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Abstract

Using an intertemporal model of saving and capital accumulation with two types of agents (workers and capitalists) we demonstrate that it is impossible for any binding minimum wage to increase the after-tax incomes of workers if the production function is Cobb-Douglas with constant returns to scale, or if there are no differences in ability among workers. We also show that it is not possible to increase the incomes of employed workers through minimum wage legislation, even under decreasing returns to scale and heterogeneity of ability among workers, unless the welfare support provided to unemployed workers is far below what they would earn in the absence of minimum wages. Moreover, we establish that in the absence of a separate class of agents (i.e. capitalists) minimum wages cannot increase the incomes of employed workers even when there are decreasing returns to scale and no welfare support is provided to the unemployed.

JEL-Codes: E210, E240, E640, H230, J230.

Keywords: minimum wage, capital accumulation, heterogeneity, unemployment.

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1. Introduction

The use of minimum wages as a way to reduce poverty and redistribute income has re-emerged forcefully in recent policy discussions. Critics of minimum-wage legislation focus usually on its disemployment effects[1] and on whether it is an effective redistributive tool.[2] While these issues are subject to intense discussion among economists, it is taken for granted by both sides of this debate that, following a minimum wage increase, the incomes of (at least some) workers that remain in employment will be higher (e.g. Card and Krueger 1995, Saint-Paul 2000, Manning 2003, Neumark and Wascher 2008).[3]

The objective of the present paper is to argue that this presumption is by no means guaranteed once we move away from static models and allow for capital accumulation.[4] To this purpose in section 2 we construct a model with two types of agents, i.e. workers and capitalists (e.g. Judd 1985, Acemoglu 2009). The latter are a homogeneous group, do all the saving and own the capital stock, whereas the workers are differentiated according to their ability. Assuming a constant-returns-to-scale Cobb-Douglas production function, we demonstrate in Section 3 that the imposition of *any* binding minimum wage, in addition to generating unemployment amongst the least able workers, will also reduce both the steady-state capital stock and the after-tax incomes of employed workers. Our result also implies that the (joint) existence of economic profits and differences in ability among workers can allow workers above an ability threshold to increase their (after-tax) incomes through the imposition of a binding minimum wage; however, they can achieve this only at the expense of low-ability workers who become unemployed. The effects of minimum-wage legislation

[1] The early consensus regarding the effects of minimum wage increases on employment in the US (e.g. Brown et al. 1982) indicated some disemployment effects – mainly concentrated on teenagers and young adults. Following Card and Krueger's (1994) study questioning the existence of disemployment effects, there have been many studies but no consensus regarding their size or existence (see, e.g. Manning 2003, Neumark and Wascher 2008, Dube et al. 2010, Meer and West 2013).

[2] The issue of whether the minimum wage is an efficient redistributive tool is not the subject of this paper; for papers discussing this issue, see, e.g. Allen (1987), Guesnerie and Roberts (1987), Marceau and Boadway (1994), Freeman (1996), Boadway and Cuff (2001), Burkhauser and Sabia (2007), Lee and Saez (2012), and Cahuc and Laroque (2014).

[3] This effect is behind some political economy explanations regarding the unwillingness of policymakers to dismantle unemployment-generating labour-market legislation (e.g. Sobel 1999, Saint-Paul 2000, Adam and Moutos 2011).

[4] The effects of minimum wages on capital accumulation and how the interaction between them may affect the demand for labour has not been much studied in the empirical literature. Sorkin (2015) provides a notable exception to this trend, and shows how the existence of putty-clay technology can generate a delayed response to increases in the real value of the minimum wage, with the implication that the measured "long-run" employment response to changes in the minimum wage can be biased as the real value of the minimum wage is eroded through time by inflation. Taking into account this consideration, he shows that previous studies may have incorrectly concluded that the short-run and the long-run effects of (nominal) minimum-wage hikes are similar, and that the value of the long-run elasticity may be three times as high as the short-run elasticity.

can thus match with what Stigler (1970) termed Director's Law – according to which public interventions are made for the primary benefit of the middle classes, and financed with taxes which are borne in considerable part by the (rich) capitalists and the poor.

In section 4 we extend the model by allowing for endogenous labour supply, and as a result, heterogeneity in the amount of labour offered by workers of different ability. These changes do not alter the main findings of section 3, and they bring into focus the role of welfare support for the unemployed. Our numerical comparisons of the long-run properties of the perfectly competitive and the minimum wage regimes indicate that it is impossible under any set of plausible parameter values to increase the incomes of employed workers under constant returns to scale even when no welfare support is provided to the unemployed. We also find that even under decreasing returns to scale it is impossible to increase the (after-tax) income of employed workers unless the unemployed workers are left with a level of welfare support far below what they would earn in the absence of minimum wages. The same conclusion emerges if we focus our comparison on utility outcomes since – in principle – reductions in consumption could be compensated by increased leisure.

The presence of two distinct groups of agents in our framework – with workers not saving and capitalists not working – raises the question of whether a Ramsey-type framework with worker heterogeneity but not with two "classes" of agents (workers versus capitalists) would be a more appropriate way to analyze the consequences of minimum wages.[5] We develop such a model in Section 5. The results we derive are similar to the ones we derive in the model with the two distinct groups of agents. In particular, we demonstrate analytically two things. First, it is impossible for the imposition of minimum wages to increase employed workers' total incomes when there are constant returns to scale. Second, in the case of diminishing returns to scale, the only possibility for the imposition of minimum wages to increase employed workers' total incomes arises when the increase in (after-tax) wage income is such that outweighs the losses they suffer from their other sources of income (capital income and dividends). However, after extensive experimentation with a wide range of plausible parameter values we have not been able to find a single case in which the total income of employed workers increases even when no welfare support is provided to the unemployed. One way to understand this finding is by noting that workers can no longer extract a larger share of output without hurting themselves. This is because a rise in the wage *rate* earned by workers reduces the profits of the firms and the dividends received by the

[5] We wish to thank an anonymous referee for bringing this possibility to our attention.

workers who are the sole owners of the firms. In this framework the inefficiency introduced by the minimum wage hurts the workers as there is no other group (i.e. the capitalists) on which the cost of inefficiency can be transferred to and from which workers can extract a larger share of output. The presence of another economic class is thus necessary for (employed) workers to be able to be made better-off from the imposition of minimum wages.

A summary and a discussion of our results are offered in Section 6.

2. The Basic Model

We consider a closed economy producing a single good under perfectly competitive conditions, and consisting of two sets of agents: workers and capitalists.

2.1 *The Perfectly Competitive Case*

2.1.1 Workers

There is a fixed number of workers in the economy (normalized to one). All worker-based households (*workers*, thereafter) are assumed to have identical preferences. However, workers differ in ability, as reflected in their endowment of effective number of labour units per unit of time (e.g. per hour, day, or year). We assume that all workers have the same endowment of *time units* at their disposal (which we also normalize to one), and that they supply inelastically their endowment of effective labour units. The distribution of effective labour units (ability) among workers is described by the Pareto distribution. Letting e denote the ability of a worker, the Pareto distribution is defined over the interval $e \geq b$, and its CDF is

$$F(e) = 1 - (b/e)^{a}, \;\; a > 1, b > 0, \tag{1}$$

Parameter b stands for the lowest ability in the population of workers, and parameter a determines the shape of the distribution (higher values of a imply greater equality in the distribution of ability). The mean of the Pareto distribution is equal to,

$$\mu = ab/(a - 1). \tag{2}$$

Workers' incomes are equal to their labour earnings, i.e. equal to worker's ability times the wage per *effective* unit of labour, $e^i w_t$. Given our simplifying assumption that workers do not save, the consumption of workers evolves according to:

$$C_t^i = e^i w_t. \tag{3}$$

2.1.2 Capitalists

There is a fixed number, N, of identical capitalists in the economy. In contrast to workers, they do not directly participate in production, but hold shares in various firms and receive as dividends the firms' profits. For simplicity, we assume that the number of capitalists is equal to the number of firms. Their preferences are:[6]

$$U^K = \sum_{t=0}^{\infty} \beta^t ln(C_t^K) \ , \tag{4}$$

whereas their budget constraint is,

$$C_t^K + K_{t+1}^K - (1-\delta)K_t^K = \Pi_t + r_t K_t^K. \tag{5}$$

In equation (4), C_t^K stands for the consumption of each capitalist, and in equation (5), $K_t^K, r_t K_t^K,$ and Π_t stand for the capital stock, capital income, and profits accruing to each capitalist. Each capitalist solves the following programme:

$$\max_{C_t^K, K_{t+1}^K} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \{lnC_t^K + \lambda_t(\Pi_t + r_t K_t^K - C_t^K - K_{t+1}^K + (1-\delta)K_t^K)\}$$

The resulting first-order conditions are:

$$\lambda_t = 1/C_t^K, \tag{6a}$$
$$\lambda_t = \beta\lambda_{t+1}(r_{t+1} + 1 - \delta). \tag{6b}$$

Combining equations (6a) and (6b) we get:

[6] For convenience we drop the subscript pertaining to each capitalist since they are identical.

$$C_{t+1}^K = \beta(r_{t+1} + 1 - \delta)C_t^K \; . \tag{7}$$

Equation (7) summarizes the optimal consumption path for the capitalists, and (implicitly), along with their budget constraint, the supply of capital in the economy.

2.1.3 Firms

Firms' technology of converting inputs into output is a Cobb-Douglas one,

$$Y_t = \left(K_t^f\right)^{\gamma}\left(L_t^f\right)^{\eta}, \;\; \gamma, \eta < 1 \tag{8}$$

where Y_t denotes output, K_t^f is the capital stock used by the firm, L_t^f is the number of *effective* units of labour used by the firm. Profit maximization implies,

$$w_t = \eta\left(K_t^f\right)^{\gamma}\left(L_t^f\right)^{\eta-1} \tag{9}$$

$$r_t = \gamma\left(K_t^f\right)^{\gamma-1}\left(L_t^f\right)^{\eta} \; . \tag{10}$$

As a result, the profits accruing to each entrepreneur are:

$$\Pi_t = (1 - \eta - \gamma)Y_t. \tag{11}$$

2.1.4 Factor Market Equilibrium

The aggregate supply of effective units of labour of all workers is,

$$L^s = \int_b^{\infty} e\left\{\alpha \frac{b^{\alpha}}{e^{\alpha+1}}\right\} de = \frac{\alpha b}{\alpha-1}, \tag{12}$$

i.e., it is just equal to the mean units of effective labour (since the number of workers is equal to one). Labour market equilibrium obtains when the aggregate demand for labour by the N firms is equal to aggregate labour supply, i.e. when,

$$NL_t^f = \frac{\alpha b}{\alpha-1}. \tag{13}$$

Similarly, equilibrium in the capital market obtains when the total supply of capital – as provided by the capitalists – is equal to the demand for capital by firms, i.e. when

$$K_t^K = K_t^f. \tag{14}$$

2.1.5 General Equilibrium

The dynamic behavior of the model is described by equations (3), (4), (7), (8), (9), (10), (11), (13), and (14), which in long-run equilibrium collapse to the following system (for ease of exposition we drop the time subscripts, and the superscripts distinguishing between capitalists and firms, since each firm is owned by a single capitalist, e.g. $L_t^f = L$):

$$C^i = e_i w \tag{LR1}$$

$$1 = \beta(r + 1 - \delta) \tag{LR2}$$

$$C^K + \delta K = \Pi + rK \tag{LR3}$$

$$Y = L^\eta K^\gamma \tag{LR4}$$

$$\Pi = (1 - \eta - \gamma)Y \tag{LR5}$$

$$NL = \frac{\alpha b}{\alpha - 1} \tag{LR6}$$

$$w = \eta K^\gamma L^{\eta - 1} \tag{LR7}$$

$$r = \gamma K^{\gamma - 1} L^\eta \tag{LR8}$$

Equations (LR2)-(LR8) determine the long-run equilibrium values of *w, r, K, L, Y, Π,* and C^K.[7] We note that once the value of the wage *rate* is found we can determine the entire distribution of workers' consumption through equation (LR1).

2.2 Minimum Wages

We now assume the existence of a government-imposed minimum wage per unit of labour *time* (e.g. per hour) equal to y, which is the minimum amount that an employer must pay in order to employ one person. This minimum wage per unit of time must be distinguished from the wage *rate* per *effective* unit of labour, which will be market-determined (i.e. as in the previous section).

2.2.1 Labour Market

The minimum wage constraint implies that firms will not be willing to employ workers whose level of ability (i.e. number of efficient units of labour per unit of time) is such that

[7] In fact, the entire system can be solved recursively in the long-run: once we find *r* from (LR2), and *L* from (LR6), (LR8) solves for *K*, then (LR7) for w, (LR4) for Y, and so on.

$y > e_i\varpi_t$, where ϖ_t stands for the market-determined wage *rate* per effective unit of labour in the presence of the minimum-wage (per unit of time) constraint at time t.[8] In order to avoid confusion in what follows we shall refer to the exogenously set, y, simply as the minimum wage, in order to differentiate it from the minimum wage *rate* , ϖ_t, and the competitive wage *rate*, w, both of which are endogenously determined. Let ε_t denote the level of ability for which it holds that:

$$y = \varepsilon_t\varpi_t \,. \tag{15}$$

It follows that only workers with $e_i \geq \varepsilon_t$ will be employed by firms, and that the individual with ability ε_t will just earn the minimum wage, y. Workers with ability smaller than ε_t will be unemployed, thus the unemployment rate will be:

$$u_t = 1 - \left\{\frac{b}{\varepsilon_t}\right\}^{\alpha} \tag{16}$$

The total number of effective units of labour possessed (and supplied) by those individuals with $e_i \geq \varepsilon_t$ is,

$$L_t^s = \int_{\varepsilon_t}^{\infty} e\left\{\alpha\frac{b^{\alpha}}{e^{\alpha+1}}\right\}de = \frac{\alpha\varepsilon_t}{\alpha-1}\left\{\frac{b}{\varepsilon_t}\right\}^{\alpha}. \tag{17}$$

We can thus describe the condition describing equilibrium in the labour market (i.e. the analogue of equation (13) as:

$$NL_t^f = \frac{\alpha\varepsilon_t}{\alpha-1}\left\{\frac{b}{\varepsilon_t}\right\}^{\alpha}. \tag{18}$$

A simple comparison of equations (13) and (18) reveals that –ceteris paribus- a binding minimum wage constraint, which implies that $b < \varepsilon$, will be associated with a higher wage *rate* per effective unit of labour than in its absence ($\varpi > w$), due to the reduction in the aggregate effective units of labour supply caused by the exclusion of the lowest-ability workers from employment**.**

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[8] We assume that the minimum wage per unit of time is such that $y > b\varpi_t$.

2.2.2 Government

In addition to setting (and enforcing) the minimum wage constraint, the government is assumed to levy a comprehensive income tax (τ) on all sources of income, in order to finance benefits for the low-ability workers that are unemployed. We assume that the level of the unemployment benefit is a fixed proportion of the minimum wage, i.e. it is equal to ϕy ($0 \leq \phi < 1$). Parameter ϕ describes the generosity of the unemployment benefit system. We note that in this model the granting of these benefits has an indefinite duration since the lowest-ability workers are permanently excluded from employment. In this sense, the income support provided to the unemployed is comparable to the real-world welfare payments (e.g. social assistance) provided to individuals whose eligibility for unemployment benefits has expired, or those who have never fulfilled the eligibility criteria for receiving them. Equation (19), i.e. the government budget constraint, just states that the net payments to the unemployed are equal to total tax receipts:

$$\phi y u_t = \tau_t N Y_t. \tag{19}$$

We assume that τ_t adjusts in every period so as to keep the budget in balance.

2.2.3 General Equilibrium

The existence of taxes implies that equations (3), (5), and (7) must be modified to:

$$C_t^i = (1-\tau)e_i\varpi_t \tag{3a}$$

$$C_t^K + K_{t+1}^K - (1-\delta)K_t^K = (1-\tau)(\Pi_t + r_t K_t^K). \tag{5a}$$

$$C_{t+1}^K = \beta((1-\tau)r_{t+1} + 1 - \delta)C_t^K \tag{7a}$$

These three equations along with equations (15)–(19) describe the dynamic evolution of the system, whose long-run equilibrium is described by the following equations:

$$C_i^t = (1-\tau)e_i\varpi \tag{LR1a}$$

$$1 = \beta((1-\tau)r + 1 - \delta) \tag{LR2a}$$

$$C^K + \delta K = (1-\tau)(\Pi + rK) \tag{LR3a}$$

$$Y = L^\eta K^\gamma \tag{LR4a}$$

$$\Pi = (1-\eta-\gamma)Y \tag{LR5a}$$

$$NL = \frac{\alpha\varepsilon}{\alpha-1}\left\{\frac{b}{\varepsilon}\right\}^{\alpha} \quad \text{(LR6a)}$$

$$\varpi = \eta K^{\gamma} L^{\eta-1} \quad \text{(LR7a)}$$

$$r = \gamma K^{\gamma-1} L^{\eta} \quad \text{(LR8a)}$$

$$u = 1 - \left\{\frac{b}{\varepsilon}\right\}^{\alpha} \quad \text{(LR9)}$$

$$y = \varepsilon\varpi \quad \text{(LR10)}$$

$$\phi y u = \tau N Y \quad \text{(LR11)}$$

These equations determine the long-run values of ϖ, ε, *r*, *u*, *K*, *L*, *Y*, *Π*, C^K, C^i, and τ. We note that unlike the perfectly competitive (PC) case, the system is no longer recursive, since equation (LR2a) does not uniquely solve for *r*. Nevertheless, we can draw some useful results by comparing the PC with the minimum wage (MW) case.

3. Comparison

Using equations (LR2), (LR6), (LR7), and (LR8) we find that the PC wage *rate* is:

$$w = \frac{\eta\gamma^{\gamma/(1-\gamma)}\left[\frac{b\alpha}{(\alpha-1)N}\right]^{(\eta+\gamma-1)/(1-\gamma)}}{[\delta-1+(1/\beta)]^{\gamma/1-\gamma}} \quad (20)$$

Similar manipulations for the MW case yield:

$$\varpi = \frac{\eta((1-\tau)\gamma)^{\gamma/(1-\gamma)}\left[\frac{b^{\alpha}\alpha}{(\alpha-1)N}\right]^{(\eta+\gamma-1)/(1-\gamma)}\varepsilon^{(\eta+\gamma-1)(1-\alpha)/(1-\gamma)}}{[\delta-1+(1/\beta)]^{\gamma/1-\gamma}} \quad (21)$$

We wish to enquire whether the workers which retain their jobs after the imposition of the MW have higher after-tax incomes than in the PC case. This will be the case if the after-tax wage *rate* (per effective unit of labour) in the MW case is larger than the PC wage *rate*, i.e. if

$$(1-\tau)\varpi > w. \quad (22)$$

Using equations (20) and (21), inequality (22) can be written as

$$1-\tau > [b/\varepsilon]^{(\alpha-1)(1-\eta-\gamma)}. \quad (23)$$

Since both τ and ε are endogenous (i.e. they depend, among other things, on the size of the imposed minimum wage, y, and on the parameter describing the generosity of the unemployment benefit system, ϕ), it is impossible to assess, in general, whether inequality (23) is satisfied.[9] For this reason, we now examine the consequences of adopting two cases which are widely employed in the macroeconomics literature.

Consider, first, the case that firms make no profits, which obtains if there are constant returns to scale ($\eta + \gamma = 1$). The required condition now becomes $1 - \tau > 1$, which is impossible for $\tau > 0$. Note that even if no income support was provided to the low-ability workers which are permanently excluded from employment ($\phi = \tau = 0$), it would still be impossible for the employed workers to (strictly) raise their incomes under the MW regime relative to the PC case.[10]

The same result obtains if $\varepsilon \to b$, which would arise if $\alpha \to \infty$, i.e. if there is complete equality in the distribution of ability among workers. Thus, we can state that:

Proposition: *With Cobb-Douglas technology, the imposition of any binding minimum wage will lead to a decrease in the after-tax incomes of the employed workers in the long run, if either*

(a) there are constant returns to scale, or

(b) all workers have the same ability.

The detrimental effects on capital accumulation that the MW regime generates can thus, under the conditions stated above, undo the presumed short-run benefits for the employed workers that the imposition of a minimum wage can generate.

The reason that employed workers may not become worse off in the presence of profits is that the capital stock will be higher than in their absence. This is because the ability to also tax profits reduces by less the incentives for capital accumulation than if the full burden of taxation falls on worker and capital income alone – since taxes on profits are less distortive than on capital. In the case of complete equality in ability among workers (i.e. a

[9] Numerical simulations indicate that it is possible to find a binding minimum wage which succeeds in increasing the after-tax incomes of employed workers but at the expense of workers of low ability who are forced into unemployment.

[10] Setting $\eta + \gamma = 1$, it can be readily ascertained from equations (20) and (21) that the wage *rate* per effective unit of labour is equal in the two regimes, i.e. $\varpi = w$. This is clearly a consequence of the reduced incentives for capital accumulation engendered by the reduction in the available effective units of labour to be used by firms due to the imposition of the minimum wage. As a result, even if no welfare support is provided for the unemployed the employed workers cannot increase their incomes in the long run by instituting any binding minimum wage.

homogeneous labour force), the MW regime decreases the aggregate units of effective labour used in proportion to the fall in the number of persons employed, thus it leads to a larger drop in output and in the income accruing to employed members – since the (pre-tax) share of output accruing to workers is fixed - out of which the taxes to support the unemployed must also be paid.

4. Endogenous Labour Supply

We proceed now to endogenize the labour supply of workers. We assume that workers' preferences are given by:

$$U_L^i = \sum_{t=0}^{\infty} \beta^t \frac{\{C_t^i + (1-h_t^i)^\nu\}^{1-\sigma}}{1-\sigma} \qquad \nu, \sigma > 0 \tag{24}$$

whereas their budget constraint is:

$$C_t^i = w_t e^i h_t^i \tag{25}$$

In equation (24) h_t^i stands for the labour supply (hours of work) of household *i* in period *t*; for simplicity the worker's time endowment has been normalized to unity. Also parameter $0 < \nu < 1$ measures the relative preference of the worker for leisure, whereas parameter σ determines the size of the elasticity of intertemporal substitution. Maximization of equation (24) subject to the budget constraint (25) results in the following labour supply function:

$$h_t^i = 1 - [\frac{\nu}{w_t e^i}]^{\frac{1}{1-\nu}}. \tag{26}$$

We note that household labour supply is increasing in both wages (per unit of effective labour) and ability.

4.1 Perfect Competition

The aggregate supply of effective units of labour in the perfectly competitive case is:

$$L_t^s = \int_b^{\infty} e \cdot h_t^i \cdot \left\{\alpha \frac{b^\alpha}{e^{\alpha+1}}\right\} de,$$

which, after using equation (26), turns to be equal to:

$$L_t^s = \frac{\alpha b}{\alpha-1} - \Theta \left[\frac{\nu}{w_t}\right]^{\frac{1}{1-\nu}}, \tag{27}$$

where $\Theta \equiv \frac{(1-\nu)\alpha b^{\frac{(\alpha-1)\nu}{1-\nu}}}{(1-\nu)\alpha+\nu}$. If leisure provides no utility to the household ($\nu = 0$), then equation (27) becomes identical to the corresponding equation (12) in the case of exogenous labour supply.

The long-run equilibrium involves now a slight modification of the system described by equations (LR1) to (LR8) in section 2.1. Equations (LR1) and (LR6) are modified to:

$$C^i = e^i h^i w \tag{LR1~}$$

$$N^K L = \frac{\alpha b}{\alpha-1} - \Theta \left[\frac{\nu}{we^i}\right]^{\frac{\nu}{1-\nu}} \quad . \tag{LR6~}$$

A new equation is also added to determine the hours worked per worker:

$$h^i = 1 - \left[\frac{\nu}{we^i}\right]^{\frac{\nu}{1-\nu}} \tag{LR12}$$

4.2 Minimum Wages

As before, with a minimum wage per unit of labour time equal to *y* in place, in each period there will be a level of ability ε_t for which it holds that $y = \varepsilon_t \varpi_t$. Workers with ability smaller than ε_t will be unemployed, thus the unemployment rate will be equal to $u_t = 1 - \left\{\frac{b}{\varepsilon_t}\right\}^{\alpha}$. The total number of effective units of labour possessed (and supplied) by those individuals with $e_i \geq \varepsilon_t$ will now be:

$$L_t^s = \int_{\varepsilon_t}^{\infty} e \cdot h_t^i \cdot \left\{\alpha \frac{b^\alpha}{e^{\alpha+1}}\right\} de.$$

Assuming that the rest of model of section 2.2 remains intact, noting that the after-tax wage *rate* per unit of labour time is $\varpi_t(1-\tau_t)$, and using the appropriately modified individual labour function by replacing w_t in equation (26) with $\varpi_t(1-\tau_t)$, we find that the aggregate labour supply of effective units of labour is:

$$L_t^s = \frac{\alpha\varepsilon_t}{\alpha-1}\left\{\frac{b}{\varepsilon_t}\right\}^{\alpha} - \Psi(\varpi_t(1-\tau_t))^{1/(\nu-1)}{\varepsilon_t}^{(\nu+\alpha(1-\nu))/(\nu-1)}, \quad (28)$$

where $\Psi \equiv \frac{\alpha b^{\alpha}(1-\nu)\nu^{1/(1-\nu)}}{\alpha(1-\nu)+\nu}$.

The long-run equilibrium involves now a slight modification of the system described by equations (LR1a) to (LR8a) plus (LR9), (LR10), and (LR11) in section 2.2. Equations (LR1a) and (LR6a) are modified to:

$$C^i = e^i h^i \varpi(1-\tau) \quad \text{(LR1a\textasciitilde)}$$

$$NL = \frac{\alpha\varepsilon}{\alpha-1}\left\{\frac{b}{\varepsilon}\right\}^{\alpha} - \Psi(\varpi(1-\tau))^{1/(\nu-1)}\varepsilon^{(\nu+\alpha(1-\nu))/(\nu-1)} \;. \quad \text{(LR6a\textasciitilde)}$$

A new equation is also added to determine the hours worked per worker:

$$h^i = 1 - \left[\frac{\nu}{\varpi(1-\tau)e^i}\right]^{\nu/(1-\nu)} \quad \text{(LR12a)}$$

Unlike the case of exogenous labour supply, it is now impossible to analytically compare the two cases, and for this reason we resort to numerical simulations.

4.3 Comparison

4.3.1 Baseline Values

Table 1 reports the baseline parameter values for policy, technology and preferences used to obtain the long-run values of the endogenous variables. The values for technology and preferences are similar to the ones used in macroeconomics (e.g. the business cycle literature), and they imply macroeconomic outcomes consistent with real world data the regarding the adjusted[11] wage (i.e. labour) share and entrepreneurial income[12], as well as empirical estimates of behavioural functions (e.g. the wage-elasticity of labour supply). Since

[11] The adjustment is made for the existence of self-employed persons.

[12] According to the Ameco database (accessed on September 14, 2015) the adjusted wage (i.e. labour) share was on average 58.2% of GDP for the (initial 12) euro area countries during 1991-2012 when evaluated at market prices, and 65.5% when evaluated at factor cost. According to Eurostat (accessed on September 14, 2015), for non-financial corporations, net entrepreneurial income (which is gross operating surplus plus all property income received minus interest and rents paid, and which approximates the concept of pre-tax corporate profits in business accounting) as a percentage of net value added was 32.7% on average for the same countries.

none of these parameter values are crucial for our comparison and are standard in the literature we provide no further discussion of their choice, and we briefly focus our attention on the rest of the assumed baseline parameter values.

In accordance with the relevant empirical studies (e.g. Creedy 1977) we set the baseline value of parameter α, which determines the shape of the Pareto distribution and is a measure of income inequality among workers, equal to 2, and its "extreme" value to 3. Parameter b, which stands for the lowest ability in the population of workers, can be chosen arbitrarily so that the model's equilibrium values of the endogenous variables match well with actual economies; we set it to 1.[13] We also assume that the number of capitalists/entrepreneurs as a share of the total population, $N/(1+N)$, is equal to 0.3. We report that the substance of our results is robust to plausible variations in the values assumed for all of the above parameters. As it will become clear below, the only parameter value that turns out to be crucial for our results is the value for parameter ϕ, which is the ratio of unemployment (or, social welfare) benefits to the minimum wage, which we initially set at 0.25.[14]

Table 1 here

4.3.2 Numerical solution and discussion of results

Using the baseline parameter values of Table 1, the long-run equilibrium values for the PC and the MW economies described earlier are presented in Tables 2-3.

In these Tables the results are displayed as follows. The PC column represents the long-run values of the perfectly competitive, full-employment case, whereas the MW columns represent the outcomes if a minimum wage (per unit of time) is imposed which is set higher than the wage (per unit of time) which the worker with the lowest ability would receive in the PC case; i.e. $y = (1+\lambda)(bw) = (1+\lambda)w, \lambda > 0$. We focus on the case that $\lambda = 0.10$ – which implies a "modest" 10% increase in the wage which the worker with the lowest ability would receive in the PC case.[15]

[13] This is just a normalization; different values of *b* would not affect the qualitative nature of the results.

[14] It is clear that the generosity of unemployment benefit schemes differs a lot among countries. Moreover, it should be remembered that in this model the granting of these benefits has an indefinite duration; in this sense, it should be compared more with the social assistance provided to individuals whose eligibility for unemployment benefits has expired, or those who have never fulfilled the eligibility criteria for receiving them.

[15] This would be the case if the minimum-ability worker remained in employment.

The imposition of a minimum wage in an otherwise perfectly competitive economy implies a cost in terms of aggregate (and, per-capita) output and consumption in the long run. (For brevity, we do not report the results for these variables; we report, however, the outcomes for the amounts of capital and labour employed in the economy.) This arises since with a binding minimum wage (per unit of time) firms will not be willing to employ the lowest ability workers, but only those whose ability times the market-determined wage *rate* (per effective unit of labour), ϖ, is larger than the institutionally set minimum wage. The resulting exclusion from employment of some low-ability workers decreases the aggregate units of effective labour employed, increases, at the initial capital stock, the marginal product of (effective) labour, and the wage *rate*, ϖ, per effective unit of labour. These developments lead to a lower capital stock in the long run, which, in combination, with the smaller labour input lead to lower aggregate output, and a lower level of aggregate profits.

As can be seen from the first two columns of Table 2, the imposition of a minimum wage per unit of time succeeds in raising the wage *rate* from w=0.406 in the PC case to $\varpi = 0.411$ under the MW regime, i.e. a rise in the wage *rate* relative to the PC case by 1.2%. The significantly smaller percentage rise in the (pre-tax) wage *rate* than the 10% minimum-wage premium over the competitive wage (per unit of time) is mostly due to the reduction in the steady-state capital stock[16] under the MW regime brought about by the reduced incentives for capital accumulation generated by the higher cost of labour and the taxes needed to support the unemployed. As a result, all workers remaining in employment become worse-off, since their (after-tax) wage *rate*, hours of work, income and consumption decrease relative to the PC case; e.g. the consumption of the worker whose ability is equal to the threshold ability level ($\varepsilon = 1.087$) decreases from 0.277 in the PC case to 0.276 in the MW case. (Since the tax rate is linear in income (and ability), the percentage decline in after-tax income is the same for all workers remaining in employment.) The corresponding decrease in the consumption of all workers below the threshold level of ability – i.e. those that become unemployed after the imposition of the MW – is very large; the consumption of the lowest ability worker ($e = b = 1$) drops from 0.238 in the PC case to 0.112 in the MW case – a reduction by 52.9%.[17] As expected, capitalists are also harmed by the imposition of the minimum wage – their consumption in the long-run drops by 6.2% relative to the PC

[16] The reduction in the capital stock is equal to 6.2% relative to the PC case (from 5.202 to 4.880).
[17] The decline is even larger for the workers who have ability levels $e \in [b, \varepsilon)$.

case. Thus, the imposition of a "moderately binding"[18] minimum wage ($\lambda = 0.10$), when combined with "moderate" support for the unemployed ($\phi = 0.25$), generates decreases in income and consumption of all groups (employed and unemployed workers, and capitalists) in the economy.[19] Table 2 shows also that despite the small reduction in hours worked (i.e. increase in leisure) under the MW regime, the utility of the worker with the threshold level of ability ($e = \varepsilon$) is reduced – due to the decline in her after-tax income and consumption. We report also that for values of $\phi > 0.25$, we have not been able to find any combination of plausible parameter values for which the MW regime offers higher income or utility to employed workers.[20]

The role of the generosity of social insurance is demonstrated in Table 2 by examining a less generous social welfare system ($\phi = 0.1$). This allows for a rise in the after-tax wage *rate* from 0.406 under PC to 0.409 ($\varpi(1-\tau) = (0.412)(1-0.005) = 0.409$) under the MW regime, and a rise in hours worked and in consumption of all employed workers; utility for all employed workers increases as well. These effects are a consequence of the smaller rise in the tax rate that the reduced need for unemployment benefits generates. It also demonstrates that even under decreasing returns to scale, the transfer of resources from capitalists to workers will not be enough to generate an increase in employed workers' income unless the unemployed workers which the minimum wage generates are left with meagre welfare support. These results are understandable since our framework involves a dynamically efficient economy - the interest rate is positive in equilibrium and there is no population growth – and it is well known that in such settings capital and labour income taxation involve efficiency losses in the form of lower aggregate consumption. Thus, any

[18] It is not clear how big is the difference between existing minimum wages (per unit of time) and the hypothetical wage (per unit of time) that the lowest ability worker would earn in a perfectly competitive market. For one thing, no actual labour market can be considered as perfectly competitive even in the absence of a national minimum wage. Nevertheless, a rough approximation may be available if one looks at the German (non-union) low-wage sector. For example, Bosch and Kalina (2008) report that in 2006, 1.9 million workers (about 6.5% of the workforce) were earning less than €5 per hour; press reports indicate that some workers were earning substantially less than that – see, e.g. http://www.reuters.com/article/2012/02/08/us-germany-jobs-idUSTRE8170P120120208). Given the small increases in wages in Germany since 2006, the €8.5 per hour minimum wage will certainly be far larger than the 10% on the *lowest-ability* workers assumed here, and for this reason we put the adjective *moderate* on the 10% premium.

[19] We have also verified that larger increases in the minimum wage will not succeed in raising the incomes (or utility) of any group in the population as long as the unemployed receive at least "moderate" support ($\phi \geq 0.25$).

[20] Obviously if welfare support were exceedingly generous so that the utility of unemployed workers was not be far below the utility of the lowest-ability employed worker, it is possible that the utility of unemployed workers would be higher than what they could attain in the PC case due to increased leisure. However, employed workers would have been far worse off in this case, and it is hard to envisage that such a policy would be politically viable since both employed workers would oppose it – see Economides and Moutos (2014) for an analysis focusing on how tax structure affects the political support for minimum wages.

increases in consumption of employed workers due to minimum wages must be associated with decreases in the consumption of other groups in the population.[21]

Table 2 here

In Table 3 we examine the potential influence on our results from modifying the assumed parameter values. We first show (first two columns) that, for the assumed baseline parameter values and under constant returns to scale $(\eta + \gamma = 1)$, there will be a decrease in the incomes (or utility) of employed workers from the imposition of a minimum wage even if very small welfare support is provided to the unemployed $(\phi = 0.1)$.[22] The decrease is driven by a fall in the (pre-tax) wage rate relative to the PC case (from 0.900 to 0.897). This fall must be contrasted with the rise in the wage rate observed in Table 2, and is associated with the far larger drop in the capital stock under constant returns to scale (9.9%) than the one observed in the first two columns of Table 2. In the next two columns we show that this result remains intact as we change parameter γ which governs the value of the wage elasticity of labour supply.[23] Finally, in the last two columns we display the effect of changes in the degree of heterogeneity in ability among workers, by setting $\alpha = 3$, which is above most estimates of this parameter; again the essence of our findings remains intact.[24]

Table 3 here

We report that the qualitative nature of the conclusions we have drawn from Tables 2 and 3 remain intact to the extensive experimentation we have conducted with various combinations of plausible parameter values. The nature of our results remains also unchanged to variations in the percentage by which the imposed minimum wage exceeds the perfectly competitive one as measured by parameter λ, i.e. by making the minimum wage more or less "binding".

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[21] We wish to thank an anonymous referee for bringing this feature of our framework to our attention.

[22] Note that if the unemployed receive no welfare support $(\phi = 0)$, then the income of employed workers is the same under the PC and MW regimes (since also $\tau = 0$).

[23] The value of the elasticity is not constant, and in addition to ν, depends also on worker ability. For the values in Table 2, when ν =0.2, the value of the elasticity for the worker with the threshold level of ability is close to 0.75. Raising the value of $\nu\ to$ 0.5 pushes the wage-elasticity of labour supply towards 1, which tends to be the upper value for most empirical estimates (see Bargain et al. 2012, for a survey of existing estimates of labour supply elasticities). We report that further increases in the value of ν produces no changes in the nature of our results.

[24] Lowering the value of α would also have no influence on the nature of our results. Note also that the relationship between parameter α and the Gini coefficient, G, is $G = 1/(2\alpha - 1)$, implying that when $\alpha = 2$, $G = 0.33$ (which is close to observed estimates for income inequality), and when $\alpha = 3$, $G = 0.2$ (implying a level of income inequality far lower than the estimated levels).

Before closing this section we discuss briefly whether minimum wages can provide a useful insurance mechanism. As mentioned by an anonymous referee, there is uninsured risk in our framework since workers face (under an ex-ante "veil of ignorance") the risk of being a low-ability type. The question then arises as to whether the imposition of minimum wages can play a useful insurance role in our framework; after all, the presumed role of the minimum wage institution is to redistribute earnings to low-ability, low-paid workers. In principle, even if the imposition of a binding minimum wage results in a fall in the after-tax earnings of employed workers, the politico-economic equilibrium could be such that the unemployment benefits received by workers of very low ability (i.e. those with $e < \varepsilon$) are at a level that under an ex-ante veil of ignorance (regarding the ability level) the expected (after-tax) income (or, utility) of a worker is higher than under the perfectly competitive case (e.g. Adam and Moutos 2006). It is also possible that the variance in incomes between being employed and being unemployed is reduced under the minimum wage regime so that, given the concavity of the utility function and the implied risk aversion, expected utility increases under the minimum wage regime even if expected income is lower. Although the above could be possible outcomes, we can report that after a wide experimentation with plausible parameter values we have been unable to uncover such cases, i.e. we find that the (ex-ante) expected utility under a veil of ignorance is lower under the minimum wage regime. This is the case for all models examined in this paper, even when the extra leisure enjoyed by unemployed individuals is taken into account. However, we note that the apparent (i.e. on the basis of the wide range of plausible parameter values we have examined) absence of an insurance role for minimum wages in our framework does not imply that such a mechanism may not be present in models with heterogeneous workers performing different tasks, e.g. skilled and unskilled workers, or in models with monopsonistic labour markets (e.g. Jones 1987, Cahuc et al. 2001). In such cases it is possible that the imposition of a minimum wage for unskilled workers can lead to a reduction in the skill premium –thus reducing the ex-ante variance of incomes, or it can result in employment increases, thus securing an increase in ex-ante expected utility.

5. A Ramsey-Type Framework

We now allow workers the possibility of saving by constructing a model that is similar to the one presented in Section 2, with one exception. Instead of two distinct groups of agents (workers and capitalists), there is only one group of agents who work, consume, save, and own the firms and the capital stock. These agents have identical preferences, differ in ability, have the same endowment of *time units* at their disposal (normalized to one), and supply inelastically their endowment of effective labour units. (The case of endogenous labour supply is discussed further below.)

5.1 The Perfectly Competitive Case

Agents' preferences are:

$$U^i = \sum_{t=0}^{\infty} \beta^t lnc_t^i \,, \tag{29}$$

whereas their budget constraint is,

$$c_t^i + k_{t+1}^i = r_t k_t^i + \mathrm{d}_t^i + w_t e^i \tag{30}$$

In equation (29), c_t^i stands for the consumption of each agent. In equation (30), $k_t^i, r_t k_t^i, d_t^i$ and $w_t e^i$ stand for the capital stock, capital income, profits and labour earnings accruing to each agent, where the latter are equal to worker's ability times the wage per *effective* unit of labour.[25]

Each agent solves the following programme:

$$\max_{C_t^i, K_{t+1}^i} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ lnc_t^i + \lambda_t (r_t k_t^i + \mathrm{d}_t^i + w_t e^i - c_t^i - k_{t+1}^i) \right\}$$

The resulting first-order conditions with respect to C_t^i and K_{t+1}^i combined give:

$$\frac{1}{c_t^i} = \beta r_{t+1} \frac{1}{c_{t+1}^i} \tag{31}$$

Equations (30) and (31) summarize the optimal behavior of each agent.

The equations pertaining to technology and firms' decisions are identical to the relevant ones in Section 2.1.3 (equations (8)-(11)).

[25] For simplicity, we assume that there is full capital depreciation. This is not important to our results.

5.1.1 *Factor Market Equilibrium*

Continuing to assume that ability is distributed according to the Pareto distribution, labour market equilibrium obtains when the aggregate demand for labour by the N firms is equal to aggregate labour supply, i.e. $NL_t^f = \frac{\alpha b}{\alpha - 1}$, which is equation (13) in Section 2.1.4. Equilibrium in the capital market obtains when the total supply of capital – as provided by the agents – is equal to the demand for capital by firms, i.e. when

$$\int_b^\infty k_t^i \left\{\alpha \frac{b^\alpha}{e^{\alpha+1}}\right\} de = NK_t^f \tag{32}$$

Finally, total dividends must be equal to total profits:

$$\int_b^\infty d_t^i \left\{\alpha \frac{b^\alpha}{e^{\alpha+1}}\right\} de = N\Pi_t \tag{33}$$

5.1.2 *Long-run General Equilibrium*

In the long-run, equation (31) becomes:

$$r = 1/\beta \tag{34}$$

Then, equations (10), (13) and (34) combined give:

$$K^f = (\beta\gamma)^{\frac{1}{1-\gamma}} \left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}} \tag{35}$$

In turn, equation (9), using equations (13) and (35), gives:

$$w = \eta(\beta\gamma)^{\frac{\gamma}{1-\gamma}} \left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta+\gamma-1}{1-\gamma}} \tag{36}$$

Now, if we plug equation (35) into equation (32), we get:

$$\int_b^\infty k^i \left\{\alpha \frac{b^\alpha}{e^{\alpha+1}}\right\} de = N(\beta\gamma)^{\frac{1}{1-\gamma}} \left(\frac{\alpha b}{N^f(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}} \tag{37}$$

To solve equation (37) for k^i, we proceed by as follows. We guess that if equation (37) (solved for k^i) has a well-defined solution, this is a function of $(e^i)^\eta$.[26] Then, it is easy to show that:

$$k^i = \frac{N(\beta\gamma)^{\frac{1}{1-\gamma}}\left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}}(\alpha-\eta)}{\alpha b^\eta}(e^i)^\eta \tag{38}$$

Repeating the same solution procedure for the dividends, and assuming that they are a function of $(e^i)^\eta$, we can show that:

$$d^i = \frac{N(1-\gamma-\eta)(\beta\gamma)^{\frac{\gamma}{1-\gamma}}\left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}}(\alpha-\eta)}{\alpha b^\eta}(e^i)^\eta \tag{39}$$

Finally, equations (30), (36), (38) and (39), imply:

$$c^i = (\beta^{-1}-1)\frac{N(\beta\gamma)^{\frac{1}{1-\gamma}}\left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}}(\alpha-\eta)}{\alpha b^\eta}\left(e^i\right)^\eta + \frac{N(1-\gamma-\eta)(\beta\gamma)^{\frac{\gamma}{1-\gamma}}\left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}}(\alpha-\eta)}{\alpha b^\eta}(e^i)^\eta + \eta(\beta\gamma)^{\frac{\gamma}{1-\gamma}}\left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta+\gamma-1}{1-\gamma}}e^i \tag{40}$$

Hence, the long-run general equilibrium is summarized by the following equations:

$$K^f = (\beta\gamma)^{\frac{1}{1-\gamma}}\left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}} \tag{LR1^}$$

$$L_t^f = \frac{\alpha b}{N(\alpha-1)} \tag{LR2^}$$

$$Y = (\beta\gamma)^{\frac{\gamma}{1-\gamma}}\left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}} \tag{LR3^}$$

$$r = \frac{1}{\beta} \tag{LR4^}$$

$$w = \eta(\beta\gamma)^{\frac{\gamma}{1-\gamma}}\left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta+\gamma-1}{1-\gamma}} \tag{LR5^}$$

$$k^i = \frac{N(\beta\gamma)^{\frac{1}{1-\gamma}}\left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}}(\alpha-\eta)}{\alpha b^\eta}(e^i)^\eta \tag{LR6^}$$

[26] The results derived are independent of the particular form of the guess function chosen as long as it is increasing in ability, e^i.

$$d^i = \frac{N(1-\gamma-\eta)(\beta\gamma)^{\frac{\gamma}{1-\gamma}}\left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}}(\alpha-\eta)}{\alpha b^\eta}(e^i)^\eta \tag{LR7^}$$

$$c^i = (\beta^{-1}-1)\frac{N(\beta\gamma)^{\frac{1}{1-\gamma}}\left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}}(\alpha-\eta)}{\alpha b^\eta}\left(e^i\right)^\eta$$

$$+\frac{N(1-\gamma-\eta)(\beta\gamma)^{\frac{\gamma}{1-\gamma}}\left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}}(\alpha-\eta)}{\alpha b^\eta}(e^i)^\eta + \eta(\beta\gamma)^{\frac{\gamma}{1-\gamma}}\left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta+\gamma-1}{1-\gamma}}e^i \tag{LR8^}$$

The above eight equations solve for $K^f, L^f, Y, r, w, k^i, d^i$ and c^i, respectively.

5.2 Minimum Wages

As in Section 2.2, we now assume the existence of a government-imposed minimum wage per unit of labour *time* (e.g. per hour) equal to y, which is the minimum amount that an employer must pay in order to employ one person. The equations describing the labour market are identical to the ones appearing in Section 2.2.1, i.e. equations (15)-(18).

5.2.1 Government

As in Section 2.2.2, the government levies a comprehensive income tax (τ) on all sources of income, in order to finance benefits for the low-ability workers that are unemployed. Here we make two assumptions: First, we assume that the average level of the unemployment benefit is a fixed proportion of the minimum wage, i.e. it is equal to ϕy, where $0 \leq \phi < 1$. Parameter ϕ describes the generosity of the unemployment benefit system. Second, we assume that each unemployed agent receives an unemployment benefit which is a fraction $\varphi(e^i)$ of the minimum wage, $\varphi(e^i)y$, but that the value of the fraction depends on her ability (below we will assume that $\varphi(e^i)$ is a linear function of e^i).[27]

Equation (41) below, i.e. the government budget constraint, just states that the net payments to the unemployed are equal to total tax receipts:

$$\phi y_t u_t = \tau_t N Y_t. \tag{41}$$

In addition to this equation we impose the condition that the (differentiated according to ability) unemployment benefits received by each of the unemployed agents must in aggregate be equal to the value of unemployment benefits disbursed by the government:

[27] We wish to thank an anonymous referee for suggesting this to us.

$$\int_b^{\varepsilon} \varphi(e^i) y \left\{\alpha \frac{b^{\alpha}}{e^{\alpha+1}}\right\} de = \phi y_t u_t \tag{42}$$

We assume that τ_t adjusts in every period so as to keep the budget in balance.

5.2.2 *Long-run General Equilibrium with minimum wages*

The existence of taxes implies that equations (30), and (31) must now be modified to:

$$c_t^i + k_{t+1}^i = (1-\tau_t)(r_t k_t^i + \mathrm{d}_t^i + \varpi_t e^i) \tag{43}$$

$$\frac{1}{c_t^i} = \beta(1-\tau_{t+1}) r_{t+1} \frac{1}{c_{t+1}^i} \tag{44}$$

These two equations along with equations (32), (9), (10), (15)-(18), (41), and (42) describe the dynamic evolution of the economy with minimum wages. The long-run equilibrium of this economy is described by the following equations:

$$K^f = (1-\tau)^{\frac{1}{1-\gamma}} (\beta\gamma)^{\frac{1}{1-\gamma}} \left(\frac{\alpha b^{\alpha} \varepsilon^{1-\alpha}}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}} \qquad \text{(LR1^)'}$$

$$L^f = \frac{\alpha b^{\alpha} \varepsilon^{1-\alpha}}{N(\alpha-1)} \qquad \text{(LR2^)'}$$

$$Y = (1-\tau)^{\frac{\gamma}{1-\gamma}} (\beta\gamma)^{\frac{\gamma}{1-\gamma}} \left(\frac{\alpha b^{\alpha} \varepsilon^{1-\alpha}}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}} \qquad \text{(LR3^)'}$$

$$r = \frac{1}{\beta(1-\tau)} \qquad \text{(LR4^)'}$$

$$\varpi = \eta(1-\tau)^{\frac{\gamma}{1-\gamma}} (\beta\gamma)^{\frac{\gamma}{1-\gamma}} \left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta+\gamma-1}{1-\gamma}} \left(\frac{b}{\varepsilon}\right)^{\frac{(\alpha-1)(\gamma+\eta-1)}{1-\gamma}} \qquad \text{(LR5^)'}$$

$$k^i = \frac{N(1-\tau)^{\frac{1}{1-\gamma}} (\beta\gamma)^{\frac{1}{1-\gamma}} \left(\frac{\alpha b^{\alpha} \varepsilon^{1-\alpha}}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}} (\alpha-\eta)}{\alpha b^{\eta}} (e^i)^{\eta} \qquad \text{(LR6^)'}$$

$$d^i = \frac{N(1-\gamma-\eta)(1-\tau)^{\frac{\gamma}{1-\gamma}} (\beta\gamma)^{\frac{\gamma}{1-\gamma}} \left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}} \left(\frac{b}{\varepsilon}\right)^{\frac{\eta(\alpha-1)}{1-\gamma}} (\alpha-\eta)}{\alpha b^{\eta}} (e^i)^{\eta} \qquad \text{(LR7^)'}$$

$$c^i =$$

$$(\beta^{-1}-1) \frac{N(1-\tau)^{\frac{1}{1-\gamma}} (\beta\gamma)^{\frac{1}{1-\gamma}} \left(\frac{\alpha b^{\alpha} \varepsilon^{1-\alpha}}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}} (\alpha-\eta)}{\alpha b^{\eta}} (e^i)^{\eta} +$$

$$+ \frac{N(1-\gamma-\eta)(1-\tau)^{\frac{\gamma}{1-\gamma}} (\beta\gamma)^{\frac{\gamma}{1-\gamma}} \left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}} \left(\frac{b}{\varepsilon}\right)^{\frac{\eta(\alpha-1)}{1-\gamma}} (\alpha-\eta)}{\alpha b^{\eta}} (e^i)^{\eta} +$$

$$+\eta(1-\tau)^{\frac{\gamma}{1-\gamma}} (\beta\gamma)^{\frac{\gamma}{1-\gamma}} \left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta+\gamma-1}{1-\gamma}} \left(\frac{b}{\varepsilon}\right)^{\frac{(\alpha-1)(\gamma+\eta-1)}{1-\gamma}} e^i \qquad \text{(LR8^)'}$$

$$u = 1 - \left\{\frac{b}{\varepsilon}\right\}^{\alpha} \tag{LR9^}$$

$$\phi y u = \tau N Y \tag{LR10^}$$

Finally, assuming that $\varphi(e^i)$ is a linear function of e^i, we are able to show that:

$$\varphi(e^i) = \frac{\phi u(\alpha-1)}{\alpha b\left[1-\left\{\frac{b}{\varepsilon}\right\}^{\alpha-1}\right]} e^i \tag{LR11^}$$

Then, the level of consumption for an unemployed agent ($e < \varepsilon$) is:

$$\begin{aligned} c^{u,i} = {} & \\ (\beta^{-1}-1) & \frac{N(1-\tau)^{\frac{1}{1-\gamma}}(\beta\gamma)^{\frac{1}{1-\gamma}}\left(\frac{\alpha b^{\alpha}\varepsilon^{1-\alpha}}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}}(\alpha-\eta)}{\alpha b^{\eta}}(e^i)^{\eta} + \\ & + \frac{N(1-\gamma-\eta)(1-\tau)^{\frac{\gamma}{1-\gamma}}(\beta\gamma)^{\frac{\gamma}{1-\gamma}}\left(\frac{\alpha b}{N(\alpha-1)}\right)^{\frac{\eta}{1-\gamma}}\left(\frac{b}{\varepsilon}\right)^{\frac{\eta(\alpha-1)}{1-\gamma}}(\alpha-\eta)}{\alpha b^{\eta}}(e^i)^{\eta} + \\ & + \frac{\phi u(\alpha-1)}{\alpha b\left[1-\left\{\frac{b}{\varepsilon}\right\}^{\alpha-1}\right]} e^i y \end{aligned} \tag{LR12^}$$

Notice that equation (LR8^)' gives the level of consumption for each one of the employed agents ($e \geq \varepsilon$).

5.3 Comparison

Careful inspection of the long-run equilibrium relationships under the perfectly competitive (PC) and the minimum wage (MW) regime reveals that:

(a) The capital income for all agents ($e \geq b$) is always higher under the PC regime (relative to the MW regime). This can be easily ascertained by comparing (LR6^) to (LR6^)'.

(b) Dividend income for all agents is always higher under the PC regime. This can be easily seen by comparing (LR7^) to (LR7^)'.

(c) Regarding the labour income of agents that remain employed under the MW regime ($e \geq \varepsilon$), the comparison does not lead to unambiguous results. In particular, using (LR5^) and (LR5^)' we can show that in order for the after-tax labour income under the MW regime to be higher relative to the perfectly competitive case the following condition must hold:

$1-\tau > [b/\varepsilon]^{(\alpha-1)(1-\eta-\gamma)}$.

This condition is exactly the same condition as the one uncovered in Section 2 (equation (23)).

The upshot of these results is that if there are either constant returns to scale or all agents have the same ability, the *total* income[28] (the sum of labour income, capital income, and dividends) of agents that are employed under both regimes ($e \geq b$) is always higher in the PC regime. Thus, the proposition stated in Section 3 holds here as well.

In the presence of decreasing returns to scale and ability heterogeneity, (after-tax) labour income may be higher in the MW regime (if the above condition holds). However, since capital income and dividend income is higher in the PC regime, it is possible for *total* income to be higher under the MW regime only if the extra labour income they may receive (relative to the PC regime) outweighs their losses from capital income and dividends. Although this possibility cannot be excluded analytically, we have not been able to find any combination of plausible parameter values for which total income of employed workers is higher in the MW regime (even if the unemployed receive no welfare support).

Table 4 depicts a case – using the previously mentioned parameter values and assuming that $\phi = 0.1$ – in which even though labour income is higher under the MW regime, the total income of employed workers is higher under the PC regime.[29] One way to understand this finding is by noting that workers can no longer extract a larger share of output without hurting themselves. This is because a rise in the wage *rate* earned by workers reduces the profits of the firms and the dividends received by the workers who are the sole owners of the firms. In this framework the inefficiency introduced by the minimum wage hurts the workers as there is no other group (i.e. the capitalists) on which the cost of inefficiency can be transferred to and from which workers can extract a larger share of output. The presence of another economic class is thus necessary for (employed) workers to be able to be made better-off from the imposition of minimum wages.

[28] We focus on total income since there is a monotonic correspondence between total income, consumption, and utility in the model.

[29] The after- tax wage rate per effective unit of labour is $0.1837(1-0.04) = 0.1829$ under MW and 0.1801 under PC. Yet, the total income and consumption for the agent with ability $e = \varepsilon$, is higher under PC, due to the much higher capital and dividend income this agent receives in the PC regime. The same holds true and for all agents with ability higher than ε.

Regarding the total income of agents that become unemployed under the MW regime ($e < \varepsilon$), we know that their capital and dividend income will be lower in the MW regime. In principle, an exceedingly generous welfare regime could provide some of them (i.e. those with the highest ability among the unemployed) with unemployment benefits above the wage income they would earn under the PC regime, so that their total income (sum of unemployment benefits, capital income, and dividends) in the MW regime. Although we have not been able to exclude such a situation analytically, we report that we have not been able to find any combination of plausible parameter values for which such a counterintuitive case would arise.

Table 4 here

We close this section by reporting that the qualitative nature of our results remains intact if we extend the model by allowing for endogenous labour supply. However, in that case it is not possible to derive any analytical results and hence our solutions are only numerical. The algebraic details of the model with endogenous labour supply, as well as the associated numerical solutions, are available upon request.

6. Conclusion

Our demonstration that it is impossible for *any* binding minimum wage to increase the after-tax incomes of workers if either the production function is Cobb-Douglas with constant returns to scale, or if there are no differences in ability among workers, is a direct application of the Chamley–Judd result.[30] This result states that any linear tax on capital will reduce the aggregate income received by workers by more than the revenue raised by the tax. By introducing heterogeneity in ability among workers, the present paper has shown that not only aggregate workers' income will decline following the imposition of a minimum wage, but also the incomes of workers remaining in employment as long as they are called to share in the cost of financing social welfare benefits for the less able who become unemployed. Nevertheless, the paper suggests that the joint existence of decreasing returns to scale and worker heterogeneity can allow a binding minimum wage to increase the incomes of workers who remain employed, but only at the expense of those who become unemployed (see Knabe

[30] See Chamley (1986), and Judd (1985).

and Schöb, 2009, for an empirical analysis of the expected impact on unemployed workers' incomes in the German context).

We find that under decreasing returns to scale the main determinant of whether employed workers can increase their incomes and utility in the long run from the imposition of a minimum wage is the generosity of welfare support provided to the unemployed. Our numerical simulations indicate that even when the unemployed receive as little as 25% of the income they would earn under the perfectly competitive regime, the employed workers' incomes and utility would be lower under the minimum wage regime. Moreover, using a Ramsey-type framework we have established that the existence of a separate class of agents (i.e. capitalists) is necessary for minimum wages to lead to increases in the incomes of employed workers even when there are decreasing returns to scale and no welfare support is provided to the unemployed.

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Table 1
Baseline parameterization

Parameters	Description	Value
γ	capital's exponent in production function	0.3
η	labour's exponent in production function	0.5
δ	capital's depreciation rate	0.08
β	rate of time preference	0.9
ν	preference parameter on leisure in utility function	0.2
N^k	number of capitalists	0.4
ϕ	parameter describing the generosity of the unemployment benefit system	0.1 or 0.25
α	shape parameter of the Pareto distribution	2
b	ability of worker with the lowest ability	1
σ	intertemporal substitution parameter in utility function	2
λ	parameter describing how much higher the minimum wage is set relative to the wage which the worker with the lowest ability would receive in the PC case	0.1

Table 2
Comparison of the PC and MW regimes under decreasing returns to scale

Variable	$\boldsymbol{\phi = 0.1}$		$\boldsymbol{\phi = 0.25}$	
	PC	**MW**	**PC**	**MW**
w	0.406	-	0.406	-
y	-	**0.446**	-	**0.446**
ϖ	-	0.412	-	0.411
ε	-	1.084	-	1.087
K	5.202	4.955	5.202	4.880
L	4.082	3.855	4.082	3.838
$C^L(e = b)$	0.238	0.044	0.238	0.112
$C^L(e = \varepsilon)$	0.276	0.280	0.277	0.276
$h(e = \varepsilon)$	0.627	0.631	0.628	0.627
$U^L(e = \varepsilon)$	-0.912	-0.910	-0.911	-0.912
C^K	1.241	1.182	1.241	1.164
u	0	0.150	0	0.154
τ	0	0.005	0	0.014

Notes: For the rest of the parameter values see Table 1**.** Numbers in bold imply exogenously set policy variables.

Table 3
Comparison of the PC and MW regimes under constant returns to scale
($\boldsymbol{\gamma = 0.35}$, $\boldsymbol{\eta = 0.65}$) and $\boldsymbol{\phi = 0.1}$

Variable	$\boldsymbol{\alpha = 2 , \nu = 0.2}$		$\boldsymbol{\alpha = 2 , \nu = 0.5}$		$\boldsymbol{\alpha = 3 , \nu = 0.2}$	
	PC	**MW**	**PC**	**MW**	**PC**	**MW**
w	0.900	-	0.900	-	0.9003	-
y	**-**	**0.990**	**-**	**0.990**	**-**	**0.990**
ϖ	-	0.897	-	0.897	-	0.892
ε	-	1.105	-	1.105	-	1.110
K	11.824	10.659	11.380	10.364	8.6199	6.869
L	4.661	4.252	4.486	4.135	3.3981	2.782
$C^L(e = b)$	0.763	0.099	0.623	0.099	0.7630	0.099
$C^L(e = \varepsilon)$	0.861	0.849	0.743	0.728	0.8660	0.838
$h(e = \varepsilon)$	0.865	0.863	0.747	0.741	0.8662	0.862
$U^L(e = \varepsilon)$	-0.654	-0.658	-0.803	-0.808	-0.6516	-0.662
C^K	1.314	1.184	1.264	1.152	0.9578	0.763
u	0	0.18	0	0.18	0	0.270
τ	0	0.01	0	0.01	0	0.02

Notes: For the rest of the parameter values see Table 1**.** Numbers in bold imply exogenously set policy variables.

Table 4
Comparison of the PC and MW regimes for the Ramsey-type economy under decreasing returns to scale ($\phi = 0.1$)

Variable	PC	MW
w	0.1801	-
y	**-**	**0.1981**
ϖ	**-**	0.1837
ε	**-**	1.0783
$c(e = b)$	0.3044	0.1406
$c(e = \varepsilon)$	0.3233	0.3189
$c(e = 1.07)$	0.3213	0.1419
$k(e = b)$	0.1459	0.1374
$k(e = \varepsilon)$	0.1515	0.1427
$k(e = 1.07)$	0.1509	0.1422
Capital income $(e = b)$	0.0162	0.0153
Capital income $(e = \varepsilon)$	0.0168	0.0159
Capital income $(e = 1.07)$	0.0167	0.0158
Dividends $(e = b)$	0.1081	0.1022
Dividends $(e = \varepsilon)$	0.1122	0.1062
Dividends $(e = 1.07)$	0.1118	0.1057
Labour income $(e = b)$	0.1801	-
Labour income $(e = \varepsilon)$	0.1942	0.1973
Labour income $(e = 1.07)$	0.1927	-
Unemployment benefit $(e = b)$	**-**	0.0191
Unemployment benefit $(e = 1.07)$	**-**	0.0204
$U(e = b)$	-1.1894	-1.9619
$U(e = \varepsilon)$	-1.1292	-1.1429
$U(e = 1.07)$	-1.1354	-1.9525
u	0	0.14
τ	0	0.004

Notes: For the parameter values see Table 1. Numbers in bold imply exogenously set policy variables. Notice that agents with ability $b = 1 < \varepsilon = 1.0783$, as well as those with $e = 1.07 < \varepsilon = 1.0783$, are unemployed in the MW regime. We present results for both of them since the benefits received depend on ability. This in turn affects the level of their total income, consumption, and utility (U).