that may mitigate informational externalities in market settings, and thus reduce the severity of price bubbles due to informational cascades."

parameters. In fact, when applying Weizsäcker's model-free approach to assess participants' responses to the empirical value of contradicting private information in prior cascade game experiments, we find that actions are significantly more informative in long decision sequences than in short ones (see Section 4.2).[8]

We have restricted our discussion of the experimental literature on observational learning to experiments which implemented Bikhchandani, Hirshleifer, and Welch's (1992) stripped-down model of information cascades. However, the overemphasis on private information and its attenuation in long herds has also been confirmed in other observational learning experiments (for example in the continuous-signal-discrete-action experiment of Çelen and Kariv, 2004 or the financial-markets-with-event-uncertainty treatment of Cipriani and Guarino, 2009).

The paper is organized as follows. Section 2 derives theoretical predictions in a simple model of observational learning with altruistic players. Section 3 describes our experimental design and procedures. Section 4 and Section 5 reports on the results in our first and second experiment, respectively. Section 6 concludes. The online supplementary material contains a series of appendices with proofs, complementary theoretical and statistical analyses, and the experimental instructions of our first experiment.

2 Theory

We consider a simple model of observational learning where, in the absence of altruism, the failure of information aggregation is spectacular in the unique sequential equilibrium outcome as the onset of information cascades is almost immediate. In the presence of altruism, we show that sequential and logit quantal response equilibria exist such that players increase their response to private information and more information is aggregated.

2.1 A Simple Game of Altruistic Observational Learning

Nature moves first and chooses a payoff-relevant state of Nature (henceforth state) $\theta \in \Theta = \{\mathcal{B}, \mathcal{O}\}$ according to the common prior $p \equiv \Pr(\theta = \mathcal{B}) \in (0.5, 1)$.[9] Each player is then endowed with a symmetric binary private signal $s_t \in S = \{b, o\}$ such that $\Pr(s_t = b \mid \theta = \mathcal{B}) = \Pr(o \mid \mathcal{O}) = 1 - \Pr(o \mid \mathcal{B}) = 1 - \Pr(b \mid \mathcal{O}) = q \in (p, 1)$. Conditional on the state, signals are independently distributed across players.

The finite set of players is $\{1, \ldots, T\}$ with generic element t. Time is discrete and, in period $t = 1, 2, \ldots, T$, player t chooses action $x_t \in X = \{B, O\}$ where B stands for "guess state $\mathcal{B}$" and O stands for "guess state $\mathcal{O}$". Before choosing her action, player t observes the history of previous actions $\mathbf{h}_t = (x_1, \ldots, x_{t-1}) \in H_t = \{B, O\}^{t-1}$ where $\mathbf{h}_1 \equiv \varnothing$ and $H \equiv \bigcup_{t=1}^{T} H_t$.

Player t's preferences depend on the *complete* profile of actions $\mathbf{x} = (x_1, \ldots, x_T)$ and the state θ, and they are represented by the von-Neumann Morgenstern utility function

$$u_t(\mathbf{x}, \theta) = \pi(x_t, \theta) + \alpha \sum_{\tau \neq t} \pi(x_\tau, \theta) \quad (1)$$

[8] In their "majority rule institution" treatment, Hung and Plott (2001) incentivize participants to exhibit altruistic behavior by rewarding them according to whether a majority of actions are right or wrong. In line with the logic of efficiency concerns, they find that more information is revealed in this treatment compared to the usual treatment where each participant is rewarded according to whether her action is right or wrong.

[9] We abstract from the non-generic case $p = 1/2$ to avoid the use of tie-breaking rules.

where $\pi(B,\mathcal{B}) = \pi(O,\mathcal{O}) = 1$, $\pi(B,\mathcal{O}) = \pi(O,\mathcal{B}) = 0$, $0 \le \alpha \le 1$ captures the level of altruism, and $\tau \in \{1,\ldots,T\}$. When $\alpha > 0$, the second component of the utility function entails that players take into account the future informational benefits of their actions because of their concern for efficient outcomes. Our functional form of prosocial preferences relates to the (utilitarian) social welfare function assumed by Smith, Sørensen, and Tian (2014) in their general welfare analysis of observational learning.

2.2 Beliefs, Strategies and Equilibrium Concepts

Denote by $\left\langle T, X, H, \Theta, p, S, q, \{u_t\}_{t=1}^{T}\right\rangle$ the (simple) game of altruistic observational learning. Without loss of generality, player t's behavior is captured by the *behavioral strategy* $\sigma_t : S \times H_t \to \mathcal{D}(X)$ where (in a slight abuse of notation) $\sigma_t(s_t, \mathbf{h}_t)$ denotes the probability that she chooses action $x_t = B$.[10] Call a behavioral strategy *pure* (completely mixed) if $\sigma_t(s_t,\mathbf{h}_t) \in \{0,1\}$ $(0 < \sigma_t(s_t,\mathbf{h}_t) < 1)$ for each $s_t \in S$ and $\mathbf{h}_t \in H_t$.

Player t forms her *belief* by combining the *public belief*, the probability of state $\mathcal{B}$ conditional on the history $\mathbf{h}_t$, with her private information, the signal s_t, in a Bayesian way. Let player t's *belief* be given by the mapping $\mu_t : S \times H_t \to \mathcal{D}(\Theta)$ and let $\mu_t(s_t,\mathbf{h}_t)$ denote the probability player t assigns to state $\mathcal{B}$ at history $\mathbf{h}_t$ given signal s_t. Given history $\mathbf{h}_t$, signal s_t, and the strategies $\boldsymbol{\sigma}_{-t} = (\sigma_1,\ldots,\sigma_{t-1},\sigma_{t+1},\ldots,\sigma_T)$ of the other players, player t's expected utility of action x_t is given by

$$U_t(x_t \mid s_t,\mathbf{h}_t,\boldsymbol{\sigma}_{-t}) = \sum_{\theta\in\Theta}\mu_t(\theta \mid s_t,\mathbf{h}_t) * \left[\pi(x_t,\theta) + \alpha\sum_{\tau<t}\pi(x_\tau,\theta) + \alpha\, C_t(x_t \mid \mathbf{h}_t,\theta,\boldsymbol{\sigma}_{-t})\right] \tag{2}$$

where

$$C_t(x_t \mid \mathbf{h}_t,\theta,\boldsymbol{\sigma}_{-t}) = \sum_{(x_{t+1},\ldots,x_T)}\left[\prod_{\tau>t}\sum_{s_\tau\in S} Pr(s_\tau \mid \theta)\,\sigma_\tau(x_\tau \mid s_\tau,\mathbf{h}_\tau)\right]\sum_{\tau>t}\pi(x_\tau,\theta)$$

with $\mathbf{h}_\tau \supseteq (\mathbf{h}_t, x_t)$ for each $\tau > t$. $C_t(x_t \mid \mathbf{h}_t,\theta,\boldsymbol{\sigma}_{-t})$ is player t's *(expected) continuation value* of action x_t at history $\mathbf{h}_t$ and state θ given strategies $\boldsymbol{\sigma}_{-t}$. Lemma A1 in Appendix A presents some useful properties of the continuation values.

To characterize players' behavior, we rely on the sequential equilibrium concept[11] since it restricts off-path beliefs more strongly than the perfect Bayesian equilibrium.[12]

Definition 1. *A sequential equilibrium of the game* $\left\langle T, X, H, \Theta, p, S, q, \{u_t\}_{t=1}^{T}\right\rangle$ *is a strategy profile* $\boldsymbol{\sigma}^*$ *and a system of beliefs* $\boldsymbol{\mu}^*$ *such that*

(i) *strategies are sequentially rational, i.e. for each* t, s_t, *and* $\mathbf{h}_t$,

$$\sigma_t^*(s_t,\mathbf{h}_t) = \begin{cases} 1 & \text{if } U_t(B \mid s_t,\mathbf{h}_t,\boldsymbol{\sigma}_{-t}^*) > U_t(O \mid s_t,\mathbf{h}_t,\boldsymbol{\sigma}_{-t}^*) \\ 0 & \text{if } U_t(B \mid s_t,\mathbf{h}_t,\boldsymbol{\sigma}_{-t}^*) < U_t(O \mid s_t,\mathbf{h}_t,\boldsymbol{\sigma}_{-t}^*) \end{cases};$$

and

[10] For a given (finite) set M, $\mathcal{D}(M)$ denotes the set of probability distributions over M.

[11] Kreps and Wilson (1982).

[12] Sequential equilibrium beliefs satisfy $\frac{1-q}{q}\frac{\mu_t(s_t,\mathbf{h}_t)}{1-\mu_t(s_t,\mathbf{h}_t)} \le \frac{\mu_t(s_t,(\mathbf{h}_t,x_t))}{1-\mu_t(s_t,(\mathbf{h}_t,x_t))} \le \frac{q}{1-q}\frac{\mu_t(s_t,\mathbf{h}_t)}{1-\mu_t(s_t,\mathbf{h}_t)}$ for each $t < T$, $\mathbf{h}_t$, x_t and s_t.

(ii) *beliefs are consistent, i.e.* $(\boldsymbol{\sigma}^*, \boldsymbol{\mu}^*) = \lim_{n\to\infty} \left(\boldsymbol{\sigma}^{(n)}, \boldsymbol{\mu}^{(n)}\right)$ *where, for each* n, $\boldsymbol{\sigma}^{(n)}$ *is a profile of completely mixed behavioral strategies and* $\boldsymbol{\mu}^{(n)}$ *is derived from* $\boldsymbol{\sigma}^{(n)}$ *by Bayes rule:*

$$\mu_t^{(n)}(s_t, \mathbf{h}_t) = \left[1 + \frac{1-p}{p}\frac{\Pr(s_t \mid \mathcal{O})}{\Pr(s_t, \mid \mathcal{B})}\prod_{\tau<t}\frac{\sum_{s_\tau\in S}\Pr(s_\tau \mid \mathcal{O})\,\sigma_\tau^{(n)}(x_\tau \mid s_\tau, \mathbf{h}_\tau)}{\sum_{s_\tau\in S}\Pr(s_\tau \mid \mathcal{B})\,\sigma_\tau^{(n)}(x_\tau \mid s_\tau, \mathbf{h}_\tau)}\right]^{-1}$$

for each t, s_t, *and* $\mathbf{h}_t$ *where* $\mathbf{h}_\tau \subset \mathbf{h}_t$ *for each* $\tau < t$.

Information externalities might give rise to herding and informational cascades in the observational learning game. Given a history $\mathbf{h}_t$, player t herds if her action does not depend on her signal i.e. $\sigma_t(b, \mathbf{h}_t) = \sigma_t(o, \mathbf{h}_t)$, thereby imitating the action of her predecessor. On the other hand, given a history $\mathbf{h}_t$, player t acts informatively if $\sigma_t(b, \mathbf{h}_t) \neq \sigma_t(o, \mathbf{h}_t)$. An informational cascade emerges after some history $\mathbf{h}_t$ if, for every player $\tau \geq t$, $\sigma_\tau(b, \mathbf{h}_\tau) = \sigma_\tau(o, \mathbf{h}_\tau)$. Finally, a herd on action x_{t-1} emerges after some history $\mathbf{h}_t$ if every player $\tau \geq t$ chooses action x_{t-1} i.e. $x_\tau = x_{t-1}$ for all $\tau \geq t$.

Further restrictions on off-path beliefs and monotonic equilibria

We pin down off-path beliefs further by focusing on two special cases. First, with *error off-path beliefs* players treat all actions off the equilibrium path as uninformative about the state. Hence, player t's belief given signal s_t at the off-path-history $\mathbf{h}_t$ is equal to the belief of player $\tau < t$ given signal $s_\tau = s_t$ at the maximal sub-history $\mathbf{h}_\tau \subset \mathbf{h}_t$ that is on the equilibrium path. Second, with *signal revealing off-path beliefs* players treat off the equilibrium path action B (resp. action O) as revealing signal b (resp. signal o). Accordingly, player t's belief given signal s_t at the off-path-history $\mathbf{h}_t$ satisfies $\mu_t(s_t, \mathbf{h}_t) / [1 - \mu_t(s_t, \mathbf{h}_t)] = \mu_\tau(s_t, \mathbf{h}_\tau) / [1 - \mu_\tau(s_t, \mathbf{h}_\tau)] \cdot (q/(1-q))^{n_B - n_O}$ where $\mathbf{h}_\tau \subset \mathbf{h}_t$ is the maximal sub-history of $\mathbf{h}_t$ that is on the equilibrium path and n_B (resp. n_O) is the number of times action B (resp. action O) is chosen by the subset of players $\{\tau, \ldots, t-1\}$. Whenever it applies, the off-path beliefs specification is assumed commonly known.

By restricting the analysis to pure strategies and either error or signal revealing off-path beliefs, we are able to capture the behavior of players by the simplified strategies $\hat{\sigma}_t(s_t, \Delta_t)$ where $\Delta_t \in \mathbb{Z}$ denotes the difference between the number of b and o signals that player t infers from history $\mathbf{h}_t$ (see Lemma A3 in Appendix A). For the sake of clarity, our main analysis focuses on *monotonic* equilibria where players adopt such simplified strategies and which require that: i) strategies are weakly increasing in the difference Δ; and ii) the *information cascade set* weakly grows over time (there is no information cascade as long as the public belief stays in a certain interval; the complement of that interval is called the information cascade set).

Definition 2. *An equilibrium* $\hat{\boldsymbol{\sigma}}^*$ *is* ***monotonic*** *if and only if*

(i) *for each* $t = 2, \ldots, T$, *each* $\Delta_t \in \{2-t, \ldots, t-1\}$ *and each* $s_t \in S$, $\hat{\sigma}_t^*(s_t, \Delta_t) \geq \hat{\sigma}_t^*(s_t, \Delta_t - 1)$;

and

(ii) *for each* $t < T$ *and each* $\Delta_t \in \{1-t, \ldots, t-1\}$, $\hat{\sigma}_t^*(b, \Delta_t) \geq \hat{\sigma}_{t+1}^*(b, \Delta_t)$ *and* $\hat{\sigma}_t^*(o, \Delta_t) \leq \hat{\sigma}_{t+1}^*(o, \Delta_t)$.

The following properties hold in every monotonic equilibrium (see Lemma A4 in Appendix A): (i) players are weakly more likely to choose action B with b-signals than with o-signals; (ii) players act informatively whenever $\Delta \in \{-1, 0\}$; (iii) for each $t = 1, \ldots, T$, $\hat{\sigma}_t^*(b, \Delta_t) = 0$ only if $\Delta_t \leq -2$ and $\hat{\sigma}_t^*(o, \Delta_t) = 1$ only if $\Delta_t \geq 1$; (iv) if players herd on action B when having inferred difference Δ, they also herd on action B when the difference is $\Delta + 1$; and similarly (v) if players herd on action O when having inferred difference Δ, they also herd on action O when the difference is $\Delta - 1$.

2.3 Onset of Cascades and Information Aggregation in Monotonic Equilibria

There are multiple equilibrium outcomes in the altruistic observational learning game. In the absence of altruism ($\alpha = 0$), the equilibrium outcome involves the spectacular failure of information aggregation as players herd on action B (resp. action O) as soon as there is an imbalance of one B action (resp. two O actions) in the history of previous actions. We refer to the informationally inefficient equilibrium outcome as to the standard equilibrium outcome. In the presence of altruism ($0 < \alpha \leq 1$), equilibrium outcomes exist where cascades are delayed and the informativeness of public information is enhanced.

2.3.1 Immediate Cascades and Poor Information Aggregation

For a richer class of prosocial preferences than the one we consider, Ali and Kartik (2012) show that the standard equilibrium outcome remains an equilibrium outcome of the altruistic observational learning game for strictly positive levels of altruism.[13]

Proposition 1 (Ali and Kartik, 2012)**.** *For any* $0 < \alpha \leq 1$ *there exists a monotonic equilibrium* $\hat{\boldsymbol{\sigma}}^*$ *which for each* $1 \leq t \leq T$ *and each* $\Delta_t \in \mathbb{Z}$ *satisfies*

$$\left(\hat{\sigma}_t^*\left(b, \Delta_t\right), \hat{\sigma}_t^*\left(o, \Delta_t\right)\right) = \begin{cases} (1,1) & \textit{if } \Delta_t \geq 1 \\ (1,0) & \textit{if } -1 \leq \Delta_t \leq 0 \\ (0,0) & \textit{if } \Delta_t \leq -2 \end{cases}.$$

The equilibrium characterization of Ali and Kartik (2012) establishes that the core insights from standard economic models of observational learning, such as the swift onset of information cascades and their inherent fragility, can be relevant even when players care *directly* about others' actions.

2.3.2 Delayed Cascades and Improved Information Aggregation

For sufficiently long sequences of players, Proposition 2 characterizes a set of monotonic equilibria where compared to the standard equilibrium the onset of information cascades is delayed and more public information is accumulated.

Proposition 2. *Assume that* T *is sufficiently large so that the lower bound* $0 < \underline{\alpha}(p,q) < 1$ *exists.*[14] *For each* $\alpha > \underline{\alpha}(p,q)$ *there exists a monotonic equilibrium* $\hat{\boldsymbol{\sigma}}^*$ *which for each* $1 \leq t \leq T$ *and each* $1 - t \leq \Delta_t \leq t - 1$ *satisfies*

$$\left(\hat{\sigma}_t^*\left(b, \Delta_t\right), \hat{\sigma}_t^*\left(o, \Delta_t\right)\right) = \begin{cases} (1,1) & \textit{if } \Delta_t \geq \overline{\Delta}_t \\ (1,0) & \textit{if } \underline{\Delta}_t + 1 \leq \Delta_t \leq \overline{\Delta}_t - 1 \\ (0,0) & \textit{if } \Delta_t \leq \underline{\Delta}_t \end{cases}$$

with $\overline{\Delta}_t \geq 1$ *and* $\underline{\Delta}_t \leq -2$ *for each* $1 \leq t \leq T$*, and either* $\overline{\Delta}_t \geq 2$ *or both* $\overline{\Delta}_t \geq 2$ *and* $\underline{\Delta}_t \leq -3$ *for some* $t < T$*. Moreover,* $\underline{\Delta}_t \leq \underline{\Delta}_{t+1}$ *and* $\overline{\Delta}_t \geq \overline{\Delta}_{t+1}$ *for each* $1 \leq t < T$ *with strict inequality for some* $t < T$*.*

[13] Ali and Kartik consider a general class of utility functions where each player has a type which specifies the strength of her preference for others to correctly guess the realized state and they term this kind of payoff interdependence *collective preferences*. They assume the *error off-path beliefs* specification.

[14] The restriction on T is weak. For instance, $T = 5$ is sufficient as long as $p < 0.74$ or $q < 0.87$, and $T = 6$ is sufficient as long as $p < 0.81$ or $q < 0.94$. See Appendix A.4 for details.

In non-standard equilibrium outcomes, players act informatively only if the monetary incentives to herd are sufficiently weak and they have sufficiently many successors who can benefit from the revelation of their private information. For example, early players in the sequence reveal their private information when facing $\Delta \in \{-2,-1,0,1\}$ as long as a few other players succeed them. On the other hand, players herd if the monetary incentives to do so are strong or if they act late in the sequence. By inducing players to rely more on their private information when choosing their actions, altruistic observational learning accumulates more public information and it enhances the expected correctness of subsequent players' guesses. Said differently, the onset of information cascades is delayed and the likelihood of a herd on the ex-post wrong action is reduced. For non-negligible levels of altruism, such equilibrium outcomes are intuitively more plausible as informative actions reflect players' concern for desirable economic outcomes.

Since our main objective has been to establish that altruism might delay the onset of cascades and aggregate more information, our analysis has focused on monotonic equilibria. As expected, the combination of forward-looking incentives and information externalities implies the existence of many non-monotonic equilibria in the altruistic observational learning game. Appendix B in the supplementary material exhibits some of these additional equilibria and offers evidence on how rapidly the number of equilibria grows with the degree of altruism.

2.4 Quantal Response Altruistic Observational Learning

Finally, we investigate the behavioral implications of the homogeneous Logit Quantal Response Equilibrium (LQRE) in the altruistic observational learning game. There is a legitimate concern that altruism has negligible influence on the response to private information and information aggregation in the presence of payoff-responsive decision errors as both extensions alter the predictions of rational observational learning in a similar way. Indeed, the standard LQRE ($\alpha = 0$) predicts that a herd-breaking action happens more frequently if the player received a private signal contradicting the herd choices and the herd is short which implies a positive relationship between the length and strength of herds and full information aggregation in the limit (Goeree, Palfrey, Rogers, and McKelvey, 2007).

Let $\sigma_t^Q(s_t,\mu_t)$ denote player t's LQRE probability to choose action B at public belief $\mu_t(\varnothing,\mathbf{h}_t)$ given signal $s_t \in S$ with $\mathbf{h}_t \in H_t$ and $1 \le t \le T$, and let $\boldsymbol{\sigma}^{\boldsymbol{Q_0}} = \left(\sigma_1^{Q_0},\dots,\sigma_T^{Q_0}\right)$ denote the standard LQRE. In the next proposition we compare the action probabilities in $\boldsymbol{\sigma}^{\boldsymbol{Q_0}}$ with the action probabilities in LQRE which are "*monotonic-within-periods*" for strictly positive degrees of altruism. In a *monotonic-within-periods* LQRE the action probabilities satisfy the following two properties: i) $\sigma_t^Q(b,\mu_t) > \sigma_t^Q(o,\mu_t)$ for each $\mu_t \in (0,1)$; and ii) $\partial\sigma_t^Q(s_t,\mu_t)/\partial\mu_t > 0$ for each $s_t \in S$ and each $\mu_t \in (0,1)$. As shown in Appendix C of the supplementary material, the existence of a *monotonic-within-periods* LQRE is guaranteed only if large degrees of altruism are assumed away.

Proposition 3. *For $\alpha > 0$ but not too large, there exists a monotonic-within-periods LQRE $\boldsymbol{\sigma}^{\boldsymbol{Q}}$ such that, for each $1 \le t < T$, $\sigma_t^Q(b,\mu_t) > \sigma_t^{Q_0}(b,\mu_t)$ if $\mu_t \in [\underline{\mu},1/2)$ and $\sigma_t^Q(o,\mu_t) < \sigma_t^{Q_0}(o,\mu_t)$ if $\mu_t \in (1/2,\overline{\mu}]$ where $0 < \underline{\mu} < 1-q$ and $1 > \overline{\mu} > q$.*

Proposition 3 shows that a LQRE exists where informative actions are more likely than in the standard LQRE if the monetary incentives to follow others are moderately weak. We therefore confirm the intuition that altruism has the potential to induce more informative observational learning even in the presence of payoff-responsive decision errors. Notice that our characterization of the LQRE is only partial since we have not been able to derive a closed-form expression of the interval $[\underline{\mu}, \overline{\mu}]$ for which informative actions are more likely in the presence of altruism. In particular, a characterization of the size of the interval *across* periods is unavailable though, obviously, $\sigma_T^Q(s,\mu) = \sigma_T^{Q_0}(s,\mu)$ for each $s \in S$ and each $\mu \in (0,1)$. Finally, we note that for negligible decision errors $0 < \underline{\mu} \leq (1-q)^2 / \left(q^2 + (1-q)^2\right)$ and $1 > \overline{\mu} \geq q^2 / \left(q^2 + (1-q)^2\right)$ which implies that altruism increases the probability of informative actions when players face herds of size 1 or 2.

We complement our analytical results with the help of numerical computations and derive LQRE predictions for the observational learning setting implemented in the laboratory. Predictions are computed for degrees of altruism $\alpha \in \{0.125, 0.25, 0.5\}$, response precisions $\lambda \in \{2.5, 5, 7.5\}$, and differences with the predictions of the standard LQRE can be summarized as follows (see Appendix C.3 of the supplementary material for more details). First, if private information contradicts the herd choices, the increase in the probability of informative actions due to altruism declines with the size of the (contrary) herd and the magnitude of decision errors, and higher degrees of altruism induce more informative actions for given herd and errors sizes. For example, if $\lambda = 5$, the probability of an informative action in the standard LQRE after a herd of size 1, 2, 3 and 4 to 6 is 0.650, 0.500, 0.375 and 0.264 respectively, and it increases to 0.870, 0.589, 0.418 and 0.273 in the LQRE with $\alpha = 0.25$. Thus, unless monetary incentives to follow others are strong, even moderate degrees of altruism increase noticeably the response to private information. Second, altruism reduces the likelihood of errors in situations where private information agrees with the herd choices. For example, if $\lambda = 5$, the probability of decision errors in the LQRE with $\alpha = 0.25$ is about one-fourth the probability of decision errors in the standard LQRE.

3 Experimental Design and Procedures

In this section, we describe the design and procedures of our two experiments which vary only in the feedback that participants received at the end of each repetition of the cascade game and we elaborate on these variations further below.

3.1 General Features

We implement a parameterized version of the observational learning setting described in Section 2.1, but with the essential modification that participants play the cascade game in two parallel sequences.[15] Each repetition of the game begins with the random selection of one of two options and the selected option is not disclosed to participants until all decisions have been made. The two options are labeled 'blue' and 'orange' with option 'blue' having a 11/20 probability to be selected and option 'orange' having a 9/20 probability to be selected. Participants obtain independent private signals that reveal information about which of the two options has been randomly selected. Then, in two randomly determined sequences, participants guess an option and they receive 1 Euro for a correct guess and nothing otherwise.

In the *observed* sequence each of the 7 participants receives a private signal of medium quality with a 2/3 probability to indicate the selected option. In the *unobserved* sequence each of the 8 participants receives a private signal of either medium, high or low quality depending on the part of the session.

[15]The "matched sequences" design is borrowed from Ziegelmeyer, Koessler, Bracht, and Winter (2010).

Once all guesses have been submitted in a given period (but the last one), the guess of one *observed* is made public knowledge and this participant stops guessing. Guesses of *unobserved* remain private. Participants keep the same role of *observed* or *unobserved* during the entire session.

3.2 The Progress of a Session

Each experimental session is partitioned into four parts. In the first part participants become familiar with the cascade game during three of its repetitions and each participant submits only one guess per repetition. In the last three parts many more guesses are collected in each of the six repetitions of the game since participants submit guesses in several situations distinguished only by the history of previous choices. Following Cipriani and Guarino (2009), this design feature allows participants to gain extensive experience with the combination of private and public information.

Part 1. In the first part of a session the procedures closely follow those used by Anderson and Holt (1997) in their baseline experiment except for the two parallel sequences of participants and the fact that guesses are collected and transmitted through computer terminals. Participants draw their private signals from a physical urn (with replacement) and they are randomly assigned to guessing periods. Each participant in the *observed* sequence obtains a single draw from an urn which has a two-thirds chance of indicating the selected option i.e. the urn contains 14 balls indicative of the selected option and 7 balls indicative of the unselected option (hereafter, simply correct and incorrect balls). Likewise, each participant in the *unobserved* sequence obtains a single draw from an urn containing 14 correct and 7 incorrect balls. In each of the first seven periods one *observed* and one *unobserved* simultaneously guess an option. The guess submitted in the *observed* sequence is then displayed on all participants' computer screens at the beginning of the next period. In the last period only the remaining *unobserved* submits a guess. From the second period on, participants may condition their guesses on the *observed* guesses submitted in previous periods.

Part 2. Repetitions in the second part of a session are identical to repetitions in the first one except that participants draw private signals from virtual urns displayed on their computer screens and submit multiple guesses. Concretely, each *unobserved* submits 8 guesses and each *observed* submits between 1 and 7 guesses. In the first period, all 15 participants guess one of the two options. The guess of one *observed* is then randomly selected to be made public at the beginning of the next period and this participant stops guessing. In the second period, each of the 14 remaining participants submits a guess. Again, the guess of one *observed* is randomly selected to be made public at the beginning of the next period and this participant stops guessing. This process continues until the last period where each of the *unobserved* submits a guess. For each participant, only one randomly selected guess is paid in each repetition. Each *observed* is paid only for the last guess she submits i.e the guess which is made public. Each *unobserved* is randomly assigned to a period at the end of the repetition and paid for the guess made in that period. Exactly one *unobserved* is assigned to any given period.

Parts 3 and 4. Repetitions in parts 3 and 4 of a session are identical to repetitions in part 2 except that participants in the *unobserved* sequence are endowed with a private signal of different quality than participants in the *observed* sequence. Concretely, each *unobserved* obtains a signal of high quality in part 3 which corresponds to a single draw from a virtual urn containing 18 correct and 3 incorrect balls whereas the signal is of low quality in part 4 which corresponds to a single draw from a virtual urn containing 12 correct and 9 incorrect balls.

Feedback screens. The feedback that participants receive at the end of each repetition of the cascade game differs in the two experiments.

In the first experiment, draws made by other participants remain private. In part 1, each participant is reminded of her draw, her guessing period, the guess she made and the sequence of *observed* guesses, and she is informed about the selected option and her earnings. In the next three parts, feedback screens are identical to those in the first part except that each participant is only reminded of the payoff-relevant guess she made and of the sequence of *observed* guesses which were made public. In all four parts, feedback screens of *observed* only display the composition of the urn used in the *observed* sequence whereas feedback screens of *unobserved* display the urn compositions of both sequences.

In addition to the information they provide in the first experiment, the feedback screens in the second experiment disclose the private signals of the *observed* guesses which were made public.

3.3 Experimental Procedures

The experimental sessions took place at the laboratory for experimental economics of the Technische Universität München (experimenTUM) in April 2014, February and March 2015. Students from the Technische Universität München and the Ludwig-Maximilians-Universität München were invited using the ORSEE recruitment system (Greiner, 2015). We conducted nine and six sessions in the first and second experiment respectively with 16 participants in each session. One participant was randomly selected to serve as the laboratory assistant and the remaining participants were randomly assigned to computer terminals. Both experiments were programmed in zTree (Fischbacher, 2007).

Each session started with short demonstrations of the option-selection procedure to small groups of participants. An experimenter shuffled a deck of 20 cards – 11 cards with a blue front and 9 cards with an orange front – and laid the cards face down on a table. The assistant then picked 1 card out of the 20 cards, and the front color of the picked card determined the randomly selected option.[16] After the demonstrations, paper instructions for part 1 were distributed and participants were given time to read them at their own pace. Instructions were then read aloud and finally participants learned about their role (*observed* or *unobserved*).

Once the three repetitions of the cascade game were over, paper instructions for part 2 were distributed and subjects were given time to read them at their own pace. A summary of the instructions was then read aloud. The paper instructions were followed by a short on-screen-demonstration of the draws from the virtual urns. Again, one of the experimenters summarized aloud the main points of the demonstration. After that, the six repetitions of part 2 were run.

The third part of the experiment was conducted in a similar way as the second one except that only short paper instructions were distributed. Part 3 was followed by a short break. Participants were offered soft drinks and water, and a paper questionnaire was distributed asking for gender, month and year of birth, academic major, mother tongue, and citizenship. Short paper instructions for part 4 were then distributed and the six repetitions of part 4 were conducted. Finally, participants privately retrieved their earnings.

In each session we collected 45 guesses from the three repetitions of the first part and 552 guesses

[16]The laboratory assistant randomly selected the option in each repetition of the cascade game. The assistant also helped with the drawing of signal realizations from the physical urns in the first part of sessions and she monitored the progress of the session on her own computer terminal.

from the six repetitions of each following part. We collected a total of 4,725 *observed* and 10,584 *unobserved* guesses in the first experiment, and a total of 3,150 *observed* and 7,056 *unobserved* guesses in the second experiment. On average, a participant in the role of *observed* earned 17.33 and 17.71 Euro in the first and second experiment respectively whereas a participant in the role of *unobserved* earned 17.96 and 18.19 Euro in the first and second experiment respectively, including a show-up fee of 3 Euro. A session lasted for about 105 minutes. In all parts of a session, participants only interacted through the computers and no other communication was permitted. Appendix E in the supplementary material contains a translated version of the instructions of our first experiment.

4 Results of Experiment 1

In the first two parts of the experiment, all participants face the same cognitive challenge when combining private and public information since all of them learn from the history of *observed* guesses once endowed with a private signal of medium quality. However, *observed* have an incentive to overweight their private information relative to public information as long as they recognize the value of signaling information to subsequent participants in any of the two sequences. *Unobserved* guesses, on the other hand, never reveal any information to others and deviations from rational herding can only be caused by cognitive biases. Thus, if the intuitive prediction that altruism induces more informative observational learning—as formalized by Propositions 2 and 3 in Section 2—holds true, *observed* should contradict their private information less often than *unobserved* in guessing situations where the monetary incentives to herd are moderately weak and *observed* have sufficiently many successors who can benefit from the revelation of their private information. By the same argument, the behavior of *observed* and *unobserved* should be similar in all other guessing situations particularly those where the public belief is strong or participants act late in the sequence.

In this section, we first summarize the aggregate properties of our data. Second, we examine the extent to which participants contradict their private information both in situations where making an informative guess is materially beneficial and in situations where it is materially costly to the participant. Finally, we measure the amount of information aggregated by *observed* guesses and we analyze participants' earnings in the two sequences. *Unobserved* guesses made in parts 3 and 4 of sessions are excluded from the analysis.

4.1 Descriptive Statistics

As an overview of the observational learning behavior in our first experiment, we report the strength distribution of the evidence conveyed by the final histories i.e. the histories composed of the seven guesses made public in each repetition of the cascade game. For each final history, the strength of the public evidence is captured by the absolute difference between the number of blue and orange guesses which we refer to as the size of the majority. Thus, the size of the majority is given by $|\ \#\text{blue} - \#\text{orange}\ |$ where #blue and #orange is the number of blue and orange guesses in the final history respectively, and it equals either 1, 3, 5 or 7. We also consider the situations where the participant's private signal and the majority of previous public guesses are conflicting pieces of information and we compare the relative frequency of herding on contrary majorities in the two sequences of participants. We examine the herding behavior of participants by considering all histories of public guesses which generate a strict majority.

The left panel of Figure 1 shows the distribution of majority sizes and the relative frequency of herding in the two sequences of participants depending on the size of the contrary majority. Black and gray bars display the fractions of majority sizes in part 1 and in parts 2 to 4, respectively. Herding frequencies of *observed* and *unobserved* are indicated by square and circle markers, respectively. Notice that the herding frequencies of *unobserved* after contrary majorities of size 7 have been omitted. The right panel of Figure 1 shows the difference between herding frequencies of *unobserved* in part 2 and *observed* in parts 2 to 4 by guessing period when participants face contrary majorities of size 1 or 2. Though for the sake of clarity we bundle parts 2 to 4 in the figure, the main differences between later parts of sessions are discussed below.

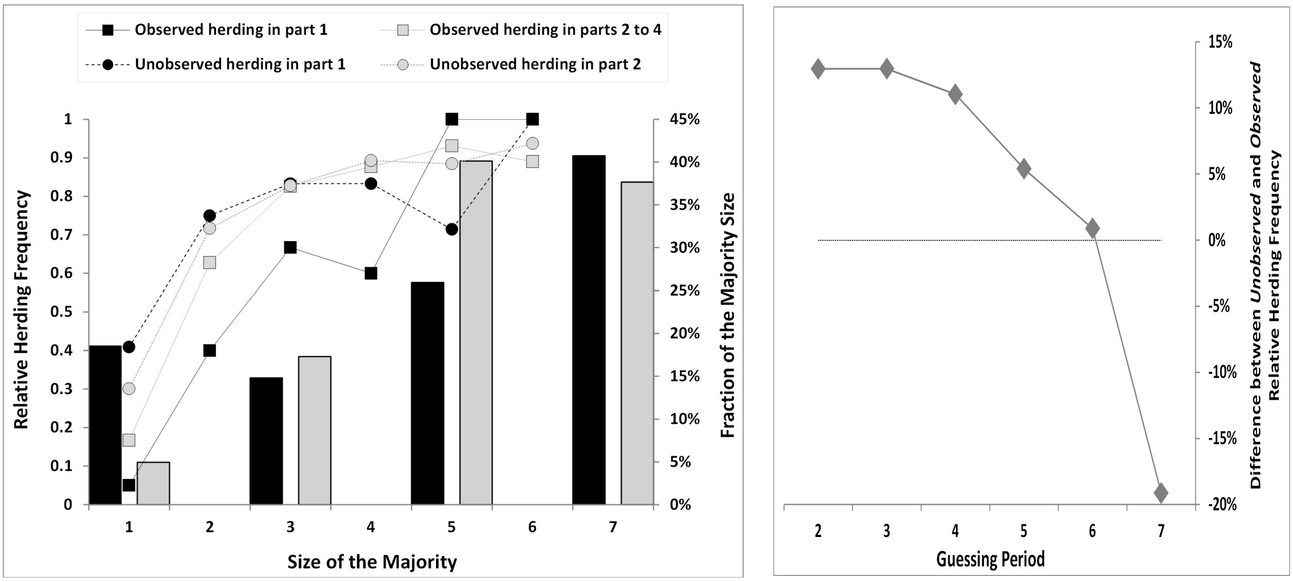


FIGURE 1. Majority Sizes and Herding Frequencies (Exp. 1)

Before commenting on Figure 1, we note that more than 97% of either *observed* or *unobserved* guesses agree with private information when participants face favoring majorities i.e. in situations where the private signal and the majority of public guesses are concordant pieces of information. Even when facing histories with an equal number of blue and orange guesses, participants guess overwhelmingly in accordance with private information (95% of *observed* and 92% of *unobserved* guesses are informative).

Across all parts, about 30% of the 106 final histories which start with two identical guesses contain both blue and orange guesses and only about 38% of the 189 final histories are full laboratory cascades.[17] The distribution of majority sizes is rather identical in the later parts of sessions with about three-quarters of the majorities being of size 5 or 7 and very few majorities of size 1. By contrast, a similar fraction of final histories are majorities of size 1, 3 or 5 in part 1 which is the consequence of low herding frequencies when *observed* face small contrary majorities.

As in prior cascade game experiments, we find that larger majorities are more stable since herding frequencies increase with the size of the contrary majority. In later parts of sessions, *observed* contradict their private information in about two-thirds (resp. four-fifths) of the situations where they face a contrary majority of size 2 (resp. 3). Most importantly, we find that when the contrary majority

[17]In previous cascade game experiments with a unique signal quality of 2/3, a prior of either 0.5 or 0.55 and sequence lengths from 6 to 10, the proportion of full laboratory cascades equals approximately 40%.

size is lower than 4, the herding frequency is lower for *observed* than for *unobserved*. The difference is particularly pronounced in part 1 (black markers), and it reduces in part 2 with *observed* (resp. *unobserved*) herding more (resp. less) than in the previous part. In the last two parts of sessions *observed* make more informative guesses when facing small contrary majorities than in part 2. There are no systematic differences in the herding frequencies of participants for contrary majorities of size 5 or 6. These descriptive results indicate that *observed* act more informatively than *unobserved* when the discordant public evidence is weak but the difference vanishes once the evidence is strong enough.

Figure 1 also shows that in situations where participants face short contrary majorities *observed* act more informatively than *unobserved* only in the early guessing periods (see right panel). In the last two guessing periods, *observed* herd to the same extent or more than *unobserved* in line with altruistic observational learning which predicts that informative guesses are made only if sufficiently many successors can benefit from them.

Finally, we note that the proportion of correct guesses increases with the reluctance of *observed* to contradict their private information when facing small contrary majorities. The proportion of guesses which match the selected option is highest in part 1 with 72% and 75% of correct guesses for *observed* and *unobserved* respectively, it decreases in part 2 to 67% in both sequences, and in the last two parts of sessions 69% of *observed* guesses are correct. Informative guesses at small contrary majorities therefore generate overall efficiency gains, and as expected these gains are largest for *unobserved* (for the sake of comparability, reported efficiency levels exclude *unobserved* guesses made in period 8). Subsection 4.3.2 examines the earning consequences of observational learning in more details.

Further data analysis excludes the few guesses made in the first part of sessions during which participants familiarized themselves with the cascade game.

4.2 Responses to the Empirical Value of Contradicting Private Information

In this subsection, we examine the extent to which participants contradict their private information in diverse guessing situations. By controlling for the monetary incentives, the regression analysis tests whether *observed* make the empirically money-maximizing guess as often as *unobserved* both in situations where following private information is materially beneficial and in situations where it is materially costly to the participant. For each guessing situation, i.e. each couple (history of *observed* guesses, private signal), we estimate the relative frequency with which the participant receives €1 if she contradicts her private signal across all observations with the same history and private signal. This empirical value of contradicting private information, denoted by *value_contra_PI*, approaches the true expected value of contradicting private information in a given guessing situation as its number of occurrences increases in the dataset. For example, averaged across histories with an equal number of blue and orange guesses (including the empty history in period 1), *value_contra_PI* equals 0.286 and 0.384 when the signal realization is 'blue' and 'orange', respectively. And *value_contra_PI* increases when histories induce contrary majorities of size 1 or 2: Averaged across observations where the contrary majority size is 1 (resp. 2), *value_contra_PI* equals 0.441 and 0.533 (resp. 0.592 and 0.615) when the signal realization is 'blue' and 'orange', respectively. Participants who perfectly assess the value of their available information and maximize their own monetary payoffs follow their signal if and only if *value_contra_PI* $\leq 1/2$.

Figure 2 plots the empirical value of contradicting private information against the proportion of contradictions collected in identical guessing situations. The abscissae of bubbles are given by levels of *value_contra_PI* and the size of a bubble reflects the number of occurrences of the situation. The ordinates of black, dark gray, light gray and white bubbles are given by the proportions of contradictions for *unobserved*, *observed* in part 2, 3 and 4, respectively. Notice that the four sets of bubbles have different abscissae since the levels of *value_contra_PI* have been estimated separately for the two sequences of participants in part 2 and for the different parts in the *observed* sequence. And each bubble corresponds to a guessing situation which occurs at least 10 times as *value_contra_PI* is likely to be far away from the true expected value of contradicting private information for rarely occurring situations. There are 139 distinct guessing situations depicted in the figure for a total of 6,329 individual observations.

Figure 2 also superimposes fitted lines from a weighted linear regression that includes a cubic polynomial in *value_contra_PI* fully interacted with indicator variables for *unobserved* and *observed* in part 3 and in part 4 of sessions.[18] To correct for the fact that *value_contra_PI* imperfectly measures the true expected value of contradicting private information, we follow the split-sample instrumental variable (IV) method described in Weizsäcker (2010) which obtains an instrument by partitioning the dataset in two subsamples. The black, dark gray, light gray and dotted line is the fitted line for *unobserved*, *observed* in part 2, 3 and 4, respectively. Appendix D in the supplementary material details the derivation of *value_contra_PI* and the split-sample instrumental variable method, it reports the regression results, and it also contains robustness checks with different subsets of data and OLS specifications. In almost all instances we find the same qualitative results and the few dissimilarities are mentioned below.

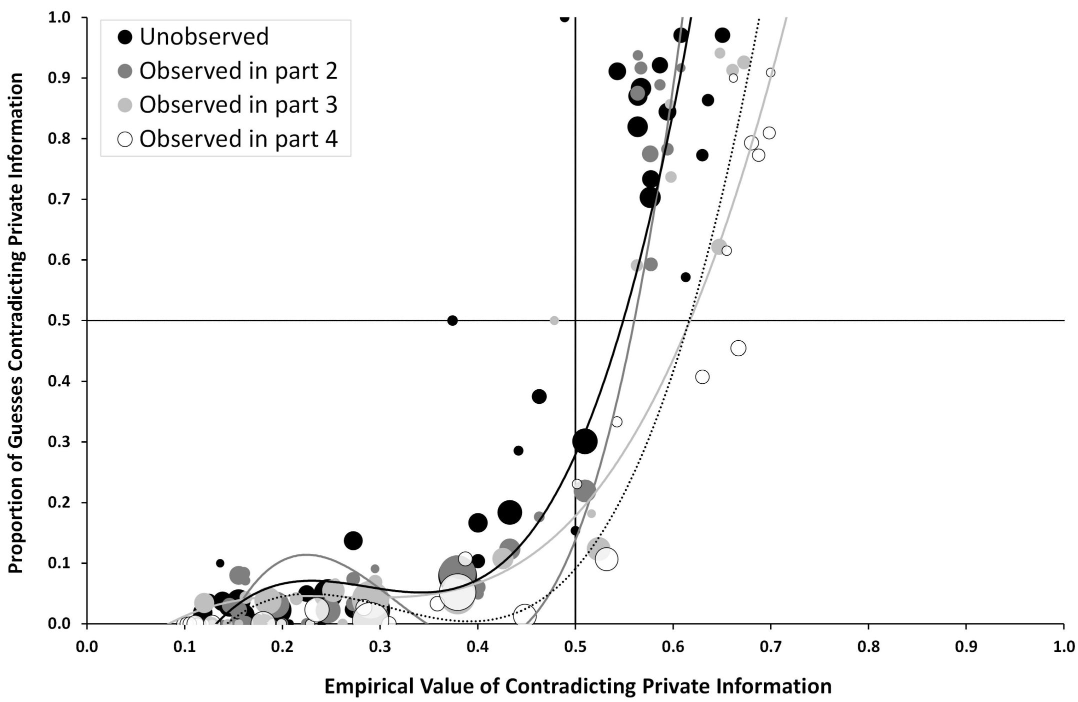


FIGURE 2. Proportion of Private Information Contradictions (Exp. 1)

[18] The regression was run using the data from our two experiments meaning that all regressors are also interacted with an indicator variable for Experiment 2.

We first discuss *unobserved* responses to the empirical value of contradicting private information as they provide a benchmark against which to assess the influence of altruism on observational learning behavior. Indeed, since their guesses never reveal any information to others, we expect *unobserved* to make money-maximizing guesses as long as they are able to use their available information successfully.

In situations where their private information happens to support the empirically optimal guess *unobserved* largely follow their signal. Averaging across observations where *value_contra_PI* ≤ 0.5, the relative frequency of *unobserved* guesses that are optimal is 0.922. Even in the more challenging situations where their private information is correct in less than half of the cases *unobserved* often guess optimally and follow others. Averaging across observations where *value_contra_PI* > 0.5, the relative frequency of *unobserved* guesses that are optimal is 0.761. Notice that incentives to act optimally are much stronger in the left than in the right half of the figure as *value_contra_PI* ranges (approximately) between 0.1 and 0.7. For similar incentive levels, the reluctance of *unobserved* to contradict private information is comparable in the two halves of the figure: Across observations where $0.3 <$ *value_contra_PI* ≤ 0.5, the relative frequency of *unobserved* guesses that are optimal is 0.785 on average. The proportions of contradictions therefore indicate that *unobserved* respond strongly to the empirical value of contradicting private information. Still, if the average *unobserved* were to make the money-maximizing guess in each situation, the fitted line for *unobserved* would be an S-shaped line through (0.5, 0.5). Actually, the dark line goes through (0.5, 0.279) and (0.549, 0.5), and we reject the hypothesis that *unobserved* probabilistically best respond to the value of their available information as the vertical distance between the dark line and (0.5, 0.5) is strongly significant (two-tailed p-value < 0.01). Only in situations where the empirical likelihood of the private signal being wrong is at least 0.549 does the average *unobserved* contradict private information more often than not. We conclude that in most situations *unobserved* make the money-maximizing guess though they fall short of successfully learning from others. When the monetary incentives to follow others are the weakest *unobserved* imperfectly assess the value of their available information and act as if they overweight their private information relative to the public information contained in the history of *observed* guesses.[19]

We now discuss the *observed* responses to the empirical value of contradicting private information. Differences between the proportions of *observed* and *unobserved* contradictions in the left half of the figure show that *observed* are more likely to follow their private information than *unobserved* in situations where private and public information are concordant. Averaging across observations where *value_contra_PI* ≤ 0.5, the relative frequency of optimal guesses is 0.952, 0.951 and 0.981 for *observed* in part 2, part 3 and part 4 of sessions, respectively. Differences in the likelihood of errors between *observed* and *unobserved* also increase as monetary incentives decrease. Averaging across observations where $0.3 <$ *value_contra_PI* ≤ 0.5, the relative frequency of guesses in line with private information is 0.909, 0.924 and 0.958 for *observed* in part 2, 3 and 4, respectively.

The main insight of our theoretical analysis is that altruism induces more informative observational learning and our data largely support this intuitive prediction. Averaging across observations where *value_contra_PI* > 0.5, the relative frequency of contradictions is 0.653, 0.542 and 0.468 for *observed* in part 2, 3 and 4, respectively. Moreover, as the monetary incentives to follow others increase, the

[19]Not all robustness checks confirm this last observation. For the OLS specification, we fail to reject the null hypothesis that the true correspondence between the value of contradicting and its frequency goes through (0.5, 0.5) even for subsets of data where the monetary incentives are precisely measured.

proportions of *observed* contradictions tend to get closer to the proportions of *unobserved* contradictions. The difference between the proportion of *unobserved* and *observed* contradictions averaged across observations where $0.5 \leq$ *value_contra_PI* < 0.6 is 0.092, 0.386 and 0.577 for *observed* in part 2, 3 and 4, respectively. The same difference averaged across observations where *value_contra_PI* ≥ 0.6 is -0.044, 0.056 and 0.210 for *observed* in part 2, 3 and 4, respectively. We test whether *observed* act more informatively than *unobserved* by comparing predicted frequencies to contradict private information at *value_contra_PI* = 0.549 which corresponds to the level of monetary incentives necessary for *unobserved* to follow others with more than probability one-half. We find that the vertical distance between the fitted line and (0.549, 0.5) is strongly significant for *observed* in all the later parts of sessions (one-tailed p-values < 0.01). The same conclusions hold for predicted frequencies to contradict private information after "full agreement" histories i.e. histories which contain either only blue or orange guesses. Our findings are therefore robust to the exclusion of the relatively few guessing situations where *value_contra_PI* is close to one-half but participants have few successors.

Observed act more informatively than *unobserved* in part 2 of sessions and *observed* also become more reluctant to contradict their private information as the session progresses. Indeed, predicted *observed* frequencies to contradict private information at *value_contra_PI* = 0.549 differ significantly between parts 2 and 3 (two-tailed p-value < 0.01) as well as between parts 3 and 4 (two-tailed p-value $= 0.04$). Our interpretation of these behavioral dynamics is that, as they accumulate experiences in the cascade game, *observed* better understand that in certain situations informative observational learning induces small monetary costs for them but large monetary benefits for their successors (*observed* monetary payoffs are discussed in the next subsection). It is doubtful that the change in the signal quality of *unobserved* across parts explains much of the *observed* behavioral dynamics. In part 3 *unobserved* are endowed with private signals of high quality which implies that they have little to gain from *observed* acting informatively. Though *unobserved* have more to gain from *observed* acting informatively in part 4, as they are endowed with private signals of low quality, LQRE predictions show that this small increase in information benefits should hardly affect *observed* responses to the empirical value of contradicting private information.[20]

To summarize, *unobserved* make the money-maximizing guess in all situations except in those where the monetary incentives to follow others are the weakest. This observation suggests that the reluctance of participants to contradict private information in cascade games is partly due to cognitive biases which prevent participants to successfully learn from others. More importantly, *observed* contradict their private information significantly less often than *unobserved* in situations where the monetary incentives to follow others are moderately weak. Future informational gains of guesses therefore enhance the overemphasis on private information in our first experiment. Once the incentives to follow others are strong enough *observed* contradict their private information to the same extent as *unobserved*.

[20]Though the standard LQRE is an imperfect benchmark to capture the average behavior of *unobserved*, we believe that LQRE predictions are helpful to understand the influence of future informational gains of guesses on the responses to monetary incentives. Appendix C.3.1 of the supplementary material illustrates the predicted differences between *observed* and *unobserved* responses to *value_contra_PI*.

Indirect Evidence of Altruistic Behavior in Previous Cascade Game Experiments

Finally, we investigate whether participants' responses to *value_contra_PI* in previous cascade game experiments are also supportive of altruistic observational learning. Though previous settings are ill-suited to cleanly measure the influence of altruism on observational learning, participants' behavior should vary with the length of the cascade game if they recognize the value of signaling information. Indeed, for any given $\alpha > 0$, longer sequences of players induce stronger responses to private information in the altruistic observational learning game as more successors benefit from informative actions. Participants should therefore contradict their private information more often in cascade games with short decision sequences than in cascade games with long decision sequences when the monetary incentives to follow others are moderately weak. We test this prediction by comparing the responses to *value_contra_PI* in the short ($T \leq 6$) and long cascade games contained in the meta-dataset of Ziegelmeyer, March, and Krügel (2013) (see Appendix D.2 for details).

The regression analysis delivers two main results. First, participants in short cascade games are reluctant to contradict their private information when monetary incentives to follow others are moderately weak. The fitted line for short cascade games goes through (0.5, 0.328) and (0.592, 0.5), and we reject the hypothesis that the average participant always makes the money-maximizing guess as the vertical distance between the fitted line and (0.5, 0.5) is strongly significant (two-tailed p-value < 0.01). Second, observational learning is significantly more informative in long than in short cascade games. The fitted line for long cascade games goes through (0.5, 0.241) and (0.662, 0.5), and we find that the vertical distance between the fitted line and (0.592, 0.5) is strongly significant (one-tailed p-value < 0.01). Thus, participants' responses to *value_contra_PI* in previous cascade game experiments provide additional support for the influence of future informational gains of guesses on observational learning.

4.3 Information Aggregation and Fractions of Correct Guesses

An increase in the response to private information at small contrary majorities is potentially beneficial for the two sequences of participants though the *observed* who acts informatively incurs a modest monetary cost. Indeed, large majorities should accumulate more public information which in turn might heighten the relative frequency of correct guesses. In this subsection we investigate whether the reluctance of *observed* to contradict their private information entails the benefits predicted by altruistic observational learning. First, we measure the amount of information aggregated by *observed* guesses in the different parts of sessions. Second, we analyze participants' earnings in the two sequences.

4.3.1 Measuring the Information Aggregated by *Observed* Guesses

Subsection 4.2 has established that in situations where the monetary incentives to follow others are moderately weak *observed* become more reluctant to contradict their private information as the session progresses. These behavioral dynamics offer the opportunity to check whether increased responses to private information enhance the informativeness of public information. If observational learning becomes more informative as the session progresses then large majorities should aggregate more information in later parts of sessions. We also assess the informational efficiency of observational learning behavior by comparing the amount of information aggregated in large majorities of *observed* guesses to the amount of information aggregated in standard equilibrium majorities.

The empirical value of contradicting private information is a natural measure of the information aggregated by a sequence of guesses. The more information guesses aggregate the lower the levels of *value_contra_PI* at large favoring majorities and the higher the levels of *value_contra_PI* at large contrary majorities. For example, when averaged over signal realizations, a sequence of guesses contains no valuable information if *value_contra_PI* = 1/3, a favoring majority aggregates one (resp. two) private signal(s) if *value_contra_PI* = 1/5 (resp. 1/9), and a contrary majority aggregates one (resp. two) private signal(s) if *value_contra_PI* = 1/2 (resp. 2/3).

To test whether more information is aggregated in later parts of sessions, we regress *value_contra_PI* on indicator variables for parts fully interacted with indicator variables for the type of majority (all regressors are also interacted with an experiment dummy as the analysis uses the data from our two experiments). We distinguish between large favoring majorities, moderate majorities, and large contrary majorities where the size of a large majority belongs to {3,4,5,6}. For the sake of conciseness, the two signal realizations are bundled together. We use an OLS specification with robust standard errors clustered at the session level and we include every guessing situation for which *value_contra_PI* can be computed. Table 1 reports the predicted levels of *value_contra_PI* by session parts and types of majorities. Appendix D.3 in the supplementary material reports the regression results as well as robustness checks where the analysis is restricted to subsamples with a more precise measurement of *value_contra_PI* and where the size of a large majority belongs to {4,5,6}. In all instances we obtain the same qualitative results in Experiment 1.

	Part 2	Part 3	Part 4
Large Favoring Majorities	0.154 (0.147, 0.160)	0.134 (0.112, 0.157)	0.108 (0.104, 0.112)
Moderate Majorities	0.330 (0.314, 0.345)	0.335 (0.322, 0.348)	0.340 (0.324, 0.357)
Large Contrary Majorities	0.578 (0.569, 0.586)	0.617 (0.598, 0.635)	0.671 (0.657, 0.685)

Every guessing situation for which *value_contra_PI* can be computed is included for a total of 7,068 individual observations. 95% robust confidence interval in brackets, clustered at the session level and constructed using the delta method.

TABLE 1. PREDICTED LEVELS OF *value_contra_PI* IN THE *Observed* SEQUENCE (EXP. 1)

The regression analysis confirms that large majorities aggregate more information in later parts of sessions. At large favoring majorities the predicted level of *value_contra_PI* equals 0.154, 0.134 and 0.108 in part 2, 3 and 4, respectively. Differences in the predicted levels of *value_contra_PI* between parts are significant (one-tailed p-value = 0.046 for part 2 versus part 3 and 0.016 for part 3 versus part 4). At large contrary majorities the predicted level of *value_contra_PI* equals 0.578, 0.617 and 0.671 in part 2, 3 and 4, respectively. Differences in the predicted levels of *value_contra_PI* between parts are always strongly significant (one-tailed p-values < 0.01). On the other hand, the predicted levels of *value_contra_PI* do not differ significantly between parts at moderate majorities which, as expected, contain no valuable information on average.

We now compare the empirical and theoretical levels of the value of contradicting private information to assess how successful *observed* are in aggregating information. In the standard sequential equilibrium, the value of contradicting private information equals $51/89 \approx 0.573$ at any contrary majority of size larger than 2 and 19/121 ≈ 0.157 at any favoring majority of size larger than 2. Table 1 shows that in each part of the sessions large majorities of *observed* guesses aggregate at least as much information as any equilibrium history, and in parts 3 and 4 significantly more information is aggregated. In fact, large majorities of *observed* guesses in part 4 aggregate significantly more information than any standard LQRE history since the value of contradicting private information belongs to the range [0.124, 0.639] for every equilibrium history and every $\lambda \geq 0$ in our laboratory cascade game.

In sum, the predicted levels of *value_contra_PI* in the different parts confirm that *observed* guesses become more informative as the session progresses. Eventually more public information is accumulated in the *observed* sequence than in the standard LQRE. Future informational gains of guesses therefore improve the information aggregation process in our first experiment.

Information Aggregation in Similar Cascade Game Experiments

In the previous subsection, we have shown that participants contradict their private information more often in cascade games with short decision sequences than in cascade games with long decision sequences when the monetary incentives to follow others are moderately weak. Long cascade games should therefore aggregate more information at large majorities than short cascade games (given the sequence length of short games, the size of a large majority belongs to {3, 4, 5}). We regress *value_contra_PI* on an indicator variable for the length of the game fully interacted with indicator variables for the type of majority, and we test the hypothesis by comparing the predicted levels of *value_contra_PI* at large majorities in short and long cascade games. For the sake of comparability with the predicted levels in our experiment, we only include previous cascade games with a unique signal quality equal to 2/3 (for subsets of data where *value_contra_PI* is measured precisely, the same qualitative results hold in the entire sample of games).

We find that the predicted level of *value_contra_PI* at moderate majorities is identical in both game lengths (0.328) and that large majorities aggregate significantly more information in long than in short games (one-tailed p-values < 0.01). Moreover, for large majorities the predicted levels of *value_contra_PI* in part 4 of our experiment are comparable to those in short games (0.101 and 0.689) but they are significantly lower than those in long games (0.077 and 0.755) as confidence intervals don't overlap.

4.3.2 Relative Frequencies of Correct Guesses

Finally, we compare the earnings of *observed* and *unobserved* in part 2 as well as the earnings of *observed* across parts 2 to 4 in Experiment 1. Earnings are measured by use of the dummy variable *correct* which takes value one if the guess is correct and zero otherwise. We first regress the relative frequency of correct guesses against indicator variables for *observed* in the different parts interacted with an indicator variable for Experiment 2. Second, we control for the monetary incentives to make the correct guess by including the variable *value_correct* which equals *value_contra_PI* if the private signal supports the wrong guess and 1 - *value_contra_PI* otherwise. The second regression follows the split-sample IV method and it uses guessing situations which occur at least ten times. In both regressions, *unobserved* guesses made in period 8 are excluded for the sake of comparability. Columns (1) and (2) of Table 2 reports the OLS and IV regression results.

As shown in the first column of the table, the earnings of *observed* and *unobserved* in part 2 are

	Dependent variable is correct guess	
	OLS	IV
Observed in part 2	0.000	0.038***
	(0.010)	(0.013)
Observed in part 3	0.018	−0.028
	(0.029)	(0.040)
Observed in part 4	0.006	0.068***
	(0.051)	(0.012)
Experiment 2	0.057	0.024
	(0.039)	(0.019)
Experiment 2 × *observed* in part 2	−0.014	−0.008
	(0.020)	(0.030)
Experiment 2 × *observed* in part 3	−0.124**	0.028
	(0.052)	(0.054)
Experiment 2 × *observed* in part 4	−0.062	−0.085**
	(0.077)	(0.030)
value_correct		1.612***
		(0.076)
Constant	0.675***	−0.345***
	(0.027)	(0.050)
Observations	12,600	9,695
Cluster	15	15
R^2	0.004	0.270

Robust standard errors in parentheses, clustered at the session level.
** (5%) and *** (1%) significance level.

TABLE 2. Frequencies of Correct Guesses (Exp. 1 and 2)

identical. At first the result seems surprising since *observed* and *unobserved* equally benefit from the increased response to private information but only the former incur the costs. However, we should remember that *observed* follow their private information more often than *unobserved* also in situations where *value_contra_PI* $\leq 1/2$ which is clearly beneficial (the relative frequency of optimal guesses is 0.922 and 0.952 for *unobserved* and *observed*, respectively). By reducing the likelihood of errors in situations where private and public information agree, the altruistic behavior of *observed* compensates for the costs of aggregating more information. Once the benefits of more information being aggregated are taken into account, altruistic observational learning enhances the average monetary payoff. Compared to part 2, *observed* make the correct guess more often in part 3 and in part 4 though the difference is never statistically significant. We also note that the earnings of participants in Experiment 1 are comparable to the highest earnings predicted by the standard LQRE.

Importantly enough, the absence of a difference in the earnings of *observed* and *unobserved* does not reflect the fact that on average participants in the two sequences use their available information equally efficiently. Indeed, in each repetition of the cascade game, there are eight *unobserved* guesses in every period but there are only $8-t$ *observed* guesses in period $t \in \{1, \dots, 7\}$. This feature of our design implies that *unobserved* make relatively more guesses at large majorities than *observed* which puts the latter at a disadvantage. The proportion of guesses made at large majorities equals 39% and 23% in the *unobserved* and *observed* sequence, respectively. Our second regression controls for the incentives to make the correct guess which enables us to assess whether the altruistic behavior of *observed* leads them to use their available information more efficiently than *unobserved*. As shown in the second column of the table, we find that once incentives are controlled for the relative frequency of correct guesses is significantly higher for *observed* than for *unobserved* at the 1% level.[21] Because *observed* become more reluctant to contradict their private information as the session progresses, the proportion of *observed* guesses made at large majorities decreases to 20% in parts 3 and 4. Once incentives are controlled for we find that the relative frequency of correct guesses is lower for *observed* in part 3 than in part 2 though not significantly so whereas the relative frequency of correct guesses is significantly higher for *observed* in part 4 than in part 2 at the 5% level. These findings are mainly driven by the fact that, averaged across observations where *value_contra_PI* $\leq 1/2$, the relative frequency of optimal guesses is 0.951 and 0.982 for *observed* in part 3 and 4, respectively.

We conclude that altruism enhances the monetary payoffs of participants in our first experiment as its associated benefits more than compensate for its associated costs.

5 Results of Experiment 2

In addition to the information they provide in Experiment 1, the feedback screens in Experiment 2 disclose the private signals of the public guesses after each repetition of the cascade game. Thus, participants in Experiment 2 are offered better opportunities to learn about the strategies played by *observed* in the cascade game. We expect the additional feedback to influence the observational learning behavior of participants differently in the two sequences. On the one hand, *unobserved* should more often extract the correct information from the public guesses. And since their guesses entail no future information benefits, *unobserved* should more frequently make the money-maximizing guess for a given value of the available information especially if inferential biases are the main driving force of their overemphasis on private information. On the other hand, though they should also better assess the informational content of public histories, *observed* are still expected to guess in accordance with their private information when the incentives to follow others are moderately weak. In fact, given the net benefits of altruistic behavior in Experiment 1, the guesses of *observed* should be even more informative in situations where the costs of following private information are negligible but the benefits for others are large. And if *observed* are more successful at identifying the strategies played by their predecessors they should more frequently make the money-maximizing guess in situations where the monetary incentives to follow others are strong. Thus, *observed* could coordinate on a more efficient outcome under reduced behavioral uncertainty.

We report the results of Experiment 2 following the same structure as in the previous section. We first provide some descriptive statistics, we then compare the proportion of private information contradictions in the two sequences of participants when incentives are controlled for, and we finally evaluate the success of observational learning in diverse guessing situations. The same data restrictions

[21] As a robustness check, we repeat the OLS regression on the subset of payoff relevant guesses as for each participant only one randomly selected guess is paid in each repetition of the cascade game. We find that the relative frequency of correct guesses is significantly higher for *observed* (0.701) than for *unobserved* (0.656) at the 10% level.

apply to the various parts of the analysis as for the first experiment.

5.1 Descriptive Statistics

Like in Experiment 1, participants guess overwhelmingly in accordance with their private information in situations where private and public information do not contradict each other. At favoring majorities 98% of *observed* and 96% of *unobserved* guesses follow private information. And when participants face histories with an equal number of blue and orange guesses 96% of *observed* and 93% of *unobserved* guesses are informative.

Moreover, herding frequencies of *unobserved* in part 2 are slightly higher than those in the first experiment with about 31%, 75% and 95% of the guesses contradicting private information after contrary majorities of size 1, 2 and more than 3, respectively. In contrast to Experiment 1, there are small differences between the herding frequencies of *observed* and *unobserved* in part 2 for any size of the contrary majority. In parts 3 to 4 *observed* herd less frequently than in part 2 and frequencies of informative guesses reach comparable levels as in the first experiment though they remain lower. Higher *observed* herding frequencies in the second than in the first experiment lead to more majorities of size 7 and less majorities of size 5 in the second than in the first experiment (46% versus 38% and 30% versus 38% when averaging over parts 2 to 4).

5.2 Responses to the Empirical Value of Contradicting Private Information

Figure 3 plots *value_contra_PI* against the proportion of contradictions collected in identical guessing situations and it superimposes IV fitted lines for *observed* in the last three parts of sessions and for *unobserved*. There are 109 distinct guessing situations depicted in the figure for a total of 3,986 individual observations.

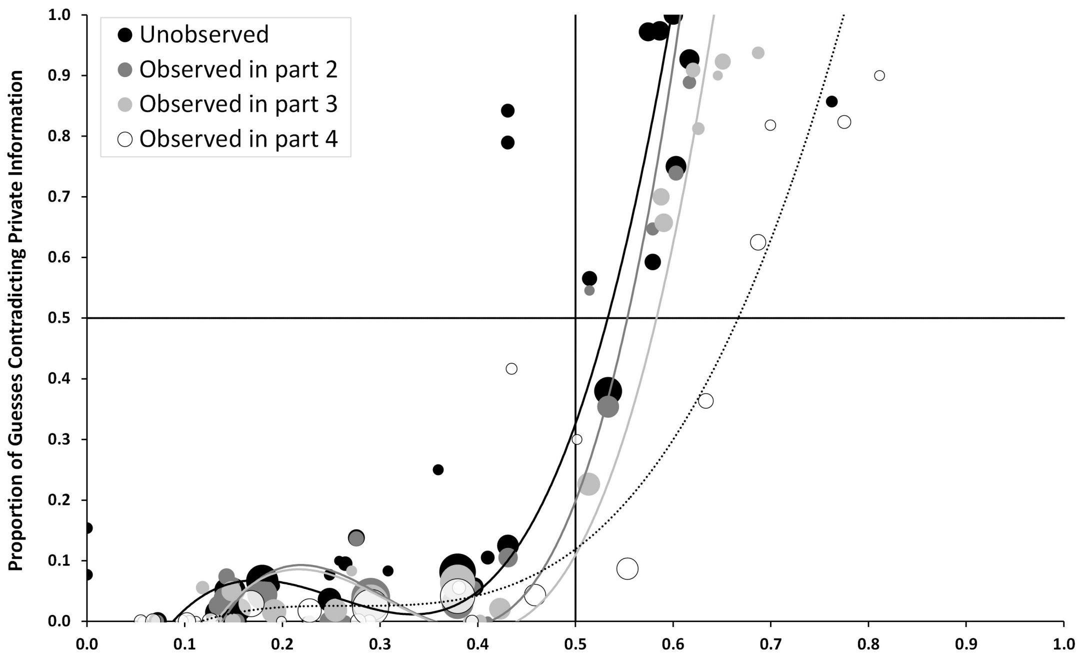


FIGURE 3. Proportion of Private Information Contradictions (Exp. 2)

The responses of *unobserved* to *value_contra_PI* are similar in our two experiments. In each half of the figure the overall proportion of *unobserved* contradictions is (almost) identical in the two experiments, and we also reject the hypothesis that the average *unobserved* systematically makes the money-maximizing guess in Experiment 2 as the vertical distance between the dark line and (0.5, 0.5) is strongly significant (two-tailed p-value < 0.01; the dark line goes through (0.5, 0.326) and (0.533, 0.5)). The evidence therefore indicates that in situations where the monetary incentives to herd are the weakest an easier identification of the strategies played by *observed* does not enable *unobserved* to assess the value of their available information significantly better. In fact, averaging across observations where *value_contra_PI* $\in [0.5, 0.6[$, the relative frequency of *unobserved* guesses that are optimal is lower in Experiment 2 than in Experiment 1 (0.695 versus 0.733). Thus, in the absence of future information benefits of guesses, the overemphasis on private information partly originates from the fact that participants incorrectly combine their private signal with the information inferred from public guesses. On the other hand, once the incentives to follow others are stronger *unobserved* make more money-maximizing guesses in Experiment 2 than in Experiment 1: Averaging across observations where *value_contra_PI* $\in [0.6, 0.65[$, 84% and 89% of the *unobserved* guesses are optimal in Experiment 1 and 2, respectively (there is no guessing situation which occurs at least 10 times for which *value_contra_PI* ≥ 0.65 in Experiment 2).

As in Experiment 1, *observed* always act more informatively than *unobserved* in Experiment 2. The vertical distance between the fitted line for *observed* and (0.533, 0.5) is significant in part 2 (one-tailed p-value = 0.038) and strongly so in parts 3 and 4 (one-tailed p-values < 0.01). More interestingly, when the strategies they play are easier to identify *observed* seem to strengthen their responses to private information. At the level of monetary incentives where the average *unobserved* response reaches 0.5—*value_contra_PI* = 0.549 and 0.533 in Experiment 1 and 2, respectively—the predicted frequencies to contradict private information for *observed* are lower in Experiment 2 than in Experiment 1 (the difference between the two predicted frequencies equals -0.047, -0.056 and -0.048 in part 2, 3 and 4, respectively). But as the monetary incentives to herd increase, the proportions of *observed* and *unobserved* contradictions also become more similar in Experiment 2. The difference between the proportion of *unobserved* and *observed* contradictions averaged across observations where $0.5 \leq$ *value_contra_PI* < 0.6 is 0.247, 0.220 and 0.570 in part 2, 3 and 4, respectively. And the same difference averaged across observations where *value_contra_PI* ≥ 0.6 is 0.078, -0.017 and 0.228 in part 2, 3 and 4, respectively. Finally, *observed* make slightly more money-maximizing guesses when *value_contra_PI* ≤ 0.5 in Experiment 2 than in Experiment 1 (across parts the average proportion equals 0.961 and 0.967 in Experiment 1 and 2, respectively) though the proportion of *unobserved* contradictions is exactly the same in the two experiments.

To summarize, in Experiment 2 *observed* increase the informativeness of their guesses whereas *unobserved* reluctance to contradict private information slightly decreases only in situations where the monetary incentives to follow others are strong enough. We conclude that reducing behavioral uncertainty amplifies the difference in the overemphasis on private information between the *observed* and *unobserved* sequence.

5.3 Information Aggregation and Fractions of Correct Guesses

5.3.1 Measuring the Information Aggregated by *Observed* Guesses

Table 3 reports the predicted levels of *value_contra_PI* in Experiment 2 by session parts and types of majorities (as explained in Subsection 4.3.1, the predicted levels in both experiments are derived from the same OLS regression).

	Part 2	Part 3	Part 4
Large Favoring Majorities	0.130 (0.116, 0.144)	0.127 (0.117, 0.136)	0.082 (0.068, 0.095)
Moderate Majorities	0.324 (0.307, 0.341)	0.340 (0.330, 0.349)	0.338 (0.324, 0.353)
Large Contrary Majorities	0.601 (0.557, 0.645)	0.644 (0.621, 0.667)	0.740 (0.699, 0.782)

Every guessing situation for which *value_contra_PI* can be computed is included for a total of 7,068 individual observations. 95% robust confidence interval in brackets, clustered at the session level and constructed using the delta method.

TABLE 3. PREDICTED LEVELS OF *value_contra_PI* IN THE *Observed* SEQUENCE (EXP. 2)

For every part of sessions, the predicted level of *value_contra_PI* is lower in Experiment 2 than in Experiment 1 at large favoring majorities and it is higher at large contrary majorities. The difference is strongly significant for both types of large majorities in part 4, it is strongly significant for large favoring majorities but insignificant for large contrary majorities in part 2, and it is weakly significant for large contrary majorities but insignificant for large favoring majorities in part 3. Overall, large majorities aggregate more information in Experiment 2 than in Experiment 1 and the effect is most pronounced in the latest part of sessions. On the other hand, the predicted levels of *value_contra_PI* do not differ significantly in the two experiments at moderate majorities for any part.[22]

We conclude that reducing the level of behavioral uncertainty improves the information aggregation process in our laboratory cascade game.

5.3.2 Relative Frequencies of Correct Guesses

Finally, based on the regression results reported in Table 2, we discuss how much participants earn and how efficiently they use their available information in Experiment 2 compared to Experiment 1.

In line with their responses to *value_contra_PI*, the second column of Table 2 shows that *unobserved* use their available information more efficiently in Experiment 2 than in Experiment 1 though the difference is statistically non-significant. And since *observed* guesses in part 2 aggregate more information in the second than in the first experiment, the earnings of *unobserved* are (non-significantly) higher in Experiment 2 than in Experiment 1.

Similarly, *observed* in parts 2 and 3 use their available information more efficiently in Experiment 2 than in Experiment 1 though the difference is never statistically significant. This finding is driven by

[22]Robustness checks always confirm that in part 4 large majorities aggregate significantly more information in the second than in the first experiment. However, differences between the two experiments in parts 2 and 3 are less robust to the subset of data used or to the minimum threshold of the large majority size. See Appendix D.3 for details.

the fact that, though *observed* in parts 2 and 3 increase the informativeness of their guesses in Experiment 2 when the monetary incentives to herd are moderately weak i.e. $value\_contra\_PI \in (0.5, 0.6)$, they make more (resp. as many) money-maximizing guesses in Experiment 2 than in Experiment 1 when $value\_contra\_PI \leq 0.5$ (resp. when $value\_contra\_PI \geq 0.6$). We therefore expect *observed* in parts 2 and 3 to receive higher earnings in Experiment 2 than in Experiment 1 given that their guesses always aggregate more information in the second than in the first experiment. This is indeed the case for *observed* in part 2 whose earnings are (non-significantly) higher in Experiment 2 than in Experiment 1 but, surprisingly enough, *observed* in part 3 earn significantly less in Experiment 2 than in Experiment 1 at the 10% level. The reason for this surprising finding is that the relative frequency of signal realizations which indicate the correct state is unfortunately much lower in Experiment 2 than in Experiment 1 (0.60 versus 0.71).[23]

On the other hand, we find that *observed* in part 4 use their available information significantly less efficiently in Experiment 2 than in Experiment 1 at the 5% level. Indeed, not only do *observed* in part 4 increase the informativeness of their guesses in Experiment 2 when $value\_contra\_PI \in (0.5, 0.6)$, they also make less money-maximizing guesses in Experiment 2 than in Experiment 1 both when $value\_contra\_PI \leq 0.5$ and when $value\_contra\_PI \geq 0.6$. Thanks to their guesses aggregating more information, the earnings of *observed* in part 4 are basically identical in the two experiments.

To summarize, when the behavioral uncertainty is reduced *unobserved* as well as *observed* in parts 2 and 3 are able to better use their available information on average which enhances their earnings (expect in the unfortunate case of *observed* in part 3). By contrast, *observed* in part 4 fail to reap the benefits of their informative guesses as they follow others too little when the monetary incentives to do so are strong.

6 Conclusion

The experimental evidence presented in this paper enriches our understanding of how people learn from the actions of others. Previous cascade game experiments concluded that the reluctance of participants to contradict their private information originates either from non-Bayesian updating or from a misperception of the informational content of observed actions. Past experiments however have not been designed to properly separate the explanation in terms of judgment or inferential biases from the intuitive explanation that participants recognize the future informational benefits of actions and behave altruistically. Our laboratory cascade setting, on the other hand, enables us to cleanly assess the relative impact of altruism and cognitive biases on observational learning. Participants play the cascade game in two parallel decision sequences, the *observed* and the *unobserved* sequence, and in each sequence they face the challenge of extracting information from the actions of others and combining it with their private information. In the *observed* sequence future informational benefits of actions are present but they are absent in the *unobserved* sequence. We report two cascade game experiments that directly test the impact of altruism on observational learning where participants in the second experiment are offered better opportunities to learn about the strategies played by *observed*.

The results of Experiment 1 confirm the main implications of altruistic observational learning which

[23] In Experiment 1, the empirical signal quality equals 0.66, 0.71 and 0.67 for *observed* in part 2, 3 and 4, respectively. In Experiment 2, the empirical signal quality equals 0.70, 0.60 and 0.65 for *observed* in part 2, 3 and 4, respectively. For *unobserved*, the empirical signal quality equals 0.65 and 0.71 in Experiment 1 and 2, respectively.

are derived in our theoretical section. Future informational benefits of actions reduce the likelihood of errors in situations where private and public information are concordant and they induce participants to significantly increase their response to private information when the monetary incentives to follow others are moderately weak. Once these incentives are strong enough, however, participants largely follow others. As a consequence, long laboratory cascades accumulate substantial public information which in turn increases the earnings of participants. In the absence of future informational benefits of actions, participants act as if they slightly overweight their private information relative to the public information only when the monetary incentives to follow others are the weakest. These findings therefore indicate that future informational benefits of actions are the main driving force of the overemphasis on private information and its attenuation in long laboratory cascades.

Reducing the level of behavioral uncertainty in Experiment 2 amplifies the impact of altruism on observational learning. The response to private information in the *observed* sequence is stronger and more public information is aggregated in the second than in the first experiment. Yet, the earnings of *observed* do not always increase when the strategies played by their predecessors are easier to identify since in part 4 they become reluctant to contradict their private information even when the monetary incentives to follow others are strong. In the *unobserved* sequence, participants are still slightly reluctant to contradict their private information when the monetary incentives to follow others are the weakest. This finding suggests that in the absence of future informational benefits of actions the overemphasis on private information is more driven by judgment biases rather than by inferential biases.

We use Weizsäcker's approach to compare the proportions of money-maximizing guesses in the *observed* and *unobserved* sequence as it controls for the monetary incentives. The approach enables us to assess the impact of altruistic behavior on participants' reluctance to contradict their private information in different guessing situations without having to rely on a structural behavioral model which would undoubtedly be an imperfect benchmark. Estimating the monetary incentives that participants face in diverse situations enables us also to shed light on another essential aspect of observational learning behavior. Indeed, the empirical value of actions is a natural measure of the information aggregated in the decision sequence. By estimating the value of actions, we are therefore able to directly measure the informational efficiency of observational learning in our laboratory cascade games.[24] The crucial requirement of the approach is to have sufficient data in a large variety of guessing situations which can be satisfied by relying on the strategy method at the history level.[25]

Our findings are important from two perspectives. First, our experimental results show that both altruism and cognitive biases influence how participants learn from others in cascade games. The methodological implication of our findings is that laboratory settings *without* future informational benefits of actions are the most appropriate for isolating and in turn understanding the influence of cognitive biases on observational learning behavior. Second, the presence of future informational benefits of actions is a contextual factor which favors the aggregation of information and heightens efficiency levels relative to rational herding. Concerned by the underinvestment in public information of rational herders, economists have designed mechanisms which release additional public informa-

[24] The strength of the underlying monetary incentives can also be used in other laboratory interactions to assess the amount of information aggregated by the decision-making institution.

[25] As noted by Cipriani and Guarino (2009), the use of the strategy-like method does not seem to induce a different herd behavior in laboratory financial markets.

tion or incentivize individuals to reveal their private information (Smith, Sørensen, and Tian, 2014). Our findings suggest that interventions which simply emphasize to individuals the value of signaling information to their successors might already improve economic welfare.

References

Acemoglu, D., D. Munther, I. Lobel, and A. Ozdaglar (2011): "Bayesian Learning in Social Networks," *Review of Economic Studies*, 78, 1201–36.

Alevy, J. E., M. S. Haigh, and J. A. List (2007): "Information Cascades: Evidence from a Field Experiment with Financial Market Professionals," *Journal of Finance*, 62, 151–80.

Ali, S. N., and N. Kartik (2012): "Herding with Collective Preferences," *Economic Theory*, 51, 601–626.

Anderson, L., and C. Holt (1997): "Information cascades in the laboratory," *American Economic Review*, 87, 847–862.

Andreoni, J., W. T. Harbaugh, and L. Vesterlund (2008): "Altruism in Experiments," in *The New Palgrave Dictionary of Economics*, ed. by S. N. Durlauf, and L. E. Blume. Palgrave Macmillan.

Banerjee, A. V. (1992): "A simple model of herd behavior," *The Quarterly Journal of Economics*, 107, 797–817.

Bernardo, A., and I. Welch (2001): "On the Evolution of Overconfidence and Entrepreneurs," *Journal of Economics & Management Strategy*, 10, 301–30.

Bikhchandani, S., D. Hirshleifer, and I. Welch (1992): "A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades," *Journal of Political Economy*, 100, 992–1026.

Bohren, J. A. (2015): "Informational Herding with Model Misspecification," *Journal of Economic Theory, forthcoming.*

Cai, H., Y. Chen, and H. Fang (2009): "Observational Learning: Evidence from a Randomized Natural Field Experiment," *American Economic Review*, 99, 864–82.

Çelen, B., and S. Kariv (2004): "Distinguishing Informational Cascades from Herd Behavior in the Laboratory," *American Economic Review*, 94, 484–498.

Chamley, C. (2004): *Rational herds: Economic models of social learning.* Cambridge, Cambridge University Press.

Cipriani, M., and A. Guarino (2009): "Herd Behavior in Financial Markets: An Experiment with Financial Market Professionals," *Journal of the European Economic Association*, 7, 206–233.

——— (2014): "Estimating a Structural Model of Herd Behavior in Financial Markets," *American Economic Review*, 104, 224–51.

Eyster, E., and M. Rabin (2010): "Naïve Herding in Rich-Information Settings," *American Economic Journal: Microeconomics*, 2, 221–43.

Fischbacher, U. (2007): "z-Tree: Zurich Toolbox for Ready-made Economic Experiments," *Experimental Economics*, 10(2), 171–8.

Goeree, J., T. Palfrey, B. Rogers, and R. McKelvey (2007): "Self-correcting information cascades," *Review of Economic Studies*, 74, 733–62.

Greiner, B. (2015): "Subject pool recruitment procedures: Organizing experiments with ORSEE," *Journal of the Economic Science Association*, 1, 114–25.

Guarino, A., and P. Jehiel (2013): "Social Learning with Coarse Inference," *American Economic Journal: Microeconomics*, 5, 147–74.

Hung, A. A., and C. R. Plott (2001): "Information Cascades: Replication and an Extension to Majority Rule and Conformity–Rewarding Institutions," *The American Economic Review*, 91, 1508–20.

Kariv, S. (2005): "Overconfidence and Informational Cascades," Working paper, University of California, Berkeley.

Knight, B., and N. Schiff (2010): "Momentum and Social Learning in Presidential Primaries," *Journal of Political Economy*, 118, 1110–50.

Kreps, D. M., and R. Wilson (1982): "Sequential Equilibria," *Econometrica*, 50, 863–894.

Kübler, D., and G. Weizsäcker (2004): "Limited depth of reasoning and failure of cascade formation in the laboratory," *Review of Economic Studies*, 71, 425–441.

Kübler, D., and G. Weizsäcker (2005): "Are Longer Cascades More Stable?," *Journal of the European Economic Association*, 3, 330–9.

Lee, I. H. (1993): "On the Convergence of Informational Cascades," *Journal of Economic Theory*, 61, 395–411.

Moretti, E. (2011): "Social Learning and Peer Effects in Consumption: Evidence from Movie Sales," *Review of Economic Studies*, 78, 356–93.

Nöth, M., and M. Weber (2003): "Information aggregation with random ordering: Cascades and overconfidence," *Economic Journal*, 113, 166–89.

Salganik, M. J., P. S. Dodds, and D. J. Watts (2006): "Experimental Study of Inequality and Unpredictability in an Artificial Cultural Market," *Science*, 311, 854–56.

Smith, L., and P. Sørensen (2000): "Pathological Outcomes of Observational Learning," *Econometrica*, 68, 371–398.

Smith, L., P. N. Sørensen, and J. Tian (2014): "Informational Herding, Optimal Experimentation, and Contrarianism," Working paper.

Stone, D. F., and B. Zafar (2014): "Do We Follow Others When We Should Outside the Lab? Evidence from the AP Top 25," *Journal of Risk and Uncertainty*, 49, 73–102.

VIVES, X. (1997): "Learning from Others: A Welfare Analysis," *Games and Economic Behavior*, 20, 177–200.

WEIZSÄCKER, G. (2010): "Do we follow others when we should? A simple test of rational expectations," *American Economic Review*, 100, 2340–2360.

ZHANG, J. (2010): "The Sound of Silence: Observational Learning in the U.S. Kidney Market," *Marketing Science*, 29, 315–35.

ZHANG, J., AND P. LIU (2012): "Rational Herding in Microloan Markets," *Management Science*, 58, 892–912.

ZIEGELMEYER, A., F. KOESSLER, J. BRACHT, AND E. WINTER (2010): "Fragility of information cascades: An experimental study using elicited beliefs," *Experimental Economics*, 13, 121–145.

ZIEGELMEYER, A., C. MARCH, AND S. KRÜGEL (2013): ""Do We Follow Others when We Should? A Simple Test of Rational Expectations": Comment," *American Economic Review*, 103, 2633–42.

Supplementary material for

Altruistic Observational Learning

Christoph March and Anthony Ziegelmeyer

[For Online Publication]

Appendix A contains the proofs of Propositions 1 and 2. Appendix B exhibits some non-monotonic equilibria and it offers evidence on how rapidly the number of equilibria grows with the degree of altruism. Appendix C outlines the Logit Quantal Response Equilibrium (LQRE) concept, it contains the proof of Proposition 3, and it provides detailed LQRE predictions for our laboratory cascade game. Appendix D complements the statistical analysis reported in the main text. Finally, Appendix E provides the instructions for Experiment 1. Instructions for Experiment 2 were adapted accordingly and they are available from the authors upon request.

Notational remark:

Throughout the paper we focus on behavioral strategies $\sigma_t : S \times H_t \to \mathcal{D}(X)$ for $t = 1, \dots, T$ where $\sigma_t(x_t \mid s, h_t)$ denotes the probability that player t picks action $x_t \in X$ at history $h_t \in H$ given signal $s \in S$. In the main text we rely on the simplified notation in which $\sigma_t(s, h_t)$ denotes the probability that the player picks action $x_t = B$ and $1 - \sigma_t(s, h_t)$ denotes the probability that she picks action $x_t = O$. In this appendix, we sometimes rely on the more rigorous notation $\sigma_t(\cdot \mid s, h_t)$, and we employ the simplified notation whenever the meaning is obvious.

Appendix A. Proofs of Propositions 1 and 2

Section A.1 contains the proof of Proposition 1. Section A.2 collects in a series of lemmas several properties of continuation values and sequential equilibria, and provides a discussion of monotonic equilibria. In Section A.3 we prove four lemmas that form the basis of Proposition 2. Finally, section A.4. completes the proof of Proposition 2.

A.1. Proof of Proposition 1

Ali and Kartik (2012) consider a simple setting of observational learning with *collective preferences*: a player's payoff depends on a binary state of nature and on the profile of any subset of all players. While players may differ in how they care about the choices of others, each player weakly prefers others to take the most profitable action. The observational learning game of Ali and Kartik is given by $\left\langle T, X, H, \Theta, p, S, q, \Xi, \phi, \{v_t\}_{t=1}^{T} \right\rangle$ where Ξ is a set of *preference types* and the vector of players' preference types $(\xi_1, \ldots, \xi_T)$ is drawn from the distribution $\phi \in \mathcal{D}\left(\Xi^T\right)$. The von-Neumann Morgenstern utility functions $v_t : X^T \times \Theta \times \Xi \to \mathbb{R}$, $t = 1, \ldots, T$, satisfy for each $t, \tau = 1, \ldots, T$, each $\boldsymbol{x}_{-\tau} = (x_1, \ldots, x_{\tau-1}, x_{\tau+1}, \ldots, x_T) \in X^{T-1}$, and each $\xi_t \in \Xi$

(Assumption 1) $v_t\left(\boldsymbol{x}_{-\tau}^{+}, \mathcal{B}, \xi_t\right) \geqslant v_t\left(\boldsymbol{x}_{-\tau}^{-}, \mathcal{B}, \xi_t\right)$ and $v_t\left(\boldsymbol{x}_{-\tau}^{+}, \mathcal{O}, \xi_t\right) \leqslant v_t\left(\boldsymbol{x}_{-\tau}^{-}, \mathcal{O}, \xi_t\right)$;

(Assumption 2) $c\left[v_t\left(\boldsymbol{x}_{-\tau}^{+}, \mathcal{B}, \xi_t\right) - v_t\left(\boldsymbol{x}_{-\tau}^{-}, \mathcal{B}, \xi_t\right)\right] = (1-c)\left[v_t\left(\boldsymbol{x}_{-\tau}^{-}, \mathcal{O}, \xi_t\right) - v_t\left(\boldsymbol{x}_{-\tau}^{+}, \mathcal{O}, \xi_t\right)\right]$ for some $c \in (0,1)$;

where $\boldsymbol{x}_{-\tau}^{+} = (x_1, \ldots, x_{\tau-1}, B, x_{\tau+1}, \ldots, x_T)$ and $\boldsymbol{x}_{-\tau}^{-} = (x_1, \ldots, x_{\tau-1}, O, x_{\tau+1}, \ldots, x_T)$.

Theorem 1 (Ali and Kartik, 2012). *For any payoff structure that satisfies Assumptions 1 and 2, the strategy profile* $(\sigma_t)_{t=1}^{T}$ *given by*

$$\sigma_t\left(s_t, \mathbf{h}_t, \xi_t\right) = \begin{cases} 1 & \text{if } \mu_t\left(s_t, \mathbf{h}_t\right) > c \\ c & \text{if } \mu_t\left(s_t, \mathbf{h}_t\right) = c \\ 0 & \text{if } \mu_t\left(s_t, \mathbf{h}_t\right) < c \end{cases}$$

is a sequential equilibrium of the observational learning game with collective preferences.

To prove Proposition 1 we show that the utility function

$$u_t\left(\mathbf{x}, \theta\right) = \pi\left(x_t, \theta\right) + \alpha \sum_{\tau \neq t} \pi\left(x_\tau, \theta\right)$$

satisfies (Assumption 1) and (Assumption 2) (there is a single preference type).

(Assumption 1) This follows from $\pi(B, \mathcal{B}) = 1 > 0 = \pi(O, \mathcal{B})$ and $\pi(B, \mathcal{O}) = 0 < 1 = \pi(O, \mathcal{O})$.

(Assumption 2) The assumption holds with $c = 1/2$ since

$$u_t\left(\boldsymbol{x}_{-\tau}^{+}, \mathcal{B}\right) - u_t\left(\boldsymbol{x}_{-\tau}^{-}, \mathcal{B}\right) = u_t\left(\boldsymbol{x}_{-\tau}^{-}, \mathcal{O}\right) - u_t\left(\boldsymbol{x}_{-\tau}^{+}, \mathcal{O}\right) = \begin{cases} 1 & \text{if } \tau = t \\ \alpha & \text{if } \tau \neq t \end{cases}.$$

Notice finally that $p > 1/2$ implies that $\mu_t\left(s_t, \mathbf{h}_t\right) = 1/2$ never occurs, so the definition of the strategy in this case is inconsequential. □

A.2. Additional Lemmas

To ease the writing of the proofs of Propositions 2 and 3, we derive in a series of lemmas some useful properties of continuation values, strategies, and sequential equilibria.

The first lemma gives a recursive statement of the continuation values.

Lemma A1. *The continuation values satisfy*

(i) $C_T(x_T \mid \mathbf{h}_T, \theta, \boldsymbol{\sigma}_{-T}) = 0$ *for each* $x_T \in X$, $\mathbf{h}_T \in H_T$, $\theta \in \Theta$, *and each* $\boldsymbol{\sigma}_{-T}$,

(ii) $C_t(x_t \mid \mathbf{h}_t, \theta, \boldsymbol{\sigma}_{-t}) = \sum_{x_{t+1} \in X} \Pr(x_{t+1} \mid \theta, \mathbf{h}_{t+1}) \left[\pi(x_{t+1}, \theta) + C_{t+1}\left(x_{t+1} \mid \mathbf{h}_{t+1}, \theta, \boldsymbol{\sigma}_{-(t+1)}\right)\right]$ *for each* $t < T$, x_t, $\mathbf{h}_t$, θ *and* $\boldsymbol{\sigma}_{-t}$ *where* $\Pr(x_{t+1} \mid \theta, \mathbf{h}_{t+1}) = \sum_{s_{t+1} \in S} \Pr(s_{t+1} \mid \theta)\, \sigma_{t+1}(x_{t+1} \mid s_{t+1}, \mathbf{h}_{t+1})$ *and* $\mathbf{h}_{t+1} = (\mathbf{h}_t, x_t)$.

Proof. The first property holds by definition. For the second property fix $\boldsymbol{\sigma}$, $t < T$, $x_t \in X$, $\mathbf{h}_t \in H_t$, and $\theta \in \Theta$, and let $\Pr(x_\tau \mid \theta, \mathbf{h}_\tau) = \sum_{s_\tau \in S} \Pr(s_\tau \mid \theta)\, \sigma_\tau(x_\tau \mid s_\tau, \mathbf{h}_\tau)$. The continuation value satisfies

$$\begin{aligned}
&C_t(x_t \mid \mathbf{h}_t, \theta, \boldsymbol{\sigma}_{-t}) \\
&= \sum_{(x_{t+1},\ldots,x_T)} \prod_{\tau > t} \Pr(x_\tau \mid \theta, \mathbf{h}_\tau) \sum_{\tau > t} \pi(x_\tau, \theta) \\
&= \sum_{x_{t+1} \in X} \sum_{(x_{t+2},\ldots,x_T)} \Pr(x_{t+1} \mid \theta, \mathbf{h}_{t+1}) \prod_{\tau > t+1} \Pr(x_\tau \mid \theta, \mathbf{h}_\tau) \left[\pi(x_{t+1}, \theta) + \sum_{\tau > t+1} \pi(x_\tau, \theta)\right] \\
&= \sum_{x_{t+1} \in X} \Pr(x_{t+1} \mid \theta, \mathbf{h}_{t+1}) \left[\pi(x_{t+1}, \theta) + \sum_{(x_{t+2},\ldots,x_T)} \prod_{\tau > t+1} \Pr(x_\tau \mid \theta, \mathbf{h}_\tau) \sum_{\tau > t+1} \pi(x_\tau, \theta)\right] \\
&= \sum_{x_{t+1} \in X} \Pr(x_{t+1} \mid \theta, \mathbf{h}_{t+1}) \left[\pi(x_{t+1}, \theta) + C_{t+1}\left(x_{t+1} \mid \mathbf{h}_{t+1}, \theta, \boldsymbol{\sigma}_{-(t+1)}\right)\right]
\end{aligned}$$

where $(\mathbf{h}_t, x_t) \subseteq \mathbf{h}_\tau$ for each $\tau > t$. The third equality follows from $\sum_{(x_{t+2},\ldots,x_T)} \prod_{\tau > t+1} \Pr(x_\tau \mid \theta, \mathbf{h}_\tau) = 1$, and we may replace $\boldsymbol{\sigma}_{-t}$ by $\boldsymbol{\sigma}_{-(t+1)}$ in the last line since the continuation values of player t only depend upon strategies σ_τ for $\tau > t$. □

The second lemma shows that the behavior of player T is uniquely determined in any equilibrium. Accordingly, equilibria can be derived backwards.

Lemma A2. *In any sequential equilibrium* $\sigma_T(s_T, \mathbf{h}_T) = 1\,(0)$ *if* $\mu_T(s_T, \mathbf{h}_T) > (<)\, 1/2$.

Proof. Since $U_T(x_T \mid s_T, \mathbf{h}_T, \boldsymbol{\sigma}_{-T})$ equals

$$\mu_T(s_T, \mathbf{h}_T) \left[\pi(x_T, \mathcal{B}) + \alpha \sum_{t<T} \pi(x_t, \mathcal{B})\right] + (1 - \mu_T(s_T, \mathbf{h}_T)) \left[\pi(x_T, \mathcal{O}) + \alpha \sum_{t<T} \pi(x_t, \mathcal{O})\right]$$

for each $x_T \in X$, $U_T(B \mid s_T, \mathbf{h}_T, \boldsymbol{\sigma}_{-T}) > (<)\, U_T(O \mid s_T, \mathbf{h}_T, \boldsymbol{\sigma}_{-T})$ if $\mu_T(s_T, \mathbf{h}_T) > (<)\, 1 - \mu_T(s_T, \mathbf{h}_T)$. □

The third lemma states that, given the focus on pure strategies and on error or signal-revealing off-path beliefs, players' behavior may be captured by (alternative) strategies $\hat{\sigma}_t(x_t \mid s_t, \Delta_t)$ which depend on the signal s_t, and the difference Δ_t between the number of b and o signals that may be inferred from the history $\mathbf{h}_t$. For instance, $\Delta_1 \equiv 0$ by definition. Accordingly, we henceforth work with alternative strategies, alternative continuation values $\hat{C}_t(\Delta_{t+1}, \theta)$,[1] and alternative beliefs

$$\hat{\mu}(s, \Delta_t) = \left[1 + \frac{1-p}{p}\frac{\Pr(s \mid \mathcal{O})}{\Pr(s \mid \mathcal{B})}\left(\frac{1-q}{q}\right)^{\Delta_t}\right]^{-1}.$$

Lemma A3. *For any pure sequential equilibrium $\boldsymbol{\sigma}^*$ with error or signal-revealing off-path beliefs there exists a profile $\hat{\boldsymbol{\sigma}}^*$ of alternative strategies $\hat{\sigma}_t^* : S \times \mathbb{Z} \to \mathcal{D}(X)$ such that for each $t = 1, \ldots, T$, each $s_t \in S$, and each $\mathbf{h}_t \in H_t$*

$$\sigma_t^*(x_t \mid s_t, \mathbf{h}_t) = \hat{\sigma}_t^*(x_t \mid s_t, \Delta_t)$$

where $\Delta_1 \equiv 0$, and for each $t > 1$

$$\Delta_{t+1} = \Delta_t + \begin{cases} \sigma_t(x_t \mid b, \Delta_t) - \sigma_t(x_t \mid o, \Delta_t) & \text{if } \sigma_t(x_t \mid s_t, \Delta_t) \neq 0 \text{ for some } s_t \in S \\ z_t & \text{if } \sigma_t(x_t \mid s_t, \Delta_t) = 0 \text{ for each } s_t \in S \end{cases}$$

with $z_t = 0$ for error off-path beliefs, and $z_t = 1\,(-1)$ if $x_t = B\,(O)$ for signal-revealing off-path beliefs.

Proof. Fix a pure sequential equilibrium $\boldsymbol{\sigma}$ with either error or signal revealing off-path beliefs. The proof shows that beliefs and continuation values only depend upon the difference Δ_t for each player t.

Beliefs: For each $t \in \{1, \ldots, T\}$, each $s_t \in S$, and each $\mathbf{h}_t \in H_t$ on the equilibrium path,

$$\mu_t(s_t, \mathbf{h}_t) = \left[1 + \frac{1-p}{p}\frac{\Pr(s_t \mid \mathcal{O})}{\Pr(s_t \mid \mathcal{B})}\prod_{\tau<t}\frac{(1-q)\,\sigma_\tau(x_\tau \mid b, \mathbf{h}_\tau) + q\,\sigma_\tau(x_\tau \mid o, \mathbf{h}_\tau)}{q\,\sigma_\tau(x_\tau \mid b, \mathbf{h}_\tau) + (1-q)\,\sigma_\tau(x_\tau \mid o, \mathbf{h}_\tau)}\right]^{-1}.$$

Since $\mathbf{h}_t$ is on the equilibrium path, $\sigma_\tau(x_\tau \mid s_\tau, \mathbf{h}_\tau) > 0$ for each $\tau < t$ and at least one $s_\tau \in S$. In addition, either $\sigma_\tau(x_\tau \mid b, \mathbf{h}_\tau) = \sigma_\tau(x_\tau \mid o, \mathbf{h}_\tau)$, or $\sigma_\tau(x_\tau \mid b, \mathbf{h}_\tau) = 1$ and $\sigma_\tau(x_\tau \mid o, \mathbf{h}_\tau) = 0$ or vice versa. It follows that

$$\frac{(1-q)\,\sigma_\tau(x_\tau \mid b, \mathbf{h}_\tau) + q\,\sigma_\tau(x_\tau \mid o, \mathbf{h}_\tau)}{q\,\sigma_\tau(x_\tau \mid b, \mathbf{h}_\tau) + (1-q)\,\sigma_\tau(x_\tau \mid o, \mathbf{h}_\tau)} = \left(\frac{1-q}{q}\right)^{\sigma_\tau(x_\tau|b,\mathbf{h}_\tau) - \sigma_\tau(x_\tau|o,\mathbf{h}_\tau)}.$$

and therefore

$$\prod_{\tau<t}\frac{(1-q)\,\sigma_\tau(x_\tau \mid b, \mathbf{h}_\tau) + q\,\sigma_\tau(x_\tau \mid o, \mathbf{h}_\tau)}{q\,\sigma_\tau(x_\tau \mid b, \mathbf{h}_\tau) + (1-q)\,\sigma_\tau(x_\tau \mid o, \mathbf{h}_\tau)} = \left(\frac{1-q}{q}\right)^{\Delta_t}.$$

On the other hand off-path beliefs are well-defined through the assumption that the interpretation of off-path choices is commonly known. This is formalized by the auxiliary variable z_t for the two specifications of off-path beliefs that we focus on.

Continuation Values: We show inductively that continuation values only depend upon the difference Δ_t for each period $t = 1, \ldots, T$. Since continuation values and beliefs uniquely determine strategies, this is

[1] Note that continuation values in period t depend upon the history $\mathbf{h}_t$ and the own action x_t. Accordingly, Δ_{t+1} is the difference between the number of b and o signals that can be inferred from history $\mathbf{h}_{t+1} = (\mathbf{h}_t, x_t)$, and therefore depends on player t's action x_t.

sufficient to prove the claim. For player T, $C_T\left(x_T \mid \mathbf{h}_T, \theta, \boldsymbol{\sigma}^*_{-T}\right) = 0$ for each x_T, $\mathbf{h}_T$, and θ implies that $C_T\left(x_T \mid \mathbf{h}_T, \theta, \boldsymbol{\sigma}^*_{-T}\right) = \hat{C}_T\left(\Delta_{T+1}, \theta\right)$ if $\hat{C}_T\left(\Delta_{T+1}, \theta\right) = 0$ for each $\Delta \in \mathbb{Z}$ and each $\theta \in \Theta$. Accordingly, assume that for each $\tau > t$

$$C_\tau\left(x_\tau \mid \mathbf{h}_\tau, \theta, \boldsymbol{\sigma}^*_{-\tau}\right) = \hat{C}_\tau\left(\Delta_{\tau+1}, \theta\right)$$

where $\Delta_{\tau+1}$ is derived from $\mathbf{h}_\tau$ and x_τ as determined above. Lemma A1 and the induction assumption imply that for each $\tau \in \{t+1, \ldots, T-1\}$, each $\Delta_\tau \in \{1-\tau, \ldots, \tau-1\}$, and each $\theta \in \Theta$

$$\hat{C}_\tau\left(\Delta_{\tau+1}, \theta\right) = \sum_{x_{\tau+1} \in X} \Pr\left(x_{\tau+1} \mid \theta, \Delta_{\tau+1}\right) \left[\pi\left(x_{\tau+1}, \theta\right) + \hat{C}_{\tau+1}\left(\Delta_{\tau+2}, \theta\right)\right]$$

with $\Pr\left(x_{\tau+1} \mid \theta, \Delta_{\tau+1}\right) = \sum_{s_{\tau+1} \in S} \Pr\left(s_{\tau+1} \mid \theta\right) \hat{\sigma}_{\tau+1}\left(x_{\tau+1} \mid s_{\tau+1}, \Delta_{\tau+1}\right)$. Using Lemma A1 we obtain for period t

$$C_t\left(x_t \mid \mathbf{h}_t, \theta, \boldsymbol{\sigma}^*_{-t}\right) = \sum_{x_{t+1} \in X} \Pr\left(x_{t+1} \mid \theta, \Delta_{t+1}\right) \left[\pi\left(x_{t+1}, \theta\right) + \hat{C}_{t+1}\left(\Delta_{t+1}, \theta\right)\right].$$

It suffices to show that Δ_{t+1} is uniquely determined at $\mathbf{h}_t$ and x_t. For choices on the equilibrium path this is true since $\Delta_{t+1} = \sum_{\tau < t}\left[\sigma^*_\tau\left(x_\tau \mid b, \mathbf{h}_\tau\right) - \sigma^*_\tau\left(x_\tau \mid o, \mathbf{h}_\tau\right)\right]$. For off-path choices it follows from the assumption that the interpretation of off-path choices is commonly known. □

The remaining lemmas collect some properties of monotonic sequential equilibria. According to Definition 2 in the main text a sequential equilibrium is monotonic if it satisfies two properties. The two conditions are minimal in the sense that none implies the other.

First, for each period $t = 1, \ldots, T$ strategies are increasing in the difference Δ_t between the number of b- and o-signals inferrable from the history $\mathbf{h}_t$. Given the focus on pure strategies this implies that a player who finds it optimal to guess B (O) regardless of her signal at difference Δ must guess B (O) at any larger (smaller) difference (part (i) of Lemma A4). Therefore, for each period t the set of possible differences $\{1-t, \ldots, t-1\}$ may by split into (up to) three subsets $D^O_t = \{1-t, \ldots, \underline{\Delta}_t\}$, $D^s_t = \{\underline{\Delta}_t + 1, \ldots, \overline{\Delta}_t - 1\}$, and $D^B_t = \{\overline{\Delta}_t, \ldots, t-1\}$ such that a player herds on action B (O) for each $\Delta_t \in D^B_t$ ($\Delta_t \in D^O_t$), and her guess strictly depends on private information for each $\Delta_t \in D^s_t$. D^B_t and D^O_t are called *cascade sets.*

The second property of monotonic equilibria implies that the third subset D^s_t is non-empty for each t, and players follow private information for $\Delta_t \in D^s_t$ (parts (ii) and (iii) of Lemma A4). Moreover the set is weakly shrinking in t or equivalently, the cascade sets grow weakly with t.

To save upon notation define for a given strategy profile $\hat{\boldsymbol{\sigma}}$, and for each $t = 1, \ldots, T$, and each $\Delta_t \in \{1-t, \ldots, t-1\}$,

$$\hat{\rho}_t\left(\Delta_t\right) = \left(\hat{\sigma}_t\left(b, \Delta_t\right), \hat{\sigma}_t\left(o, \Delta_t\right)\right).$$

Lemma A4. *In each monotonic sequential equilibrium it holds for each $t = 1, \ldots, T$, and each $\Delta_t \in \{1-t, \ldots, t-1\}$,*

(i) $\hat{\rho}_t(z) = \hat{\rho}_t\left(\Delta_t\right)$ *for each* $z > (<)\, \Delta_t$ *if* $\hat{\rho}_t\left(\Delta_t\right) = (1,1)$ $\left(\hat{\rho}_t\left(\Delta_t\right) = (0,0)\right)$*,*

(ii) $\hat{\sigma}_t\left(b, \Delta_t\right) \geqslant \hat{\sigma}_t\left(o, \Delta_t\right)$*,*

(iii) $\hat{\rho}_t\left(\Delta_t\right) = (1,0)$ *for each* $\Delta_t \in \{-1, 0\}$*.*

(iv) $\hat{\sigma}_t\left(o, \Delta_t\right) = 1$ *only if* $\Delta_t \geqslant 1$*, and* $\hat{\sigma}_t\left(b, \Delta_t\right) = 0$ *only if* $\Delta_t \leqslant -2$*.*

Proof. (i) The properties follow directly from the first part of Definition 2 (main text).

(ii) The proof is by induction: First, the property holds for $t = T$ since $\hat{\sigma}_T(b, \Delta_T) = 1\,(0)$ for $\Delta_T \geqslant (<) -1$, and $\hat{\sigma}_T(o, \Delta_T) = 1\,(0)$ for $\Delta_T \geqslant (<)\,1$ by Lemma A2. Second, the property holds for t if it holds for $t+1$, since $\hat{\sigma}_t(b, \Delta_t) \geqslant \hat{\sigma}_{t+1}(b, \Delta_t) \geqslant \hat{\sigma}_{t+1}(o, \Delta_t) \geqslant \hat{\sigma}_t(o, \Delta_t)$ for each $\Delta_t \in \{1-t, \ldots, t-1\} \subset \{1-(t+1), (t+1)-1\}$ using part (ii) of Definition 2 (main text).

(iii) The property follows from Lemma A2 and the second part of Definition 2 (main text).

(iv) By part (iii) and the first part of Definition 2 (main text) $\hat{\sigma}_t(o, z) \leqslant \hat{\sigma}_t(o, 0) = 0$ for each $z \leqslant -1$, and $\hat{\sigma}_t(b, z) \geqslant \hat{\sigma}_t(b, -1) = 1$ for each $z \geqslant -1$.

□

Lemma A5 states that continuation values are weakly increasing (decreasing) in the difference Δ under state $\mathcal{B}$ ($\mathcal{O}$).

Lemma A5. *In any monotonic sequential equilibrium it holds for each $t = 2, \ldots, T$, and each $1-t \leqslant \Delta_t \leqslant t-2$, (i) $\hat{C}_t(\Delta_t, \mathcal{B}) \leqslant \hat{C}_t(\Delta_t + 1, \mathcal{B})$, and (ii) $\hat{C}_t(\Delta_t, \mathcal{O}) \geqslant \hat{C}_t(\Delta_t + 1, \mathcal{O})$.*

Proof. The proof is by induction. For $t = T$ both claims are trivially true, since $\hat{C}_T(\Delta_T, \theta) = 0$ for each $\Delta_T \in \{1-T, \ldots, T-1\}$, and each $\theta \in \Theta$. Assume the claims are true for $t+1$. By Lemma A4 we need to distinguish 5 cases:

(A) $\hat{\rho}_{t+1}(\Delta+1) = \hat{\rho}_{t+1}(\Delta) = (1,1)$
(B) $\hat{\rho}_{t+1}(\Delta+1) = (1,1)$ and $\hat{\rho}_{t+1}(\Delta) = (1,0)$
(C) $\hat{\rho}_{t+1}(\Delta+1) = \hat{\rho}_{t+1}(\Delta) = (1,0)$
(D) $\hat{\rho}_{t+1}(\Delta+1) = (1,0)$ and $\hat{\rho}_{t+1}(\Delta) = (0,0)$
(E) $\hat{\rho}_{t+1}(\Delta+1) = \hat{\rho}_{t+1}(\Delta) = (0,0)$

To prove these cases we employ Lemma A1.

Ad (A): $\hat{C}_t(\Delta, \mathcal{B}) = 1 + \hat{C}_{t+1}(\Delta, \mathcal{B}) \leqslant 1 + \hat{C}_{t+1}(\Delta+1, \mathcal{B}) = \hat{C}_t(\Delta+1, \mathcal{B})$, and $\hat{C}_t(\Delta, \mathcal{O}) = \hat{C}_{t+1}(\Delta, \mathcal{O}) \geqslant \hat{C}_{t+1}(\Delta+1, \mathcal{O}) = \hat{C}_t(\Delta+1, \mathcal{O})$.

Ad (B): $\hat{C}_t(\Delta, \mathcal{B}) = q + q\,\hat{C}_{t+1}(\Delta+1, \mathcal{B}) + (1-q)\,\hat{C}_{t+1}(\Delta-1, \mathcal{B}) \leqslant q + \hat{C}_{t+1}(\Delta+1, \mathcal{B}) \leqslant 1 + \hat{C}_{t+1}(\Delta+1, \mathcal{B}) = \hat{C}_t(\Delta+1, \mathcal{B})$, and $\hat{C}_t(\Delta, \mathcal{O}) = q + q\,\hat{C}_{t+1}(\Delta-1, \mathcal{O}) + (1-q)\,\hat{C}_{t+1}(\Delta+1, \mathcal{O}) \geqslant q + \hat{C}_{t+1}(\Delta+1, \mathcal{O}) \geqslant \hat{C}_{t+1}(\Delta+1, \mathcal{O}) = \hat{C}_t(\Delta+1, \mathcal{O})$.

Ad (C): $\hat{C}_t(\Delta, \mathcal{B}) = q + q\,\hat{C}_{t+1}(\Delta+1, \mathcal{B}) + (1-q)\,\hat{C}_{t+1}(\Delta-1, \mathcal{B}) \leqslant q + q\,\hat{C}_{t+1}(\Delta+2, \mathcal{B}) + (1-q)\,\hat{C}_{t+1}(\Delta, \mathcal{B}) = \hat{C}_t(\Delta+1, \mathcal{B})$, and $\hat{C}_t(\Delta, \mathcal{O}) = q + q\,\hat{C}_{t+1}(\Delta-1, \mathcal{O}) + (1-q)\,\hat{C}_{t+1}(\Delta+1, \mathcal{O}) \geqslant q + q\,\hat{C}_{t+1}(\Delta, \mathcal{O}) + (1-q)\,\hat{C}_{t+1}(\Delta+2, \mathcal{O}) = \hat{C}_t(\Delta+1, \mathcal{O})$.

Ad (D): $\hat{C}_t(\Delta, \mathcal{B}) = \hat{C}_{t+1}(\Delta, \mathcal{B}) \leqslant q\,\hat{C}_{t+1}(\Delta, \mathcal{B}) + (1-q)\,\hat{C}_{t+1}(\Delta+2, \mathcal{B}) \leqslant q + q\,\hat{C}_{t+1}(\Delta, \mathcal{B}) + (1-q)\,\hat{C}_{t+1}(\Delta+2, \mathcal{B}) = \hat{C}_t(\Delta+1, \mathcal{B})$, and $\hat{C}_t(\Delta, \mathcal{O}) = 1 + \hat{C}_{t+1}(\Delta, \mathcal{O}) \geqslant q + q\,\hat{C}_{t+1}(\Delta, \mathcal{O}) + (1-q)\,\hat{C}_{t+1}(\Delta+2, \mathcal{O}) = \hat{C}_t(\Delta+1, \mathcal{O})$.

Ad (E): $\hat{C}_t(\Delta, \mathcal{B}) = \hat{C}_{t+1}(\Delta, \mathcal{B}) \leqslant \hat{C}_{t+1}(\Delta+1, \mathcal{B}) = \hat{C}_t(\Delta+1, \mathcal{B})$, and $\hat{C}_t(\Delta, \mathcal{O}) = 1 + \hat{C}_{t+1}(\Delta, \mathcal{O}) \geqslant 1 + \hat{C}_{t+1}(\Delta+1, \mathcal{O}) = \hat{C}_t(\Delta+1, \mathcal{O})$. □

A.3. Lemmas Relevant for the Proof of Proposition 2

The subsequent lemmas are the building blocks of Proposition 2. Lemma A6 establishes that a player finds it optimal to herd on action B (O), if her direct successor finds it optimal to herd on this action whatever the player's choice, and if beliefs are sufficiently high (low). Conversely, according to Lemma A7, a player finds it optimal to follow private information at Δ, if her direct successor does so at $\Delta + 1$ and $\Delta - 1$. Finally, lemmas A8 and A9 consider the boundary of the *cascade sets*. For instance, Lemma A8 implies that player t is weakly more inclined to follow an o-signal at each $\Delta \in \{\overline{\Delta}_{t+1} - 1, \overline{\Delta}_{t+1}\}$ than player $t+1$. Accordingly, player t must follow her o-signal at $\overline{\Delta}_{t+1} - 1$, and she may do so at $\overline{\Delta}_{t+1}$. Furthermore, Lemma A9 shows that following a b-signal is always favorable for player t at $\Delta \in \{\overline{\Delta}_{t+1} - 1, \overline{\Delta}_{t+1}\}$. Similar considerations apply to $\Delta \in \{\underline{\Delta}_{t+1}, \underline{\Delta}_{t+1} + 1\}$.

Lemma A6. *In any monotonic sequential equilibrium it holds for each $t < T$, and each $1-t \leqslant \Delta \leqslant t-1$*

(i) $\hat{\sigma}_t(s_t, \Delta) = 1$ for each $s_t \in S$ if $\hat{\sigma}_{t+1}(s_{t+1}, \Delta - 1) = 1$ for each $s_{t+1} \in S$ and $\hat{\mu}(o, \Delta) > 1/2$,

(ii) $\hat{\sigma}_t(s_t, \Delta) = 0$ for each $s_t \in S$ if $\hat{\sigma}_{t+1}(s_{t+1}, \Delta + 1) = 0$ for each $s_{t+1} \in S$ and $\hat{\mu}(b, \Delta) < 1/2$.

Proof. $\hat{\sigma}_{t+1}(s_{t+1}, \Delta - 1) = 1$ for each $s_{t+1} \in S$ implies that $\hat{\sigma}_{t+1}(s_{t+1}, z) = 1$ for each $s_{t+1} \in S$ and each $z \geqslant \Delta - 1$ (part (i) of Lemma A4). Hence, $\hat{\sigma}_\tau(s_\tau, z) = 1$ for each $\tau > t$, each $s_\tau \in S$, and each $z \geqslant \Delta - 1$, since $\hat{\sigma}_\tau(b, z) \geqslant \hat{\sigma}_\tau(o, z) \geqslant \hat{\sigma}_{t+1}(o, z) = 1$ for each $z \geqslant \Delta - 1$ where the first inequality follows from Lemma A4(ii), and the second inequality follows from part (ii) of Definition 2 (main text). Accordingly, $\hat{C}_t(z, \mathcal{B}) = T - t$ and $\hat{C}_t(z, \mathcal{O}) = 0$ for each $z \geqslant \Delta - 1$. Since any choice of player t leads to a difference no smaller than $\Delta - 1$, $\hat{U}_t(B \mid s_t, \Delta) > \hat{U}_t(O \mid s_t, \Delta)$ provided

$$\begin{aligned} \hat{\mu}(s_t, \Delta)\,[1 + \alpha\,(T-t)] &> \hat{\mu}(s_t, \Delta)\,\alpha\,(T-t) + [1 - \hat{\mu}(s_t, \Delta)] \\ \Leftrightarrow \qquad \hat{\mu}(s_t, \Delta) &> 1 - \hat{\mu}(s_t, \Delta) \end{aligned}$$

which follows from $\hat{\mu}(b, \Delta) > \hat{\mu}(o, \Delta) > 1/2$. Therefore only $\hat{\sigma}_t(s_t, \Delta) = 1$ for each $s_t \in S$ can be sustained in equilibrium.

The second part of the lemma follows analogously from $\hat{\sigma}_{t+1}(s_{t+1}, \Delta + 1) = 0$ for each $s_{t+1} \in S$ which implies that $\hat{\sigma}_\tau(s_\tau, z) = 0$ for each $\tau > t$, $s_\tau \in S$, and $z \leqslant \Delta + 1$, and hence $\hat{C}_t(z, \mathcal{B}) = 0$ and $\hat{C}_t(z, \mathcal{O}) = T - t$ for each $z \leqslant \Delta + 1$. □

Lemma A7. *For each pure sequential equilibrium $\hat{\boldsymbol{\sigma}}$, each $t < T$, and each $\Delta \in \{1-t, \ldots, t-1\}$, $\hat{\rho}_{t+1}(\Delta) = (1, 0)$ if $\hat{\rho}_{t+1}(\Delta + 1) = \hat{\rho}_{t+1}(\Delta - 1) = (1, 0)$.*

Proof. Notice first that if player t is expected to follow private information at Δ

$$\begin{aligned} \hat{U}_t(B \mid s_t, \Delta) &= \hat{\mu}(s_t, \Delta)\left[1 + \alpha\,\hat{C}_t(\Delta + 1, \mathcal{B})\right] + [1 - \hat{\mu}(s_t, \Delta)]\,\alpha\,\hat{C}_t(\Delta + 1, \mathcal{O}) \\ \text{and } \hat{U}_t(O \mid s_t, \Delta) &= \hat{\mu}(s_t, \Delta)\,\alpha\,\hat{C}_t(\Delta - 1, \mathcal{B}) + [1 - \hat{\mu}(s_t, \Delta)]\left[1 + \alpha\,\hat{C}_t(\Delta - 1, \mathcal{O})\right]. \end{aligned}$$

Hence, $\hat{U}_t(B \mid s_t, \Delta) > (<)\, \hat{U}_t(O \mid s_t, \Delta)$ iff

$$\begin{aligned} &\alpha\left\{\hat{\mu}(s_t, \Delta)\left[\hat{C}_t(\Delta + 1, \mathcal{B}) - \hat{C}_t(\Delta - 1, \mathcal{B})\right] - [1 - \hat{\mu}(s_t, \Delta)]\left[\hat{C}_t(\Delta - 1, \mathcal{O}) - \hat{C}_t(\Delta + 1, \mathcal{O})\right]\right\} \\ &> (<)\ [1 - \hat{\mu}(s_t, \Delta)] - \hat{\mu}(s_t, \Delta)\,. \end{aligned}$$

By Lemma A1, and since $\hat{\rho}_{t+1}(z) = (1,0)$ for each $z \in \{\Delta+1, \Delta-1\}$ the LHS equals

$$\begin{aligned}
& \alpha \left\{ \hat{\mu}\left(s_t,\Delta\right) \left[q \,+\, q\,\hat{C}_{t+1}\left(\Delta+2,\mathcal{B}\right) \,+\, (1-q)\;\hat{C}_{t+1}\left(\Delta,\mathcal{B}\right) \,-\, q \,-\, q\,\hat{C}_{t+1}\left(\Delta,\mathcal{B}\right), -\,(1-q)\;\hat{C}_{t+1}\left(\Delta-2,\mathcal{B}\right) \right] \right. \\
& \left. -\; \left[1-\hat{\mu}\left(s_t,\Delta\right)\right] \left[q \,+\, q\,\hat{C}_{t+1}\left(\Delta-2,\mathcal{O}\right) \,+\, (1-q)\;\hat{C}_{t+1}\left(\Delta,\mathcal{O}\right) \,-\, q \,-\, q\,\hat{C}_{t+1}\left(\Delta,\mathcal{O}\right) \,-\, (1-q)\;\hat{C}_{t+1}\left(\Delta+2,\mathcal{O}\right) \right] \right\} \\
= \;\; & \alpha \left\{ q\,\hat{\mu}\left(s_t,\Delta\right) \left[\hat{C}_{t+1}\left(\Delta+2,\mathcal{B}\right) \,-\, \hat{C}_{t+1}\left(\Delta,\mathcal{B}\right) \right] \,-\, (1-q)\;\left[1-\hat{\mu}\left(s_t,\Delta\right)\right] \left[\hat{C}_{t+1}\left(\Delta,\mathcal{O}\right) \,-\, \hat{C}_{t+1}\left(\Delta+2,\mathcal{O}\right) \right] \right\} \\
& +\; \alpha \left\{ (1-q)\;\hat{\mu}\left(s_t,\Delta\right) \left[\hat{C}_{t+1}\left(\Delta,\mathcal{B}\right) \,-\, \hat{C}_{t+1}\left(\Delta-2,\mathcal{B}\right) \right] \,-\, q\;\left[1-\hat{\mu}\left(s_t,\Delta\right)\right] \left[\hat{C}_{t+1}\left(\Delta-2,\mathcal{O}\right) \,-\, \hat{C}_{t+1}\left(\Delta,\mathcal{O}\right) \right] \right\}.
\end{aligned} \tag{1}$$

Consider $s_t = b$. $\hat{\sigma}_{t+1}\left(b,\Delta+1\right) = 1$ implies that $\hat{U}_{t+1}\left(B \mid b,\Delta+1\right) > \hat{U}_{t+1}\left(O \mid b,\Delta+1\right)$ or equivalently

$$\begin{aligned}
& \alpha \left\{ \hat{\mu}\left(b,\Delta+1\right) \left[\hat{C}_{t+1}\left(\Delta+2,\mathcal{B}\right) \,-\, \hat{C}_{t+1}\left(\Delta,\mathcal{B}\right) \right] \,-\, \left[1-\hat{\mu}\left(b,\Delta+1\right)\right] \left[\hat{C}_{t+1}\left(\Delta,\mathcal{O}\right) \,-\, \hat{C}_{t+1}\left(\Delta+2,\mathcal{O}\right) \right] \right\} \\
& >\; \left[1-\hat{\mu}\left(b,\Delta+1\right)\right] \,-\, \hat{\mu}\left(b,\Delta+1\right)
\end{aligned}$$

Using

$$\begin{aligned}
\hat{\mu}\left(b,\Delta+1\right) &= \frac{p\,q^{\Delta+2}}{D_{\Delta+2}} = q\,\frac{D_{\Delta+1}}{D_{\Delta+2}}\,\frac{p\,q^{\Delta+1}}{D_{\Delta+1}} = \frac{D_{\Delta+1}}{D_{\Delta+2}}\;q\,\hat{\mu}\left(b,\Delta\right), \\
1-\hat{\mu}\left(b,\Delta+1\right) &= \frac{(1-p)\,(1-q)^{\Delta+2}}{D_{\Delta+2}} = (1-q)\;\frac{D_{\Delta+1}}{D_{\Delta+2}}\;\frac{(1-p)\,(1-q)^{\Delta+1}}{D_{\Delta+1}} = \frac{D_{\Delta+1}}{D_{\Delta+2}}\;(1-q)\;\left[1-\hat{\mu}\left(b,\Delta\right)\right]
\end{aligned}$$

where $D_\Delta = p\,q^\Delta \,+\, (1-p)\,(1-q)^\Delta$ we obtain

$$\begin{aligned}
& \alpha \left\{ q\,\hat{\mu}\left(b,\Delta\right) \left[\hat{C}_{t+1}\left(\Delta+2,\mathcal{B}\right) \,-\, \hat{C}_{t+1}\left(\Delta,\mathcal{B}\right) \right] \,-\, (1-q)\;\left[1-\hat{\mu}\left(b,\Delta\right)\right] \left[\hat{C}_{t+1}\left(\Delta,\mathcal{O}\right) \,-\, \hat{C}_{t+1}\left(\Delta+2,\mathcal{O}\right) \right] \right\} \\
& >\, (1-q)\;\left[1-\hat{\mu}\left(b,\Delta\right)\right] \,-\, q\,\hat{\mu}\left(b,\Delta\right)
\end{aligned} \tag{2}$$

Analogously, $\hat{\sigma}_{t+1}\left(b,\Delta-1\right) = 1$ implies that

$$\begin{aligned}
& \alpha \left\{ (1-q)\,\hat{\mu}\left(b,\Delta\right) \left[\hat{C}_{t+1}\left(\Delta,\mathcal{B}\right) \,-\, \hat{C}_{t+1}\left(\Delta-2,\mathcal{B}\right) \right] \,-\, q\;\left[1-\hat{\mu}\left(b,\Delta\right)\right] \left[\hat{C}_{t+1}\left(\Delta-2,\mathcal{O}\right) \,-\, \hat{C}_{t+1}\left(\Delta,\mathcal{O}\right) \right] \right\} \\
& >\, q\;\left[1-\hat{\mu}\left(b,\Delta\right)\right] \,-\, (1-q)\,\hat{\mu}\left(b,\Delta\right)
\end{aligned} \tag{3}$$

since

$$\begin{aligned}
\hat{\mu}\left(b,\Delta-1\right) &= \frac{p\,q^{\Delta}}{D_{\Delta}} = \frac{1-q}{q\,(1-q)}\;\frac{D_{\Delta+1}}{D_{\Delta}}\;\frac{p\,q^{\Delta+1}}{D_{\Delta+1}} = \frac{D_{\Delta+1}}{q\,(1-q)\,D_{\Delta+2}}\;(1-q)\,\hat{\mu}\left(b,\Delta\right), \\
1-\hat{\mu}\left(b,\Delta-1\right) &= \frac{(1-p)\,(1-q)^{\Delta}}{D_{\Delta}} = \frac{q}{q\,(1-q)}\;\frac{D_{\Delta+1}}{D_{\Delta}}\;\frac{(1-p)\,(1-q)^{\Delta+1}}{D_{\Delta+1}} = \frac{D_{\Delta+1}}{q\,(1-q)\,D_{\Delta}}\;q\;\left[1-\hat{\mu}\left(b,\Delta\right)\right].
\end{aligned}$$

Combining equations (1), (2), and (3) yields

$$\begin{aligned}
& \alpha \left\{ \hat{\mu}\left(b,\Delta\right) \left[\hat{C}_{t+1}\left(\Delta+1,\mathcal{B}\right) \,-\, \hat{C}_{t+1}\left(\Delta-1,\mathcal{B}\right) \right] \,-\, \left[1-\hat{\mu}\left(b,\Delta\right)\right] \left[\hat{C}_{t+1}\left(\Delta-1,\mathcal{O}\right) \,-\, \hat{C}_{t+1}\left(\Delta+1,\mathcal{O}\right) \right] \right\} \\
& >\, (1-q)\;\left[1-\hat{\mu}\left(b,\Delta\right)\right] \,-\, q\,\hat{\mu}\left(b,\Delta\right) \,+\, q\;\left[1-\hat{\mu}\left(b,\Delta\right)\right] \,-\, (1-q)\,\hat{\mu}\left(b,\Delta\right) \\
& =\, \left[1-\hat{\mu}\left(b,\Delta\right)\right] \,-\, \hat{\mu}\left(b,\Delta\right)
\end{aligned}$$

which implies that $\hat{U}_t\left(B \mid b,\Delta\right) > \hat{U}_t\left(O \mid b,\Delta\right)$.

The case $s_t = o$ follows analogously from $\hat{\sigma}_{t+1}\left(o,z\right) = 0$ for each $z \in \{\Delta-1, \Delta+1\}$. □

Lemma A8. *Let* $\mu_\Delta = \frac{p\,q^\Delta}{p\,q^\Delta + (1-p)\,(1-q)^\Delta}$ *and define* $\varphi_t(\Delta) \equiv \mu_\Delta\, \hat{C}_t(\Delta, \mathcal{B}) \;+\; (1-\mu_\Delta)\, \hat{C}_t(\Delta, \mathcal{O})$. *Any monotonic sequential equilibrium satisfies for each* $t < T$ *and each* $1-t \leqslant \Delta \leqslant t-1$

$$\varphi_t(\Delta) \geqslant \varphi_{t+1}(\Delta) \;+\; \max\{\mu_\Delta, 1-\mu_\Delta\}\,.$$

Proof. The proof is by induction.

Period $T-1$: Since $\hat{C}_T(\Delta) = 0$ for each $1-T \leqslant \Delta \leqslant T-1$ and each $\theta \in \Theta$ it is sufficient to prove that $\varphi_{T-1}(\Delta) \geqslant \mu_\Delta$ for each $0 \leqslant \Delta \leqslant T-2$, and $\varphi_{T-1}(\Delta) \geqslant 1-\mu_\Delta$ for each $2-T \leqslant \Delta \leqslant -1$. By Lemma A2

$$\hat{\rho}_T(\Delta) = \begin{cases} (1,1) & \text{if } \Delta > 0 \\ (1,0) & \text{if } \Delta \in \{-1,0\} \\ (0,0) & \text{if } \Delta < -1 \end{cases}\,.$$

Accordingly, Lemma A1 implies that

$$\hat{C}_{T-1}(\Delta, \mathcal{B}) = \begin{cases} 1 & \text{if } \Delta > 0 \\ q & \text{if } \Delta \in \{-1,0\} \\ 0 & \text{if } \Delta < -1 \end{cases}\,, \quad \text{and} \quad \hat{C}_{T-1}(\Delta, \mathcal{O}) = \begin{cases} 0 & \text{if } \Delta > 0 \\ q & \text{if } \Delta \in \{-1,0\} \\ 1 & \text{if } \Delta < -1 \end{cases}\,.$$

Hence, $\varphi_{T-1}(\Delta) = \mu_\Delta$ for $\Delta > 0$, $\varphi_{T-1}(\Delta) = 1-\mu_\Delta$ for $\Delta < -1$, and $\varphi_{T-1}(\Delta) = q$ for $\Delta \in \{-1,0\}$ which proves the claim since $q > \mu_0 = p$ and $q > 1-\mu_{-1} = \frac{(1-p)\,q}{p\,(1-q) + (1-p)\,q}$.

Period $t < T$*:* Assume the claim is true for period $t+1$ (i.e. $\varphi_{t+1}(\Delta) \geqslant \varphi_{t+2}(\Delta) \;+\; \max\{\mu_\Delta, 1-\mu_\Delta\}$ for each $-t \leqslant \Delta \leqslant t$). Part (ii) of Definition 2 (main text) implies that 5 cases must be distinguished:

(A) $\hat{\rho}_{t+1}(\Delta) = \hat{\rho}_{t+2}(\Delta) = (1,1)$

(B) $\hat{\rho}_{t+1}(\Delta) = \hat{\rho}_{t+2}(\Delta) = (0,0)$

(C) $\hat{\rho}_{t+1}(\Delta) = \hat{\rho}_{t+2}(\Delta) = (1,0)$

(D) $\hat{\rho}_{t+1}(\Delta) = (1,0)$ and $\hat{\rho}_{t+2}(\Delta) = (1,1)$

(E) $\hat{\rho}_{t+1}(\Delta) = (1,0)$ and $\hat{\rho}_{t+2}(\Delta) = (0,0)$

Ad (A): $\varphi_t(\Delta) = \mu_\Delta \;+\; \varphi_{t+1}(\Delta)$ since $\hat{C}_t(\Delta, \mathcal{B}) = 1 \;+\; \hat{C}_{t+1}(\Delta, \mathcal{B})$ and $\hat{C}_t(\Delta, \mathcal{O}) = \hat{C}_{t+1}(\Delta, \mathcal{O})$. Moreover $\mu_\Delta > 1-\mu_\Delta$ since $\Delta \geqslant 1$ by Lemma A4 (iv).

Ad (B): $\varphi_t(\Delta) = 1-\mu_\Delta \;+\; \varphi_{t+1}(\Delta)$ since $\hat{C}_t(\Delta, \mathcal{B}) = \hat{C}_{t+1}(\Delta, \mathcal{B})$ and $\hat{C}_t(\Delta, \mathcal{O}) = 1 \;+\; \hat{C}_{t+1}(\Delta, \mathcal{O})$. Moreover $1-\mu_\Delta > \mu_\Delta$ since $\Delta \leqslant -2$ by Lemma A4 (iv).

For cases **(C)** $-$ **(E)**, $\hat{\rho}_{t+1}(\Delta) = (1,0)$. By Lemma A1 this implies

$$\begin{aligned} \varphi_t(\Delta) = q \;&+\; q\,\mu_\Delta\, \hat{C}_{t+1}(\Delta+1, \mathcal{B}) \;+\; (1-q)\,\mu_\Delta\, \hat{C}_{t+1}(\Delta-1, \mathcal{B}) \\ &+\; (1-q)\;(1-\mu_\Delta)\; \hat{C}_{t+1}(\Delta+1, \mathcal{O}) \;+\; q\;(1-\mu_\Delta)\; \hat{C}_{t+1}(\Delta-1, \mathcal{O})\,. \end{aligned} \tag{4}$$

Ad (C): Using equation (4) we obtain

$$\begin{aligned}
\varphi_t(\Delta) &= q + \frac{D_{\Delta+1}}{D_\Delta}\varphi_{t+1}(\Delta+1) + q(1-q)\frac{D_{\Delta-1}}{D_\Delta}\varphi_{t+1}(\Delta-1) \\
&\geqslant q + \frac{D_{\Delta+1}}{D_\Delta}\left[\varphi_{t+2}(\Delta+1) + \max\{\mu_{\Delta+1}, 1-\mu_{\Delta+1}\}\right] \\
&\quad + q(1-q)\frac{D_{\Delta-1}}{D_\Delta}\left[\varphi_{t+2}(\Delta-1) + \max\{\mu_{\Delta-1}, 1-\mu_{\Delta-1}\}\right] \\
&= \varphi_{t+1}(\Delta) + \max\{q\,\mu_\Delta, (1-q)\,(1-\mu_\Delta)\} + \max\{(1-q)\,\mu_\Delta, q\,(1-\mu_\Delta)\}.
\end{aligned}$$

with $D_\Delta = p\,q^\Delta + (1-p)(1-q)^\Delta$ where the inequality follows from the induction assumption, and the last line employs equation (4) for period $t+1$ since $\hat{\rho}_{t+2}(\Delta) = (1,0)$. For $\Delta \geqslant 1$ the claim follows from $q\,\mu_\Delta > (1-q)\,(1-\mu_\Delta)$, $(1-q)\,\mu_\Delta > q\,(1-\mu_\Delta)$, and $\mu_\Delta > 1-\mu_\Delta$. Equivalently, for $\Delta \leqslant -2$ we have $q\,\mu_\Delta < (1-q)\,(1-\mu_\Delta)$, $(1-q)\,\mu_\Delta < q\,(1-\mu_\Delta)$, and $\mu_\Delta < 1-\mu_\Delta$. Finally, if $-1 \leqslant \Delta \leqslant 0$, $q\,\mu_\Delta > (1-q)\,(1-\mu_\Delta)$ and $(1-q)\,\mu_\Delta < q\,(1-\mu_\Delta)$ imply that $\varphi_t(\Delta) \geqslant \varphi_{t+1}(\Delta) + q$ which proves the claim since $q > p = \mu_0 > 1-\mu_0$ and $q > \frac{(1-p)\,q}{p\,(1-q) + (1-p)\,q} = 1-\mu_{-1} > \mu_{-1}$.

Ad (D): By Lemma A4 and Definition 2 (main text) $\hat{\rho}_{t+2}(\Delta) = (1,1)$ implies that $\hat{\rho}_\tau(z) = (1,1)$ for each $\tau > t+1$, and each $z \geqslant \Delta$. Therefore $\hat{C}_{t+1}(\Delta,\theta) = \hat{C}_{t+1}(\Delta+1,\theta)$ for each $\theta \in \Theta$. Furthermore $\Delta \geqslant 1$ by Lemma A4 (iv). Hence, by equation (4) $\varphi_t(\Delta) \geqslant \varphi_{t+1}(\Delta) + \max\{\mu_\Delta, 1-\mu_\Delta\}$ is equivalent to

$$\begin{aligned}
&\mu_\Delta(1-q)\left[\hat{C}_{t+1}(\Delta+1,\mathcal{B}) - \hat{C}_{t+1}(\Delta-1,\mathcal{B})\right] - (1-\mu_\Delta)\,q\left[\hat{C}_{t+1}(\Delta-1,\mathcal{O}) - \hat{C}_{t+1}(\Delta+1,\mathcal{O})\right] \\
&\leqslant q - \max\{\mu_\Delta, 1-\mu_\Delta\} = q - \mu_\Delta.
\end{aligned}$$

On the other hand $\hat{\sigma}_{t+1}(o,\Delta) = 0$ implies that $\hat{U}_{t+1}(B \mid o,\Delta) < \hat{U}_{t+1}(O \mid o,\Delta)$ or equivalently

$$\begin{aligned}
&\mu_\Delta(1-q)\left[\hat{C}_{t+1}(\Delta+1,\mathcal{B}) - \hat{C}_{t+1}(\Delta-1,\mathcal{B})\right] - (1-\mu_\Delta)\,q\left[\hat{C}_{t+1}(\Delta-1,\mathcal{O}) - \hat{C}_{t+1}(\Delta+1,\mathcal{O})\right] \\
&= q(1-q)\frac{D_{\Delta-1}}{D_\Delta}\left\{\begin{array}{l}\hat{\mu}(o,\Delta)\left[\hat{C}_{t+1}(\Delta+1,\mathcal{B}) - \hat{C}_{t+1}(\Delta-1,\mathcal{B})\right] \\ -(1-\hat{\mu}(o,\Delta))\left[\hat{C}_{t+1}(\Delta-1,\mathcal{O}) - \hat{C}_{t+1}(\Delta+1,\mathcal{O})\right]\end{array}\right\} \\
&< q(1-q)\frac{D_{\Delta-1}}{D_\Delta}\,\frac{1-\hat{\mu}(g,\Delta) - \hat{\mu}(g,\Delta)}{\alpha} \leqslant q\,(1-\mu_\Delta) - (1-q)\,\mu_\Delta = q - \mu_\Delta
\end{aligned}$$

where the last inequality follows from $\alpha \leqslant 1$ and $1-\mu_\Delta < \mu_\Delta$ (since $\Delta \geqslant 1$).

Ad (E): As in the previous case $\hat{\rho}_{t+2}(\Delta) = (0,0)$ implies that $\hat{C}_{t+1}(\Delta,\theta) = \hat{C}_{t+1}(\Delta-1,\theta)$ for each $\theta \in \Theta$ and $\Delta \leqslant -2$. Accordingly, by equation (4) $\varphi_t(\Delta) \geqslant \varphi_{t+1}(\Delta) + \max\{\mu_\Delta, 1-\mu_\Delta\}$ iff

$$\begin{aligned}
&\mu_\Delta\,q\left[\hat{C}_{t+1}(\Delta+1,\mathcal{B}) - \hat{C}_{t+1}(\Delta-1,\mathcal{B})\right] - (1-\mu_\Delta)\,(1-q)\left[\hat{C}_{t+1}(\Delta-1,\mathcal{O}) - \hat{C}_{t+1}(\Delta+1,\mathcal{O})\right] \\
&\geqslant \max\{\mu_\Delta, 1-\mu_\Delta\} - q = (1-\mu_\Delta) - q.
\end{aligned}$$

This inequality follows from $\hat{\sigma}_{t+1}(b,\Delta)=1$ since $\hat{U}_{t+1}(B\mid b,\Delta)>\hat{U}_{t+1}(O\mid b,\Delta)$ is equivalent to

$$\begin{aligned}
&\mu_\Delta\, q\left[\hat{C}_{t+1}(\Delta+1,\mathcal{B})\;-\;\hat{C}_{t+1}(\Delta-1,\mathcal{B})\right]\;-\;(1-\mu_\Delta)\;(1-q)\;\left[\hat{C}_{t+1}(\Delta-1,\mathcal{O})\;-\;\hat{C}_{t+1}(\Delta+1,\mathcal{O})\right]\\
&=\frac{D_{\Delta+1}}{D_\Delta}\left\{\begin{array}{c}\hat{\mu}(b,\Delta)\left[\hat{C}_{t+1}(\Delta+1,\mathcal{B})\;-\;\hat{C}_{t+1}(\Delta-1,\mathcal{B})\right]\\ -(1-\hat{\mu}(b,\Delta))\left[\hat{C}_{t+1}(\Delta-1,\mathcal{O})\;-\;\hat{C}_{t+1}(\Delta+1,\mathcal{O})\right]\end{array}\right\}\\
&>\frac{D_{\Delta+1}}{D_\Delta}\,\frac{1-\hat{\mu}(b,\Delta)\;-\;\hat{\mu}(b,\Delta)}{\alpha}\;\geqslant\;(1-q)\;(1-\mu_\Delta)\;-\;q\,\mu_\Delta=1-\mu_\Delta\;-\;q
\end{aligned}$$

where the last inequality follows from $\alpha\leqslant 1$ and $1-\mu_\Delta>\mu_\Delta$ (since $\Delta\leqslant -2$). □

Lemma A9. *There exists a lower bound $0<\underline{\alpha}(p,q)<1$ s.t. for each $\underline{\alpha}(p,q)<\alpha\leqslant 1$ any monotonic sequential equilibrium in which $\hat{\rho}^*_t(\Delta_t)=(1,0)$ whenever possible satisfies for each $t<T$ and each $1-t\leqslant\Delta\leqslant t-1$*

(i) $\hat{\sigma}_t(b,\Delta)=1$ *if* $\hat{\rho}_{t+1}(\Delta-1)=(1,0)$ *and* $\hat{\rho}_{t+1}(\Delta+1)=(1,1)$,

and (ii) $\hat{\sigma}_t(o,\Delta)=0$ *if* $\hat{\rho}_{t+1}(\Delta+1)=(1,0)$ *and* $\hat{\rho}_{t+1}(\Delta-1)=(0,0)$,

Proof. We focus on property (i) as (ii) follows from similar arguments. Assume first that subsequent players expect player t to follow private information, i.e. identify player t's action B (O) with $s_t=b$ ($s_t=o$). In this case $\hat{U}_t(B\mid b,\Delta)>\hat{U}_t(O\mid b,\Delta)$ iff

$$\begin{aligned}
&1-\hat{\mu}(b,\Delta)\;-\;\hat{\mu}(b,\Delta)\\
&<\alpha\left\{\hat{\mu}(b,\Delta)\left[\hat{C}_t(\Delta+1,\mathcal{B})\;-\;\hat{C}_t(\Delta-1,\mathcal{B})\right]\;-\;[1-\hat{\mu}(b,\Delta)]\left[\hat{C}_t(\Delta-1,\mathcal{O})\;-\;\hat{C}_t(\Delta+1,\mathcal{O})\right]\right\}\\
&=\alpha\left\{\begin{array}{c}\hat{\mu}(b,\Delta)\left[1\;+\;\hat{C}_{t+1}(\Delta+1,\mathcal{B})\;-\;q\;-\;q\,\hat{C}_{t+1}(\Delta,\mathcal{B})\;-\;(1-q)\,\hat{C}_{t+1}(\Delta-2,\mathcal{B})\right]\\ -[1-\hat{\mu}(b,\Delta)]\left[q\;+\;q\,\hat{C}_{t+1}(\Delta-2,\mathcal{O})\;+\;(1-q)\,\hat{C}_{t+1}(\Delta,\mathcal{O})\;-\;\hat{C}_{t+1}(\Delta+1,\mathcal{O})\right]\end{array}\right\}\\
&=\alpha\left\{\hat{\mu}(b,\Delta)\;(1-q)\;\left[\hat{C}_{t+1}(\Delta,\mathcal{B})\;-\;\hat{C}_{t+1}(\Delta-2,\mathcal{B})\right]-[1-\hat{\mu}(b,\Delta)]\;q\;\left[\hat{C}_{t+1}(\Delta-2,\mathcal{O})\;-\;\hat{C}_{t+1}(\Delta,\mathcal{O})\right]\right\}\\
&\quad+\;\alpha\left\{\hat{\mu}(b,\Delta)\left[\hat{C}_{t+1}(\Delta+1,\mathcal{B})\;-\;\hat{C}_{t+1}(\Delta,\mathcal{B})\right]\;-\;[1-\hat{\mu}(b,\Delta)]\left[\hat{C}_{t+1}(\Delta,\mathcal{O})\;-\;\hat{C}_{t+1}(\Delta+1,\mathcal{O})\right]\right\}\\
&\quad+\;\alpha\;\{(1-q)\,\hat{\mu}(b,\Delta)\;-\;q\;[1-\hat{\mu}(b,\Delta)]\}
\end{aligned}$$

where we employ lemma A1 and $\hat{\rho}_{t+1}(\Delta-1)=(1,0)$ on the third line. $\hat{\sigma}_{t+1}(b,\Delta-1)=1$ implies that $\hat{U}_{t+1}(B\mid b,\Delta-1)>\hat{U}_{t+1}(O\mid b,\Delta-1)$, or equivalently

$$\begin{aligned}
&\alpha\left\{\hat{\mu}(b,\Delta)\;(1-q)\;\left[\hat{C}_{t+1}(\Delta,\mathcal{B})\;-\;\hat{C}_{t+1}(\Delta-2,\mathcal{B})\right]\;-\;[1-\hat{\mu}(b,\Delta)]\;q\;\left[\hat{C}_{t+1}(\Delta-2,\mathcal{O})\;-\;\hat{C}_{t+1}(\Delta,\mathcal{O})\right]\right\}\\
&=q\,(1-q)\,\frac{D_\Delta}{D_{\Delta+1}}\,\alpha\left\{\begin{array}{c}\hat{\mu}(b,\Delta-1)\left[\hat{C}_{t+1}(\Delta,\mathcal{B})\;-\;\hat{C}_{t+1}(\Delta-2,\mathcal{B})\right]\\ -\;[1-\hat{\mu}(b,\Delta-1)]\left[\hat{C}_{t+1}(\Delta-2,\mathcal{O})\;-\;\hat{C}_{t+1}(\Delta,\mathcal{O})\right]\end{array}\right\}\\
&>q\,(1-q)\,\frac{D_\Delta}{D_{\Delta+1}}\;\{1-\hat{\mu}(b,\Delta-1)\;-\;\hat{\mu}(b,\Delta-1)\}\\
&=q\;[1-\hat{\mu}(b,\Delta)]\;-\;(1-q)\,\hat{\mu}(b,\Delta)\;\;=\;\;q\;-\;\hat{\mu}(b,\Delta)
\end{aligned}$$

Hence, it suffices to show that

$$\begin{aligned}
&(1-q)\,[1-\hat{\mu}\,(b,\Delta)]\;-\;q\,\hat{\mu}\,(b,\Delta)\\
&<\alpha\,\Big\{\hat{\mu}\,(b,\Delta)\,\Big[\hat{C}_{t+1}\,(\Delta+1,\mathcal{B})\;-\;\hat{C}_{t+1}\,(\Delta,\mathcal{B})\Big]\;-\;[1-\hat{\mu}\,(b,\Delta)]\,\Big[\hat{C}_{t+1}\,(\Delta,\mathcal{O})\;-\;\hat{C}_{t+1}\,(\Delta+1,\mathcal{O})\Big]\Big\} \qquad (5)\\
&\;+\;\alpha\,\{(1-q)\,\hat{\mu}\,(b,\Delta)\;-\;q\,[1-\hat{\mu}\,(b,\Delta)]\}\,.
\end{aligned}$$

If $\hat{\rho}_{t+1}\,(\Delta)\;=\;(1,1)$, it follows that $\hat{C}_{t+1}\,(\Delta,\theta)\;=\;\hat{C}_{t+1}\,(\Delta+1,\theta)$ for each $\theta\in\Theta$ by Lemma A4 and Definition 2 (main text). Hence, (5) is equivalent to $1-\hat{\mu}\,(b,\Delta)\;-\;q\;<\;\alpha\,\{\hat{\mu}\,(b,\Delta)\;-\;q\}$ which is true since $\hat{\mu}\,(b,\Delta)>q>1-\hat{\mu}\,(b,\Delta)$ ($\Delta\geqslant 1$ by Lemma A4 (iv)).

If $\hat{\rho}_{t+1}\,(\Delta)\;=\;(1,0)$, we may use $\hat{C}_{t+1}\,(\Delta+1,\theta)\;=\;\hat{C}_{t+1}\,(\Delta+2,\theta)$ since $\hat{\sigma}_{t+1}\,(o,\Delta+1)\;=\;1$ to show that (5) is equivalent to

$$\frac{1-\hat{\mu}(b,\Delta)}{\hat{\mu}(b,\Delta)}\;<\;\frac{q\;+\;\alpha\,(1-q)\;+\;\alpha\,Z_{\mathcal{B}}}{1-q\;+\;\alpha\,q\;+\;\alpha\,Z_{\mathcal{O}}} \qquad (6)$$

where $Z_\theta\;=\;\Big|\hat{C}_{t+1}\,(\Delta+2,\theta)\;-\;\hat{C}_{t+1}\,(\Delta,\theta)\Big|\;>\;0$ for each $\theta\in\Theta$. Moreover by the assumption that $\hat{\rho}^*_t\,(\Delta_t)=(1,0)$ whenever possible, $\hat{\sigma}_{t+1}\,(o,\Delta+1)=1$ also implies that $\hat{U}_{t+1}\,(B\mid o,\Delta+1)>\hat{U}_{t+1}\,(O\mid o,\Delta+1)$, or equivalently

$$\frac{1-\hat{\mu}\,(o,\Delta+1)}{\hat{\mu}\,(o,\Delta+1)}\;=\;\frac{q}{1-q}\,\frac{1-\hat{\mu}\,(b,\Delta)}{\hat{\mu}\,(b,\Delta)}\;<\;\frac{1\;+\;\alpha\,Z_{\mathcal{B}}}{1\;+\;\alpha\,Z_{\mathcal{O}}}.$$

Equation (6) is therefore true if

$$\frac{1-q}{q}\,\frac{1\;+\;\alpha\,Z_{\mathcal{B}}}{1\;+\;\alpha\,Z_{\mathcal{O}}}\;<\;\frac{q\;+\;\alpha\,(1-q)\;+\;\alpha\,Z_{\mathcal{B}}}{1-q\;+\;\alpha\,q\;+\;\alpha\,Z_{\mathcal{O}}}$$

or equivalently

$$\begin{aligned}
&(1-q)^2\;+\;\alpha\,q\,(1-q)\;+\;(1-q)\,\alpha\,Z_{\mathcal{O}}\;+\;(1-q)^2\,\alpha\,Z_{\mathcal{B}}\;+\;q\,(1-q)\,\alpha^2\,Z_{\mathcal{B}}\;+\;(1-q)\,\alpha^2\,Z_{\mathcal{B}}\,Z_{\mathcal{O}}\\
&<q^2\;+\;\alpha\,q\,(1-q)\;+\;q^2\,\alpha\,Z_{\mathcal{O}}\;+\;q\,(1-q)\,\alpha^2\,Z_{\mathcal{O}}\;+\;q\,\alpha\,Z_{\mathcal{B}}\;+\;q\,\alpha^2\,Z_{\mathcal{B}}\,Z_{\mathcal{O}}.
\end{aligned}$$

This holds for any $\alpha\;>\;\frac{1-q-q^2}{q\,(1-q)}$. Accordingly, if subsequent players expect player t to follow private information, following signal b is optimal under the conditions of the theorem.

Assume second that subsequent players expect player t to herd on action B and interpret the off-path action O as evidence of $s_t=o$. Accordingly, if player t chooses action B (O), the relevant difference for player $t+1$ is Δ ($\Delta-1$). If $\hat{\rho}_{t+1}\,(\Delta)=(1,0)$ player $t+1$ follows private information regardless of player t's action, and therefore $\hat{\sigma}_t\,(b,\Delta)=1$ by Lemma A7. If $\hat{\rho}_{t+1}\,(\Delta)\;=\;(1,1)$, $\hat{C}_{t+1}\,(\Delta,\theta)\;=\;\hat{C}_{t+1}\,(\Delta+1,\theta)$, and $\hat{U}_t\,(B\mid b,\Delta)>\hat{U}_t\,(O\mid b,\Delta)$ follows from $1-\hat{\mu}\,(b,\Delta)\;-\;q\;<\;\alpha\,\{\hat{\mu}\,(b,\Delta)\;-\;q\}$ (see above).

Finally, if subsequent players expect player t to herd on action B and interpret the off-path action O as an uninformative error, player t's action does not influence player $t+1$'s belief, and thus continuation values. Consequently, $U_t\,(B\mid b,\mathbf{h}_t)>U_t\,(O\mid b,\mathbf{h}_t)$ iff $\hat{\mu}\,(b,\Delta)>1/2$ which is true since $\hat{\sigma}_{t+1}\,(o,\Delta+1)=1$ implies that $\Delta\geqslant 0$ by Lemma A4 (iv). □

A.4. Proof of Proposition 2

Proposition 2 follows straightforwardly from the series of lemmas presented in the previous section. We first establish existence of an equilibrium for sufficiently large α (such that Lemma A9 holds) which satisfies for each $t = 1, \ldots, T$

$$\hat{\rho}_t(\Delta) = \begin{cases} (1,1) & \text{if } \Delta \geqslant \overline{\Delta}_t \\ (1,0) & \text{if } \underline{\Delta}_t < \Delta < \overline{\Delta}_t \\ (0,0) & \text{if } \Delta \leqslant \underline{\Delta}_t \end{cases} \tag{7}$$

where $1 - t \leqslant \underline{\Delta}_t \leqslant -2$ and $1 \leqslant \overline{\Delta}_t \leqslant t-1$ for each $t = 1, \ldots, T$, and $\underline{\Delta}_t \leqslant \underline{\Delta}_{t+1}$ and $\overline{\Delta}_t \geqslant \overline{\Delta}_{t+1}$ for each $t < T$. It is easily seen that such an equilibrium is monotonic. Second, we show that $\underline{\Delta}_t < \underline{\Delta}_{t+1}$ or $\overline{\Delta}_t > \overline{\Delta}_{t+1}$ for some $t < T$ provided T and α are sufficiently large.

To prove the first claim, we proceed backwards, starting with player T. Lemma A2 shows that player T's strategy satisfies (7) with $\underline{\Delta}_T = -2$ and $\overline{\Delta}_T = 1$. Assume therefore that strategies for players $\tau > t$ satisfy (7). We proceed in three steps.

(A) Lemma A7 shows that $\hat{\rho}_t(\Delta) = (1,0)$ if $\hat{\rho}_{t+1}(z) = (1,0)$ for each $z \in \{\Delta - 1, \Delta + 1\}$, and Lemma A6 shows that for each $x \in \{0,1\}$, $\hat{\rho}_t(\Delta) = (x,x)$ if $\hat{\rho}_{t+1}(z) = (x,x)$ for each $z \in \{\Delta - 1, \Delta + 1\}$. Accordingly, it remains to show that (i) $\hat{\rho}_t\left(\overline{\Delta}_{t+1} - 1\right) = (1,0)$ and $\hat{\rho}_t\left(\overline{\Delta}_{t+1}\right) \in \{(1,1),(1,0)\}$, and (ii) $\hat{\rho}_t\left(\underline{\Delta}_{t+1} + 1\right) = (1,0)$ and $\hat{\rho}_t\left(\underline{\Delta}_{t+1}\right) \in \{(0,0),(1,0)\}$.

(B) For $\Delta \in \left\{\overline{\Delta}_{t+1} - 1, \overline{\Delta}_{t+1}\right\}$ we have $\hat{\rho}_{t+1}(\Delta + 1) = (1,1)$ and $\hat{\rho}_{t+1}(\Delta - 1) = (1,0)$. By Lemma A9 $\hat{\sigma}_t(b,\Delta) = 1$. Consider signal $s_t = o$. If subsequent players expect player t to follow her private information, $\hat{U}_t(O \mid o,\Delta) > \hat{U}_t(B \mid o,\Delta)$ is equivalent to

$$\begin{aligned} &\alpha\left\{\hat{\mu}(o,\Delta)\left[\hat{C}_t(\Delta+1,\mathcal{B}) - \hat{C}_t(\Delta-1,\mathcal{B})\right] - \left[1-\hat{\mu}(o,\Delta)\right]\left[\hat{C}_t(\Delta-1,\mathcal{O}) - \hat{C}_t(\Delta+1,\mathcal{O})\right]\right\} \\ &< 1 - \hat{\mu}(o,\Delta) - \hat{\mu}(o,\Delta). \end{aligned}$$

By $\hat{\rho}_{t+1}(\Delta+1) = (1,1)$, Lemma A4 and Definition 2 (main text) $\hat{C}_t(\Delta+1,\mathcal{B}) = 1 + \hat{C}_{t+1}(\Delta+1,\mathcal{B})$ and $\hat{C}_t(\Delta+1,\mathcal{O}) = \hat{C}_{t+1}(\Delta+1,\mathcal{O})$. Consequently, the LHS is equivalent to

$$\begin{aligned} &\alpha\left\{\hat{\mu}(o,\Delta)\left[\hat{C}_{t+1}(\Delta+1,\mathcal{B}) - \hat{C}_{t+1}(\Delta-1,\mathcal{B})\right] - \left[1-\hat{\mu}(o,\Delta)\right]\left[\hat{C}_{t+1}(\Delta-1,\mathcal{O}) - \hat{C}_{t+1}(\Delta+1,\mathcal{O})\right]\right\} \\ &\quad + \alpha\left\{\hat{\mu}(o,\Delta) + \varphi_{t+1}(\Delta-1) - \varphi_t(\Delta-1)\right\} \\ &\leqslant \alpha\left\{\hat{\mu}(o,\Delta)\left[\hat{C}_{t+1}(\Delta+1,\mathcal{B}) - \hat{C}_{t+1}(\Delta-1,\mathcal{B})\right] - \left[1-\hat{\mu}(o,\Delta)\right]\left[\hat{C}_{t+1}(\Delta-1,\mathcal{O}) - \hat{C}_t(\Delta+1,\mathcal{O})\right]\right\}. \end{aligned}$$

where the inequality follows from Lemma A8. Therefore, $\hat{\sigma}_t(o,\Delta) = 0$ if $\hat{\sigma}_{t+1}(o,\Delta) = 0$, and thus $\overline{\Delta}_t \geqslant \overline{\Delta}_{t+1}$. On the other hand it is possible that $\hat{\sigma}_{t+1}(o,\Delta) = 1$ while $\hat{\sigma}_t(o,\Delta) = 0$ (see below). If $\hat{\sigma}_t(o,\Delta) = 0$ cannot hold in equilibrium, there exists an equilibrium in which player t herds on action B at Δ under either specification of off-path beliefs. For signal revealing off-path beliefs this follows since

$$\begin{aligned} &\alpha\left\{\hat{\mu}(o,\Delta)\left[\hat{C}_t(\Delta,\mathcal{B}) - \hat{C}_t(\Delta-1,\mathcal{B})\right] - \left[1-\hat{\mu}(o,\Delta)\right]\left[\hat{C}_t(\Delta-1,\mathcal{O}) - \hat{C}_t(\Delta,\mathcal{O})\right]\right\} \\ &= \alpha\left\{\hat{\mu}(o,\Delta)\left[\hat{C}_t(\Delta+1,\mathcal{B}) - \hat{C}_t(\Delta-1,\mathcal{B})\right] - \left[1-\hat{\mu}(o,\Delta)\right]\left[\hat{C}_t(\Delta-1,\mathcal{O}) - \hat{C}_t(\Delta+1,\mathcal{O})\right]\right\}. \end{aligned}$$

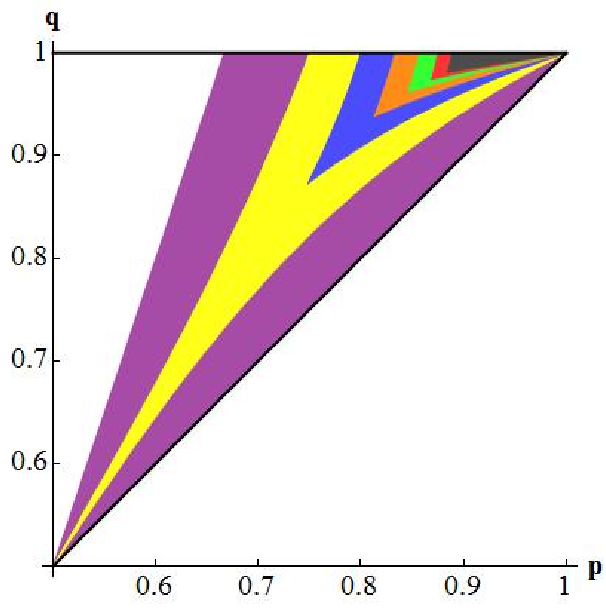
For error off-path beliefs it follows from $\hat{\mu}(o,\Delta) > 1/2$ (since $\hat{\sigma}_{t+1}(o,\Delta) = 1$ implies $\Delta \geqslant 1$), and since continuation values do not depend on player t's choice.

(C) $\Delta \in \{\underline{\Delta}_{t+1}+1, \underline{\Delta}_{t+1}\}$ implies that $\hat{\rho}_{t+1}(\Delta+1) = (1,0)$ and $\hat{\rho}_{t+1}(\Delta-1) = (0,0)$. This case is similar to (B): Lemmas A9 shows that $\hat{\sigma}_t(o,\Delta) = 0$. On the other hand $\hat{\rho}_{t+1}(\Delta-1) = (0,0)$, Lemmas A4 and A8, and Definition 2 (main text) imply that

$$\alpha\left\{\hat{\mu}(b,\Delta)\left[\hat{C}_t(\Delta+1,\mathcal{B}) - \hat{C}_t(\Delta-1,\mathcal{B})\right] - [1-\hat{\mu}(b,\Delta)]\left[\hat{C}_t(\Delta-1,\mathcal{O}) - \hat{C}_t(\Delta+1,\mathcal{O})\right]\right\}$$
$$\geqslant \alpha\left\{\hat{\mu}(b,\Delta)\left[\hat{C}_{t+1}(\Delta+1,\mathcal{B}) - \hat{C}_{t+1}(\Delta-1,\mathcal{B})\right] - [1-\hat{\mu}(b,\Delta)]\left[\hat{C}_{t+1}(\Delta-1,\mathcal{O}) - \hat{C}_t(\Delta+1,\mathcal{O})\right]\right\}.$$

Therefore $\hat{\sigma}_t(b,\Delta) = 1$ if $\hat{\sigma}_{t+1}(b,\Delta) = 1$, while possibly $\hat{\sigma}_{t+1}(b,\Delta) = 0$ and $\hat{\sigma}_t(b,\Delta) = 1$. If $\hat{\sigma}_t(b,\Delta) = 1$ is not possible similar arguments as in (B) establish existence of an equilibrium in which player t herds on action O at Δ for either specification of off-path beliefs.

We finally show that $\underline{\Delta}_t < \underline{\Delta}_{t+1}$ or $\overline{\Delta}_t > \overline{\Delta}_{t+1}$ is possible for some $t < T$ provided T and α are sufficiently large. For given values of p and q the lower bounds on α and T may be calculated backwards by starting from the (arbitrary) period T and proceeding to earlier periods $t < T$ until either $\underline{\Delta}_t < 2$, or $\overline{\Delta}_t > 1$. These calculations (programmed in Mathematica) are available from the authors upon request. Figure 1 and Table 1 give an overview of the conditions on the parameters (T,p,q) under which NO lower bound $0 < \underline{\alpha}(T,p,q) \leqslant 1$ exists.



Note: Colors represent respectively $T=3$ (purple), $T=4$ (yellow), $T=5$ (blue), $T=6$ (orange), $T=7$ (green), $T=8$ (red), and $T=9$ (black).

Figure 1: (p,q)-values for which NO more revealing equilibrium exists.

	$T=3$	$T=4$	$T=5$	$T=6$	$T=7$	$T=8$	$T=9$
% of (p,q)-space	0.667	0.277	0.083	0.037	0.020	0.012	0.008

Table 1: Relative size of the subsets for which NO more revealing equilibrium exists.

Appendix B. Complementary Theoretical Results

In this appendix we first address the multiplicity of sequential equilibria in the observational learning game when prosocial preferences are present (B.1). Second, we study the equilibria of the laboratory cascade game which features two parallel sequences of players (B.2).

B.1. A Large Variety of Equilibrium Outcomes

Proposition 4 establishes that there is a large variety of equilibrium outcomes. We call a property generic if it holds for a subset of parameters which is open and dense (Smith and Sørensen, 2000).

Proposition 4. *Generically there exist sequential equilibria satisfying one of the following properties:*

(i) $\hat{\rho}_t^*(\Delta) \in \{(1,1),(0,0)\}$ *while* $\hat{\rho}_{t+1}^*(\Delta) = (1,0)$ *for some* $t < T$ *and some* $1-t \leqslant \Delta \leqslant t-1$,

(ii) $\hat{\rho}_t^*(\Delta_t) = (0,1)$ *for some* $t < T$ *and some* $1-t \leqslant \Delta_t \leqslant t-1$,

where $\hat{\rho}_t^*(\Delta_t) = (\hat{\sigma}_t^*(b,\Delta_t), \hat{\sigma}_t^*(o,\Delta_t))$.

Equilibria for which either one of the properties characterized in Proposition 4 holds are non-monotonic. In case (i) player t picks an action (the one supported by the public belief) regardless of her signal, which is why player t's action does not convey new information. Still, player $t+1$ who is in the same position as player t finds it optimal to follow her private signal. Notice that in such a situation there usually exists another equilibrium in which players $\tau > t$ behave similarly, and player t follows her private information. This is possible since different continuation values (at different beliefs) are relevant depending on whether subsequent players expect player t to follow private information, or to disregard it. On the other hand if property (ii) holds in equilibrium, player t finds it optimal to *mirror* her private information, i.e. to choose action B (resp. O) given signal o (resp. b). Intuitively, while this strategy is clearly suboptimal for the player's expected own monetary payoff, it is sustained by the fact that in equilibrium subsequent players interpret actions conversely.

As becomes clear from Proposition 4 the combination of information and payoff externalities induces multiple equilibria. Moreover the number of equilibria grows with α. Section B.2 below provides several concrete equilibrium outcomes, and collects some statistics about the number of equilibria for the experimental social learning game.

Proof of Proposition 4. **Part (i):** We show that the property may hold in period $t = T-2$ for $\Delta_t = 1$ provided $\frac{1}{2} < p < \frac{2}{3}$, $p < q < 3p-1$, and $\underline{\alpha}(p,q) = \frac{2p-1}{q-p} < \alpha \leqslant 1$. Moreover we show that in this case player $T-3$'s strategy is monotonic, and $\varphi_{T-3}(\Delta) \geqslant \varphi_{T-2}(\Delta) + \max\{\mu_\Delta, 1-\mu_\Delta\}$ for each $1-(T-3) \leqslant \Delta \leqslant (T-3)-1$ which implies that there exists a sequential equilibrium as in Proposition 2 which is monotonic *from this point backwards.*

Part (ii): We continue the construction of the equilibrium by showing that player $T-4$ may find it optimal to reverse her private information at $\Delta = 0$ (i.e. $\hat{\rho}_{T-4}(0) = (0,1)$) for generic values of p and q. Hence, a sequential equilibrium satisfying the second property of Proposition 2 exists for those values and $T=5$. This completes the proof since any singleton of the natural numbers is open and dense.

As before let $\hat{\rho}_t(\Delta_t) = (\hat{\sigma}_t(b,\Delta_t), \hat{\sigma}_t(o,\Delta_t))$.

Player T: By definition $\hat{C}_T(\Delta_{T+1}, \theta) = 0$ for each $\theta \in \Theta$ and each $\Delta_{T+1} \in \mathbb{Z}$. Hence, by Lemma A2

$$\hat{\rho}_T(\Delta_T) = \begin{cases} (1,1) & \text{if } \Delta_T \geqslant 1 \\ (1,0) & \text{if } \Delta_T \in \{-1, 0\} \\ (0,0) & \text{if } \Delta_T \leqslant -2 \end{cases}.$$

Player $T-1$: $\hat{\sigma}_T$ and Lemma A1 imply that continuation values are given by

	$\Delta_T \geqslant 1$	$\Delta_T \in \{-1, 0\}$	$\Delta_T \leqslant -2$
$\hat{C}_{T-1}(\Delta_T, \mathcal{B}) =$	1	q	0
$\hat{C}_{T-1}(\Delta_T, \mathcal{O}) =$	0	q	1

Lemma A6 implies that $\hat{\rho}_{T-1}(\Delta_{T-1}) = (1,1)$ for $\Delta_{T-1} > 1$ and $(\hat{\rho}_{T-1}(\Delta_{T-1}) = (0,0))$ for $\Delta_{T-1} < -2$. Second, if subsequent players expect player $T-1$ to follow private information

$$\hat{U}_{T-1}(B \mid s_{T-1}, \Delta_{T-1}) > \hat{U}_{T-1}(O \mid s_{T-1}, \Delta_{T-1})$$

is equivalent to

$$\frac{\hat{\mu}(s_{T-1}, \Delta_{T-1})}{1 - \hat{\mu}(s_{T-1}, \Delta_{T-1})} > \frac{1 + \alpha \left[\hat{C}_{T-1}(\Delta_{T-1} - 1, \mathcal{O}) - \hat{C}_{T-1}(\Delta_{T-1} + 1, \mathcal{O})\right]}{1 + \alpha \left[\hat{C}_{T-1}(\Delta_{T-1} + 1, \mathcal{B}) - \hat{C}_{T-1}(\Delta_{T-1} - 1, \mathcal{B})\right]}. \tag{8}$$

Hence, $\hat{\rho}_{T-1}(\Delta_{T-1}) = (1,0)$ for each $\Delta_{T-1} \in \{0, -1\}$ as

$$\frac{\hat{\mu}(o,0)}{1-\hat{\mu}(o,0)} = \frac{p}{1-p}\frac{1-q}{q} < \frac{1+\alpha q}{1+\alpha(1-q)} < \frac{p}{1-p}\frac{q}{1-q} = \frac{\hat{\mu}(b,0)}{1-\hat{\mu}(b,0)}$$

$$\text{and} \quad \frac{\hat{\mu}(o,-1)}{1-\hat{\mu}(o,-1)} = \frac{p}{1-p}\left(\frac{1-q}{q}\right)^2 < \frac{1+\alpha(1-q)}{1+\alpha q} < \frac{p}{1-p} = \frac{\hat{\mu}(b,-1)}{1-\hat{\mu}(b,-1)}.$$

Third, for $\Delta_{T-1} = 1$, $\frac{p}{1-p}\left(\frac{q}{1-q}\right)^2 > \frac{1+\alpha q}{1+\alpha(1-q)}$ implies that $\hat{\sigma}_{T-1}(b,1) = 1$, while $\hat{\sigma}_{T-1}(o,1) = 0$ can only hold if

$$\frac{p}{1-p} < \frac{1+\alpha q}{1+\alpha(1-q)} \quad \Leftrightarrow \quad \alpha > \underline{\alpha}_1(p,q) = \frac{2p-1}{q-p}.$$

In particular, $\underline{\alpha}_1(p,q) < 1$ if and only if $q > 3p - 1$. We will focus on this case henceforth. Finally, at $\Delta_{T-1} = -2$, $\hat{\sigma}_{T-1}(o,-2) = 0$ since $\frac{p}{1-p}\left(\frac{1-q}{q}\right)^3 < \frac{1+\alpha(1-q)}{1+\alpha q}$, while $\hat{\sigma}_{T-1}(b,-2) = 1$ can only hold if

$$\frac{p}{1-p}\frac{1-q}{q} > \frac{1+\alpha(1-q)}{1+\alpha q} \quad \Leftrightarrow \quad \alpha > \underline{\alpha}_2(p,q) = \frac{q-p}{q(1-q)(2p-1)}.$$

Since $q > 3p - 1$ implies $\underline{\alpha}_2(p,q) > 1$ and since in addition $\hat{C}_{T-1}(-2,\theta) = \hat{C}_{T-1}(-3,\theta)$ for each $\theta \in \Theta$ we obtain

$$\hat{\rho}_{T-1}(\Delta_{T-1}) = \begin{cases} (1,1) & \text{if } \Delta_{T-1} \geqslant 2 \\ (1,0) & \text{if } \Delta_{T-1} \in \{-1, 0, 1\} \\ (0,0) & \text{if } \Delta_{T-1} \leqslant -2 \end{cases}.$$

Player $T-2$: $\hat{\sigma}_{T-1}$ and Lemma A1 imply that

	$\Delta_{T-2} \geqslant 2$	$\Delta_{T-2} \in \{0,1\}$	$\Delta_{T-2} = -1$	$\Delta_{T-2} \leqslant -2$
$\hat{C}_{T-2}(\Delta_{T-2}, \mathcal{B}) =$	2	$q(3-q)$	$q(1+q)$	0
$\hat{C}_{T-2}(\Delta_{T-2}, \mathcal{O}) =$	0	$q(1+q)$	$q(3-q)$	2

One can directly see that $\hat{\rho}_{T-2}(1) = (1,1)$ can be sustained in equilibrium under either specification of off-path beliefs. With error off-path beliefs this is trivial since player $T-2$'s action has no influence on subsequent players' beliefs, and since $\hat{\mu}(o,1) = p > 1/2$. With signal revealing off-path beliefs it follows from $\hat{C}_{T-2}(1,\theta) = \hat{C}_{T-2}(0,\theta)$ for each $\theta \in \Theta$. The full equilibrium strategy at $t = T-2$ is given by

$$\hat{\rho}_{T-2}(\Delta_{T-2}) = \begin{cases} (1,1) & \text{if } \Delta_{T-2} \geqslant 1 \\ (1,0) & \text{if } \Delta_{T-2} \in \{-1,0,1\} \\ (0,0) & \text{if } \Delta_{T-2} \leqslant -2 \end{cases}.$$

For $\Delta_{T-2} \geqslant 3$ and $\Delta_{T-2} \leqslant -3$ this follows from Lemma A6. For $\Delta_{T-2} \in \{-2,-1,0,2\}$ it follows from equation (8) (adapted to period $T-2$), using $3p-1 < q < 1$ for $\Delta_{T-2} = -2$.

Player $T-3$: Applying Lemma A1 and $\hat{\sigma}_{T-2}$ we obtain

	$\Delta_{T-3} \geqslant 2$	$\Delta_{T-3} = 1$	$\Delta_{T-3} = 0$	$\Delta_{T-3} = -1$	$\Delta_{T-3} \leqslant -2$
$\hat{C}_{T-3}(\Delta_{T-3}, \mathcal{B}) =$	3	$1+3q-q^2$	$q(2-q)(2q+1)$	$q\left(1+3q-q^2\right)$	0
$\hat{C}_{T-3}(\Delta_{T-3}, \mathcal{O}) =$	0	$q(1+q)$	$q(2-q)(2q+1)$	$q\left(4-q^2\right)$	3

Straightforward algebraic manipulations then imply that $\varphi_{T-3}(\Delta) \geqslant \varphi_{T-2}(\Delta) + \max\{\mu_\Delta, 1-\mu_\Delta\}$ for each $4-T \leqslant \Delta \leqslant T-4$ (the difficult cases are $\Delta \in \{-1,0\}$ since $\hat{C}_{T-3}(\Delta,\mathcal{B}) = 1 + \hat{C}_{T-2}(\Delta,\mathcal{B})$ and $\hat{C}_{T-3}(\Delta,\mathcal{O}) = \hat{C}_{T-2}(\Delta,\mathcal{O})$ for each $\Delta > 0$, and $\hat{C}_{T-3}(\Delta,\mathcal{B}) = \hat{C}_{T-2}(\Delta,\mathcal{B})$ and $\hat{C}_{T-3}(\Delta,\mathcal{O}) = 1 + \hat{C}_{T-2}(\Delta,\mathcal{O})$ for each $\Delta < -1$). One may thus show that

$$\hat{\rho}_{T-3}(\Delta_{T-3}) = \begin{cases} (1,1) & \text{if } \Delta_{T-3} \geqslant 2 \\ (1,0) & \text{if } \Delta_{T-3} \in \{-1,0,1\} \\ \in \{(1,0),(0,0)\} & \text{if } \Delta_{T-3} = -2 \\ (0,0) & \text{if } \Delta_{T-3} \leqslant -3 \end{cases}$$

where for $\Delta_{T-3} \in \{0,1\}$ this follows from $1/2 < p < 2/3$ and $3p-1 < q < 1$, and $\hat{\rho}_{T-3}(-2) = (1,0)$ if and only if (additionally to $\frac{2p-1}{q-p} < \alpha \leqslant 1$) $\frac{q-p}{q(1-q)(3-q-q^2-2p(2-q)(1+q))} < \alpha \leqslant 1$. However, regardless of $\hat{\rho}_{T-3}(-2)$ it is easily seen that $\hat{\sigma}_{T-3}$ satisfies properties (i) and (ii) of the definition of monotonic equilibria. Hence, as in the proof of Proposition 2 we may construct an equilibrium inductively from this point backwards.

Player $T-4$ Let $\hat{\rho}_{T-3}(-2) = (0,0)$. Lemma A1 and $\hat{\sigma}_{T-3}$ imply

	$\Delta \geqslant 2$	$\Delta = 1$	$\Delta = 0$	$\Delta = -1$	$\Delta \leqslant -2$
$\hat{C}_{T-4}(\Delta, \mathcal{B}) =$	4	$q\left(6+q-5q^2+2q^3\right)$	$q(3-q)\left(1+2q-q^2\right)$	$q\left(1+2q+3q^2-2q^3\right)$	0
$\hat{C}_{T-4}(\Delta, \mathcal{O}) =$	0	$q\left(1+2q+3q^2-2q^3\right)$	$q\left(2+4q-q^2-q^3\right)$	$q\left(6+q-5q^2+2q^3\right)$	4

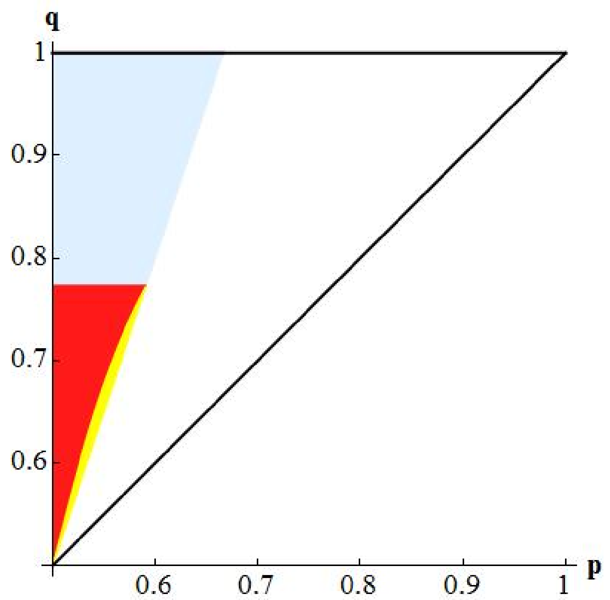
The following equilibrium strategies may thus be identified

	$\Delta \geqslant 2$	$\Delta = 1$	$\Delta = 0$	$\Delta = -1$	$\Delta = -2$	$\Delta \leqslant -3$
$\hat{\rho}_{T-4}(\Delta) =$	$(1,1)$	$\in \{(1,0),(1,1)\}$	$\in \{(1,0),(0,1)\}$	$(1,0)$	$\in \{(1,0),(0,0)\}$	$(0,0)$

Notice first, that $\hat{\rho}_{T-4}(1) = (1,0)$ is possible for each p, q, and α satisfying the assumptions we made $\left(1/2 < p < 2/3,\ 3p-1 < q < 1,\ \text{and}\ \frac{2p-1}{q-p} < \alpha \leqslant 1\right)$, while $\hat{\rho}_{T-4}(1) = (1,1)$ can only hold for a more restricted set of values. Second, $\hat{\rho}_{T-4}(-2) = (1,0)$ requires $\frac{q-p}{q(1-q)(4-5p-2q-3q^2+2q^3)} < \alpha \leqslant 1$ in addition to our assumptions. Finally, while $\hat{\rho}_{T-4}(0) = (1,0)$ is always possible under our assumptions, $\hat{\rho}_{T-4}(0) = (0,1)$ requires additionally

$$\frac{1}{5q-q^2-8q^3+4q^4} < \alpha \leqslant 1.$$

Figure 2 depicts the subset of (p,q)-values for which $\hat{\rho}_{T-4}(0) = (0,1)$ is possible.



Note: The yellow and the red area capture the set of values such that player $T-4$ reverses her private information at $\Delta = 0$. The blue area is the set of values we have assumed throughout. For values of p and q in the yellow area the condition on α is implied by $(2p-1)/(q-p) < \alpha \leqslant 1$

Figure 2: (p,q)-values for which player $T-4$ reverses her private information.

□

B.2. Sequential Equilibria of the Laboratory Cascade Game

In this appendix we provide some details about the sequential equilibria of the cascade game we played in the laboratory. The major difference between the observational learning setting considered so far and the cascade game is the presence of two sequences of players, an *observed* and an *unobserved* sequence. In each period $t = 1, \dots, 7$ one *observed* and one *unobserved* player simultaneously predict which of two options has been randomly chosen. At the end of the period only the decision of the *observed* player is publicly revealed. The predictions of the *unobserved* players remain private. There is one more *unobserved* player since in the last decision period $t = 8$ only one *unobserved* player makes a prediction. The cascade game features a binary state of nature and and binary private signals with parameters $p = 11/20$ and $q = 2/3$.

While it is clear that *unobserved* players – just like the last player in the standard game – follow the unique equilibrium strategy of the standard observational learning game, the behavior of *observed* players changes slightly due to the presence of the *unobserved* sequence. In particular given an *observed* strategy profile $\hat{\boldsymbol{\sigma}}$ continuation values (for the *observed*) are given by

$$\hat{C}_7\left(\Delta_8, \mathcal{B}\right) = \begin{cases} 1 & \text{if } \Delta_8 \geqslant 1 \\ q & \text{if } \Delta_8 \in \{-1, 0\} \\ 0 & \text{if } \Delta_8 \leqslant -2 \end{cases}, \qquad \hat{C}_7\left(\Delta_8, \mathcal{O}\right) = \begin{cases} 0 & \text{if } \Delta_8 \geqslant 1 \\ q & \text{if } \Delta_8 \in \{-1, 0\} \\ 1 & \text{if } \Delta_8 \leqslant -2 \end{cases},$$

and

$$\hat{C}_t\left(\Delta_{t+1}, \theta\right) = \sum_{x_{t+1} \in X} \Pr\left(x_{t+1} \mid \Delta_{t+1}, \theta\right) \left[\pi\left(x_{t+1}, \theta\right) + \hat{C}_{t+1}\left(\Delta_{t+2} + z\left(x_{t+1}, \Delta_{t+1}, \hat{\sigma}_{t+1}\right), \theta\right)\right] + \hat{C}_7\left(\Delta_{t+1}, \theta\right)$$

for each $t = 1, \dots, 7$ and each $\theta \in \Theta$ where $\Pr\left(x_{t+1} \mid \Delta_{t+1}, \theta\right) = \sum_{s_{t+1} \in S} \Pr\left(s_{t+1} \mid \theta\right) \hat{\sigma}_{t+1}\left(x_{t+1} \mid s_{t+1}, \Delta_{t+1}\right)$ and $z\left(x_{t+1}, \Delta_{t+1}, \hat{\sigma}_{t+1}\right) = \hat{\sigma}_{t+1}\left(x_{t+1} \mid b, \Delta_{t+1}\right) - \hat{\sigma}_{t+1}\left(x_{t+1} \mid o, \Delta_{t+1}\right)$, since the *observed* player deciding in period t must also take into account her influence on the *unobserved* player deciding in period $t + 1$.

We focus on sequential equilibria where off-path choices are interpreted signal revealing and this is commonly known. Table B1 presents sequential equilibria and equilibrium outcomes for small values of the social preference parameter α. Only the strategies of the *observed* are presented as arrays where columns capture periods, rows capture the difference between b and o signals inferred from the history, and (partial) strategies are given by B (O) if the player picks action B (O) regardless of her signal, s if the player follows private information, and $\bar{s}$ if the player *reverses* private information. Entries in curly brackets denote off-path choices. If more than two (partial) strategies are given, each can be supported in different equilibria.

As the table demonstrates even moderate social preferences may induce early players to follow private information at more histories. On the other hand more complex equilibria involving e.g. reversing of private information may also occur. Notice that due to the presence of the *unobserved* sequence, moderate altruism looms larger than in the standard game with a single sequence of (*observed*) players. Our utility function encompasses the case that players only care about the average monetary payoff of the others, by assuming that $\alpha \leqslant 1/14 \approx 0.071$.

In general the number of equilibria and equilibrium outcomes grows with α, and it becomes extraordinary large. Table B2 provides the complete partition of the parameter space ($\alpha \in [0, 1]$) together with the number of (sequential) equilibria and equilibrium outcomes. As $\alpha \to 1$ there are 11,781 distinct equilibria and 742 equilibrium outcomes.

TABLE B1
Sequential equilibria of the experimental social learning game for small values of α.

Range of α	Equilibria
$(0, 0.059)$	$\begin{pmatrix} & -3 & -2 & -1 & 0 & 1 & 2^{+} \\ 1 & & & & s & & \\ 2 & & & s & \{s\} & B & \\ 3 & & O & \{s\} & s & B & \{B\} \\ 4 & \{O\} & O & s & \{s\} & B & \{B\} \\ 5 & \{O\} & O & \{s\} & s & B & \{B\} \\ 6 & \{O\} & O & s & \{s\} & B & \{B\} \\ 7 & \{O\} & O & \{s\} & s & B & \{B\} \end{pmatrix}$
$(0.059, 0.074)$	$\begin{pmatrix} & -3 & -2 & -1 & 0 & 1 & 2 & 3^{+} \\ 1 & & & & s & & & \\ 2 & & & s & \{s\} & s & & \\ 3 & & O & \{s\} & s & \{B\} & B & \\ 4 & \{O\} & O & s & \{s\} & B & B & \{B\} \\ 5 & \{O\} & O & \{s\} & s & B & B & \{B\} \\ 6 & \{O\} & O & s & \{s\} & B & B & \{B\} \\ 7 & \{O\} & O & \{s\} & s & B & B & \{B\} \end{pmatrix}$
$(0.074, 0.099)$	$\begin{pmatrix} & -3 & -2 & -1 & 0 & 1 & 2 & 3^{+} \\ 1 & & & & s & & & \\ 2 & & & s & \{s\} & s & & \\ 3 & & O & \{s\} & s & \{s\} & B & \\ 4 & \{O\} & O & s & \{s\} & B & B & \{B\} \\ 5 & \{O\} & O & \{s\} & s & B & B & \{B\} \\ 6 & \{O\} & O & s & \{s\} & B & B & \{B\} \\ 7 & \{O\} & O & \{s\} & s & B & B & \{B\} \end{pmatrix}$ $\begin{pmatrix} & -3 & -2 & -1 & 0 & 1 & 2 & 3^{+} \\ 1 & & & & s & & & \\ 2 & & & s & \{s\} & B & & \\ 3 & & O & \{s\} & s & s & \{B\} & \\ 4 & \{O\} & O & s & s & B & B & \{B\} \\ 5 & \{O\} & O & s & s & B & B & \{B\} \\ 6 & \{O\} & O & s & s & B & B & \{B\} \\ 7 & \{O\} & O & s & s & B & B & \{B\} \end{pmatrix}$
$(0.099, 0.116)$	$\begin{pmatrix} & -3 & -2 & -1 & 0 & 1 & 2 & 3^{+} \\ 1 & & & & s & & & \\ 2 & & & s & \{s\} & s & & \\ 3 & & O & \{s\} & s & \{B, s\} & B & \\ 4 & \{O\} & O & s & \{s\} & s & B & \{B\} \\ 5 & \{O\} & O & \{s\} & s & \{B\} & B & \{B\} \\ 6 & \{O\} & O & s & \{s\} & B & B & \{B\} \\ 7 & \{O\} & O & \{s\} & s & B & B & \{B\} \end{pmatrix}$ $\begin{pmatrix} & -3 & -2 & -1 & 0 & 1 & 2 & 3^{+} \\ 1 & & & & s & & & \\ 2 & & & s & \{s\} & B & & \\ 3 & & O & \{s\} & s & B, s & \{B\} & \\ 4 & \{O\} & O & s & s & s & B & \{B\} \\ 5 & \{O\} & O & s & s & B & B & \{B\} \\ 6 & \{O\} & O & s & s & B & B & \{B\} \\ 7 & \{O\} & O & s & s & B & B & \{B\} \end{pmatrix}$

Continued on next page

Table B1 (ctd.)

Range of α	Equilibria
...	

$(0.285, 0.308)$

t	-3	-2	-1	0	1	2	3^+
1				s			
2			s	$\{\bar{s}, s\}$	s		
3		s	$\{s\}$	s	$\{s\}$	B	
4	O	$\{O\}$	s	$\{s\}$	B	B	$\{B\}$
5	O	O	$\{s\}$	s	s	B	$\{B\}$
6	O	O	s	s	s	B	$\{B\}$
7	O	O	s	s	B	B	$\{B\}$

t	-3	-2	-1	0	1	2	3^+
1				s			
2			s	$\{\bar{s}, s\}$	s		
3		s	$\{s\}$	s	$\{B, s\}$	B	
4	O	$\{O\}$	s	$\{s\}$	s	B	$\{B\}$
5	O	O	$\{s\}$	s	$\{B, s\}$	B	$\{B\}$
6	O	O	s	$\{s\}$	s	B	$\{B\}$
7	O	O	$\{s\}$	s	$\{B\}$	B	$\{B\}$

$(0.308, 0.317)$

t	-3	-2	-1	0	1	2	3^+
1				s			
2			s	$\{\bar{s}, s\}$	s		
3		s	$\{s\}$	s	$\{s\}$	B	
4	O	$\{O\}$	s	$\{s\}$	B	B	$\{B\}$
5	O	O	$\{s\}$	s	s	B	$\{B\}$
6	O	O	s	s	s	B	$\{B\}$
7	O	O	s	s	B	B	$\{B\}$

t	-3	-2	-1	0	1	2	3^+
1				s			
2			s	$\{\bar{s}, s\}$	s		
3		s	$\{s\}$	s	$\{B, s\}$	B	
4	O	$\{O\}$	s	$\{s\}$	s	B	$\{B\}$
5	O	O	$\{s\}$	s	$\{B, s\}$	B	$\{B\}$
6	O	O	s	$\{s\}$	s	B	$\{B\}$
7	O	O	$\{s\}$	s	$\{B\}$	B	$\{B\}$

$(0.317, 0.326)$

t	-3	-2	-1	0	1	2	3^+
1				s			
2			s	$\{\bar{s}, s\}$	s		
3		s	$\{s\}$	s	$\{s\}$	B	
4	O	$\{O\}$	s	$\{s\}$	B	B	$\{B\}$
5	O	O	$\{s\}$	s	s	B	$\{B\}$
6	O	O	s	s	s	B	$\{B\}$
7	O	O	s	s	B	B	$\{B\}$

t	-3	-2	-1	0	1	2	3^+
1				s			
2			s	$\{\bar{s}, s\}$	s		
3		s	$\{s\}$	s	$\{B, s\}$	B	
4	O	$\{O\}$	s	$\{s\}$	s	B	$\{B\}$
5	O	O	$\{s\}$	s	$\{B, s\}$	B	$\{B\}$
6	O	O	s	$\{s\}$	s	B	$\{B\}$
7	O	O	$\{s\}$	s	$\{B\}$	B	$\{B\}$

$(0.326, 0.347)$

t	-3	-2	-1	0	1	2	3^+
1				s			
2			s	$\{\bar{s}, s\}$	s		
3		s	$\{s\}$	s	$\{B, s\}$	B	
4	O	$\{O\}$	s	$\{s\}$	s	B	$\{B\}$
5	O	O	$\{s\}$	s	$\{B, s\}$	B	$\{B\}$
6	O	O	s	$\{s\}$	s	B	$\{B\}$
7	O	O	$\{s\}$	s	$\{B\}$	B	$\{B\}$

Notes: Equilibrium strategies of the *observed* are presented as matrices where rows denote periods t, columns denote the difference Δ between the number of b and o signals inferred from the history (columns labeled Δ^- (resp. Δ^+) denote differences "$\leqslant \Delta$" (resp. "$\geqslant \Delta$"), B (O) denotes choosing action B (O) regardless of the private signal, and s ($\bar{s}$) denotes following (reversing) private information. Entries in curly brackets are off the equilibrium path. Multiple entries in a given cell of the array summarize various equilibria which differ in the given cell. Since all situations such that $\Delta \leqslant -4$ are off-path, and all players cascade on action O in those situations, we omit the column $\Delta = -4^-$ to enhance readability.

TABLE B2:
Number of Equilibria and Equilibrium Outcomes in the Experimental Social Learning Game.

Range of α	Number of Equilibria	Number of Equilibrium Outcomes	Range of α	Number of Equilibria	Number of Equilibrium Outcomes
$(0, 0.059))$	1	1	$(0.726, 0.727)$	616	98
$(0.059, 0.074)$	1	1	$(0.727, 0.75)$	617	99
$(0.074, 0.099)$	2	2	$(0.75, 0.751)$	1,628	218
$(0.099, 0.116)$	4	3	$(0.751, 0.756)$	1,629	219
$(0.116, 0.148)$	3	2	$(0.756, 0.776)$	1,839	224
$(0.148, 0.228)$	5	4	$(0.776, 0.789)$	1,857	225
$(0.228, 0.231)$	4	4	$(0.789, 0.794)$	2,964	284
$(0.231, 0.236)$	5	5	$(0.794, 0.796)$	5,018	473
$(0.236, 0.243)$	6	6	$(0.796, 0.804)$	4,940	469
$(0.243, 0.258)$	5	5	$(0.804, 0.816)$	4,930	468
$(0.258, 0.269)$	6	6	$(0.816, 0.819)$	4,924	467
$(0.269, 0.278)$	6	3	$(0.819, 0.825)$	4,926	467
$(0.278, 0.285)$	9	3	$(0.825, 0.828)$	4,959	475
$(0.285, 0.285)$	10	3	$(0.828, 0.829)$	6,088	539
$(0.285, 0.308)$	8	2	$(0.829, 0.833)$	6,109	539
$(0.308, 0.317)$	7	2	$(0.833, 0.847)$	5,995	525
$(0.317, 0.326)$	8	2	$(0.847, 0.852)$	5,996	525
$(0.326, 0.347)$	6	1	$(0.852, 0.854)$	7,438	535
$(0.347, 0.371)$	72	25	$(0.854, 0.857)$	7,439	535
$(0.371, 0.382)$	78	26	$(0.857, 0.857)$	7,440	535
$(0.382, 0.388)$	78	35	$(0.857, 0.871)$	4,522	372
$(0.388, 0.42)$	52	28	$(0.871, 0.881)$	4,835	383
$(0.42, 0.424)$	50	27	$(0.881, 0.884)$	4,838	383
$(0.424, 0.447)$	48	26	$(0.884, 0.885)$	4,844	383
$(0.447, 0.469)$	36	19	$(0.885, 0.89)$	4,874	386
$(0.469, 0.473)$	38	20	$(0.89, 0.899)$	4,880	387
$(0.473, 0.475)$	40	21	$(0.899, 0.911)$	4,884	388
$(0.475, 0.493)$	60	22	$(0.911, 0.913)$	6,525	471
$(0.493, 0.496)$	74	23	$(0.913, 0.921)$	6,528	472
$(0.496, 0.535)$	94	24	$(0.921, 0.93)$	6,530	473
$(0.535, 0.564)$	196	38	$(0.93, 0.95)$	8,194	547
$(0.564, 0.58)$	214	45	$(0.95, 0.962)$	8,197	547
$(0.58, 0.593)$	231	46	$(0.962, 0.966)$	8,200	547
$(0.593, 0.602)$	171	36	$(0.966, 0.972)$	8,206	547
$(0.602, 0.606)$	352	65	$(0.972, 0.973)$	8,209	547
$(0.606, 0.617)$	321	63	$(0.973, 0.984)$	8,205	546
$(0.617, 0.625)$	322	64	$(0.984, 0.987)$	10,150	676
$(0.625, 0.633)$	326	65	$(0.987, 0.988)$	10,253	678
$(0.633, 0.637)$	327	65	$(0.988, 0.992)$	11,875	753
$(0.637, 0.647)$	347	66	$(0.992, 0.996)$	11,921	753
$(0.647, 0.657)$	354	66	$(0.996, 0.998)$	11,903	751
$(0.657, 0.705)$	378	67	$(0.998, 1.)$	11,781	742
$(0.705, 0.726)$	618	99			

Appendix C. Quantal Response Equilibria

In this appendix, we investigate the logit quantal response equilibrium of the altruistic observational learning game (McKelvey and Palfrey, 1995, 1998, LQRE henceforth). Section C.1 outlines the equilibrium concept. Section C.2 provides a proof for Proposition 3. Finally, Section C.3 derives detailed predictions based on numerical computations.

C.1. Logit Quantal Response Equilibrium

LQRE assumes that each player i privately observes a random payoff disturbance ϵ_i^a for each action a. In line with the bulk of the literature we consider payoff disturbances which are i.i.d. and follow an extreme-value distribution. This implies that choice probabilities are determined by a logit quantal response function, i.e. given expected utilities U_B and U_O of the two actions a player picks action B with probability

$$f_\lambda\left(U_B - U_O\right) \;=\; \frac{1}{1 \,+\, \exp\left(-\lambda\,\left(U_B \,-\, U_O\right)\right)}.$$

The parameter $\lambda > 0$ measures subjects' payoff sensitivity. Choices become completely random as $\lambda \to 0$, and approach sequentially rational choices as $\lambda \to \infty$. LQRE assumes in addition that the distribution of payoff disturbances and thus the quantal response function and the payoff sensitivity λ are commonly known.

Consider a player acting in period $t \in \{1, \ldots, T\}$ at history $\mathbf{h}_t$ given signal s_t, and let $\mu_t\left(s_t, \mathbf{h}_t\right) \in [0, 1]$ denote the belief of the player, i.e. the conditional probability she assigns to state $\mathcal{B}$. Her expected utility of action $x \in \{B, O\}$ is given by

$$U_t\left(x_t \mid s_t, \mathbf{h}_t\right) = \mu_t\left(s_t, \mathbf{h}_t\right) \cdot \left[\pi(x_t, \mathcal{B}) \,+\, \alpha \cdot C_t\left((\mathbf{h}_t, x_t), \mathcal{B}\right)\right] + \left[1 - \mu_t\left(s_t, \mathbf{h}_t\right)\right] \cdot \left[\pi(x_t, \mathcal{O}) \,+\, \alpha \cdot C_t\left((\mathbf{h}_t, x_t), \mathcal{O}\right)\right]$$

where $\pi(B, \mathcal{B}) = \pi(O, \mathcal{O}) = 1$, $\pi(B, \mathcal{O}) = \pi(O, \mathcal{B}) = 0$, and $C_t\left((\mathbf{h}_t, x_t), \theta\right)$ denotes the continuation value at history $\mathbf{h}_{t+1} = (\mathbf{h}_t, x_t)$ and state θ. The continuation value is the expected number of correct decisions in subsequent periods, i.e. decisions $x_\tau = B$ if $\theta = \mathcal{B}$ or $x_\tau = O$ if $\theta = \mathcal{O}$ for $\tau > t$. By convention, $C_T\left(\mathbf{h}_{T+1}, \theta\right) \equiv 0$. Continuation values depend on the history $\mathbf{h}_{t+1}$ solely through the public belief $\mu_{t+1}\left(\varnothing, \mathbf{h}_{t+1}\right)$ where $\mu_1(\varnothing, \mathbf{h}_1) = p$ and

$$\mu_{t+1}\left(\varnothing, \mathbf{h}_{t+1}\right) = \left[1 \,+\, \frac{1 - \mu_t\left(\varnothing, \mathbf{h}_t\right)}{\mu_t\left(\varnothing, \mathbf{h}_t\right)} \cdot \frac{(1-q)\,\sigma_t\left(x_t \mid s_t = b, \mathbf{h}_t\right) \,+\, q\,\sigma_t\left(x_t \mid s_t = o, \mathbf{h}_t\right)}{q\,\sigma_t\left(x_t \mid s_t = b, \mathbf{h}_t\right) \,+\, (1-q)\,\sigma_t\left(x_t \mid s_t = o, \mathbf{h}_t\right)}\right]^{-1}.$$

For each period t and history $\mathbf{h}_t$, equilibrium action probabilities $\sigma_t^Q(b, \mathbf{h}_t)$ and $\sigma_t^Q(o, \mathbf{h}_t)$ constitute a fixed-point in the space $[0, 1]^2$ as they are determined by expected utilities via the quantal response function and expected utilities depend upon continuation values. More generally, an LQRE $\boldsymbol{\sigma}^Q$ is a fixed-point in the space $[0, 1]^{2^T - 2}$. As the fixed-point problem may have multiple solutions, multiple QRE may exist for any given $\lambda > 0$ and $\alpha > 0$. Notice that the fixed-point problem is absent in period T, i.e. $\sigma_T^Q\left(s_T, \mathbf{h}_T\right) = f_\lambda\left(2\mu_T\left(s_T, \mathbf{h}_T\right) - 1\right)$ for each $\mathbf{h}_T \in H_T$ and each $s_T \in S$.

C.2. Proof of Proposition 3

For the sake of expositional brevity we employ the following notation. As continuation values $C_t(\mathbf{h}_{t+1}, \theta)$ depend on the history $\mathbf{h}_{t+1}$ solely through the public belief $\mu_{t+1}(\varnothing, \mathbf{h}_{t+1})$, we let $C_t(\mu_{t+1}, \theta)$ for each $t = 1, \ldots, T$ denote the continuation value at state θ given public belief μ_{t+1}. Similarly, we denote by $\sigma_t(s_t, \mu_t)$ the probability that player t picks action $x_t = B$ at public belief μ_t given signal s_t. Finally, given public belief $\mu_t = \mu_t(\varnothing, \mathbf{h}_t)$ we let $r(\mu_t, s) = \mu_t(s, \mathbf{h}_t)$ denote the Bayesian posterior belief given signal $s \in S$, and we let $r(\mu_t, x) = \mu_{t+1}(\varnothing, (\mathbf{h}_t, x))$ denote the updated public belief in period $t+1$ given action $x \in X$ where

$$\frac{r(\mu_t, x)}{1 - r(\mu_t, x)} = \frac{\mu_t}{1 - \mu_t} \frac{q\,\sigma_t(x \mid b, \mu_t) + (1-q)\,\sigma_t(x \mid o, \mu_t)}{(1-q)\,\sigma_t(x \mid b, \mu_t) + q\,\sigma_t(x \mid o, \mu_t)}.$$

The proof is organized in a series of lemmas.

Lemma C1. *For sufficiently small $\alpha > 0$ there exists a (logit) QRE satisfying for each $t = 1, \ldots, T$*

(i) $\frac{\partial C_t(\mu_{t+1}, \mathcal{B})}{\partial \mu_{t+1}} \geqslant 0$ *and* $\frac{\partial C_t(\mu_{t+1}, \mathcal{O})}{\partial \mu_{t+1}} \leqslant 0$ *for each* $\mu_{t+1} \in (0,1)$,

(ii) $\sigma_t^Q(b, \mu_t) > \sigma_t^Q(o, \mu_t)$ *for each* $\mu_t \in (0,1)$,

(iii) $\frac{\partial \sigma_t^Q(s, \mu_t)}{\partial \mu_t} > 0$ *for each* $\mu_t \in (0,1)$ *and each* $s \in S$.

Proof. The proof is by induction, starting in period T and proceeding backwards.

For period T (i) follows from $C_T(\mu_{T+1}, \theta) = 0$ for each $\mu_{T+1} = [0,1]$ and each $\theta \in \Theta$. Furthermore, $\sigma_T^Q(s_T, \mu_T) = f_\lambda(2\,\mu_T - 1)$ and $f'_\lambda(x) > 0$ jointly imply that (ii) is equivalent to $\mu_T^b > \mu_T^o$, and (iii) is equivalent to $\frac{\partial r(\mu_t, s)}{\partial \mu_t} > 0$ for each $s \in S$ which are both easily seen.

Consider period $t < T$ and assume that (i)–(iii) are true for each period $\tau > t$.

Ad. (i): Continuation values in period t satisfy

$$C_t(\mu_{t+1}, \mathcal{B}) = \Pr(x_{t+1} = B \mid \mu_{t+1}, \mathcal{B})\,[1 + C_{t+1}(r(\mu_{t+1}, B), \mathcal{B}) - C_{t+1}(r(\mu_{t+1}, O), \mathcal{B})] + C_{t+1}(r(\mu_{t+1}, O), \mathcal{B}),$$
$$C_t(\mu_{t+1}, \mathcal{O}) = \Pr(x_{t+1} = O \mid \mu_{t+1}, \mathcal{O})\,[1 + C_{t+1}(r(\mu_{t+1}, O), \mathcal{O}) - C_{t+1}(r(\mu_{t+1}, B), \mathcal{O})] + C_{t+1}(r(\mu_{t+1}, B), \mathcal{O})$$

where $\Pr(x_{t+1} = B \mid \mu_{t+1}, \theta) = 1 - \Pr(x_{t+1} = O \mid \mu_{t+1}, \theta) = \sum_{s \in S} \Pr(s \mid \theta)\, \sigma_{t+1}^Q(s, \mu_{t+1})$ for $\theta \in \Theta$. Therefore,

$$\begin{aligned}\frac{\partial C_t(\mu_{t+1}, \mathcal{B})}{\partial \mu_{t+1}} = {} & \frac{\partial \Pr(x_{t+1} = B \mid \mu_{t+1}, \mathcal{B})}{\partial \mu_{t+1}}\,[1 + C_{t+1}(r(\mu_{t+1}, B), \mathcal{B}) - C_{t+1}(r(\mu_{t+1}, O), \mathcal{B})] \\ & + \Pr(x_{t+1} = B \mid \mu_{t+1}, \mathcal{B})\, \frac{\partial C_{t+1}(r(\mu_{t+1}, B), \mathcal{B})}{\partial \mu_{t+2}} \frac{\partial r(\mu_{t+1}, B)}{\partial \mu_{t+1}} \\ & + \Pr(x_{t+1} = O \mid \mu_{t+1}, \mathcal{B})\, \frac{\partial C_{t+1}(r(\mu_{t+1}, O), \mathcal{B})}{\partial \mu_{t+2}} \frac{\partial r(\mu_{t+1}, O)}{\partial \mu_{t+1}}.\end{aligned}$$

By induction assumption, properties (i)–(iii) are true for period $t+1$. From (ii) it follows that $r(\mu_{t+1}, B) > r(\mu_{t+1}, O)$. Applying (i) implies $C_{t+1}(r(\mu_{t+1}, B), \mathcal{B}) \geqslant C_{t+1}(r(\mu_{t+1}, O), \mathcal{B})$ and $\partial C_{t+1}(\mu_{t+2}, \mathcal{B}) / \partial \mu_{t+2} \geqslant 0$ for each $\mu_{t+2} \in [0,1]$. Finally, $\frac{\partial \Pr(x_{t+1} = B \mid \mu_{t+1}, \mathcal{B})}{\partial \mu_{t+1}} = \sum_{s \in S} \Pr(s \mid \mathcal{B})\, \frac{\partial \sigma_{t+1}^Q(s, \mu_{t+1})}{\partial \mu_{t+1}} > 0$ by (iii). Hence, $\frac{\partial C_t(\mu_{t+1}, \mathcal{B})}{\partial \mu_{t+1}} \geqslant 0$.

Similarly,

$$\begin{aligned}\frac{\partial C_t(\mu_{t+1},\mathcal{O})}{\partial\mu_{t+1}} &= \frac{\partial\Pr(x_{t+1}=O\mid\mu_{t+1},\mathcal{O})}{\partial\mu_{t+1}}\left[1+C_{t+1}(r(\mu_{t+1},O),\mathcal{O})-C_{t+1}(r(\mu_{t+1},B),\mathcal{O})\right]\\ &\quad+\Pr(x_{t+1}=B\mid\mu_{t+1},\mathcal{O})\,\frac{\partial C_{t+1}(r(\mu_{t+1},B),\mathcal{O})}{\partial r(\mu_{t+1},B)}\frac{\partial r(\mu_{t+1},B)}{\partial\mu_{t+1}}\\ &\quad+\Pr(x_{t+1}=O\mid\mu_{t+1},\mathcal{O})\,\frac{\partial C_{t+1}(r(\mu_{t+1},O),\mathcal{O})}{\partial r(\mu_{t+1},O)}\frac{\partial r(\mu_{t+1},O)}{\partial\mu_{t+1}}\end{aligned}$$

is negative where $\frac{\partial\Pr(x_{t+1}=O\mid\mu_{t+1},\mathcal{O})}{\partial\mu_{t+1}}=-\frac{\partial\Pr(x_{t+1}=B\mid\mu_{t+1},\mathcal{B})}{\partial\mu_{t+1}}<0$ follows from (iii), $\partial C_{t+1}(\mu_{t+2},\mathcal{O})/\partial\mu_{t+2}\leqslant 0$ for each $\mu_{t+2}\in[0,1]$ follows from (i), and $C_{t+1}(r(\mu_{t+1},O),\mathcal{O})\geqslant C_{t+1}(r(\mu_{t+1},B),\mathcal{O})$ follows from (i) and (ii).

Ad. (ii): In equilibrium, $\sigma_t^Q(s,\mu_t)=f_\lambda(\Delta U_t(s,\mu_t))$ where

$$\begin{aligned}\Delta U_t(s,\mu_t) \;=\; & \quad r(\mu_t,s)\left\{1+\alpha\left[C_t(r(\mu_t,B),\mathcal{B})-C_t(r(\mu_t,O),\mathcal{B})\right]\right\}\\ & -\left[1-r(\mu_t,s)\right]\left\{1+\alpha\left[C_t(r(\mu_t,O),\mathcal{O})-C_t(r(\mu_t,B),\mathcal{O})\right]\right\}.\end{aligned} \tag{9}$$

Fix $\mu_t\in(0,1)$ and consider the function $f_\lambda\circ\Delta U(\cdot,\mu_t)$ mapping the set of choice probabilities $(\sigma_t(b,\mu_t),\sigma_t(o,\mu_t))$ into itself. For a given pair of choice probabilities satisfying $\sigma_t(b,\mu_t)>\sigma_t(o,\mu_t)$ it follows that $r(\mu_t,B)>r(\mu_t,O)$, and by applying (i) $C_t(r(\mu_t,B),\mathcal{B})>C_t(r(\mu_t,O),\mathcal{B})$ and $C_t(r(\mu_t,O),\mathcal{O})>C_t(r(\mu_t,B),\mathcal{O})$. Equation (9) and $r(\mu_t,b)>r(\mu_t,o)$ thus imply $\Delta U_t(b,\mu_t)>\Delta U_t(o,\mu_t)$ and therefore $f_\lambda(\Delta U_t(b,\mu_t))>f_\lambda(\Delta U_t(o,\mu_t))$. Hence, $f_\lambda\circ\Delta U(\cdot,\mu_t)$ maps the halfspace $\Psi=\left\{(x,y)\in[0,1]^2\mid y\geqslant x\right\}$ into itself. Since Ψ is compact and convex and $f_\lambda\circ\Delta U(\cdot,\mu_t)$ is continuous, there exists a fixed point $\psi^Q\in\Psi$ by Brouwer's fixed point theorem. Moreover, the fixed point lies in the interior since for a pair of choice probabilities satisfying $\sigma_t(b,\mu_t)=\sigma_t(o,\mu_t)$ it follows that $\Delta U_t(s,\mu_t)=2\mu(s,\mu_t)-1$ and therefore $f_\lambda(\Delta U_t(b,\mu_t))>f_\lambda(\Delta U_t(o,\mu_t))$ as $r(\mu_t,b)>r(\mu_t,o)$.

Ad. (iii): From $f_\lambda'(x)>0$ it follows that $\sigma_t^Q(s,\mu_t)$ is increasing in μ_t if and only if $\Delta U_t(s,\mu_t)$ is. The derivative of the latter is given by

$$\begin{aligned}\frac{\partial\Delta U_t(s,\mu_t)}{\partial\mu_t} &= \frac{\partial r(\mu_t,s)}{\partial\mu_t}\left\{1+\alpha\left[C_t(r(\mu_t,B),\mathcal{B})-C_t(r(\mu_t,O),\mathcal{B})\right]\right\}\\ &\quad+\frac{\partial r(\mu_t,s)}{\partial\mu_t}\left\{1+\alpha\left[C_t(r(\mu_t,O),\mathcal{O})-C_t(r(\mu_t,B),\mathcal{O})\right]\right\}\\ &\quad+\alpha\,r(\mu_t,s)\left\{\frac{\partial C_t(r(\mu_t,B),\mathcal{B})}{\partial r(\mu_t,B)}\frac{\partial r(\mu_t,B)}{\partial\mu_t}-\frac{\partial C_t(r(\mu_t,O),\mathcal{B})}{\partial r(\mu_t,O)}\frac{\partial r(\mu_t,O)}{\partial\mu_t}\right\}\\ &\quad-\alpha\left[1-r(\mu_t,s)\right]\left\{\frac{\partial C_t(r(\mu_t,O),\mathcal{O})}{\partial r(\mu_t,O)}\frac{\partial r(\mu_t,O)}{\partial\mu_t}-\frac{\partial C_t(r(\mu_t,B),\mathcal{O})}{\partial r(\mu_t,B)}\frac{\partial r(\mu_t,B)}{\partial\mu_t}\right\}.\end{aligned}$$

While the first two terms are clearly positive, the sign of the latter terms is not easily derived. However, each continuation value is essentially a sum of products of logit choice probabilities. It is straightforward to show that $f_\lambda'(x)\leqslant\lambda/4$. In addition, $\frac{\partial r(\mu_t,x)}{\partial\mu_t}$ is bounded for each $x\in X$. Therefore, the bracketed terms on the last two lines are bounded and (iii) holds for α not too large. □

For each $t < T$ let

$$Q_t(\mu_t) \equiv \frac{C_t(r(\mu_t, O), \mathcal{O}) \; - \; C_t(r(\mu_t, B), \mathcal{O})}{C_t(r(\mu_t, B), \mathcal{B}) \; - \; C_t(r(\mu_t, O), \mathcal{B})}. \tag{10}$$

Lemma C2. *There exists a logit QRE such that* $Q_t(\mu_t) = 1/Q_t(1-\mu_t)$ *for each* $t < T$*, and each* $\mu_t \in [0,1]$.

Proof. The proof is by induction. We note first that for each $t < T$, each $\mu_t \in [0,1]$, each $s \in S$, and $\bar{s} \neq s$

$$r(1-\mu_t, s) = 1 - r(\mu_t, \bar{s}). \tag{11}$$

In period $T-1$ continuation values satisfy

$$\begin{aligned} C_{T-1}(\mu_T, \mathcal{B}) &= q\, \sigma_T^Q(b, \mu_T) \; + \; (1-q)\, \sigma_T^Q(o, \mu_T) \\ \text{and } C_{T-1}(\mu_T, \mathcal{O}) &= (1-q) \; \left[1 - \sigma_T^Q(b, \mu_T)\right] \; + \; q \; \left[1 - \sigma_T^Q(o, \mu_T)\right] \end{aligned}$$

where $\sigma_T^Q(s, \mu_T) = f_\lambda(2\, r(\mu_T, s) - 1)$. By the symmetry of the logistic function and (11) $\sigma_T^Q(s, \mu_T) = 1 - \sigma_T^Q(\bar{s}, 1-\mu_T)$ for each $s \in S$ and each $\mu_T \in [0,1]$. Therefore,

$$C_{T-1}(\mu_T, \mathcal{O}) = (1-q) \; \left[\sigma_T^Q(o, 1-\mu_T)\right] \; + \; q \; \left[\sigma_T^Q(b, 1-\mu_T)\right] = C_{T-1}(1-\mu_T, \mathcal{B}).$$

Let $t < T$ and assume that for each $\tau \geqslant t$

$$C_\tau(\mu_{\tau+1}, \mathcal{O}) \; = \; C_\tau(1-\mu_{\tau+1}, \mathcal{B}).$$

Fix public belief $\mu_t \in [0,1]$ and consider the fixed point problem at public belief $1-\mu_t$. Let $\sigma_t^Q(s, \mu_t)$, $s \in S$, denote the equilibrium choice probabilities of action B at μ_t and let $\Delta U_t^Q(s, \mu_t)$ denote the associated expected utility differences. We show that the choice probabilities

$$\sigma_t^Q(s, 1-\mu_t) = 1 - \sigma_t^Q(\bar{s}, \mu_t) \tag{12}$$

for $s \in S$ and $\bar{s} \neq s$ solve the fixed-point problem at $1-\mu_t$. Given these choice probabilities and

$$\begin{aligned} r(\mu_t, O) &= \left\{1 + \frac{1-\mu_t}{\mu_t} \, \frac{q\,[1-\sigma_t(o,\mu_t)] \; + \; (1-q)\,[1-\sigma_t(b,\mu_t)]}{(1-q)\,[1-\sigma_t(o,\mu_t)] \; + \; q\,[1-\sigma_t(b,\mu_t)]}\right\}^{-1}, \\ 1 - r(1-\mu_t, B) &= \left\{1 + \frac{1-\mu_t}{\mu_t} \, \frac{q\,\sigma_t(b, 1-\mu_t) \; + \; (1-q)\,\sigma_t(o, 1-\mu_t)}{(1-q)\,\sigma_t(b, 1-\mu_t) \; + \; q\,\sigma_t(o, 1-\mu_t)}\right\}^{-1} \end{aligned}$$

it follows that

$$r(\mu_t, O) = 1 - r(1-\mu_t, B). \tag{13}$$

Therefore, the induction assumption and (11) jointly imply that expected utility differences at $1-\mu_t$ satisfy $\Delta U_t^Q(s, 1-\mu_t) = -\Delta U_t^Q(\bar{s}, \mu_t)$ for each $s \in S$ and $\bar{s} \neq s$. The fixed point problem at $1-\mu_t$ is thus given by

$$\sigma_t^Q(s, 1-\mu_t) = f_\lambda\left(\Delta U_t^Q(s, 1-\mu_t)\right) \quad \Leftrightarrow \quad 1 - \sigma_t^Q(\bar{s}, \mu_t) = 1 - f_\lambda\left(-\Delta U_t^Q(\bar{s}, \mu_t)\right)$$

for each s. Since the choice probabilities $\sigma_t^Q(s,\mu_t)$, $s \in S$, solve the fixed point problem at μ_t, the choice probabilities $1-\sigma_t^Q(\bar{s},\mu_t)$, $\bar{s} \in S$, must solve the fixed point problem at $1-\mu_t$. Therefore, (12) and (13) hold for each $\mu_t \in [0,1]$. Combining this with the induction assumption that $C_\tau(\mu_{\tau+1},\mathcal{O}) = C_\tau(1-\mu_{\tau+1},\mathcal{B})$ for each $\tau \geqslant t$ yields

$$\begin{aligned} Q_t(\mu_t) &= \frac{C_t(r(\mu_t,O),\mathcal{O}) - C_T(r(\mu_t,B),\mathcal{O})}{C_t(r(\mu_t,B),\mathcal{B}) - C_t(r(\mu_t,O),\mathcal{B})} = \frac{C_t(1-r(\mu_t,O),\mathcal{B}) - C_t(1-r(\mu_t,B),\mathcal{B})}{C_t(1-r(\mu_t,B),\mathcal{O}) - C_t(1-r(\mu_t,O),\mathcal{O})} \\ &= \frac{C_t(r(1-\mu_t,B),\mathcal{B}) - C_t(r(1-\mu_t,O),\mathcal{B})}{C_t(r(1-\mu_t,O),\mathcal{O}) - C_t(r(1-\mu_t,B),\mathcal{O})} = 1/Q_t(1-\mu_t) \end{aligned}$$

as desired. Finally, Lemma A1 in Appendix A implies

$$C_{t-1}(\mu_t,\mathcal{O}) = \Pr(x_t = B \mid \mu_t,\mathcal{O})\; C_t(r(\mu_t,B),\mathcal{O}) \;+\; \Pr(x_t = O \mid \mu_t,\mathcal{O})\;[1 + C_t(r(\mu_t,O),\mathcal{O})]$$

By induction assumption and (13)

$$\begin{aligned} C_t(r(\mu_t,B),\mathcal{O}) &= C_t(1-r(\mu_t,B),\mathcal{B}) = C_t(r(1-\mu_t,O),\mathcal{B}), \\ C_t(r(\mu_t,O),\mathcal{O}) &= C_t(1-r(\mu_t,O),\mathcal{B}) = C_t(r(1-\mu_t,B),\mathcal{B}). \end{aligned}$$

Moreover for each $x \in X$ and $\bar{x} \neq x$

$$\Pr(x_t = x \mid \mu_t,\mathcal{O}) \;=\; \sum_{s\in S}\Pr(s \mid \mathcal{O})\;\sigma_t^Q(s,\mu_t) \;=\; \sum_{s\in S}\Pr(\bar{s} \mid \mathcal{B})\;\left[1-\sigma_t^Q(\bar{s},1-\mu_t)\right] = \Pr(x_t = \bar{x} \mid 1-\mu_t,\mathcal{B})$$

where the second equality follows from (12) and since $\Pr(s \mid \mathcal{B}) = \Pr(\bar{s} \mid \mathcal{O})$ for each $s \in S$. It follows that

$$\begin{aligned} C_{t-1}(\mu_t,\mathcal{O}) \;&=\; \Pr(x_t = O \mid 1-\mu_t,\mathcal{B})\; C_t(r(1-\mu_t,O),\mathcal{B}) \;+\; \Pr(x_t = B \mid 1-\mu_t,\mathcal{B})\;[1 + C_t(r(1-\mu_t,B),\mathcal{B})] \\ &=\; C_{t-1}(1-\mu_t,\mathcal{B}). \end{aligned}$$

which completes the inductive step and therefore the proof. □

Lemma C3. *For each $t < T$, $Q_t(\mu_t) \;<\; (>)\, 1$ if $\mu_t \;<\; (>)\, \frac{1}{2}$.*

Proof. The proof is by induction. Consider first period $T-1$. $C_{T-1}(\mu_T,\mathcal{B}) \;=\; \Pr(x_T = B \mid \mu_T,\mathcal{B})$ and $C_{T-1}(\mu_T,\mathcal{O}) \;=\; \Pr(x_T = O \mid \mu_T,\mathcal{O})$ jointly imply that

$$Q_{T-1}(\mu_{T-1}) \;=\; \frac{(1-q)\left[\sigma_T^Q(b,r(\mu_{T-1},B)) \;-\; \sigma_T^Q(b,r(\mu_{T-1},O))\right] \;+\; q\left[\sigma_T^Q(o,r(\mu_{T-1},B)) \;-\; \sigma_T^Q(o,r(\mu_{T-1},O))\right]}{q\left[\sigma_T^Q(b,r(\mu_{T-1},B)) \;-\; \sigma_T^Q(b,r(\mu_{T-1},O))\right] \;+\; (1-q)\left[\sigma_T^Q(o,r(\mu_{T-1},B)) \;-\; \sigma_T^Q(o,r(\mu_{T-1},O))\right]}$$

Let $\Delta\sigma_T(s,\mu) \equiv \sigma_T(s,r(\mu,B)) - \sigma_T(s,r(\mu,O))$ for $s \in S$. Then $Q_{T-1}(\mu_{T-1}) \;<\; (>)\, 1$ if and only if $\Delta\sigma_T(b,\mu_{T-1}) \;>\; (<)\, \Delta\sigma_T(o,\mu_{T-1})$. Using

$$\begin{aligned} r(r(\mu_{T-1},B),b) - r(r(\mu_{T-1},O),b) &= \frac{q(1-q)\;(r(\mu_{T-1},B) - r(\mu_{T-1},O))}{[q\,r(\mu_{T-1},B) + (1-q)(1-r(\mu_{T-1},B))]\;[q\,r(\mu_{T-1},O) + (1-q)(1-r(\mu_{T-1},O))]}, \\ r(r(\mu_{T-1},B),o) - r(r(\mu_{T-1},O),o) &= \frac{q(1-q)\;(r(\mu_{T-1},B) - r(\mu_{T-1},O))}{[(1-q)\,r(\mu_{T-1},B) + q\;(1-r(\mu_{T-1},B))]\;[(1-q)\,r(\mu_{T-1},O) + q\;(1-r(\mu_{T-1},O))]} \end{aligned}$$

it follows that $r(r(\mu_{T-1},B),b) - r(r(\mu_{T-1},O),b) \;>\; (<)\, r(r(\mu_{T-1},B),o) - r(r(\mu_{T-1},O),o)$ iff $1 - r(\mu_{T-1},O) \;>\; (<)\, r(\mu_{T-1},B)$, or by symmetry iff $\mu_{T-1} \;<\; (>)\, \frac{1}{2}$. Since the logit function $f_\lambda(x)$ is convex for $0 \leqslant x < \frac{1}{2}$ and concave for $\frac{1}{2} < x \leqslant 1$ it follows that $\Delta\sigma_T(b, \mu_{T-1}) > \Delta\sigma_T(o, \mu_{T-1})$ if $r(r(\mu_{T-1}, B), b) < \frac{1}{2}$, and $\Delta\sigma_T(b, \mu_{T-1}) < \Delta\sigma_T(o, \mu_{T-1})$ if $r(r(\mu_{T-1}, O), o) > \frac{1}{2}$. In addition, symmetry implies that $\Delta\sigma_T\left(b, \frac{1}{2}\right) = \Delta\sigma_T\left(o, \frac{1}{2}\right)$. However, decreasing (increasing) μ_{T-1} below (above) $\frac{1}{2}$ increases (decreases) $\Delta\sigma_T(b, \mu_{T-1})$ and decreases (increases) $\Delta\sigma_T(o, \mu_{T-1})$. Accordingly, $\Delta\sigma_T(b, \mu_{T-1}) > (<) \Delta\sigma_T(o, \mu_{T-1})$ if $\mu_{T-1} < (>) \frac{1}{2}$.

Assume that the property holds for each $\tau > t$. Because of symmetry we only prove that $Q_t(\mu_t) < \frac{1}{2}$ if $\mu_t < \frac{1}{2}$. Define $\Delta C_t(\mu_t, \mathcal{B}) = C_t(r(\mu_t, B), \mathcal{B}) - C_t(r(\mu_t, O), \mathcal{B})$ and $\Delta C_t(\mu_t, \mathcal{O}) = C_t(r(\mu_t, O), \mathcal{O}) - C_t(r(\mu_t, B), \mathcal{O})$. By Lemma A1 in Appendix A differences in continuation values in period t may be decomposed as follows:

$$\begin{aligned}
\Delta C_t(\mu_t, \mathcal{O}) = {} & \Pr(x_{t+1} = B \mid r(\mu_t, B), \mathcal{O})\ \Delta C_{t+1}(r(\mu_t, B), \mathcal{O}) \\
& + \Pr(x_{t+1} = O \mid r(\mu_t, O), \mathcal{O})\ \Delta C_{t+1}(r(\mu_t, O), \mathcal{O}) \\
& + \Pr(x_{t+1} = O \mid r(\mu_t, O), \mathcal{O}) - \Pr(x_{t+1} = O \mid r(\mu_t, B), \mathcal{O}) \\
& + C_{t+1}(r(r(\mu_t, O), B), \mathcal{O}) - C_{t+1}(r(r(\mu_t, B), O), \mathcal{O}),
\end{aligned}$$

and

$$\begin{aligned}
\Delta C_t(\mu_t, \mathcal{B}) = {} & \Pr(x_{t+1} = B \mid r(\mu_t, B), \mathcal{B})\ \Delta C_{t+1}(r(\mu_t, B), \mathcal{B}) \\
& + \Pr(x_{t+1} = O \mid r(\mu_t, O), \mathcal{B})\ \Delta C_{t+1}(r(\mu_t, O), \mathcal{B}) \\
& + \Pr(x_{t+1} = B \mid r(\mu_t, B), \mathcal{B}) - \Pr(x_{t+1} = B \mid r(\mu_t, O), \mathcal{B}) \\
& + C_{t+1}(r(r(\mu_t, B), O), \mathcal{B}) - C_{t+1}(r(r(\mu_t, O), B), \mathcal{B}).
\end{aligned}$$

Consider the case $r(\mu_t, B) < \frac{1}{2}$. First, by induction assumption

$$\Delta C_{t+1}(r(\mu_t, B), \mathcal{O}) < \Delta C_{t+1}(r(\mu_t, B), \mathcal{B}) \text{ and } \Delta C_{t+1}(r(\mu_t, O), \mathcal{O}) < \Delta C_{t+1}(r(\mu_t, O), \mathcal{B}).$$

Second, $\frac{\Pr(x_{t+1}=B|r(\mu_t,B),\mathcal{O})}{\Pr(x_{t+1}=B|r(\mu_t,B),\mathcal{B})} \in \left[\frac{1-q}{q}, 1\right]$. Third, $\frac{\Pr(x_{t+1}=O|r(\mu_t,O),\mathcal{O})}{\Pr(x_{t+1}=O|r(\mu_t,O),\mathcal{B})} \approx 1$ since $r(\mu_t, O)$ is small. Fourth,

$$\begin{aligned}
& \Pr(x_{t+1} = O \mid r(\mu_t, O), \mathcal{O}) - \Pr(x_{t+1} = O \mid r(\mu_t, B), \mathcal{O}) \\
& \quad = (1-q)\ [\sigma_{t+1}(b, r(\mu_t, B)) - \sigma_{t+1}(b, r(\mu_t, O))] + q\ [\sigma_{t+1}(o, r(\mu_t, B)) - \sigma_{t+1}(o, r(\mu_t, O))]
\end{aligned}$$

and

$$\begin{aligned}
& \Pr(x_{t+1} = B \mid r(\mu_t, B), \mathcal{B}) - \Pr(x_{t+1} = B \mid r(\mu_t, O), \mathcal{B}) \\
& \quad = q\ [\sigma_{t+1}(b, r(\mu_t, B)) - \sigma_{t+1}(b, r(\mu_t, O))] + (1-q)\ [\sigma_{t+1}(o, r(\mu_t, B)) - \sigma_{t+1}(o, r(\mu_t, O))].
\end{aligned}$$

Similar arguments as employed for $t = T-1$ therefore imply that

$$\begin{aligned}
& \Pr(x_{t+1} = O \mid r(\mu_t, O), \mathcal{O}) - \Pr(x_{t+1} = O \mid r(\mu_t, B), \mathcal{O}) \\
& \qquad < \Pr(x_{t+1} = B \mid r(\mu_t, B), \mathcal{B}) - \Pr(x_{t+1} = B \mid r(\mu_t, O), \mathcal{B}).
\end{aligned}$$

For the last term there are two possibilities: If $r(r(\mu_t, O), B) \approx r(r(\mu_t, B), O)$ or

$$C_{t+1}(r(r(\mu_t, O), B), \mathcal{O}) - C_{t+1}(r(r(\mu_t, B), O), \mathcal{O}) < C_{t+1}(r(r(\mu_t, B), O), \mathcal{B}) - C_{t+1}(r(r(\mu_t, O), B), \mathcal{B})$$

it follows directly that $Q_t(\mu_t) < 1$ whenever $r(\mu_t, B) < \frac{1}{2}$.

Assume therefore that

$$C_{t+1}\left(r\left(r\left(\mu_t,O\right),B\right),\mathcal{O}\right)-C_{t+1}\left(r\left(r\left(\mu_t,B\right),O\right),\mathcal{O}\right)>C_{t+1}\left(r\left(r\left(\mu_t,B\right),O\right),\mathcal{B}\right)-C_{t+1}\left(r\left(r\left(\mu_t,O\right),B\right),\mathcal{B}\right). \tag{14}$$

Using the fact that $\Pr\left(O \mid r\left(\mu_t,O\right),\theta\right)+\Pr\left(B \mid r\left(\mu_t,B\right),\theta\right)\geqslant 1$ for each $\theta\in\Theta$ differences in continuation values may be rewritten as

$$\begin{aligned}\Delta C_t\left(\mu_t,\mathcal{O}\right) = {} & \Pr\left(x_{t+1}=B \mid r\left(\mu_t,B\right),\mathcal{O}\right)\left[C_{t+1}\left(r\left(r\left(\mu_t,O\right),B\right),\mathcal{O}\right)-C_{t+1}\left(r\left(r\left(\mu_t,B\right),B\right),\mathcal{O}\right)\right]\\ & +\Pr\left(x_{t+1}=O \mid r\left(\mu_t,O\right),\mathcal{O}\right)\left[C_{t+1}\left(r\left(r\left(\mu_t,O\right),O\right),\mathcal{O}\right)-C_{t+1}\left(r\left(r\left(\mu_t,B\right),O\right),\mathcal{O}\right)\right]\\ & +\Pr\left(x_{t+1}=O \mid r\left(\mu_t,O\right),\mathcal{O}\right)-\Pr\left(x_{t+1}=O \mid r\left(\mu_t,B\right),\mathcal{O}\right)\\ & +\left[\Pr\left(x_{t+1}=B \mid r\left(\mu_t,B\right),\mathcal{O}\right)+\Pr\left(x_{t+1}=O \mid r\left(\mu_t,O\right),\mathcal{O}\right)-1\right]\\ & \quad *\left[C_{t+1}\left(r\left(r\left(\mu_t,B\right),O\right),\mathcal{O}\right)-C_{t+1}\left(r\left(r\left(\mu_t,O\right),B\right),\mathcal{O}\right)\right],\end{aligned}$$

and

$$\begin{aligned}\Delta C_t\left(\mu_t,\mathcal{B}\right) = {} & \Pr\left(x_{t+1}=B \mid r\left(\mu_t,B\right),\mathcal{B}\right)\left[C_{t+1}\left(r\left(r\left(\mu_t,B\right),B\right),\mathcal{B}\right)-C_{t+1}\left(r\left(r\left(\mu_t,O\right),B\right),\mathcal{B}\right)\right]\\ & +\Pr\left(x_{t+1}=O \mid r\left(\mu_t,O\right),\mathcal{B}\right)\left[C_{t+1}\left(r\left(r\left(\mu_t,B\right),O\right),\mathcal{B}\right)-C_{t+1}\left(r\left(r\left(\mu_t,O\right),O\right),\mathcal{B}\right)\right]\\ & +\Pr\left(x_{t+1}=B \mid r\left(\mu_t,B\right),\mathcal{B}\right)-\Pr\left(x_{t+1}=B \mid r\left(\mu_t,O\right),\mathcal{B}\right)\\ & +\left[\Pr\left(x_{t+1}=B \mid r\left(\mu_t,B\right),\mathcal{B}\right)+\Pr\left(x_{t+1}=O \mid r\left(\mu_t,O\right),\mathcal{B}\right)-1\right]\\ & \quad *\left[C_{t+1}\left(r\left(r\left(\mu_t,O\right),B\right),\mathcal{O}\right)-C_{t+1}\left(r\left(r\left(\mu_t,B\right),O\right),\mathcal{O}\right)\right],\end{aligned}$$

respectively. Consider the function

$$\hat{Q}_t\left(\mu^H,\mu^L\right)=\frac{C_t\left(\mu^L,\mathcal{O}\right)-C_t\left(\mu^H,\mathcal{O}\right)}{C_t\left(\mu^H,\mathcal{B}\right)-C_t\left(\mu^L,\mathcal{B}\right)}$$

defined for each $t<T$ and for beliefs $0\leqslant\mu^L<\mu^H\leqslant 1$. As $\hat{Q}_t$ is a generalization of Q_t for each $t<T$, it may be shown inductively using the arguments put forward in this proof that $\hat{Q}_t\left(\mu^H,\mu^L\right)<1$ if $\mu^H<\frac{1}{2}$, and $\hat{Q}_t\left(\mu^H,\mu^L\right)>1$ if $\mu^L>\frac{1}{2}$. Hence, for μ_t sufficiently small

$$\begin{aligned}C_{t+1}\left(r\left(r\left(\mu_t,O\right),B\right),\mathcal{O}\right)-C_{t+1}\left(r\left(r\left(\mu_t,B\right),B\right),\mathcal{O}\right)<C_{t+1}\left(r\left(r\left(\mu_t,B\right),B\right),\mathcal{B}\right)-C_{t+1}\left(r\left(r\left(\mu_t,O\right),B\right),\mathcal{B}\right),\\ C_{t+1}\left(r\left(r\left(\mu_t,O\right),O\right),\mathcal{O}\right)-C_{t+1}\left(r\left(r\left(\mu_t,B\right),O\right),\mathcal{O}\right)<C_{t+1}\left(r\left(r\left(\mu_t,B\right),O\right),\mathcal{B}\right)-C_{t+1}\left(r\left(r\left(\mu_t,O\right),O\right),\mathcal{B}\right).\end{aligned}$$

Moreover,

$$\begin{aligned}&\Pr\left(x_{t+1}=B \mid r\left(\mu_t,B\right),\mathcal{O}\right)+\Pr\left(x_{t+1}=O \mid r\left(\mu_t,O\right),\mathcal{O}\right)\\ &\qquad<\Pr\left(x_{t+1}=B \mid r\left(\mu_t,B\right),\mathcal{B}\right)+\Pr\left(x_{t+1}=O \mid r\left(\mu_t,O\right),\mathcal{B}\right).\end{aligned}$$

Jointly with assumption (14) this implies $Q_t\left(\mu_t\right)<1$ if $r\left(\mu_t,B\right)<\frac{1}{2}$.

As μ_t increases towards $\frac{1}{2}$ (and thus $r\left(\mu_t,B\right)$ grows larger than $\frac{1}{2}$), $Q_{t+1}\left(r\left(\mu_t,B\right)\right)$ becomes larger than one and $\frac{\Pr(x_{t+1}=B|r(\mu_t,B),\mathcal{O})}{\Pr(x_{t+1}=B|r(\mu_t,B),\mathcal{B})}$ increases towards one. In addition, $\frac{\Pr(x_{t+1}=O|r(\mu_t,O),\mathcal{O})}{\Pr(x_{t+1}=O|r(\mu_t,O),\mathcal{B})}$ and $\frac{\Pr(x_{t+1}=O|r(\mu_t,O),\mathcal{O})-\Pr(x_{t+1}=O|r(\mu_t,B),\mathcal{O})}{\Pr(x_{t+1}=B|r(\mu_t,B),\mathcal{B})-\Pr(x_{t+1}=B|r(\mu_t,O),\mathcal{B})}$ increase. On the other hand $Q_t\left(r\left(\mu_t,O\right)\right)<1$ since $r\left(\mu_t,O\right)\leqslant\mu_t<\frac{1}{2}$ and $r\left(r\left(\mu_t,O\right),B\right)\to r\left(r\left(\mu_t,B\right),O\right)$. Hence, there exists $\hat{\mu}_t$ such that $Q_t\left(\mu_t\right)>1$ for $\mu_t>\hat{\mu}_t$. It follows from Lemma C2 that $\hat{\mu}_t=\frac{1}{2}$. □

Remainder of the Proof

First, Lemma C1 shows that unless for $\alpha > 0$ not too large there exists a *monotonic-within-periods* LQRE in which continuation values under state $\mathcal{B}$ ($\mathcal{O}$) are increasing (decreasing) in the public belief.

Second, it follows from (9) that altruism increases the equilibrium probability to select action B (O) in period $t < T$ at public belief μ_t if and only if

$$Q_t(\mu_t) \;<\; (>)\, \frac{\mu_t(s, \mathbf{h}_t)}{1 - \mu_t(s, \mathbf{h}_t)}.$$

Lemmas C2 and C3 establish that for any $t < T$ $Q_t(\cdot)$ is smaller (larger) than 1 if $\mu_t(\varnothing, \mathbf{h}_t) < (>) \frac{1}{2}$. On the other hand $\frac{r(\mu_t,b)}{1-r(\mu_t,b)} = \frac{q}{1-q}\frac{\mu_t}{1-\mu_t} > 1$ if $\mu_t > 1-q$ and $\frac{\mu_t^o}{1-\mu_t^o} = \frac{1-q}{q}\frac{\mu_t}{1-\mu_t} < 1$ if $\mu_t < q$. This completes the proof of the proposition.

□

Corollary. $\underline{\mu} \leqslant \frac{(1-q)^2}{q^2+(1-q)^2}$ *and* $\overline{\mu} \geqslant \frac{q^2}{q^2+(1-q)^2}$ *for* λ *sufficiently large.*

Proof. We first show by induction in t that for each $t < T$ $Q_t(\mu_t) \leqslant \frac{1-q}{q}$ for $\frac{(1-q)^2}{q^2+(1-q)^2} < \mu_t < 1-q$ and $Q_t(\mu_t) \geqslant \frac{q}{1-q}$ for $\frac{1}{2} < \mu_t < \frac{q^2}{q^2+(1-q)^2}$.

For period $T-1$, $\frac{(1-q)^2}{q^2+(1-q)^2} < \mu_{T-1} < 1-q$ implies that for each $\epsilon > 0$

$$r(r(\mu_{T-1}, O), o) < r(r(\mu_{T-1}, B), o) \leqslant r(r(\mu_{T-1}, O), b) < \frac{1}{2} - \epsilon \leqslant r(r(\mu_{T-1}, B), b)$$

if λ is sufficiently large. Therefore for sufficiently large λ, $\sigma_T(o, r(\mu_{T-1}, O)) < \sigma_T(o, r(\mu_{T-1}, B)) < \sigma_T(b, r(\mu_{T-1}, O)) < \epsilon$ whereas $\sigma_T(b, r(\mu_{T-1}, B)) > \frac{1}{2} - \epsilon$ which implies that $Q_{T-1}(\mu_{T-1}) \approx \frac{1-q}{q}$. Because of symmetry $q < \mu_{T-1} < \frac{q^2}{q^2+(1-q)^2}$ implies $Q_{T-1}(\mu_{T-1}) \approx \frac{q}{1-q}$ by similar arguments.

Let $t < T-1$ and assume that the property holds for each $\tau > t$. Consider a public belief $\frac{(1-q)^2}{q^2+(1-q)^2} < \mu_t < 1-q$. For sufficiently large $\lambda > 0$ it holds

(a) $\frac{\Delta C_{t+1}(r(\mu_t,B),\mathcal{O})}{\Delta C_{t+1}(r(\mu_t,B),\mathcal{B})} \approx \frac{1-q}{q}$ since $r(\mu_t, B) < \frac{1}{2}$, and $\frac{\Pr(x_{t+1}=B|r(\mu_t,B),\mathcal{O})}{\Pr(x_{t+1}=B|r(\mu_t,B),\mathcal{B})} < 1$[2];

(b) $\frac{\Delta C_{t+1}(r(\mu_t,O),\mathcal{O})}{\Delta C_{t+1}(r(\mu_t,O),\mathcal{B})} \leqslant \frac{1-q}{q}$, and $\frac{\Pr(x_{t+1}=O|r(\mu_t,O),\mathcal{O})}{\Pr(x_{t+1}=O|r(\mu_t,O),\mathcal{B})} \approx 1$;

(c) $\frac{\Pr(x_{t+1}=O|r(\mu_t,O),\mathcal{O}) - \Pr(x_{t+1}=O|r(\mu_t,B),\mathcal{O})}{\Pr(x_{t+1}=B|r(\mu_t,B),\mathcal{B}) - \Pr(x_{t+1}=B|r(\mu_t,O),\mathcal{B})} \approx \frac{1-q}{q}$.

In addition, either $r(r(\mu_{T-1}, B), O) \approx r(r(\mu_{T-1}, O), B)$ (if $\Pr(x_t = O \mid \mu_t, \theta) \approx \Pr(s = o \mid \theta)$ for each $\theta \in \Theta$) which implies the desired property by (a)–(c), or $r(r(\mu_{T-1}, B), O) \approx r(r(\mu_{T-1}, O), O)$ (if $\Pr(x_t = O \mid \mu_t, \theta) \approx 1$ for each $\theta \in \Theta$) which implies that

$$Q_t(\mu_t) \approx \frac{\Pr(x_{t+1} = O \mid r(\mu_t, O), \mathcal{O}) - \Pr(x_{t+1} = O \mid r(\mu_t, B), \mathcal{O}) + \Pr(x_{t+1} = B \mid r(\mu_t, B), \mathcal{O})\, \Delta C_{t+1}(r(\mu_t, B), \mathcal{O})}{\Pr(x_{t+1} = B \mid r(\mu_t, B), \mathcal{B}) - \Pr(x_{t+1} = B \mid r(\mu_t, O), \mathcal{B}) + \Pr(x_{t+1} = B \mid r(\mu_t, B), \mathcal{B})\, \Delta C_{t+1}(r(\mu_t, B), \mathcal{B})}$$

and the property follows by (a) and (c). The case $q < \mu_t < \frac{q^2}{q^2+(1-q)^2}$ follows from similar arguments.

The result follows by noting that $\frac{r(\mu_t,b)}{1-r(\mu_t,b)} = \frac{q}{1-q} \geqslant \frac{1-q}{q}$ if $\frac{\mu_t}{1-\mu_t} \geqslant \left(\frac{1-q}{q}\right)^2$ and $\frac{\mu_t^o}{1-\mu_t^o} = \frac{1-q}{q} \leqslant \frac{q}{1-q}$ if $\frac{\mu_t}{1-\mu_t} \leqslant \left(\frac{q}{1-q}\right)^2$. □

[2] In fact, $\frac{\Pr\left(x_{t+1}=B|r(\mu_t,B),\mathcal{O}\right)}{\Pr\left(x_{t+1}=B|r(\mu_t,B),\mathcal{B}\right)} \approx \frac{1-q}{q}$ if $\mu_t < 1-q-\delta$ for some small $\delta > 0$.

C.3. LQRE Predictions in the Laboratory Cascade Game

To further characterize the logit QRE for $\alpha > 0$ we calculate the QRE numerically for the laboratory cascade game for $\lambda \in \{2.5, 5, 7.5\}$ and $\alpha \in \{0, 0.125, 0.25, 0.5\}$.[3] Recall that the cascade game is played by two parallel sequences of players, *observed* and *unobserved*. Since guesses made by *unobserved* never reveal any information to others *unobserved* follow the standard logit QRE strategy profile $\boldsymbol{\sigma}^{Q_0}$ for $\alpha = 0$, i.e. $\sigma_t^{Q_0}(s_t, \mathbf{h}_t) = f_\lambda(2\,\mu_t(s_t, \mathbf{h}_t) - 1)$ for each t, s_t, and $\mathbf{h}_t$. On the other hand *observed* take into account the presence of *unobserved*. Continuation values (for *observed*) are thus given by

$$C_T(\mu_T, \theta) = \sum_{s \in S} \Pr(s \mid \theta)\; f_\lambda\left(2\,\mu_{T+1}^s - 1\right)$$

$$\text{and } C_{t-1}(\mu_t, \theta) = \sum_{x \in X} \Pr_t^Q(x_t = x \mid \mu_t, \theta)\,\left[\pi(x, \theta) + \hat{C}_t(r(\mu_t, x), \theta)\right] + \sum_{s \in S} \Pr(s \mid \theta)\; f_\lambda(2\,r(\mu_t, s) - 1)$$

where $T = 7$, $t = 2, \ldots, T$, $\theta \in \Theta$, and $\Pr_t^Q(x_t = x \mid \mu_t, \theta) = \sum_{s \in S} \Pr(s \mid \theta)\, \sigma_t(s, \mu_t)$. We focus on *monotonic* QRE in which (i) for each $t = 1, \ldots, T$, σ_t is increasing in the public belief, and (ii) for each $t < T$ and each public belief $\mu_t \in [0, 1]$, $\sigma_t(b, \mu_t) \geqslant \sigma_{t+1}(b, \mu_t)$ and $\sigma_t(o, \mu_t) \leqslant \sigma_{t+1}(o, \mu_t)$.

Our numeric computations rely on the absence of the fixed-point problem for the strategies of the *unobserved*. We first employ an algorithm to determine choice probabilities $\sigma_t(s_t, \mu_t)$ for each $t = 1, \ldots, T$, each $s_t \in S$, and each $\mu_t \in [0, 1]$. The algorithm repeats the following two steps for $t = T, \ldots, 1$:

Step 1: For each $\mu_{t+1} \in [0, 1]$ and each $\theta \in \Theta$ the continuation values $C_t(\mu_{t+1}, \theta)$ are calculated from the choice probabilities $\sigma_{t+1}(s_{t+1}, \mu_{t+1})$, $s_{t+1} \in S$ where σ_{T+1} refers to the strategy of the *unobserved* acting in period $T + 1$;

Step 2: For each $\mu_t \in [0, 1]$ the vector of choice probabilities $(\sigma_t(b, \mu_t), \sigma_t(o, \mu_t))$ is determined by solving the fixed-point problem given the continuation values $C_t(\mu_{t+1}, \theta)$.

Second, we calculate the QRE by calculating from period 1 on the public belief for each history, selecting the corresponding action probabilities, and using the latter for the calculation of public beliefs in the subsequent period.

In our numerical algorithm we discretize the space of public beliefs by calculating action probabilities and continuation values for each public belief in the set $G = \{0.005, 0.010, \ldots, 0.995\}$. Furthermore, for each period and each public belief choice probabilities are calculated by conducting a grid search in the space G^2: For each vector $(P_b, P_o) \in G^2$ we compute the updated public belief assuming that P_s, $s \in S$, are the choice probabilities, we use the continuation values at the closest grid point in G to calculate expected utilities, and we calculate choice probabilities $\hat{P}_s$, $s \in S$ based on these expected utilities using the logit quantal response function. Accordingly, we map each vector $(P_b, P_o) \in G^2$ onto $[0, 1]^2$. The fixed-point of this mapping is then given by $(P_b^*, P_o^*) = \arg\min_{(P_b, P_o) \in G^2} \max_{s \in S} \left|P_s - \hat{P}_s\right|$. For all results reported below $\left|P_s^* - \hat{P}_s^*\right| < 0.005$ for each s.

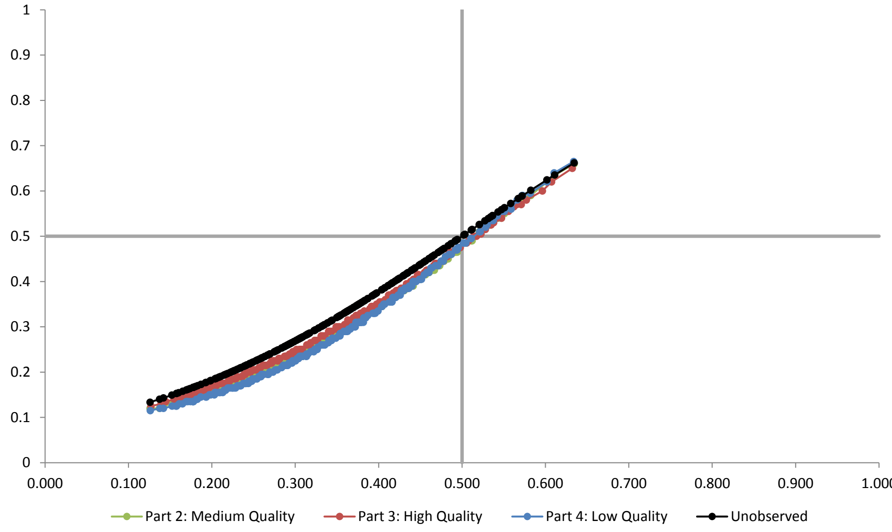
Our numerical results clearly establish that altruism affects behavior even if players make noisy decisions. Yet, altruism does not always have a sufficient impact, as demonstrated by the results for $\lambda = 2.5$. In the following we focus in our discussion on values of λ for which the impact of altruism is non-negligible.

[3] The values of λ have been selected to match estimated values commonly found in the literature. For instance, Goeree, Palfrey, Rogers, and McKelvey (2007) report an estimate of $\lambda = 6.12$ when treatments are pooled, and a value of $\lambda = 6.36$ for the data from Anderson and Holt (1997). Results for $\lambda = 10$ are available from the authors upon request.

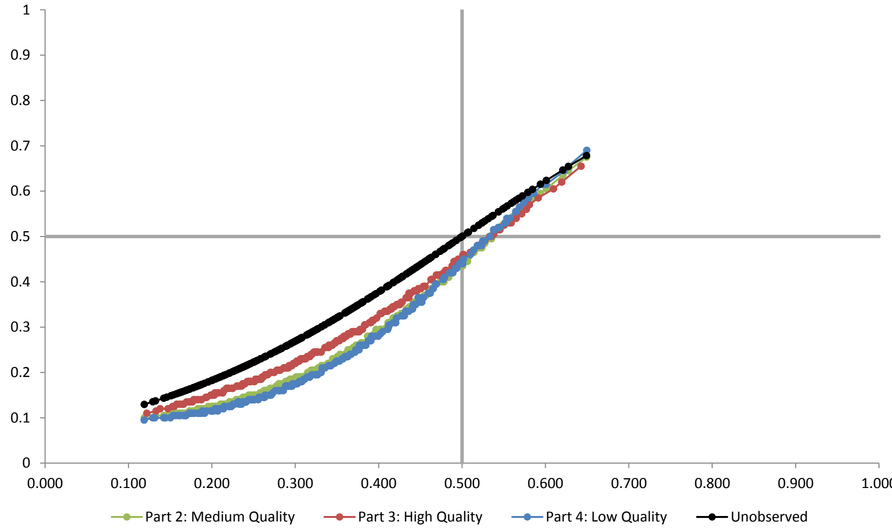
C.3.1 Response to the Expected Payoff of Contradicting Private Information

Figures 3, 4, and 5 plot the predicted probability to contradict private information (*prob_contradict*) against the expected payoff from contradicting (*value_contra_PI*) for $\lambda = 2.5$, $\lambda = 5$, and $\lambda = 7.5$, respectively. In each of the figures the top (middle and bottom) panel presents the results for $\alpha = 0.125$ ($\alpha = 0.25$ and $\alpha = 0.5$). Each marker in the scatterplots reflects one distinct decision situation in the laboratory cascade game with x-value the expected payoff from contradicting the private signal and y-value the predicted probability to contradict. Blue (red and green) markers indicate decision situations in part 2 (part 3 and part 4) of the experiment where *unobserved* observe signals of medium (high and low) quality. For comparison, black markers indicate the probability to contradict the private signal for the *unobserved* (or selfish players). For the sake of clarity, we superimpose lines on each set of markers although clearly some values of *value_contra_PI* are not reached.

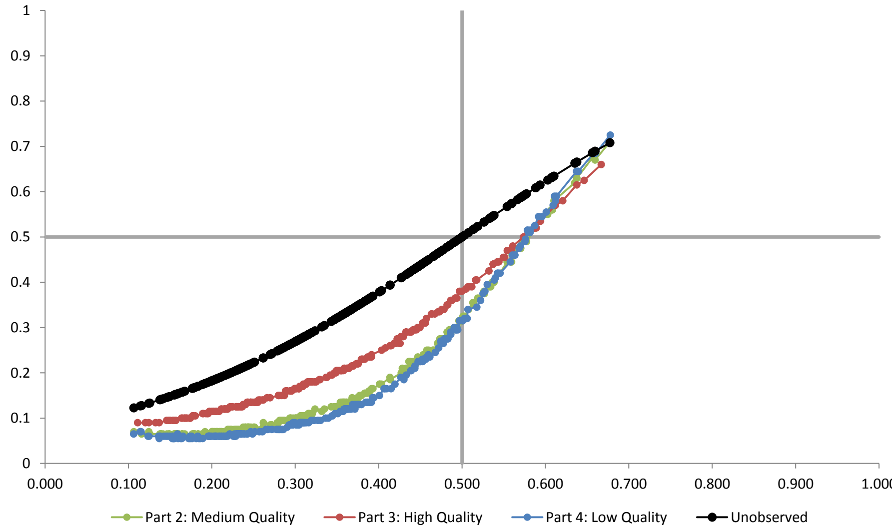
The results can be summarized as follows: First, altruism distorts incentives towards following private information as markers for the *observed* lie systematically below markers for the *unobserved*. Second, the distortion is strongest when incentives to follow others are moderate. In particular, altruism strongly reduces the probability to contradict private information for *value_contra_PI* around 0.5. Furthermore, the value of *value_contra_PI* such that *observed* follow others with probability larger than one half is strictly larger than 0.5. In contrast, differences between *observed* and *unobserved* are small when incentives to follow others are low or high. Fourth, the effects are larger, the larger are α and λ. Finally, altruism has a smaller effect on behavior of the *observed* when *unobserved* receive private signals of high signal quality, and a slightly larger effect when *unobserved* receive signals of low signal quality.



(a) $\alpha = 0.125$

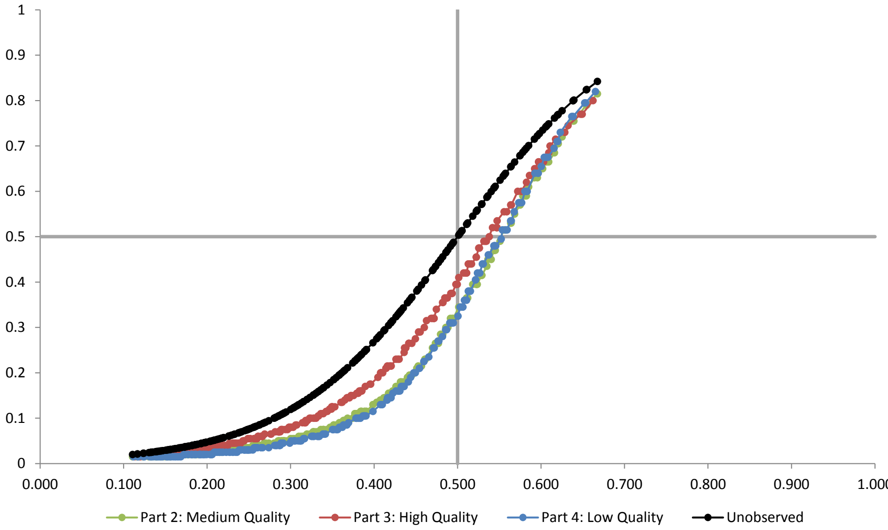


(b) $\alpha = 0.25$

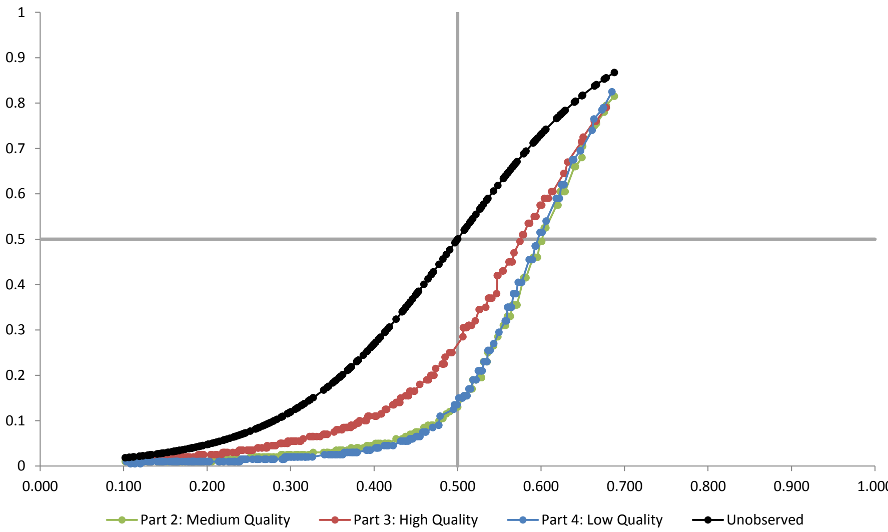


(c) $\alpha = 0.5$

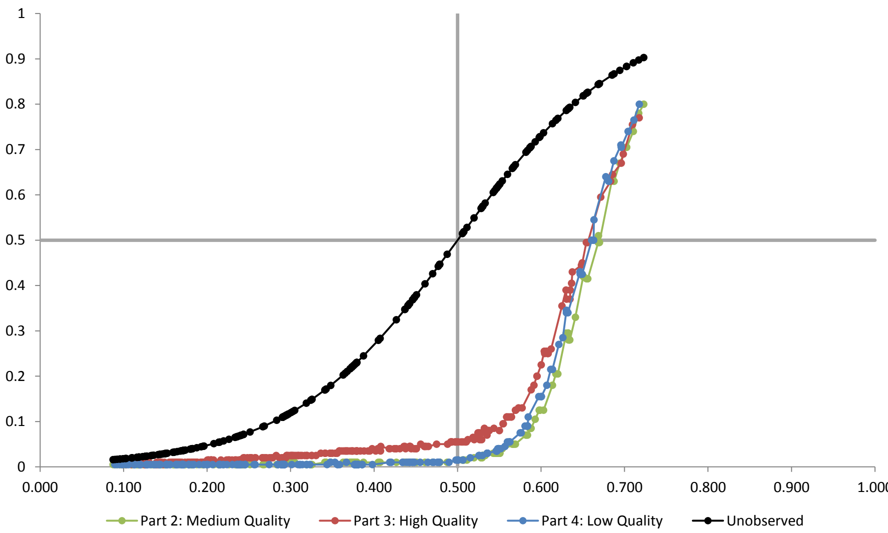
Figure 3: Expected Payoff and Probability to Contradict Private Information for $\lambda = 2.5$.



(a) $\alpha = 0.125$

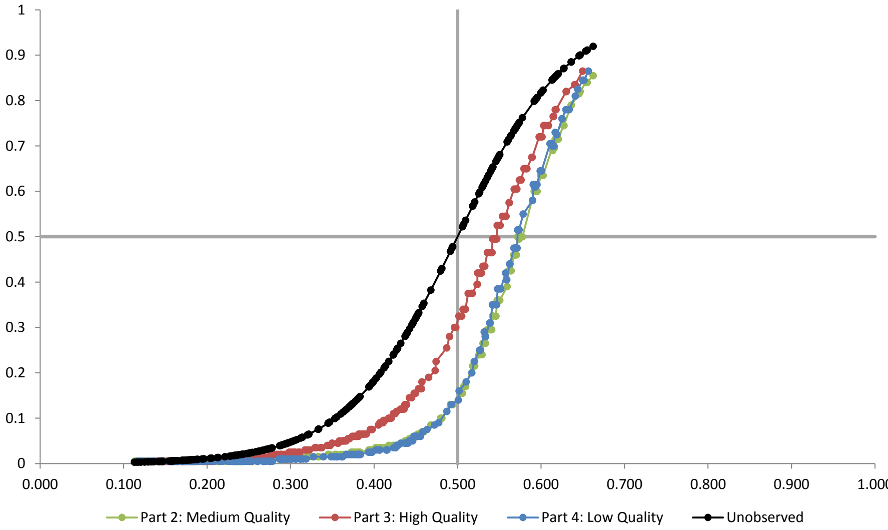


(b) $\alpha = 0.25$

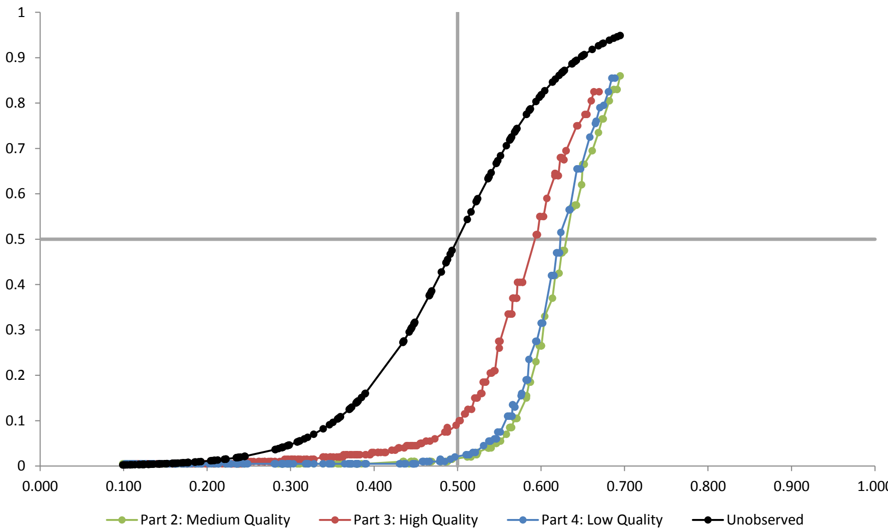


(c) $\alpha = 0.5$

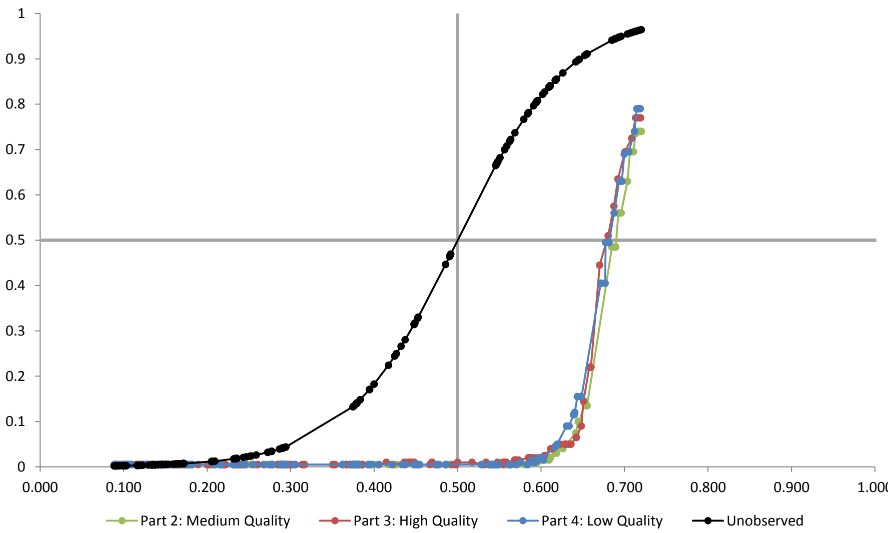
Figure 4: Expected Payoff and Probability to Contradict Private Information for $\lambda = 5$.



(a) $\alpha = 0.125$



(b) $\alpha = 0.25$



(c) $\alpha = 0.5$

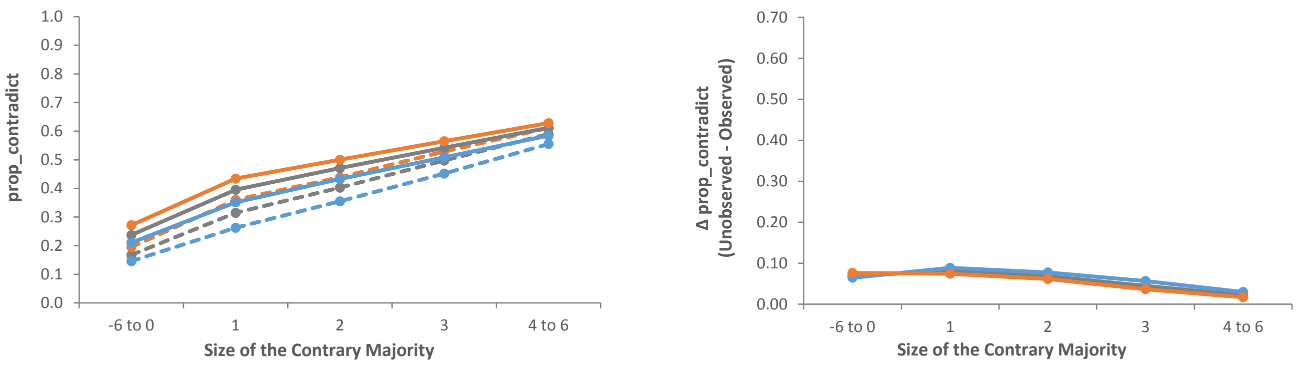
Figure 5: Expected Payoff and Probability to Contradict Private Information for $\lambda = 7.5$.

C.3.2 Response to the Size of the Contrary Majority

Figures 6, 7, and 8 illustrate players' responses to the size of the contrary majority for $\alpha = 0.25$ and $\lambda = 2.5$, $\lambda = 5$, and $\lambda = 7.5$, respectively. For a player with a blue (orange) signal, the size of the contrary majority is given by the number of orange (blue) guesses less the number of blue (orange) guesses in the history. If the size if the contrary majority is positive, the player's private signal and the majority of previous public guesses are conflicting pieces of information. In contrast, if the size of the contrary majority is negative, the player faces a favoring majority, i.e. her private signal and the majority of previous public guesses are concordant.

For each figure the left panel plots the probability to contradict private information against the size of the contrary majority for *unobserved* (solid lines) and *observed* in part 2 (dashed lines). Grey lines represent probabilities pooled across signals, while blue (orange) lines depict probabilities for a blue (orange) signal. In addition, the right panel plots the differences between *unobserved* and *observed* for both signals, and for each signal separately. Tables 2, 3, and 4 contain the probabilities to follow others for $\lambda = 2.5$, $\lambda = 5$, and $\lambda = 7.5$, respectively, and all values of α.

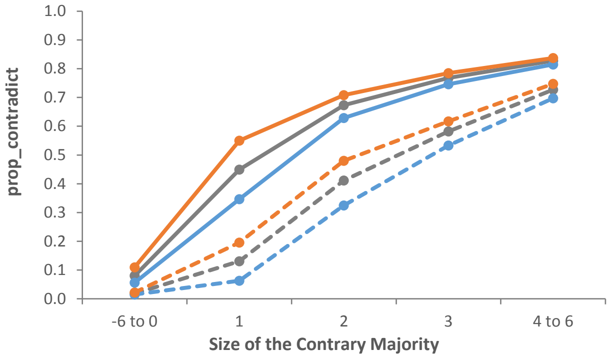
The results can be summarized as follows: The probability to contradict private information is systematically smaller for *observed* than for *unobserved*. Differences are largest for a contrary majority of size 1, decreasing in the size of the contrary majority, and increasing (decreasing) in λ for a contrary (favoring) majority. In addition, the probability to contradict private information is larger with an orange than with a blue signal for any value of α. For $\lambda > 2.5$, differences are smaller for a blue than for an orange signal at a contrary majority of size one, and larger with a blue than with an orange signal at any larger contrary majority.



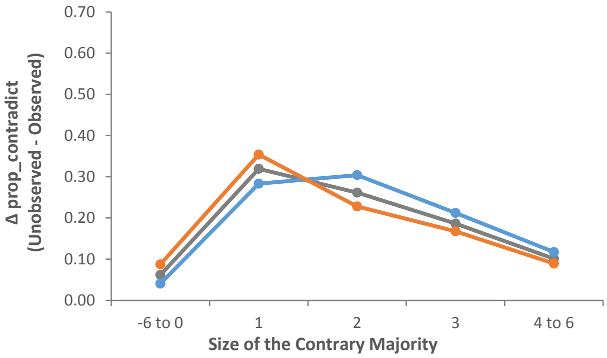
(a) Probability to Contradict

(b) Differences between *Unobserved* and *Observed*

Figure 6: Response to the Size of the Contrary Majority for $\lambda = 2.5$.

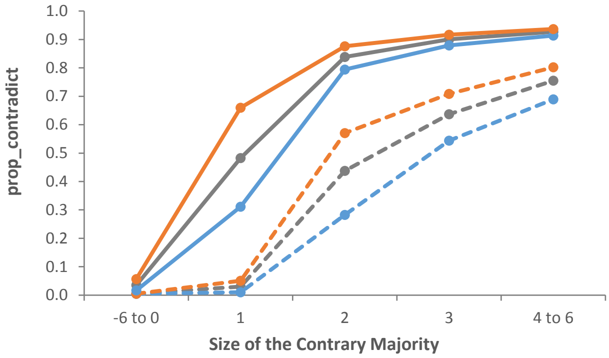


(a) Probability to Contradict

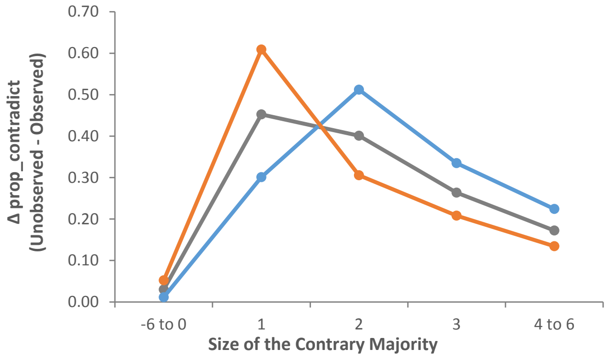


(b) Differences between *Unobserved* and *Observed*

Figure 7: Response to the Size of the Contrary Majority for $\lambda = 5$.



(a) Probability to Contradict



(b) Differences between *Unobserved* and *Observed*

Figure 8: Response to the Size of the Contrary Majority for $\lambda = 7.5$.

Contrary	*Unobserved*			*Observed* Part 2			*Observed* Part 3			*Observed* Part 4		
Majority	All s	blue	orange	All s	blue	orange	All s	blue	orange	All s	blue	orange
$\alpha = 0.125$												
-6 to 0	0.245	0.217	0.280	0.211	0.184	0.245	0.227	0.200	0.261	0.205	0.179	0.239
1	0.382	0.338	0.420	0.346	0.299	0.387	0.352	0.307	0.391	0.347	0.298	0.389
2	0.450	0.409	0.481	0.421	0.375	0.456	0.420	0.377	0.453	0.423	0.377	0.459
3	0.517	0.479	0.543	0.496	0.453	0.525	0.491	0.447	0.521	0.503	0.457	0.533
4 to 6	0.589	0.557	0.608	0.581	0.546	0.601	0.568	0.531	0.589	0.584	0.548	0.606
$\alpha = 0.25$												
-6 to 0	0.237	0.211	0.271	0.168	0.146	0.195	0.200	0.176	0.231	0.158	0.137	0.185
1	0.396	0.351	0.435	0.315	0.263	0.360	0.333	0.287	0.373	0.311	0.257	0.359
2	0.471	0.433	0.501	0.403	0.355	0.439	0.407	0.364	0.439	0.410	0.359	0.449
3	0.542	0.508	0.565	0.498	0.452	0.529	0.483	0.443	0.511	0.507	0.460	0.538
4 to 6	0.612	0.585	0.628	0.590	0.555	0.611	0.567	0.529	0.589	0.601	0.568	0.622
$\alpha = 0.5$												
-6 to 0	0.223	0.197	0.254	0.089	0.079	0.102	0.146	0.128	0.168	0.077	0.068	0.088
1	0.430	0.383	0.473	0.232	0.176	0.282	0.274	0.230	0.313	0.228	0.164	0.285
2	0.520	0.484	0.548	0.359	0.299	0.406	0.364	0.316	0.401	0.372	0.315	0.417
3	0.592	0.566	0.612	0.494	0.443	0.530	0.459	0.414	0.491	0.509	0.463	0.542
4 to 6	0.654	0.635	0.667	0.606	0.570	0.630	0.560	0.521	0.584	0.622	0.588	0.646

Table 2: Probability to Contradict Private Information and Contrary Majority Size for $\lambda = 2.5$.

Contrary	*Unobserved*			*Observed* Part 2			*Observed* Part 3			*Observed* Part 4		
Majority	All s	blue	orange	All s	blue	orange	All s	blue	orange	All s	blue	orange
$\alpha = 0.125$												
-6 to 0	0.083	0.060	0.113	0.041	0.030	0.055	0.060	0.044	0.080	0.037	0.027	0.049
1	0.408	0.322	0.485	0.248	0.164	0.322	0.286	0.212	0.351	0.253	0.164	0.333
2	0.596	0.550	0.632	0.464	0.404	0.510	0.464	0.409	0.504	0.474	0.413	0.520
3	0.708	0.680	0.728	0.614	0.578	0.639	0.599	0.562	0.624	0.622	0.588	0.646
4 to 6	0.790	0.773	0.800	0.738	0.713	0.755	0.722	0.698	0.737	0.745	0.720	0.762
$\alpha = 0.25$												
-6 to 0	0.080	0.056	0.110	0.018	0.015	0.022	0.038	0.029	0.049	0.014	0.012	0.017
1	0.450	0.346	0.550	0.130	0.063	0.196	0.208	0.137	0.272	0.139	0.058	0.216
2	0.673	0.629	0.708	0.411	0.325	0.480	0.415	0.352	0.464	0.427	0.357	0.481
3	0.768	0.746	0.785	0.582	0.534	0.617	0.565	0.522	0.597	0.595	0.550	0.628
4 to 6	0.828	0.815	0.837	0.727	0.697	0.748	0.699	0.664	0.722	0.729	0.695	0.753
$\alpha = 0.5$												
-6 to 0	0.079	0.054	0.109	0.006	0.005	0.007	0.016	0.014	0.019	0.005	0.005	0.005
1	0.483	0.369	0.601	0.019	0.010	0.029	0.057	0.044	0.070	0.023	0.010	0.036
2	0.758	0.708	0.802	0.278	0.131	0.410	0.240	0.131	0.334	0.304	0.160	0.428
3	0.837	0.813	0.856	0.501	0.404	0.577	0.446	0.349	0.520	0.521	0.445	0.580
4 to 6	0.873	0.857	0.884	0.666	0.603	0.711	0.629	0.566	0.674	0.666	0.610	0.706

Table 3: Probability to Contradict Private Information and Contrary Majority Size for $\lambda = 5$.

Contrary	*Unobserved*			*Observed* Part 2			*Observed* Part 3			*Observed* Part 4		
Majority	All s	blue	orange	All s	blue	orange	All s	blue	orange	All s	blue	orange
$\alpha = 0.125$												
-6 to 0	0.034	0.017	0.056	0.009	0.007	0.012	0.017	0.011	0.026	0.008	0.007	0.010
1	0.457	0.302	0.606	0.162	0.055	0.264	0.262	0.140	0.373	0.180	0.051	0.305
2	0.763	0.729	0.789	0.555	0.494	0.600	0.556	0.502	0.596	0.569	0.517	0.607
3	0.844	0.828	0.856	0.701	0.660	0.730	0.698	0.668	0.718	0.702	0.668	0.726
4 to 6	0.891	0.882	0.897	0.799	0.784	0.810	0.803	0.785	0.815	0.801	0.784	0.813
$\alpha = 0.25$												
-6 to 0	0.035	0.017	0.057	0.005	0.005	0.005	0.009	0.007	0.012	0.005	0.005	0.005
1	0.482	0.311	0.659	0.030	0.010	0.051	0.113	0.045	0.180	0.036	0.005	0.067
2	0.838	0.794	0.876	0.438	0.282	0.570	0.489	0.403	0.555	0.461	0.329	0.569
3	0.900	0.879	0.917	0.637	0.544	0.709	0.652	0.605	0.686	0.643	0.571	0.698
4 to 6	0.927	0.914	0.937	0.755	0.689	0.802	0.761	0.739	0.777	0.765	0.711	0.805
$\alpha = 0.5$												
-6 to 0	0.036	0.017	0.059	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
1	0.485	0.315	0.673	0.005	0.005	0.005	0.008	0.010	0.005	0.005	0.005	0.005
2	0.883	0.837	0.927	0.267	0.026	0.503	0.206	0.028	0.370	0.265	0.039	0.478
3	0.942	0.928	0.953	0.532	0.388	0.642	0.400	0.275	0.501	0.518	0.363	0.640
4 to 6	0.955	0.948	0.960	0.654	0.581	0.705	0.584	0.465	0.671	0.655	0.543	0.736

Table 4: Probability to Contradict Private Information and Contrary Majority Size for $\lambda = 7.5$.

C.3.3 Information Aggregation and Fraction of Correct Guesses

We finally investigate how altruism affects the payoffs of players in the LQRE. Indeed, an increase in the response to private information is potentially beneficial to players in both sequences. We first analyze the amount of information accumulated in equilibrium. Second, we calculate the players' expected earnings, i.e. the expected fraction of correct guesses.

Since payoff-responsive decision errors imply that more information is aggregated in the standard LQRE than in sequential equilibrium, there is a legitimate concern that altruism has a negligible impact on information aggregation and players' earnings in LQRE. In fact, it is not clear whether altruism increases the relative frequency of correct guesses in the presence of decision errors. To account for this possibility, we analyze the impact of altruism on information aggregation and players' earnings for the value of λ which maximizes information aggregation and earnings in the standard LQRE of the laboratory cascade game.

We measure information aggregation by the average level of *value_contra_PI* at large majorities of size 3, 4, 5, or 6. Indeed, the more information is aggregated the lower the level of *value_contra_PI* at large favoring majorities and the higher the level of *value_contra_PI* at large contrary majorities. We find that guesses aggregate the largest amount of information in the standard LQRE when $\lambda = 7.5$. Table 5 reports the average levels of *value_contra_PI* in the LQRE for $\lambda = 7.5$ and $\alpha \in \{0, 0.125, 0.25, 0.5\}$ separately for large favoring majorities (contrary majorities of size -6, -5, -4, or -3), moderate majorities (contrary majorities of size -2, -1, 0, 1, or 2), and large contrary majorities (of size 3, 4, 5, or 6). The results demonstrate that altruism increases information aggregation considerably beyond the levels in standard LQRE. The average level of *value_contra_PI* at large favoring (contrary) majorities is lower (higher) for larger values of α. The signal quality in the *unobserved* sequence has a small impact. Finally, there is no difference in the average level of *value_contra_PI* at moderate majorities. In fact, the average level of *value_contra_PI* equals the average level in period 1 for all values of α and all signal qualities of the *unobserved*.

		$\alpha = 0.125$			$\alpha = 0.25$			$\alpha = 0.5$		
	$\alpha = 0$	Part 2	Part 3	Part 4	Part 2	Part 3	Part 4	Part 2	Part 3	Part 4
Large Favoring Majorities	0.146	0.126	0.134	0.128	0.111	0.123	0.113	0.096	0.103	0.099
Moderate Majorities	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333
Large Contrary Majorities	0.593	0.635	0.617	0.631	0.667	0.641	0.662	0.702	0.685	0.695

Table 5: Average Levels of *value_contra_PI* for $\lambda = 7.5$.

We finally analyze the earnings of players in the LQRE measured by the average fraction of correct guesses. In the standard LQRE, this fraction is largest for $\lambda \approx 60$. Though less information is aggregated for this value of λ, players do not seem to be able to fully reap the benefits of enhanced information aggregation for smaller values of λ due to the larger likelihood of decision errors. Yet, differences in earnings are negligible for sufficiently large values of λ. Indeed, players in the standard LQRE with $\lambda = 7.5$ earn 97% of the average earnings in the standard LQRE with $\lambda = 60$. To facilitate the discussion we therefore focus on $\lambda = 7.5$.

Table 6 reports the earnings for $\lambda = 7.5$ and different values of α. We note first that altruism improves welfare, as average earnings of *observed* and *unobserved* are always higher than average earnings in the standard LQRE. Second, *unobserved* are predicted to make the correct guess more often than *observed* across all values of α. The difference is more pronounced the larger is α. Finally, compared to part 2 *observed* are predicted to earn less in part 3 where *unobserved* receive private signals of high signal quality, and they earn a similar amount in part 4 where *unobserved* receive private signals of low signal quality.

	Observed			
	Part 2	Part 3	Part 4	*Unobserved*
$\alpha = 0$	0.686			
$\alpha = 0.125$	0.706	0.698	0.706	0.707
$\alpha = 0.25$	0.709	0.704	0.709	0.719
$\alpha = 0.5$	0.706	0.699	0.705	0.728

Table 6: Earnings as Fraction of the High Payoff for $\lambda = 7.5$.

Appendix D. Complements to the Main Statistical Analysis

In this appendix we first detail the derivation of the empirical value of contradicting private information. Then we report the regression results on the responses to *value_contra_PI* and the amounts of information aggregated in Section D.2 and Section D.3, respectively.

D.1. The Empirical Value of Contradicting Private Information

Following the approach introduced by Weizsäcker (2010) and refined by Ziegelmeyer, March, and Krügel (2013), we detail below the derivation of the empirical value of contradicting private information. In period $t \in \{1, \ldots, T\}$ and given history $\mathbf{h}_t$ and signal s_t, *value_contra_PI* $(s_t, \mathbf{h}_t)$ equals

$$\begin{cases} \left[1 + \frac{p}{1-p} \frac{q}{1-q} \prod_{\tau<t} \frac{q \widehat{\Pr}(x_\tau|\mathbf{h}_\tau,b,\mathcal{B}) + (1-q) \widehat{\Pr}(x_\tau|\mathbf{h}_\tau,o,\mathcal{B})}{(1-q) \widehat{\Pr}(x_\tau|\mathbf{h}_\tau,b,\mathcal{O}) + q \widehat{\Pr}(x_\tau|\mathbf{h}_\tau,o,\mathcal{O})} \right]^{-1} & \text{if } s_t = b \\ \left[1 + \frac{1-p}{p} \frac{1-q}{q} \prod_{\tau<t} \frac{(1-q) \widehat{\Pr}(x_\tau|\mathbf{h}_\tau,b,\mathcal{O}) + q \widehat{\Pr}(x_\tau|\mathbf{h}_\tau,o,\mathcal{O})}{q \widehat{\Pr}(x_\tau|\mathbf{h}_\tau,b,\mathcal{B}) + (1-q) \widehat{\Pr}(x_\tau|\mathbf{h}_\tau,o,\mathcal{B})} \right]^{-1} & \text{if } s_t = o \end{cases}$$

where $\widehat{\Pr}(x_\tau \mid \mathbf{h}_\tau, s_\tau, \theta)$ is the relative frequency with which action x_τ is chosen in period $\tau < t$ across all *observed* choices where the history is $\mathbf{h}_\tau \subset \mathbf{h}_t$, the private signal is $s_\tau \in S$, and the state of nature is $\theta \in \Theta$. In our laboratory cascade game, $p = 11/20$ and $q = 2/3$.

We derive the empirical value of contradicting private information separately in the different session parts. Concretely, the empirical value of contradicting private information in a given part is derived from the *observed* choices made only in that part. We take such a conservative approach as *unobserved* are endowed with different signal qualities in the different parts and if *observed* care about the correctness of *unobserved* actions then the informativeness of their own actions might vary across parts.

We would like to emphasize that *value_contra_PI* cannot be derived in every guessing situation. Indeed, the derivation of *value_contra_PI* $(s_t, \mathbf{h}_t)$ is impossible whenever the relative action frequencies cannot be computed for each couple (signal, state) at each sub-history $\mathbf{h}_\tau \subset \mathbf{h}_t$. In Experiment 1, we are able to calculate *value_contra_PI* for 268 out of the 504 distinct guessing situations encountered by *observed* in parts 2 to 4 or *unobserved* in part 2.[4] This covers 6,906 out of the 7,992 guesses meaning that the omitted guessing situations occur rather infrequently. In Experiment 2, *value_contra_PI* can be calculated for 227 out of the 437 guessing situations which covers 4,466 out of the 5,328 guesses.

More importantly, *value_contra_PI* is an imperfect measure of the true underlying incentives whose precision depends on the number of observations from which the relative action probabilities are estimated. We relate the precision of *value_contra_PI* $(s_t, \mathbf{h}_t)$ to the number of occurrences of the guessing situation $(s_t, \mathbf{h}_t)$ and we denote this number by *sitcount*$(s_t, \mathbf{h}_t)$. Note that *sitcount* is calculated separately for *observed* and *unobserved* which ensures that bubble sizes in Figures 2 and 3 of the main text accurately reflect the weight of each guessing situation in the two sequences of participants.

The fact that *value_contra_PI* imperfectly measures the true expected value of contradicting private information could invalidate the inferences on observational learning behavior. We address this inference problem in two (non-exclusive) ways. First, the statistical analysis is conducted on different subsets of data with more and more stringent minimum thresholds for *sitcount*. Below we check that the analysis reported in the main text is robust to variations in the minimum threshold for *sitcount*. The second

[4] We distinguish guessing situations according to whether they were encountered by *observed* or *unobserved* and in part 2, 3, or 4 of the session. Accordingly, a guessing situation is fully determined by the tuple (sequence, part, history, signal).

approach uses an instrumental variable (IV) to correct for measurement error in statistical analyses where *value_contra_PI* is an explanatory variable (see e.g. Cameron and Trivedi, 2005, Chapter 4, 6). A valid instrument can be obtained by randomly splitting the dataset in two subsets of approximately equal size, deriving *value_contra_PI* separately on each subset, and using one of the estimates as an instrument for the other. Since the derivation of *value_contra_PI* $(s_t, \mathbf{h}_t)$ requires the calculation of relative action frequencies for each triple $(\mathbf{h}_\tau, s_\tau, \theta)$ where $\tau < t$ and $\mathbf{h}_\tau \subset \mathbf{h}_t$, the two subsets are obtained by splitting the set of *repetitions* of the cascade game for each part.[5] However, a considerable efficiency loss occurs because only half of the sample is used to derive the empirical value of contradicting private information (Cameron and Trivedi, 2005, p.192). The efficiency loss takes two forms. First, *value_contra_PI* can often not be derived in both subsets though it can be derived in the entire dataset. This results in a smaller number of observations that can be used in IV regressions. Second, the split-sample method increases the measurement error in monetary incentives as the control variable included in IV regressions is *value_contra_PI*$_1$, the empirical value of contradicting private information in the first subset. To alleviate the efficiency loss, we repeat the random splitting 100 times and we select the split which minimizes the loss of efficiency along the two dimensions just mentioned. First, the selected split permits *value_contra_PI* to be derived in both subsets for 9,579 observations (across the 100 randomly generated splits the number of available observations ranges from 7,866 to 9,861). Second, for each split we regress *value_contra_PI*$_1$ on *value_contra_PI*. The resulting R^2 (roughly) assesses the additional measurement error generated by the splitting. The R^2 of the selected split equals 0.9801 (across the 100 splits R^2 ranges from 0.8257 to 0.9833). Among the 4 splits which satisfy $R^2 > 0.98$, the selected split uses the largest number of observations by far. For Experiment 1 we are able to calculate *value_contra_PI* in both subsets for 125 guessing situations and 5,854 guesses in total. For Experiment 2 *value_contra_PI* can be calculated in both subsets for 109 guessing situations and 3,725 guesses in total.[6]

Clustering

It is likely that subjects' behavior is influenced by individual characteristics and session dynamics. We therefore have a nested hierarchy of potential levels of clustering (across which residuals are likely to be dependent). As is recommended in this case we rely on cluster-robust standard errors computed at the most aggregated level of clustering, i.e. at the session level (see e.g. Cameron and Miller, 2010). A potential problem in this case is that few clusters are available. In order to correct for the small number of clusters, we apply a finite-cluster correction to the cluster-robust estimate of the variance matrix, and we rely on the T(G-1)-distribution and the F(h,G-1)-distribution, respectively, to compute p-values of one- and two-tailed hypothesis tests.

Demographics

During the experiment we collected information on subjects' age, gender, field of studies, mother tongue, and citizenship. Though not reported below, all results are robust to controlling for these demographic variables. The results are available from the authors upon request.

[5] There are 54 (36) distinct repetitions of the game in each part of Experiment 1 (2).

[6] Note that *value_contra_PI* can be calculated for the entire dataset whenever it can be calculated in both subsets.

D.2. Responses to *value_contra_PI*

Here we report the regression results discussed in Sections 4.2 and 5.2 of the main text along with robustness checks. We regress the proportion to contradict private information against a cubic polynomial in *value_contra_PI* fully interacted with indicator variables for *unobserved* and *observed* in part 3 and in part 4, and all regressors are interacted with an indicator variable for Experiment 2. Table 7 reports the regression results based on the IV specification for *sitcount* $\geqslant 10$ (minimum threshold considered in the main text) as well as for *sitcount* $\geqslant 1$ and for *sitcount* $\geqslant 20$.

We obtain the same qualitative results for each subset of data in the first experiment. On the other hand, two qualitative findings in Experiment 2 are sensitive to the precision with which *value_contra_PI* is measured. First, the hypothesis that the average *unobserved* systematically makes the money-maximizing guess is only rejected for *sitcount* $\geqslant 10$ and *sitcount* $\geqslant 20$. Second, there is a significant increase in the *observed* reluctance to contradict private information from part 2 to part 3 only for *sitcount* $\geqslant 1$ and *sitcount* $\geqslant 10$, and there is a significant increase in the *observed* reluctance to contradict private information from part 3 to part 4 only for *sitcount* $\geqslant 20$.

Table 8 reports the regression results based on the OLS specification for *sitcount* $\geqslant 10$ (minimum threshold considered in the main text) as well as for *sitcount* $\geqslant 20$ and for *sitcount* $\geqslant 30$. The OLS regression results confirm those obtained with the IV specification except for the fact that in Experiment 1 the vertical distance between the *unobserved* fitted line and (0.5, 0.5) is insignificant for all *sitcount* minimum thresholds.

	$sitcount \geqslant 1$	$sitcount \geqslant 10$	$sitcount \geqslant 20$
Constant	-0.895***	-1.173***	-1.291***
	(0.173)	(0.269)	(0.255)
$value\_contra\_PI_1$	11.013***	14.055***	15.291***
	(1.813)	(2.935)	(2.742)
$(value\_contra\_PI_1)^2$	-38.949***	-48.636***	-52.696***
	(5.674)	(9.348)	(8.808)
$(value\_contra\_PI_1)^3$	42.339***	51.549***	55.815***
	(5.399)	(8.907)	(8.569)
Observed in part 3	0.809***	1.058***	1.092***
	(0.180)	(0.258)	(0.266)
Observed in part 3 × $value\_contra\_PI_1$	-9.370***	-12.095***	-12.377***
	(1.910)	(2.790)	(2.876)
Observed in part 3 × $(value\_contra\_PI_1)^2$	32.124***	40.831***	41.835***
	(6.036)	(8.788)	(9.094)
Observed in part 3 × $(value\_contra\_PI_1)^3$	-33.072***	-41.433***	-42.959***
	(5.748)	(8.264)	(8.595)
Observed in part 4	0.462**	0.710**	0.925***
	(0.183)	(0.286)	(0.271)
Observed in part 4 × $value\_contra\_PI_1$	-5.791**	-8.471**	-10.847***
	(1.988)	(3.182)	(2.960)
Observed in part 4 × $(value\_contra\_PI_1)^2$	20.764***	29.257**	37.206***
	(6.116)	(10.014)	(9.400)
Observed in part 4 × $(value\_contra\_PI_1)^3$	-22.584***	-30.692***	-38.959***
	(5.550)	(9.291)	(8.961)
Unobserved	0.378	0.656*	1.095***
	(0.224)	(0.345)	(0.312)
Unobserved × $value\_contra\_PI_1$	-4.424*	-7.465*	-12.366***
	(2.458)	(3.822)	(3.350)
Unobserved × $(value\_contra\_PI_1)^2$	14.996*	24.683*	40.544***
	(7.990)	(12.478)	(10.909)
Unobserved × $(value\_contra\_PI_1)^3$	-14.422*	-23.633*	-39.023*
	(7.818)	(12.207)	(10.739)
Experiment 2	1.621***	0.289	0.951***
	(0.210)	(0.288)	(0.268)
Experiment 2 × $value\_contra\_PI_1$	-17.850***	-3.037	-10.583***
	(2.184)	(3.106)	(2.968)
Experiment 2 × $(value\_contra\_PI_1)^2$	56.678***	9.191	34.754***
	(6.485)	(9.961)	(9.696)
Experiment 2 × $(value\_contra\_PI_1)^3$	-52.933***	-8.064	-34.404***
	(5.763)	(9.633)	(9.562)
Experiment 2 × *Observed* in part 3	-2.200***	-0.864**	-1.608***
	(0.330)	(0.384)	(0.420)
Experiment 2 × *Observed* in part 3 × $value\_contra\_PI_1$	24.623***	9.824**	17.668***
	(3.766)	(4.265)	(4.511)
Experiment 2 × *Observed* in part 3 × $(value\_contra\_PI_1)^2$	-79.785***	-32.498**	-57.968***
	(11.973)	(13.471)	(13.980)
Experiment 2 × *Observed* in part 3 × $(value\_contra\_PI_1)^3$	76.085***	31.590**	57.525***
	(10.970)	(12.556)	(12.925)
Experiment 2 × *Observed* in part 4	-1.291***	0.058	-0.803**
	(0.223)	(0.322)	(0.305)
Experiment 2 × *Observed* in part 4 × $value\_contra\_PI_1$	14.051***	-0.942	8.998**
	(2.439)	(3.617)	(3.501)
Experiment 2 × *Observed* in part 4 × $(value\_contra\_PI_1)^2$	-43.929***	4.187	-29.461**
	(7.445)	(11.611)	(11.416)
Experiment 2 × *Observed* in part 4 × $(value\_contra\_PI_1)^3$	40.169***	-5.317	28.816**
	(6.601)	(10.976)	(11.022)
Experiment 2 × *Unobserved*	-0.573**	-0.055	-2.039***
	(0.237)	(0.393)	(0.522)
Experiment 2 × *Unobserved* × $value\_contra\_PI_1$	5.907**	1.332	23.063***
	(2.600)	(4.321)	(5.819)
Experiment 2 × *Unobserved* × $(value\_contra\_PI_1)^2$	-18.308**	-6.562	-75.971***
	(8.399)	(13.859)	(18.903)
Experiment 2 × *Unobserved* × $(value\_contra\_PI_1)^3$	17.186*	8.135	74.739***
	(8.053)	(13.326)	(18.188)
Observations	9,579	9,365	8,516
R^2	0.404	0.399	0.428

Robust standard errors in parentheses, clustered at the session level.
* (10%); ** (5%); and *** (1%) significance level.

Table 7: Frequency to contradict private information (IV)

	$sitcount \geqslant 10$	$sitcount \geqslant 20$	$sitcount \geqslant 30$
Constant	-0.334*	-0.344	-0.488***
	(0.176)	(0.207)	(0.158)
value_contra_PI	4.765**	4.916**	6.421***
	(1.866)	(2.138)	(1.584)
$(value\_contra\_PI)^2$	-19.455***	-19.926***	-24.801***
	(5.971)	(6.691)	(4.990)
$(value\_contra\_PI)^3$	25.155***	25.489***	30.368***
	(5.649)	(6.260)	(4.892)
Observed in part 3	0.171	0.093	0.255**
	(0.179)	(0.166)	(0.121)
Observed in part 3 × *value_contra_PI*	-2.256	-1.342	-3.130**
	(1.935)	(1.632)	(1.246)
Observed in part 3 × $(value\_contra\_PI)^2$	9.389	6.581	12.826***
	(6.284)	(5.086)	(4.108)
Observed in part 3 × $(value\_contra\_PI)^3$	-12.053*	-9.779*	-16.673***
	(5.930)	(4.814)	(4.189)
Observed in part 4	0.139	0.102	0.325*
	(0.190)	(0.198)	(0.164)
Observed in part 4 × *value_contra_PI*	-2.155	-1.777	-4.433**
	(2.069)	(2.046)	(1.661)
Observed in part 4 × $(value\_contra\_PI)^2$	9.648	8.570	17.753***
	(6.770)	(6.476)	(5.278)
Observed in part 4 × $(value\_contra\_PI)^3$	-13.512*	-12.672*	-22.243***
	(6.455)	(6.113)	(5.222)
Unobserved	0.737***	0.639***	0.686***
	(0.185)	(0.173)	(0.158)
Unobserved × *value_contra_PI*	-8.693***	-7.534***	-7.882***
	(2.059)	(1.782)	(1.605)
Unobserved × $(value\_contra\_PI)^2$	29.671***	25.486***	26.405***
	(6.785)	(5.560)	(5.409)
Unobserved × $(value\_contra\_PI)^3$	-29.161***	-24.829***	-25.700***
	(6.425)	(5.119)	(5.687)
Experiment 2	0.150	0.062	0.290
	(0.182)	(0.220)	(0.205)
Experiment 2 × *value_contra_PI*	-1.595	-0.769	-3.246
	(2.011)	(2.416)	(2.299)
Experiment 2 × $(value\_contra\_PI)^2$	5.676	3.431	11.450
	(6.407)	(7.855)	(8.040)
Experiment 2 × $(value\_contra\_PI)^3$	-6.669	-4.909	-12.857
	(5.978)	(7.652)	(8.587)
Experiment 2 × *Observed* in part 3	0.015	-0.014	-0.510*
	(0.212)	(0.291)	(0.265)
Experiment 2 × *Observed* in part 3 × *value_contra_PI*	-0.188	0.260	5.740*
	(2.388)	(3.092)	(2.928)
Experiment 2 × *Observed* in part 3 × $(value\_contra\_PI)^2$	-0.851	-2.892	-20.983*
	(7.824)	(9.817)	(9.942)
Experiment 2 × *Observed* in part 3 × $(value\_contra\_PI)^3$	3.263	6.129	24.444**
	(7.398)	(9.319)	(10.243)
Experiment 2 × *Observed* in part 4	0.021	0.012	-0.152
	(0.198)	(0.234)	(0.218)
Experiment 2 × *Observed* in part 4 × *value_contra_PI*	-0.490	0.115	1.712
	(2.271)	(2.726)	(2.531)
Experiment 2 × *Observed* in part 4 × $(value\_contra\_PI)^2$	1.567	-1.446	-5.881
	(7.521)	(9.156)	(8.888)
Experiment 2 × *Observed* in part 4 × $(value\_contra\_PI)^3$	-0.653	2.795	6.579
	(7.180)	(8.963)	(9.438)
Experiment 2 × *Unobserved*	-0.290	-0.521**	-0.635**
	(0.192)	(0.220)	(0.257)
Experiment 2 × *Unobserved* × *value_contra_PI*	2.458	6.222**	7.401**
	(2.262)	(2.567)	(3.118)
Experiment 2 × *Unobserved* × $(value\_contra\_PI)^2$	-6.214	-21.643**	-25.616**
	(7.734)	(8.580)	(11.156)
Experiment 2 × *Unobserved* × $(value\_contra\_PI)^3$	5.541	22.441**	26.606**
	(7.664)	(8.317)	(11.793)
Observations	10,315	9,041	7,982
R^2	0.517	0.525	0.511

Robust standard errors in parentheses, clustered at the session level.
* (10%); ** (5%); and *** (1%) significance level.

Table 8: Frequency to contradict private information (OLS)

Previous Cascade Game Experiments

As explained in the main text, we compare the responses to *value_contra_PI* in the short ($T \leqslant 6$) and long cascade games contained in the meta-dataset of Ziegelmeyer, March, and Krügel (2013). The meta-dataset includes 14 information cascade experiments which are variations of Anderson and Holt's (1997) seminal experiment. Using the split-sample IV method, we regress the proportion to contradict private information against a cubic polynomial in *value_contra_PI* fully interacted with an indicator variable for long sequences of participants in the cascade game. Robust standard errors are clustered at the group level since several distinct groups of participants might play the cascade game in a given experimental session. Table 9 reports the regression results for three different *sitcount* levels.

	sitcount $\geqslant 1$	*sitcount* $\geqslant 10$	*sitcount* $\geqslant 20$
Constant	0.059*	0.090***	0.109***
	(0.033)	(0.017)	(0.016)
$value\_contra\_PI_1$	-0.917**	-1.000***	-1.271***
	(0.379)	(0.229)	(0.203)
$(value\_contra\_PI_1)^2$	3.651***	3.447***	4.367***
	(1.108)	(0.742)	(0.612)
$(value\_contra\_PI_1)^3$	-1.469*	-0.994	-1.813***
	(0.885)	(0.627)	(0.482)
Long sequences	-0.010	-0.036	-0.068***
	(0.037)	(0.023)	(0.020)
Long sequences $\times$ $value\_contra\_PI_1$	-0.265	-0.131	0.523*
	(0.499)	(0.399)	(0.318)
Long sequences $\times$ $(value\_contra\_PI_1)^2$	0.481	0.432	-1.722*
	(1.500)	(1.265)	(1.016)
Long sequences $\times$ $(value\_contra\_PI_1)^3$	-0.602	-0.746	0.987
	(1.152)	(0.980)	(0.793)
Observations	21,775	16,697	13,872
R^2	0.214	0.325	0.342

Robust standard errors in parentheses, clustered at the group level.
* (10%); ** (5%); and *** (1%) significance level.

Table 9: Frequency to contradict private information in prior cascade game experiments

The fitted line for short sequences of participants goes through (0.5, 0.329) and (0.597, 0.5) if *sitcount* $\geqslant 1$, it goes through (0.5, 0.328) and (0.592, 0.5) if *sitcount* $\geqslant 10$, and it goes through (0.5, 0.339) and (0.587, 0.5) if *sitcount* $\geqslant 20$. For each *sitcount* level, the vertical distance between the fitted line and (0.5, 0.5) is strongly significant which indicates that participants in short games are reluctant to contradict their private information when monetary incentives to follow others are weak. Moreover, the fitted line for long games goes through (0.5, 0.231) and (0.678, 0.5) if *sitcount* $\geqslant 1$, it goes through (0.5, 0.241) and (0.662, 0.5) if *sitcount* $\geqslant 10$, and it goes through (0.5, 0.226) and (0.682, 0.5) if *sitcount* $\geqslant 20$. And the vertical distance between the fitted line for long games and (0.597, 0.5) (respectively (0.592, 0.5) and (0.587, 0.5)) is strongly significant if *sitcount* $\geqslant 1$ (respectively *sitcount* $\geqslant 10$ and *sitcount* $\geqslant 20$). Thus, observational learning is significantly more informative in long than in short sequences of participants in previous cascade experiments.

D.3. Information Aggregation

In the main text we consider the case where the size of large majorities equals 3 or more and every guessing situation for which *value_contra_PI* can be computed is included (i.e. $sitcount \geqslant 1$). Table 10 reports the regression results discussed in the main text as well as robustness checks with $sitcount \geqslant 10$ and $sitcount \geqslant 20$. The restriction to subsets of data where *value_contra_PI* is measured more precisely has the unfortunate consequence that guessing situations which generate large majorities are sometimes missing in the second experiment.

	$sitcount \geqslant 1$	$sitcount \geqslant 10$	$sitcount \geqslant 20$
Constant	0.330*** (0.007)	0.328*** (0.007)	0.332*** (0.007)
Large Favoring Majorities	-0.176*** (0.005)	-0.173*** (0.007)	-0.176*** (0.007)
Large Contrary Majorities	0.248*** (0.010)	0.250*** (0.008)	0.241*** (0.007)
Part 3 × Moderate Majorities	0.005 (0.009)	0.002 (0.009)	-0.001 (0.010)
Part 3 × Large Favoring Majorities	-0.019* (0.011)	-0.038*** (0.002)	-0.045*** (0.001)
Part 3 × Large Contrary Majorities	0.039*** (0.008)	0.063*** (0.009)	0.094*** (0.001)
Part 4 × Moderate Majorities	0.011 (0.012)	0.011 (0.010)	0.004 (0.010)
Part 4 × Large Favoring Majorities	-0.045*** (0.002)	-0.048*** (0.002)	-0.050*** (0.001)
Part 4 × Large Contrary Majorities	0.093*** (0.007)	0.109*** (0.003)	0.114*** (0.001)
Experiment 2 × Part 2 × Moderate Majorities	-0.006 (0.011)	-0.007 (0.010)	-0.025** (0.009)
Experiment 2 × Part 2 × Large Favoring Majorities	-0.023*** (0.007)	-0.026*** (0.007)	-0.018*** (0.001)
Experiment 2 × Part 2 × Large Contrary Majorities	0.023 (0.021)	0.039*** (0.002)	—
Experiment 2 × Part 3 × Moderate Majorities	0.005 (0.008)	0.012 (0.009)	0.011 (0.009)
Experiment 2 × Part 3 × Large Favoring Majorities	-0.008 (0.011)	0.002 (0.003)	—
Experiment 2 × Part 3 × Large Contrary Majorities	0.027* (0.014)	0.004 (0.010)	-0.030*** (0.003)
Experiment 2 × Part 4 × Moderate Majorities	-0.002 (0.010)	-0.003 (0.009)	-0.004 (0.010)
Experiment 2 × Part 4 × Large Favoring Majorities	-0.026*** (0.007)	-0.026*** (0.005)	-0.039*** (0.000)
Experiment 2 × Part 4 × Large Contrary Majorities	0.069*** (0.020)	0.076*** (0.011)	—
Observations	7,068	6,224	5,288
Cluster	15	15	15
R^2	0.428	0.447	0.320

Robust standard errors are clustered at the session level.
* (10%); ** (5%); and *** (1%) significance level.

Table 10: The Empirical Value of Contradicting Private Information (size of large majorities is 3 or more)

The regression results show that, for each subset of data in the first experiment, large majorities aggregate significantly more information in later parts of sessions whereas moderate majorities never contain any valuable information on average. On the other hand, the level of *sitcount* slightly affects the difference between the two experiments in the amount of information aggregated by large contrary majorities. Compared to part 2 in Experiment 1, large contrary majorities in part 2 of Experiment 2 aggregate non-significantly more information when *sitcount* $\geqslant 1$ and they aggregate significantly more information when *sitcount* $\geqslant 10$ at the 1% level (there is no guessing situation which generates large contrary majorities in part 2 of Experiment 2 when *sitcount* $\geqslant 20$). Compared to part 3 in Experiment 1, large contrary majorities in part 3 of Experiment 2 aggregate significantly more information when *sitcount* $\geqslant 1$ at the 10% level, they aggregate non-significantly more information when *sitcount* $\geqslant 10$, and they aggregate significantly less information when *sitcount* $\geqslant 20$ at the 1% level.

Table 11 reports robustness checks where the size of large majorities equals 4 or more. With a higher threshold on the size of large majorities, even less guessing situations which generate large majorities are available as *value_contra_PI* is measured more precisely.

	sitcount $\geqslant 1$	*sitcount* $\geqslant 10$	*sitcount* $\geqslant 20$
Constant	0.332*** (0.009)	0.329*** (0.009)	0.333*** (0.008)
Large Favoring Majorities	-0.181*** (0.007)	-0.175*** (0.009)	-0.173*** (0.008)
Large Contrary Majorities	0.248*** (0.010)	0.249*** (0.010)	0.234*** (0.008)
Part 3 × Moderate Majorities	0.005 (0.011)	0.003 (0.012)	-0.001 (0.011)
Part 3 × Large Favoring Majorities	-0.018* (0.010)	-0.039*** (0.002)	-0.047*** (0.000)
Part 3 × Large Contrary Majorities	0.042*** (0.006)	0.062*** (0.007)	0.094*** (0.000)
Part 4 × Moderate Majorities	0.013 (0.015)	0.014 (0.013)	0.008 (0.013)
Part 4 × Large Favoring Majorities	-0.043*** (0.003)	-0.045*** (0.002)	-0.047*** (0.000)
Part 4 × Large Contrary Majorities	0.101*** (0.010)	0.112*** (0.007)	0.131*** (0.000)
Experiment 2 × Part 2 × Moderate Majorities	-0.011 (0.012)	-0.016 (0.011)	-0.036*** (0.011)
Experiment 2 × Part 2 × Large Favoring Majorities	-0.006 (0.004)	-0.009*** (0.002)	-0.018*** (0.000)
Experiment 2 × Part 2 × Large Contrary Majorities	0.011** (0.004)	—	—
Experiment 2 × Part 3 × Moderate Majorities	0.011 (0.010)	0.018 (0.012)	0.032*** (0.011)
Experiment 2 × Part 3 × Large Favoring Majorities	-0.013 (0.009)	-0.003 (0.003)	—
Experiment 2 × Part 3 × Large Contrary Majorities	0.034*** (0.011)	0.014 (0.009)	—
Experiment 2 × Part 4 × Moderate Majorities	-0.005 (0.013)	-0.006 (0.012)	-0.019 (0.013)
Experiment 2 × Part 4 × Large Favoring Majorities	-0.029*** (0.007)	-0.029*** (0.007)	—
Experiment 2 × Part 4 × Large Contrary Majorities	0.072*** (0.023)	0.121*** (0.005)	—
Observations	7,068	6,224	5,288
Cluster	15	15	15
R^2	0.227	0.214	0.102

Robust standard errors are clustered at the session level.
* (10%); ** (5%); and *** (1%) significance level.

Table 11: The Empirical Value of Contradicting Private Information (size of large majorities is 4 or more)

Like in Table 10, the regression results in Table 11 show that, for each subset of data in the first experiment, large majorities aggregate significantly more information in later parts of sessions whereas moderate majorities never contain any valuable information on average. And in terms of differences between the two experiments the results are also qualitatively similar for the two thresholds except that for the higher threshold the difference is non-significant for large favoring majorities in part 2 when *sitcount* $\geqslant 1$ and it is significant for large contrary majorities in part 2 when *sitcount* $\geqslant 1$.

In sum, we obtain the same qualitative results for each subset of data and the two minimum thresholds of the large majority size in the first experiment. Similarly, large majorities in part 4 always aggregate significantly more information in Experiment 2 than in Experiment 1. However, differences between the two experiments in parts 2 and 3 are less robust to the minimum thresholds of the large majority size or *sitcount*.

Appendix E. Instructions

E.1. General Instructions

Welcome to the experiment!

Please do not touch the mouse and do not open the envelope until you are instructed to do so.

This is an experiment in decision-making and all your decisions will be treated in an anonymous way. From now on, we ask you to remain seated quietly at your computer desk. Please do not talk, exclaim, or try to communicate with other participants during the experiment. Participants who intentionally violate this rule will be asked to leave the experiment without being financially compensated. If you have any questions during the experiment, please raise your hand and wait for an experimenter to come to you.

Your earnings will depend partly on your decisions and partly on chance. In addition to the earnings from your decisions, you will receive 3 Euros. This payment is to compensate you for showing up on time. At the end of the experiment the total amount of money that you have earned will be paid to you privately in cash.

Setting of the experiment

In the experiment, there are two roles: *observed* and *unobserved*.

7 participants have been assigned randomly to the role of observed. All 8 remaining participants have been assigned to the role of unobserved. Each participant remains in the same role for the entire duration of the experiment.

The experiment consists of 4 parts. Instructions for the first part of the experiment will be distributed in a few moments. We ask you to read the instructions for the first part of the experiment carefully, and once each participant has done so an experimenter will read them aloud. After the instructions for the first part of the experiment have been read aloud, you will be informed about the role you have been assigned to, *observed* or *unobserved*. Instructions for the second, third, and fourth part of the experiment will be made available before each of the respective parts begins.

E.2. Instructions for Part 1

Part 1 of the experiment consists of 3 independent rounds and each round is conducted in the same way.

A. How a round progresses

1. The *assistant* picks either BLUE or ORANGE at random.

Each round begins with the *assistant* picking either the color **BLUE** or the color **ORANGE** at random. You and all other participants have just been instructed about the picking procedure which is as follows:

1. An experimenter shuffles a deck of 20 cards and lays them down on a table with the back of the cards facing the *assistant*. **11** cards have a **blue front** and **9** cards have an **orange front**.
2. The *assistant* picks 1 card out of the 20 cards.
    - If the picked card has a **blue front** then the color picked at random is **BLUE**.
    - If the picked card has an **orange front** then the color picked at random is **ORANGE**.

In each round your task, which is also the task of each of the other participants, is to guess which color has been picked at random by the *assistant*.

2. The *assistant* selects the "OBSERVED" and "UNOBSERVED" urns

Once a color has been picked at random, the *assistant* selects an urn labeled "OBSERVED" and an urn labeled "UNOBSERVED" from a collection of urns containing **blue** and **orange** balls.

The composition of the urn labeled "OBSERVED" depends only on the color which has been picked at random by the *assistant*. The composition of the urn labeled "OBSERVED" is

In case the color **BLUE** has been picked, the "OBSERVED" urn contains **14 blue** and **7 orange** balls.	In case the color **ORANGE** has been picked, the "OBSERVED" urn contains **7 blue** and **14 orange** balls.

The composition of the urn labeled "UNOBSERVED" also depends only on the color picked at random by the *assistant*. The composition of the urn labeled "UNOBSERVED" is

In case the color **BLUE** has been picked, the "UNOBSERVED" urn contains **14 blue** balls and **7 orange** balls.	In case the color **ORANGE** has been picked, the "UNOBSERVED" urn contains **7 blue** balls and **14 orange** balls.

3. Each participant learns the color of 1 ball

Once the "OBSERVED" and "UNOBSERVED" urns have been selected by the *assistant*, each *observed* is informed about the color of a ball drawn from the "OBSERVED" urn whereas each *unobserved* is informed about the color of a ball drawn from the "UNOBSERVED" urn. Concretely,

- one of the experimenters approaches each *observed*, one at a time, to draw a ball from the "OBSERVED" urn. Each *observed* draws a ball without being able to see the composition of the "OBSERVED" urn. After each draw, the ball is returned to the urn before making the next draw. Apart from the participant who draws the ball, no other participant sees its color. Thus, each *observed* is informed about the color of 1 and only 1 ball drawn from the "OBSERVED" urn.

- another experimenter approaches each *unobserved*, one at a time, to draw a ball from the "UNOBSERVED" urn. Each *unobserved* draws a ball without being able to see the composition of the "UNOBSERVED" urn. After each draw, the ball is returned to the urn before making the next draw. Apart from the participant who draws the ball, no other participant sees its color. Thus, each *unobserved* is informed about the color of 1 and only 1 ball drawn from the "UNOBSERVED" urn.

4. Each participant makes a guess

Each round consists of 8 guessing periods with one *observed* making a guess in each of the first seven periods and one *unobserved* making a guess in each of the eight periods. Thus, each participant makes one and exactly one guess in each round.

In each round the order in which *observed* make their guesses is randomly determined. If you have been assigned to the role of an *observed* then, in a given round, you might be the first *observed* to make a guess, or you might guess in any period from period 2 to period 6, or you might be the last *observed* to make a guess.

Similarly, in each round the order in which *unobserved* make their guesses is randomly determined. If you have been assigned to the role of an *unobserved* then, in a given round, you might be the first *unobserved* to make a guess, or you might guess in any period from period 2 to period 7, or you might be the last *unobserved* to make a guess.

First guessing period. In period 1, 1 *observed* and 1 *unobserved* are asked to guess which color has been picked at random by the *assistant*. Once both guesses have been made, period 2 starts. The *observed* and the *unobserved* who made a guess in period 1 do not make any further guess in the current round.

Guessing period 2 to 7. In period 2 to 7, the guess made by the *observed* in the previous period is made public meaning that all other *observed* as well as all *unobserved* are informed of that guess. After that, 1 *observed* and 1 *unobserved* are asked to guess which color has been picked at random by the *assistant*. Both participants do not make any further guess in the current round. Once both guesses have been made, the next period starts.

Last guessing period. In period 8, the guess made by the *observed* in period 7 is made public and only the *unobserved* who did not make a guess yet is asked to guess which color has been picked at random by the *assistant*.

Please note that the guess made by each of the *unobserved* is kept private meaning that no other *unobserved* and no *observed* is informed of the guess made by any of the *unobserved*.

Once each participant has made a guess, you and each of the other participants are informed of the color that was actually picked at random by the *assistant* at the beginning of the round. Once all participants have been informed, the round is over.

B. Earnings

In each of the 3 independent rounds, participants get paid for the guess they make. If the participant's guess matches the color picked at random by the *assistant*, the participant earns 1 Euro. If the participant's guess does not match the color picked at random by the *assistant*, the participant earns nothing.

Once the 3 independent rounds have been completed, participants are informed of the total amount of euros they earned in the first part of the experiment.

E.3. Instructions for Part 2

The second part of the experiment shares many similarities with the first part of the experiment. Still, the two parts of the experiment differ in some respects.

Hereafter, we explain thoroughly the aspects of the second part of the experiment which were not present in the first part of the experiment. On the other hand, the aspects of the second part of the experiment which were already present in the first part of the experiment are merely mentioned without much detail.

Part 2 of the experiment consists of 6 independent rounds and each round is conducted in the same way.

A. How a round progresses

1. The *assistant* picks either BLUE or ORANGE at random

Each round begins with the *assistant* picking either the color **BLUE** or the color **ORANGE** at random. The picking procedure used in part 2 of the experiment is identical to the picking procedure used in part 1 of the experiment.

In each round your task, which is also the task of each of the other participants, is to guess which color has been picked at random by the *assistant*.

2. Each participant learns the color of 1 ball

Once a color has been picked at random by the *assistant*, each *observed* is informed about the color of a ball drawn from the "OBSERVED" *virtual* urn whereas each *unobserved* is informed about the color of a ball drawn from the "UNOBSERVED" *virtual* urn.

Detailed explanations about the drawing of balls from *virtual* urns will be displayed on the screen of your computer after all participants have finished reading these two pages.[7]

3. Each participant makes guesses

In each round, after having learned the color of 1 ball, each participant has to guess which color has been picked at random by the assistant. Each round consists of 8 guessing periods.

- In each round, an *observed* makes between 1 and 7 guesses.
- In each round, each *unobserved* makes 8 guesses.

First guessing period. In period 1, all 7 *observed* and all 8 *unobserved* are asked to guess which color has been picked at random by the *assistant*. Once all 15 guesses have been made, period 2 starts.

Guessing period 2. At the beginning of period 2, the guess made by 1 of the 7 *observed* in period 1 is selected at random and this guess is shown to all 15 participants. The *observed* whose guess is randomly selected does not make any further guess in the current round. Therefore, only 6 *observed* remain who can guess in period 2. Afterwards, all 6 remaining *observed* and all 8 *unobserved* are asked to guess which color has been picked at random by the *assistant*. Once all 14 guesses have been made, period 3 starts.

Guessing periods 3, 4, 5, and 6. At the beginning of the period, the guess made by 1 of the *observed* in the previous period is selected at random and this guess is shown to all 15 participants. The *observed* whose guess is randomly selected does not make any further guess in the current round.

[7]The short on-screen demonstration of the draws from the virtual urns is available from the authors upon request.

Afterwards, all remaining *observed* and all 8 *unobserved* are asked to guess which color has been picked at random by the *assistant*. Once all guesses have been made, the next period starts.

Guessing period 7. At the beginning of period 7, the guess made by 1 of the 2 *observed* in period 6 is selected at random and this guess is shown to all 15 participants. The *observed* whose guess is randomly selected does not make any further guess in the current round. Therefore, only 1 *observed* remains who can guess in period 7. Afterwards, the *observed* and all 8 *unobserved* are asked to guess which color has been picked at random by the *assistant*. Once all 9 guesses have been made, period 8 starts.

Last guessing period. At the beginning of period 8, the guess made by the *observed* in period 7 is shown to all 15 participants. The *observed* who guessed in period 7 does not make a guess in period 8. Therefore, only the 8 *unobserved* are asked to guess which color has been picked at random by the *assistant*.

Please note that the guesses made by each of the *unobserved* are kept private meaning that no other *unobserved* and no *observed* is informed of the guesses made by any of the *unobserved*.

Once all participants have made all their guesses, you and each of the other participants are informed of the color that was actually picked at random by the *assistant* at the beginning of the round. Once all participants have been informed, the round is over.

B. Earnings

In each of the 6 independent rounds, each participant gets paid for 1 and only 1 of the guesses made. If the participant's guess matches the color picked at random by the *assistant*, the participant earns 1 Euro. If the participant's guess does not match the color picked at random by the *assistant*, the participant earns nothing.

1. **For each *observed*, only the last guess is paid.**
   Each *observed* gets paid only for the last guess he/she made in the round. Said differently, the guess of an observed is paid only in case the guess is made public meaning that it is observed by all 15 participants. Obviously, at the time a guess is made, an *observed* does not know whether the guess is going to be made public or not. So, for each guess that an *observed* makes, there is a chance that this guess is the one which is going to be paid.

2. **For each *unobserved*, only the guess of the assigned period is paid.**
   In each of the 6 independent rounds, each *unobserved* makes a guess in each period for a total of 8 guesses. Once each of the *unobserved* has made 8 guesses, the round is over. As soon as the round is over, each of the 8 *unobserved* is assigned a period number from 1 to 8. Concretely, one of the *unobserved* is assigned to period 1, another *unobserved* is assigned to period 2, ..., and another *unobserved* is assigned to period 8. The assignment is completely random meaning that the guesses made by the *unobserved* do not influence the period numbers assigned to them. An *unobserved* gets paid only for the guess made in the assigned period. Obviously, before having made all 8 guesses, an *unobserved* does not know which period number is assigned to her/him. So, each guess that an *unobserved* makes has an equal chance of being paid.

Once the 6 independent rounds have been completed, participants are informed of the total amount of euros they earned in the second part of the experiment.

E.4. Instructions for Part 3

Part 3 of the experiment consists of 6 independent rounds. Each round proceeds the same way as in part 2 except that

In case the color **BLUE** has been picked, the "UNOBSERVED" urn contains **18 blue** balls and **3 orange** balls.	In case the color **ORANGE** has been picked, the "UNOBSERVED" urn contains **3 blue** balls and **18 orange** balls.

Once the 6 independent rounds have been completed, participants are informed of the total amount of euros they earned in the third part of the experiment.

E.5. Demographic Questionnaire

1. What is your field of study?
2. When were you born? (Month/Year)
3. Your gender: ☐ Female ☐ Male

To know our subject pool better, it would be helpful to learn about your cultural background. We thus ask you to also answer the following questions.

4. What is your first language?
   (By first language we mean the language you have mainly spoken during your childhood or at your family home.)
5. What is your nationality?

E.6. Instructions for Part 4

Part 4 of the experiment consists of 6 independent rounds. Each round proceeds the same way as in part 3 except that

In case the color **BLUE** has been picked, the "UNOBSERVED" urn contains **12 blue** balls and **9 orange** balls.	In case the color **ORANGE** has been picked, the "UNOBSERVED" urn contains **9 blue** balls and **12 orange** balls.

Once the 6 independent rounds have been completed, participants are informed of the total amount of euros they earned in the course of the experiment.

References

Ali, S. N., and N. Kartik (2012): "Herding with Collective Preferences," *Economic Theory*, 51, 601–626.

Anderson, L., and C. Holt (1997): "Information cascades in the laboratory," *American Economic Review*, 87, 847–862.

Cameron, A. C., and D. L. Miller (2010): "Robust Inference with Clustered Data," in *Handbook of Empirical Economics and Finance*, ed. by A. Ullah, and D. E. Giles, pp. 1–28. Chapman & Hall/CRC.

Cameron, A. C., and P. K. Trivedi (2005): *Microeconometrics*. New York: Cambridge University Press.

Goeree, J., T. Palfrey, B. Rogers, and R. McKelvey (2007): "Self-correcting information cascades," *Review of Economic Studies*, 74, 733–762.

McKelvey, R. D., and T. R. Palfrey (1995): "Quantal Response Equilibria for Normal Form Games," *Games and Economic Behavior*, 10, 6–38.

——— (1998): "Quantal Response Equilibria for Extensive Form Games," *Experimental Economics*, 1, 9–41.

Smith, L., and P. Sørensen (2000): "Pathological Outcomes of Observational Learning," *Econometrica*, 68, 371–398.

Weizsäcker, G. (2010): "Do we follow others when we should? A simple test of rational expectations," *American Economic Review*, 100, 2340–2360.

Ziegelmeyer, A., C. March, and S. Krügel (2013): ""Do We Follow Others when We Should? A Simple Test of Rational Expectations": Comment," *American Economic Review*, 103, 2633–2642.