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Abstract

This paper analyzes how to allocate experts into committees that use the unanimity rule to make decisions. We show that an optimal allocation of experts is extremely asymmetric. To reach the optimal allocation, therefore, one needs only to rank the experts in terms of their abilities and then allocate adjacent experts such that an expert's ability tends to vary inversely with the size of his committee. In the special case of three-member committees, we show that the optimal allocation maximizes the sum of the products of the experts' skills in each committee.

JEL-Codes: D710.

Keywords: unanimity rule, extremely asymmetric committees, optimal composition of committees.

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1 Introduction

It is often the case that the unanimous approval of all members of a decision-making body is necessary for implementing a certain action. For example, in the US, the Supreme Court has ruled that the Sixth Amendment mandates unanimity for a guilty verdict in a federal court criminal law jury trial. Also other jurisdictions often require a guilty verdict by a jury to be unanimous. In the UK, the common-law Duomatic principle requires unanimous consent among shareholders in order for their power to be exercised informally. In addition, many organizations have a hierarchical structure where successively higher ranks need to approve an action in order for it to be implemented, which is essentially a requirement of sequential unanimity. It should also be mentioned that international organizations such as NATO, the European Union, and WTO use the unanimity rule to decide on sensitive issues, and that essentially the veto power of the permanent members of the UN Security Council is the same as requiring unanimity in the approval of any action.[1]

From a theoretical perspective, the unanimity rule is strongly biased toward the status quo since any change requires the approval of all the committee members. The rule therefore may be optimal in an asymmetric environment, as, for instance, when there is a significant difference between the net benefit from changing the status quo if this is the correct decision, and the net benefit from not changing the status quo if this is the correct decision, where the net benefit is defined as the difference between the gains from the correct and incorrect decision.[2]

The purpose of this paper is to examine how to allocate experts with different abilities

[1] See Maggi and Morelli (2006), Payton (2010), and Blake and Payton (2013).

[2] Sah and Stiglitz (1988) and Ben-Yashar and Nitzan (1997, 2001) give the exact conditions under which the unanimity rule is preferred to all other voting rules. See also Feddersen and Pesendorfer (1998), Romme (2004), Ali et al. (2008), and Rijnbout and McKimmie (2014).

into committees that make dichotomous decisions using the unanimity rule.[3] We also compare the ensuing allocation to what would be optimal under the simple majority rule. The latter has recently been considered in Ben-Yashar and Danziger (2011) where it was shown that, even if feasible, committees that have the same size and use the simple majority rule should not generally be symmetric, i.e., should not have the same composition in terms of the experts' abilities. Ben-Yashar and Danziger (2014) have furthermore shown that in the case of three-member committees that use the simple majority rule,[4] the allocation is optimal if and only if it *minimizes* the sum of the product of the experts' skills in each committee.[5] One implication of this finding is that extreme asymmetry of committees, i.e., for every two committees having the three best experts in one committee and the three worst in another, is never optimal.

In the present paper we show that for any size of committees that use the unanimity rule, and even if the committees have different sizes, the opposite is true: Extremely asymmetric committees are optimal. Thus, in the special case that the committees have the same size, N', then the N' experts with the highest abilities should be allocated to one committee, the N' experts with the next highest abilities to another committee, and so on until the N' experts with the lowest abilities are allocated to one committee. More generally, we will refer to an allocation of experts to committees as extremely asymmetric if a committee with the smallest number of members consists of the best experts, a committee with the same or the next smallest number of members consists of the best of the remaining experts, and so

[3] The problem of how to aggregate the opinions of independent experts in dichotomous choice situations has been extensively researched. See Young (1995), Baharad and Nitzan (2002), Austen-Smith and Feddersen (2006), Berend and Sapir (2007), Dietrich and List (2008, 2013), and Bozbay et al. (2014).

[4] The United States Court of Appeals is an example of a system that randomly assigns dichotomous decisions to three-member committees.

[5] An expert's skill is defined as the difference between his ability and random choice.

on until all N experts are allocated to committees.

For the case of three-member committees that use the unanimity rule, the allocation of experts is optimal if and only if it *maximizes* the sum of the product of the experts' skills in each committee. In other words, while extreme asymmetry of three-member committees is never optimal with the simple majority rule, it is always optimal with the unanimity rule.

2 The Model

We consider the question of how to divide a given number of experts, N, into $Z > 1$ disjoint committees of given sizes, $N_1, N_2, \cdots, N_Z \geq 3$, where N_z is the number of experts in a committee $z = 1, 2, \cdots, Z$ and $\sum_{z=1}^{Z} N_z = N$. The same number of different proposals is assigned to each committee. Each committee uses a unanimity rule for each proposal to decide whether it should be accepted; that is, committee z accepts a proposal if all its N_z members are in favor, and rejects it otherwise. For each proposal, the correct decision is either acceptance or rejection, where the prior probability that acceptance is the correct decision is $\frac{1}{2}$. An expert must indicate whether he favors acceptance or rejection for each proposal assigned to his committee, and, following the literature back to Condorcet (1985), we assume that one expert's choice is independent of the other experts' choices.[6] The ability of expert $i = 1, 2, \cdots, N$ is represented by the probability $p_i \in (\frac{1}{2}, 1)$ that he favors the correct decision for a proposal. At most $N - 2$ experts have the same level of ability.

Let c_z denote the particular N_z experts allocated to committee z, and $c = \{c_1, ..., c_Z\}$ the ensuing composition of the committees that partitions the N experts into the Z disjoint committees. Further, let C denote the set of all such possible partitions, and $\Gamma(c_z)$

[6] See, among others, Sah and Stiglitz (1986, 1988). Ladha (1992) and Berg (1993) provide a tentative discussion of the consequences of relaxing the independence assumption.

the probability that the committee consisting of c_z makes the correct decision when using the unanimity rule on a proposal assigned to it. An optimal allocation of experts to the committees maximizes the average probability $(1/Z)\sum_{c_z \in c} \Gamma(c_z)$ that the committees make correct decisions on their proposals.

3 An Optimal Allocation of Experts

For committee z with composition c_z, if the correct decision is to accept a proposal, then the probability that the committee reaches the correct decision is equal to the probability that all its N_z experts support acceptance, i.e., $\prod_{i \in c_z} p_i$. If the correct decision is to reject a proposal, then the probability that the committee reaches the correct decision is equal to the probability that at least one expert supports rejection, i.e., $1 - \prod_{i \in c_z}(1 - p_i)$. Hence, the probability that the committee makes the correct decision is

$$\tfrac{1}{2}\left[\prod_{i \in c_z} p_i + 1 - \prod_{i \in c_z}(1 - p_i)\right].$$

Accordingly, the average probability that the committees make the correct decisions is

$$\begin{aligned}\frac{1}{Z}\sum_{c_z \in c} \Gamma(c_z) &= \tfrac{1}{2}\frac{1}{Z}\sum_{z=1}^{Z}\left[\prod_{i \in c_z} p_i + 1 - \prod_{i \in c_z}(1 - p_i)\right] \\ &= \tfrac{1}{2} + \tfrac{1}{2}\frac{1}{Z}\sum_{z=1}^{Z}\left[\prod_{i \in c_z} p_i - \prod_{i \in c_z}(1 - p_i)\right].\end{aligned}$$

We will refer to an allocation of experts to committees as extremely asymmetric if experts with the highest abilities are allocated to a committee with the smallest number of members, of the remaining experts those with the highest abilities are allocated to a committee with the same or the next smallest number of members, and so on until the experts with the lowest ability are allocated to a committee with the largest number of members. We now prove

Theorem: *For a given $N_1, N_2, \cdots, N_Z$, a composition of the Z disjoint committees $c \in C$ is optimal if and only if the allocation of experts is extremely asymmetric.*

Proof: The proof has four steps. In step 1 we assume that there are only two committees and that an expert with the highest ability and an expert with the lowest ability are in different committees. We show that unless the ability of every expert in one committee is at least at high as the ability of every expert in the other committee, will it be possible to increase the average probability of making the correct decision by switching between an expert in one committee and one expert in the other. In step 2 we continue to assume that there are only two committees and show that if it is possible that an expert with the highest ability is not in the same committee as an expert with the lowest ability, then it will not be optimal to allocate these experts to the same committee. In step 3 we use the results established in steps 1 and 2 to show that with two committees, an optimal allocation of experts is extremely asymmetric. Finally, in step 4 we extend the result of step 3 to show that with any number of committees an optimal allocation of experts is also extremely asymmetric. In step 4 we show that an allocation of experts is optimal only if it is extremely asymmetric.

In steps 1 and 2 where there are only two committees, we let h_1 denote the expert with the highest ability (or, if there are more than one expert with the highest ability, denote a particular one of these experts) and p_{h_1} his ability level, and ℓ_1 denote the expert with the lowest ability (or, if there are more than one expert with the lowest ability, denote a particular one of these experts) and p_{ℓ_1} his ability level.

Step 1: If $Z = 2$, and h_1 and ℓ_1 are in different committees, a necessary condition for optimality is that the highest ability experts are in one committee and the lowest ability *experts in the other.*

The average probability that the two committees make the correct decisions is

$$\tfrac{1}{2} + \tfrac{1}{4}\sum_{z=1}^{2}\left[\prod_{i\in c_z} p_i - \prod_{i\in c_z}(1-p_i)\right].$$

Consider any particular division of all the experts with the exception of h_1 and ℓ_1 into two disjoint groups: a_1 consisting of $N_1 - 1$ members and b_1 consisting of $N_2 - 1$ members. We assume wlog that the probability of unanimity for or against a proposal is at least as high in group a_1 as in group b_1, i.e., that

$$\prod_{i\in a_1} p_i + \prod_{i\in a_1}(1-p_i) \geq \prod_{i\in b_1} p_i + \prod_{i\in b_1}(1-p_i). \tag{1}$$

If h_1 is added to a_1 and ℓ_1 is added to b_1, then the average probability that the committees consisting of $a_1 \bigcup \{h_1\}$ and $b_1 \bigcup \{\ell_1\}$ make the correct decisions is

$$\tfrac{1}{2} + \tfrac{1}{4}\left[p_{h_1}\prod_{i\in a_1} p_i - (1-p_{h_1})\prod_{i\in a_1}(1-p_i) + p_{\ell_1}\prod_{i\in b_1} p_i - (1-p_{\ell_1})\prod_{i\in b_1}(1-p_i)\right], \tag{2}$$

while if ℓ_1 is added to a_1 and h_1 is added to b_1, then the average probability that the committees consisting of $a_1 \bigcup \{\ell_1\}$ and $b_1 \bigcup \{h_1\}$ make the correct decisions is

$$\tfrac{1}{2} + \tfrac{1}{4}\left[p_{\ell_1}\prod_{i\in a_1} p_i - (1-p_{\ell_1})\prod_{i\in a_1}(1-p_i) + p_{h_1}\prod_{i\in b_1} p_i - (1-p_{h_1})\prod_{i\in b_1}(1-p_i)\right]. \tag{3}$$

The difference between (2) and (3) equals

$$\tfrac{1}{4}(p_{h_1} - p_{\ell_1})\left[\prod_{i\in a_1} p_i + \prod_{i\in a_1}(1-p_i) - \prod_{i\in b_1} p_i - \prod_{i\in b_1}(1-p_i)\right].$$

Since the bracketed term is assumed to be nonnegative by (1), adding h_1 to a_1 and ℓ_1 to b_1 does not decrease the average probability that the two committees make the correct decisions.[7]

[7] If the bracketed term is zero, then adding h_1 to b_1 and ℓ_1 to a_1 would yield the same average probability.

Now, let ℓ_2 be an expert with the lowest ability in a_1, and h_2 be an expert with the highest ability in b_1. Thus, $p_{\ell_2} \leq p_{h_1}$ and $p_{h_2} \geq p_{\ell_1}$. If $p_{\ell_2} \geq p_{h_2}$, then the highest ability experts are allocated to one committee and the lowest ability experts to the other, and we make no switches. But if $p_{\ell_2} < p_{h_2}$, we let $a_2 \equiv a_1 \bigcup\{h_1\}\backslash\{\ell_2\}$ and $b_2 \equiv b_1 \bigcup\{\ell_1\}\backslash\{h_2\}$, and then, the average probability that the committees $a_1 \bigcup\{h_1\}$ and $b_1 \bigcup\{\ell_1\}$ make the correct decisions is[8]

$$\tfrac{1}{2} + \tfrac{1}{4}\left[p_{\ell_2}\prod_{i\in a_2} p_i - (1-p_{\ell_2})\prod_{i\in a_2}(1-p_i) + p_{h_2}\prod_{i\in b_2} p_i - (1-p_{h_2})\prod_{i\in b_2}(1-p_i)\right]. \tag{4}$$

Switching h_2 to a_2 and ℓ_2 to b_2, the average probability that the committees make the correct decisions is

$$\tfrac{1}{2} + \tfrac{1}{4}\left[p_{h_2}\prod_{i\in a_2} p_i - (1-p_{h_2})\prod_{i\in a_2}(1-p_i) + p_{\ell_2}\prod_{i\in b_2} p_i - (1-p_{\ell_2})\prod_{i\in b_2}(1-p_i)\right]. \tag{5}$$

The difference between (5) and (4) equals

$$\tfrac{1}{4}\left(p_{h_2} - p_{\ell_2}\right)\left[\prod_{i\in a_2} p_i + \prod_{i\in a_2}(1-p_i) - \prod_{i\in b_2} p_i - \prod_{i\in b_2}(1-p_i)\right]. \tag{6}$$

Since $p_{h_2} - p_{\ell_2} > 0$, (6) has the same sign as the bracketed term. Furthermore, for any $g \subset I$ with $|g| = N-1$, we have that

$$\prod_{i\in g} p_i + \prod_{i\in g}(1-p_i)$$

increases with each of the p_i's. Recalling that $p_{\ell_2} \leq p_{h_1}$ and $p_{h_2} \geq p_{\ell_1}$, it follows that

$$\begin{aligned}
\prod_{i\in a_2} p_i + \prod_{i\in a_2}(1-p_i) &\geq \prod_{i\in a_1} p_i + \prod_{i\in a_1}(1-p_i),\\
\prod_{i\in b_2} p_i + \prod_{i\in b_2}(1-p_i) &\leq \prod_{i\in b_1} p_i + \prod_{i\in b_1}(1-p_i),
\end{aligned}$$

[8] Note that $a_1 \bigcup\{h_1\}$ is the same committee as $a_2 \bigcup\{\ell_2\}$, and that $b_1 \bigcup\{\ell_1\}$ is the same committee as $b_2 \bigcup\{h_2\}$.

so that (6) is nonnegative.

Suppose we switch h_2 and ℓ_2. The resulting committees would then consist of $a_2 \bigcup \{h_2\}$ and $b_2 \bigcup \{\ell_2\}$. If the average probability that the committees make the correct decisions cannot be further increased by switching between an expert with the lowest ability in $a_2 \bigcup \{h_2\}$ and an expert with the highest ability in $b_2 \bigcup \{\ell_2\}$, then we do not make any further switches. If the average probability can be further increased, then we make the switch. If we did make the switch, we proceed to examine whether a further switch between an expert with the lowest ability in the committee originating from a_1 and an expert with the highest ability in the committee originating from b_1 can increase the average probability. If it cannot, we make no further switches, and if it can, we make the switch. This procedure is repeated until the switch of one more expert cannot increase the average probability. The upshot is that as long as one committee is not composed of the experts with the highest abilities and the other committee not of the experts with the lowest abilities, it is possible to increase the average probability by reallocating the experts. Since there is a finite number of possible committee compositions and the above reasoning is true for any possible initial division of the $N_1 + N_2 - 2$ experts (that does not include an expert with the highest ability and an expert with the lowest ability) into two disjoint groups with $N_1 - 1$ and $N_2 - 1$ members, we conclude that with two committees where two experts with the most different abilities are in different committees, an optimal allocation requires that the highest ability experts be in one committee and the lowest ability experts be in the other.

Step 2: If $Z = 2$ and it is possible that an expert with ability p_{h_1} is not in a committee together with an expert with ability p_{ℓ_1}, then it is not optimal that these experts are in the same committee.

We assume that h_1 and ℓ_1 are in the same committee and then show that the average probability of making correct decisions can be increased by switching either h_1 or ℓ_1 to the other committee. Therefore, starting with group a_1 and b_1, assume that both h_1 and ℓ_1 are added to one group, say group a_1, and that one member of group a_1, denoted by i_{a_1} and whose ability is represented by the probability $p_{i_{a_1}}$, is moved to group b_1. Assume that $p_{\ell_1} < p_{i_{a_1}} < p_{h_1}$.[9] We will show that the resulting average probability of making correct decisions is less with h_1 and ℓ_1 together in one committee than with h_1 and ℓ_1 in different committees, i.e., that

$$\begin{aligned} &\tfrac{1}{2} + \tfrac{1}{4}\Bigg[p_{\ell_1} p_{h_1} \prod_{i \in a_1 \setminus \{i_{a_1}\}} p_i - (1-p_{\ell_1})(1-p_{h_1}) \prod_{i \in a_1 \setminus \{i_{a_1}\}} (1-p_i) \\ &+ p_{i_{a_1}} \prod_{i \in b_1} p_i - (1-p_{i_{a_1}}) \prod_{i \in b_1} (1-p_i)\Bigg] \\ < \ & \tfrac{1}{2} + \tfrac{1}{4}\Bigg[p_{h_1} \prod_{i \in a_1} p_i - (1-p_{h_1}) \prod_{i \in a_1} (1-p_i) + p_{\ell_1} \prod_{i \in b_1} p_i - (1-p_{\ell_1}) \prod_{i \in b_1} (1-p_i)\Bigg]. \end{aligned}$$

This inequality is true if

$$\begin{aligned} &(p_{\ell_1} p_{h_1} - p_{i_{a_1}} p_{h_1}) \prod_{i \in a_1 \setminus \{i_{a_1}\}} p_i - \big[(1-p_{\ell_1})(1-p_{h_1}) - (1-p_{i_{a_1}})(1-p_{h_1})\big] \prod_{i \in a_1 \setminus \{i_{a_1}\}} (1-p_i) \\ &+ (p_{i_{a_1}} - p_{\ell_1}) \prod_{i \in b_1} p_i - \big[(1-p_{i_{a_1}}) - (1-p_{\ell_1})\big] \prod_{i \in b_1} (1-p_i) < 0 \\ \Leftrightarrow \quad & \big(p_{\ell_1} - p_{i_{a_1}}\big) \Bigg[p_{h_1} \prod_{i \in a_1 \setminus \{i_{a_1}\}} p_i + (1-p_{h_1}) \prod_{i \in a_1 \setminus \{i_{a_1}\}} (1-p_i) - \prod_{i \in b_1} p_i - \prod_{i \in b_1} (1-p_i)\Bigg] < 0, \end{aligned}$$

which is equivalent to

$$p_{h_1} \prod_{i \in a_1 \setminus \{i_{a_1}\}} p_i + (1-p_{h_1}) \prod_{i \in a_1 \setminus \{i_{a_1}\}} (1-p_i) - \prod_{i \in b_1} p_i - \prod_{i \in b_1} (1-p_i) > 0.$$

[9] If $p_{i_{a_1}} = p_{\ell_1}$ or $p_{i_{a_1}} = p_{h_1}$, then the situation is similar to the one examined in step 1 and, therefore, the average probability is higher if the experts with the highest ability are in one committee and the experts with the lowest ability are in the other.

This last inequality is true since $p_{h_1} > p_{i_{a_1}}$ implies that[10]

$$
\begin{aligned}
& p_{h_1} \prod_{i \in a_1 \setminus \{i_{a_1}\}} p_i + (1 - p_{h_1}) \prod_{i \in a_1 \setminus \{i_{a_1}\}} (1 - p_i) - \prod_{i \in b_1} p_i - \prod_{i \in b_1} (1 - p_i) \\
> & \prod_{i \in a_1} p_i + \prod_{i \in a_1} (1 - p_i) - \prod_{i \in b_1} p_i - \prod_{i \in b_1} (1 - p_i),
\end{aligned}
$$

and the last term is nonnegative due to the assumption in (1).

If instead h_1 and ℓ_1 were added to group b_1 and one of the members of group b_1 were switched to group a_1, the proof would be analogous. Hence, if it can be avoided, it is not optimal that h_1 and ℓ_1 are allocated to the same committee.

Step 3: If $Z = 2$, a composition of the committees is optimal if the allocation of experts is extremely asymmetric.

Step 1 and step 2 imply that with $Z = 2$, an optimal allocation of experts will have the highest ability experts in one committee and the lowest ability experts in the other. Hence, if $N_1 = N_2$, an optimal allocation of experts is extremely asymmetric.[11]

If $N_1 \neq N_2$, then an optimal allocation will have either the highest ability experts in the smaller committee and the lowest ability experts in the larger committee, or vice versa, i.e., the highest ability experts in the larger committee and the lowest ability experts in the smaller committee. In order to show that an optimal allocation of experts is extremely asymmetric, we need to show that the first of these possibilities is optimal while the second

[10] As mentioned earlier, if $g \subset I$ satisfies $|g| = N - 1$, then

$$\prod_{i \in g} p_i + \prod_{i \in g} (1 - p_i)$$

increases with each of the p_i's.

[11] The committee with the most skilled experts originates from group a_1. If the inequality in (1) would go the other way, a similar process would still lead to the N_1 most-skilled experts being allocated to one committee and the N_1 least-skilled experts to the other. The only difference would be that the committee with the most skilled experts would originate from group b_1.

is not.

To do so, we assume wlog that $N_1 < N_2$ and $p_1 \geq p_2 \geq \cdots \geq p_{N_1+N_2}$. We want to prove that

$$\begin{aligned}
&\tfrac{1}{2}+\tfrac{1}{4}\left[\prod_{i=1,\cdots,N_1} p_i - \prod_{i=1,\cdots,N_1}(1-p_i) + \prod_{i=N_1+1,\cdots,N_1+N_2} p_i - \prod_{i=N_1+1,\cdots,N_1+N_2}(1-p_i)\right] \\
\geq\ &\tfrac{1}{2}+\tfrac{1}{4}\left[\prod_{i=1,\cdots,N_2} p_i - \prod_{i=1,\cdots,N_2}(1-p_i) + \prod_{i=N_2+1,\cdots,N_1+N_2} p_i - \prod_{i=N_2+1,\cdots,N_1+N_2}(1-p_i)\right].
\end{aligned}$$

This is true if and only if

$$\begin{aligned}
&\left(1-\prod_{i=N_1+1,\cdots,N_2} p_i\right)\prod_{i=1,\cdots,N_1} p_i - \left[1-\prod_{i=N_1+1,\cdots,N_2}(1-p_i)\right]\prod_{i=1,\cdots,N_1}(1-p_i) \\
\geq\ &\left(1-\prod_{i=N_1+1,\cdots,N_2} p_i\right)\prod_{i=N_2+1,\cdots,N_1+N_2} p_i - \left[1-\prod_{i=N_1+1,\cdots,N_2}(1-p_i)\right]\prod_{i=N_2+1,\cdots,N_1+N_2}(1-p_i),
\end{aligned}$$

and hence true if and only if

$$\begin{aligned}
&\left(1-\prod_{i=N_1+1,\cdots,N_2} p_i\right)\left(\prod_{i=1,\cdots,N_1} p_i - \prod_{i=N_2+1,\cdots,N_1+N_2} p_i\right) \\
&-\left[1-\prod_{i=N_1+1,\cdots,N_2}(1-p_i)\right]\left[\prod_{i=1,\cdots,N_1}(1-p_i) - \prod_{i=N_2+1,\cdots,N_1+N_2}(1-p_i)\right] \\
\geq\ &0. \qquad (7)
\end{aligned}$$

Due to the assumption that $p_1 \geq p_2 \geq \cdots \geq p_{N_1+N_2}$ and there being the same number of multipliers, namely N_1, in the four products $\prod_{i=1,\cdots,N_1} p_i$, $\prod_{i=N_2+1,\cdots,N_1+N_2} p_i$, $\prod_{i=1,\cdots,N_1}(1-p_i)$, and $\prod_{i=N_2+1,\cdots,N_1+N_2}(1-p_i)$, it follows that $\prod_{i=1,\cdots,N_1} p_i - \prod_{i=N_2+1,\cdots,N_1+N_2} p_i \geq 0$ and $\prod_{i=1,\cdots,N_1}(1-p_i) - \prod_{i=N_2+1,\cdots,N_1+N_2}(1-p_i) \leq 0$. Accordingly, inequality (7) is true so that if $Z = 2$, the composition of the the committees is optimal if the allocation of experts is extremely asymmetric.

Step 4: If $Z \geq 2$, a composition of the committees is optimal if and only if the allocation of experts is extremely asymmetric.

The arguments for two committees in the previous three steps imply that the average probability of any two of the Z committees, say, z and z' with $N_z \leq N_{z'}$, is maximized by allocating the N_z experts with the highest abilities of the pertinent $N_z + N_{z'}$ experts to committee z and the other $N_{z'}$ experts with the lowest abilities to committee z'. As a consequence, the average probability is maximized by an extremely asymmetric allocation of experts. That is, an optimal allocation of experts is extremely asymmetric. Furthermore, since a non-optimal allocation cannot be extremely asymmetric, it follows that an extremely asymmetric allocation is optimal. □

The optimality of extremely asymmetric committees is a consequence of the fact that if decisions are made by the unanimity rule, then the experts' abilities are complements in producing the correct decisions.[12] The theorem highlights the fact that the characterization of an optimal allocation of experts with different abilities to committees is simple: One just needs to rank the experts in terms of their abilities and then allocate the best experts to a committee with the smallest number of members, the next best experts to one of the remaining committees that now have the smallest number of members, and so on. Accordingly, in order to allocate the experts optimally, one does not need to know the precise abilities of the experts, but only their ranking.

Since there may be more than one extremely asymmetric allocation, an optimal allocation of experts to committees is generally not unique. The reason is that different experts may have the same ability level, and that if two committees have the same size, then switching all the experts between the two committees would not affect the average probability that the committees make the correct decisions. Nevertheless, the allocation of the expert's abilities

[12] Since $\partial^2\Gamma(c_z)/(\partial p_i \partial p_{i'}) > 0$, the abilities of experts i and i' are complements in a committee that makes its decisions by the unanimity rule.

is unique in the sense that the exchange of two experts with the same ability between two committees does not change the allocation of the experts' abilities, and if two committees have the same size, then switching all the experts between the two committees does not change the allocation of the experts' abilities to committee sizes.

4 Three-Member Committees

We now consider the special case of three-member committees, $N_1 = N_2 = \cdots = N_Z = 3$ and $3Z = N$. As in Ben-Yashar and Danziger (2014), we let $q_i \equiv p_i - \frac{1}{2}$ denote the skill of expert i; that is, how much the probability that he favors the correct decision exceeds that of a random choice. We then have

Corollary: *A composition of three-member committees $c \in C$ is optimal if and only if it maximizes*

$$\sum_{c_z \in c} \prod_{i \in c_z} q_i. \tag{8}$$

Proof: Suppose that $c_z = \{\alpha, \beta, \gamma\}$. Then the probability that a committee makes the correct decision is

$$\begin{aligned}
\Gamma(c_z) &= \tfrac{1}{2} \{p_\alpha p_\beta p_\gamma + [1 - (1 - p_\alpha)(1 - p_\beta)(1 - p_\gamma)]\} \\
&= \tfrac{1}{2} \left\{(q_\alpha + \tfrac{1}{2})(q_\beta + \tfrac{1}{2})(q_\gamma + \tfrac{1}{2}) + \left[1 - (\tfrac{1}{2} - q_\alpha)(\tfrac{1}{2} - q_\beta)(\tfrac{1}{2} - q_\gamma)\right]\right\} \\
&= \tfrac{1}{2} + \tfrac{1}{4} (q_\alpha + q_\beta + q_\gamma) + q_\alpha q_\beta q_\gamma.
\end{aligned}$$

Accordingly, the average probability that the committees make the correct decisions is

$$\frac{1}{Z} \sum_{c_z \in c} \Gamma(c_z) = \tfrac{1}{2} + \frac{1}{4Z} \sum_{i=1}^{3Z} q_i + \frac{1}{Z} \sum_{c_z \in c} \prod_{i \in c_z} q_i.$$

Since the skills of the experts and hence $\sum_{i=1}^{3Z} q_i$ are given, choosing $c \in C$ to maximize $(1/Z)\sum_{c_z \in c} \Gamma(c_z)$ is equivalent to choosing $c \in C$ to maximize $\sum_{c_z \in c} \prod_{i \in c_z} q_i$. That is, an optimal c maximizes $\sum_{c_z \in c} \prod_{i \in c_z} q_i$. □

Thus, with the unanimity rule it is optimal to allocate the experts so as to maximize the sum of the products of the skills in each committee, and only such allocations are optimal. This is exactly the opposite of the optimality criterion for when decisions are made by a simple majority rule. In that case, an allocation of experts is optimal if and only if it minimizes the sum of the products of the skills in each committee (Ben-Yashar and Danziger, 2014). Indeed, with the unanimity rule, the average probability that the committees make correct decisions would be minimized by allocating the experts so as to minimize the sum of the products of the skills in each committee, and vice versa for the simple majority rule. The reason for this difference is that if decisions are made by the unanimity rule, then with any committee size the experts' abilities (skills) are complements in achieving the correct decisions, while if decisions are made by a simple majority rule, then with three-member committees the experts' abilities (skills) are substitutes in achieving the correct decisions.[13]

Due to the oppositeness of the optimality criterion for the unanimity rule and the simple majority rule, in the case of disjoint three-member committees, many characteristics of an optimal allocation of experts with the unanimity rule are opposite to those with the simple majority rule. In particular, while the extremely asymmetric allocation of experts is optimal with the unanimity rule, this is never the case with the simple majority rule. For example, if there are three experts at each of three different skill levels, then under the unanimity rule each of the three committees should be composed of only one type of expert, that is,

[13] Since $\partial^2 G(c_z)/(\partial p_i \partial p_{i'}) < 0$, where $G(c_z)$ denotes the probability that a three-member committee makes the correct decision by the simple majority rule, the abilities (skills) of i and i' are substitutes in a committee that makes its decision by this rule.

they should be extremely asymmetric. But with the simple majority rule each of the three committees should be composed of one of each type of experts, that is, they should be symmetric.

If the model were extended to include useless "experts," i.e., some of the experts have zero skill, the corollary would still remain valid. With the unanimity rule it would then be optimal to concentrate these useless experts into the smallest possible number of committees, while with the simple majority rule it would be optimal to spread these useless experts into as many committees as possible.

5 Conclusion

This paper has analyzed how experts should be allocated into committees with given, and generally different, sizes that use the unanimity rule to make decisions. We have shown that an optimal allocation of experts is extremely asymmetric: The experts with the highest abilities should be allocated to the smallest committee (or any particular one of the smallest committees), the experts with the next highest abilities to the smallest remaining committee (or any particular one of the smallest remaining committees), and so on until the experts with the lowest abilities are allocated to a committee that has the most members. To reach an optimal allocation, therefore, one needs only to rank the experts in terms of their abilities and then allocate adjacent experts such that an expert's ability tends to vary inversely with the size of his committee. This result reflects that the experts' abilities are complements in making the correct decision. In the special case of three-member committees, we have shown that an optimal allocation of experts maximizes the sum of the products of the experts' skills in each committee.

If the number of committees and experts needs to be reduced (perhaps due to a reduction in the number of proposals that need to be decided), it is always preferable to dismiss the experts with the lowest abilities. This is relatively easily done with the unanimity rule as the reallocation of the remaining experts is simple. In contrast, with other voting rules such as the simple majority rule, the disbanding of some of the committees and the dismissal of the experts with the lowest abilities might require a major reshuffling of the remaining experts.

In the same vein, suppose that the experts' abilities increase with experience and that experts with the same seniority have the same ability. With the unanimity rule, an optimal allocation of experts to the committees will not change under these conditions. In contrast, with many other voting rules, there is a need to reoptimize in order to determine whether the committees should be reshuffled.

We have assumed that the objective is to maximize the average probability of making correct decisions. However, suppose that the model is modified so that the objective is to maximize the average net benefit of making correct decisions and that the net benefit from making a correct decision is not the same for different proposals. Since proposals with higher net benefits will then be assigned to committees consisting of experts with higher abilities, it will still be optimal for the committees to be extremely asymmetric.

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