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Abstract

We consider optimal monetary policy in a model that integrates credit frictions in the standard New Keynesian model with sticky prices and wages as well as adjustment costs of capital. Different from traditional models with credit frictions such as Carlstrom and Fuerst (1998), the model is able to generate an anti-cyclical external finance premium as observed empirically in the US economy. Monetary policy is characterized by a Taylor rule according to which the nominal interest rate is set as a function of the deviation of the inflation rate from its target rate, the output gap, and Tobin's q. The latter is measured by the relative price of newly installed capital. We show that monetary policy should optimally decrease interest rates with higher capital prices. However, the consideration of Tobin's q implies only small welfare effects. These results are robust with respect to a more general Epstein and Zin (1989) welfare specification and to exogenous shifts to both the atemporal marginal rate of substitution between consumption and leisure as well as the households' discounting behavior.

JEL-Codes: E120, E320, E520, G120.

Keywords: asset prices, monetary policy, New Keynesian model, q targeting.

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1 Introduction

The financial crises of 2007 has triggered renewed interest into a debate which started in the late 1990s: should central banks target asset prices? Bernanke and Gertler (1999, 2001) were among the first to ask how central bankers should react to asset price volatility. They argue that there is no need for concern if asset price movements reflect changes in economic fundamentals. In this case, asset prices are only relevant with regard to monetary policy if they convey additional information about the state of the econommy. However, if asset prices were driven by nonfundamental factors, their influence could be destabilizing. For this case, they consider a bursting asset price bubble in a version of the model developed in Bernanke, Gertler, and Gilchrist (1999) and show that asset price targeting may even destabilize the economy.

More recently, the question of asset price targeting has been analyzed in models that introduce financial frictions in the standard New Keynesian business cycle. For instance, Carlstrom and Fuerst (2007) argue that asset price targeting may increase the parameter region within which the rational expectations equilibrium is not unique so that sunspot equilibria arise. Machado (2012) considers learning in the model of Carlstrom and Fuerst (2007) and shows that asset price targeting may hamper the convergence to the rational expectations equilibrium. Christiano, Rostagno, and Motto (2010) study a purely news-driven upswing in a model with the financial accelerator of Bernanke, Gertler, and Gilchrist (1999). The central bank can moderate the effects of this kind of shock if it also reacts to the increased credit demand of borrowers. Faia and Monacelli (2007) introduce a nominal price rigidity into the financial accelerator model of Carstrom and Fuerst (1997). The interaction between the nominal friction and the financial friction requires a negative response of the nominal interest rate set by the central bank and the relative price of capital. Moreover, the welfare gains of targeting the price of capital in addition to inflation are very small as compared to a strict anti-inflation policy.

In this paper, we also consider the desirability of asset price targeting with respect to its effect on the welfare of the representative household. Our study is most closely related to Faia and Monacelli (2007). With regard to the methodology, we employ the approach pioneered by Schmitt-Grohé and Uribe (2005, 2007) and compute the welfare effects of an extended Taylor rule relative to a simple Taylor rule that just reacts to the deviation of inflation from the central bank's target. As in these papers we disregard rules that i) lead to indeterminacy and ii) may hurt the zero lower bound. Different from Faia and Monacelli (2007), we consider i) both additively separable and non-separable preferences and ii) a richer structure of shocks as a reduced form of capturing market incompleteness and saving behavior in presence of possible severe economic downturns as described in the Gourio (2012) disaster framework. In addition, iii) we merge adjustment costs of capital and financial frictions so that our model is closer in spirit to Bernanke, Gertler and Gilchrist (1999).

The paper is structured as follows. In the next section, we introduce a first model with the usual shock to total factor productivity and a government spending shock. The model features two nominal frictions (price and wage staggering as in Calvo (1983)) and a financial friction in the production of primary goods as proposed by Carlstrom and Fuerst (1998). Section 3 presents the calibration of the model. In Section 4, we present our results. Section 5 studies the robustness of these results with respect to the specification of the household's preferences and with respect to the interaction of supply and demand shocks. The main conclusion of Faia and Monacelli (2007) remains intact in all our setups: the welfare gain of targeting the relative price of capital are negligible. Section 6 concludes.

2 The Model

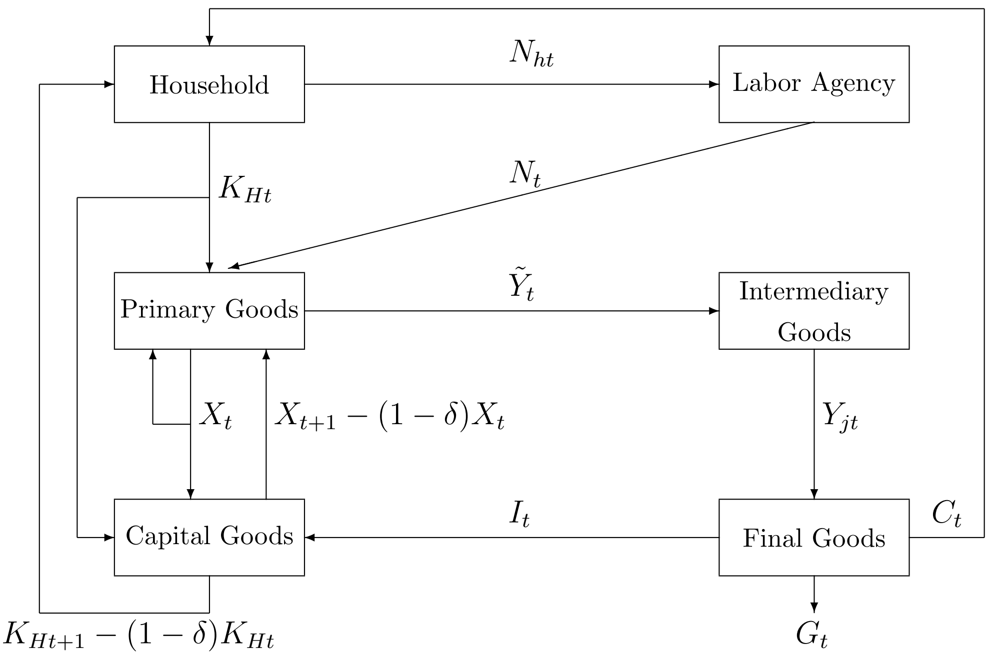
The basic model merges a standard New Keynesian model with sticky nominal prices and wages as, e.g., in Erceg, Henderson, and Levin (2000) and Christiano, Eichenbaum, and Evans (2005), and adjustment costs of capital as, e.g., in Jermann (1998) and Bernanke, Gertler, and Gilchrist (1999) with the credit friction model of Carlstrom and Fuerst (1998).

2.1 Structure of the Model

The model consists of a household, the government, a labor agency, a sector of primary goods producers, a wholesale sector, a final goods sector, and a capital goods producing sector. Time is discrete and denoted by t. Figure 2.1 illustrates the flows of factor services and goods between the household and the various sectors of the economy.

The household has a unit mass of members who rent their labor services N_{ht} to the labor agency. The agency sells a composite N_t of these services to the primary goods

Figure 2.1: Structure of the Model



producers. Each firm $f \in [0, 1]$ in this sector employs labor N_t and capital services from the household sector K_{Ht} and from other firms X_t to produce $\tilde{Y}_t$ units of a good, which serves as input in the production of intermediary goods. Each firm $j \in [0, 1]$ in this sector produces a differentiated good Y_{jt} and sells it to the final goods sector. This sector bundles the intermediary goods and sells consumption goods C_t to the household, investment goods I_t to the capital goods sector and public goods G_t to the government. New capital goods are produced from capital services, rented from the household and primary goods producers and from investment goods. They are sold to primary goods producers, $X_{t+1} - (1 - \delta)X_t$, and to the household, $K_{Ht+1} - (1 - \delta)K_{Ht}$.

2.2 Final Goods

The firm in this sector buys the brands Y_{jt}, $j \in [0, 1]$ at the nominal price P_{jt} from the intermediary goods sector and combines them to a final good Y_t, which is sold at the nominal price P_t to the household as consumption good C_t, to the capital goods production sector as investment good I_t, and to the government, G_t. The technology is given by

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{\epsilon_y - 1}{\epsilon_y}} dj \right]^{\frac{\epsilon_y}{\epsilon_y - 1}}, \quad \epsilon_y > 1. \tag{2.1}$$

Profit maximization implies the usual demand function for intermediary good j:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\epsilon_y} Y_t \tag{2.2}$$

and the zero-profit condition

$$P_t Y_t = \int_0^1 P_{jt} Y_{jt} dj$$

determines the prise index

$$P_t = \left(\int_0^1 P_{jt}^{1-\epsilon_y} dj\right)^{\frac{1}{1-\epsilon_y}}. \tag{2.3}$$

2.3 Capital Goods

We implement adjustment costs of capital as in Bernanke, Gertler, and Gilchrist (1999). New capital goods are produced from capital services $K_t = K_{Ht} + X_t$ rented at the price r_{Kt} and from investment goods I_t according to the function $\Psi(I_t/K_t)K_t$. They are sold at the price q_t. The function Ψ is increasing in its argument and strictly concave. As usual, we assume that it is costless to keep the capital stock constant: $\Psi(\delta) = \delta$ and $\Psi'(\delta) = 1$, where δ is the rate of capital depreciation. In our numerical simulations we follow Jermann (1998) and employ the functional form

$$\Psi(I_t/K_t) = \frac{a_1}{1-\zeta}\left(\frac{I_t}{K_t}\right)^{1-\zeta} + a_2. \tag{2.4}$$

Profit maximization,

$$\max_{K_t, I_t} \quad q_t \Psi(I_t/K_t)K_t - r_{Kt}K_t - I_t$$

implies

$$q_t = \frac{1}{\Psi'(I_t/K_t)}, \tag{2.5a}$$

$$r_{Kt} = q_t \Psi(I_t/K_t) - (I_t/K_t) \tag{2.5b}$$

so that profits are zero in equilibrium.

2.4 Intermediary Goods and Price Setting

A firm $j \in [0,1]$ in the intermediary sector buys goods at the nominal price P_{yt} from the primary production sector, brands it and sells it at the price P_{jt} to the final goods sector. Its profit in units of the final product equals

$$D^I_{jt} = \left(\frac{P_{jt}}{P_t} - g_t\right) Y_{jt}, \quad g_t = \frac{P_{yt}}{P_t} \tag{2.6}$$

and is distributed to the household sector. We apply the Calvo (1983) mechanism for sticky price setting: In each period t, a randomly selected fraction $1 - \varphi_y$ of firms in this sector receives the signal to optimally choose their price P_{At} and thus their relative price $p_{At} := P_{At}/P_t$. The remaining fraction is allowed to raise their nominal price P_{Nt} according to the inflation rate observed in the previous period:

$$P_{Nt} = \pi_{t-1} P_{Nt-1}, \quad \pi_t := \frac{P_t}{P_{t-1}}. \tag{2.7}$$

2.5 Primary Production

Primary production is organized in a sector with a unit mass of firms $f \in [0,1]$. In Carlstrom and Fuerst (1998) these firms are owned by risk-neutral entrepreneurs. We follow Chugh (2013) and assume that the household owns the firms but that firms are more impatient than the household. This reflects an un-modeled principal agent problem that drives a wedge between the interest of the household and the management of the firm. Its effect is to prevent full self-financing of firms (see Carlstrom and Fuerst (1997)).

Firm Assets. The firms need credit to pay for their factor services in advance. In order to get credit they have to accumulate assets. Let X_{ft} denote the stock of capital owned by firm f at the beginning of period t. The firm rents this capital at the price r_{Yt} to other primary goods producing firms. When production in this sector has taken place it rents the same amount to the capital goods sector at the rate r_{Kt}. In addition to its factor income the firm receives a small transfer Δ_{ft} from the household. This ensures that the firm will be able to continue operating even in the case of credit default. The transfer is deducted from the firms dividend payment to the household. The net worth NW_{ft} of the firm, therefore, is equal to

$$NW_{ft} = (q_t(1-\delta) + r_{Yt} + r_{Kt})X_{ft} + \Delta_{ft}. \tag{2.8}$$

Production and Factor Demand. The firm f employs labor N_{ft} and capital K_{ft} to produce the amount

$$\tilde{Y}_{ft} = \omega_{ft} Z_t N_{ft}^{1-\alpha} K_{ft}^{\alpha}, \quad \alpha \in (0,1), \tag{2.9}$$

where ω_{ft} is an idiosyncratic shock, distributed iid with density ϕ and mean $\mathbb{E}(\omega_{ft}) = \Omega_t$. Z_t is an aggregate technology shock that is governed by

$$\ln Z_t = \rho_Z \ln Z_{t-1} + \sigma_Z \epsilon_t, \quad \rho_Z \in [0,1), \ \epsilon_t \sim \text{iidN}(0,1). \tag{2.10}$$

The firm must pay for its factor services the real amount M_{ft} in advance. The firm observes Z_t but not ω_{ft} before it decides on the size of its credit $M_{ft} - NW_{ft}$. After it has observed ω_{ft} the firm maximizes

$$g_t \omega_{ft} Z_t N_{ft}^{1-\alpha} K_{ft}^{\alpha}$$

subject to $M_{ft} \geq w_t N_{ft} + r_{Yt} K_{ft}$, where w_t denotes the wage rate in units of the final good. The first-order conditions

$$\begin{aligned} \lambda_{ft} w_t &= (1-\alpha) g_t \omega_{ft} Z_t N_{ft}^{-\alpha} K_{ft}^{\alpha}, \\ \lambda_{ft} r_{Yt} &= \alpha g_t \omega_{ft} Z_t N_{ft}^{1-\alpha} K_{ft}^{\alpha-1}, \end{aligned}$$

imply $w_t / r_{Yt} = ((1-\alpha)/\alpha)(K_{ft}/N_{ft})$ so that all firms employ the same capital-labor-ratio $k_t := (K_{ft}/N_{ft})$. As a consequence, the scaled Lagrange multiplier of the constraint $v_t := \lambda_{ft}/\omega_{ft}$ is independent of the firm index f, and

$$v_t M_{ft} = g_t Z_t N_{ft}^{1-\alpha} K_{ft}^{\alpha} = g_t Z_t k_t^{\alpha} N_{ft}. \tag{2.11}$$

Thus, in terms of the final good, v_t is a mark-up on factor costs M_{ft}. For later reference note that (2.11) can be integrated to

$$v_t M_t = g_t Z_t k_t^{\alpha} N_t, \tag{2.12}$$

where $x_t = \int_0^1 x_{ft} df$, for $x \in \{M, N, K\}$, and that the first-order conditions for factor demand can be written in terms of aggregate variables only:

$$w_t = (1-\alpha)(g_t/v_t)\tilde{Y}_t/N_t, \tag{2.13a}$$

$$r_{Yt} = \alpha (g_t/v_t)\tilde{Y}_t/K_t, \tag{2.13b}$$

$$\tilde{Y}_t = Z_t N_t^{1-\alpha} K_t^{\alpha}. \tag{2.13c}$$

Since both M_{ft} and k_t are independent of the realization of ω_{ft}, equation (2.11) also implies that N_{ft} is independent of the idiosyncratic shock. This allows us to compute the aggregate output of the primary sector:[1]

$$\int_0^1 \omega_{ft} Z_t N_{ft}^{1-\alpha} K_{ft}^{\alpha} df = Z_t k_t^{\alpha} \int_0^1 \omega_{ft} N_{ft} df = Z_t k_t^{\alpha} \Omega_t N_t = \Omega_t \tilde{Y}_t. \tag{2.14}$$

The Credit Contract. The firm borrows the amount $M_{ft} - NW_{ft}$ intra-periodically from the household. The realization of ω_{ft} is private information. If the creditor wishes to see the firm's production, he must pay a screening cost. This cost is assumed to be proportional to marked-up real factor costs in terms of the final good $v_t M_{ft}$ with factor of proportionality κ. This is the costly state verification framework of Townsend (1979), Gale and Hellwig (1985), and Williamson (1987) as employed in Carlstrom and Fuerst (1997, 1998).

The credit contract specifies M_{ft}, the lending rate r_{Lt}, and a bankruptcy threshold $\bar{\omega}_{ft}$, given by

$$\bar{\omega}_{ft} := (1 + r_{Lt}) \frac{M_{ft} - NW_{ft}}{g_t Z_t N_{ft}^{1-\alpha} K_{ft}^{\alpha}}, \tag{2.15}$$

so that for $\omega_{ft} < \bar{\omega}_{ft}$ firm f defaults and the creditor seizes the firm's output less the screening costs. Otherwise the firm redeems the loan, pays the interest and keeps all of its production. Because the household lends to all firms, he can fully diversify the risk and acts as if he was risk-neutral. The expected return for the firm equals

$$\int_{\bar{\omega}_{ft}} \omega_t g_t Z_t N_{ft}^{1-\alpha} K_{ft}^{\alpha} \phi(\omega_t) d\omega_t) - (1 - \Phi(\bar{\omega}_{ft}))(1 + r_{Lt})(M_{ft} - NW_{ft}),$$
$$\Phi(\bar{\omega}_{ft}) = \int^{\bar{\omega}_{ft}} \phi(\omega_t) d\omega_t.$$

Using (2.15) this expression can be written as $g_t Z_t N_{ft}^{1-\alpha} K_{ft}^{\alpha} f(\bar{\omega}_{ft})$, where

$$f(\bar{\omega}_{ft}) = \int_{\bar{\omega}_{ft}} \omega_t \phi(\omega_t) d\omega_t - (1 - \Phi(\bar{\omega}_{ft}))\bar{\omega}_{ft}. \tag{2.16}$$

Note that from (2.11) the expected return to the borrower can also be written as a fraction of the factor costs in terms of final output, $v_t M_{ft} f(\bar{\omega}_{ft})$. The expected return of the creditor equals

$$\int^{\bar{\omega}_{ft}} \omega_t g_t Z_t N_{ft}^{1-\alpha} K_{ft}^{\alpha} \phi(\omega_t) d\omega_t + (1 - \Phi(\bar{\omega}_{ft}))(1 + r_{Lt})(M_{ft} - NW_{ft}) - \Phi(\bar{\omega}_{ft}) \kappa v_t M_{ft}.$$

[1] The last part of the equation follows from the independence of ω_{ft} and N_{ft}, so that $\int_0^1 \omega_f N_f df = \int_0^1 N_f df \int_0^1 \omega_f df$, and from a law of large numbers, i.e., $\int_0^1 \omega_f df = \mathbb{E}(\omega_f) \equiv \Omega$.

Using (2.15) and (2.13c), the expected return is equal to $v_t M_{ft} g(\bar{\omega}_t)$ with

$$g(\bar{\omega}_{ft}) = \int^{\bar{\omega}_{ft}} \omega_t \phi(\omega_t) d\omega_t + (1 - \Phi(\bar{\omega}_{ft}))\bar{\omega}_{ft} - \Phi(\bar{\omega}_{ft})\kappa. \tag{2.17}$$

Finally, note that

$$f(\bar{\omega}_{ft}) + g(\bar{\omega}_{ft}) = \Omega_t - \Phi(\bar{\omega}_{ft})\kappa. \tag{2.18}$$

The optimal pair $(M_{ft}, \bar{\omega}_t)$ maximizes the expected return of the firm $v_t M_{ft} f(\bar{\omega}_t)$ subject to the participation constraint of the household. Since the loan is intra-period, the household will be indifferent between lending to a producer or keeping his funds, if he will at least get back his loan: $v_t M_{ft} g(\bar{\omega}_t) \geq M_{ft} - NW_{ft}$. This optimal pair thus solves

$$\begin{aligned} 1 &= v_t \left[\Omega_t - \Phi(\bar{\omega}_{ft})\kappa - \frac{f(\bar{\omega}_{ft})\phi(\bar{\omega}_{ft})\kappa}{1 - \Phi(\bar{\omega}_{ft})} \right], \\ M_{ft} &= \frac{NW_{ft}}{1 - v_t g(\bar{\omega}_{ft})}. \end{aligned}$$

Condition (2.19a) determines the bankruptcy threshold as a function of the markup on factor costs v_t. Since the share $f(\bar{\omega}_t)$ depends only on the (cumulative) probability density function, all firms face the same threshold $\bar{\omega}_t$. Further note that equations (2.19b) and (2.8) can be aggregated over all firms in the primary production sector. Therefore,

$$1 = v_t \left[\Omega_t - \Phi(\bar{\omega}_t)\kappa - \frac{f(\bar{\omega}_t)\phi(\bar{\omega}_t)\kappa}{1 - \Phi(\bar{\omega}_t)} \right], \tag{2.19a}$$

$$M_t = \frac{NW_t}{1 - v_t g(\bar{\omega}_t)}. \tag{2.19b}$$

Eventually, equations (2.11) and (2.19b) imply that the external finance premium r_{Lt} in equation (2.15) is determined by

$$r_{Lt} = \frac{\bar{\omega}_t}{g(\bar{\omega}_t)} - 1. \tag{2.20}$$

Asset Accumulation of the Firm. We assume that the firm f distributes

$$D^P_{ft} = v_t M_{ft} f(\bar{\omega}_t) - \Delta_{ft} - q_t X_{ft+1} \tag{2.21}$$

as dividends to the household. As we shall demonstrate subsequently, the household's discount factor for returns from period $t + s$ is equal to $\beta^s \Lambda_{t+s}/\Lambda_t$, where Λ_t is the multiplier of the household's budget constraint. The firm is more impatient than the household and employs the discount factor $(\beta\gamma)^s \Lambda_{t+s}/\Lambda_t$ with $\gamma \in (0,1)$. Therefore, the value of the firm is given by

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\gamma\beta)^s \frac{\Lambda_{t+s}}{\Lambda_t} D^P_{ft+s}.$$

Substituting for D^P_{ft} from (2.21), for M_{ft} from (2.19b), and for NW_{ft} from (2.8) and maximizing with respect to X_{ft+1} yields the Euler equation

$$q_t = \gamma\beta\mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[q_{t+1}(1-\delta) + r_{Yt+1} + r_{Kt+1} \right] \frac{\upsilon_{t+1} f(\bar{\omega}_{t+1})}{1 - \upsilon_{t+1} g(\bar{\omega}_{t+1})}. \tag{2.22}$$

2.6 Labor Demand

The household has a unit mass of members $h \in [0,1]$ who sell their labor services N_{ht} at the wage rate W_{ht} to an agency. The agency bundles them into a single service,

$$N_t = \left[\int_0^1 N_{ht}^{\frac{\epsilon_n - 1}{\epsilon_n}} dh \right]^{\frac{\epsilon_n}{\epsilon_n - 1}}, \quad \epsilon_n > 1, \tag{2.23}$$

and sells this service at the nominal wage W_t to the primary good producers. Profit maximization implies the demand function

$$N_{ht} = \left(\frac{W_{ht}}{W_t} \right)^{-\epsilon_n} N_t, \tag{2.24}$$

and the zero-profit condition

$$W_t N_t = \left(\int_0^1 W_{ht} N_{ht} dh \right)$$

determines the wage index

$$W_t = \left[\int_0^1 W_{ht}^{1-\epsilon_n} \right]^{\frac{1}{1-\epsilon_n}}. \tag{2.25}$$

2.7 Wage Setting

The current period utility u of household member h depends on his consumption C_{ht}, labor supply N_{ht} and the consumption habit $\mathscr{C}_t$. We parameterize u as follows:

$$u(C_{ht}, N_{ht}) = \frac{(C_{ht} - \mathscr{C}_t)^{1-\eta} - 1}{1-\eta} - \frac{\nu_0}{1+\nu_1} N_{ht}^{1+\nu_1}, \quad \eta, \nu_0, \nu_1 \geq 0, \tag{2.26}$$

where $\mathscr{C}_t := \chi C_{t-1}, \chi \in [0, 1)$. In equilibrium, $\mathscr{C}_t$ thereby equals a fraction χ of previous period's aggregate consumption $C_{t-1} = \int_0^1 C_{ht-1} dh$.

We again apply the Calvo (1983) framework, i.e. in each period a random fraction $1 - \varphi_n$ of the household members receive a signal to choose their nominal wage W_{At} optimally. The remaining fraction φ_n is allowed to increase their wage W_{Nt} according to the price inflation observed in the previous period:

$$W_{Nt} = a\pi_{t-1} W_{Nt-1}, \quad \pi_t = \frac{P_t}{P_{t-1}}. \tag{2.27}$$

Those who receive a signal choose the optimal real wage $\tilde{w}_t := W_{At}/P_t$ to maximize their individual (standard) lifetime utility

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_n)^s u(C_{ht+s}, N_{ht+s})$$

subject to labor demand (2.24) and the budget constraint

$$\frac{W_{ht}}{P_t} N_{ht} + RMT_t \geq 0,$$

where RMT_t is a stand-in for the remaining terms of this constraint, which are independent of the optimal wage and will be introduced in the next subsection.

2.8 Consumption and Portfolio Allocation

As usual we assume that the members of the household pool their income so that their decision to consume and save is subject to a budget constraint in which we can ignore the index h. The representative household owns two different kinds of assets:[2] nominal bonds B_t and physical capital K_{Ht}. The former pays the predetermined nominal interest rate $Q_t - 1$. The latter yields a factor income of $(r_{Yt} + r_{Kt})K_{Ht}$ because capital is first employed in the production of primary goods and subsequently in the production of capital goods. In addition to interest income, rental income, and wage income $w_t N_t$, the household receives dividends from the primary goods producers $\int D^I_{ft} df$ and dividends from the intermediary goods producers $\int D^P_{jt} dj$. He pays taxes T_t to the government, and spends the remaining income on consumption C_t and asset accumulation.

[2]In addition, he lends intra-periodically to firms in the primary sector. Since – as noted above – he receives his loan back at the end of the period, we ignore the loan in the budget constraint.

His budget constraint in terms of the final goods, therefore, reads:

$$\begin{aligned} & w_t N_t + (r_{Yt} + r_{Kt}) K_{Ht} + \int_0^1 D^P_{jt} dj + \int_0^1 D^I_{ft} df + (Q_t - 1)\frac{B_t}{P_t} - T_t \\ & \geq C_t + q_t(K_{Ht+1} - (1-\delta)K_{Ht}) + \frac{B_{t+1} - B_t}{P_t}. \end{aligned} \tag{2.28}$$

In our basic framework, per capita consumption $C_{ht} = C_t$, the future stock of capital K_{Ht+1}, and optimal bond holdings B_{t+1} are determined from maximizing the (standard) welfare specification

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \int_0^1 u(C_{ht+s}, N_{ht+s}) dh$$

subject to the budget constraint (2.28). The respective first-order conditions are

$$\Lambda_t = (C_t - \mathscr{C}_t)^{-\eta}, \tag{2.29a}$$

$$q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left(q_{t+1}(1-\delta) + r_{Yt+1} + r_{Kt+1} \right), \tag{2.29b}$$

$$1 = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{Q_{t+1}}{\pi_{t+1}}. \tag{2.29c}$$

2.9 Government

The government's budget constraint is presented by

$$\frac{B_{t+1} - B_t}{P_t} + T_t = (Q_t - 1)\frac{B_t}{P_t} + G_t. \tag{2.30}$$

We assume $B_t = 0$ for all t and that government spending G_t is exogenously governed by

$$\ln G_t = (1-\rho_G)\ln G + \rho_G \ln G_{t-1} + \sigma_G \epsilon_t, \quad \rho_G \in [0,1), \sigma_G \geq 0, \; \epsilon_t \sim \text{iidN}(0,1). \tag{2.31}$$

2.10 Monetary Authority

The central bank sets the nominal interest rate Q_{t+1} according to a Taylor rule. This rule includes the previously set interest rate Q_t to account for sluggish adjustment, the deviation of the inflation π_t rate from the target rate π, the deviation of Tobin's q (q_t) from its (efficient) steady state value of $q = 1$, and the deviation of output Y_t from its stationary level Y:

$$Q_{t+1} = Q_t^{\delta_1} \left(\frac{\pi}{\beta}\right)^{1-\delta_1} \left(\frac{\pi_t}{\pi}\right)^{\delta_2} (q_t)^{\delta_3} (Y_t/Y)^{\delta_4}, \quad \delta_1 \in [0,1). \tag{2.32}$$

The choice of the parameters δ_2, δ_3, and δ_4 must satisfy two requirements: (i) the equilibrium dynamics of the economy must be determinate and (ii) the Taylor rule is subject to the zero lower bound, i.e. $Q_t \geq 1$.

2.11 Equilibrium Dynamics

In equilibrium all markets clear. Capital services employed in the production of primary goods equal

$$K_t = K_{Ht} + X_t, \quad X_t = \int_0^1 X_{ft} df \tag{2.33}$$

and accumulate according to

$$K_{t+1} - (1-\delta)K_t = \Psi(I_t/K_t)K_t. \tag{2.34}$$

Equation (2.21) implies

$$q_t X_{t+1} = f(\bar{\omega}_t) g_t \tilde{Y}_t - \int_0^1 (D_{ft}^P + \Delta_{ft}) df, \tag{2.35}$$

where the right-hand side of equation (2.12) was used to replace $v_t M_t$. Aggregating equation (2.8) over all primary production firms yields

$$NW_t = [q_t(1-\delta) + r_{Yt} + r_{Kt}]\, X_t + \int_0^1 \Delta_{ft} df. \tag{2.36}$$

Condition (2.19b) and equation (2.12) imply

$$\tilde{Y}_t = \frac{v_t}{g_t} \frac{NW_t}{1 - v_t g(\bar{\omega}_t)}. \tag{2.37}$$

Consolidating the budget constraints of the household, the government, and the definition of dividend payments to the household yields

$$g_t \tilde{Y}_t(\Omega_t - \Phi(\bar{\omega}_t)\kappa) + \int_0^1 \left(\frac{P_{jt}}{P_t} - g_t \right) Y_{jt} dj = C_t + I_t + G_t.$$

Market clearing for intermediary goods requires $\int_0^1 Y_{jt} dj = \Omega_t \tilde{Y}_t$ and the first part of the integral term equals Y_t (see (2.2)). Hence, the preceding equation reduces to the resource constraint:

$$Y_t = C_t + I_t + G_t + \Phi(\bar{\omega}_t)\kappa g_t \tilde{Y}_t. \tag{2.38}$$

The last term on the right-hand-side reflects the resource costs of monitoring insolvent firms in the primary production sector.

We present the full system of equations that determine the dynamics of the model in the Appendix.

2.12 Welfare Analysis

Our goal is to determine whether or not the inclusion of Tobin's q in the Taylor rule (2.32) does improve monetary policy. Our point of reference is the welfare associated with the simple rule $\delta_1 = \delta_3 = \delta_4 = 0$ and $\delta_2 = 1.5$. Let

$$\begin{aligned} V_t &= V_t^C - V_t^N, \text{ with} \\ V_t^C &:= \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{(C_{t+s} - \chi C_{t+s-1})^{1-\eta} - 1}{1-\eta} \right], \\ V_t^N &:= \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{\nu_0}{1+\nu_1} \tilde{N}_{t+s}^{1+\nu_1} \right], \end{aligned} \tag{2.39}$$

denote the welfare associated with this solution, where $\tilde{N}_t^{1+\nu_1} = \int_0^1 N_{ht}^{1+\nu_1} dh.$

We solve the model for non-zero coefficients δ_i, $i = 1, \dots, 4$ on a four-dimensional grid. Let

$$\begin{aligned} \bar{V}_t &= \bar{V}_t^C - \bar{V}_t^N, \text{ with} \\ \bar{V}_t^C &:= \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{(\bar{C}_{t+s} - \chi \bar{C}_{t+s-1})^{1-\eta} - 1}{1-\eta} \right], \\ \bar{V}_t^N &:= \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{\nu_0}{1+\nu_1} \bar{\tilde{N}}_{t+s}^{1+\nu_1} \right]. \end{aligned} \tag{2.40}$$

be life-time utility implied by any of these solutions. Accordingly, we implicitly define our measure of welfare enhancement λ by[3]

$$\bar{V}_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{((1-\lambda)C_{t+s} - \chi(1-\lambda)C_{t+s-1})^{1-\eta} - 1}{1-\eta} - \frac{\nu_0}{1+\nu_1} \tilde{N}_{t+s}^{1+\nu_1} \right]. \tag{2.41}$$

Like the policy functions that solve the model, λ is a function of the given initial state of the system. In our model the vector of state variables consists of the vector of shocks $\mathbf{z}_t = [\ln Z_t, \ln(G_t/G)]'$ and the endogenous states $\mathbf{x}$, which comprise the aggregate stock of capital K_t, the capital of primary producers X_t, the nominal interest rate factor Q_t, and, from the previous period, consumption C_{t-1}, the real wage w_{t-1}, inflation π_{t-1}, the measure of price dispersion s_{t-1}^y, and the measure of wage dispersion s_{t-1}^n.[4] We

[3] Schmitt-Grohé and Uribe (2004a) do not compensate for consumption at time $t-1$. Their definition yields a smaller welfare measure since the household's utility is a decreasing function of the habit. However, because the ranking of different monetary policy rules is independent of the scale of the welfare measure, we use the analytically more convenient definition.

[4] See the Appendix for the definition of these variables.

approximate $\lambda(\mathbf{x}, \mathbf{z})$ at the stationary point $(\mathbf{x}, \mathbf{y})$ of the deterministic counterpart of the model. In the Appendix we show that by a second-order approximation

$$\lambda \approx \frac{1-\beta}{1+(1-\eta)(1-\beta)V^C}\left[V^C_{\sigma\sigma} + \bar{V}^N_{\sigma\sigma} - \bar{V}^C_{\sigma\sigma} - V^N_{\sigma\sigma}\right]. \tag{2.42}$$

In this expression, $V^i_{\sigma\sigma}$ are the second partial derivatives of V^i, $i = C, N$, with respect to the scaling parameter in the driving process of the shocks, $\mathbf{z}_t = \Pi\mathbf{z}_{t-1} + \sigma\Omega\boldsymbol{\epsilon}_t$.[5]

3 Calibration

We calibrate the model with respect to the U.S. economy. The length of a period is one quarter. Table 3.1 summarizes the model's parameters and the values assigned to them. We thereby, for the most part, follow Christiano, Eichenbaum, and Evans (2005) and Carlsrom and Fuerst (1997). In particular, with respect to the credit friction we use a log-normal distribution of the idiosyncratic shock ω, and determine the parameters of this distribution as well as the bankruptcy threshold $\bar{\omega}$ from three targets: a mean of one, a quarterly bankruptcy rate of 0.974 percent, and an annual external finance premium of 187 basis points. Given $\bar{\omega}$, equation (2.19a) determines the mark-up v, and the value of the additional discount factor γ follows from the steady-state versions of equations (2.22) and (2.29b) as:

$$\gamma = \frac{1 - vg(\bar{\omega})}{vf(\bar{\omega})}.$$

The steady state share of government spending in output $G/Y = 0.16$ as well as the parameters of the TFP shock and the government spending shock stem from Schmitt-Grohé and Uribe (2007). We also follow Schmitt-Grohé and Uribe (2005) and set the steady-state inflation rate equal to the average growth rate of the U.S. GDP deflator over the period 1960-1998, which gives $\pi = 1.042^{0.25}$.

Finally, to consider the potential of our model to produce a counter-cyclical external finance premium, we disregard the spillover from the aggregate shock to the mean of the distribution of the idiosyncratic productivity modeled in Faia and Monacelli (2007) and set $\Omega_t = 1$ for all periods.

[5] See Schmitt-Grohé and Uribe (2004b) for this representation.

Table 3.1

Parameter	Value	Description
β	$1.03^{-0.25}$	Subjective discount factor
$1/\eta$	1	Intertemporal elasticity of substitution
$1/\nu_1$	1	Frisch elasticity of labor supply
χ	0.65	Habit parameter
N	1	Steady state labor supply
α	0.36	Share of capital in value added
δ	0.025	Rate of capital depreciation
ζ	$\{0.5, 2.5\}$	Elasticity of marginal adjustment cost function Ψ'
ρ_Z	0.856	Autocorrelation of TFP shock
σ_Z	0.0064	Standard deviation of innovations of TFP shock
$E(\omega)$	1	Mean of distribution of idiosyncratic productivity shock
κ	0.25	Costs of bankruptcy
$\Phi(\bar{\omega})$	0.00974	Steady state bankruptcy rate
$1 + r_L$	$1.0187^{0.25}$	Gross external finance premium
ϵ_y	6	Price elasticity of demand for intermediary goods
ϵ_n	21	Wage elasticity of labor demand
φ_y	0.60	Fraction of intermediary goods firms not setting their prices optimally
φ_n	0.64	Fraction of household members not setting their wages optimally
G/Y	0.16	Share of government spending in steady state production
ρ_G	0.87	Autocorrelation parameter in government spending shock
σ_G	0.016	Standard deviation of innovations in government spending shock
π	$1.042^{0.25}$	Steady state inflation factor

4 Results

In this section, we present our results on how the introduction of a q-target in the Taylor rule affects the utility of the households. In particular, we search for the optimal monetary policy rule and analyze if monetary policy should respond to higher asset prices by lowering or increasing interest rates. Our benchmark is the Taylor rule (2.32) with zero coefficients on the past interest rate $\delta_1 = 0$, a coefficient of $\delta_2 = 1.5$ on the inflation gap, and zero coefficients on capital price $\delta_3 = 0$ and the output gap $\delta_4 = 0$. We compute the welfare gains of policies with non-zero δ_i, $i = 1, 2, 3, 4$ over the grid

$$\begin{aligned} \delta_1 &\in [0, 0.95], \\ \delta_2 &\in [1.2, 2.5], \\ \delta_3 &\in [-2.5, 2.5], \\ \delta_4 &\in [0, 2.5], \end{aligned}$$

for two different values of the parameter ζ, indicating small and medium size costs of capital accumulation.

Table 4.1 presents the results obtained for the benchmark model without the financial friction. Apart from the monopoly power in product and labor markets, this model embeds three kinds of distortions: 1) The variable mark-up (the inverse of the variable g_t) over marginal costs and the variable mark-up over the marginal rate of substitution between leisure and consumption introduces inefficient fluctuations of hours and production. The combined effect of both distortions is reflected by the gap between the (aggregate) marginal product of labor (MPL) and the marginal rate of substitution between consumption and leisure (MRS) defined as

$$gap_t := \frac{(1-\alpha)\tilde{Y}_t/N_t}{\nu_0 N_t^{\nu_1}/\Lambda_t}.$$

Additionally, 2) the price and 3) wage dispersion forces the household members to spread consumption and labor supply unevenly over the continuum of consumption goods and labor services, respectively.[6] The Taylor rule that maximizes the welfare gain of the household places a negative coefficient on the price of capital, $\delta_3 = -1.41$ ($\delta_3 = -0.46$) for the case of high (low) adjustment cost, $\zeta = 5.0$ ($\zeta = 2.5$). Compared to a policy which ignores this variable (columns iii and v), there is a welfare loss of about 0.018 ($\zeta = 0.5$) and 0.01 ($\zeta = 2.5$) percentage points, respectively.

[6] The latter two effects are not present in the model of Faia and Monacelli (2007), because they

Table 4.1

Welfare Effects: Benchmark Model Without Financial Frictions

	$\zeta = 0.5$		$\zeta = 2.5$	
	ii	iii	iv	v
δ_1	0.78	0.0	0.47	0.0
δ_2	2.38	2.5	1.20	2.5
δ_3	−1.41	0.0	−0.46	0.0
δ_4	1.79	0.75	0.75	0.75
λ	−0.0628	−0.0451	−0.0341	−0.0240

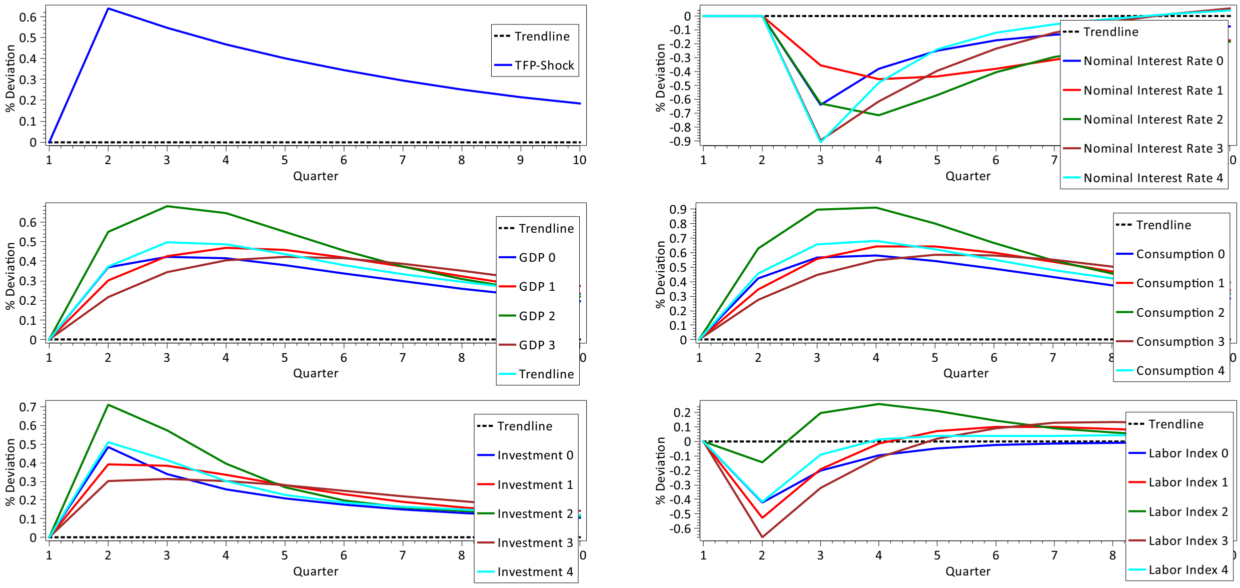
Notes: ζ is the elasticity of Tobin's q with respect to the investment-capital ratio I/K. δ_i, $i = 1, 2, 3, 4$ denote the coefficients of the Taylor rule (2.32) on the past interest rate, the inflation gap, the price of capital, and the output gap, respectively. λ is the percentage of consumption that must be given (taken if positve) to the household in the pure inflation target regime with $\delta_2 = 1.5$ and $\delta_i = 0$, $i = 1, 3, 4$, to make him equally well-off as under the rule specified in columns ii-v.

The intuition behind this result rests on the observation that the cycle is mainly driven by the supply shock. Figures 4.1 and 4.2 display the response of the economy to a one-time shock in quarter $t = 2$ for different specifications of the model. The case "0" refers to the model without nominal rigidities and without the financial friction. Cases "1" ("2") denote the response of the model with the nominal frictions and the simple (the optimal) Taylor rule, while cases "3" ("4") show the behavior of the model with nominal and financial frictions for the simple (the optimal) Taylor rule.

A positive supply shock increases labor productivity. It is well-known that the increase in the real wage is not sufficient to off-set the implicit labor tax implied by the adjustment costs of capital so that labor supply declines (see Panel 6 in Figure 4.1). In economies with nominal frictions this effect is more pronounced since the real wage responds sluggishly. Accordingly, the gap between the MPL and the MRS widens (see Panel 5 in Figure 4.2). If the central bank reduces its interest rate in response to the decreasing inflation the real rate of interest declines, since the nominal frictions prevent a full adjustment of expected inflation. The household increases consumption and thus fuels the upswing. This effect is more pronounced if the central bank does not only react to inflation but also (inversely) to the increase in the price of capital.

assume convex costs of price adjustment so that, in the symmetric equilibrium of the product market, all firms will choose the same price. In addition, they do not model wage stickiness.

Figure 4.1: Impulse responses 1



In this way, the monetary authority is able to temporarily lower the gap between the MPL and MRS and, accordingly, reduces the welfare distortions of price and wage setting. Therefore, (negative) q-targeting is welfare enhancing in economies with nominal rigidities.

In the model with financial frictions there is an interplay between adjustment costs of capital and the size of the financial friction. The technology shock boosts the demand for new capital and Tobin's q increases. In this way, the net wealth of producers increases and, therefore, reduces their demand for external funds. As a consequence, both the mark-up on factor costs and the external finance premium decline (see Panel 6 in Figure 4.2). Our model is thus able to explain the observed counter-cyclicity of the external finance premium without the additional assumption in Faia and Monacelli (2007) of a spill-over from aggregate technology shifts to the idiosyncrtatic shock. Eventually, this effect also introduces a channel between monetary policy and the financial friction: If the central bank stimulates the boom by lowering its interest rate it also indirectly reduces the financial friction.

Table 4.2 presents the results for the benchmark model with the financial friction.[7] The basic intuition behind the small welfare gain remains intact: it rests on the temporary

[7]Since the search for the optimal policy is relatively time-consuming in this model, we have not computed the welfare for policies that neglect the price of capital. The low speed of computation is caused by the repeated numerical evaluation of the Hessian matrix of the dynamic system of equations, some of which require numeric integration.

Figure 4.2: Impulse responses 2

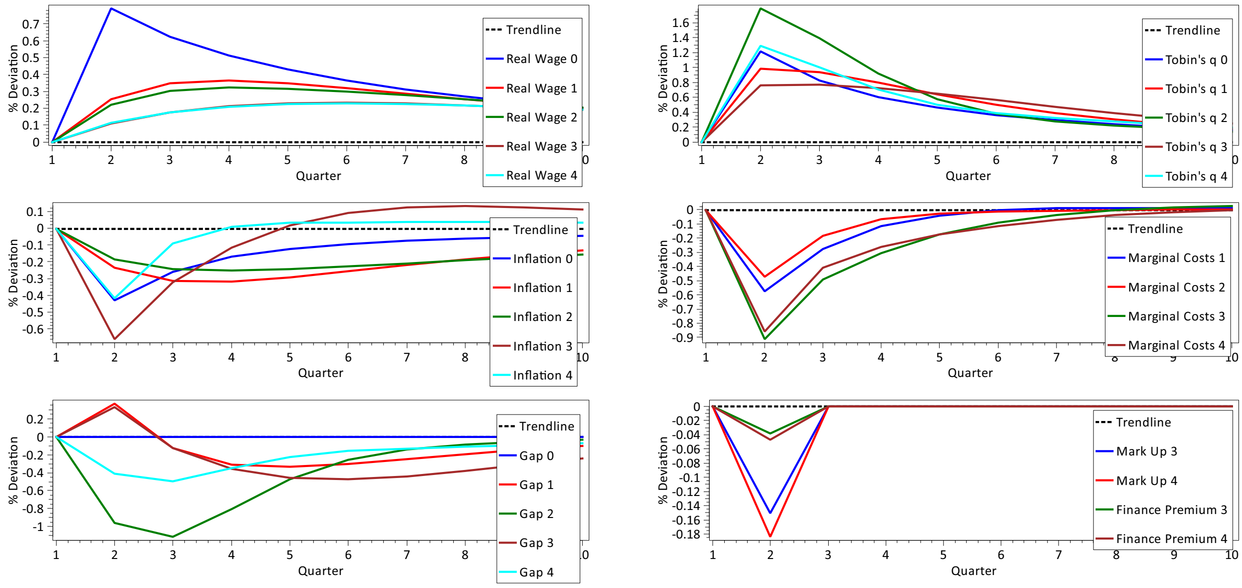


Table 4.2

Welfare Effects: Benchmark Model With Financial Friction

	$\zeta = 0.5$	$\zeta = 2.5$
δ_1	0.36	0.52
δ_2	1.79	1.22
δ_3	-0.81	-0.48
δ_4	0.97	0.82
λ	-0.0617	-0.0355

Notes: See Table 4.1.

alleviation of the welfare distortion of monopoly power from fueling the upswing triggered by the technology shock. In the case of $\zeta = 2.5$ we also observe a small additional effect which stems from the inverse relation between the external finance premium and the relative price of capital. The fact that this effect is tiny is explained by the size of the potential welfare gain: to make the household in the financial friction economy equally well-off as in an economy without this friction would require an increase of consumption of less than one-hundredth of one percent ($\lambda \times 100 = 0.00975$).

5 Robustness

In this section, we examine the robustness of our main results with respect to the household's preference representation. On the one hand, we generalize our economy's welfare specification towards the Epstein and Zin (1989), henceforth EZ, utility recursion. On the other hand, we consider two additional demand shocks as we allow for exogenous shifts to the household's marginal rate of substitution (MRS) between consumption and leisure as well as to his discounting parameter. While such preference shocks have been demonstrated to be able to significantly improve on DSGE models' reconcilability with empirical macro-evidence,[8] researchers have cast doubts on the theoretical soundness of such "dubiously structural" elements.[9] The two considered preference shifts are, however, solidly founded in recent theoretical work. In particular, as shown by Nakajima (2005), the MRS shock can be interpreted as an aggregation of market incompleteness on the micro-level within a heterogenous agent economy, while Gourio (2012) demonstrates that the discounting shock is a reduced form of capturing the household's savings behavior in presence of a perceived (time-varying) positive probability of an economic disaster.

Formal Implementation The household's (centralized) value function is now required to (only) satisfy

$$V_t = \left[(1-\beta)\mathscr{U}_t^{1-\frac{1}{\psi}} + \beta\vartheta_t \left(\mathbb{E}_t\left[V_{t+1}^{1-\eta}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\eta}} \right]^{\frac{1}{1-\frac{1}{\psi}}}, 1 \neq \psi > 0.$$

Thereby, in order to still be able to apply the above outlined welfare analysis, the composite good is now of the Cobb-Douglas type, i.e.

$$\mathscr{U}_t := (C_t - \mathscr{C}_t)^{\nu}(1-N_t)^{\theta_t(1-\nu)}, \nu \in (0,1),$$

and the preference shifts follow stationary AR(1) processes, i.e.

$$\ln\vartheta_{t+1} = \rho_\vartheta \ln\vartheta_t + \sigma_\vartheta\epsilon_t^{\text{BETA}}, \rho_\vartheta \in [0,1), \sigma_\vartheta \geq 0, \epsilon_t^{\text{BETA}} \sim \text{iidN}(0,1),$$

$$\ln\theta_{t+1} = \rho_\theta \ln\theta_t + \sigma_\theta\epsilon_t^{\text{MRS}}, \rho_\theta \in [0,1), \sigma_\theta \geq 0, \epsilon_t^{\text{MRS}} \sim \text{iidN}(0,1).$$

[8] See, e.g., Hall (1997).

[9] See, e.g., Chari, Kehoe, and McGrattan (2009), p. 244.

Note that ψ directly measures the elasticity of intertemporal substitution (EIS) of the composite good and that η independently parameterizes risk aversion (RA) such that attitudes towards risk and towards intertemporal substitution are disentangled.[10]

These changes directly affect the studied economy through two channels. First, its stochastic discount factor from period t until τ now reads

$$\Pi_{j=0}^{\tau}\left(\beta\vartheta_t\left(\frac{V_{t+j}}{\left(\mathbb{E}_{t+j-1}\left[V_{t+j}^{1-\eta}\right]\right)^{\frac{1}{1-\eta}}}\right)^{\frac{1}{\psi}-\eta}\right)\left(\frac{\mathscr{U}_{t+\tau}}{\mathscr{U}_t}\right)^{1-\frac{1}{\psi}}\frac{C_t-\mathscr{C}_t}{C_{t+\tau}-\mathscr{C}_{t+\tau}}.$$

Second, the model's (optimal) wage equations necessarily also reflect the household's generalized objective.

Numerical Results We use the additional degree of freedom associated with the EZ representation to further confirm robustness with respect to the parametrization. In particular, we consider two different risk aversion scenarios, namely $\eta \in \{2, 12\}$. In both cases, RA is parameterized at (roughly) 2: in the first case RA is measured with respect to the composite good, in the second case directly with respect to consumption.[11] With regard to the intertemporal elasticity of substitution, we follow Gourio (2012) and choose $\psi = 2$ (i.e. larger than unity) in order to keep the above outlined microfoundation behind the discounting shock intuitively intact. Note that we therefore implicitly assume the household to have a preference for earlier resolution of composite good uncertainty. With regard to the preference shift processes, we proceed as follows: First, we follow Basu and Bundick (2012) and fix $\rho_\theta = \rho_\vartheta = 0.9$. Second, we calibrate the two remaining shock volatility parameters so that the model optimally replicates some stylized facts of the real US economy such as the equity premium and business cycle statistics. In particular, we find $\sigma_\vartheta = 0.0009$ and $\sigma_\theta = 0.01$, where the small magnitude of the calibrated discount shock volatility mainly reflects the model's zero lower bound sensitivity with respect to this parameter.

Again applying the above outlined welfare approach, we find that the main conclusion from this preference generalization is that the central results are robust (see Table

[10]Note, however, that the deviation from the reciprocity of EIS and RA within our basic framework of additively separable expected utility comes with the implicit assumption of non-indifference with respect to the timing of uncertainty resolution which is tricky to calibrate, cf. Epstein, Farhi, and Strzalecki (2014).

[11]Cf. Swanson (2012).

Table 5.1

Welfare Effects: EZ Model With Financial Friction and Preference Shocks

	$\eta = 2$	$\eta = 12$
δ_1	0	0
δ_2	1.85	1.2
δ_3	-0.13	-0.13
δ_4	0.28	0
λ	-0.009	-0.008

Notes: See Table 4.1.

5.1): While the central bank is advised to react negatively to its asset price target, the welfare effect of additionally considering such a target is negligible.

6 Conclusion

We have considered a model that merges adjustment costs of capital, a financial friction in the production of primary goods and Calvo-type fictions in the adjustment of nominal prices and wages. In this environment we have asked whether or not the cental bank can increase the welfare of the representative household if – in addition to inflation and the output gap – it also links its interest rate to the price of new capital. Our answer to this question is: yes, but with negligible effects. The intuition behind this result rests on two observations: first, the financial friction imposes only a very small welfare loss. Second, the main way in which the central bank achieves welfare gains is not by reducing the financial friction but via the temporary reduction of the welfare distortions implied by monopoly power.

We regard these results as robust, since our analysis shows that they do not depend on the number and the source (demand or supply) of shocks that drive the business cycle. However, the (additional) presence of other types of financial frictions may still change the picture. E.g., as pointed out by Christiano, Eichenbaum, and Trabandt (2015), a shock to the economy's credit spread plays a major role in explaining the most recent great recession. We, therefore, plan to further extend our analysis with respect to a more general description that (simultaneously) allows for a number of different sources of financial frictions.

References

Basu, Susanto, and Brent Bundick. 2012. Uncertainty Shocks in a Model of Effective Demand. *NBER Working Paper*. No. 18420.

Bernanke, Ben S., and Mark Gertler. 1999. Monetary policy and asset price volatility. *Economic Review, Federal Reserve Bank of Kansas City*. Quarter IV. pp 17-51.

Bernanke, Ben S., and Mark Gertler. 2001. Should Central Banks Respond to Movements in Asset Prices? *American Economic Review*. Vol. 91(2). pp. 253-257.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist. 1999. The Financial Accelerator in a Quantitative Business Cycle Framework. in: John B. Taylor and Michael Woodford (eds.), Handbook of Macroeconomics, Volume 1C. pp. 1341-1393.

Calvo, Guillermo A. 1983. Staggered Prices in a Utility-Maximizing Framework. *Journal of Monetary Economics*. Vol. 12. pp. 383-98.

Carlstrom, Charles T. and Timothy S. Fuerst. 1997. Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis. *American Economic Review*. Vol. 87. pp. 893-910.

Carlstrom, Charles T. and Timothy S. Fuerst. 1998. Agency Costs and Business Cycles. *Economic Theory*. Vol. 12. pp. 583-597.

Carlstrom, Charles T. and Timothy S. Fuerst. 2007. Asset Prices, Nominal Rigidities, and Monetary Policy. *Review of Economic Dynamics*. Vol. 10. pp. 256-275.

Chari V.V., Patrick J. Kehoe, and Ellen R. McGrattan. 2009. New Keynesian Models: Not Yet Useful for Policy Analysis. *American Economic Journal: Macroeconomics*. Vol. 1(1). pp. 242-266.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. 2005. Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*. Vol. 113. pp. 1-45.

Christiano, Lawrence J., Martin S. Eichenbaum, and Mathias Trabandt. 2015. Understanding the Great Recession. *American Economic Journal: Macroeconomics*. Vol. 7. pp. 110-167.

Christiano, Lawrence J., Massimo Rostagno, and Roberto Motto. 2010. Financial factors in economic fluctuations. *European Central Bank (ECB) Working Paper Series* No. 1192.

Chugh, Sanjay K. 2013. Costly External Finance and Labor Market Dynamics. *Journal of Economic Dynamics and Control*. Vol. 37. pp. 2882-2912.

Epstein, Larry G., Emmanuel Farhi, and Tomasz Strzalecki. 2014. How Much Would You Pay to Resolve Long Run Risk? *American Economic Review*. Vol. 104 (9). pp. 2680-2697.

Epstein, Larry G., and Stanley Zin. 1989. Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. *Econometrica*. Vol. 57. pp. 937-969.

Erceg, Christopher J., Henderson, Dale W., Levin, Andrew D. 2000. Optimal Monetary Policy with Staggered Wage and Price Contracts. *Journal of Monetary Economics*. Vol. 46. pp. 281-313.

Faia, Ester, and Tomasso Monacelli. 2007. Optimal interest rate rules, asset prices, and credit frictions. *Journal of Economic Dynamics and Control*. Vol. 31. pp. 3228-3254.

Gale, Douglas, and Martin Hellwig. 1985. Incentive-Compatible Debt Contracts: The One-Period Problem. *Review of Economic Studies*. Vol. 52. pp. 647-664.

Gourio, Francios. 2012. Disaster Risk and Business Cycles. *American Economic Review*. Vol. 102 (6). pp. 2734-2766.

Hall, Robert E. 1997. Macroeconomic Fluctuations and the Allocation of Time. *Journal of Labor Economics*. Vol. 15(1). pp. S223-S250.

Jermann, Urban J. 1998. Asset Pricing in Production Economies. *Journal of Monetary Economics*. Vol. 41. pp. 257-275.

Lütkepohl, Helmut. 2005. New Introduction to Multiple Time Series Analysis. Springer. Berlin.

Machado, Vicente da Gama. 2012. Monetary Policy and Asset Price Volatility. *Working Paper Series, Banco Central do Brasil*. No. 274.

Nakajima, Tomoyuki. 2005. A Business Cycle Model With Variable Capacity Utilization and Demand Disturbances. *European Economic Review*. Vol. 49. pp. 1331-1360.

Schmitt-Grohé, Stephanie and Martin Uribe. 2004a. Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function. *Journal of Economic Dynamics and Control*. Vol. 28. pp. 755-775.

Schmitt-Grohé, Stephanie and Martin Uribe. 2004b. Optimal Operational Monetary Policy in the Christiano-Eichenbaum-Evans Model of the US Business Cycle. *Centre for Economic Policy Research (CEPR) Discussion Paper* No. 4554.

Schmitt-Grohé, Stephanie and Martin Uribe. 2005. Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model: Expanded Version. *National Bureau of Economic Research (NBER) Working Paper* No. W11417.

Schmitt-Grohé, Stephanie and Martin Uribe. 2007. Optimal Simple and Implementable Monetary and Fiscal Rules. *Journal of Monetary Economics*. Vol. 54. pp. 1702-1725.

Swanson, Eric T. 2012. Risk Aversion and the Labor Margin in Dynamic Equilibrium Models. *American Economic Review*. Vol. 102 (4). pp. 1663-1691.

Townsend, Robert M. 1979. Optimal Contracts and Competitive Markets With Costly State Verification. *Journal of Economic Theory*. Vol. 21. pp. 265-293.

Williamson, Stephen D. 1987. Costly Monitoring, Optimal Contracts, and Equilibrium Credit Rationing. *Quarterly Journal of Economics*. Vol. 102. pp. 135-145.

Appendix

A Analysis of the Model

A.1 Price Setting

Consider the relative price P_{jt+s}/P_{t+s} of an intermediary goods producer j receiving the signal to choose its optimal relative price $p_{At} = P_{At}/P_t$ in period t who has not been able to reset its price up to period $t+s$:

$$\frac{P_{jt+s}}{P_{t+s}} = \frac{\pi_{t+s-1}\cdots\pi_t}{\pi_{t+s}\cdots\pi_{t+1}} p_{At} = \frac{\pi_t}{\pi_{t+s}} p_{At}.$$

Accordingly, the firm will choose p_{At} in period t to maximize

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_y)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[\left(\frac{\pi_t}{\pi_{t+s}} p_{At} \right)^{-\epsilon_y} Y_{t+s} - g_{t+s} \left(\frac{\pi_t}{\pi_{t+s}} p_{At} \right)^{1-\epsilon_y} Y_{t+s} \right].$$

The first-order condition for this problem is:

$$0 = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_y)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[(1-\epsilon_y) \left(\frac{\pi_t}{\pi_{t+s}} \right)^{1-\epsilon_y} Y_{t+s} p_{At}^{-\epsilon_y} + \epsilon_y g_{t+s} \left(\frac{\pi_t}{\pi_{t+s}} \right)^{-\epsilon_y} Y_{t+s} p_{At}^{-\epsilon_y - 1} \right]$$

and can be written as

$$p_{At} = \frac{\epsilon_y}{\epsilon_y - 1} \frac{\Gamma_{1t}}{\pi_t \Gamma_{2t}}, \tag{A.1a}$$

$$\Gamma_{1t} = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_y)^s \pi_{t+s}^{\epsilon_y} g_{t+s} \Lambda_{t+s} Y_{t+s} = \pi_t^{\epsilon_y} g_t \Lambda_t Y_t + (\beta\varphi_y) \mathbb{E}_t \Gamma_{1t+1}, \tag{A.1b}$$

$$\Gamma_{2t} = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_y)^s \pi_{t+s}^{\epsilon_y - 1} \Lambda_{t+s} Y_{t+s} = \pi_t^{\epsilon_y - 1} \Lambda_t Y_t + (\beta\varphi_y) \mathbb{E}_t \Gamma_{2t+1}. \tag{A.1c}$$

The price index (2.3) implies

$$P_t^{1-\epsilon_y} = (1-\varphi_y) P_{At}^{1-\epsilon_y} + \varphi_y P_{Nt}^{1-\epsilon_y} = (1-\varphi_y) P_{At}^{1-\epsilon_y} + \varphi_y (\pi_{t-1} P_{t-1})^{1-\epsilon_y}.$$

The second equality follows from the updating rule (2.7) and the fact that the non-optimizers in the present period are a random sample of optimizers and non-optimizers of the previous period. Dividing by P_t on both sides delivers:

$$1 = (1-\varphi_y) p_{At}^{1-\epsilon_y} + \varphi_y (\pi_{t-1}/\pi_t)^{1-\epsilon_y}. \tag{A.1d}$$

Market clearing requires

$$\tilde{Y}_t = \int_0^1 Y_{jt} dj = \int_0^1 \left(\frac{P_{jt}}{P_t}\right)^{-\epsilon_y} Y_t dj = \underbrace{\left(\frac{\tilde{P}_t}{P_t}\right)^{-\epsilon_y}}_{\equiv s_t^y} Y_t, \quad \tilde{P}_t^{-\epsilon_y} \equiv \int_0^1 P_{jt}^{-\epsilon_y} dj,$$

so that

$$s_t^y Y_t = \tilde{Y}_t. \tag{A.1e}$$

Using the same reasoning for $\tilde{P}_t$ as for the price index P_t yields:

$$s_t^y = (1-\varphi_y) p_{At}^{-\epsilon_y} + \varphi_y (\pi_{t-1}/\pi_t)^{-\epsilon_y} s_{t-1}^y. \tag{A.1f}$$

A.2 Wage Setting

Consider the real wage W_{ht}/P_t of a household member who has set his wage optimally in period t to $\tilde{w}_t = W_{At}/P_t$ and who has not been able to do so again until period $s = 1, 2, \ldots$. In this case, his real wage in period $t+s$ is given by

$$\frac{W_{Nt+s}}{P_{t+s}} = \frac{\prod_{i=1}^{s} \pi_{t+i-1} W_{At}}{\prod_{i=1}^{s} \pi_{t+s} P_t} = \frac{\pi_t}{\pi_{t+s}} \tilde{w}_t,$$

and the demand for his type of labor service equals

$$N_{ht+s} = \left(\frac{(\pi_t/\pi_{t+s})\tilde{w}_t}{w_{t+s}}\right)^{-\epsilon_n} N_{t+s},$$

where w_{t+s} denotes the real wage prevailing in period $t+s$. Accordingly, the Lagrangian for the optimal real wage reads:

$$\begin{aligned}\mathscr{L} =& \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_n)^s \Bigg\{ \frac{(C_{ht+s} - \chi \bar{C}_{t+s})^{1-\eta} - 1}{1-\eta} - \frac{\nu_0}{1+\nu_1} \left[\left(\frac{(\pi_t/\pi_{t+s})\tilde{w}_t}{w_{t+s}}\right)^{-\epsilon_n} N_{t+s} \right]^{1+\nu_1} \\ &+ \Lambda_{ht+s} \left[\frac{\pi_t}{\pi_{t+s}} \tilde{w}_t \left(\frac{(\pi_t/\pi_{t+s})\tilde{w}_t}{w_{t+s}}\right)^{-\epsilon_n} N_{t+s} + RMT \right] \Bigg\}.\end{aligned}$$

The first-order condition with respect to $\tilde{w}_t$ is

$$\begin{aligned}0 = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_n)^s \Bigg\{ & \epsilon_n \nu_0 \tilde{w}_t^{-\epsilon_n(1+\nu_1)-1} \left(\frac{(\pi_t/\pi_{t+s})}{w_{t+s}}\right)^{-\epsilon_n(1+\nu_1)} N_{t+s}^{1+\nu_1} \\ &+ (1-\epsilon_n)\Lambda_{ht+s} \tilde{w}_t^{-\epsilon_n} w_{t+s}^{\epsilon_n} \left(\frac{\pi_t}{\pi_{t+s}}\right)^{1-\epsilon_n} N_{t+s} \Bigg\}.\end{aligned}$$

Using $\Lambda_{ht+s} = \Lambda_{t+s}$ this can be arranged to read

$$\tilde{w}_t = \frac{\epsilon_n}{\epsilon_n - 1}\frac{\Delta_{1t}}{\Delta_{2t}}, \tag{A.2a}$$

where

$$\begin{aligned}
\Delta_{1t} &= \nu_0 \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_n)^s \left(\frac{\pi_t \tilde{w}_t}{\pi_{t+s} w_{t+s}}\right)^{-\epsilon_n(1+\nu_1)} N_{t+s}^{1+\nu_1}, \\
&= \nu_0 \left(\frac{\tilde{w}_t}{w_t}\right)^{-\epsilon_n(1+\nu_1)} N_t^{1+\nu_1} + (\beta\varphi_n)\mathbb{E}_t \left(\frac{\pi_t \tilde{w}_t}{\tilde{\pi}_{t+1} w_{t+1}}\right)^{-\epsilon_n(1+\nu_1)} \Delta_{1t+1}, && \text{(A.2b)} \\
\Delta_{2t} &= \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_n)^s \Lambda_{t+s} \left(\frac{\tilde{w}_t}{w_{t+s}}\right)^{-\epsilon_n} \left(\frac{\pi_t}{\pi_{t+s}}\right)^{1-\epsilon_n} N_{t+s}, \\
&= \Lambda_t \left(\frac{\tilde{w}_t}{w_t}\right)^{-\epsilon_n} N_t + (\beta\varphi_n)\mathbb{E}_t \left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}}\right)^{-\epsilon_n} \left(\frac{\pi_t}{\pi_{t+1}}\right)^{1-\epsilon_n} \Delta_{2t+1}. && \text{(A.2c)}
\end{aligned}$$

The wage index (2.25) implies

$$W_t^{1-\epsilon_n} = (1-\varphi_n) W_{At}^{1-\epsilon_n} + \varphi_n (\pi_{t-1} W_{t-1})^{1-\epsilon_n}$$

so that the real wage equals

$$w_t^{1-\epsilon_n} = (1-\varphi_n)\tilde{w}_t^{1-\epsilon_n} + \varphi_n \left(\frac{\pi_{t-1}}{\pi_t} w_{t-1}\right)^{1-\epsilon_n}. \tag{A.2d}$$

Finally consider the index

$$\tilde{N}_t^{1+\nu_1} = \int_0^1 N_{ht}^{1+\nu_1} dh,$$

in the families current-period utility function. Using (2.24), this can be written

$$\tilde{N}_t^{1+\nu_1} = N_t^{1+\nu_1} \int_0^1 \left(\frac{W_{ht}}{W_t}\right)^{-\epsilon_n(1+\nu_1)} dh.$$

Let

$$\bar{W}_t^{-\epsilon_n(1+\nu_1)} = \int_0^1 W_{ht}^{-\epsilon_n(1+\nu_1)} dh = (1-\varphi_n)(W_{At})^{-\epsilon_n(1+\nu_1)} + \varphi_n(\pi_{t-1} W_{Nt-1})^{-\epsilon_n(1+\nu_1)}$$

and

$$(s_t^n)^{1+\nu_1} = \left(\frac{\bar{W}_t}{W_t}\right)^{-\epsilon_n(1+\nu_1)} = \left(\frac{\bar{W}_t/P_t}{W_t/P_t}\right)^{-\epsilon_n(1+\nu_1)} = \left(\frac{\bar{w}_t}{w_t}\right)^{-\epsilon_n(1+\nu_1)}.$$

Using the same line of argument employed to derive (A.1f) yields the dynamic equation for the measure of wage dispersion s_t^n:

$$(s_t^n)^{1+\nu_1} = (1-\varphi_n)\left(\frac{\tilde{w}_t}{w_t}\right)^{-\epsilon_n(1+\nu_1)} + \varphi_n \left(\frac{\pi_{t-1} w_{t-1}}{\pi_t w_t}\right)^{-\epsilon_n(1+\nu_1)} (s_{t-1}^n)^{1+\nu_1} \tag{A.2e}$$

so that

$$\tilde{N}_t = s_t^n N_t. \tag{A.2f}$$

Note that we must track the variable $\tilde{N}_t$ in order to compute our welfare measure.

A.3 Dynamics

The full model consists of equations (A.1), (A.2), (2.5), (2.13), (2.19a), (2.20), (2.22), (2.29), (2.35), (2.36), (2.37), the resource constraint (2.38), the capital accumulation equation (2.34), the Taylor rule (2.32), and the dynamics of the shocks, (2.10) and (2.31), respectively. In order to compute our welfare measure we have to add the recursive definitions of V_t^C and V_t^N implied by (2.39). These are presented by

$$V_t^C = \left[\frac{(C_t - \chi C_{t-1})^{1-\eta} - 1}{1-\eta}\right] + \beta \mathbb{E}_t V_{t+1}^C, \tag{A.3a}$$

$$V_t^N = \frac{\nu_0}{1+\nu_1}\tilde{N}_t^{1+\nu_1} + \beta \mathbb{E}_t V_{t+1}^N. \tag{A.3b}$$

For convenience, we summarize the entire system of equilibrium conditions:

$$\Lambda_t = (C_t - \chi C_{t-1})^{-\eta}, \tag{A.4.1}$$

$$q_t = \frac{1}{\Psi'(I_t/K_t)}, \tag{A.4.2}$$

$$r_{Kt} = q_t \Psi(I_t/K_t) - (I_t/K_t), \tag{A.4.3}$$

$$w_t = (1-\alpha)(g_t/v_t)\tilde{Y}_t/N_t, \tag{A.4.4}$$

$$r_{Yt} = \alpha(g_t/v_t)\tilde{Y}_t/K_t, \tag{A.4.5}$$

$$\tilde{Y}_t = Z_t N_t^{1-\alpha} K_t^{\alpha}, \tag{A.4.6}$$

$$1 = v_t\left[\Omega_t - \Phi(\bar{\omega}_t)\kappa - \frac{f(\bar{\omega}_t)\phi(\bar{\omega}_t)\kappa}{1-\Phi(\bar{\omega}_t)}\right], \tag{A.4.7}$$

$$r_{Lt} = \frac{\bar{\omega}_t}{g(\bar{\omega}_t)}, \tag{A.4.8}$$

$$g_t\tilde{Y}_t = \frac{v_t}{1 - v_t g(\bar{\omega}_t)}\left[q_t(1-\delta) + r_{Yt} + r_{Kt}\right] X_t, \tag{A.4.9}$$

$$p_{At} = \frac{\epsilon_y}{\epsilon_y - 1}\frac{\Gamma_{1t}}{\pi_t \Gamma_{2t}}, \tag{A.4.10}$$

$$1 = (1-\varphi_y)p_{At}^{1-\epsilon_y} + \varphi_y(\pi_{t-1}/\pi_t)^{1-\epsilon_y}, \tag{A.4.11}$$

$$s_t^y = (1-\varphi_y)p_{At}^{-\epsilon_y} + \varphi_y(\pi_{t-1}/\pi_t)^{-\epsilon_y} s_{t-1}^y, \tag{A.4.12}$$

$$s_t^y Y_t = \tilde{Y}_t, \tag{A.4.13}$$

$$\tilde{w}_t = \frac{\epsilon_n}{\epsilon_n - 1}\frac{\Delta_{1t}}{\Delta_{2t}}, \tag{A.4.14}$$

$$w_t^{1-\epsilon_n} = (1-\varphi_n)\tilde{w}_t^{1-\epsilon_n} + \varphi_n \left(\frac{\pi_{t-1}}{\pi_t} w_{t-1}\right)^{1-\epsilon_n}, \tag{A.4.15}$$

$$(s_t^n)^{1+\nu_1} = (1-\varphi_n)\left(\frac{\tilde{w}_t}{w_t}\right)^{-\epsilon_n(1+\nu_1)} + \varphi_n \left(\frac{\pi_{t-1} w_{t-1}}{\pi_t w_t}\right)^{-\epsilon_n(1+\nu_1)} (s_{t-1}^n)^{1+\nu_1}, \tag{A.4.16}$$

$$\tilde{N}_t = s_t^n N_t, \tag{A.4.17}$$

$$Y_t = C_t + I_t + G_t + \Phi(\bar{\omega}_t)\kappa g_t \tilde{Y}_t, \tag{A.4.18}$$

$$K_{t+1} = \Psi(I_t/K_t)K_t + (1-\delta)K_t, \tag{A.4.19}$$

$$q_t X_{t+1} = f(\bar{\omega}_t) g_t \tilde{Y}_t - D_t^P - \Delta_t^P, \tag{A.4.20}$$

$$Q_{t+1} = Q_t^{\delta_1}\left(\frac{\pi}{\beta}\right)^{1-\delta_1}\left(\frac{\pi_t}{\pi}\right)^{\delta_2}(q_t)^{\delta_3}(Y_t/Y)^{\delta_4}, \tag{A.4.21}$$

$$q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}\left(q_{t+1}(1-\delta) + r_{Yt+1} + r_{Kt+1}\right), \tag{A.4.22}$$

$$1 = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}\frac{Q_{t+1}}{\pi_{t+1}}, \tag{A.4.23}$$

$$q_t = \gamma\beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}\left[q_{t+1}(1-\delta) + r_{Yt+1} + r_{Kt+1}\right]\frac{\upsilon_{t+1} f(\bar{\omega}_{t+1})}{1 - \upsilon_{t+1} g(\bar{\omega}_{t+1})}, \tag{A.4.24}$$

$$\Gamma_{1t} = \mathbb{E}_t \sum_{s=0}^{\infty}(\beta\varphi_y)^s \pi_{t+s}^{\epsilon_y} g_{t+s}\Lambda_{t+s}Y_{t+s} = \pi_t^{\epsilon_y} g_t \Lambda_t Y_t + (\beta\varphi_y)\mathbb{E}_t\Gamma_{1t+1}, \tag{A.4.25}$$

$$\Gamma_{2t} = \mathbb{E}_t \sum_{s=0}^{\infty}(\beta\varphi_y)^s \pi_{t+s}^{\epsilon_y - 1}\Lambda_{t+s}Y_{t+s} = \pi_t^{\epsilon_y - 1}\Lambda_t Y_t + (\beta\varphi_y)\mathbb{E}_t\Gamma_{2t+1}, \tag{A.4.26}$$

$$\Delta_{1t} = \nu_0 \left(\frac{\tilde{w}_t}{w_t}\right)^{-\epsilon_n(1+\nu_1)} N_t^{1+\nu_1} + (\beta\varphi_n)\mathbb{E}_t\left(\frac{\pi_t \tilde{w}_t}{\tilde{\pi}_{t+1} w_{t+1}}\right)^{-\epsilon_n(1+\nu_1)}\Delta_{1t+1}, \tag{A.4.27}$$

$$\Delta_{2t} = \Lambda_t \left(\frac{\tilde{w}_t}{w_t}\right)^{-\epsilon_n} N_t + (\beta\varphi_n)\mathbb{E}_t\left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}}\right)^{-\epsilon_n}\left(\frac{\pi_t}{\pi_{t+1}}\right)^{1-\epsilon_n}\Delta_{2t+1}, \tag{A.4.28}$$

$$V_t^C = \left[\frac{(C_t - \chi C_{t-1})^{1-\eta} - 1}{1-\eta}\right] + \beta\mathbb{E}_t V_{t+1}^C, \tag{A.4.29}$$

$$V_t^N = \frac{\nu_0}{1+\nu_1}\tilde{N}_t^{1+\nu_1} + \beta\mathbb{E}_t V_{t+1}^N. \tag{A.4.30}$$

A.4 Stationary Solution and Calibration

The model is solved via a second-order approximation of the decision rules at the stationary solution of the deterministic version of the model. This solution follows from the model's equations if we set the shocks equal to $Z_t = 1$, and $G_t = G$ and cancel the time indices.

In the first step we determine v and $\bar{\omega}$. We proceed as Carlstrom and Fuerst (1997, 1998) and employ a log-normal distribution for ϕ with parameters μ_ω and σ_ω. We determine these parameters and the stationary bankruptcy threshold $\bar{\omega}$ from three conditions:

i. We assume a mean of one: $\mathbb{E}(\omega) = \Omega = 1$,

ii. a bankruptcy rate of $\Phi(\bar{\omega}) = 0.00974$ (taken from Carlstrom and Fuerst (1998), p. 590),

iii. and an external finance premium of $\frac{\bar{\omega}}{g(\bar{\omega})} - 1 = r_L = 1.0187^{0.25} - 1$ (also taken from Carlstrom and Fuerst (1998), p. 590)

Given $\bar{\omega}$ we can solve (A.4.7) for v.

In the second step we determine the additional discount parameter γ. The stationary versions of (A.4.22) and (A.4.24) imply

$$\gamma = \frac{1 - vg(\bar{\omega})}{vf(\bar{\omega})}.$$

In the third step, we solve the stationary wage and price equations. It is immediate from equation (A.4.11) that $p_A = 1$ so that equation (A.4.12) implies $s^y = 1$ and equation (A.4.13) can be solved for $Y = \tilde{Y}$. Equations (A.4.10), (A.4.25), and (A.4.26) deliver

$$g = \frac{\epsilon_y - 1}{\epsilon_y}, \tag{A.5a}$$

$$\Gamma_1 = \frac{g\Lambda Y \pi^{\epsilon}}{1 - \beta\varphi_y}, \tag{A.5b}$$

$$\Gamma_2 = \frac{\Lambda Y \pi^{\epsilon - 1}}{1 - \beta\varphi_y}. \tag{A.5c}$$

Equation (A.4.15) implies $\tilde{w} = w$ so that $\tilde{s}^n = 1$ via (A.4.16) and $N = \tilde{N}$ from (A.4.17). The stationary values of the auxiliary variables follow from (A.4.27) and (A.4.28) as

$$\Delta_1 = \nu_0 \frac{N^{1+\nu_1}}{1 - \beta\varphi_n}, \tag{A.5d}$$

$$\Delta_2 = \frac{\Lambda N}{1 - \beta\varphi_n} \tag{A.5e}$$

so that (A.4.14) implies

$$\nu_0 N^{\nu_1} = \frac{\epsilon_n - 1}{\epsilon_n}\Lambda w. \tag{A.5f}$$

In the fourth step we solve for Y/K. Our assumption with respect to the function Ψ in (2.34) imply $q = 1$ (see (2.5a)) and $r_K = 0$ (see (2.5b)) so that equations (A.4.5) and (A.4.22) can be solved for

$$\frac{Y}{K} = \frac{1 - \beta(1-\delta)}{\alpha\beta(g/v)}. \tag{A.5g}$$

The production function (A.4.6) yields

$$\frac{K}{N} = \left(\frac{Y}{K}\right)^{\frac{1}{\alpha-1}}. \tag{A.5h}$$

Given N, this allows us to compute K, Y, $I = \delta K$. The solution for consumption follows from (A.4.18):

$$C = Y(1 - g\Phi(\bar{\omega})\kappa) - I - G \tag{A.5i}$$

so that Λ is determined by (A.4.1):

$$\Lambda = [(1-\chi)C]^{-\eta}. \tag{A.5j}$$

Equation (A.4.4) determines the stationary real wage w. We are now able to determine the parameter ν_0 from condition (A.5f) and the auxiliary variables Γ_1, Γ_2, Δ_1 and Δ_2 from (A.5b)-(A.5e).

In the last step we can compute the stock of capital owned by firms in the primary sector from (A.4.9)

$$X = \frac{gY(1 - vg(\bar{\omega}))}{v(1 - \delta + r_Y)}, \tag{A.5k}$$

and dividends distributed from primary production firms to the household from (2.35)

$$D^P = f(\bar{\omega})g\tilde{Y} - X - \Delta^P. \tag{A.5l}$$

In our simulations we follow Carlstrom and Fuerst (1998) and set $\Delta^P = 0$.[12] The stationary values of the life-time utility associated with consumption V^C and working hours V^N equal

$$V^C = \frac{1}{1-\beta}\frac{[(1-\chi)C]^{1-\eta} - 1}{1-\eta}, \tag{A.5m}$$

[12]Carlstrom and Fuerst (1997) assume that Δ_t^P equals the wage income of entrepreneurs $\alpha_e \tilde{Y}_t$ with α_e close to zero and ignore this term in their 1998 paper.

$$V^N = \frac{1}{1-\beta}\frac{\nu_0}{1+\nu_1}N^{1+\nu_1}. \tag{A.5n}$$

Finally, the stationary version of the Euler equation (A.4.23) determines the nominal interest rate

$$Q = \frac{\pi}{\beta}. \tag{A.5o}$$

B Approximation of λ

Note that

$$(1-\lambda)^{1-\eta}V_t^C + \frac{(1-\lambda)^{1-\eta}-1}{(1-\eta)(1-\beta)} = \mathbb{E}_t\sum_{s=0}^{\infty}\beta^s\frac{(1-\lambda)^{1-\eta}(C_{t+s}-\chi C_{t+s-1})^{1-\eta}-1}{1-\eta}$$

so that condition (2.41) can be written

$$\tilde{V}_t = \tilde{V}_t^C - \tilde{V}_t^N = (1-\lambda)^{1-\eta}V_t^C + \frac{(1-\lambda)^{1-\eta}-1}{(1-\eta)(1-\beta)} - V_t^N. \tag{B.1}$$

This equation can be solved for λ, yielding

$$\lambda = 1 - \left[\frac{1+(1-\eta)(1-\beta)[\tilde{V}_t^C + V_t^N - \tilde{V}_t^N]}{1+(1-\eta)(1-\beta)V_t^C}\right]^{\frac{1}{1-\eta}}.$$

Thus, with $\sigma = 1$, we get

$$\lambda \simeq \lambda(\mathbf{x}) + \lambda_\sigma(\mathbf{x}) + \frac{1}{2}\lambda_{\sigma\sigma}$$

With identical initial conditions $\lambda(\mathbf{x}) = 0$. As shown by Schmitt-Grohé and Uribe (2004b), the first-order effect of the scaling factor σ on the policy functions of the model is nil. As a consequence, $\lambda_\sigma(\mathbf{x}) = 0$. Using this and differentiating (B.1) twice yields the equation (2.42) in the body of the paper.

C Zero Lower Bound

The Taylor rules which we consider must satisfy the non-negativity constraint on the nominal interest rate: $Q_t \geq 1$. Since our solution rests on perturbation methods, we cannot directly take care of this constraint. We, thus, follow Schmitt-Grohé and Uribe (2004b), p.31, who propose to disregard solutions which entail a significant probability to violate this constraint. Assume $Q_t - Q$ is distributed normally with mean zero and variance σ_Q so that $\bar{z} \equiv (Q_t - Q)/\sigma_Q$ is a standard normal random variable. For $\bar{z} = -2.05$ the probability of the event $z \leq \bar{z}$ is 2 percent. Therefore, we disregard solutions for which $\sigma_Q \geq (Q-1)/2.05$.

To determine whether or not a particular monetary policy violates this condition, we must compute the unconditional variance σ_Q^2 of the deviation of the interest factor Q_t from its non-stochastic stationary solution Q.

Let

$$\mathbf{x}_t = \Big[K_t, \quad X_t \quad C_{t-1}, \quad Q_t, \quad w_{t-1}, \quad s^y_{t-1}, \quad s^n_{t-1}, \quad \pi_{t-1}\Big]'$$

denote the vector of endogenous state variables, $\bar{\mathbf{x}}_t = \mathbf{x}_t - \mathbf{x}$ the deviation of the states from the non-stochastic steady state, and $\mathbf{z}_t = [\ln Z_t, \ln(G_t/G)]'$ the vector of exogenous state variables. The first-order solution of the model is given by

$$\bar{\mathbf{x}}_{t+1} = L^x\bar{\mathbf{x}}_t + L^z\mathbf{z}_t, \tag{C.1}$$

$$\mathbf{z}_{t+1} = \Pi\mathbf{z}_t + \boldsymbol{\epsilon}_{t+1}, \quad \mathbb{E}(\boldsymbol{\epsilon}_{t+1}\boldsymbol{\epsilon}'_{t+1}) = \Sigma^\epsilon = \Omega\Omega'. \tag{C.2}$$

We seek to determine $\Sigma^x \equiv \mathbb{E}(\bar{\mathbf{x}}_t\bar{\mathbf{x}}_t')$. Since $\mathbf{z}_t$ is a stationary stochastic process and since the eigenvalues of L^x are within the unit circle, Σ^x exists and is independent of the time index t. Multiplying both sides of (C.1) with $\bar{\mathbf{x}}_{t+1}$ yields:

$$\begin{aligned}
\mathbb{E}(\bar{\mathbf{x}}_{t+1}\bar{\mathbf{x}}'_{t+1}) &= \mathbb{E}(L^x\bar{\mathbf{x}}_t + L^z\mathbf{z}_t)(L^x\bar{\mathbf{x}}_t + L^z\mathbf{z}_t)' \\
&= \mathbb{E}(L^x\bar{\mathbf{x}}_t\bar{\mathbf{x}}_t'(L^x)') + \mathbb{E}(L^z\mathbf{z}_t\mathbf{z}_t'(L^z)') + \mathbb{E}(L^x\bar{\mathbf{x}}_t\mathbf{z}_t'(L^z)') + \mathbb{E}(L^z\mathbf{z}_t\bar{\mathbf{x}}_t'(L^x)'), \\
\Sigma^x &= L^x\Sigma^x(L^x)' + L^z\Sigma^z(L^z)' + L^x\Sigma^{xz}(L^z)' + L^z(\Sigma^{xz})'(L^x)'.
\end{aligned}$$

Applying the vec-operator on both sides of the previous equation yields:[13]

$$\operatorname{vec}\Sigma^x = \left(I_{n(x)^2} - L^x \otimes L^x\right)^{-1}\operatorname{vec}\left(L^z\Sigma^z(L^z)' + L^x\Sigma^{xz}(L^z)' + L^z(\Sigma^{xz})'(L^x)'\right). \tag{C.3a}$$

The matrices Σ^{xz} and Σ^z in this expression follow from the same reasoning. Consider $\Sigma^{xz} = \mathbb{E}(\bar{\mathbf{x}}_t\mathbf{z}_t')$:

$$\begin{aligned}
\Sigma^{xz} &= \mathbb{E}(\bar{\mathbf{x}}_{t+1}\mathbf{z}'_{t+1}) = \mathbb{E}(L^x\bar{\mathbf{x}}_t + L^z\mathbf{z}_t)(\Pi\mathbf{z}_t + \boldsymbol{\epsilon}_{t+1})', \\
&= \mathbb{E}(L^x\bar{\mathbf{x}}_t\mathbf{z}_t'\Pi') + \mathbb{E}(L^z\mathbf{z}_t\mathbf{z}_t'\Pi') + \mathbb{E}(L^x\bar{\mathbf{x}}_t\boldsymbol{\epsilon}'_{t+1}) + \mathbb{E}(L^z\mathbf{z}_t\boldsymbol{\epsilon}'_{t+1}), \\
\Sigma^{xz} &= L^x\Sigma^{xz}\Pi' + L^z\Sigma^z\Pi',
\end{aligned}$$

[13] The respective rule is $\operatorname{vec}(ABC) = (C' \otimes A)\operatorname{vec}B$, where $\otimes$ denotes the Kronecker product of the matrices C' and A. Since the eigenvalues of $C' \otimes A$ are equal to the product of the eigenvalues of C' and A, the eigenvalues of $\mathrm{L}^x \otimes L^x$ are within the unit circle and $I - L^x \otimes L^x$ is invertible. See Lütkepohl (2005), p. 661-662, for these results.

because the expectation of the terms that involve $\boldsymbol{\epsilon}_{t+1}$ is zero, since $\mathbf{z}_t$ and $\bar{\mathbf{x}}_t$ are predetermined when $\boldsymbol{\epsilon}_{t+1}$ is realized. Therefore,

$$\operatorname{vec}\Sigma^{xz} = \left(I_{n(x)n(z)} - \Pi \otimes L^x\right)^{-1} \operatorname{vec}\left(L^z \Sigma^z \Pi'\right). \tag{C.3b}$$

Finally:

$$\begin{aligned}
\Sigma^z &\equiv \mathbb{E}(\mathbf{z}_{t+1}\mathbf{z}'_{t+1}) = \mathbb{E}(\Pi\mathbf{z}_t + \boldsymbol{\epsilon}_{t+1})(\Pi\mathbf{z}_t + \boldsymbol{\epsilon}_{t+1})', \\
&= \mathbb{E}(\Pi\mathbf{z}_t\mathbf{z}'_t\Pi') + \mathbb{E}(\boldsymbol{\epsilon}_{t+1}\boldsymbol{\epsilon}'_{t+1}) + \mathbb{E}(\Pi\mathbf{z}_t\boldsymbol{\epsilon}'_{t+1}) + \mathbb{E}(\boldsymbol{\epsilon}_{t+1}\mathbf{z}'_t\Pi'), \\
\Sigma^z &= \Pi\Sigma^z\Pi' + \Sigma^\epsilon
\end{aligned}$$

so that

$$\operatorname{vec}\Sigma^{z} = \left(I_{n(z)^2} - \Pi \otimes \Pi\right)^{-1} \operatorname{vec}\Sigma^\epsilon. \tag{C.3c}$$

Equations (C.3) allow us to compute σ_Q as the square root of the third diagonal element of Σ^x, given the model's first order solution L^x and L^z.