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Minimum Wages and Relational Contracts

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Abstract

The need to give incentives is usually absent in the literature on minimum wages. However, especially in the service sector it is important how well a job is done, and employees must be incentivized to perform accordingly. Furthermore, many aspects regarding service quality cannot be verified, which implies that relational contracts have to be used to provide incentives. The present article shows that in this case, a minimum wage increases implemented effort, i.e., realized service quality, as well as the efficiency of an employment relationship. Hence, it can be explained why productivity and service quality went up after the introduction of the British National Minimum Wage, and that this might actually have caused a more efficient labor market. Furthermore, if workers have low bargaining power, a higher minimum wage also increases firm profits and consequently employment. Therefore, the present article presents a new perspective on reasons for why minimum wages often have no or only negligible employment effects.

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1   Introduction

Minimum wage laws and its positive or negative effects are one of the most contro-
versially debated issues in economics. When trying to understand its consequences,
though, only limited attention has been paid to how a minimum wage affects the
provision of incentives.

In this article, I show that a minimum wage has a crucial impact on a firm´s
optimal choice of incentives and consequently on efficiency, profits and employment.
In a dynamic setting where wages are determined by a bargaining process between
firms and workers, firms will require their workers to do a better job in return for
high wage payments induced by a binding minimum wage. Given performance is
not verifiable, an appropriate minimum wage can then increase the surplus of an
employment relationship. If a worker's bargaining power is low, a binding mini-
mum wage will furthermore increase profits and consequently also employment –
because the minimum wage serves as a commitment device to pay higher wages in
the future. I therefore show that via the channel of increasing a worker's effective
bargaining power and implementable effort, a higher minimum wage can induce a
Pareto improvement yielding higher profits, worker rents and employment.

Minimum wages are especially relevant in the service sector.[1] There in particular
what matters is *how well* - and not only *that* - a job is done. The degree of service
quality provided by employees is important for customer satisfaction and will have
an impact on a firm's profits. Take employees of a fast food restaurant, who are
supposed to be friendly to customers and careful when preparing the food. A cleaner
can do a superficial job or clean everything thoroughly, and a nightwatchman might
be more or less attentive. Then, it is necessary to incentivize workers, and the
question arises how this should and can be done. As many aspects of service quality
are highly subjective and cannot be precisely measured, it will generally be difficult
to capture all relevant dimensions in an explicit, i.e., court enforceable, contract.
Hence, relational contracts are needed, which generally are used in settings where
employees need to be given incentives to perform a desired task, but where it is
impossible or at least very difficult to verify effort and output.

The present article analyzes the impact of a minimum wage on the optimal choice
of incentives within a relational contracting framework. A labor market with many
homogeneous firms and workers exists, where entry is costly for firms. In every pe-

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[1]As an example, in an overview on minimum wages in Canada, Sussman and Tabi (2004, p. 9)
state that "[a]lmost all minimum wage workers were employed in the service sector".

riod of an infinite horizon game, the terms of an employment relationship consisting of one firm and one worker are determined by a bargaining process between the two. As a result, each gets a fixed share of the resulting relationship surplus. In order to create some surplus, though, workers must exert effort, which they will only do if they believe to be sufficiently compensated. Since no formal contracts are feasible, their willingness to exert effort depends on the future rents they expect to achieve within the relationship, and these rents are increasing in their bargaining power. Low bargaining power of workers is hence associated with low effort and a low relationship surplus. Then, an (exogenous) increase of workers' bargaining power can even yield higher profits for firms – namely if the associated surplus increase more than offsets the higher rents workers collect. Firms are not able to imitate this effect and implement higher effort by promising workers a larger share of the future surplus – because they cannot credibly commit to pay workers more in the future.

A binding minimum wage is basically equivalent to an increase in a worker's bargaining power: Knowing they will earn higher rents in the future, workers are willing to increase effort today in order to keep their job. A binding minimum wage hence increases effort and generally the surplus of an employment relationship. If workers' bargaining power has initially been rather low (which seems to be true for many jobs where a minimum wage is relevant), a binding minimum wage can further increase profits and consequently – as the number of firms and hence jobs is determined by a zero-profit condition – also total employment.

There is evidence that a minimum wage increases productivity, and that this is driven by higher effort levels of employees. Galinda-Rueda and Pereira (2004) and Rizov and Croucher (2011) analyze the effects of the introduction of a National Minimum Wage in Britain in 1999 on labor productivity. Both find a positive and significant effect - in particular in the service sector. In addition, several surveys attempt to provide a better understanding of the specific channels that induced the observed increase in productivity. These surveys find that a substantial amount of firms responded to the minimum wage by inducing higher effort of workers or by providing higher service quality (Low Pay Commission, 2001, or Heyes and Gray, 2003).

Hirsch et.al (2011) show that these effects could also be observed in the US. They state that managers responded to a minimum wage increase with – inter alia – higher prices and higher performance standards. Furthermore, the workforce in their sample was supposed to improve service quality in order to become more productive, where managers in particular used approaches to boost the morale of employees.

Concerning the relationship between a minimum wage and profits (which in my setting is positive for high and negative for low values of worker bargaining power), Harasztosi and Lindner (2015) observe a non-negative effect. They show that this has been driven by a substantial increase in sales revenues, which indicates that their results are consistent with my story: An increased service quality might have allowed firms to charge higher prices and generate higher sales revenues. Moreover, the positive effect of a minimum wage on (net) sales was mainly observed in the service industry.

Finally, their empirical outcomes support my prediction that employment effects of a minimum wage should be driven by its impact on profits if relational contracts are relevant: Harasztosi and Lindner (2015) not only find that the minimum wage increase did not affect profits, but also observe no negative employment effects in the service industry.

Related Literature

An important and considerable amount of research deals with employment effects of minimum wages. The hypothesis derived from the standard textbook model of a labor market - that a binding minimum wage leads to job losses - is now seriously questioned. Empirical studies like Katz and Krueger (1992), Card and Krueger (1994), Machin and Manning (1994) and more recently Dube et al. (2010) or Harasztosi and Lindner (2015) suggest that the employment effect of a minimum wage is not necessarily negative and might even be slightly positive. Other articles (for overviews see Brown, 1999, or Neumark and Wascher, 2007) still claim that a minimum wage destroys jobs.

Several theoretical models have been developed to explain the observed patterns. Bhashkar and To (1999), for example, develop a model of monopsonistic competition where a minimum wage raises employment per firm but causes firms to exit the market. Generally rent-creating search frictions are used as an explanation for the seemingly counterintuitive outcome that a minimum wage does not necessarily destroy jobs (based on Burdett and Mortensen, 1998, see also Card and Krueger, 1995, Flinn, 2006, or Dube et al., forthcoming). These approaches, though, would generally predict a negative effect of a minimum wage on profits, which is not found by Harasztosi and Lindner (2015). There, a higher minimum wage neither has a negative effect on employment nor on profits in the service industry, a result my model can generate.

Furthermore, these articles abstract from incentives, which have been given almost no attention in the relevant literature. Exceptions are Kadan and Swinkels (2010, 2013) and Rebitzer and Taylor (1995). Kadan and Swinkels (2010, 2013) analyze the effect of a wage floor in a standard moral hazard setting. They show that a minimum wage generally has a negative impact on induced effort levels. Different from my setting, they assume that workers are risk averse, effort cannot be observed, and an explicit contract is feasible. Then, a higher wage floor (i.e. payments that have to be made for the lowest output realization) generally increases the marginal costs of inducing effort, reducing total incentives given to employees. However, the non-verfiability of certain activities will often render explicit contracts infeasible, especially in the service sector where minimum wage laws are particularly important. Rebitzer and Taylor (1995) develop an efficiency wage model where a minimum wage makes it easier for firms to prevent a given number of employees from shirking. Thereby, the authors can explain positive employment effects of a minimum wage, which however is also associated with a profit reduction[2]. Furthermore, they do not take the impact on a worker's productivity into account, which in my model is the driving factor of potential positive employment effects of a minimum wage.

Concluding, I derive a new potential driving force for positive employment effects of a minimum wage: It increases workers' effort and potentially has a positive impact on profits, namely when a firm's commitment to compensate workers is insufficient. The consequence that higher profits then imply more demand for workers naturally follows.

2 Model Setup

2.1 Environment and Production

The market I consider consists of a mass 1 of identical workers ("he") and a potentially infinite mass of identical firms ("it"). Principals and workers are risk neutral. The time horizon is infinite, time is discrete (with periods $t = 1, 2, ...$), and all players share a common discount factor $\delta \in (0, 1)$. To become active in the market, though, a firm has to pay one-time entry costs $k > 0$ (after which it can stay in the market forever). The decision whether to enter or not is made by each unactive firm

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[2]Georgiadis (2013) empirically analyzes how efficiency wages respond to a minimum wage. Although he finds evidence in favor of an efficiency wage model, he does not observe a negative effect on profitability, which a standard efficiency wage model would predict.

at the beginning of every period. The mass of active firms in period t is denoted by M_t.

In every period, all workers and all active firms either are part of a firm-worker match or not. The matching process for unmatched parties is random and frictionless and takes place after M_t has been determined: If $M_t < 1$ (i.e., there are more workers than firms), every *unmatched* firm is randomly matched with exactly one *unmatched* worker. Then, all firms are part of a match, while $1 - M_t$ workers remain unmatched. If $M_t > 1$, every *unmatched* worker is randomly matched with exactly one *unmatched* firm. In the following I focus on symmetric matches, hence omit indices.

Afterwards, every matched pair (which can either be a new match or one with longer tenure) starts a bargaining process, which is further described below and – if successful – determines a wage payment w_t from firm to worker, an effort level $\tilde{e}_t \geq 0$ the worker is supposed to choose, and intended continuation play. Workers then consume w_t and exert actual effort e_t, generating output $y_t = e_t\theta$, with $\theta > 0$. The size of θ can be a function of M_t, the number of firms active in the market, where I naturally assume $d\theta/dM \leq 0$, reflecting a negative effect of competition on an individual firm's revenues. While output is directly consumed by the firm, a worker faces effort costs $c(e_t)$, with $c(0) = c'(0) = 0$ and c', $c'' > 0$. Hence, first-best effort e^{FB} – maximizing the total per-period surplus in a match – is characterized by

$$\theta - c'(e^{FB}) = 0,$$

with $e^{FB}\theta - c(e^{FB}) > 0$. Note that I exclude the possibility to pay an additional bonus after effort has been exerted. I show below, in section 5.1, that this assumption is without loss of generality.

All unmatched players and those who are part of a match but where bargaining has failed consume their exogenous outside utilities in the respective period, and potentially re-enter the matching market in the subsequent period. For simplicity, I set all players's *exogenous* outside utility levels to zero (note that an unmatched player's *endogenous* reservation utility - which reflects the possibility of finding a match with a positive rent and is further described below - can be positive).

Therefore, a firm's per-period profit in period t given it is part of a match and given bargaining has been successful is $e_t\theta - w_t$, whereas it is equal to 0 otherwise. A worker's per-period utility given he is part of a match and given bargaining has been successful is $w_t - c(e_t)$, whereas it is equal to 0 otherwise.

Finally, all matches decide whether they want to remain matched for another

period, in which case they again start the bargaining process in period $t+1$. If any player decides to leave, both players re-enter the matching market in the subsequent period.

The timing within a period t is summarized in the following graph:

| Entry and matching | Bargaining | W supplies e_t | Separation decisions |
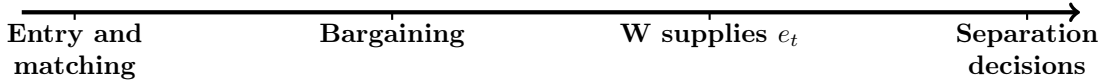
2.2 Observability and Contractibility

Effort as well as output can be observed by both, firm and worker, but not by anyone outside the respective match. Hence no verifiable measure of the agent's performance exists, and relational contracts must be used to provide incentives.

This also implies that effort in my setting should not be mistaken for working hours – those are verifiable and can hence be enforced with the use of formal contracts. Effort rather reflects issues like an employee's motivation or provided service quality. Therefore, my model arguably mainly applies to the service sector where it seems very difficult to pin down aspects like provided service quality in formal, court-enforceable contracts.[3]

2.3 Bargaining

Within a match, wages and intended effort levels, but also planned continuation play, are determined by a bargaining process between firm and worker. There, I dispense with a full formal description of the bargaining process but assume that players arrive at a Generalized Nash bargaining outcome. As for example, Ramey and Watson (1997) and Den Haan et.al (1999), I hence analyze a somehow hybrid model which is non-cooperative at heart but where the bargaining outcome is cooperative.[4]

The bargaining outcome distributes the net value created within a given match (in the current as well as all future periods) according to fixed shares, where the worker pockets $\alpha \in [0, 1]$ and the firm $1 - \alpha$ of this value. The share α reflects respective bargaining powers and is determined by the number of firms active in

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[3]I further discuss this statement in the conclusion.

[4]Using an axiomatic approach in a related setting, Miller and Watson (2013) show that this will actually be the outcome in a purely non-cooperative environment.

the market, with $d\alpha/dM \geq 0$. The net value is defined as the difference between payoffs after an agreement has been reached and payoffs under disagreement, i.e. after bargaining has failed.

An agreement is sealed by a formal short-term contract. This contract states that firm and worker have formed an employment relationship in the respective period and determines a wage payment w_t.[5] Furthermore, the assumption that players arrive at a Generalized Nash bargaining outcome implies that players never agree on a Pareto-dominated outcome.

Under disagreement, no contract is signed for the respective period, no payment is made and no effort exerted. I furthermore assume that the match breaks up, both players consume their exogenous outside options of zero and have to wait until the next period before they can re-enter the matching market.

Note that I exclude the possibility of bargaining at later points in time within a period, for example after the agent has exerted effort. In section 5.2, I explore the implications of having another bargaining round.

2.4   Strategies and Equilibrium Concept

Strategies are assumed to be *contract-specific* in the sense of Board and Meyer-ter-Vehn (2015). This implies that actions of firms and workers, as well as bargaining and disagreement outcomes, do not depend on the identity of the worker, calendar time, or history outside the current relationship.

Following Milller and Watson (2013) and extending it to a setting with many players, the equilibrium concept I apply is a so-called *contractual social equilibrium*. This concept describes a subgame perfect equilibrium (SPE), which is restricted by the assumptions that strategies are contract-specific, disagreement triggers a separation, and that agreement does not involve Pareto dominated outcomes.

The equilibrium concept is called *social* because – although strategies are contract-specific – a player's strategy will still depend on the strategies of all market participant, since the possibility of a re-match determines everyone's endogenous outside option.[6]

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[5]Although intended effort e_t is part of a bargaining agreement as well, it cannot be specified by the formal contract because of its assumed non-verifiability.

[6]See Ghosh and Ray (1996), Kranton (1996), or MacLeod and Malcomson (1998) for descriptions of social equilibria in related settings.

3 Results Without a Minimum Wage

3.1 Steady-State Payoffs for a Given M

In the following, I describe payoffs in the steady state of the game absent a minimum wage, and where no one expects a minimum wage to be introduced in the future. Because there are no matching frictions, this steady state is immediately reached. In addition, matches are persistent on the equilibrium path, and no turnover is observed (this turns out to be optimal). In an extension in section 5.3, I show that my results are (qualitatively) unaffected by the introduction of exogenous turnover.

Furthermore, the restriction to contract-specific strategies implies that calendar time is irrelevant. The tenure of a match potentially matters, though, which will be indicated by the subscript τ. With a slight abuse of notation, I will use the term "period" when describing the tenure of a relationship (however, note that because no turnover is observed, $\tau = t$ on the equilibrium path).

Then, an employed worker's discounted payoff stream in the τ's period (of the tenure of his current match) is denoted by U_τ and equals

$$U_\tau = \sum_{j=\tau}^{\infty} \delta^{j-\tau} \left(w_j - c(e_j^*) \right),$$

where e_j^* is equilibrium effort in period j.

A firm's payoff in any period τ in which it is in an employment relationship is denoted by Π_τ and equals

$$\Pi_\tau = \sum_{j=\tau}^{\infty} \delta^{j-\tau} \left(e_j^* \theta - w_j \right) .$$

The payment w_τ is determined by the outcome of the bargaining process that takes place at the beginning of every period and splits the net surplus of an employment relationship accordingly. There, note that the *gross* surplus generated within a period of a given match is $e_\tau^* \theta - c(e_\tau^*)$, yielding a total gross surplus of a given relationship, $S_\tau^G = \sum_{j=\tau}^{\infty} \delta^{j-\tau} \left(e_j^* \theta - c(e_j^*) \right)$. The *net* surplus of an employment relationship equals the difference between S^G and disagreement payoffs. Because disagreement implies that no employment contract is signed for the current period and that players split up and enter the matching market in the subsequent period, disagreement payoffs are equal to $\delta \overline{\Pi}_\tau$ (firm) and $\delta \overline{U}_\tau$ (worker). $\overline{\Pi}_\tau$ and $\overline{U}_\tau$ are defined as players' *endogenous* outside utilities – in contrast to their *exogenous* outside utilities which are set to zero. The values $\overline{U}_\tau$ and $\overline{\Pi}_\tau$ depend on the number of active

firms and are determined by the likelihood with which an unmatched player finds a new match. Hence they are independent of τ, and I can omit τ-subscripts when describing outside utilities.

If $M < 1$, firms find a new match with probability 1 after a breakup, and $\overline{\Pi} = \Pi_1$. Unemployed workers, on the other hand, are (re-) matched with probability zero (because existing matches do not break up on the equilibrium path), and $\overline{U} = 0$. If $M > 1$, firms with free vacancies will not find a match in periods $t > 1$, whereas workers are always matched; then $\overline{U} = U_1$ and $\overline{\Pi} = 0$.

Therefore, the net surplus generated within a given match is $S_\tau = S_\tau^G - \delta\overline{\Pi} - \delta\overline{U}$. Since the bargaining process allocates the share α of the net surplus to the worker, and $1 - \alpha$ to the firm, payoffs can also be written as

$$U_\tau = \delta\overline{U} + \alpha S_\tau = (1 - \alpha)\delta\overline{U} + \alpha\left(\sum_{j=\tau}^\infty \delta^{j-\tau}\left(e_j^*\theta - c(e_j^*)\right) - \delta\overline{\Pi}\right)$$

$$\Pi_\tau = \delta\overline{\Pi} + (1 - \alpha)S_\tau = \alpha\delta\overline{\Pi} + (1 - \alpha)\left(\sum_{j=\tau}^\infty \delta^{j-\tau}\left(e_j^*\theta - c(e_j^*)\right) - \delta\overline{U}\right).$$

3.2 Equilibrium Outcome for a Given M

Each match has the objective to maximize the relationship surplus

$S_\tau = \sum_{j=\tau}^\infty \delta^{j-\tau}\left(e_j\theta - c(e_j)\right) - \delta\overline{\Pi} - \delta\overline{U}$ in every period τ. Disagreement outcomes must be taken as given, though (those involve a separation and strategies are restricted to be contract specific). Therefore, the problem is equivalent to maximizing $\sum_{j=\tau}^\infty \delta^{j-\tau}\left(e_j\theta - c(e_j)\right)$ in every period, which implies sequential efficiency and hence boils down to maximizing the stage-game surplus $e_\tau\theta - c(e_\tau)$ in every period. However, although I assume that players never agree on a Pareto-dominated outcome, the following potentially profitable *unilateral* deviations must be taken care of and might restrict the efficiency of an employment relationship.

First, players are always able to leave their current relationship and go for their endogenous outside options. This implies that in every period τ, the following non-reneging (NR) conditions, one for the worker and one for the firm, must hold:

$$U_\tau \geq \overline{U} \tag{NRW}$$

$$\Pi_\tau \geq \overline{\Pi}. \tag{NRF}$$

Note that because $\overline{U}, \overline{\Pi} \geq 0$ (the surplus cannot be negative), additional individual rationality constraints $U_\tau, \Pi_\tau \geq 0$, are implied by (NR) conditions and therefore

do not have to be considered explicitly.

Second, the worker cannot be forced to select a certain effort level (since no external enforcement mechanism exists), hence must be incentivized by a relational contract to choose equilibrium effort e_τ^*. In the present setting, implementable effort levels are determined by the difference between a worker's continuation payoff after choosing equilibrium effort e_τ^* and his continuation payoff after choosing any other effort level. Therefore, equilibrium effort e_τ^* can be implemented if and only if a worker's incentive compatibility (IC) constraint,

$$-c(e_\tau^*) + \delta U_{\tau+1} \geq -c(\tilde{e}_\tau) + \delta \tilde{U}_{\tau+1}(\tilde{e}_\tau), \tag{IC}$$

holds for all $\tilde{e}_\tau$, where $\tilde{U}_{\tau+1}(\tilde{e}_\tau)$ is a worker's continuation payoff after choosing effort $\tilde{e}_\tau$.

Concluding, the problem is to maximize stage-game payoffs $e_\tau \theta - c(e_\tau)$ in every period τ, subject to (NRW), (NRF) and (IC), and taking into account that the total net relationship surplus is allocated according to shares $(\alpha, 1 - \alpha)$ in every period τ.

First, I can show that relational contracts are stationary in a sense that effort and payoffs are the same in every period:

Lemma 1: *In any contractual social equilibrium, equilibrium effort e_τ^* is the same in every period τ. Furthermore, in every period τ within-match payoffs on and off the equilibrium path are the same for worker and firm, respectively. Finally, payoffs are constant over time.*

The proof can be found in the Appendix.

Lemma 1 implies that – because $\overline{\Pi} \in \{0, \Pi_1\}$ and $\overline{U} \in \{0, U_1\}$, and because $U_2 = ... \equiv U$ and $\Pi_1 = \Pi_2 = ... \equiv \Pi$ – (NR) constraints automatically hold and can hence be omitted. It is driven by the stationary structure of the game, as well as by the assumptions that players never agree on a Pareto-dominated outcome, and that players receive the same share of the surplus in every period. Besides stationarity (which allows me to omit τ-subscripts), the Lemma implies that within a relationship it is not possible to punish players for deviations, because such punishment would either have to involve a reduction of future surplus or a re-allocation of surplus shares. Therefore, the worker's continuation value can only change after a deviation

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if he is fired afterwards, and a termination threat consequently is the only means to provide incentives.

Because of the requirements imposed by subgame perfection, though, a termination after a deviation must also be optimal for at least one player: If $M < 1$, a firm will induce the separation, because then $\overline{\Pi} = \Pi_1$ and it is indifferent between starting a new or sticking to its old relationship. If $M > 1$, a separation is weakly optimal for the worker. Concluding, $\tilde{U}(\tilde{e}) = \overline{U} \ \forall \tilde{e} \neq e^*$, which also implies that if a worker deviates, he will optimally select zero-effort. Finally, note that using a termination after a deviation does not destroy surplus because the short side of the market can always find an immediate replacement in the next period.

Taking this into account, the agent's (IC) constraint which determines implementable effort levels becomes

$$-c(e^*) + \delta \left(U - \overline{U} \right) \geq 0. \tag{IC}$$

The (IC) constraint implies that positive effort is only feasible if firms are on the short side of the market:

Lemma 2: *No effort can be implemented for $M \geq 1$.*

Proof: $M \geq 1$ implies $\overline{U} = U$, giving the (IC) constraint $-c(e^*) \geq 0$, which has $e^* = 0$ as a unique solution. ∎

Slightly anticipating the analysis of the equilibrium number of firms in section 3.3, Lemma 2 also implies that $M \geq 1$ cannot determine an equilibrium: $e^* = 0$ would mean that firms make no profits. Given strictly positive entry costs k, though, no firm would enter if it expected the number of firms to be larger than 1, contradicting that this can be an equilibrium. Therefore, from now on I can restrict attention to $M < 1$, implying that $\overline{\Pi} = \Pi_1$ and $\overline{U} = 0$.

The next Lemma gives a characterization of equilibrium effort e^*.

Lemma 3: *Equilibrium effort e^* is unique and satisfies the following conditions:*

- *If $-c(e^{FB}) + \delta \alpha e^{FB} \theta \geq 0$, $e^* = e^{FB}$.*

- *Otherwise, $e^* < e^{FB}$ and is characterized by the largest effort level such that $-c(e^*) + \delta \alpha e^* \theta = 0$ holds.*

The proof can be found in the Appendix.

Summarizing, the bargaining agreement selects the highest attainable joint pay-off, given the parties' continuation payoffs from the matching pool. Furthermore, the worker's (IC) constraint must be satisfied, and the best punishment for the firm must be to separate following any deviation. The contractual social equilibrium is found by computing the highest fixed point of the function that takes the continuation value of the match to itself (given separation values). Because of the concavity of the objective function, and because of the stationarity of the setting, this process yields a unique and constant effort level which is in particular restricted by the worker's (IC) constraint. This constraint equals

$$-c(e^*) + \delta\alpha\frac{e^*\theta - c(e^*)}{1 - \alpha\delta} \geq 0,$$

and can be further simplified to $-c(e^*) + \delta\alpha e^*\theta \geq 0$.

As is standard in relational contracting settings, a certain effort level can be implemented if today's effort costs are offset by the discounted future stream of the relationship surplus. Different from those standard approaches (such as MacLeod and Malcomson, 1989, Levin, 2003), though, in my setting the effective factor with which future surplus streams are discounted is $\delta\alpha$. This is because the worker can only be motivated by his own continuation payoffs (which he is bound to lose after a deviation), and those only represent a share of the future (gross) relationship surplus.

Note that even if I allowed for a bonus payment from firm to worker – to be paid after the latter exerted equilibrium effort – implementable effort would not be higher[7]: The firm would only be willing to pay such a bonus if its continuation payoff upon reneging was smaller than upon complying. This, however, is not possible because the firm can always find a new match after leaving its current one and hence cannot be punished for a deviation (neither within nor outside its current match). Therefore, the firm does not have to remain within its current relationship to collect its share in the future, and this share cannot be used to provide incentives.

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[7]See section 5.1 for a more detailed discussion of this aspect.

Comparative Statics Fixing the number of firms M, I now explore how the allocation of bargaining power affects equilibrium effort and payoffs. I merely focus on *implementable* effort, which is determined by the maximum effort level where the (IC) constraint binds. As shown in Lemma 2, if e^{FB} violates (IC), then equilibrium effort e^* is equal to implementable effort. Otherwise, implementable effort is irrelevant and $e^* = e^{FB}$.

Proposition 1 shows that a higher α always increases implementable effort and a worker's utility U. Moreover, firms might also benefit from workers having higher bargaining power:

Proposition 1 : Implementable effort and workers' utilities increase in α. Furthermore, there exists an $\overline{\alpha} \in (0, 1)$ such that profits Π increase in α for $\alpha \leq \overline{\alpha}$ and decrease in α for $\alpha > \overline{\alpha}$.

The proof can be found in the Appendix.

Implementable effort increases in $U = \alpha \frac{e^*\theta - c(e^*)}{1 - \alpha\delta}$ (note that $\overline{U} = 0$) and therefore in α.[8] Hence, as long as $e^* < e^{FB}$, a higher α increases equilibrium effort and consequently the total surplus of an employment relationship. Furthermore, the firm can even benefit from workers having a higher bargaining power, namely if α and hence implementable effort are rather small. To grasp the intuition for this result, note that in the extreme case of $\alpha = 0$, no effort at all can be implemented: Plugging $\alpha = 0$ into the (IC) constraint gives $-c(e^*) \geq 0$, which only holds for $e^* = 0$. But zero effort also implies zero surplus and hence zero profits. Therefore, a strictly positive $\alpha(< 1)$ which is associated with strictly positive effort yields strictly positive profits and is hence preferred by a firm. More generally, a higher α has a direct and an indirect effect on profits Π. Whereas the direct effect is always negative (for a given surplus, a worker receives a larger share), the indirect effect is positive if the (IC) constraint binds. Then, α increases the total surplus which increases profits as well. If α and hence e^* are rather small, the latter effect is likely to dominate. For larger values of effort and α, the negative direct effect dominates. Then, inducing additional effort becomes more costly (since effort costs are convex), and the increasing surplus does not make up for the smaller share the firm can pocket.

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[8]The result that implementable effort increases in a worker's bargaining power if effort is not verifiable has first been derived by Den Haan et.al (1999).

This implies that for low levels of α, the firm would like to promise the worker a larger share of the future surplus but cannot commit to do so: Bargaining takes place in every period, and the firm can always capture a share $1 - \alpha$ of the relationship surplus. Since deviations cannot be punished within a match, a promise to grant the worker a higher surplus share in the future is not credible. In addition, even if the firm were able to promise the worker a higher share of the future surplus (for example because players could commit to only bargain once, at the beginning of a relationship, and then determine all future actions), such a promise would still not be credible. In this case, the firm would have an incentive to fire the worker after the first employment period and then enter a new match where it can secure a share $1 - \alpha$ of the relationship surplus. All this is driven by the assumption that players never agree on a Pareto-dominated outcome. Without this assumption, other kinds of equilibria are feasible (I further explore this aspect in section 5.4).

Concluding, the firm's situation in case of a low α resembles a standard hold-up problem: The worker's effort benefits the firm. From an ex-ante point of view, the firm would like to promise the worker to compensate him accordingly but cannot commit to do so. Ex post, the firm has the opportunity to break its promise and will do so because it cannot be punished for a deviation.

3.3   Equilibrium Number of Firms

In this section, I do not take M as given and allow firms to freely enter (and leave) the market. Entry is associated with one-time costs $k > 0$ (exit is costless). Furthermore, I assume $d\alpha/dM \geq 0$, i.e. more firms being active in the market (weakly) increases each worker's bargaining power, and $d\theta/dM \leq 0$. Hence, more firms are associated with a more intense competition on (not further modelled) product markets. This also indicates that first-best effort e^{FB} generally is a function of M. However, for expositional convenience and because it is of no relevance when deriving results for a given M, I generally do not explicitly describe e^{FB} (or α or θ) as functions of M. Note that since α and θ are defined as differentiable functions of α, those are also continuous.

I focus on equilibria where a strictly positive number of firms is active. This is implied by the two following properties: First, k is not too large compared to potential profits. Second, I abstract from coordination issues that might initially be present: An equilibrium with $M = 0$ could exist even in the presence of high potential profits. A single firm might not have an incentive to enter if it did not

expect others to do so as well, because this firm would have a very high bargaining power (α would likely be close to zero). Then, hardly any effort could be enforced with the resulting negative consequences on profits.

Finally, I assume that at least one $M^* > 0$ exists that satisfies the zero-profit condition

$$-k + \delta\Pi(M^*) = 0, \tag{1}$$

and where in addition $d\Pi/dM < 0$ holds.[9]

4 Minimum Wage

Asssume the market faces a minimum wage, which I define as a lower bound on per-period wages, $\overline{w} \geq 0$. I first explore the effect of changes of $\overline{w}$ on implementable effort and on payoffs for a given market structure, i.e. for fixed values of α and M. Then, I analyze the effect of a minimum wage on the entry and exit of firms and consequently on equilibrium employment. Note that I consider the effects of unanticipated changes on a minimum wage. Even if changes were anticipated, though, the qualitative effects would remain and only magnitudes would change.

First, it is important to assess whether a minimum wage actually has an impact on equilibrium outcomes. This will only be the case if it is larger than the equilibrium wage derived in the previous section. There, note that the worker's utility stream if no minimum wage is present, $U = \alpha\left(e^*\theta - c(e^*)\right)/(1 - \alpha\delta)$, can also be written as $\left(w^* - c(e^*)\right)/(1 - \delta)$. Hence equilibrium per-period wages without a minimum wage are $w^* = c(e^*) + \alpha\left(1 - \delta\right)\left(e^*\theta - c(e^*)\right)/(1 - \alpha\delta)$. This indicates that a minimum wage does only affect outcomes if it binds, i.e. if $\overline{w} > w^*$. What is also needed for this to hold, though, is that a non-binding minimum wage does not affect disagreement outcomes. This is the case in my setting, where an agreement has to be sealed by a contract specifying a wage for the respective period. Under disagreement, no employment contract is signed, no payments are made and the match breaks up. Hence, a minimum wage solely affects bargaining in a sense that the contract sealing the bargaining phase cannot specify a wage below $\overline{w}$.

Before presenting my results, note that having a binding minimum wage implies that it might (and generally will) be impossible for the firm to get a share $1 - \alpha$ of the surplus of an employment relationship, but that its maximum feasible payoff

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[9]If M^* satisfied the zero-profit condition, however with $d\Pi/dM > 0$, this could not constitute an equilibrium. Then, additional entry would yield higher, i.e., strictly positive, profits.

is equivalent to a strictly lower share of the surplus. I assume that in this case, the firm's payoff is as high as feasible and that the firm is still willing to enter an employment relationship as long as this is profitable.

4.1 Minimum Wage and Effort

My first result establishes a positive relationship between a minimum wage and equilibrium effort, where e^* and w^* represent equilibrium outcomes in the setting without a minimum wage:

Proposition 2 : Assume a binding minimum wage, i.e., $\overline{w} > w^*$. Holding M and α fixed, equilibrium effort with the minimum wage is strictly larger than e^* and further increasing in $\overline{w}$.

The proof can be found in the Appendix.

A binding minimum wage increases equilibrium effort, which implies that it also increases the efficiency of an employment relationship as long as effort is inefficiently low. This is driven by two implications of a mandatory wage raise. First, a worker's future benefit of keeping the job (i.e., U) goes up, which increases the effort costs he is willing to bear today in order to not get fired. Second, higher effort is not only feasible but will also be agreed upon in the bargaining process and consequently be implemented. This is because a (binding) minimum wage ceteris paribus increases a worker's payoff above the level indicated by his bargaining power α. Then, a higher effort is the only way to have an effective sharing rule that comes closer to "fair" levels. The higher implementable effort cannot completely make up for the additional transfer, though, and additional rents are shifted from firms to workers. Consequently, maximum feasible effort will generally be implemented, which is characterized by $c(\overline{e}) = \delta\overline{w}$, with $d\overline{e}/d\overline{w} = \delta/c' > 0$.

Only if efficient effort e^{FB} can already be implemented if a minimum wage is not present, equilibrium effort might be below maximum feasible effort $\overline{e}$ and effective bargaining shares might not be affected by the presence of a binding minimum wage. Even in this instance the minimum wage has a positive effect on effort, though, which then will actually turn out to be inefficiently high. This case seems less interesting, though, because a rather high effort level without a minimum wage (for example because α is large) implies that high wages are paid anyway, making

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it less likely that a minimum wage actually binds. Furthermore, markets where a minimum wage is relevant supposedly are characterized by low levels of workers' bargaining power. This conjecture is supported by Manning (2003), who presents evidence that employers set wages in markets where a minimum wage is relevant. He states that "for the average worker in a non-union setting, this does seem to be the appropriate assumption" (p. 4). Concluding, a binding minimum wage is expected to have a positive effect on the efficiency of employment relationships if those are characterized by low wages and consequently low employee effort, and where effort is hard to verify as for example in the service industry.

Indeed, there is evidence that the productivity of firms has gone up after the introduction of a minimum wage, and that these productivity gains were particularly significant for firms in the service industry. Galindo-Rueda and Pereira (2004) analyze how British firms responded to the introduction of a National Minimum Wage in 1999. They find a positive one-off effect on labor productivity (measured as gross output relative to employment), which in addition is only observed in the service sector and not in manufacturing.

Rizov and Croucher (2011) conduct a further study on the effect of the British National Minimum Wage. They compute a structural estimation of production functions within disaggregate 4-digit industries, controlling for supply and demand factors that affect firms. They find that productivity substantially went up after the introduction - and subsequent increases - of the minimum wage, again with a substantially higher impact in service industries than in manufacturing.

Both studies can only speculate on the factors that caused the observed productivity increases, though. In general, productivity might go up because of reductions in employment or working hours (which however was not observed in both studies), the adjustment of prices, or issues like training, changes in the organizational structure of firms, or - as is the point of this paper - the provision of more effort and hence a higher service quality. Several studies attempt to fill this gap, conducting extensive surveys in which managers were asked how they responded to the introduction of the British National Minimum Wage. Manning et. al (2003) focus on workers in the residential care homes industry. They find that the effect of the minimum wage on worker effort is positive, however not significantly different from zero. The British Low Pay Commission (Low Pay Commission, 2001) - which is supposed to analyze the impact of the British National Minimum Wage and make recommendation concerning potential increases - initiated several research projects to study the exact impact of the National Minimum Wage. They find that 30 % of

all firms in the surveys responded by improving the quality of provided services. In one of the involved projects, Heyes and Gray (2003) conduct a survey of small-scale enterprises in the Yorkshire and Humberside region, with a special focus on service industries (motor services, retail, care homes, hairdressing and hospitality). There, 61 % of the firms state that "Increasing workers' level of effort" was an important or very important response to the minimum wage. The point "Improving quality of products and/or services" is regarded as important or very important by 63% of the respondents.

Hirsch et.al (2011) show that these effects could also be observed in the US. They analyze increases of the US federal minimum wage between 2007 and 2009, using a sample of 81 quick-service restaurants in Georgia and Alabama. Their data includes a written survey of restaurant managers and qualitative data collected in interviews with restaurant managers. They observe that managers responded to the minimum wage increase with – amongst others – higher prices and higher performance standards. Furthermore, the workforce was supposed to become more productive and increase sales through improved service, where managers in particular used approaches to boost the morale of employees to increase their productivity.

4.2   Minimum Wage and Payoffs

For a given level of M, a minimum wage has two implications: it shifts rents from firms to workers, but generally also increases the efficiency of an employment relationship. Unless effort is inefficiently high, workers hence always benefit from a higher (binding) minimum wage. The case is less straightforward for firms. Although they are directly harmed by the rent-shifting effect, they also benefit from the increased efficiency of an employment relationship. If effort is rather low, the latter effect dominates, and firms' profits can actually go up.

Corollary 1: *Holding M and α fixed, an employed worker's payoff U increases in a binding minimum wage as long as effort is inefficiently low, and might or might not increase otherwise; a firm's payoff Π increases in a binding minimum wage as long as effort is below $\hat{e}$, where $\hat{e}$ is characterized by $\delta\theta - c'(\hat{e}) = 0$, and decreases otherwise.*

Proof: In Proposition 2, I have shown that if a minimum wage binds and effort is inefficiently low, $U = (\overline{w} - c(\overline{e}))/(1-\delta)$ and $\Pi = (\overline{e}\theta - \overline{w})/(1-\delta)$,

where $\overline{e}$ is characterized by $\overline{w} - c(\overline{e})/\delta = 0$. Therefore, $dU/d\overline{w} = 1 > 0$ and $d\Pi/d\overline{w} = (\delta\theta - c') / [(1-\delta) c']$, which is positive for $\delta\theta - c' \geq 0$ and negative otherwise. If effort is inefficiently high, then either $U = \frac{\overline{w}-c(\overline{e})}{1-\delta}$ or $U = \frac{\overline{w}-c(\tilde{e})}{1-\delta}$, where $\tilde{e}$ is characterized by $\overline{w} = c(\tilde{e}) + \alpha\frac{(1-\delta)(\tilde{e}\theta-c(\tilde{e}))}{(1-\alpha\delta)}$ (see the proof to Proposition 2). In the first case, $dU/d\overline{w} > 0$; in the second case, $\frac{dU}{d\overline{w}} = \frac{\alpha(\theta-c')}{[c'(1-\alpha)+\alpha(1-\delta)\theta]}$, which is negative for inefficiently high effort levels. ∎

Whereas the intuition for the mostly positive effect of a minimum wage on U for a given market structure is straightforward, the driver of a potentially positive value of $d\Pi/d\overline{w}$ is less obvious. It is equivalent, though, to the one of a potentially positive impact of a worker's bargaining power α on Π: Larger (future) rents for workers conditional on keeping their jobs increase their willingness to exert effort today, because the non-verifiability of effort requires effort costs to be covered by future rents. If effort has been rather low, the associated efficiency increase can more than offset the pure rent-shifting effect. Which effect is expected to dominate is an empirical question. However, since markets where a minimum wage binds are supposedly associated with workers having rather small bargaining power (see Manning, 2003), the impact of a minimum wage on profits should at least be not "too" negative.

Whereas there is vast evidence that employees benefit from a higher minimum wage[10] (at least those who keep their jobs; potential employment effects are analyzed in the next section), only little research exists that assesses the interaction between minimum wages and profits (theoretically, search-and-matching models that try to explain non-negative employment effects such as Dube et. al, forthcoming, but also Manning, 2003, or Flinn, 2006, would generally predict a negative effect on profits). Exceptions for empirical research are Draca et. al (2011) and Harasztosi and Lindner (2015):

Draca et.al (2011) find a negative effect of the British National Minimum Wage on profits. However, due to a lack of more detailed data, they use the profit-to-sales ratio as a measure for profits, although both are not identical. In my setting, for example, the profit-to-sales ratio would correspond to $(e\theta - w)/e\theta$. This measure would *always* decrease in a higher (binding) minimum wage, even if profits Π were increasing in $\overline{w}$[11].

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[10]See Holzer et al. (1991), or Harasztosi and Lindner (2015).

[11]To see that, take $\frac{d(\overline{e}\theta-\overline{w})/\overline{e}\theta}{d\overline{w}} = -\frac{(\overline{e}c'-c(\overline{e}))/c'}{\overline{e}^2\theta}$, where I took into account that in the case of

Harasztosi and Lindner (2015) analyze the impact of a large and persistent increase in the minimum wage in Hungary in 2001, utilizing detailed information on firms' balance-sheets and income statements. They find that the higher minimum wage had large positive effects on labor costs and earnings, however that profits did not decline. Instead, sales substantially went up. Harasztosi and Lindner (2015) claim that higher costs must have been passed on to (final) customers via higher prices. I present a complementary story, claiming that in order to being able to increase prices, firms must also have increased service quality. This claim is further supported by their result that the positive effect of the minimum wage on (net) sales is almost entirely driven by the service industry.

4.3   Minimum Wage and Employment

Finally, I assess the impact of a minimum wage on total employment, i.e. the number of firms M in the economy. Because M is determined by a zero-profit condition, there is a strong link between the effect of $\overline{w}$ on profits and the effect it has on employment.

Proposition 3 *: Equilibrium employment M^* is increasing in a binding minimum wage as long as effort is below $\hat{e}$, where $\hat{e}$ is characterized by $\delta\theta - c'(\hat{e}) = 0$. For higher effort levels, there exists a threshold $\hat{\hat{e}}$ such that employment $dM^*/d\overline{w} = 0$ for $e \leq \hat{\hat{e}}$ and $dM^*/d\overline{w} < 0$ for $e > \hat{\hat{e}}$.*

The proof can be found in the Appendix.

If a relatively low minimum wage binds in a given industry (indicating that without the minimum wage, wages and hence implemented effort would also be rather low), a moderate increase is likely to have a positive employment effect. The intuition for this result is straighforward, given the previously derived effects of a minimum wage on effort and profits. If workers' bargaining power α is low, a higher minimum wage effectively not only increases effort, but also a firm's profits. Starting from a steady state where M^*, the total employment level in the industry under consideration, has been generated by a zero-profit condition, higher profits naturally trigger an entry of additional firms. Because $\theta(M)$ is assumed to be

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inefficiently low effort $\overline{e}$ is defined by $\overline{w}\delta - c(\overline{e}) = 0$. Because $c(\cdot)$ is a strictly convex function with $c(0) = 0$, $\overline{e}c' - c(\overline{e}) > 0$, and $\frac{d(\overline{e}\theta - \overline{w})/\overline{e}\theta}{d\overline{w}} < 0$.

continuous, this holds in any case despite the potentially negative effect of M on $\theta(M)$.

At intermediate levels, the minimum wage will not have direct negative effects on employment, even though firms' profits go down. This is due to entry costs k: Firms that have entered the market make positive profits, and entry costs k are sunk. So a reduction in profits does not automatically make them give up and leave the market. In the long run, though, employment effects should also be negative: I do not model the possibility of an exogenous exit of firms. If this was possible, then in a steady state without a minimum wage, exiting firms would be replaced by new entries, not affecting equlibrium employment. If a minimum wage reduced profits, though, some exiting firms would not be replaced by new entries, triggering a negative employment effect in the long run. Finally, if the profit reduction is so large that firms even make ex-post losses, some of them will give up, triggering an immediate negative effect on employment.

There exists a large empirical literature on (often non-existent) employment effects of a minimum wage (as examples, take Card and Krueger (1994), Katz and Krueger (1992), Machin and Manning (1994), or more recently Dube et al. (2010) and Harasztosi and Lindner (2015)). Several theoretical explanations have been offered to explain this apparent puzzle, where the predominant approach involves a labor market with matching frictions. Based on the Burdett and Mortensen (1998) search-and-matching model frame, Card and Krueger (1995) or Dube et al. (forthcoming), among others, show that a minimum wage can reduce matching frictions by reducing separations: Because a higher minimum wage induces less wage dispersion, the likelihood of finding a better job is lower. This results in fewer job-to-job transitions and less occupancies firms must fill. In a similar vein, Flinn (2006) uses the positive effect of a minimum wage on workers' bargaining power. Abstracting from on-the-job search, a higher bargaining power increases workers' participation in a given market and induces a larger search intensity, which might eventually increase employment. Generally, if the reduction of matching frictions is sufficiently large to overcome the negative effects of a minimum wage (lower profits make some low-productivity jobs disappear, low-productivity workers have lower chances to find a job), the total employment effects do not have to be negative.

Whereas these models can explain the (partial) absence of negative employment effects of a minimum wage, strong assumptions are needed – especially on the matching process – to generate the desired outcomes. I offer a complementary explanation, where the labor market friction is grounded in an agency problem between a firm

and its workers.

Furthermore, in addition to delivering a new perspective on the impact of a minimum wage on labor markets, my approach also links employment outcomes to the observed empirical positive connection between a minimum wage and effort of workers, in particular in the service sector. Besides delivering a theoretical explanation for this link, I state that it might further be a driving force behind employment effects of a minimum wage – vis-a-vis its impact on a firm's profits. Support for this view is provided by Harasztosi and Lindner (2015). In their sample, they do not observe a decline in profits following a minimum wage increase. Furthermore, they only detect small (if any) disemployment effect, and then mostly in manufacturing and exporting industries, and not in the service industry, where my model mainly applies to. These connections cannot be made by search-and-matching models. First, those do not consider the importance of how well a job is done, and efficiency only depends on firm and worker characteristics. Second, given an occupation is filled, a minimum wage there has a negative effect on profits.

5 Discussion & Extensions

In the following, I discuss some assumptions that have been imposed and discuss the implications of relaxing them.

5.1 Bonus Payments

I do not allow for discretionary bonus payments from firm to worker. Here, I show that this assumption is without loss of generality – because it would not be possible to enforce a positive bonus (to be paid after the worker chose equilibrium effort e^*). This is driven by players' inability to write formal contracts based on effort (or output), hence it must be in the firm's interest to actually pay the bonus. Put differently, the firm's continuation payoff after paying the bonus must be larger than its continuation payoff after reneging. Therefore, a dynamic enforcement constraint must be satisfied,

$$-b + \delta\Pi \geq \delta\tilde{\Pi},$$

where $\tilde{\Pi}$ is the firm's continuation payoff after reneging on paying the bonus. A strictly positive bonus could only be enforced if $\tilde{\Pi} < \Pi$ which however is not possible because the firm can always secure to make profits Π, for example by leaving its current match and going for a new one. As mentioned before, this is because the

net relationship surplus which determines the maximum power of incentives does not contain the firm's profits – those can also be generated outside the current relationship.

However, a negative bonus paid from worker to firm in case the former did not exert effort would be feasible (then one only would have to be careful that in case a minimum wage is present, total per-period wages cannot be below $\overline{w}$) – without extending the set of equilibrium outcomes, though. Consider the following arrangement. After a deviation, the worker is not fired but instead has to pay an amount p to the firm. The firm is willing to retain the worker in this case (it is indifferent between staying in its current or starting a new relationship), whereas the worker needs sufficient incentives to pay p. The maximum feasible p is also determined by a dynamic enforcement constraint, $\delta U \geq -p + \delta \overline{U}$, taking into account that the worker is fired after refusing to pay p. The worker's incentives to exert effort are solely given by the size of p (because he is not fired when paying p, his continuation payoffs are independent of effort), hence the (IC) constraint becomes $-c(e^*) \geq -p$. Adding both constraints gives a new condition which is necessary and sufficient for the enforceability of equilibrium payoff e^*,

$$-c(e^*) + \delta \left(U - \overline{U} \right) \geq 0,$$

which is equivalent to the (IC) constraint in the case without bonus payment. Since p would never materialize on the equilibrium path, equilibrium outcomes would hence be the same as before.

5.2   Bargaining at Intermediate Stages

I assume that bargaining only occurs at the beginning of a period. One could argue, though, that players should also be able to bargain at later stages, for example after the worker has exerted effort. The firm would have an incentive to do so, because it is indifferent between retaining and firing its current worker, whereas the worker is better off within the current relationship. Hence, the firm would be able to extract some of the worker's utility from keeping his current job. In this section, I allow for a second bargaining round after the worker has exerted effort. I show that in this case, cooperation totally breaks down in case no minimum wage is present – because the firm does not fire the worker anymore after the latter has not exerted effort which destroys incentives. With a minimum wage, though, the second bargaining round is irrelevant, hence positive effects of a minimum wage are even more pronounced.

I impose the following assumptions: The allocation of bargaining power in the second bargaining round is the same as in the first, with the firm pocketing a share $1 - \alpha$ of the net relationship surplus and the worker α. Furthermore, in the first bargaining round players take the outcome of the second bargaining round into account and adjust payments accordingly[12]. However, disagreement in the first bargaining round still leads to an immediate termination of the current match, without players reaching the second bargaining round. Therefore, payoffs at the beginning of a period (without a minimum wage) still amount to $\Pi = \frac{1-\alpha}{1-\alpha\delta} \left(\frac{e^*\theta - c(e^*)}{1-\delta} \right)$ and $U = \alpha \frac{e^*\theta - c(e^*)}{1-\alpha\delta}$, respectively. In the following, I denote the payoffs from the perspective of the second bargaining round Π^2 and U^2, and stick to Π and U when describing the outcomes of the first round. Then, disagreement payoffs in the second bargaining round are $\delta\overline{\Pi}(= \delta\Pi)$ and $\delta\overline{U}(= 0)$, whereas the net relationship surplus at this stage is $\delta\frac{e^*\theta - c(e^*)}{1-\delta} - \delta\overline{\Pi} - \delta\overline{U}$. Therefore, $\Pi^2 = \delta\overline{\Pi} + (1-\alpha)\left(\delta\frac{e^*\theta - c(e^*)}{1-\delta} - \delta\overline{\Pi} - \delta\overline{U} \right)$ and $U^2 = \delta\overline{U} + \alpha \left(\delta\frac{e^*\theta - c(e^*)}{1-\delta} - \delta\overline{\Pi} - \delta\overline{U} \right)$. Plugging the values for Π and U into the respective expressions yields

$$\Pi^2 = \delta(1-\alpha)\frac{1+\alpha-\alpha\delta}{1-\alpha\delta}\frac{e^*\theta - c(e^*)}{1-\delta}$$
$$U^2 = \alpha^2\delta\frac{e^*\theta - c(e^*)}{1-\alpha\delta}.$$

The second bargaining round allows the firm to capture some of the worker's utility of remaining in the current relationship. Since $\Pi^2 > \delta\Pi$, the firm now is strictly better off within its current match compared to starting a new one in the next period. Therefore, it is not willing to fire the worker in case the latter did not exert equilibrium effort e^* anymore. Because players never agree on a Pareto-dominated outcome, the firm cannot commit to not enter the second bargaining round (or ask for less than a share $1 - \alpha$). Naturally, this will be anticipated by the worker who will consequently not exert effort, leading to a complete breakdown of production because no effort can be implemented.

This is different with a minimum wage, though, because total payments from firm to worker now must not fall below $\overline{w}$ (assuming that private, undetectable side payments are not feasible), and because the second bargaining round basically results in a payment from worker to firm. Recall that with a binding minimum

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[12]This is because an immediate replacement in the same period is not feasible. If players disagree in the first bargaining stage, the firm has to wait until the next period before it can enter a new match.

wage, the worker's effective share of the net relationship was generally higher than α, and the wage $\overline{w}$ was paid at the beginning of the period. The latter will still be the case here (now for any positive minimum wage because no effort can be enforced without), therefore players cannot agree on a transfer from worker to firm in the second bargaining round because such an agreement would violate the minimum wage.

Concluding, a minimum wage makes the second bargaining round irrelevant and hence has unambigously positive effects on outcomes (unless it was so high that firms would make losses). Of course, it is debatable whether the setting leading to a complete breakdown of cooperation is particularly realistic. However, it makes the benefits of a minimum wage that have been worked out in this paper even more explicit – a minimum wage can serve as a commitment device for firms not to exploit their workers ex-post. Like in standard hold-up problems, this commitment mitigates the negative ex-ante effects such an exploitation opportunity has on a worker's incentives.

5.3   Exogenous Turnover

Many papers that analyze agency problems in market settings similar to mine (such as Ghosh and Ray, 1996, Kranton, 1996, or MacLeod and Malcomson, 1998) assume some exogenous turnover because otherwise, no turnover at all would be observed on the equilibrium path. Then, *any* turnover would reveal a deviation from equilibrium behavior and could be punished accordingly. Because the main results in these papers are driven by players' ability to replace their partners, exogenous turnover is necessary.

Although my results also (partially) depend on the firm's ability to replace workers, I do not need exogenous turnover. This is because of my assumption that players do not agree on Pareto-dominated outcomes, which excludes surplus-reducing off-path punishments. In the following, I briefly sketch that my results prevail in a setting with exogenous turnover. Outcomes would only change qualitatively because exogenous turnover gives workers the chance to find a new job after a separation, i.e., their endogenous outside option goes up.

Consider the following model extension: At the end of every period, each worker - no matter whether part of a match or not - leaves the market with probability $(1 - \gamma)$, and remains for another period with probability γ. If a worker exits the market for exogenous reasons, he leaves for good and receives a payoff of zero from

then on. Furthermore, the number of employees remains fixed over time, hence $(1 - \gamma)$ new workers enter the market in every period.

Then, the total gross surplus created in a given relationship is $S^G = \frac{e^*\theta - c(e^*)}{1 - \delta\gamma}$. The disagreement payoff of a firm is still $\delta\overline{\Pi}$, whereas it amounts $\delta\gamma\overline{U}$ for a worker. Therefore, equilibrium payoffs are

$$U = (1 - \alpha)\delta\gamma\overline{U} + \alpha\left(\frac{e^*\theta - c(e^*)}{1 - \delta\gamma} - \delta\overline{\Pi}\right)$$

$$\Pi = \alpha\delta\overline{\Pi} + (1 - \alpha)\left(\frac{e^*\theta - c(e^*)}{1 - \delta\gamma} - \delta\gamma\overline{U}\right),$$

and the (IC) constraint equals

$$-c(e^*) + \delta\gamma\left(U - \overline{U}\right) \geq 0. \tag{IC}$$

Generally, exogenous turnover makes it more difficult to enforce a given effort level: Because a worker might leave for exogenous reasons and consequently not enjoy next period's utility, his rent conditional on staying must be higher. Furthermore, his increased outside option reduces the cost of a deviation.

Still, (IC) implies that only $M < 1$ can be part of an equilibrium (otherwise, $\overline{U} = U$, and no positive effort level can be enforced), giving $\overline{\Pi} = \Pi$. Furthermore, workers who are unemployed at the beginning of a period are matched with probability $\mu \equiv \frac{(1-\gamma)M}{(1-\gamma)+\gamma(1-M)} = \frac{(1-\gamma)M}{1-\gamma M}$, where the nominator gives the number of available jobs at the beginning of a period (consisting of matches that broke up at the end of the previous period because the respective worker left the market), and the denominator the number of workers looking for a job (consisting of those who newly entered the market as well as those who have been unemployed in the previous period and remained on the market). Therefore, $\overline{U} = \mu U + \delta(1 - \mu)\gamma\overline{U}$.

Using these results allows to rewrite the (IC) constraint, which becomes

$$-c(e^*) + \delta\gamma\left(\frac{\alpha(1 - \delta)(e^*\theta - c(e^*))(1 - \mu)}{(1 - \alpha\delta)(1 - \delta\gamma) + \alpha\delta\gamma\mu(1 - \delta)}\right) \geq 0. \tag{IC}$$

This is a more complicated expression than before, because exogenous turnover and the worker's positive outside option also affect the worker's net payoff from continuing the relationship. Still, the positive effect of α on implementable effort remains. Furthermore, for low initial levels of α an increase in α has a positive impact on profits – because the positive effect of a higher relationship surplus dominates

the negative rent-shifting effect.

Therefore, the effect of a minimum wage on efficiency and payoffs remains (qualitatively) the same. A higher (binding) minimum wage always increases implemented effort, which yields a higher surplus given the (IC) constraint binds and effort is inefficiently low. For low initial effort levels, the effect on profits and employment can also be positive, for higher initial effort levels the effect of a minimum wage on profits is negative[13].

5.4 Pareto Dominated Outcomes Feasible

Firms can benefit from a minimum wage and equivalently from a higher worker bargaining power α because they are not able to credibly promise workers a higher (future) share of the relationship surplus. This is driven by the assumption that players never agree on a Pareto-dominated outcome. Otherwise, a social equilibrium would exist in which the worker captures a higher share of the surplus (however also in the first and not only in later periods of an employment relationship), despite the firm's ability to fire the worker and start a new relationship in any period. Here, I present a brief sketch of such an equilibrium and refer to an earlier version of this paper (Fahn, 2013) for a more detailed analysis.

Take an arbitrary wage level that is (supposed to be) paid in every period. Furthermore, if a worker is offered a lower wage in any period of an employment relationship (or if the firm tries to "bargain down" the worker's wage), he selects an effort level of zero (this would not happen in my main setting for $\alpha > 0$ because players would always agree upon an agreement that is not Pareto-dominated). Only if the equilibrium wage is offered, he is willing to exert effort. Equilibrium effort is still determined by an (IC) constraint, where the worker's effort costs have to be covered by his discounted net continuation payoff. In such an equilibrium, any wage where the firm does not make negative profits can be sustained, hence also the one that maximizes the firm's profits. If such an equilibrium was played (for example because of the firm's high bargaining power), a minimum wage could not trigger larger profits and positive employment effects. In case of exogenous turnover (as analyzed in the previous section), though, a minimum wage could still increase the efficiency of an employment relationship even if the profit-maximizing equilib-

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[13]However, the threshold above which the effect on profits is negative is lower. Now, a firm's payoff Π increases in a binding minimum wage as long as effort is below $\hat{e}$, where $\hat{e}$ is characterized by $\delta\gamma(1-\mu)\theta - c'(\hat{e}) = 0$, and decreases otherwise. Without exogenous turnover, $\hat{e}$ is characterized by $\delta\theta - c'(\hat{e}) = 0$.

rium was played. This is because the equilibrium wage also has to be paid in the first period of a new employment relationship (otherwise, the firm would have an incentive to fire the worker after the first period and then go for a new match). But the first-period wage cannot be used to provide incentives (the worker is solely motivated by the prospect of future rents), hence is a mere cost factor for the firm. Therefore, exogenous turnover is costly for the firm, and it would rather sacrifice some effort in order to decrease wages and consequently reduce those turnover costs. In a profit-maximizing equilibrium, a binding minimum wage can then still increase implemented effort and consequently the efficiency of employment relationships.

In this vein, it is also interesting to compare this setting to a Shapiro/Stiglitz efficiency wage (Shapiro and Stiglitz, 1984). Their environment is similar, with firms having all bargaining power and workers getting fired when caught shirking. Although effort can be implemented with efficiency wages in Shapiro and Stiglitz (1984), it is not straightforward to state which version of my model mostly resembles their setting – because strategy space and equilibrium concept have been left rather vague in Shapiro and Stiglitz (1984). It seems, though, that their results are driven by the *assumption* of a stationary wage structure. Without this assumption, it would be optimal to reduce first-period wages (or demand some signing fee from workers) in order to extract the workers' rents. But then, firms would have an incentive to immediately fire workers after collecting their rents and enjoying some of their effort, and consequently no effort could be implemented.

6 Conclusion

Incentives should not be neglected when analyzing the impact of a minimum wage. If relevant aspects of performance like the friendliness towards customers cannot be verified, relational contracts must be used to give incentives. As firms cannot commit to pay workers more in the future than given by their bargaining power, they enforce inefficiently low service quality. If forced to pay a higher wage than actually intended, they also require higher levels of effort. Thus, a minimum wage can increase service quality and even the efficiency of many occupations. If workers' bargaining power has initially been low, a minimum wage might even increase profits and consequently employment.

I repeatedly claimed that my model mainly applies to the service sector because there it seems particularly difficult to pin down aspects like provided service quality in formal, court-enforceable contracts. This is *not* to say, though, that relational

contracts generally are more relevant in the service sector than in other industries. On the contrary, relational contracts seem to be important in many sectors, governing various relationships within and between firms (see Gibbons and Henderson, 2013, or Malcomson, 2013, for surveys). What I argue is that for low-paying jobs where a minimum wage is potentially relevant, formal performance measures exist to a lesser degree in the service sector than, for example, in manufacturing (in addition, the number of low-end jobs has become rather small in developed countries[14]). Examples for the importance of relational contracts in the service sector are provided by Gibbons and Henderson (2013), who discuss the example of Nordstrom employees who are expected to exercise "good judgement in all situations", which arguably is very difficulty to precisely define in formal contracts. Furthermore, Shemwell et.al (1994) find that aspects such as trust and commitment are very important in customer service-provider relationships.

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[14]I thank an anonymous referee for bringing up this point.

Appendix

Proof of Lemma 3.

I first show that $\tilde{U}_{\tau+1}(\tilde{e}_\tau) = U_{\tau+1}$ if the match continues in period $\tau + 1$. There, note that $\tilde{U}_{\tau+1}(\tilde{e}_\tau) = U_{\tau+1}$ is *feasible* because continuation play after both histories can be equalized. Now, assume to the contrary that $\tilde{U}_{\tau+1}(\tilde{e}_\tau) < U_{\tau+1}$. Because bargaining shares in any period are independent of the history of the game, it follows that $\tilde{S}_{\tau+1}(\tilde{e}_\tau) < S_{\tau+1}$. However this contradicts the assumption that players never agree on a Pareto-dominated outcome. It follows that $\tilde{U}_{\tau+1}(\tilde{e}_\tau) > U_{\tau+1}$ cannot be an equilibrium as well, giving $\tilde{U}_{\tau+1}(\tilde{e}_\tau) = U_{\tau+1}$ if the match continues in period $\tau + 1$.

It remains to show that effort is constant over time. To the contrary, assume there is a contractual social equilibrium where effort is not constant. Take a period τ^* where the stage-game surplus, $e_{\tau^*}\theta - c(e_{\tau^*})$ is maximized within this contractual social equilibrium (note that because $e_\tau\theta - c(e_\tau)$ is a strictly concave function, e_{τ^*} is unique). For any τ with $e_\tau \neq e_{\tau^*}$, replace the subsequent history with the history following τ^*. This is feasible and optimal because it does not tighten any constraint but relaxes some. ∎

Proof of Lemma 3.

The objective is to maximize $S = \frac{e^*\theta - c(e^*)}{1-\delta} - \delta\overline{\Pi} - \delta\overline{U}$, where the following aspects have to be taken into account:

1. Disagreement payoffs $\delta\overline{\Pi}$ and $\delta\overline{U}$ are taken as given because they are obtained after a separation of the current match, and because strategies are assumed to be contract specific

2. The optimal punishment for the firm after a deviation by the worker must be a separation of the current match

3. On the equilibrium path, worker and firm must prefer to remain in the current match

4. The (IC) constraint, $-c(e^*) + \delta\left(U - \overline{U}\right) \geq 0$, must hold

Point 1. states that the problem is equivalent to maximizing $e^*\theta - c(e^*)$ in every period. Regarding point 2, note that when the firm continues the present relationship after a deviation, its next period's profits are Π (because of my assumption that players never agree on a Pareto dominated outcome). If it terminates the present relationship, the firm's next-period profits are $\overline{\Pi}$, hence the condition $\overline{\Pi} \geq \Pi$ must hold. Since $M < 1$ implies $\overline{\Pi} = \Pi$, this condition is satisfied. Point 3. requires

$\Pi \geq \overline{\Pi}$ and $U \geq \overline{U}$ to hold, which is the case since $\overline{\Pi} = \Pi$ and $\overline{U} = 0$. Furthermore, it must be optimal to stay in the current match, compared to consuming one's exogenous outside option, i.e. $U, \Pi \geq 0$. There, note that given $M < 1$ payoffs are

$$
\begin{aligned}
U &= \alpha \frac{e^* \theta - c(e^*)}{1 - \alpha \delta} \\
\Pi &= \frac{1 - \alpha}{1 - \alpha \delta} \left(\frac{e^* \theta - c(e^*)}{1 - \delta} \right).
\end{aligned}
$$

Since $e^* \theta - c(e^*) \geq 0$, and given $\alpha \in [0, 1]$, equilibrium payoffs are non-negative.

Concerning point 4., note that plugging $U = \alpha \left(e^* \theta - c(e^*) \right) / (1 - \alpha \delta)$ and $\overline{U} = 0$ into (IC) and rearranging yields

$$
-c(e^*) + \delta \alpha e^* \theta \geq 0, \tag{IC}
$$

and the problem and has become to maximize $e\theta - c(e)$, subject to (IC). By construction, $e^* \theta - c(e^*)$ is (uniquely) maximized by e^{FB}. Hence, if $-c(e^{FB}) + \delta \alpha e^{FB} \theta \geq 0$, e^{FB} is the equilibrium effort level.

Now assume $-c(e^{FB}) + \delta \alpha e^{FB} \theta < 0$. It remains to show that e^* is characterized by the binding (IC) constraint. First, note that $-c(e) + \delta \alpha e \theta$ is a strictly concave function, and define $\tilde{e} \equiv \mathrm{argmax}\left(-c(e) + \delta \alpha e \theta \right)$. $\tilde{e}$ is unique and positive, and $\tilde{e} < e^{FB}$.

For $\alpha = 0$, $\tilde{e} = 0$ and the only effort level that actually satisfies (IC). Hence, $e^* = 0$ for $\alpha = 0$, with the (IC) constraint holding as an equality.

For $\alpha > 0$, $\tilde{e} > 0$. Take the largest effort level that satisfies (IC), i.e., where it holds as an equality. This effort level is smaller than e^{FB} (because of the concavity of $-c(e) + \delta \alpha e \theta$ and because $-c(e^{FB}) + \delta \alpha e^{FB} \theta < 0$), furthermore it is optimal because the objective function $e\theta - c(e)$ is strictly increasing for $e < e^{FB}$. Therefore, equilibrium effort e^* is the largest effort level such that $-c(e^*) + \delta \alpha e^* \theta = 0$. ∎

Proof of Proposition 1. First, I show that implementabe effort increases in α. If the (IC) constraint does not bind, effort is at its first-best and not affected by marginal changes in α. Now, assume that (IC) binds. Then, $\frac{de^*}{d\alpha} = -\frac{\delta \frac{(1-\delta)(e^* \theta - c(e^*))}{((1-\alpha\delta)(1-\delta))^2}}{-c' + \delta \frac{\alpha(\theta - c')}{(1-\alpha\delta)}}$, which is positive. This is because the denominator – which reflects the partial derivative of the left-hand side of the (IC) constraint with respect to effort – must be negative. If it were not negative, higher effort would relax the (IC) constraint,

contradicting that it binds. To show that U is increasing in α, note that $\frac{dU}{d\alpha} = \frac{\partial U}{\partial \alpha} + \frac{\partial U}{\partial e} \frac{de^*}{d\alpha}$, and $U = \frac{\alpha(e^*\theta - c(e^*))}{1 - \alpha\delta}$. The direct effect of α on U, $\frac{\partial U}{\partial \alpha} = \frac{(e^*\theta - c(e^*))}{(1 - \alpha\delta)^2} > 0$. Naturally, $\partial U/\partial e \geq 0$ as long as effort is not at its first-best. Finally, $de^*/d\alpha > 0$ for $e^* < e^{FB}$ and $de^*/d\alpha = 0$ for $e^* = e^{FB}$, establishing $dU/d\alpha > 0$.

Regarding the impact of α on profits Π, $\frac{d\Pi}{d\alpha} = \frac{\partial \Pi}{\partial \alpha} + \frac{\partial \Pi}{\partial e} \frac{de^*}{d\alpha}$ and $\Pi = \frac{(1 - \alpha)(e^*\theta - c(e^*))}{(1 - \alpha\delta)(1 - \delta)}$. There $\partial \Pi/\partial \alpha < 0$, whereas $\frac{\partial \Pi}{\partial e} \frac{de^*}{d\alpha} \geq 0$. Hence, $d\Pi/d\alpha < 0$ if the (IC) constraint does not bind. For a binding (IC) constraint and $e^* < e^{FB}$, $\frac{d\Pi}{d\alpha} = -\frac{(e^*\theta - c(e^*))(1 - \delta)}{[(1 - \delta)(1 - \alpha\delta)]^2} \left\{ \frac{\delta\theta - c'}{-c' + \delta\alpha\frac{(\theta - c')}{(1 - \alpha\delta)}} \right\}$. This is positive for $\delta\theta - c' > 0$ and negative for $\delta\theta - c' < 0$ (the denominator of the second term again represents the partial derivative of the left-hand side of the (IC) constraint with respect to effort and must hence be negative). To establish the existence of $\overline{\alpha} \in (0, 1)$, with $d\Pi/d\alpha > 0$ for $\alpha < \overline{\alpha}$ and $d\Pi/d\alpha < 0$ for $\alpha > \overline{\alpha}$, note that for $\alpha = 0$, the (IC) constraint becomes $-c(e^*) \geq 0$, which only holds for $e^* = 0$. Hence, $\Pi = 0$ for $\alpha = 0$, whereas $\Pi > 0$ for $0 < \alpha < 1$ (because of $de^*/d\alpha \geq 0$, with a strict inequality for $e^* < e^{FB}$, hence $e^* > 0$ for $\alpha > 0$). Furthermore, for $\alpha = 1$ the (IC) constraint becomes $-c(e^*) + e^*\theta\delta \geq 0$. If it binds, $\delta\theta - c' < 0$ because otherwise, a larger effort level would relax the (IC) constraint. If it does not bind, $\delta\theta - c' < 0$ holds as well because then, effort is characterized by $\theta - c' = 0$. This establishes the existence of $\overline{\alpha} \in (0, 1)$, with $d\Pi/d\alpha > 0$ for $\alpha < \overline{\alpha}$ and $d\Pi/d\alpha < 0$ for $\alpha > \overline{\alpha}$. ∎

Proof of Proposition 2. Note that absent a minimum wage, equilibrium wages are $w^* = c(e^*) + \alpha\frac{(1 - \delta)(e^*\theta - c(e^*))}{(1 - \alpha\delta)}$, where e^* is equilibrium effort absent a minimum wage. If $\overline{w} \leq w^*$, the minimum wage is not binding and irrelevant, in since it does not affect disagreement outcomes. Therefore, assume that $\overline{w} > w^*$. This has two effects, comparing the situations with and without the minimum wage. First, it represents a redistribution from firm to worker, hence Π goes down and U increases. Second, the increase in U also increases implementable effort. In the following, denote by $\overline{e}$ the maximum implementable effort given the minimum wage is paid to the worker, which characterized by

$$c(\overline{e}) = \overline{w}\delta \tag{2}$$

with

$$\frac{d\overline{e}}{d\overline{w}} = \frac{\delta}{c'} > 0. \tag{3}$$

Next, I show that $\overline{e}$ will actually be implemented – at least if $e^* < e^{FB}$. $\overline{e}$ would

not be implemented if it resulted in workers getting a share of the surplus that is below α. Then, bargaining would yield a different (lower) effort level. Only if the worker's effective surplus is at least α, maximum effort $\bar{e}$ will be implemented (note that it is not feasible to reduce the worker's effective surplus share – payments cannot be reduced below $\overline{w}$, and effort cannot be increased above $\bar{e}$).

Put differently, I have to compare $(\overline{w} - c(\bar{e}))/(1 - \delta)$, the worker's payoff given a binding minimum wage and given $\bar{e}$ is implemented, , to $\alpha(\bar{e}\theta - c(\bar{e}))/(1 - \alpha\delta)$, which represents the worker's "fair" payoff given $\bar{e}$ is implemented, i.e. given he receives a share α of the total net surplus when effort is $\bar{e}$.

If $(\overline{w} - c(\bar{e}))/(1 - \delta) \geq \alpha(\bar{e}\theta - c(\bar{e}))/(1 - \alpha\delta)$, then $\bar{e}$ is implemented, and the worker is paid $\overline{w}$ in every period. Otherwise, implemented effort is set to a level such that this condition holds as an equality. This condition can be rewritten to $\overline{w} \geq c(\bar{e}) + \frac{\alpha(1-\delta)(\bar{e}\theta - c(\bar{e}))}{(1-\alpha\delta)}$. Using (2) which gives $\overline{w} = c(\bar{e})/\delta$, and plugging it into the condition and re-arranging gives

$$-c(\bar{e}) + \alpha\delta\bar{e}\theta \leq 0. \tag{4}$$

Recall that the (IC) constraint without a minimum wage is

$$-c(e^*) + \alpha\delta e^*\theta \geq 0.$$

The left-hand-side of the (IC) constraint is concave in effort, and – in case it binds – furthermore decreasing for levels above e^*. In addition $\bar{e} > e^*$ for a binding minimum wage, hence (4) holds if (IC) binds absent a minimum wage. Therefore, $\bar{e}$ is implemented if $e^* < e^{FB}$.

Now, assume that $e^* = e^{FB}$. Then, condition (4) might or might not hold for a given value of $\overline{w}$. Assume it does not hold (otherwise, I am done). Then, the worker would get less than his "fair" share when exerting maximum feasible effort $\bar{e}$, hence effort e^{**} is set such that $U = \frac{\alpha(e^{**}\theta - c(e^{**}))}{(1-\alpha\delta)}$ and $\overline{w} = c(e^{**}) + \alpha\frac{(1-\delta)(e^{**}\theta - c())}{(1-\alpha\delta)}$. Still, though, equilibrium effort increases in the minimum wage because $\frac{de^{**}}{d\overline{w}} = \frac{1}{c' + \alpha\frac{(1-\delta)(\theta - c')}{(1-\alpha\delta)}} = \frac{(1-\alpha\delta)}{c'(1-\alpha) + \alpha(1-\delta)\theta} > 0$. ∎

Proof of Proposition 3. Assume we are in a steady state equilibrium where equilibrium employment is characterized by $-k + \delta\Pi = 0$. The unexpected introduction/increase of a binding minimum wage $\overline{w}$ has the following effect: If $-k + \delta\Pi$ is increased, employment goes up. Otherwise, M^* is reduced if and only if the minimum wage increase leads to $\Pi < 0$, and remains unaffected as long as $\Pi \geq 0$.

In the first case, I compute $\frac{dM^*}{d\overline{w}} = -\frac{d\Pi/\overline{w}}{d\Pi/dM}$. Since the denominator has to be negative (otherwise, the situation before would not have constituted an equilibrium), the sign of $dM^*/d\overline{w}$ is the same as the sign of $d\Pi/\overline{w}$. In Corollary 1, I established $\frac{d\Pi}{d\overline{w}} = \frac{\delta\theta - c'}{(1-\delta)c'}$, giving the first part of the proposition. Because $\theta(M)$ is continuous, a ceteris-paribus-increase in profits will always trigger an additional entry of firms, despite the counteracting effect on θ. Concerning the remainder of the proposition, note that if the minimum wage binds, Π is concave in $\overline{w}$. This follows from effort $\overline{e}$ being characterized by $\overline{w} - c(\overline{e})/\delta = 0$, and the convexity of $c(\cdot)$. ∎

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