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Abstract

We study the dynamics of competition in a model with network effects, an incumbent and entry. We propose a new way of representing the strategic advantages of incumbency in a static model. We then embed this static analysis in a dynamic framework with heterogeneous consumers. We completely identify the conditions under which inefficient equilibria with two platforms will emerge at equilibrium; explore the reasons why these inefficient equilibria arise; and compute the profits of the incumbent when there is only one platform at equilibrium.

JEL-Codes: L860.

Keywords: network effect, network externalities, platforms.

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1 Introduction

> "A standard concern is that if a platform becomes dominant, there may be dynamic inefficiencies because users are coordinated and locked-in to a single platform. It may be difficult for an innovative new platform to gain market share, even if its underlying attributes and technology are better. This concern has helped to motivate antitrust actions in industries such as operating systems and payment cards.... Indeed, even in industries such as social networking, where one might expect positive feedback effects to generate agglomeration, it is easy to point to examples of successful entry (Twitter) or rapid decline (MySpace)."
>
> Levin (2013)

Although the importance of competition *for* as opposed to *in* the market has been stressed by economists studying the new information technologies, relatively little work has been done to explore the strategies used by platforms to maintain their dominant positions in the presence of network externalities, and, when they are able to do so, to determine the value of these dominant positions. To help remedy this gap, we study a dynamic market with network effects and free entry (comparisons with the extent literature can be found in Section 8). We are, in effect, analyzing the following conversation between competition authorities and a dominant platform:

> *— Because of the network externalities in your industry, you yield enormous market power and can extract large profits from your clients.*
> *— You forget that if I try to use this market power, entrants will be able to convince my consumers to join their platform.*
> *— This is your standard argument, but according to your reasoning, these entrants will be scared of entry in future periods, and therefore will not be aggressive.*

In order to examine the argumentation of the two parties to this dialogue, we construct a model in which at the outset a single platform controls the market. There are positive network externalities so that consumers prefer to be on the same platform as other consumers. We study dynamic competition, assuming that there are (at least two) potential entrants in each of an infinite sequence of periods,[1] both in models where consumers are all similar

[1] We make this assumption for expositional reasons. All our results hold if are at least two infinitely lived entrants at the start of the game.

to each other, and in models where consumers are heterogenous, with two types of consumers. We derive bounds on the intertemporal profit of the incumbent and compare it to the profit which would be computed using a static one period model. When consumers are heterogenous, we characterize the conditions under which there is one platform or (inefficiently) several platforms in equilibrium.

This analysis yields rich insights about the strategies that platforms use to defend their incumbency advantages. In particular, we are able to pinpoint the roles of consumers whose willingness to pay is low, but who are valuable as their presence increases what other consumers are willing to pay.

We are also able to throw light on some of the difficulties of competition policy when network externalities are present. Competition has a beneficial consequence: it lowers prices which benefits consumers. It potentially also has a harmful consequence: it can lead to the choice of different platforms by different groups of consumers, which is inefficient in the presence of positive network externalities. Our full characterization of the circumstances under which two platforms can co-exist at equilibrium in a dynamic model (Proposition 3) provides some useful lessons. First, static models have a tendency to exaggerate the extent of inefficiencies: they predict the separation of consumers on different platforms more often than do dynamic models. Thus, it is important to think through the long run consequences of competition. Second, in our framework, the presence of entrants generally increases social welfare. Not only does it decrease prices and ensures that more consumers join a platform, it also makes it more likely that they join the same platform. This seems to indicate that entry deterring strategies by an incumbent typically cannot be justified on the grounds that they help consumers coordinate on the same platform.

Our analysis also allows us to compare the value of incumbency in static and dynamic settings. As in switching cost models, we find that if consumers are all identical to each other, or if their preferences are not too different, then the profits which would be computed for the incumbent in a one period model are exactly the same as those obtained from a fully dynamic model. In the first period, entrants are willing to price low enough in order to attract clients that competition "eats up" all the incumbent's profits. On the other hand, when consumers are heterogenous, the one period model underestimates the profits obtained through a fully dynamic model. However, this difference is relatively limited and in any case the dynamic profits are always strictly less than the value of a flow of one period profit. The value of incumbency is more limited than what a naive analysis would predict. This should give policy makers pause before they react too aggressively in markets with network externalities.

For the purposes of conducting this analysis, we develop a new and, we believe, more convenient way to represent the reluctance of consumers to migrate from one platform to the other. In policy discussions economists often argue that network effects make consumers reluctant to migrate and hence provide a strong advantage to incumbents: Levin's statement that "It may be difficult for an innovative new platform to gain market share, even if its underlying attributes and technology are better" provides a typical example. However, formal models of competition between platforms do not naturally lead to this conclusion. Absent switching costs, there is no reason for all the members of an incumbent platform *not* to purchase from a new entrant who would offer better conditions. If all consumers are willing to pay a premium of more than 10€ to belong to the same platform as the other consumers, and an incumbent platform charges 25€ while an entrant offers its service at a price of 15€, it is an equilibrium for the consumers to all purchase from the incumbent, but it is also a (Pareto better from their point of view) equilibrium for all of them to purchase from the entrant. The characterization of network externalities as "social switching costs" is due to our intuition that in many such cases the most likely equilibrium is for the consumers to purchase from the incumbent.

Without formalizing this intuition it is impossible to study the constraints that potential entry puts on the strategies of incumbents, but, as we discuss in our literature review in section 8, much of the small amount of work which has been conducted on this issue tackles the problem by modelling the belief of consumers. This approach is not feasible (or, at least, we do not know how to use it) in our case, as we would have to model the belief formation about the whole sequence of future periods. It is also not clear how it allows for the presence of heterogenous consumers.

In section 2, we propose to represent the coordination of consumers through what we call *Attached Consumers* (AC) Equilibria. We do so through a very simple model where we essentially assume that consumers only change platforms when it is individually rational for them to do so — they are very bad at coordinating their moves even when it would be Pareto efficient for them to do so. This enables us to select an equilibrium of the game played by the consumers when they choose which platform to join; this equilibrium depends on the prices charged by the platforms and also on the initial allocation of consumers among the various platforms; our equilibrium is tractable even when there are several types of consumers. We show that AC Equilibria always exist and that they are often unique. This equilibrium concept gives a great deal of power to the incumbent(s) and can be viewed as choosing the best equilibrium from the incumbent's point of view. This makes our results that an incumbent's profit are limited in the dynamic model more striking. In order to show that the concept of AC equilibrium is reasonable, in section 3 we illustrate its application to simple one period games with one incumbent.

Starting in section 4, we turn to the core of the paper, the study of an infinite horizon model with free entry. In section 5, we present simple versions of our dynamic model, among them one where consumers all have identical preferences and one where some consumers are insensitive to network externalities. These allow us to identify situations where the incumbent has the same profit in the one period and the infinite number of period models. We also show that the presence of consumers who derive little utility from network externalities can increase the profit of the incumbent, despite the fact that they never join its platform.

We provide a complete characterization of the circumstances where there are inefficiently several platforms in equilibrium in section 6, while in section 7 we characterize the circumstances where there is, efficiently, only one platform in equilibrium. We discuss the literature in section 8. Markets with network externalities include such diverse markets such as social networks, games, and market sites such as eBay. The focus of our paper is on general principles and effects that are present in many markets with network externalities. In the conclusion we discuss how our insights may apply in some specific market settings.

2 Modeling incumbency

The main focus of the paper is on dynamic multi-period models. However, in this section we study a one period model and propose a new way to represent incumbency advantage. Starting in section 4, we will embed it in dynamic models.

We use the following strategy. We first define *unattached consumers* (UC) equilibria, in which there are no incumbency advantages (because consumers move easily from platform to platform) — these are the standard Nash equilibria of the games played by the consumers. We then define *attached consumers* (AC) equilibria, which are the outcome of a migration process between the incumbent(s) and the entrants, and which are essentially unique. We finally show that AC equilibria are UC equilibria. Using the concept of AC equilibrium yields in the static model predictions which are not very different from those of Caillaud and Jullien (2003), but it is much easier than their selection concept to use in the dynamic games that we will be considering starting in section 4.

In order to avoid introducing useless complications, we assume that there are two types of consumers, but the results of this section extend to any number of types. We also assume that the only choice that the consumers face is which platform to join, but there is no difficulty extending the definitions to situations where one of the choices is to join no platform (and indeed is what we do so in section 3).

There is a mass α_h of *high network effects* (HNE) consumers and a mass α_ℓ of *low network effects* (LNE) consumers.[2] We will refer to h or ℓ as the types of the consumers. A consumer of type θ derives utility $\psi_\theta(\gamma_{i\theta}, \gamma_{i\theta'})$ from belonging to platform i to which $\gamma_{i\theta}$ consumers of the same type and $\gamma_{i\theta'}$ consumers of type $\theta' \neq \theta$ also belong. The functions ψ_θ are strictly increasing in both arguments and satisfy $\psi_\theta(0,0) = 0$.[3]

Even though consumers like to have more consumers of both types on the platform to which they belong, they prefer consumers of their own type more:

$$\partial\psi_\theta(\gamma_{i\theta}, \gamma_{i\theta})/\partial\gamma_{i\theta} > \partial\psi_\theta(\gamma_{i\theta}, \gamma_{i\theta'})/\partial\gamma_{i\theta'} \geq 0. \tag{1}$$

(Here, as in the rest of the paper, θ' will always be taken to be "the other type," different from θ.) We will need no other hypothesis on the utility functions, in particular no concavity or convexity assumptions.

Assume that there are m platforms, indexed by i. An *allocation* γ of consumers is a $2 \times m$ vector of nonnegative numbers $\{\gamma_{ih}, \gamma_{i\ell}\}_{i=1,\ldots,m}$ with $\sum_i \gamma_{i\theta} = \alpha_\theta$ and $\sum_i \gamma_{i\ell} = \alpha_\ell$ where γ_{ih} and $\gamma_{i\ell}$ are the number of HNE and LNE consumers on platform i. Let p_i be the price charged by platform i. An allocation γ is an *unattached consumers* (UC) equilibrium (that is an equilibrium in which incumbency plays no role) if and only if

$$\gamma_{i\theta} > 0 \Longrightarrow \psi_\theta(\gamma_{i\theta}, \gamma_{i\theta'}) - p_i = \max_j \psi_\theta(\gamma_{j\theta}, \gamma_{j\theta'}) - p_j.$$

The definition of UC equilibrium treats all platforms in the same way and is the standard definition of equilibrium in the economics of network externalities: there is no incumbency advantage. We now turn to a definition of equilibrium which depends on the initial allocation of consumers, and show in lemma 1 that it selects a UC equilibrium. This definition is illustrated on Figure 1.

Let β be an initial allocation of consumers among the platforms. The allocation γ is on a *migration path* from β if there exists an integer $T \geq 0$ and

[2]For the purpose of this section, the fact that some consumers derive more utility than the others from the presence of other consumers play no role.

[3]All our results still hold true if $\psi_h(0,0) = \psi_\ell(0,0) > 0$. On the other hand we would have to change our analysis, but in non-essential ways, if the consumers had different stand alone utilities for the platform, *i.e.,* if $\psi_h(0,0) \neq \psi_\ell(0,0)$.

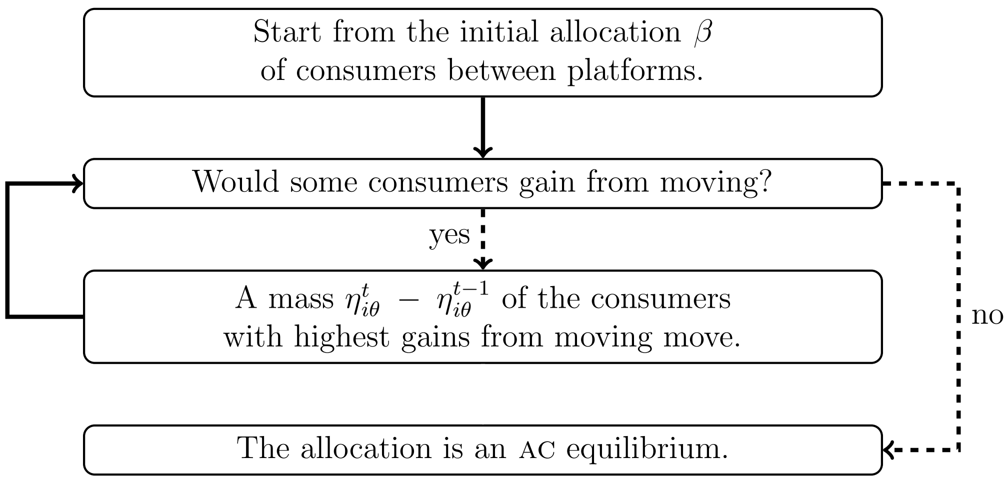

Figure 1: This figure represents in algorithmic form the definition of AC equilibria.

sequences $\{\eta^t\}_{t=0,1\ldots,T}$ of allocations of consumers to platforms ($\sum_i \eta^t_{i\theta} = \alpha_\theta$ for all $t = 1, 2, \ldots$ and all θ) which *lead from* β to γ:

$$\eta^0_{i\theta} = \beta_{i\theta} \text{ and } \eta^T_{i\theta} = \gamma_{i\theta}, \text{ for all } i \text{ and } \theta,$$

and which satisfy the following property:

> for all $t = 1, 2, \ldots, T-1$ there exists a *type transferred* $\theta(t)$, a *source platform* $s(t)$, and a *destination platform* $d(t)$ such that
>
> $$\eta^t_{d(t)\theta(t)} - \eta^{t-1}_{d(t)\theta(t)} = \eta^{t-1}_{s(t),\theta(t)} - \eta^t_{s(t)\theta(t)} > 0;$$
>
> $$\eta^t_{i\theta} = \eta^{t-1}_{i\theta} \text{ if } \{i, \theta\} \notin \Big\{\{d(t), \theta(t)\}, \{s(t), \theta(t)\}\Big\};$$
>
> and
>
> $$\begin{aligned} &\left[\psi_{\theta(t)}\left(\eta^{t-1}_{d(t)\theta(t)}, \eta^{t-1}_{d(t)\theta'(t)}\right) - p_{d(t)}\right] - \left[\psi_{\theta(t)}\left(\eta^{t-1}_{s(t)\theta(t)}, \eta^{t-1}_{s(t)\theta'(t)}\right) - p_{s(t)}\right] \\ &\quad = \max_{\widetilde{\theta}, j, j'} \left\{\left[\psi_{\widetilde{\theta}}\left(\eta^{t-1}_{j\widetilde{\theta}}, \eta^{t-1}_{j,\widetilde{\theta}'}\right) - p_j\right] - \left[\psi_{\widetilde{\theta}}\left(\eta^{t-1}_{j'\widetilde{\theta}}, \eta^{t-1}_{j',\widetilde{\theta}'}\right) - p_{j'}\right]\right\} > 0. \end{aligned} \tag{2}$$

It may be worthwhile restating what we are doing. At each step, we check whether some consumer would find it optimal "to move on its own" to another platform. If there is, then we move a strictly positive mass of the consumers who have the greatest incentives to move.[4] Equation (2) expresses the fact that it is the consumer with the highest gain in utility who changes platform, if this *strictly* increases his utility.[5]

The allocation γ is a *final allocation* if there is no migration path leading from γ to another allocation.

Definition 1. *An allocation γ is an* UC *equilibrium if it is on a migration path from the original allocation and is a final allocation.*

If an initial allocation is a final allocation, then there can be no other allocation that can be reached by a migration path of length 1. It is straightforward to see that this implies that the allocation is an AC equilibrium and therefore proves the following lemma.

Lemma 1. *All* AC *equilibria are also* UC *equilibria. An initial allocation is an* AC *equilibrium if and only if it is a* UC *equilibrium. Furthermore, if an initial allocation is a* UC *allocation, it is the only* AC *equilibrium.*

Lemma 1 has two consequences for the interpretation of AC equilibria. First, trivially from a technical viewpoint (this is a consequence of the fact that there is a continuum of consumers) but importantly for interpretation, the sequence of moves from an initial allocation to a UC equilibrium is a (perfect Nash) equilibrium of the dynamic game played by the consumers.[6] Secondly, we have presented the migration of consumers between platforms as happening in "real time". Thanks to lemma 1, it can also be interpreted

[4]An alternative assumption would have any group of consumers with a strictly positive gain from moving move, *i.e,* (2) would be rewritten under the simpler form

$$\left[\psi_{\theta(t)}\left(\eta^{t-1}_{d(t)\theta(t)},\eta^{t-1}_{d(t)\theta'(t)}\right)-p_{d(t)}\right]-\left[\psi_{\theta(t)}\left(\eta^{t-1}_{s(t)\theta(t)},\eta^{t-1}_{s(t)\theta'(t)}\right)-p_{s(t)}\right]>0.$$

This is not sufficient to prove our results. Indeed, we have built an example using this relaxed assumption where in the initial allocation the HNE consumers are on one platform and the LNE consumers on another. The migration leads to an UC equilibrium where all the HNE consumers and some of the LNE consumers migrate to one platform and the rest of the LNE consumers to another one. Lemma 3 does not hold.

[5]This assumption considerably simplifies the reasoning below. In an AC equilibrium, consumers will stay on the incumbent platform when they are indifferent between doing so and joining an entrant. We could do without the assumption, but this would require that when studying competition between platforms, we use the type of limit pricing arguments standard in, for instance, the study of Bertrand competition with different marginal costs. In our framework, this would make the proofs much more complicated without changing the equilibrium payoffs. We relax this condition in subsection E.4 for off the equilibrium path events to guarantee existence for a certain class of equilibria.

[6]A precise statement of this fact would require a formal description of the dynamic game, which is beyond the scope of this paper.

in terms of fictitious play, where all consumers move at the same time. In that interpretation, consumers think about how their fellow consumers will react to the price offerings that they all face in a subgame. They would all correctly predict how the consumers will choose which platform to belong to. Of course, no consumer will have an incentive to deviate from their platform choice.The description of moves in Figure 1 then represents the way in which consumers think about the choices of the other consumers.

We will call a migration path a *migration path through large steps* if at every step all the consumers of type $\theta(t)$ on platform $s(t)$ migrate to platform $d(t)$: $\eta^t_{s(t)\theta(t)} = 0$ for all t. The following lemma makes easier both the identification and proof of existence of AC equilibria.

Lemma 2. *The set of* AC *equilibria is not changed if we impose the restriction that the migration path is a migration path through large steps.*

The proof of Lemma 2 is presented in appendix A; once some consumers have begun migrating a 'snowballing' effect arises. The idea is very simple: if in any migration process not all the consumers of type $\theta(t)$ migrate from platform $s(t)$ to platform $d(t)$, then we must have $\theta(t+1) = \theta(t)$, $s(t+1) = s(t)$ and $d(t+1) = d(t)$. Indeed, for consumers of type $\theta(t)$, the utility of being of platform $d(t)$ has strictly increased while those of being on platform $s(t)$ has decreased; there is no other move between two platforms which would yield a greater increase the utility of a consumer of type $\theta(t)$. Condition 1 shows that this also holds for consumers of type $\theta'(t)$, but for a smaller gain than for type $\theta(t)$.

This implies the following lemma, which we will use extensively in the sequel.

Lemma 3. *If all the consumers of type θ belong to the same platform in the initial allocation (i.e., if $\beta_{i\theta} = \alpha_\theta$ for some i), then they also belong to the same platform in any* AC *equilibrium.*

The following lemma, whose proof can be found on page A — 1 in appendix A, shows that we can also interpret our migration paths as a sequence of "individual moves".

Lemma 4. *The set of* AC *equilibria is not changed if we add the restriction that $\eta^t_{d(t)\theta(t)} - \eta^{t-1}_{d(t)\theta(t)} = \eta^{t-1}_{s(t),\theta(t)} - \eta^t_{s(t)\theta(t)}$ must be smaller than some $\epsilon > 0$ for all t.*

Lemma 4 is proved by "cutting" each step of a large step algorithm into smaller steps with the same source and destination platforms and the same migrating type. It shows that we can think of migration paths as approximating a process in which the consumers move "one by one" from one platform to the other; in each stage it is the consumer with the greatest gain from moving who moves.

It is easy to show, and we prove formally on page A — 2 in appendix A, that large step migrations must eventually stop at an AC equilibrium, which proves the following lemma.

Lemma 5. *Whatever the initial allocation* $\{\beta_{iH}, \beta_{iL}\}_{i=1,\dots,m}$ *and prices* p_i *charged by the platforms, there exists an* AC *equilibrium.*

3 One Period Games

3.1 Notation and assumptions

In the models which we study in the rest of this paper, there is at the start an Incumbent platform from which all consumers purchased in the past. By lemma 3, this implies that in any subsequent period consumers of the same type will all join the same platform, and the following shorthand notation will prove useful:

$$u_\theta = \psi_\theta(\alpha_\theta, 0),\ v_\theta = \psi_\theta(0, \alpha_{\theta'}),\ w_\theta = \psi_\theta(\alpha_\theta, \alpha_{\theta'}).$$

We assume that the consumers prefer to be with consumers of the same type:

$$w_\theta > u_\theta > v_\theta \text{ for } \theta \in \{h, \ell\}. \tag{3}$$

Condition (1) implies $w_\theta > v_\theta$ and $w_\theta > u_\theta$. It also implies $u_\theta > v_\theta$ if $\alpha_h = \alpha_\ell$. If, for instance, α_ℓ were much larger than α_h, HNE consumers might rather belong to the same platform as LNE consumers than belong to the same platform as other HNE consumers. The right most inequality in (3) assumes this away.

We are representing the fact that the HNE consumers value network effects more than LNE consumers by the following conditions:

$$w_h > w_\ell \text{ and } u_h > u_\ell. \tag{4}$$

Finally, unless we explicitly state the opposite, we assume

$$w_\ell < u_h - v_h. \tag{SmallCE}$$

The right hand side is the amount an HNE consumer would be willing to pay to move from a platform to which all the LNE consumers belong to another platform to which all the other HNE consumers belong. The left hand side is what LNE consumers are willing to pay to be on the same platform as all other consumers. Written $v_h < u_h - w_\ell$, it puts an upper bound on v_h, hence the name "Small Cross Effects". When it does not hold there is only one platform in equilibrium whether in a one period or in an infinite horizon model (see section 5.2).

3.2 Equilibrium in one period models

In the rest of this section, we study AC equilibria in one period models. This will both prepare the study of dynamic games and demonstrate that AC equilibria are intuitive.

First, as a benchmark, suppose that there is no entry: the Incumbent announces a price and the consumers decide whether or not to stay on its platform. They all stay if the Incumbent charges w_ℓ, and its profit is then $(\alpha_h + \alpha_\ell)w_\ell$. If the Incumbent charges u_h, which is greater than w_ℓ by (SMALLCE), its only clients are the HNE consumers and its profit is $\alpha_h u_h$. All other prices are dominated by one of these two, and we have therefore proved the following lemma.

Lemma 6. *In a one period model with no entrants, the incumbent sells only to* HNE *consumers if*

$$\alpha_h u_h > (\alpha_h + \alpha_\ell)w_\ell, \tag{5}$$

and to all consumers if $\alpha_h u_h < (\alpha_h + \alpha_\ell)w_\ell$. It charges α_h and its profit is $\alpha_h u_h$ in the first case; in the second, it charges w_ℓ and its profit is $(\alpha_h + \alpha_\ell)w_\ell$.

Let us now add entry, and assume that there are least two entrants. Although it is easy to see that Nash timing would give exactly the same results, for simplicity we assume Stackelberg timing where the Incumbent first chooses its price p_I followed by the entrants; afterwards, the consumers decide which platforms to join. Entrants will never charge less than 0, and competition among them implies that any entrant who attract consumers will do so at a price of 0. The incumbent either charges w_ℓ and keeps all the consumers or $u_h - v_h$ and keeps only the HNE consumers. This implies the following lemma.

Lemma 7. *In the one period model with entry, the incumbent sells only to the* HNE *customers if*

$$\alpha_h(u_h - v_h) > (\alpha_h + \alpha_\ell)w_\ell. \tag{6}$$

and to all consumers if $\alpha_h(u_h - v_h) < (\alpha_h + \alpha_\ell)w_\ell$. It charges $u_h - v_h$ and its profit is $\alpha_h(u_h - v_h)$ in the first case; in the second it charges w_ℓ and its profit is $(\alpha_h + \alpha_\ell)w_\ell$.

Lemmas 6 and 7 together imply the following corollaries, which follow immediately from (5) and (6). The first is stated for future reference (see Lemma 9).

Corollary 1. *In a one period model with or without entry, if* (SMALLCE) *does not hold, the Incumbent sells to all consumers.*

The second corollary is important from an economic viewpoint.

Corollary 2. *Entry makes the separation of* LNE *and* HNE *consumers less likely and therefore improves efficiency.*[7] *When the incumbent sells to both types of consumers the price it charges and its profit are the same with or without entry.*

Note the reason why entry improves efficiency: the incumbent finds it more costly to let the LNE consumers "go" to an entrant, since it will be more costly to keep the HNE consumers. This result has ramifications for policy, since it says, somewhat counter-intuitively, that entry keeps more consumers on an incumbent's platform despite network effects. This is a gain in efficiency in two senses. One, it has all consumers on the same platform. Second, without entry, instead of joining another platform LNE consumers opt out of the market. We will review the role of entry and find more ambiguous results for intermediate values of the discount factor in Section 6.2 — see Corollary 8. Fudenberg and Tirole (2000) also discuss the role of entry in a dynamic model and find that "in equilibrium, the welfare effects of inducing additional entry are ambiguous".

4 The dynamic model

4.1 Equilibria in the dynamic game with entry

The dynamic model which we will use is represented in Figure 2. At the beginning of period 1 there is one incumbent, which, as in Section 3, we will denote the "Incumbent". In each subsequent period, there will be one or more incumbents: the firms that sold to a strictly positive measure of consumers in the previous period. There will also be $n_E \geq 2$ entrants in each period.

[7]There are some parameter values for which separation occurs with entry but not without entry, and none for which the opposite is true.

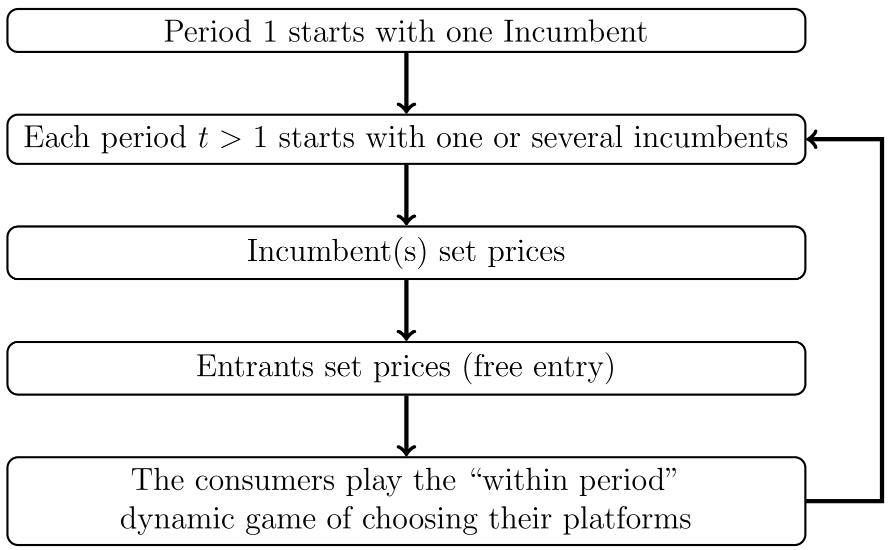

Figure 2: The dynamic model.

Alternatively, we would obtain exactly the same results if we assumed that there are at least 2 entrants at the start of the game and that we examine only equilibria where the distribution of consumers on each platform matters and that players do not condition their strategies on the names of firms.[8]

For simplicity, we assume Stackelberg timing where all the incumbents first set prices simultaneously and then the entrants, having seen these prices, choose their own prices.[9] Afterwards, the consumers choose their platforms, and the game moves to period $t+1$. We assume that firms with no consumer at the end of a period "drop out"of the game.[10]

Note that there are two dynamics in the game which we are describing: the "large scale" dynamics from period to period and the "small scale" or "within period" dynamics, when consumers choose which platform to join according to the process described in section 2. We assume there is no dis-

[8]This is to prevent the possibility that firms can implement equilibria that are seemingly collusive. See Biglaiser and Crémer (2011) for details in a switching cost framework.

[9]We can also obtain the same basic results regarding profits using a Nash timing, where firms simultaneously set prices. There would only exist mixed strategy equilibria, but the equilibrium profits of the platforms would be the same as with Stackelberg timing (see Biglaiser, Crémer, and Dobos (2013) and Biglaiser, Crémer, and Dobos (2015) for discussion of similar issues in a model of switching costs). Along the equilibrium path, we would observe more switching between platforms by consumers than under Stackelberg timing.

[10]Formally, this would be done by assuming that in any period $\tau > t$, their strategy set is a singleton, and that purchasing from these firms is not in the consumers strategy set.

counting within a period, as in the one period model, but there is a common discount factor, $\delta < 1$, between periods. This formalizes the assumption that consumers move very quickly between platforms, and then have a full period to consume the benefits of belonging to a platform.

We focus on symmetric and measurable equilibria. This implies that along the equilibrium paths, in any period t the price charged by the incumbents i depends only on the $\beta^t_{j\theta}$s; the prices charged by the entrants depend only on the $\beta^t_{j\theta}$s and on the prices charged by the incumbents; the equilibrium of the game played between the consumers depend only of $\beta^t_{j\theta}$s and on the prices charged by the firms. Furthermore, we are assuming anonymity: all that matters is the $\beta^t_{j\theta}$s and not the name of the platform.

When the horizon is infinite, we will restrict ourselves to Markov equilibria, which we define more precisely in 4.3. The results of 4.2 do not depend on stationarity.

4.2 The myopia principle

We now turn to the extension of the notion of AC equilibrium appropriate for dynamic games. In each period t, there is a set of incumbents $\{1, 2, \ldots, n^t_I\}$ (in equilibrium, n^t_I will actually be equal to either 1 or 2), and a set of entrants $\{1, 2, \ldots, n_E\}$. In period 1, the initial allocation allocates all consumers to the Incumbent. In future periods, the initial allocation is the allocation of consumers at the end of the previous period. For incumbent i in period t, we call β^t_{ih} and $\beta^t_{i\ell}$ the mass of LNE and LNE consumers in his initial clientele; because it is an incumbent, we must have $\beta^t_{ih} + \beta^t_{i\ell} > 0$.

The purchasing decisions of the consumers depend on the $\beta^t_{j\theta}$s, on the prices charged by the firms, and on their expectations of the decisions of other consumers. We call $W_{i\theta,t+1}\left(\beta^{t+1}\right)$ the expected discounted utility measured at the beginning of period $t+1$, before incumbents have chosen their prices of a consumer of type θ who has purchased from platform i in period t.

Because consumers are "small" and do not affect the market through their individual choices and there are no switching costs, $W_{i\theta,t+1}\left(\beta^{t+1}\right)$ does not depend on i, and can therefore be written $W_{\theta,t+1}\left(\beta^{t+1}\right)$. If the equilibrium allocation of consumers in period t has γ^t_{jh} HNE consumers and $\gamma^t_{j\ell}$ LNE consumers in platform j, the utility of a consumer of type θ who purchases from platform i which charges p^t_i will be

$$\psi_\theta(\gamma^t_{i\theta}, \gamma^t_{i\theta'}) - p^t_i + \delta W_{\theta,t+1}\left(\{\gamma_{jh}, \gamma_{j\ell}\}_{j \in \mathcal{I}(t+1)}\right),$$

where $\mathcal{I}(t+1)$ is the set of incumbents at stage t.

We can apply the same reasoning as in section 2 to define migration paths within period t. At each step τ, the consumers who change platforms are those consumers of type $\theta(\tau)$ such that there exists source and destination platforms, $s(\tau)$ and $d(\tau)$, which are solution of

$$\max_{\theta',i,i'} \left\{ \left[\psi_{\theta'}(\eta^{\tau-1}_{i\theta'}, \eta^{\tau-1}_{i,-\theta'}) + W_{\theta',t+1}\left(\left\{ \eta^{\tau-1}_{j\theta'}, \eta^{\tau-1}_{j,-\theta'} \right\}_{j\in\mathcal{I}(t)} \right) - p^t_i \right] - \left[\psi_{\theta'}(\eta^{\tau-1}_{i'\theta'}, \eta^{\tau-1}_{i',-\theta'}) + W_{\theta',t+1}\left(\left\{ \eta^{\tau-1}_{j'\theta'}, \eta^{\tau-1}_{j',-\theta'} \right\}_{j\in\mathcal{I}(t)} \right) - p^t_{i'} \right] \right\}, \quad (7)$$

as long as the value of this solution is strictly positive. The same W term appears in both terms of this expression and therefore solving (7) is equivalent to solving (2). We obtain the following "myopia principle" which plays a very important role in the sequel.

Lemma 8 (Myopia principle)**.** *Given the prices chosen by the firms, the set of equilibria of the game played by the consumers in any period t of a dynamic game is the same as if the game were a one period game.*

The myopia principle *does not imply* that the prices charged by the platforms will be the same in a multi-period game as in a one period game — it is only the consumers who act as if they are "myopic", not the firms.

The myopia principle does not hold in switching cost models unless future switching costs are uncorrelated with current switching costs: as shown in Biglaiser et al. (2015) high switching cost consumers try to "hide among" low switching cost consumers who induce firms to charge low prices — the high switching cost consumers are willing to incur higher costs in the current period in order to do so. The fact that there are no switching cost and, as in the one period model, that there is a continuum of consumers, and strategies and continuation values are measurable are necessary for lemma 8 to hold.

4.3 Markov equilibria

We focus our attention on Markov equilibria, defined as follows. Along the equilibrium path, the price charged by incumbent i depends only on the $\beta^t_{j\theta}$s and not on t; the prices charged by entrants depend only on the $\beta^t_{j\theta}$s and the prices charged by the incumbents; the equilibrium of the game played between the consumers depend only of $\beta^t_{j\theta}$s and the prices charged by the firms. Furthermore, we are assuming anonymity: all that matters is the $\beta^t_{j\theta}$s and not the name of the platform (this is one of the important differences between our model and the dynamic model of Hałaburda, Jullien, and Yehezkel (2016)).

By Lemmas 3 and 8, consumers of the same type will all "stay together"; therefore, along the equilibrium path, there will be only either one or two incumbents in every period. Therefore, we need only distinguish the following equilibrium prices and profits for the incumbents:

p_h (Π_h), the price charged by and the total discounted profit of a firm whose clients in the previous period were (only) the HNE consumers;

p_ℓ (Π_ℓ), the price charged by and the total discounted profit of a firm whose clients in the previous period were (only) the LNE consumers;

p_2 (Π_2), the price charged by and the total discounted profit of a firm who sold to both types of clients in the previous period.

The main focus of the paper will be on *two platform equilibria* where along the equilibrium path, after the first period, consumers purchase from two different platforms (this implies that in the first period at least one type of consumers purchase from an entrant). By stationarity, if after a deviation there is only one incumbent platform, consumers will reallocate themselves among two different platforms. We study these equilibria, in section 6. In section 7, we study the equilibria with only one platform.

Before studying our main model, section 5 presents some simple versions of our dynamic model that will illustrate the use of our equilibrium concept and develop intuition.

5 Simple infinite horizon models with free entry

In this section, we examine a series of very dynamic models with free entry. They both illustrate how our solution concept works in dynamic models and show that the value of incumbency can be quite limited.

5.1 Identical Consumers

We begin by examining the equilibrium when there is only one type of consumer — for definitiveness only HNE consumers. Our main result will be that the profits in the infinite horizon are exactly the same as in the static model.

Let Π denote the equilibrium profits of the incumbent when all consumers are on the same platform. Entrants are willing to "price down to" $-\delta\Pi/\alpha_h$ and no further, as any lower price would yield negative profits[11]. The Incumbent chooses the largest price p_I which enables him to keep all the consumers:

[11] We allow entrants to offer negative prices. They can be thought of as a discount below the cost of providing service or as the value of goods that the entrants give away in addition to access to the platform.

it satisfies $w_h - p_I = \delta\Pi/\alpha_h$. Because $\Pi = \alpha_h p_I/(1-\delta)$, this proves the following proposition.

Proposition 1. *If all consumers are of the same type, then the unique equilibrium has a single platform. The Incumbent keeps all the consumers and charges $p_I = (1-\delta)w_h$ in each period. Its profit is $\alpha_h w_h$, the same as in a static model.*

Biglaiser et al. (2013) establish the same result of equality of static and dynamic profit with homogenous consumers in the case of switching costs. Competition from the entrants prevents the incumbent from enjoying the rents of incumbency more than once: it can take only one bite from the apple. The result would also hold in a model with a finite number of periods. In the infinite horizon case, it requires the stationarity assumption (Biglaiser and Crémer (2011)).

5.2 Equilibrium when condition (SmallCE) does not hold

The aim of this subsection is to show the following proposition.

Proposition 2. *If Condition* (SmallCE) *does not hold, there exists an essentially*[12] *unique equilibrium. It is a single platform equilibrium. where the Incumbent charges $(1-\delta)w_\ell$ in every period and its discounted profit is $(\alpha_h + \alpha_\ell)w_\ell$, the same as in the one period model.*

We begin by proving the following lemma, which is stronger than we need to prove Proposition 2 and which is of independent interest. It mirrors Lemma 1 in the one period case. In the proof (and in the rest of the paper), we will use the notation p_E for the lowest price charged by an entrant.

Lemma 9. *Condition* (SmallCE) *is necessary for the existence of a two platform equilibrium.*

Proof. Because $w_\ell < w_h$, a first period entrant cannot attract the HNE consumers without also attracting the LNE consumers.[13] In the first period of a

[12] The "essentiality" refers to the fact that the prices charged by the entrants, who acquire no consumers, are not uniquely determined.

[13] The Incumbent will have some sales in the first period. If in period 1 it sells only to the LNE consumers, the price p_I charged by the Incumbent and p_E satisfy such that $w_h - p_2 \leq -p_E$ (HNE consumers purchase from the entrant), and $u_\ell - p_2 \geq v_\ell - p_E$ (LNE consumers purchase from the incumbent). This implies $w_h \leq p_2 - p_E \leq u_\ell - v_\ell$, which contradicts (3) and (4).

two platform equilibrium, we have $w_\ell - p_2 < -p_E$ because the LNE consumers purchase from the entrant and $u_h - p_2 \geq v_h - p_E$ because the HNE consumers purchase from the Incumbent. These two conditions imply (SMALLCE). $\square$

Trivially, Lemma 9 implies that when (SMALLCE) does not hold, any equilibrium is a single platform equilibrium. In such an equilibrium, the Incumbent sells to all the consumers in all periods at price p_2. If $-p_E \leq w_\ell - p_2 < w_h - p_2$, both LNE and HNE consumers purchase from the Incumbent. On the other hand, by the myopia principle, if $-p_E > w_\ell - p_2$, the LNE consumers will migrate to one of the entrants who charges p_E. HNE consumers will follow, as $u_h - p_2 < w_\ell + v_h - p_2 < v_h - p_2$. Therefore, the Incumbent keeps all the consumers if $p_2 - p_E \leq w_\ell$ and loses them all otherwise.

The lowest price that entrants are willing to price to attract all consumers is $-\delta\Pi_2/(\alpha_\ell + \alpha_h)$, where Π_2 is the discounted intertemporal profit of the incumbent starting from any period. Therefore the profit maximizing p_2 satisfies $p_2 = -\delta\Pi_2/(\alpha_h + \alpha_\ell) + w_\ell$. Along with the equality $\Pi_2 = (\alpha_h + \alpha_\ell)p_2/(1-\delta)$, this proves Proposition 2.

Intuitively, Proposition 2 shows that if SmallCE does not hold, the HNE consumers will follow the LNE consumers whenever they migrate to an entrant, and everything happens as if there were only one type of consumers. We obtain results very similar to those of 5.1.

5.3 Equilibria when LNE consumers do not derive any utility from belonging to a platform[14]

We now turn to a case where the LNE consumers derive no utility from belonging to a platform: $w_\ell = u_\ell = v_\ell = 0$. HNE consumers only derive utility from the presence of other HNE consumers and do not care about the presence of LNE consumers: $v_h = 0$ and $w_h = u_h > 0$ (but see footnotes 18 and 19). We will show that, even under these circumstances, LNE consumers affect the equilibrium by dampening the aggressiveness of entrants.

As proved formally in section 6, there are two[15] platforms at equilibrium: this is obvious as the Incumbent must charge 0 in order to keep the LNE consumers while he can make a strictly positive profit by selling only to the HNE consumers.

[14]Formally, the results of this subsection are a special case of those of section 6; our aim here is to bring out some of the economics of competition between the incumbents and the entrants which might not be as transparent in the analysis of the general case.

[15]We are assuming that the LNE consumers join a platform. There could be an equilibrium where the LNE consumers do not join a platform.

Let Π be the discounted profits of the Incumbent measured from the start of a period (in section 6, we show that this profit is the same whether all the consumers or only the HNE consumers were its clients in the last period). If an entrant attracts[16] the HNE consumers, it will also attract the LNE consumers and the lowest price that it is willing to offer is $-\delta\Pi/(\alpha_h + \alpha_\ell)$. To keep the HNE consumers, the Incumbent chooses a price p_I that makes them just indifferent between staying on its platform and purchasing from the entrant at that price: we must have

$$u_h - p_I = \delta\Pi/(\alpha_h + \alpha_\ell) \implies \Pi = \frac{p_I}{1-\delta} = \frac{(\alpha_h + \alpha_\ell)\alpha_h u_h}{\alpha_h + (1-\delta)\alpha_\ell}. \tag{8}$$

The profit of the Incumbent is increasing in the number of LNE consumers: because they accept the offers destined to attract HNE consumers but do not contribute to profits, they make the entrants less aggressive. This is especially striking when δ converges to 1, as Π converges to $(\alpha_h + \alpha_\ell)u_h$: a LNE consumer, who never purchases from the Incumbent, contributes as much to its profit as a HNE consumer! This result is similar to the results presented by Biglaiser et al. (2013) and Biglaiser et al. (2015) in the framework of a switching cost model, where with δ close to 1, a consumer which has low switching cost is worth as much to the Incumbent as a consumer with larger switching cost.

It may be worthwhile noting that in a one period model, the incumbent would charge u_h and its profit would be $\alpha_h u_h$. With δ close to 1, it is as if in the dynamic model, LNE consumers had been transformed into HNE consumers.[17] When α_ℓ is equal to zero, the derivative of this profit with respect to α_ℓ or to α_h is u_h.[18,19]

[16] This assumes that the LNE consumers all coordinate on the same entrant. They need not do so if there are indifferent to network effects. This coordination must either be assumed or the results which we present here can be considered as limit results when LNE consumers are close to indifferent to network effects.

[17] This result will also arise in the more general model below. See equation (12).

[18] The reasoning of the preceding paragraph assumes that u_h is not affected by changes in α_h. If we make explicit the dependence of the utility of the HNE consumers on the size of the platforms, the profit Π becomes equal to $(\alpha_h + \alpha_\ell)\psi_h(\alpha_h, 0)$ when δ is close to 1. When $\alpha_\ell = 0$, the derivative of this profit with respect to α_ℓ is $\psi_h(\alpha_h, 0)$ as $\partial\psi_h/\partial\alpha_\ell = 0$. The derivative with respect to α_h is equal to $\psi_h(\alpha_h, 0) + \alpha_h \partial\psi_h(\alpha_h, 0)/\partial\alpha_h$. The second term, $\alpha_h \partial\psi_h(\alpha_h, 0)/\partial\alpha_h$, is the increase in the value of the platform for the other HNE consumers. A LNE consumer is worth as much as the "direct" effect of a HNE consumer. If u_h is concave, then $\alpha_h \partial\psi_h(\alpha_h, 0)/\partial\alpha_h > \psi_h(\alpha_h, 0)$ and therefore a LNE consumer is worth *less* than half a HNE consumer.

[19] Very similar results hold if HNE consumers do care about the presence of LNE consumers; *i.e.*, if $v_h > 0$. Entrants will still be willing to price down to $-\delta\Pi/(\alpha_h + \alpha_\ell)$,

6 Two platform equilibria

6.1 Main results

In this section, we study the conditions under which two platforms coexist at equilibrium. Given our Markov assumption, and given the results of section 2 which show that consumers of the same type always purchase from the same platform if they are initially together, such an equilibrium must look as follows. In the first period, the Incumbent charges p_2. After the first period, there will be two platforms on the equilibrium path. This can only happen if in the first period an entrant charges p_E and attracts one type of consumers. Because $w_\ell < w_h$ implies

$$-p_E - (w_\ell - p_2) > -p_E - (w_h - p_2),$$

LNE consumers gain the most from purchasing from the entrant. Therefore, the entrant will attract the LNE consumers and the Incumbent will sell to the HNE consumers. In subsequent periods, along the equilibrium path, there will be two incumbents: the Incumbent, who sells to the HNE consumers and the successful first period entrant, who sells to the LNE consumers. This implies that, using the notation introduced on page 15, we have $\Pi_h = \alpha_h p_h/(1-\delta)$, $\Pi_\ell = \alpha_\ell p_\ell/(1-\delta)$ and $\Pi_2 = \alpha_h p_2 + \delta \Pi_h$.

If, off equilibrium, in some period all the consumers bought from one firm, by the Markov hypothesis in the subsequent period they would again split among two platforms as described in the previous paragraph.

The following proposition summarizes our results, which we prove in the rest of this section.

Proposition 3. *There exists a two platform equilibrium if and only if*

$$u_h - v_h \geq \frac{(1-\delta)\alpha_\ell + \alpha_h}{(1-\delta)\alpha_h}(w_\ell - \delta u_\ell). \tag{2NtwCond}$$

In this equilibrium L incumbents charge the same price and have the same profit as if there were only LNE *consumers:*

$$p_\ell = u_\ell(1-\delta) \tag{9}$$

but the first part of (8) becomes $u_h - p_I = v_h + \delta\Pi/(\alpha_h + \alpha_\ell)$: the presence of the LNE consumers increase the attractiveness of the entrant. Then Π is given by (12). An increase in v_h decreases profits: it makes it easier for entrants to attract the HNE consumers by first attracting the LNE consumers.

It may also be worthwhile noticing that the separation of the consumers in two different platforms is now inefficient. Otherwise, the same comments as in the previous two paragraphs hold.

and

$$\Pi_\ell = \alpha_\ell u_\ell. \tag{10}$$

H incumbents and firms which, after a deviation, have sold to every consumers in the preceding period charge the same price,

$$p_2 = p_h = \frac{(1-\delta)(\alpha_h + \alpha_\ell)(u_h - v_h)}{(1-\delta)\alpha_\ell + \alpha_h}, \tag{11}$$

and have the same profit,

$$\Pi_2 = \Pi_h = \frac{\alpha_h(\alpha_h + \alpha_\ell)(u_h - v_h)}{(1-\delta)\alpha_\ell + \alpha_h}. \tag{12}$$

This profit, which is also the profit of the Incumbent,

1. *is greater than the profit of the Incumbent of the two platform equilibrium in the one period model, $\alpha_h(u_h - v_h)$, and smaller than the value of a flow of this one period profit, $\alpha_h(u_h - v_h)/(1-\delta)$;*

2. *is less than $(\alpha_\ell + \alpha_h)(u_h - v_h)$;*

3. *is increasing in u_h, decreasing in v_h and independent of w_h, w_ℓ, u_ℓ and v_ℓ;*

4. *is increasing in α_h and in α_ℓ.*

The difficult part of the proof is proving that (2NtwCond) is a necessary and sufficient condition and that equations (10) to (12) hold — this is done in 6.3. The rest of the proposition is an immediate consequence of (12).

As stated in claim 2 below, the binding deviation for the existence of a two platform equilibrium is the attempt by the Incumbent to keep all the consumers. By (10) the lowest price that entrants are willing to charge is $-\delta u_\ell$. Because the LNE consumers are the most eager to change platforms, the Incumbent has to charge at most $w_\ell - \delta u_\ell$ if it wants to keep all the consumers. The profits resulting from repeating this strategy forever are

$$\Pi_D = \frac{(\alpha_\ell + \alpha_h)(w_\ell - \delta u_\ell)}{1-\delta}. \tag{13}$$

Condition (2NtwCond) is equivalent to $\Pi_D \leq \Pi_2$.

Point 3 shows that the profits of the Incumbent are independent of the preferences of the LNE consumers. These preferences do play a role in the existence of a two platform equilibrium. Once such an equilibrium exists, it is only the strength of attraction that LNE consumers hold for HNE consumers that affect the Incumbent's profit.

Point 4 states that, as in section 5.3, LNE consumers have value for the Incumbent even though they never join its platform, since they get in the way of an entrant who would try to attract high value consumers. Points 1 and 2 put bounds on its profit. In particular, the profit of the Incumbent is increasing in δ, and, as stated in point 2, is always smaller than $(\alpha_h + \alpha_\ell)(u_h - v_h)$. By Lemma 6, the Incumbent charges $u_h - v_h$ when there are two platforms in the one period model. Therefore in the dynamic infinite horizon model, its profit is always inferior to what it would be in the one period model if all the consumers were HNE consumers. Section 7 will show that this result also holds true when there is only one platform in equilibrium.

In 6.2, we examine in greater details the positive and normative consequences of Proposition 3.

6.2 Welfare and policy implications

We turn to a detailed discussion of the existence of two platform equilibria; this is of policy importance as in our setup with positive network externalities, it is always more efficient to have one rather than two platforms. We will show that two platform equilibria are, in general, more likely in the static than in the dynamic setting: more precisely, under most parameter values, if a two platform equilibrium exists in the dynamic model, then a two platform equilibrium will also in the static model, while the reverse is not true. Thus, a policymaker who uses the static model as a prediction the inefficiency of market outcomes, will in general reach overly pessimistic conclusions.

Condition (2NtwCond) immediately yields the following corollary.

Corollary 3. *Fix the parameters except for δ. There exist $\widetilde{\delta}$ such that there exist no two platform equilibrium for $\delta \geq \widetilde{\delta}$.*

As we will prove below, for large δ there always exists a one platform equilibrium (see Corollary 9). In industrial organization economics, the discount factor is thought off as being influenced both by the interest rate and the probability of the "end of the world", which for our model would be interpreted as the appearance of a new disruptive technology. Our results indicate that efficiency, under the form of the existence of a single platform, is more likely in a more stable world.

We continue by the following, easy to prove, corollaries of Proposition 3, which shows that there is a fundamental difference between the cases $u_\ell < w_\ell$ and $w_\ell = u_\ell$: that is, the case where the presence of HNE consumers add utility to LNE consumers and the case where it does not. Corollary 6 provides more details for the case $u_\ell < w_\ell$.

Corollary 4. *Assume $u_\ell < w_\ell$ and consider parameter values such that there exists a two platform equilibrium in the static model (condition* (6) *holds), then there exists $\delta_2 < 1$ such that there exists a two platform equilibrium if and only if $\delta < \delta_2$.*

Corollary 5. *Assume $w_\ell = u_\ell$ and consider parameter values such that there does not exist a two platform equilibrium in the static model (condition* (6) *does not hold), then there exists a δ_1 such that a two platform equilibrium exists in the dynamic model if and only if $0 \leq \delta \leq \delta_1$.*

From (13), Π_D is increasing in δ when $u_\ell < w_\ell$ and independent of δ when $u_\ell = w_\ell$. Combined with the fact that condition (2NtwCond) is equivalent to $\Pi_D \leq \Pi_2$, this explains the sharp contrast between the two cases. For intermediate values of $u_h - v_h$ and small cross effects for the LNE consumers, a two platform equilibrium may exist in the dynamic model even though none exists in the static model. This is stated more precisely in the following corollary.

Corollary 6. *Assume $u_\ell < w_\ell$ and consider parameter values such that there does not exist a two platform equilibrium in the static model (condition* (6) *does not hold), then there exists a $\widetilde{V}$ such that*

1. *if either $\alpha_h w_\ell \geq u_\ell(\alpha_h + \alpha_\ell)$ or $u_h - v_h > \widetilde{V}$, then, whatever δ, there is no two platform equilibrium in the dynamic model;*

2. *otherwise, there exists a two platform equilibrium in the dynamic model if and only if $\delta \in [\delta_1, \delta_2]$ for some $0 < \delta_1 < \delta_2 < 1$.*

The second part of the corollary, when $\alpha_h w_\ell < u_\ell(\alpha_h + \alpha_\ell)$, is illustrated in Figure 3 while the proof is presented in the appendix on page A — 2. For small δ, the right hand side of (2NtwCond) is decreasing in δ when $\alpha_h w_\ell < u_\ell(\alpha_h + \alpha_\ell)$. Therefore, for intermediate values of $u_h - v_h$ there can exist a two platform equilibrium in the dynamic model, but not the static model.

The following corollary follows:

Corollary 7. *The per period welfare in the dynamic version of the model, $\Pi_2 \times (1 - \delta)$, is greater than the profit in the static version unless $u_\ell < w_\ell$, $\alpha_h w_\ell < u_\ell(\alpha_h + \alpha_\ell)$, and $u_h - v_h < \widetilde{V}$.*

It is also possible to prove the following corollary, which is similar to Corollary 2, but in the dynamic case.

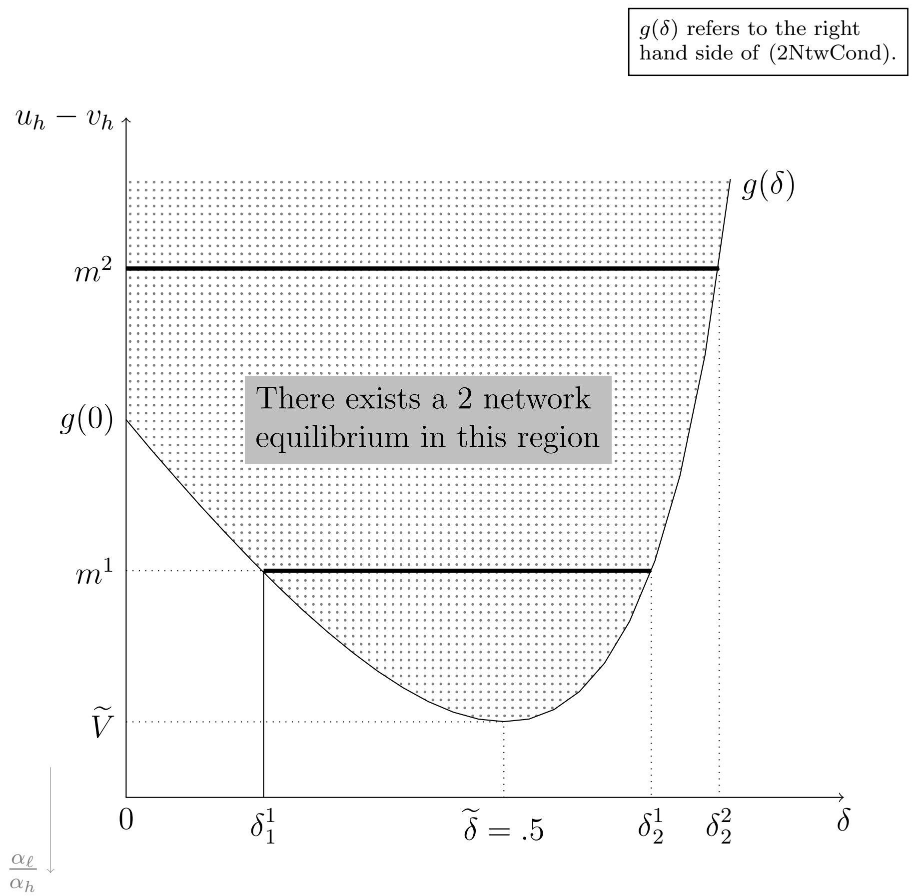

Figure 3: This figure illustrates part 2 of corollary 6 for $\alpha_\ell = \alpha_h$, $w_\ell = 1$ and $u_\ell = .8$. The right hand side of (2NtwCond) is then to $(2-\delta)(1-.8\delta)/(1-\delta)$, which is equal to 2 when $\delta = 0$. Its minimum, $\widetilde{V}$ is equal to 1.8 and is obtained for $\widetilde{\delta} = .5$. For $u_h - v_h = m^1 \in [1.8, 2]$, there exists an interval $[\delta_1^1, \delta_2^1]$ such that there exists a two network equilibrium. For $u_h - v_h = m^2 > 2$, there exists a two network equilibrium if δ is small enough.

Corollary 8. *For δ close to 0 or 1, entry makes the separation of* LNE *and* HNE *consumers less likely in the dynamic model and therefore entry improves efficiency.*

In the static model, entry improves welfare as it increases the cost to the Incumbent of letting an entrant capture the LNE consumers who are valuable to the HNE consumers (see Corollary 2). The situation is more complicated in the dynamic model. Without entry, Condition 5 must be satisfied both in the static and dynamic model for the Incumbent to keep only HNE consumers, while with entry Condition (2NtwCond) is necessary and sufficient for a two network equilibrium to exist in the dynamic model. When δ is equal either to 0 or 1, then entry makes it more likely that all consumers remain on the Incumbent network, just as in the static model. For intermediate values of δ, however, for some values of the parameters there exist a two network equilibrium in the dynamic model without entry but not in the dynamic model with entry. On one hand, the Incumbent finds it more costly to keep the LNE consumers in the dynamic model where entrants are willing to price down to $-\delta u_l$ while in the static model they are not willing to offer a negative price. On the other hand, without entry the profit in the two network dynamic model is equal to $1/(1-\delta)$ times the profit in the static model, but with entry when a two network equilibrium exists in the dynamic model, the profit is smaller than $1/(1-\delta)$ times the static profit. This leads to the corollary.

The rest of this section is devoted to the analysis of the strategies of the platforms and to the proof of Proposition 3. The reader who is interested mostly in the results can skip forward to Section 7 for discussion of one platform equilibria.

6.3 Condition (2NtwCond) is necessary for existence of a two platform equilibrium

As one would expect, in any equilibrium, the net surplus of the HNE consumers is larger than the net surplus of the LNE consumers:

$$u_\ell - p_\ell \leq u_h - p_h. \tag{14}$$

This implies that, along the equilibrium path, an entrant cannot attract the HNE consumers without having first attracted the LNE consumers.[20] In the

[20] If (14) holds as an equality, then an entrant charging $-p_E = v_h - p_\ell$, will obtain no consumers. If $-p_E > v_h - p_\ell$, the entrant will attract all consumers. It will not matter which type of consumers moves first in the migration to the entrant.

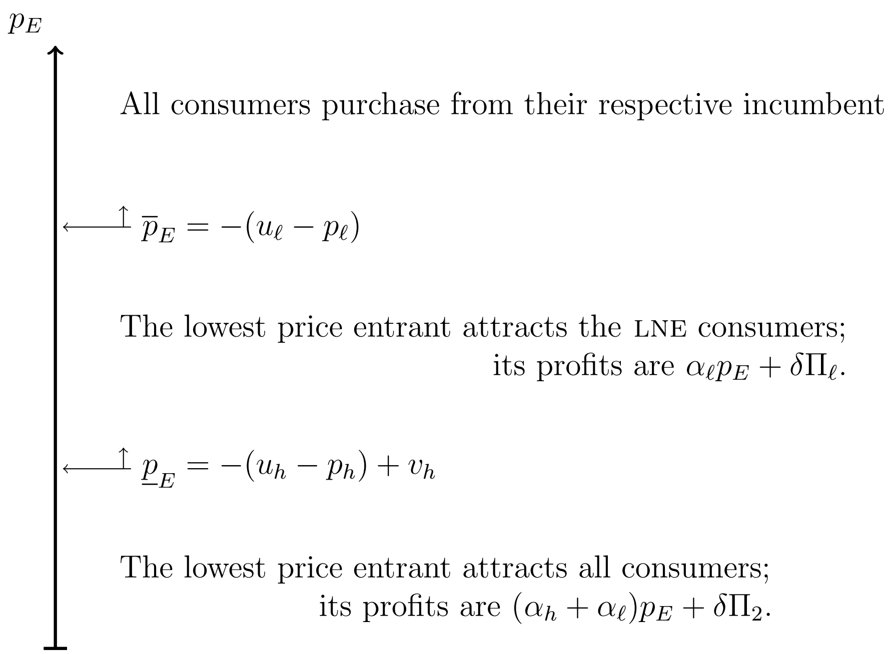

Figure 4: The response of consumers to entry when (15) holds.

main text, we assume that (14) holds and show in section D of the appendix that this must indeed be the case whenever a two platform equilibrium exists.

We first show that we can strengthen (14). The proof of all the claims that follow in this subsection can be found in section C of the appendix.

Claim 1. *If* (14) *holds, then in any two platform equilibrium*

$$v_h + u_\ell - p_\ell < u_h - p_h. \tag{15}$$

Condition (15) obviously implies $v_h - p_\ell < u_h - p_h$: along the equilibrium path, HNE consumers *strictly* prefer to purchase from the H incumbent than from the L incumbent.

Condition (15) implies that the continuation equilibria in the consumers' game as a function of p_E are as represented on Figure 4.

From the definition of the cutoff prices $\underline{p}_E$ and $\overline{p}_E$ in Figure 4, the following conditions are necessary to ensure that there is no profitable entry

$$\alpha_\ell \overline{p}_E + \delta\Pi_\ell \leq 0,$$

and

$$(\alpha_h + \alpha_\ell)\underline{p}_E + \delta\Pi_2 \leq 0. \tag{16}$$

The first equation states that an entrant cannot profitably attract only the LNE consumers, while the second states that an entrant cannot profitably attract all consumers. This enables us to prove the following claim.

Claim 2. *In a two platform equilibrium*

$$-(\alpha_h + \alpha_\ell)[u_h - p_h - v_h] + \delta\Pi_2 = 0, \tag{17}$$

$$-\alpha_\ell(u_\ell - p_\ell) + \delta\Pi_\ell = 0. \tag{18}$$

It is relatively intuitive, and proved in claim A-1 of the appendix, that (18) is binding at equilibrium: otherwise, the L incumbent could raise its price and keep its consumers. Because $\Pi_\ell = \alpha_\ell p_\ell/(1-\delta)$, equations (9) and (10) hold. Thus, once the two groups are separated, the L incumbent behaves in the same way and obtain the same profit as if it where the incumbent with only the LNE consumers present (see Proposition 1). Similarly, the fact that (17) is binding follows; if not, the H incumbent could raise its price and increase its profit.

The constraint that the L incumbent does not try to attract the HNE consumers is not included in Claim 2, because it is not binding. Indeed the L incumbent finds it less attractive to attract the HNE consumers than do the entrants, as its opportunity cost to do so is greater because it attracts a positive profit from the LNE consumers.

The reasoning which precedes show the "necessity" part of the following lemma. The sufficiency part, which is quite straightforward, is proved in the appendix.

Lemma 10. *Equations* (9) *and* (11) *are sufficient and necessary for the fact that once* LNE *and* HNE *consumers have purchased from different platforms then will continue to do so in the continuation equilibrium.*

This lemma provides conditions for the fact that once there are two platforms in a period, there are also two platforms in subsequent periods. Its proof relies on deviations in period 2 and subsequent periods, when the consumers have already split between the two platforms.

We now turn to the study of the first period and on the incentives of the agents to create two platforms out of one. First, we must have

$$-(\alpha_h + \alpha_\ell)\left[p_2 - (u_h - v_h)\right] + \delta\Pi_2 \le 0. \tag{19}$$

Otherwise, in the first period an entrant could attract all the consumers by charging a price "slightly below" $p_2 - (u_h - v_h)$ and make strictly positive profit. Claim A-2, presented on page A — 4 in the appendix, shows that in equilibrium this constraint must be binding: otherwise the Incumbent could profitably increase its price in period 1. Along with (17) this implies $p_2 = p_h$ and therefore $\Pi_2 = \Pi_h$. Then, $\Pi_h = \alpha_h p_h/(1-\delta)$ implies (11) and (12).

Summarizing the discussion so far, we have proved that if there is an equilibrium satisfying (15), then the prices must satisfy equations (9) and (11). Furthermore, if the prices satisfy these equations, then there is no profitable entry by (17) and (18).

In the first period, the entrant who attracts the LNE consumers must charge less than $p_2 - w_\ell$ and its profit will be less than $-\alpha_\ell(w_\ell - p_2) + \delta\Pi_\ell = \alpha_\ell(p_2 - (w_\ell - \delta u_\ell))$. No entrant will be willing to attract the LNE consumers unless $p_2 > w_\ell - \delta u_\ell$, which is therefore a necessary condition for the existence of a two platform equilibrium.

If $(\alpha_h + \alpha_\ell)(w_\ell - \delta u_\ell) > \alpha_h p_2$, the Incumbent would find it profitable to charge $w_\ell - \delta u_\ell + \varepsilon$ and keep all the consumers. Hence, $(\alpha_h + \alpha_\ell)(w_\ell - \delta u_\ell) \leq \alpha_h p_2$, which is equivalent to (2NtwCond).

We have used (14) and (SMALLCE) to prove that (2NtwCond) is necessary for the existence of a two platform equilibrium. It is easy to show (see claim A-3 on page A — 5 in the appendix) that both of these conditions hold whenever (2NtwCond) holds, we have therefore proved (2NtwCond) to be a necessary conditions for the existence of a two platform equilibrium.

6.4 Condition (2NtwCond) is a sufficient condition for a two platform equilibrium.

We now show that (2NtwCond) is sufficient for the existence of a two platform equilibrium. Much of the construction of the equilibrium in the preceding subsection can be used in this proof. We will proceed by going through the possible deviations showing that they are not profitable.

First period and any subsequent period where, off equilibrium, there is only one incumbent

Incumbent: By the reasoning leading to (19), if it increased its price an entrant would find it profitable to attract all the consumers. Decreasing the price to $p_2' \geq w_\ell - \delta u_\ell$ would not change demand and hence would lead to lower profit. Decreasing the price below $w_\ell - \delta u_\ell$ would enable the Incumbent to keep all the consumers, but, because (2NtwCond) holds, at the cost of lower profits.

Entrants: Competition between the entrants will lead them to charge a price equal to $-\delta\Pi_\ell/\alpha_\ell = -\delta u_\ell$. At that price, LNE consumers find it profitable to purchase from the entrants. The proof of proposition 3 shows that no entrant will find it profitable to attract all the consumers.

Subsequent periods with two incumbents

Incumbents The same reasoning as for period 1 shows that the H incumbent has no incentive to deviate. The incentives of the L incumbent are the same as in 5.1 as far as competing for the LNE consumers. In order to attract the HNE consumers it would have to choose a price smaller to $p_2 - (u_h - v_h)$, which is unprofitable for the same reason that it would be unprofitable for an entrant to attract all the consumers in the first period.

Entrants For the same reasons as in 5.1 they cannot profitably attract the LNE consumers. For the same reason as in the first period, they cannot attract profitably all the consumers.

7 Analysis of equilibria with one platform

In section E of the Appendix, we study the existence and the properties of single network equilibria. In this section, we survey these results as they apply when δ is close enough to 1 — other results, in particular those who pertain to "small" δ, are presented in the Appendix.

The main result is the following corollary, which requires some additional assumptions that will be presented later.

Corollary 9. *There exists a* $\overline{\delta}$ *such that for any* $\delta \geq \overline{\delta}$*:*

a) there exists at least one single platform equilibrium;

b) the Incumbent's profit in all single platform equilibria is $(\alpha_h + \alpha_\ell)(u_h - v_h)$*;*[21]

c) the profit of the Incumbent is larger than in the static model, but smaller than the value of a flow of one period profit.

Recall that for large δ, there exist no two platform equilibrium (see Corollary 3 and the comment that follows).[22]

From item b), the Incumbent's profit is equal to the product of the total number of consumers, $\alpha_h + \alpha_\ell$, and the price it would charge to maximize its profit from HNE consumers in the static model, $u_h - v_h$. This quantity is also an upper bound on the profits of the Incumbent for any type of equilibrium and any discount factor. This shows that the ratio of long run profits to profits in the one period model is quite small.

[21] This corollary is an easy consequence of Lemmas A-2, A-3, A-6 and A-5: for δ close to 1, only type S2 and T2 equilibria exist and in all these equilibria the Incumbent's profit is $(\alpha_h + \alpha_\ell)(u_h - v_h)$.

[22] More generally, whatever δ, for no set of parameters does there exist at the same time a two platform equilibrium and an "S type" one platform equilibrium (see definition below). On the other hand, for a small set of parameters there exist both a two platform equilibrium and a "T type" equilibrium.

Finally, we obtain the same result as in the dynamic switching cost model of Biglaiser et al. (2013): for large discount factors LNE consumers are as valuable to the Incumbent as HNE consumers: the derivative of the profit with respect to α_ℓ is the same as the derivative with respect to α_h (the same caveat discussed in footnote 18 also applies here). This is for the same reason: the LNE consumers will accept aggressive offers from entrants but also be very footloose in future periods. Thus, entrants will be reluctant to offer low prices.

We now turn to a discussion of the structure of one platform equilibria. The analysis of these equilibria is complicated by the fact that they can differ along two dimensions. The first dimension describes what happens off the equilibrium path if the consumers ever get "separated" in two different platforms: consumers can either stay separated in subsequent periods — the S (for **S**eparated) equilibria, or they can all purchase from the same platform in the period after they have split so that two platforms coexist for only one period — the T (for **T**ogether) equilibria.

The second dimension is which entry constraint binds on the Incumbent along the equilibrium path when it sells to both types of consumers: preventing profitable entry which would attract only the LNE consumers or preventing profitable entry which would attract all consumers. As we discuss in more detail and prove in the Appendix, for large δ the binding constraint comes from platforms which try to attract both LNE and HNE consumers. To attract both types of consumers, an entrant must charge a price p_E which satisfies $v_h - p_E > u_h - p_2$.[23] This is unprofitable only if $(\alpha_h + \alpha_\ell)p_E + \delta\Pi_2 < 0$, which is equivalent to $p_E + \delta p_2/(1-\delta) < 0$ because $\Pi_2 = (\alpha_h + \alpha_\ell)p_2/(1-\delta)$. Therefore, to prevent this type of entry p_2 must satisfy

$$p_2 \leq (1-\delta)(u_h - v_h). \tag{20}$$

Along the equilibrium path, constraint (20) is binding and this yields the profits of Corollary 9.

The study of the existence of T equilibria raises some difficulties. In Section 2, we assumed that consumers left their current platform only if this *strictly* increased their utility (this is the strict inequality in (2)). If we maintain this assumption, no T equilibrium exist: after, out of equilibrium, some consumers have purchased from an entrant, the Incumbent would have to choose the highest possible price that makes them strictly prefer to come back to its platform, but no such highest price exists.[24] To finesse this issue,

[23] By (SMALLCE), this condition is sufficient for the entrant to attract first the LNE consumers and then the HNE consumers.

[24] The set of prices that make consumers prefer to purchase from the entrant than from the incumbent has a supremum, but no maximum.

only in the analysis of T equilibria, we assume that, when the consumers are separated and indifferent between moving and not, they move whenever the platform they are purchasing from would generate negative profits if it lowered its price, whereas the destination platform would still make positive profits if it decreased its price by a small enough amount.[25]

8 Literature

Since the beginning of the literature on network externalities (Katz and Shapiro, 1985, 1986, 1994), it has been well understood that the externalities generate multiple equilibria in the subgame where the consumers choose a network to join.[26] Most policy analyses and research papers have assumed that consumers would "naturally" coordinate on the incumbent's platform. Few authors have tried to analyze more formally the way in which consumers coordinate. Among those who have done so, some have relied on the global games approach pioneered by Carlsson and Van Damme (1993). Hałaburda and Yehezkel (2013) and Jullien and Pavan (2013) use it to analyze models of two sided markets, focussing on different types of asymmetric information between agents while Argenziano (2008) and Gunay Bendas (2013) use the global games approach with network effects. Others have used equilibrium selection arguments, for instance in two sided markets, Caillaud and Jullien (2003) assume that the consumers coordinate on the equilibrium the less favorable to the entrant while Ambrus and Argenziano (2009) use the concept of coalitional rationalizability to solve the consumer coordination problem. Hałaburda et al. (2016) in a dynamic duopoly model with heterogenous firms assumes, essentially, that last period's incumbent is "focal" in the beliefs of the consumers, who all have the same preferences.

Other authors use strategies closer to those of the present paper, modelling either the way in which consumers would migrate from an incumbent to an entrant or the way in which they would choose to join a platform where none existed previously. For instance, Ochs and Park (2010) assume that firms are uncertain about the tastes of others and go through several rounds of choosing whether or not to join a firm. Farrell and Saloner (1985, 1986, 1988) study games where consumers choose one after the other whether or not to join a network. Biglaiser, Crémer, and Veiga (2016) studies the way in which consumers would attempt to free ride on each other while attempting

[25] A more fundamentalist approach would conduct a full Bertrand game analysis, including the continuation game played by the consumers.

[26] Farrell and Klemperer (2007) present a very complete survey of the literature up to the mid 2000's.

not to be the first to abandon the incumbent.

Weyl (2010) analyzes a static monopoly multi-sided platform and solves the multiplicity problem by assuming that a monopoly platform can offer "insulating tariff" which insure consumers against the risk of mis-coordination, while Weyl and White (2016) study the consequences the effects of these tariffs when platforms compete.

Some of the earlier literature simply assumed that the consumers coordinate on the platform which they prefer (see, for instance, Katz and Shapiro, 1986; Fudenberg and Tirole, 2000)). This assumption, which only makes sense when consumers agree on the identity of the best platform, would by itself negate incumbency advantage. The source of the incumbency advantage comes from the fact that consumers, who live for two periods, cannot change platform and therefore new customers prefer the incumbent platform if the price difference is not too large. A similar approach is taken in a static model by Crémer, Rey, and Tirole (2000) who simply assume that there is a mass of "trapped" consumers who cannot leave the Incumbent.

Of course, for the purposes of this paper the discussion of the coordination between consumers is not an end in itself, but a step necessary to the study of dynamic competition between networks. The literature on the topic is vast, and we cannot do it justice here. The aim of the rest of this section is simply to pinpoint where we believe our contributions to lie.

Most of the recent literature on dynamic competition has concentrated on oligopoly models where platforms are in stable competition with each other. Cabral (2011) provides an interesting and representative example. In every period, a new consumer arrives who chooses to join one of two differentiated platforms; once a consumer has joined a platform he stays with this platform until "death" — this is equivalent to assuming infinite switching costs. The dynamics of the model are driven by the interplay of two forces: platforms would like to price low to be more likely to attract consumers and increase their future attractiveness. On the other hand, as they become larger, they have incentives to increase their prices in order to reap high profits. The analysis focuses on the dynamics of dominance and on the fact that convergence to monopoly is unlikely. Some authors have used similar models to study compatibility between platforms (Chen, Doraszelski, and Harrington, 2009).

Contrary to this strand of literature, we are interested in the dynamics of competition *for* the market rather than competition *in* the market. Consumers can switch networks and the networks do try to attract each other's customers.

Some of the older network externalities literature has concerns closest to ours. For instance, Katz and Shapiro (1986) build a two period model where consumers are unattached at the start. They show that firms will compete aggressively in the first period, to benefit of the incumbency advantage in the second period — this phenomenon also arises in the switching cost literature under the name “invest and harvest” (Klemperer, 1995). In Fudenberg and Tirole (2000) consumers live for two periods and new consumers coordinate on the best network from their own viewpoint. In some periods all the consumers will purchase from the incumbent. In others, a low cost entrant will price low enough to attract the new consumers. Combined with the different equilibrium selection discussed above, this yields a view of incumbency very different from ours.

In our model, it is entrants that will create new platforms and separate consumers. Some authors have been studied the incentives for a monopolist to do so, allocating its customers among several platforms that it owns (see Board, 2009; Veiga, 2013).

Finally, we note that two companion papers, Biglaiser et al. (2013) and Biglaiser et al. (2015), examine models with free entry, an incumbent, and consumer switching costs. As in the current paper, the Incumbent’s profit does not grow very much when expanding the time horizon from 1 period to an infinite number of periods. Network externalities have often been called “social switching costs”, however there are subtle but important differences in their consequences for the strategies of firms. These differences are analyzed in Crémer and Biglaiser (2012).

9 Conclusion

In this paper, we study the long run incumbency value of a market where consumer network externalities are present with free entry. Competition *for* the market greatly limits the additional profits in a dynamic model relative to the static market outcome. Consumer heterogeneity can have great strategic value and even consumers that never join the incumbent’s platform can greatly enhance the incumbent’s profits. In order to study incumbency value we define a criterion for equilibrium selection which is based on a model of migration between platforms. solution concept that is based on consumers’ beliefs about what other consumers will do that is very favorable for incumbent platforms.

In order to identify the main economic forces at play, we have purposefully used a very sparse model. In the rest of this conclusion, we discuss two dimensions in which it could be fruitfully expanded: two sided markets and the addition of switching costs.

Many markets with network externalities are two-sided markets. Using our selection approach in two sided settings introduces some interesting possibilities. The 'snowballing' effect discussed in Section 2 when discussing migration through large steps would not necessarily arise. Also, many two sided platforms offer multiple functionalities on at least one side. They may therefore compete on some dimensions and not others (for instance, eBay and Amazon compete for the sales of some goods, but not on the ebook market). Combined with the fact that consumers often multi-home, this opens up a very rich area for investigation which has not been sufficiently explored.

Levin (2013) states "In traditional industries with network effects, high switching costs are often an important compounding factor" but argues that for Internet platforms switching costs are small. However, we know very little about the interaction between switching costs and network externalities and more theoretical and empirical work needs to be done on the topic. Until then, it will be impossible, for instance, to know whether relatively small switching costs can have large effects. In Crémer and Biglaiser (2012), we present an example that shows that switching costs and network externalities do not necessarily have additive effects. We plan to investigate further this interaction in future work. The loss of the myopia principle should make the study of dynamic models quite challenging.

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Appendix

A Proofs of lemmas in section 2

Proof of lemma 2. Assume that a migration path which leads to AC equilibrium is not a migration path through large steps. Then, there exists a t such that $\eta^t_{\theta(t)s(t)} > 0$. It is easy to see that $\{\theta(t+1), d(t+1), s(t+1)\} = \{\theta(t), d(t), s(t)\}$: at step $t+1$, the migration will involve the same type of consumers moving from the same platform to the same platform as at step $t+1$. Indeed, in all platforms the utility of agents of type $\theta(t)$ is the same at the end and at the beginning of step t, except for the fact that it is strictly higher in $d(t)$ and strictly smaller in $s(t)$. For agents of the other type, the same property holds true; however by (1), the increase in the utility they derive from $d(t)$ and the the decrease in the utility they derive from $s(t)$ are smaller than for agents of type $\theta(t)$. Hence, condition (2) holds true for $\{\theta(t), d(t), s(t)\}$ when the superscript $t-1$ is replaced by t. We can therefore construct a new migration path, which will lead to the same final allocation by replacing steps t and $t+1$ by one "larger" step with the same θ, d and s. Iterating on this procedure will lead to a migration path through large steps which leads to the same allocation as the original path. □

Proof of lemma 4. It is easy to see that a migration path through large steps can be replaced by a migration path with $\mu(t) < \varepsilon$ for all t. Let $\bar{\theta}(t)$, $\bar{\mu}(t)$, $\bar{s}(t)$, and $\bar{d}(t)$ be respectively the type of transferred customers, the mass of migrating customers, the source platform and the destination platform in the large step migration path. We construct a new migration path in the following way. Let $\underline{\theta}(1) = \bar{\theta}(1)$, $\underline{d}(1) = \bar{d}(1)$, $\underline{s}(1) = \bar{s}(1)$, and $\underline{\eta}$ such that

$$0 < \underline{\eta}^1_{\underline{\theta}(1)\underline{d}(1)} - \underline{\eta}^0_{\underline{\theta}(1)\underline{d}(1)} = \underline{\eta}^0_{\underline{\theta}(1)\underline{s}(1)} - \underline{\eta}^1_{\underline{\theta}(1)\underline{s}(1)} < \varepsilon.$$

At the end of step 1 on the new migration path, by the same reasoning as in the proof of lemma 2,

$$\psi_{\underline{\theta}(1)}\left(\underline{\eta}^1_{h\underline{d}(1)}, \underline{\eta}^1_{\ell\underline{d}(1)}\right) - p_{\underline{d}(1)} > \psi_{\theta'}\left(\underline{\eta}^1_{j\theta'}, \underline{\eta}^1_{j,-\theta'}\right) - p_j - \left[\psi_{\theta'}\left(\underline{\eta}^1_{j'\theta'}, \underline{\eta}^1_{j',-\theta'}\right) - p_{j'}\right]$$

for all $(\theta', j, j') \neq (\underline{\theta}(1), \underline{d}(1), \underline{s}(1))$. Therefore

$$\{\underline{\theta}(2), \underline{d}(2), \underline{s}(2)\} = \{\underline{\theta}(1), \underline{d}(1), \underline{s}(1)\},$$

and by an easy recurrence it is possible to build a new migration path which after a finite number t_1^* of steps will rejoin the original migration path: $\underline{\eta}_{\theta j}^{t_1^*} = \overline{\eta}_{\theta j}^1$ for all θ and j. We can then take $\underline{\theta}(t_1^*+1) = \overline{\theta}(2)$, $\underline{d}(t_1^*+1) = \overline{d}(2)$ and $\underline{s}(t_1^*+1) = \overline{s}(2)$. By the same reasoning as in the previous paragraph there will exist t_2^* such that after t_2^* steps the new migration path will have the same allocation as the original migration path at $t = 2$. The result is proved by noticing that we can repeat the process until convergence to the final allocation along the original path. □

Proof of lemma 5 (page 9). We have defined migration paths by the fact that they lead from one initial allocation to a final allocation. To show that there exists a final allocation define the following procedures, inspired by large steps migration paths, but without guarantee that they lead to a final allocation. At every step, check whether there exist a $\{\theta(t), d(t), s(t)\}$ satisfying (2). If there is move all the consumers of type $\theta(t)$ from $s(t)$ to $d(t)$. If there is not, we have identified a AC equilibrium. To finish the proof, we only need to show that any such procedure will eventually find itself at a stage when this happens. At every step, either the destination platform already has clients or it charges a strictly lower price that the source platform, or both. To each platform which has a strictly positive mass of consumers, associate an index equal to the number of platforms which charge strictly lower prices multiplied by either 1 if it has a positive mass of only one type of consumers and 2 if it has a positive mass of both types of consumers. The sum of these platform indexes decreases by at least one at each stage of the migration. Given that this sum cannot be smaller than 1, the result is proved. □

B Proofs of Corollaries 4 and 6

Proof of Corollary 4, page 22. Condition (2NtwCond) is equivalent to

$$h(\delta) \stackrel{\text{def}}{=} (1-\delta)\alpha_h(u_h - v_h) - ((1-\delta)\alpha_\ell + \alpha_h)(w_\ell - \delta u_\ell) > 0.$$

If $\alpha_h(u_h - v_h) > (\alpha_h + \alpha_\ell)w_\ell$, the function h is positive for $\delta = 0$ and strictly negative for $\delta = 1$. Furthermore, it is a quadratic function for which the coefficient of δ^2 is equal to $-\alpha_\ell u_\ell$; it is therefore concave and has exactly one zero for $\delta \in (0,1)$, which proves the first part of the corollary. When $w_\ell = u_\ell$ condition (2NtwCond) is equivalent to $\alpha_h(u_h - v_h) \geq \big((1-\delta)\alpha_\ell + \alpha_h\big)w_\ell$, which, together with (SmallCE), proves the result. □

Proof of Corollary 6. The derivative of the right hand side of (2NtwCond) with respect to δ has the same sign as

$$\begin{aligned}[-\alpha_\ell(w_\ell - \delta u_\ell) + ((1-\delta)\alpha_\ell + \alpha_h)(-u_\ell)](1-\delta) \\ + ((1-\delta)\alpha_\ell + \alpha_h)(w_\ell - \delta u_\ell) \\ = -\alpha_\ell u_\ell (1-\delta)^2 + \alpha_h(w_\ell - u_\ell).\end{aligned}$$

If $\alpha_h(w_\ell - u_\ell) > \alpha_\ell u_\ell$, this derivative is positive for all δ; the maximum of the right hand side of (2NtwCond) is obtained for $\delta = 1$ and is equal to $\alpha_\ell(w_\ell - u_\ell)$. The first part of the corollary is a direct consequence of these facts. If $\alpha_h(w_\ell - u_\ell) < \alpha_\ell u_\ell$, this derivative is negative for all

$$\delta < \widetilde{\delta} \stackrel{\text{def}}{=} 1 - \sqrt{\frac{\alpha_h(w_\ell - u_\ell)}{\alpha_\ell u_\ell}} < 1.$$

Calling $\widetilde{V}$ the corresponding value of the right hand side, the second part of the corollary follows. (See figure 3.) □

C Proof of Proposition 3

In this appendix, we provide the formal proofs of the lemmas and claims of section 6.3 of the main text.

Proof of Claim 1. Assume that the H incumbent charges a price p_h which satisfies (14) and $v_h + u_\ell - p_\ell \geq u_h - p_h$. If an entrant charges $p_E \geq -(u_\ell - p_\ell)$, it attracts no consumer as, by (14), we have $-p_E \leq u_\ell - p_\ell \leq u_h - p_h$.

On the other hand if the entrant charges $p_E < -(u_\ell - p_\ell)$, it attracts all the consumers: the LNE consumers as $-p_E > u_\ell - p_\ell$, and, once it has attracted the LNE consumers, the HNE consumers as $v_h - p_E > v_h + u_\ell - p_\ell \geq u_h - p_h$. In equilibrium, this must not be profitable and we must therefore have $-(\alpha_h + \alpha_\ell)(u_\ell - p_\ell) + \delta\Pi_2 \leq 0$. Then, the H incumbent would find it profitable to increase p_h until the left hand side and the right hand side of (14) are equal, which proves the claim. □

Claim A-1. *If* (14) *holds, then* $-\alpha_\ell(u_\ell - p_\ell) + \delta\Pi_\ell = 0$.

Proof. Assume $-\alpha_\ell(u_\ell - p_\ell) + \delta\Pi_\ell < 0$. In any period after the first, the L incumbent could increase its profit by charging $p'_\ell \in (p_\ell, u_\ell - \delta\Pi_\ell/\alpha_\ell)$. Indeed, in order to attract the LNE consumers an entrant would have to charge at most $\overline{p}'_E = -(u_\ell - p'_\ell) < -\delta\Pi_\ell/\alpha_\ell$ and would therefore make negative profits, $\alpha_\ell \overline{p}'_E + \delta\Pi_\ell$. □

Proof of Claim 2. By (16), if (17) does not hold, it is possible to find $p'_h > p_h$ which satisfies both

$$-(\alpha_h + \alpha_\ell)(u_h - p'_h - v_h) + \delta\Pi_2 < 0 \tag{A-1}$$

and

$$u_h - p'_h > v_h + u_\ell - p_\ell \Longrightarrow u_h - p'_h > v_h - p_\ell. \tag{A-2}$$

We will show that a deviation by the H incumbent to such a p'_h would be profitable.

The H and L incumbents announce their prices simultaneously; therefore the deviation by the H incumbent would not affect p_ℓ. By (A-2), after such a deviation the LNE consumers would respond by purchasing either from the lowest price entrant or from the L incumbent, as in Figure 4 (replacing, of course, p_h by p'_h). Therefore, the deviation would be unprofitable for the H incumbent only if an entrant could profitably attract *all* the consumers. It could do this only by charging a price p'_E which satisfies $v_h - p'_E > u_h - p_h$, which by (A-1) implies $p'_E \leq -(u_h - p'_h - v_h) < -\delta\Pi_2/(\alpha_h + \alpha_\ell)$. The profits of the entrant, $(\alpha_h + \alpha_\ell)p'_E + \delta\Pi_2$, would be strictly negative, which proves the result. □

Proof of Lemma 10. Only the sufficiency part is left to prove. From Figure 4 an entrant could try either to a) attract only the LNE consumers by charging a price strictly smaller than $-(u_\ell - p_\ell)$, but, by claim A-1, this is not profitable as $\alpha_\ell(-(u_\ell - p_\ell)) + \delta\Pi_\ell = \alpha_\ell u_\ell(-1 + (1-\delta) + \delta) = 0$, or b) attract all consumers by charging a price strictly smaller that $-(u_h - p_h) + v_h$, but this is not profitable by (19). □

Claim A-2. *If* (14) *holds, then*

$$-(\alpha_h + \alpha_\ell)(u_h - p_2 - v_h) + \delta\Pi_2 = 0. \tag{A-3}$$

Proof. Because (19) holds, it is sufficient to show that if $-(\alpha_h + \alpha_\ell)(u_h - p_2 - v_h) + \delta\Pi_2 < 0$, then a deviation by the period 1 incumbent to a price $p'_2 > p_2$ satisfying

$$-(\alpha_h + \alpha_\ell)(u_h - p'_2 - v_h) + \delta\Pi_2 < 0 \tag{A-4}$$

would be profitable. At the original p_2, there was profitable entry by attracting only the LNE consumers; *a fortiori*, it will also be profitable to attract the LNE consumers when the price is p'_2. Therefore, the deviation by the period 1 incumbent is unprofitable only if an entrant could profitably attract *all* the consumers when the price is p_2'. By (A-4), in order to attract the HNE consumers as well as the LNE consumers, a entrant needs to charge a price p_E' which satisfies $p_E' < -(u_h - p_2') + v_h < -\delta\Pi_2/(\alpha_h + \alpha_\ell)$. The profits of the entrant, $(\alpha_h + \alpha_\ell)p_E' + \delta\Pi_2$, would be strictly negative, which proves the result. □

Claim A-3. *If* (2NtwCond) *holds, then a)* (SMALLCE) *holds and b) the prices defined by* (9) *and* (11) *satisfy condition* (14).

Proof. a) Because $w_\ell - \delta u_\ell \geq (1-\delta)w_\ell$, (2NtwCond) implies (SMALLCE).

b)
$$u_h - p_h = u_h - \frac{(1-\delta)(\alpha_h+\alpha_\ell)(u_h - v_h)}{(1-\delta)\alpha_\ell + \alpha_h} \geq \delta\frac{\alpha_h}{(1-\delta)\alpha_\ell + \alpha_h}(u_h - v_h)$$
$$\geq \delta\frac{\alpha_h}{(1-\delta)\alpha_\ell + \alpha_h} \times \frac{(1-\delta)\alpha_\ell + \alpha_h}{(1-\delta)\alpha_h}(w_\ell - \delta u_\ell) \text{ (by (2NtwCond))}$$
$$= \frac{\delta}{1-\delta}(w_\ell - \delta u_\ell) \geq \delta u_\ell = u_\ell - p_\ell.$$
□

D Proof that (14) holds

In the main text, we have assumed that condition (14) holds. In this part of the appendix, we show that this must indeed be the case whenever a two platform equilibrium exists.

We proceed by contradiction. If (14) did not hold, we would have $u_h - p_h < u_\ell - p_\ell$. We first show that the results which we obtained in 6.3 based on the fact that consumers do not change networks from period 2 onwards hold with h and ℓ inverted. Then, we show that these results are incompatible with the separation of the consumers in two different platforms in the first period.

The proof of claim 1 can be reproduced with h and ℓ inverted[27] and therefore

$$v_\ell + u_h - p_h < u_\ell - p_\ell. \tag{A-5}$$

Similarly, adapting the reasoning which leads to claim 2 we obtain:

$$-(\alpha_h + \alpha_\ell)(u_\ell - p_\ell - v_\ell) + \delta\Pi_2 = 0; \tag{A-6}$$
$$-\alpha_h(u_h - p_h) + \delta\Pi_h = 0 \tag{A-7}$$

Equation (A-7) implies $p_h = u_h(1-\delta)$. Along with (A-5) and (A-6), this implies

$$u_h < \frac{\Pi_2}{\alpha_h + \alpha_\ell}. \tag{A-8}$$

[27]It is sufficient to note that the proof of Claim 1 depends on the relative sizes of the network effect only through (14).

To compute Π_2, we eliminate p_2 from the system composed of the two equations a) (A-3), which still holds as Claim A-2, whose proof is based on period 1 deviations which attract all consumers, is still valid as it stands, and b) $\Pi_2 = \alpha_h p_2 + \delta\alpha_h u_h$. Substituting into (A-8), we obtain

$$u_h((1+\delta)\alpha_h + \alpha_\ell) < \alpha_h(1+\delta)u_h - \alpha_h v_h \Longleftrightarrow \alpha_\ell u_h + \alpha_h v_h < 0,$$

which establishes the contradiction.

E One platform equilibria

E.1 Main results

In this appendix, we provide a fuller analysis of one platform equilibria than in the text that will also hold for lower values of δ. For ease of exposition, we will assume that (14) holds, where a sufficient condition for this to hold is $\delta > u_\ell/u_h$.[28] The main result in terms of Incumbent profits is that while profits maybe higher in the dynamic than in the static versions of the model, there still is a quite limited improvement in them. In particular, for the smaller values of δ, the Incumbent profit can be lower than $(\alpha_\ell+\alpha_h)(u_h-v_h)$.

We begin by stating some results about the Incumbent's equilibrium profits, then we discuss the four types of one platform equilibria and discuss their properties and their existence.

For small δ the profits of the incumbent can be smaller than expressed in Corollary 9:

Corollary 10. *The Incumbent's equilibrium profit in the dynamic model does not exceed*

$$(\alpha_h + \alpha_\ell) \min\left[u_h - v_h, \frac{w_\ell}{1-\delta}\right].$$

There is a small set of parameters for which there exist both a single platform and a two platform equilibrium. As the following corollary states, the Incumbent always prefers the one platform equilibrium, which is also welfare maximizing.

[28]If $u_h - p_h < u_\ell - p_\ell$, then by the same argument as in the proof of claim A-1, $p_h = u_h(1-\delta)$ and therefore $u_h - p_h = \delta u_h$. Since $p_\ell \geq 0$, $u_h - p_h < u_\ell - p_\ell$ is possible only if $u_\ell > \delta u_h$.

Assuming that condition (14) holds is only relevant in the analysis of S equilibria.

From Claim 2, when (14) holds the profits of an entrant who attracts the LNE consumers are $\alpha_\ell u_\ell$. From Appendix D, when (14) fails, they would be greater. We conjecture that this will make entrants more aggressive along the equilibrium path and lead to lower profits for the Incumbent.

		Entry constraints	
		LNE consumers	both types
After separation	keep separated	Sℓ	S2
	back together	Tℓ	T2

Figure 5: The type of equilibria in the one platform case.

Corollary 11. *For parameter values such that both a single platform equilibrium and a two platform equilibrium exist, the profit of the incumbent is larger in the single platform equilibrium.*

E.2 Classification of Equilibria

As stated in section 7, there are four types of single platform equilibria represented in Figure 5; they differ along two dimensions. The first dimension describes what happens off the equilibrium path if the consumers ever get "separated" in two different platforms: consumers can either stay separated in subsequent periods — the S (for **S**eparated) equilibria; or they can all purchase from the same platform in the period after they have split so that two platforms coexist for only one period — the T (for **T**ogether) equilibria.

As long as it sells to both types of consumers, the Incumbent platform faces two entry constraints: preventing profitable entry which would attract only the LNE consumers and preventing profitable entry which would attract all consumers. This second dimension describes which one of these two constraints is binding.

In E.3, we examine the S type equilibria in which consumers keep on purchasing from different platforms after out-of-equilibrium moves in which they do so. In E.4, we study T type equilibria.

E.3 S equilibria: consumers stay separated after they split

In S equilibria, if, off the equilibrium path, HNE and LNE consumers join different platforms in some period, then they stay on these platforms in subsequent periods.

As discussed just after the statement of Claim 2, when condition (14) holds $\Pi_\ell = \alpha_\ell u_\ell$ and $p_\ell = u_\ell(1-\delta)$. Along the equilibrium path, in order to attract only the LNE consumers, an entrant must charge a price p_E which satisfies $-p_E > w_\ell - p_2$ as well as $v_h - p_E \leq u_h - p_2$; such a p_E exists by (SMALLCE). This is profitable if $\alpha_\ell p_E + \delta\Pi_\ell > 0$, which is equivalent to $-p_E < \delta\Pi_\ell/\alpha_\ell = \delta u_\ell$.

To make this type of entry impossible the incumbent must ensure that if $p_E = -\delta u_\ell$ the LNE consumers choose not to purchase from the entrant. Therefore, it must choose p_2 such that

$$p_2 \leq w_\ell - \delta u_\ell. \tag{A-9}$$

To attract both types of consumers, an entrant must charge a price p_E which satisfies $v_h - p_E > u_h - p_2$.[29] This is unprofitable only if $(\alpha_h + \alpha_\ell)p_E + \delta\Pi_2 < 0$, which is equivalent to $p_E + \delta p_2/(1-\delta) < 0$ because $\Pi_2 = (\alpha_h + \alpha_\ell)p_2/(1-\delta)$. Therefore, to prevent this type of entry p_2 must satisfy

$$p_2 \leq (1-\delta)(u_h - v_h). \tag{A-10}$$

Along the equilibrium path, both constraints (A-9) and (A-10) must be met, with at least one binding. This implies the following lemma.

Lemma A-1. *In S type single platform equilibria the profit of the incumbent is*

$$(\alpha_h + \alpha_\ell)\min\left[\frac{w_\ell - \delta u_\ell}{1-\delta}, u_h - v_h\right].$$

When (A-9) is binding, we have an Sℓ equilibrium; when (A-10) is binding we have an S2 equilibrium. As we show below, a full characterization of the conditions under which these equilibria exist are rather complicated. However, we do find that a) S equilibria exist only for "small" $u_h - v_h$ and b) no S equilibrium exists when (2NtwCond) holds, *i.e.*, when a two platform equilibrium exists. Furthermore, the set of parameters for which there exist either a two platform equilibrium or an S equilibrium is quite large. S2 equilibria are described in Lemma A-2, while Sℓ equilibria are described in Lemma A-3.

Lemma A-2. *If $\delta\alpha_\ell - (1-\delta)\alpha_h > 0$, then an S2 equilibrium exists if and only if*

$$u_h - v_h \leq \min\left[\frac{w_\ell - \delta u_\ell}{1-\delta}, \frac{(\alpha_h + \alpha_\ell)(\delta u_\ell - v_\ell)}{\delta\alpha_\ell - (1-\delta)\alpha_h}\right].$$

[29] By (SMALLCE), this condition is sufficient for the entrant to attract first the LNE consumers and then the HNE consumers.

If $\delta\alpha_\ell - (1-\delta)\alpha_h < 0$, then an S2 equilibrium exists if and only if

$$\frac{(\alpha_h + \alpha_\ell)(\delta u_\ell - v_\ell)}{\delta\alpha_\ell - (1-\delta)\alpha_h} \leq u_h - v_h \leq \frac{w_\ell - \delta u_\ell}{1-\delta}.$$

In both cases, the profit of the Incumbent is $(\alpha_h + \alpha_\ell)(u_h - v_h)$.

Proof. The fact that Condition (A-10) is binding implies $\Pi_2 = (\alpha_h+\alpha_\ell)(u_h - v_h)$. Along with (A-9) it also implies $u_h - v_h \leq (w_\ell - \delta u_\ell)/(1-\delta)$. By the discussion on page 26, after LNE and HNE consumers are separated Claim 2 holds. This implies $p_h = p_2$ and $\Pi_h = \alpha_h(u_h - v_h)$. In order to attract the LNE consumers the H incumbent would have to charge a price p'_h which satisfies $v_\ell - p'_h > u_\ell - p_\ell = \delta u_\ell$. This is unprofitable only if $\Pi_h \geq (\alpha_h+\alpha_\ell)(v_\ell - \delta u_\ell) + \delta\Pi_2$, which is equivalent to $(\alpha_h + \alpha_\ell)(\delta u_\ell - v_\ell) \geq (\delta\alpha_\ell - (1-\delta)\alpha_h)(u_h - v_h)$, which proves the lemma. □

Lemma A-3. *An $S\ell$ type equilibrium exists if and only if $u_h - v_h \geq (w_\ell - \delta u_\ell)/(1-\delta)$ and*

$$\frac{(1-\delta)(\alpha_\ell+\alpha_h)}{\alpha_h}(v_\ell - \delta u_\ell) + \frac{\delta[\alpha_h(2-\delta)+\alpha_\ell(1-\delta)]}{(1-\delta)\alpha_h}(w_\ell - \delta u_\ell) \\ \leq u_h - v_h \leq \frac{(1-\delta)\alpha_\ell + \alpha_h}{(1-\delta)\alpha_h}(w_\ell - \delta u_\ell). \quad \text{(A-11)}$$

The profit of the Incumbent is $(\alpha_h + \alpha_\ell)(w_\ell - \delta u_\ell)/(1-\delta)$.

Proof. The fact that (A-9) is binding immediately yields $p_2 = (w_\ell - \delta u_\ell)/(1-\delta)$ and Π_2.

The right most inequality is the consequence of the fact that the Incumbent must have no incentive to increase the price on the equilibrium path in such a way that it sells only to the HNE consumers. The lowest price that an entrant would be willing to charge in order to attract all the consumers is $-\delta\Pi_2/(\alpha_h + \alpha_\ell)$, and therefore the incumbent can price up to $(u_h - v_h) - \delta\Pi_2/(\alpha_h + \alpha_\ell)$ and sell only to the HNE consumers. In subsequent periods, it will set the same price by Claim 2. Therefore, this deviation is unprofitable, only if

$$\Pi_2 \geq \frac{1}{1-\delta} \times \alpha_h \times \left(u_h - v_h - \delta \frac{\Pi_2}{\alpha_\ell + \alpha_h} \right), \quad \text{(A-12)}$$

which is equivalent to the right most inequality of (A-11).

Off the equilibrium path, consumers stay separated. In order to attract the LNE consumers away from the L incumbent, the H incumbent would have to announce a price not larger than $v_\ell - u_\ell + p_\ell = v_\ell - \delta u_\ell$. This is not profitable only if $\Pi_h \geq (\alpha_h + \alpha_\ell)(v_\ell - \delta u_\ell) + \delta\Pi_2$, where Π_h, the profit of the H incumbent is equal to the right hand side of (A-12). This inequality is equivalent to the left most inequality in (A-11).[30]

Out of equilibrium, once the consumers are separated, the profit of either the L incumbent or of an entrant which would charge p' and attract all the consumers would be $(\alpha_h + \alpha_\ell)p' + \delta\Pi_2$. For such a strategy to be profitable, we must have $p' \geq -\delta\Pi_2/(\alpha_h + \alpha_\ell)$. It is only feasible if $p' + (u_h - v_h) < 0$ — otherwise the H incumbent can profitably ensure the fidelity of the HNE consumers. By (A-10), these two bounds on p' cannot hold simultaneously and therefore no such deviation is possible. □

[30] The other possible deviations are not profitable. If the H incumbent increases its price it looses all its consumers. Claim 2 shows that the L incumbent cannot deviate.

E.4 T equilibria: consumers come back together after they split

In a T equilibrium, we must have

$$p_2 \leq w_\ell. \tag{A-13}$$

Otherwise, by charging, for instance, $(p_2 - w_\ell)/2 > 0$, an entrant would attract the LNE consumers and make positive profits even if it "lost" all these consumers in the following period. An entrant must also find it unprofitable to attract all the consumers. This occurs if and only if $u_h - p_2 \geq v_h + \delta\Pi_2/(\alpha_\ell + \alpha_h)$, which is equivalent to

$$p_2 \leq (1-\delta)(u_h - v_h). \tag{A-14}$$

because $\Pi_2 = (\alpha_h + \alpha_\ell)p_2/(1-\delta)$.

When (A-13) is binding, we have a Tℓ equilibrium; when (A-14) is binding we have a T2 equilibrium. T equilibria are described in the lemmas below.

For some parameters, there exist both a two platform equilibrium and a T equilibrium: this is true, for instance, when $\delta = 1/2$, $\alpha_\ell = 3\alpha_h/2$ and $u_\ell = v_\ell = w_\ell = 8(u_h - v_h)/15$. Then, (SmallCE), (2NtwCond) and the conditions of lemma A-5 below are satisfied. This is impossible for S type one platform equilibria.

The following fact is both economically interesting and technically important for the characterization of T equilibria: if, off the equilibrium path, the consumers are separated in two platforms, in the next period they will all purchase from the H incumbent, not from the L incumbent. The L incumbent firm profitably attracts all the consumers only if $p_\ell \geq -\delta\Pi_2$. It will attract the HNE consumers if for all non negative prices the consumers prefer to "leave" the H incumbent (otherwise, the H incumbent could deviate and profitably keep its consumers). Thus, we must have $p_\ell < -(u_h - v_h)$. But, from Lemma A-4, $-(\alpha_h + \alpha_\ell)(u_h - v_h) + \delta\Pi_2 < 0$, and these two conditions cannot be met simultaneously. A similar argument shows that no entrant can attract all consumers to its platform.

Lemma A-4. *In the T type single platform equilibria the profit of the incumbent is*

$$(\alpha_h + \alpha_\ell)\min\left[\frac{w_\ell}{1-\delta}, u_h - v_h\right].$$

The following two lemmas provide the conditions for existence and the profits for each of the types of T equilibria.

Lemma A-5. *A T2 equilibrium exists if and only if*

$$\frac{(\alpha_\ell + \alpha_h)(u_\ell - v_\ell)}{\delta\alpha_\ell - (1-\delta)\alpha_h} \leq u_h - v_h \leq w_\ell/(1-\delta).$$

The equilibrium profit of the Incumbent is $(\alpha_h + \alpha_\ell)(u_h - v_h)$.

Proof. In a T2 equilibrium, the Incumbent cannot profitably raise its price and sells only to the HNE consumers at price $p_2 = (1-\delta)(u_h - v_h) \leq w_\ell$. Thus, it immediately follows that $\Pi_2 = (\alpha_h + \alpha_\ell)(u_h - v_h)$ and this gives us the right hand side of the necessary and sufficient condition.

Off the equilibrium path, we need to find conditions for the Incumbent to be willing and able to attract the LNE consumers if the consumers are ever separated. The Incumbent attracts the LNE consumers, by charging a p_h smaller than or equal to $v_\ell - u_\ell$, since the L incumbent is willing to charge any positive price to keep the LNE consumers.[31,32] Furthermore, the Incumbent must choose a price that will induce the HNE consumers to stay on its platform instead of joining an entrant platform. Entrants are willing to price down to $-\delta(u_h - v_h)$ in order to attract all the consumers and the incumbent must therefore charge a price p_h smaller than or equal to $(u_h - v_h)(1-\delta)$ to keep the HNE consumers. Because $v_\ell - u_\ell \leq 0 \leq (1-\delta)(u_h - v_h)$, the binding constraint is $p_h \leq v_\ell - u_\ell$.

The most profitable deviation which allows the H to keep only the HNE consumers is to charge $(1-\delta)(u_h - v_h)$. Thus, for the H incumbent to prefer to attract the LNE consumers to deviating and keeping only the HNE consumers we must have $(\alpha_\ell + \alpha_h)(v_\ell - u_\ell + \delta(u_h - v_h)) \geq \alpha_h(u_h - v_h)$ which is equivalent to the left hand side of the necessary and sufficient condition. □

Lemma A-6. *A Tℓ equilibrium exists if and only if:*

$$\begin{aligned} \frac{w_\ell}{1-\delta} \leq u_h - v_h \leq \\ \min\Bigg[\frac{(\alpha_\ell + \alpha_h)\left\{(1-\delta)(v_\ell - u_\ell) + \delta w_\ell\right\}}{\alpha_h} + \frac{\delta w_\ell}{1-\delta}, \\ \frac{(\alpha_\ell + \alpha_h)\{w_\ell(1+\delta) - \delta(v_\ell - u_\ell)\}}{\alpha_h}\Bigg]. \end{aligned} \tag{A-15}$$

The Incumbent's equilibrium profit is $(\alpha_\ell + \alpha_h)w_\ell/(1-\delta)$.

[31] We assume that firms do not use weakly dominated strategies and it is clear that if the L firm charged a positive price and lost consumers that it could profitably deviate and lower its price.

[32] Recall that we are using the weak inequality definition of AC equilibria for this class of equilibria.

Proof. The binding pricing constraint when the incumbent has all the consumers is (A-13) and the left hand side of condition (A-15) reflects this. It follows immediately that $p_2 = w_\ell/(1-\delta)$ and $\Pi_2 = (\alpha_\ell + \alpha_h)w_\ell/(1-\delta)$.

Off the equilibrium path, in order to attract back the LNE consumers, the Incumbent must offer a price less than or equal to $v_\ell - u_\ell$, since the L incumbent will price at 0.[33] The lowest price an entrant is willing to offer to attract all the consumers is $-\delta w_\ell/(1-\delta)$. Thus, the Incumbent's price must not exceed $u_h - v_h - \delta w_\ell/(1-\delta)$. Since $v_\ell - u_\ell < 0 < u_h - v_h - \delta w_\ell/(1-\delta)$, it is the first constraint which is binding. The most profitable deviation which would allow the Incumbent to keep only the HNE consumers is to charge $u_h - v_h - \delta w_\ell/(1-\delta)$. Thus, for the Incumbent to prefer to bring all consumers onto its platform we must have

$$(\alpha_\ell + \alpha_h)\left[v_\ell - u_\ell + \delta w_\ell/(1-\delta)\right] \geq \frac{\alpha_h}{1-\delta}\left[u_h - v_h - \frac{\delta w_\ell}{1-\delta}\right], \tag{A-16}$$

which is the first term of the right hand side of (A-15).

On the equilibrium path, the Incumbent must prefer to keep all consumers to just keeping the HNE consumers and then bringing them back the LNE onto its platform the following period at a price of $v_\ell - u_\ell$. Hence, we must have

$$(\alpha_\ell + \alpha_h)w_\ell/(1-\delta) \geq \alpha_h(u_h - v_h) + \delta(\alpha_\ell + \alpha_h)\left[v_\ell - u_\ell + \delta w_\ell/(1-\delta)\right],$$

where the left hand side is the equilibrium profit, and the right hand side is the sum of profits in the defection period plus the discounted left hand side of expression (A-16). This can be rewritten

$$\frac{(\alpha_\ell + \alpha_h)w_\ell(1+\delta) - \delta(\alpha_\ell + \alpha_h)\left[v_\ell - u_\ell\right]}{\alpha_h} \geq u_h - v_h,$$

which is the second term of the right hand side of (A-15). □

[33] See proof of Lemma A-5.