The international platform of Ludwig-Maximilians University's Center for Economic Studies and the Ifo Institute

CESifo Working Papers
www.cesifo.org/wp

A Neglected Semi-Stylized Fact of Daily Stock Returns

Walter Krämer

CESIFO WORKING PAPER NO. 5806
CATEGORY 7: MONETARY POLICY AND INTERNATIONAL FINANCE
MARCH 2016

An electronic version of the paper may be downloaded
- *from the SSRN website:* *www.SSRN.com*
- *from the RePEc website:* *www.RePEc.org*
- *from the CESifo website:* *www.CESifo-group.org/wp*

ISSN 2364-1428

CESifo
Center for Economic Studies & Ifo Institute

CESifo *Working Paper No. 5806*

A Neglected Semi-Stylized Fact of Daily Stock Returns

Abstract

We plot aggregated daily stock returns with absolute value less than x against x and show empirically that this often produces a typical spoon-shaped pattern which indicates a special type of asymmetry which has not been widely discussed before. This pattern disappears when individual returns are averaged; it is also absent in stock price indices, which points to explanations based on firm-specific drivers of returns.

JEL-Codes: C400, G120, G140.

Keywords: stock returns, skewness, symmetry.

Walter Krämer
Faculty of Statistics
TU Dortmund
Dortmund / Germany
walterk@statistik.tu-dortmund.de

Research supported by DFG-Sonderforschungsbereich 823. I am grateful to Simon Neumärker and Kaya Miah for expert computational assistance and to Joachim Grammig, André Güttler and Günter Bamberg for helpful discussions and comments.

1 Introduction and Summary

Let r_t be some (time-continuous) daily stock return (adjusted for dividends, stock-splits and so on). This paper is concerned with distributional (as opposed to time-series) properties of r_t. Typical among these are heavy tails as evidenced by a curtosis larger than 3, i.e. larger than for normality, or the near independence of $|r_t|$ and $sign(r_t)$ (with equal probabilities for positive and negative), see e.g. Granger and Ding (1995), Granger et al. (2000), Rydén et al. (1998), Cont (2001) or Teräsvirta and Zhao (2011), among many others.

Below we are concerned with the relationship between $|r_t|$ and $sign(r_t)$. That these cannot be exactly independent, at least under the standard assumption that the return density is univariate and symmetric, follows at once from the fact that $\sum_1^T r_t > 0$ for large T and almost all stocks (otherwise, there would be no incentive to hold them). In technical terms,

$$E(r_t) = \mu > 0 \tag{1}$$

where, for daily data, μ_t is small and often neglected, but still positive. This then implies that, under standard assumptions, returns with absolute values less than μ are more likely to be positive than negative.

The present paper argues that, even after taking (1) into account and considering $|r_t - \mu|$ instead, independence of $|r_t - \mu|$ and $sign(r_t - \mu)$ does not obtain in practice; in fact, it is violated in a very peculiar fashion. Sticking to the assumption that the unconditional density of r_t is time invariant, unimodal and symmetric, it is immediately obvious from the form of the density function that

$$0 < E(r_t|\ |r_t| < x) < \mu \tag{2}$$

and that $E(r_t|\ |r_t| < x)$ is increasing in x. Therefore $E\left(\sum_{t=1}^T r_t\ I_{|r_t|\leq x}\right)$ is likewise increasing in x and, by plotting

$$\sum_{t=1}^{T} r_t\ I_{|r_t|<x} \tag{3}$$

for some preselected and fixed values of x and joining points by straight lines, one should, on average (i.e. by averaging over independent draws from the random

variables (3)); obtain a monotonically increasing function. In practice, however, what one often observes looks like this:

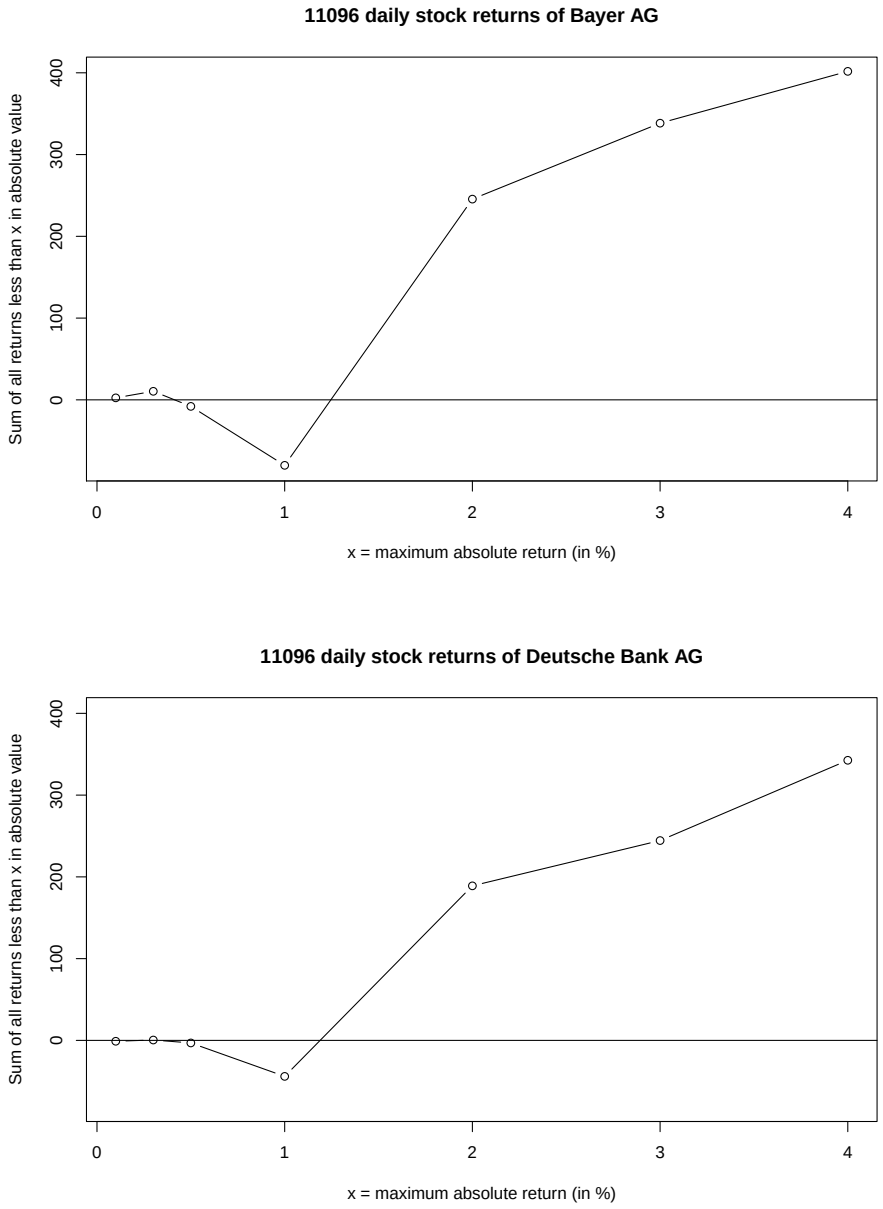


Figure 1: Aggregrated stock returns of Bayer and Deutsche Bank plotted against maximum absolute value.

Or more formally: The distribution of r_t cannot be exactly symmetric around

some nonnegative mean. And the spoon shaped-pattern from figure 1 indicates a particular form of asymmetry; it repeats itself for many other constituents of the German stock price index DAX, for half the constituents of the Dow Jones Industrial average and for many other individual stocks. In particular, the sum of all returns less than 1% in absolute value is very often negative. We call this a semi-stylized fact because it is not as universal as excess kurtosis but too frequent to be easily explainable by chance (given symmetrically distributed returns).

Our analysis below is mainly descriptive, with the aim of soliciting a more thorough discussion both from the viewpoint of economics and statistics. For an in-depth statistical discussion of various stylized facts of stock returns and how to model them see Davies and Krämer (2015).

2 Additional empirical evidence

Figure 2 plots 256, 356 daily returns, covering the years 1973-2015 of all companies which are currently covered by the German stock price index DAX, from 1973 to 2015.

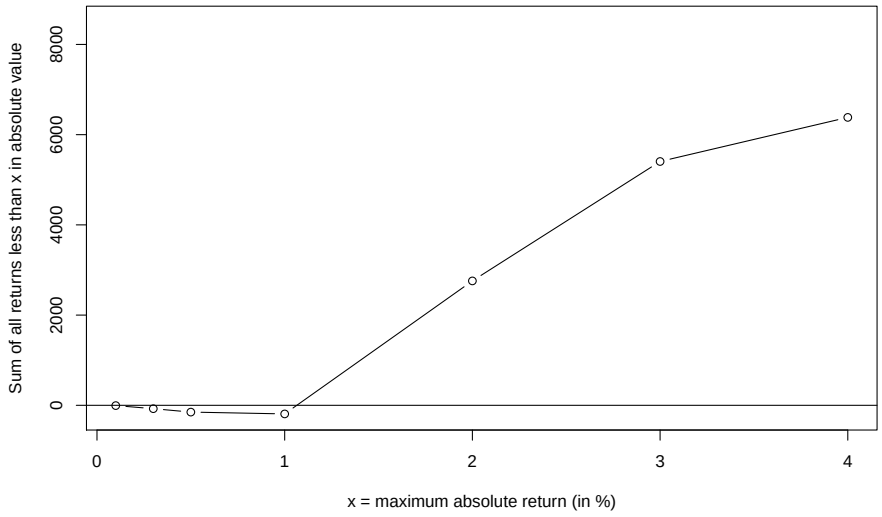


Figure 2: 256356 daily German stock returns.

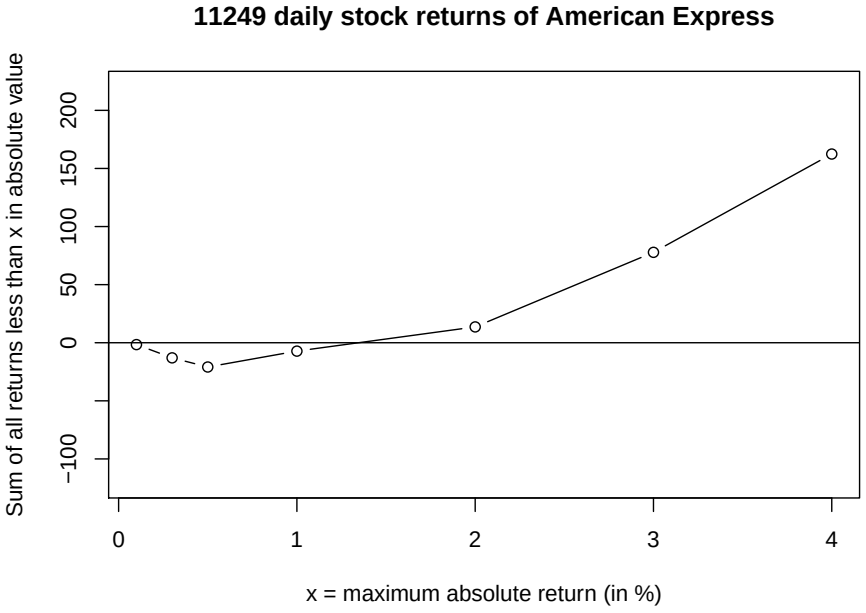
The pattern shown above for Bayer and Deutsche Bank persists, albeit less pronounced. Figure A1 in the appendix repeats this exercise for the remaining 28 DAX-constituents. It shows that for 21 of the remaining 28 companies, cumulated returns with absolute value less than x are negative for x in some range between 0.5 and 2.

Table 1 reports a count of positive and negative returns - excluding zeros - with absolute values less than some threshold for all DAX-companies. It shows that, overall, the percentage of negatives is slightly smaller than that of positives for most companies (23 out of 30), with both numbers clustering around 50%, as one would expect from a return distribution which is symmetric around some small nonnegative mean (small in relation to the spread). However, for small thresholds, the percentage of negatives increases, with 21 companies each reporting more negatives for a threshold of 0.5% and 1%.

We also checked the spoon effect for an independent data set composed of the constituents of the Dow Jones Industrial Average. The results are in the appendix and show that there exists a spoon effect for may shares, but less than for the constituents of the DAX. This might point to liquidity - which is larger for constituents of the Dow Jones than for constituents of the DAX - as a possible explanation; see below. As an example, figure 3 exhibits the spoon effect for Pfizer and American Express.

Table 1: Percentage of negative returns among returns with absolute value less than some threshold

	threshold											
	0.5%			1%			2%			∞		
Company	+	-	%	+	-	%	+	-	%	+	-	%
Adidas	596	634	0.52	1137	1204	0.51	1809	1886	0.51	2436	2432	0.50
Allianz	1600	1549	0.49	2843	2743	0.49	4204	3990	0.49	5193	4940	0.49
BASF	1691	1733	0.51	3062	3102	0.50	4522	4325	0.49	5320	5098	0.49
Bayer	1678	1641	0.49	2998	3058	0.50	4441	4278	0.49	5308	5091	0.49
Beiersdorf	1564	1535	0.50	2848	2746	0.49	4015	3862	0.49	4793	4557	0.49
BMW	1444	1497	0.51	2676	2702	0.50	4122	4042	0.50	5197	5065	0.49
Commerzbank	1434	1423	0.50	2642	2719	0.51	3981	4024	0.50	5116	5185	0.50
Continental	1244	1219	0.49	2333	2387	0.51	3739	3805	0.50	5154	5074	0.50
Daimler	482	479	0.50	926	899	0.49	1551	1501	0.49	2112	2079	0.50
Deutsche Bank	1676	1710	0.51	2888	2998	0.51	4295	4267	0.50	5231	5201	0.50
Deutsche Boerse	485	481	0.50	892	878	0.50	1389	1366	0.50	1844	1764	0.49
Deutsche Post	499	497	0.50	942	920	0.49	1480	1430	0.49	1860	1790	0.49
Deutsche Telekom	626	657	0.51	1153	1207	0.51	1754	1764	0.50	2305	2333	0.50
E.ON	1711	1759	0.51	3060	3046	0.50	4487	4302	0.49	5249	5084	0.49
Fresenius Medical Care	661	680	0.51	1241	1255	0.50	1854	1878	0.50	2341	2321	0.50
Fresenius	639	678	0.51	1297	1301	0.50	2050	1999	0.49	2754	2611	0.49
HeidelbergCement	1206	1320	0.52	2274	2461	0.52	3474	3641	0.51	4645	4782	0.51
Henkel	638	676	0.51	1190	1231	0.51	1846	1897	0.51	2340	2321	0.50
Infineon Technologies	330	348	0.51	653	657	0.50	1114	1147	0.51	1887	1946	0.51
K+S	1186	1307	0.52	2376	2543	0.52	3741	3904	0.51	4973	4996	0.50
Lanxess	266	270	0.50	490	546	0.53	856	907	0.51	1316	1322	0.50
Linde	1683	1728	0.51	3021	3088	0.51	4333	4321	0.50	5139	5029	0.49
Deutsche Lufthansa	1038	1070	0.51	2115	2206	0.51	3596	3672	0.51	4984	5002	0.50
Merck	664	716	0.52	1187	1253	0.51	1885	1870	0.50	2460	2404	0.49
Munich Re	1357	1316	0.49	2562	2455	0.49	3855	3653	0.49	4920	4641	0.49
RWE	1791	1803	0.50	3100	3055	0.50	4369	4252	0.49	5153	5039	0.49
SAP	830	856	0.51	1612	1550	0.49	2527	2334	0.48	3401	3132	0.48
Siemens	1772	1780	0.50	3105	3027	0.49	4502	4232	0.48	5345	5065	0.49
ThyssenKrupp	1301	1326	0.50	2437	2471	0.50	3953	3933	0.50	5187	5095	0.50
Volkswagen	1267	1365	0.52	2449	2567	0.51	3958	4055	0.51	5204	5180	0.50



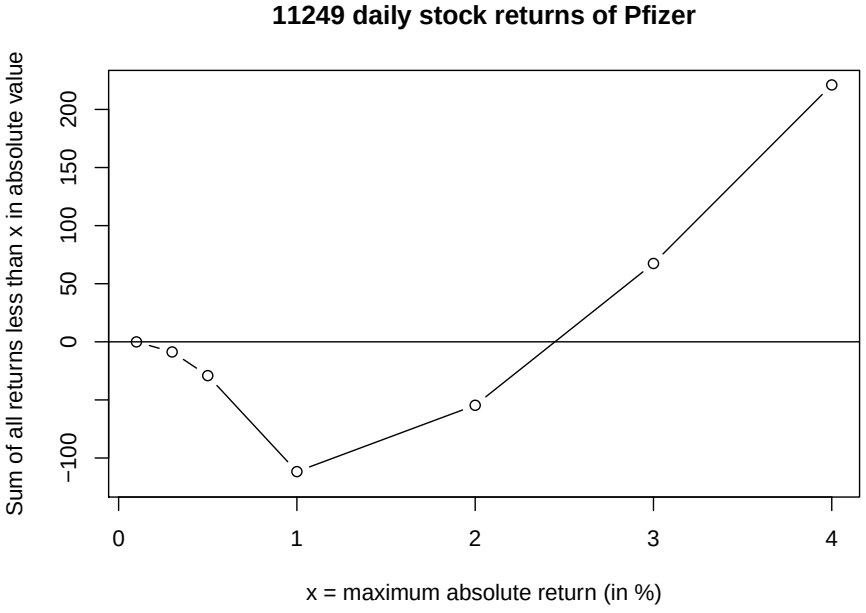
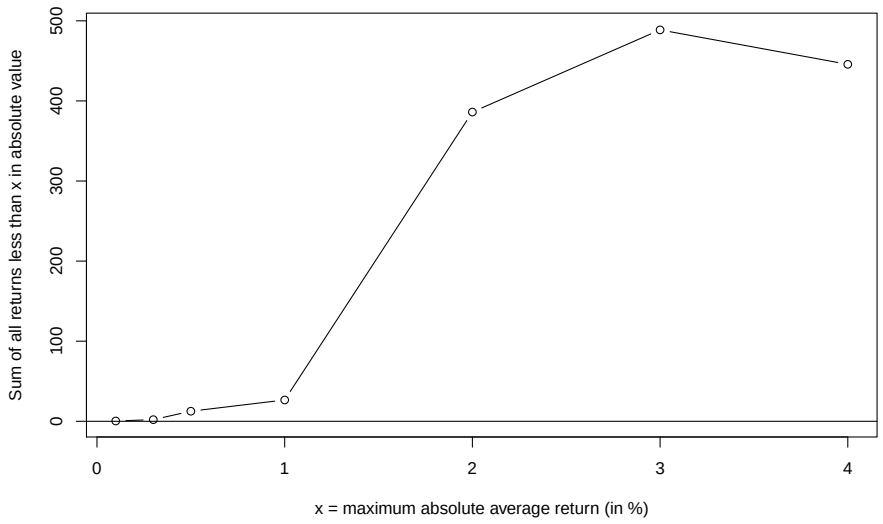


Figure 3: Spoon effect for Pfizer and American Express.

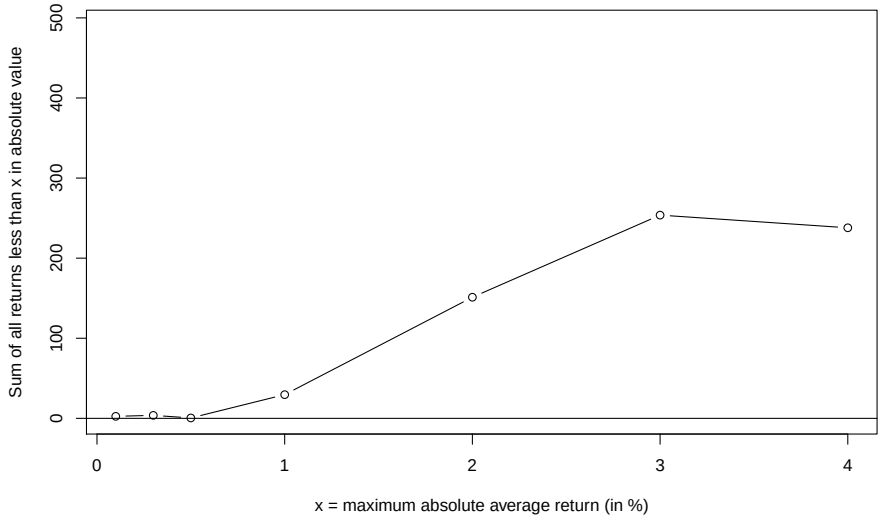
3 Possible explanations

The major purpose of the present note is to point out the effect as such. We are not concerned with its consistency with more elaborate models for returns, as for instance discussed in Davies and Krämer (2015). Among possible economic explanations, one might think of transaction costs and small differences between "true" (=efficient) prices and the market price induced by them: to the extent that efficient prices are more often slightly above than slightly below the market price, and changes of efficient prices cluster around zero, small negative changes in the market price are more frequent than they otherwise would be. Likewise, other market microstructure specifics such as short selling restrictions might induce a prevalence of small negative changes, along the lines of Diamond and Verrechia (1987): As traders who perceive a reduction in the efficient price are not allowed to sell, it takes longer for negative information to affect the market price, which then adjusts downwards not at once but in consecutive smaller steps. This is in line with the observation from the microstructure literature that bad news and trading volume are often negatively correlated.

Another simple explanation that comes to mind is what in marketing is called a threshold effect (Bemmaor (1984)): If we keep to the basic assumption that stock prices are moved by news, then one might hypothesize that information pertaining to the value of a stock has to cross some importance-threshold to be recognized in the first place by either the media or the investor or both. And according to the old saying "only bad news is good news", this threshold might be lower for bad news on the margin of general importance. A cursory perusal of two leading German business papers, Frankfurter Allgemeine Zeitung and Handelsblatt, indeed provides empirical evidence that firm specific borderline-news - defined as appearing in only one of the papers, but not in both - are predominantly negative. Therefore, the spoon effect should be weaker or disappear if returns are averaged across firms. Figure 4 shows that this is indeed the case for both Bayer and Deutsche Bank and Pfizer and American Express. Each individual return exhibits a marked spoon effect, but the respective averages do not.



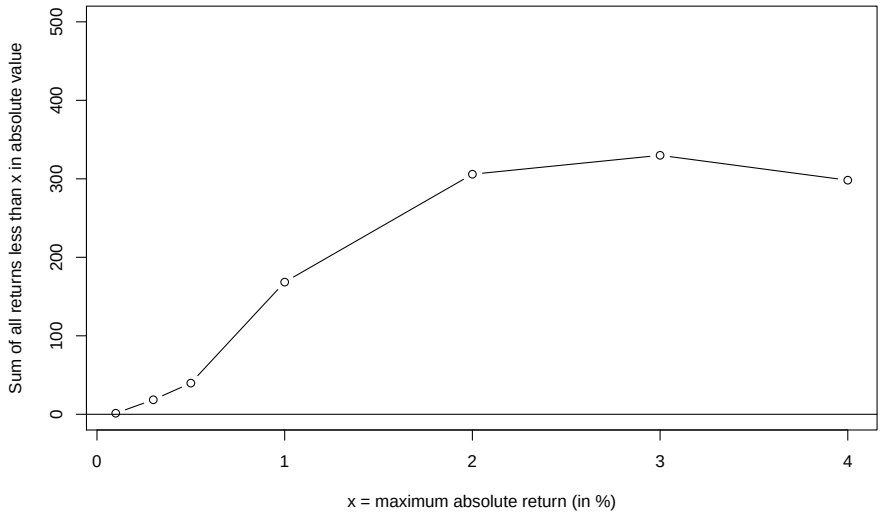
(a) Average of 11096 daily stock returns for Deutsche Bank and Bayer.



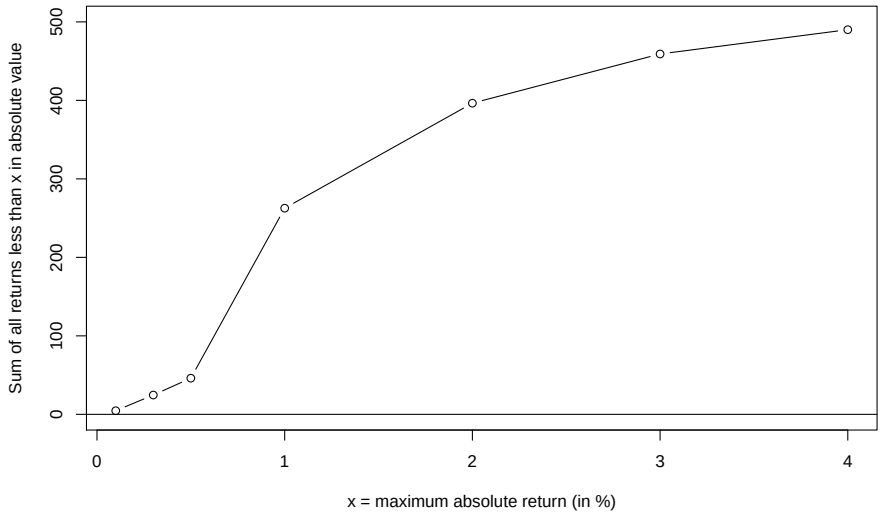
(b) Average of 11249 daily stock returns for American Express and Pfizer.

Figure 4: No spoon effect for average returns.

Similarly, the returns of the DAX and the Dow Jones do not show any spoon effect (see figure 5).



(a) 6350 daily DAX returns.



(b) 15133 daily Dow Jones returns.

Figure 5: No spoon effect for price indices.

Still another avenue which might be worth exploring is the observation by Kudryavtsev (2013) that returns are lower following days with relatively large

low-to-close price differences. While this so far is only correlation, it might help to identify a common factor responsible for both.

Finally, it might be worth noting that the spoon effect presented here is not an artifact of using discrete time returns: If time-continuous returns are symmetric and normal with mean zero, is is easily seen that discrete-time returns must then exhibit some spoon effect due to the skewness of the lognormal distribution. But returns used in the examples above are already in continuous time.

References

Bemmaor, A. C. (1984). Testing alternative econometric models on the existence of advertising threshold effect. *Journal of Marketing Research*, **21**, 289-308.

Cont, R. (2001). Empirical properties of asset returns: stylized facts and statistical issues. *Quantitative Finance*, **1**, 223-236.

Davies, L. and Krämer, W. (2015). Stylized facts and simulating long range financial data. *SFB 823 discussion paper*, 48/15, Dortmund `https://www.statistik.tu-dortmund.de/fileadmin/user_upload/DP_4815_SFB823_Davies_Kraemer.pdf`.

Diamond, D. W. and Verrechia, R. E. (1987): Constraints on short-selling and asset price adjustment to private information. *Journal of Financial Economics*, **18(2)**, 277-311.

Granger, C. W. J. and Ding, Z. (1995). Some properties of absolute returns: An alternative measure of risk, *Annales d'Economie et de Statistique*, **40**, 67-91.

Granger, C. W. J., Spear, S. A., and Ding, Z. (2000). Stylized facts on the temporal and distributional properties of absolute returns: An update, in Chan W. et al.: Statistics and Finance: An interface, 97-120.

Kudryavtsev, A. (2013). Stock price reversals following end-of-the-day price moves, *Economics Letters*, **118(1)**, 203-205.

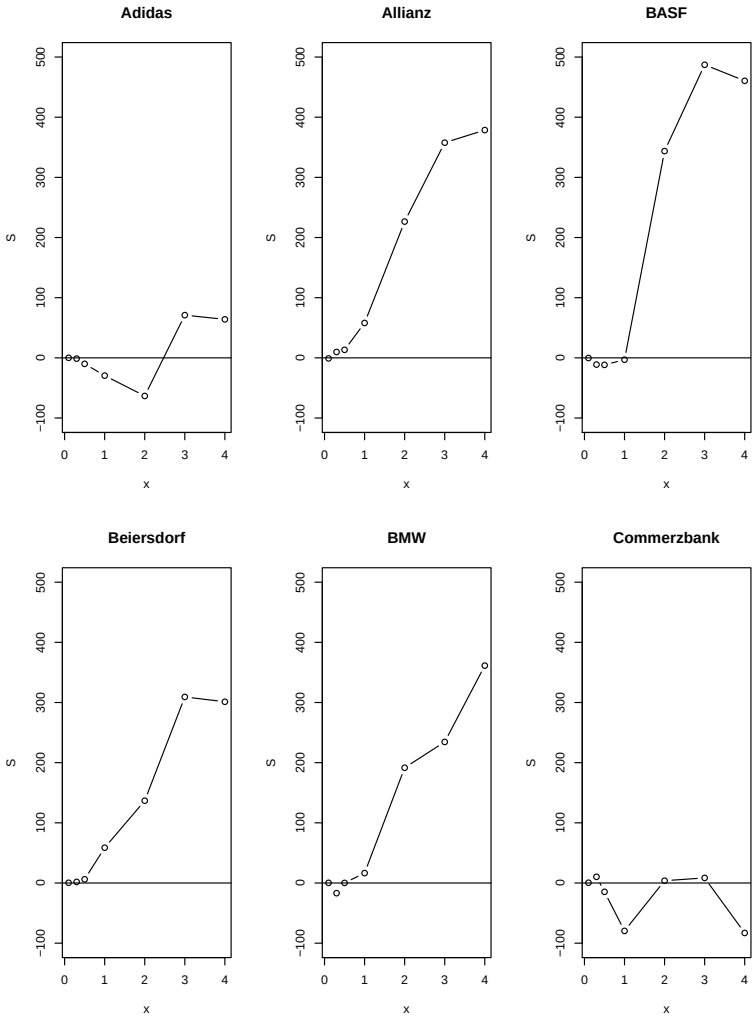
Malmsten, H. and Teräsvirta, T. (2010). Stylized facts of financial time series and three popular models of volatilty. *European Journal of Pure and Applied Mathematics*, **3(3)**, 443-477.

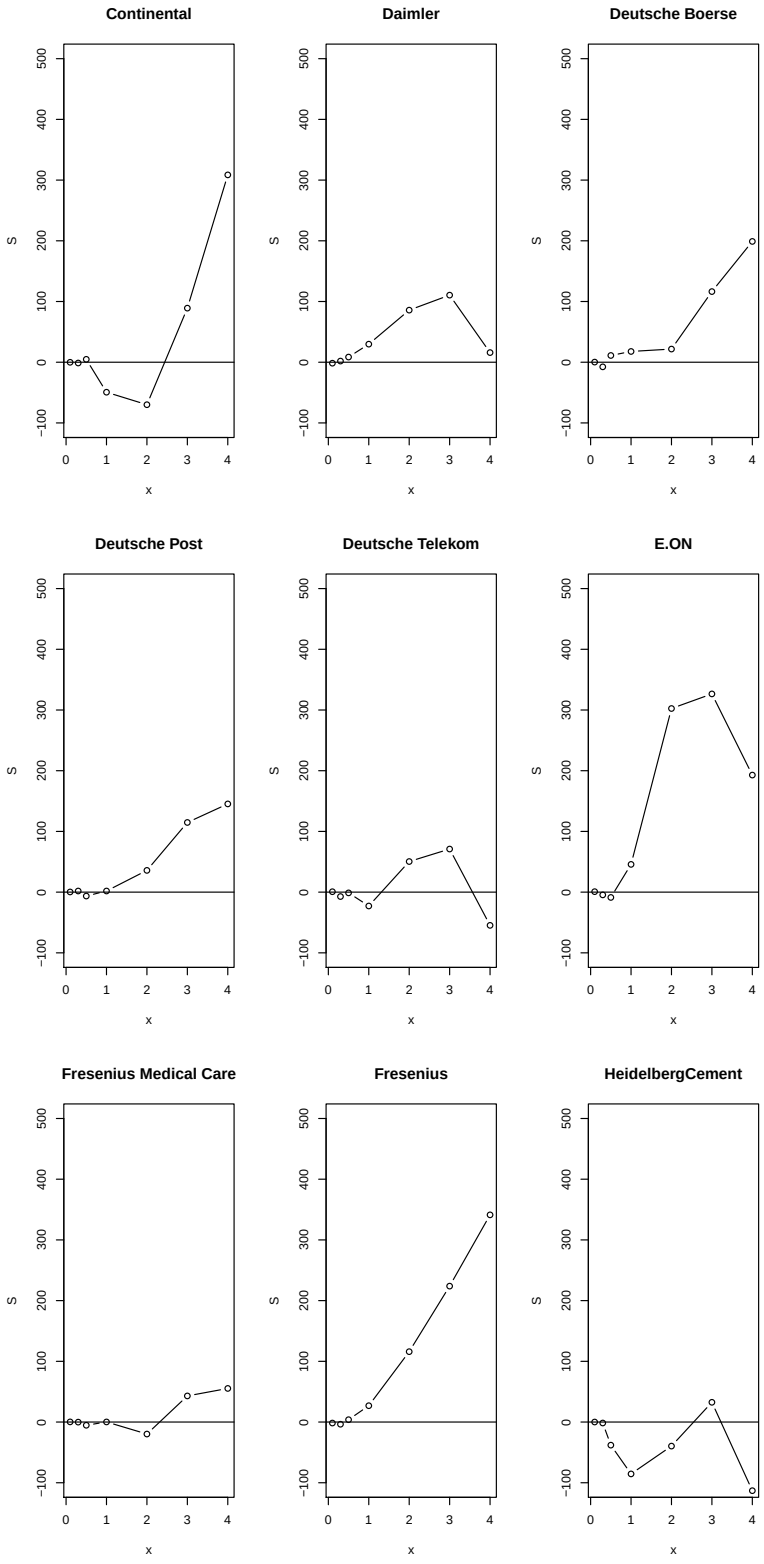
Rydén, T., Teräsvirta, T. and Åsbrink, S. (1998). Stylized facts of daily return series and the hidden markov model. *Journal of Applied Econometrics*, **13**, 217-244.

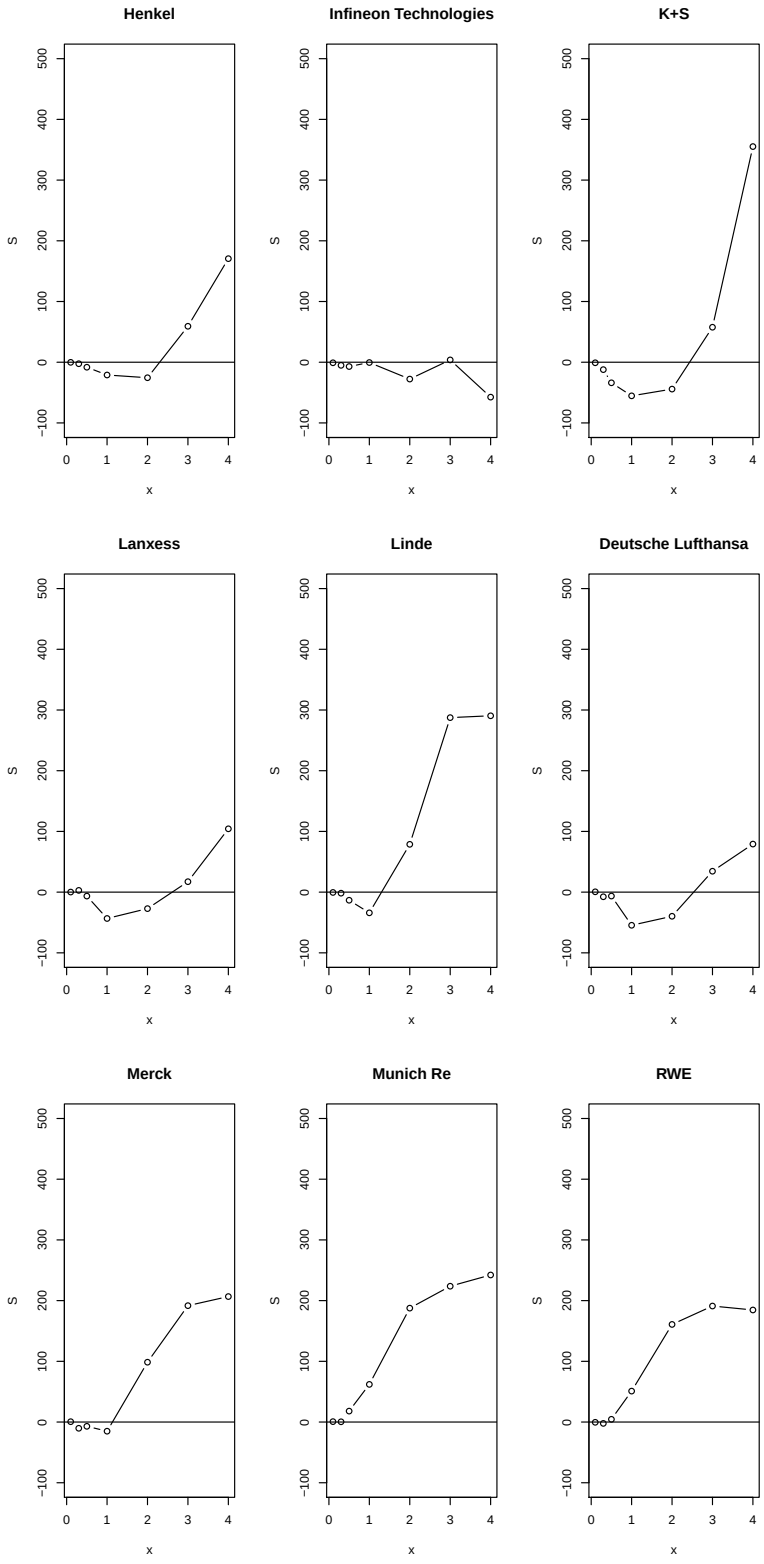
Teräsvirta, T. and Zhao, Z. (2010). Stylized facts of return series, robust estimates and three popular models of volatility. *Applied Financial Economics*, **21(1)**, 67-94.

Appendix

Figure A1: Daily stock returns for remaining DAX-constituents, where x denotes the returns (in %) and S the sum of all returns less than x in absolute value (also in %).




Continental
Daimler
Deutsche Boerse
Deutsche Post
Deutsche Telekom
E.ON
Fresenius Medical Care
Fresenius
HeidelbergCement
S
x
−100
0
100
200
300
400
500
0
1
2
3
4


Henkel
Infineon Technologies
K+S
Lanxess
Linde
Deutsche Lufthansa
Merck
Munich Re
RWE
S
x

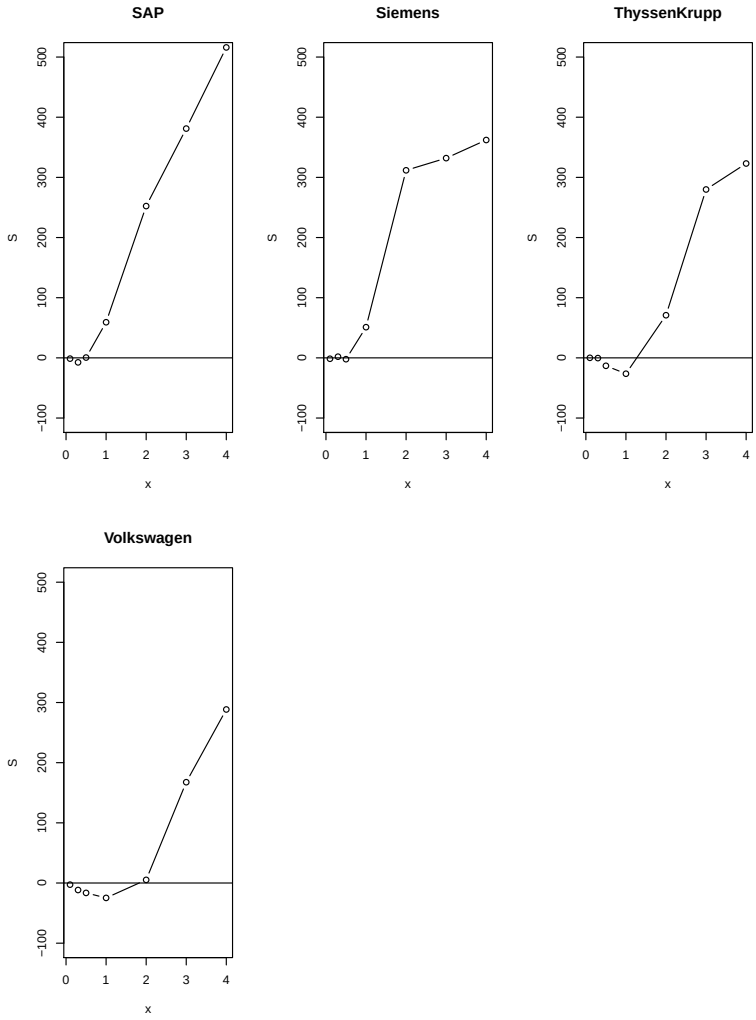
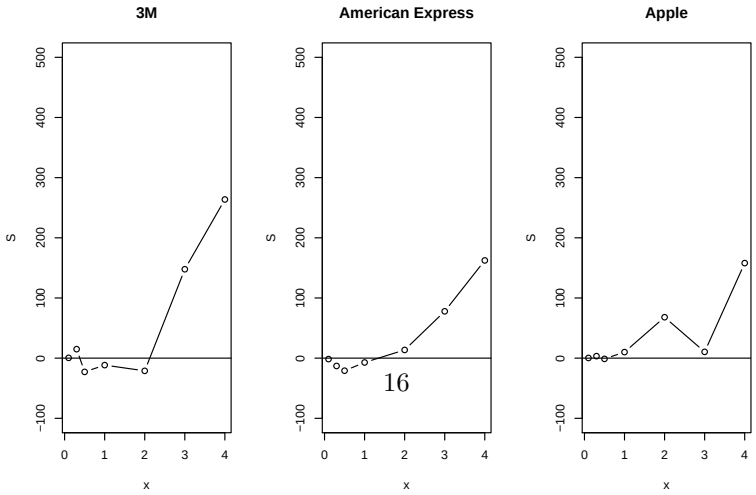
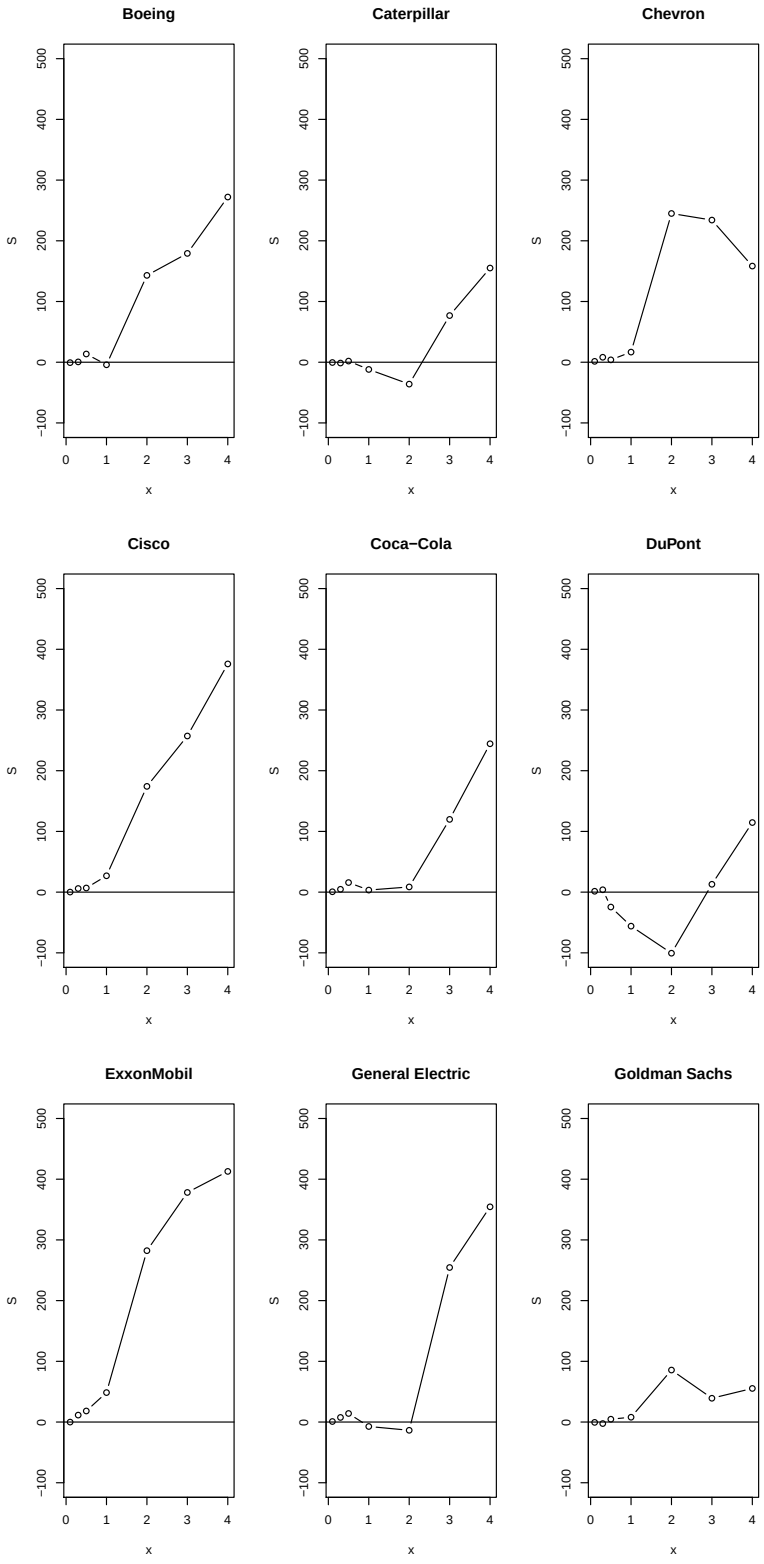
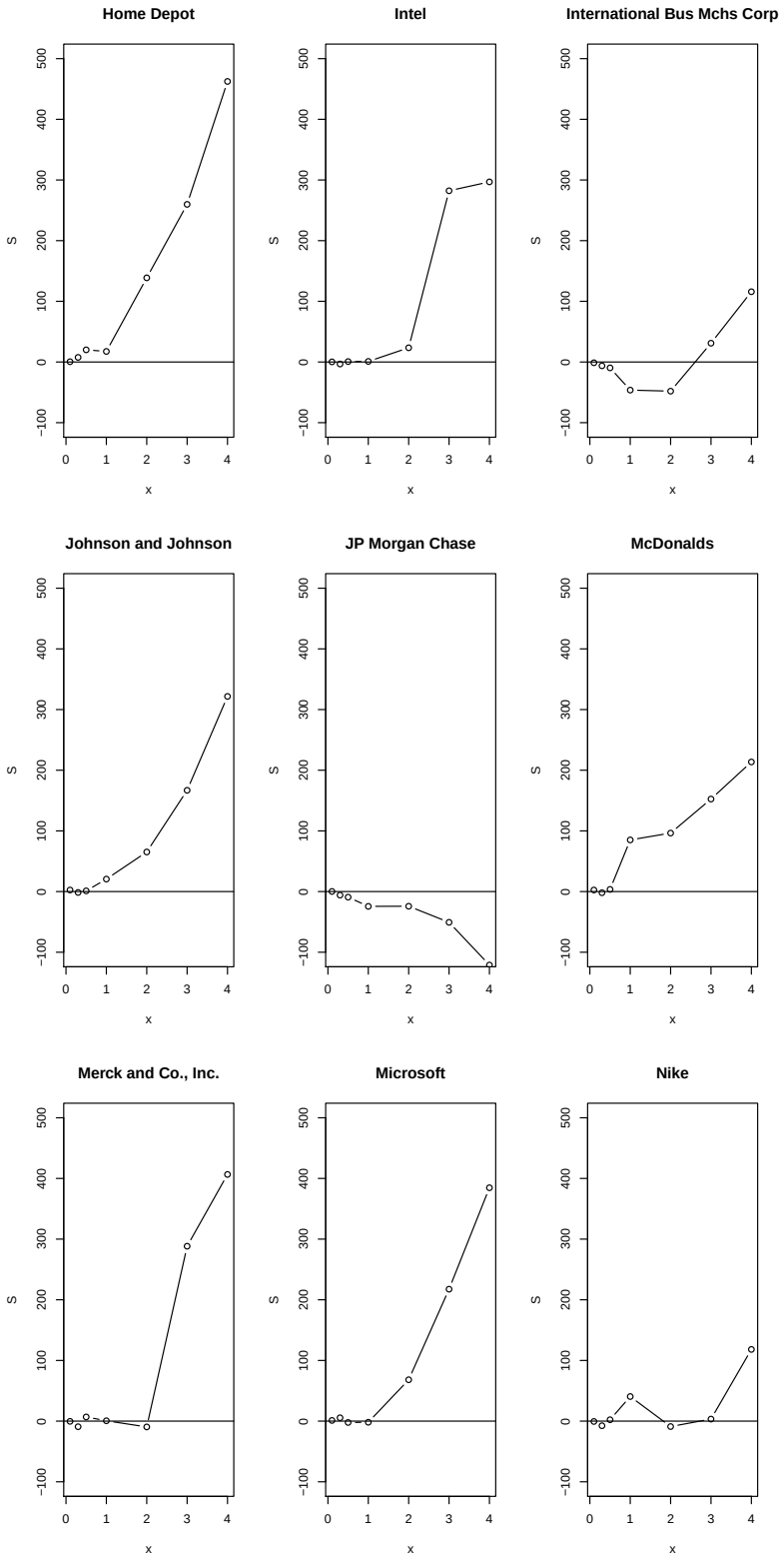
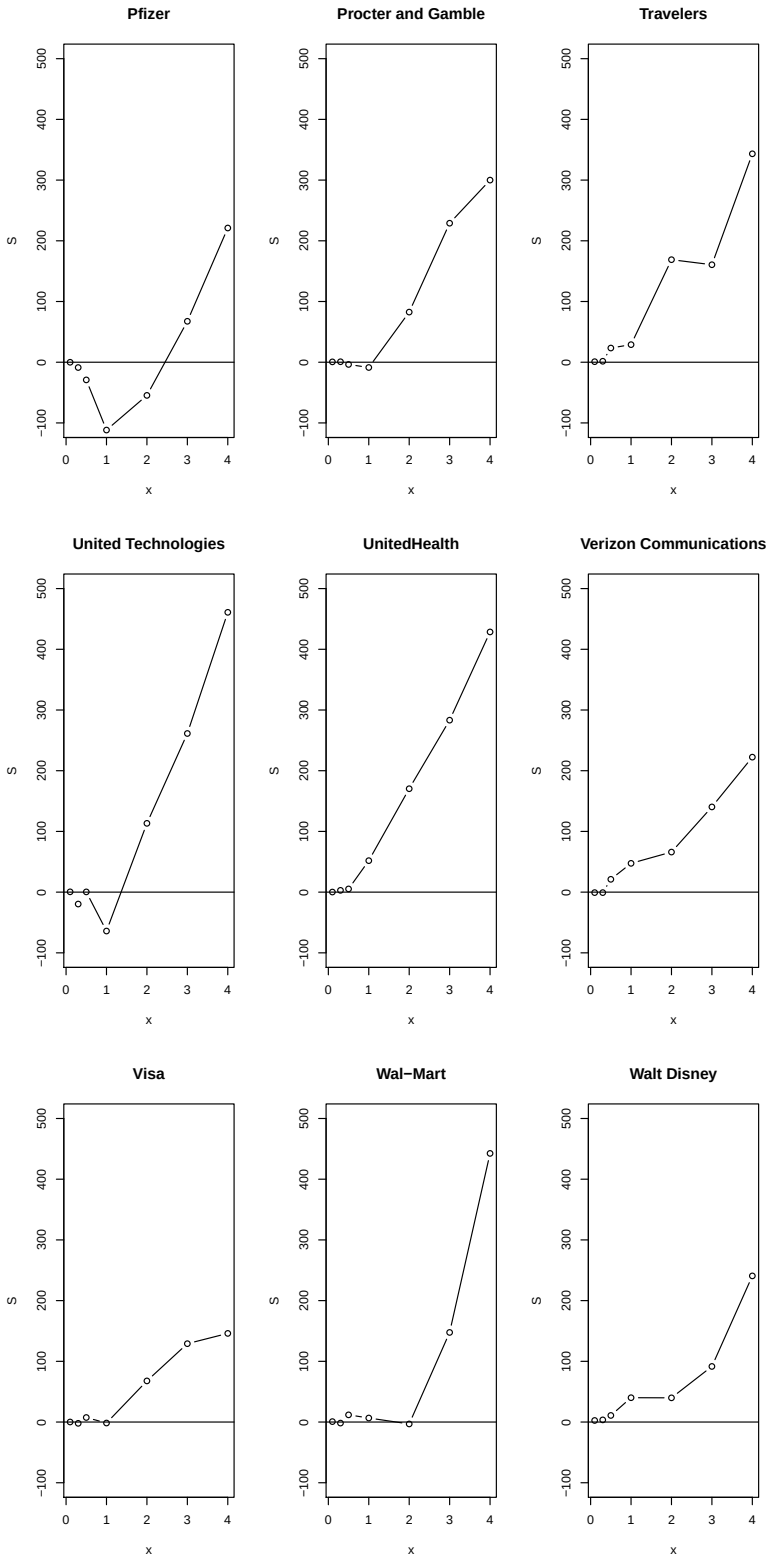


Figure A2: Daily stock returns for the Dow Jones-constituents, where x denotes the returns (in %) and S the sum of all returns less than x in absolute value (also in %).




Boeing
Caterpillar
Chevron
Cisco
Coca−Cola
DuPont
ExxonMobil
General Electric
Goldman Sachs
S
x
−100
0
100
200
300
400
500
0
1
2
3
4


Home Depot
Intel
International Bus Mchs Corp
Johnson and Johnson
JP Morgan Chase
McDonalds
Merck and Co., Inc.
Microsoft
Nike
S
x
−100
0
100
200
300
400
500
0
1
2
3
4


Pfizer
Procter and Gamble
Travelers
United Technologies
UnitedHealth
Verizon Communications
Visa
Wal−Mart
Walt Disney
S
x
−100
0
100
200
300
400
500
0
1
2
3
4