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Abstract

We build a new Keynesian DSGE model consisting of two heterogeneous countries participating in a monetary union. We study how public debt consolidation in a country with high debt (like Italy) affects welfare in a country with solid public finances (like Germany). Our results show that debt consolidation in the high-debt country benefits the country with solid public finances over all time horizons. By constrast, in Italy, namely the country that takes the consolidation measures, such a policy is productive only in the medium and long term. Thus, although there is a conflict of national interests in shorter horizons, there is a common interest in the medium and long term. All this is with optimized feedback policy rules. By contrast, debt consolidation is welfare inferior to non-consolidation for both countries and all the time, if it is implemented in an ad hoc way, like an increase in income taxes. Therefore, the policy mix is important.

JEL-Codes: E600, F300, H600.

Keywords: debt consolidation, country spillovers, feedback policy rules, new Keynesian.

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1 Introduction

Since the global shock of 2008, several eurozone periphery countries have been in a multiple crisis.[1] In view of sustainability concerns and loss of confidence, these countries have been forced, among other things, to take restrictive fiscal policy measures which have further dampened demand in the short term. It is thus not surprising that fiscal consolidation has been one of the most debated policy areas over the past years and this controversy has been intensified by the lasting recession experienced by these countries.[2] On the other hand, fiscal policy in eurozone center countries, like Germany, has been neutral.[3] Nevertheless, the recession in the crisis countries has also affected the German economy, which is another reminder of the importance of spillovers in an integrated area like the euro area.

In this paper, we study how public debt consolidation in a country with high debt and sovereign premia affects welfare in other countries with solid public finances. In particular, we study how public debt consolidation in a country like Italy affects welfare in a country like Germany and how these cross-border effects depend on the fiscal policy mix chosen to bring public debt down.[4]

The setup is a new Keynesian DSGE model consisting of two heterogeneous countries forming a currency union. An international asset market allows private agents across countries to borrow from, or lend to, each other and the same market allows national governments to sell their bonds to foreign private agents. Regarding macroeconomic policy, being in a monetary union, there is a single monetary policy. On the other hand, the two countries are free to follow independent or national fiscal policies. Following most of the literature on debt consolidation (see below), we assume that policy is conducted via simple and implementable feedback rules, meaning that the single monetary policy is conducted via a standard Taylor rule, while national fiscal policy instruments can respond to the gap between public debt and target public debt as shares of output, as well as to the output gap. We experiment with various public debt policy targets depending on whether national policymakers aim just to stabilize the economy around its status quo (defined as the solution consistent with the recent data), or whether they also want to move the economy to a new reformed steady state (defined as a solution

[1] See e.g. the EEAG Report on the European Economy (2013) by CESifo and EMU-Public Finances (2015) by the European Commission.

[2] This is in particularly true in Cyprus, Greece, Italy, Portugal and Spain.

[3] See e.g. EMU-Public Finances (2015) by the European Commission.

[4] Italy's (public and foreign) debt position, although sizeable in absolute terms, is not one of the worst in the euroarea. Greece, Portugal, Spain, Ireland and Cyprus, are in a worse position (see e.g. the EEAG Report on the European Economy, 2012, by CESifo). However, since these countries have received financial aid from the EC-ECB-IMF, we prefer to use Italy as our euroarea periphery country.

with lower public debt than in the recent data). We compute optimized feedback policy rules where the welfare criterion is a weighted average of households' expected discounted lifetime utility in the two countries; thus, this can be thought of as a cooperative policy. In particular, adopting the methodology of Schmitt-Grohé and Uribe (2004 and 2007), we compute the welfare-maximizing values of feedback (monetary and fiscal) policy coefficients by taking a second-order approximation to both the equilbrium conditions and the welfare criterion. By computing optimized feedback policy rules for various fiscal policy instruments used at the same time, we do not have to make any arbitrary assumptions about which instrument to use to react to economic conditions (like public debt, output, or inflation) and/or how strong this reaction is. But, for comparison, we will also study ad hoc debt consolidation scenarios resembing recently observed fiscal policies.

We solve the above model numerically employing commonly used parameter values and fiscal policy data from Germany (called the home country) and Italy (called the foreign country). As we shall see, the steady state solution of this model can mimic relatively well the key features of the two countries over the euro years and, in particular, the current account deficits in Italy financed by current account surpluses in Germany over 2001-2011 (this is believed to be one of the most important macroeconomic imbalances in Europe today). It is useful to stress that this is achieved by simply allowing for differences in fiscal policy and the degree of patience; the latter means that Italians have been less patient than Germans during the euro period.[5] In turn, we use this solution as a point of departure to study the dynamic evolution of endogenous variables in response to policy reforms, focusing on debt consolidation in the high-debt country, namely, Italy.

Our main results are as follows. First, as expected, had tax-spending policy in Italy remained unchanged as in the data averages over 2001-2011, the model would be dynamically unstable. In other words, some fiscal reaction (spending cuts and/or tax rises) to public debt imbalances would be necessary for dynamic stability.

Second, debt consolidation in the high-debt country (Italy) benefits the country with solid public finances (Germany) over all time horizons. By constrast, in Italy, namely the country that takes the consolidation measures, such a policy is productive only in the medium and long term. Thus, in Italy, although the benefits outweigh the costs when the criterion is lifetime utility, debt consolidation comes at a short-term pain relative to non-consolidation. To put it differently, fiscal consolidation in a high debt country is a common interest over longer horizons but, in shorter horizons, there seems to be a conflict of national interests. It is interesting to

[5] See subsection 2.1 below for these assumptions. See also Economides et al. (2016) for further details.

add that the long-term benefits from fiscal consolidation become more substantial for both countries when debt reduction is such that sovereign premia are also eliminated in the new reformed steady state; but such elimination requires an equalization of time discount factors, or an equal degree of patience, across countries in the new reformed steady state (see section 2.1 for further details). All this holds with optimized feedback policy rules.

Third, the least distorting fiscal policy mix from the point of view of both countries is the one which, during the early phase of pain, Italy cuts public spending to address its public debt problem and, at the same time, reduces (labor and capital) income tax rates in order to mitigate the recessionary effects of spending cuts, while, once its public debt has been reduced in the later phase, it uses the fiscal space created to further cut capital taxes which are particularly distorting. Note that the anticipation of cuts in capital taxes in the future, once debt consolidation has been achieved, plays a key role even in the short term. Use of public spending (and not taxes) is also recommended in Germany, where the policy aim is cyclical stabilization only. It is also interesting to report that the higher the say of Germany in policy setting, the stronger the fiscal consolidation in Italy should be during the early period of pain. As said above, all this holds with optimized feedback policy rules, so these are normative results.

The fourth result is about ad hoc policies. The implications of ad hoc policies are very different from the normative implications listed above. In particular, we experiment with an ad hoc scenario of debt consolidation that resembles fiscal policy after 2012; this means that, in Italy, the tax revenue to GDP ratio rises by around two percentage points, while the spending ratio remains practically unchanged, and, in Germany, fiscal policy is kept neutral. In this ad hoc case, debt consolidation in Italy is harmful for both countries and across all time horizons, always relative to non-consolidation. Therefore, the way public debt is brought down is important.

Our work is related to at least two literatures. First, it is related to the literature on how monetary and fiscal policy instruments react, or should react, to the business cycle (see e.g. Leeper (1991), Schmitt-Grohé and Uribe (2005 and 2007), Leith and Wren-Lewis (2008), Batini et al. (2008), Leeper et al. (2009), Kirsanova et al. (2009), Malley et al. (2009), Bi and Kumhof (2011), Kirsanova and Wren-Lewis (2012), Cantore et al. (2015) and Philippopoulos et al. (2015)). Second, it is related to the literature on fiscal consolidation that usually compares spending cuts versus tax rises needed for debt reduction in DSGE models (see e.g. Coenen et al. (2008), Forni et al. (2010), Erceg and Lindé (2013), Cogan et al. (2013), Bi et al. (2013), Benigno and Romei (2014) and Benigno et al. (2014)).

Nevertheless, as far as we know, there have not been any previous attempts to search for the best possible use of the tax-spending policy instruments (when we depart from their values in the recent data) in a new Keynesian DSGE model consisting of two heterogeneous countries (one with weak public finances and external debt and the other with neutral public finances and external assets) and study the cross-border implications of debt consolidation policies in a high-debt country. We also focus on optimized feedback policy rules so, as already pointed out above, our results are not driven by arbitrary assumptions about which policy instruments to use to react to macroeconomic indicators and/or how strong these reactions are.

The rest of the paper is organized as follows. Section 2 presents the model. The status quo solution is in section 3. Section 4 explains our policy experiments. Results are reported in sections 5 and 6. Section 7 closes the paper. An appendix provides technical details.

2 A two-country model of a monetary union

This section sets up a New Keynesian DSGE model consisting of two heterogeneous countries forming a monetary union. We start with an informal description of the model.[6]

2.1 Informal description of the model and discussion of key assumptions

Two countries form a closed system in a new Keynesian setup. In each country, there are households, firms and a national fiscal authority or government. In a regime of a currency union, there is a single monetary authority or central bank.

Households in each country can save in the form of physical capital, domestic government bonds and internationally traded assets. The market for internationally traded assets allows private agents across countries to borrow from, or lend to, each other and it also allows national governments to sell their bonds to foreign private agents.[7] In other words, the government in each country can sell its bonds to domestic and foreign households, where the latter, namely, government's borrowing from abroad, takes place via the international asset market. We assume that all international borrowing/lending takes place through a financial intermediary or bank. This financial intermediation requires a transaction, or monitoring, cost proportional to the amount of the nation's debt.[8] This cost creates, in turn, a wedge between the borrowing

[6] The model is similar to that in Economides et al (2016). However, here we study debt consolidation within a currency union, while that paper compares a currency union to flexible exchange rates and a fiscal union.

[7] See also Forni et al. (2010), Cogan et al. (2013), Erceg and Lindé (2013) and many others.

[8] Instead of using the device of a financial intermediary or bank, we could just assume transaction costs incurred upon borrowers (see e.g. Forni et al. (2010), Cogan et al. (2013), Erceg and Lindé (2013) and many

and the lending interest rate. As a result, when they participate in the international asset market, agents (private and public) in the debtor country face a higher interest rate than agents (private and public) in the creditor country.[9] To the extent that the bank makes a profit, this profit is rebated lump-sum to households in the creditor country.

As is well-known, systematic borrowing and lending cannot occur in an homogeneous world. Some type of heterogeneity is needed. A popular way of producing borrowers and lenders has been to assume that agents differ in their patience to consume or, equivalently, in their discount factors; specifically, the discount factor of lenders is higher than that of borrowers or, equivalently, borrowers are more impatient than lenders.[10] It is also well-known that such differences in discount factors need to be combined with an imperfection in the capital market in order to get a well-defined solution;[11] in our model, the capital market imperfection is the transaction, or monitoring, cost of the loan, as said above. Therefore, in our model, the international transcation cost ensures, not only stationarity of foreign asset positions as is typically the case in the literature (see e.g. Schmitt-Grohe and Uribe (2003)), but it also allows for a well-defined solution with different discount factors across different countries.

The solution of the above described model will imply that one country is a net lender and the other is a net borrower in the international asset market and that interest rates are higher in the net debtor country. Given the current account data over the euro years, we will think of the lender country as Germany and the debtor country as Italy. In this case, in equilibrium, the relatively impatient Italians will finance their current account deficits by borrowing funds from the patient Germans who run current account surpluses. This scenario, as said above, is as in the euro period data. It is also consistent with the literature on the interpretation of current accounts in the sense that systematic low saving rates and current account deficits are believed to reflect relatively low patience.[12]

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others who assume a transcation cost when agents trade in the international asset market). We prefer however the bank device because we find it to be more intuitive (see also e.g. Curdia and Woodford (2009, 2010) and Benigno et al (2014) although in a closed economy).

[9] That is, here, differences in interest rates across countries are produced by transcation or monitoring costs incurred by the bank. As is known such differences can be produced in various ways including the probability of sovereign default (see subsection 2.5 below for details).

[10] See also e.g. Benigno et al. (2014). Kiyotaki and Moore (1997) also use a general equilibrium model with two types of agents, creditors and borrowers, who discount the future differently. Note that we could further enrich our model so as the discount factors are formed endogenously; see e.g. Becker and Mulligan (1997) and Doepke and Zilibotti (2008) for an endogenous formation of discount factors depending on income, education, effort, religion, etc. See also e.g. Schmitt-Grohe and Uribe (2003) and Choi et al. (2008) for calibrated models where the discount factor depends on consumption changes.

[11] See also e.g. Benigno et al. (2014). Similarly, in Doepke and Zilibotti (2008), some financial market imperfections are necessary for getting differences in patience across different social classes.

[12] See e.g. Choi et al. (2008).

On other dimensions, the model is a standard new Keynesian currency union model.[13] In particular, each country produces an array of differentiated goods and, in both countries, firms act monopolistically facing Calvo-type nominal fixities. Nominal fixities give a real role to monetary and exchange rate policy, at least in the transition path. In a monetary union, we assume a single monetary policy but independent national fiscal policies. Policy (both monetary and fiscal) is conducted by optimized state-contingent policy rules.

In the home economy, there are N identical households and N firms each one of them producing a differentiated domestically produced tradable good. Similarly, in the foreign economy. For simplicity, population in both countries, N and N^*, is constant over time and the two countries are of equal size, $N = N^*$.

The rest of this section formalizes the above story. We will present the domestic country. The structure of the foreign country will be analogous except otherwise said. A star will denote the counterpart of a variable or a parameter in the foreign country.

2.2 Households

This subsection presents households in the domestic country. There are N identical households indexed by $i = 1, 2, ..., N$.

2.2.1 Consumption bundles

The quantity of each variety h produced at home by domestic firm h and consumed by each domestic household i is denoted as $c_{i,t}^H(h)$. Using a Dixit-Stiglitz aggregator, the composite of domestic goods consumed by each domestic household i, $c_{i,t}^H$, consists of h varieties and is given by:[14]

$$c_{i,t}^H = \left[\sum_{h=1}^{N} \kappa [c_{i,t}^H(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{1}$$

where $\phi > 0$ is the elasticity of substitution across goods produced in the domestic country and $\kappa = 1/N$ is a weight chosen to avoid scale effects in equilibrium.

Similarly, the quantity of each imported variety f produced abroad by foreign firm f and consumed by each domestic household i is denoted as $c_{i,t}^F(f)$. Using a Dixit-Stiglitz aggregator, the composite of imported goods consumed by each domestic household i, $c_{i,t}^F$, consists of f

[13] See Okano (2014) for a review of the related literature dating back to Galí and Monacelli (2005, 2008).

[14] As in e.g. Blanchard and Giavazzi (2003), here we work with summations rather than with integrals.

varieties and is given by:

$$c_{i,t}^{F} = \left[\sum_{f=1}^{N} \kappa [c_{i,t}^{F}(f)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{2}$$

In turn, having defined $c_{i,t}^{H}$ and $c_{i,t}^{F}$, domestic household i's consumption bundle, $c_{i,t}$, is:

$$c_{i,t} = \frac{\left(c_{i,t}^{H}\right)^{\nu} \left(c_{i,t}^{F}\right)^{1-\nu}}{\nu^{\nu}(1-\nu)^{1-\nu}} \tag{3}$$

where ν is the degree of preference for domestic goods (if $\nu > 1/2$, there is a home bias).

2.2.2 Consumption expenditure, prices and terms of trade

Domestic household i's total consumption expenditure is:

$$P_t c_{i,t} = P_t^H c_{i,t}^H + P_t^F c_{i,t}^F \tag{4}$$

where P_t is the consumer price index (CPI), P_t^H is the price index of home tradables, and P_t^F is the price index of foreign tradables (expressed in domestic currency).

Each domestic household's total expenditure on home goods and foreign goods are respectively:

$$P_t^H c_{i,t}^H = \sum_{h=1}^{N} \kappa P_t^H(h) c_{i,t}^H(h) \tag{5}$$

$$P_t^F c_{i,t}^F = \sum_{f=1}^{N} \kappa P_t^F(f) c_{i,t}^F(f) \tag{6}$$

where $P_t^H(h)$ is the price of each variety h produced at home and $P_t^F(f)$ is the price of each variety f produced abroad, both denominated in domestic currency.

We assume that the law of one price holds meaning that each tradable good sells at the same price at home and abroad. Thus, $P_t^F(f) = S_t P_t^{H*}(f)$, where S_t is the nominal exchange rate (where an increase in S_t implies a depreciation) and $P_t^{H*}(f)$ is the price of variety f produced abroad denominated in foreign currency. As said above, a star denotes the counterpart of a variable or a parameter in the rest-of-the world. Note that the terms of trade are defined as $\frac{P_t^F}{P_t^H}$ $(= \frac{S_t P_t^{H*}}{P_t^H})$, while the real exchange rate is defined as $\frac{S_t P_t^*}{P_t}$. In a currency union model, we will exogenously set $S_t \equiv 1$ at all t.

2.2.3 Household's optimization problem

Each domestic household i acts competitively to maximize expected discounted lifetime utility, V_0, defined as:

$$V_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_{i,t}, n_{i,t}, m_{i,t}, g_t\right) \tag{7}$$

where $c_{i,t}$ is i's consumption bundle as defined above, $n_{i,t}$ is i's hours of work, $m_{i,t}$ is i's real money holdings, g_t is per capita public spending, $0 < \beta < 1$ is domestic agents' discount factor, and E_0 is the rational expectations operator.

For our numerical solutions, the period utility function will be (see also e.g. Galí, 2008):

$$u_{i,t}\left(c_{i,t}, n_{i,t}, m_{i,t}, g_t\right) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_{i,t}^{1+\varphi}}{1+\varphi} + \chi_m \frac{m_{i,t}^{1-\mu}}{1-\mu} + \chi_g \frac{g_t^{1-\zeta}}{1-\zeta} \tag{8}$$

where χ_n, χ_m, χ_g, σ, φ, μ, ζ are standard preference parameters. Thus, $1/\sigma$ is the elasticity of substitution between consumption at two points in time and $1/\varphi$ is the Frisch labour elasticity.

The period budget constraint of household i in the domestic country written in real terms is:

$$\begin{gathered}
\left(1+\tau_t^c\right)\left[\tfrac{P_t^H}{P_t} c_{i,t}^H + \tfrac{P_t^F}{P_t} c_{i,t}^F\right] + \tfrac{P_t^H}{P_t} x_{i,t} + b_{i,t} + m_{i,t} + \tfrac{S_t P_t^*}{P_t} f_{i,t}^h = \\
= \left(1-\tau_t^k\right)\left[r_t^k \tfrac{P_t^H}{P_t} k_{i,t-1} + \widetilde{\omega}_{i,t}\right] + \left(1-\tau_t^n\right) w_t n_{i,t} + R_{t-1} \tfrac{P_{t-1}}{P_t} b_{i,t-1} + \\
+ \tfrac{P_{t-1}}{P_t} m_{i,t-1} + Q_{t-1} \tfrac{S_t P_t^*}{P_t} \tfrac{P_{t-1}^*}{P_t^*} f_{i,t-1}^h - \tau_{i,t}^l + \pi_{i,t}
\end{gathered} \tag{9}$$

where $x_{i,t}$ is i's domestic investment, $b_{i,t}$ is the real value of i's end-of-period domestic government bonds, $m_{i,t}$ is i's end-of period real domestic money holdings, $f_{i,t}^h$ is the real value of i's end-of-period internationally traded assets denominated in foreign currency (if $f_{i,t}^h < 0$, it denotes private foreign debt), r_t^k denotes the real return to the beginning-of-period domestic capital, $k_{i,t-1}$, $\widetilde{\omega}_{i,t}$ denotes i's real dividends received by domestic firms, w_t is the real wage rate, $R_{t-1} \geq 1$ denotes the gross nominal return to domestic government bonds between $t-1$ and t, $Q_{t-1} \geq 1$ denotes the gross nominal return to international assets between $t-1$ and t, $\tau_{i,t}^l$ are real lump-sum taxes/transfers to each household, $\pi_{i,t}$ is profits distributed in a lump-sum fashion to the domestic household by the financial intermediary (see below) in a lump-sum fashion and $0 \leq \tau_t^c$, τ_t^k, $\tau_t^n \leq 1$ are the tax rates on consumption, capital income and labour income respectively. Note that small letters denote real values, namely, $m_{i,t} \equiv \frac{M_{i,t}}{P_t}$, $b_{i,t} \equiv \frac{B_{i,t}}{P_t}$, $f_{i,t}^h \equiv \frac{F_{i,t}^h}{P_t^*}$, $w_t \equiv \frac{W_t}{P_t}$, $\widetilde{\omega}_{i,t} \equiv \frac{\widetilde{\Omega}_{i,t}}{P_t}$, $\tau_{i,t}^l \equiv \frac{T_{i,t}^l}{P_t}$, where capital letters denote nominal values.

The law of motion of physical capital for each household i is:

$$k_{i,t} = (1-\delta) k_{i,t-1} + x_{i,t} - \frac{\xi}{2}\left(\frac{k_{i,t}}{k_{i,t-1}} - 1\right)^2 k_{i,t-1} \tag{10}$$

where $0 < \delta < 1$ is the depreciation rate of capital and $\xi \geq 0$ is a parameter capturing adjustment costs related to physical capital.

Further details on the household's problem, its first-order conditions and implications for the price bundles are in Appendix 1.

2.3 Firms

This subsection presents firms in the domestic economy. There are N domestic firms indexed by $h = 1, 2, ..., N$. Each firm h produces a differentiated tradable good of variety h under monopolistic competition and Calvo-type nominal fixities.

2.3.1 Demand for firm's product

The demand for each domestic firm h's product, $y_t^H(h)$, is (see Appendix 2 for details):

$$y_t^H(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} y_t^H \tag{11}$$

where y_t^H is total product in the domestic country.

2.3.2 Firm's optimization problem

Nominal profits of each domestic firm h are defined as:

$$\widetilde{\Omega}_t(h) \equiv P_t^H(h)y_t^H(h) - r_t^k P_t^H(h)k_{t-1}(h) - W_t n_t(h) \tag{12}$$

where $k_{t-1}(h)$ and $n_t(h)$ denote respectively the current capital and labor inputs chosen by the firm.

Maximization is subject to the demand function, (11), and the production function:

$$y_t^H(h) = A_t[k_{t-1}(h)]^{\alpha}[n_t(h)]^{1-\alpha} \tag{13}$$

where A_t is an exogenous stochastic TFP process whose motion is defined below and $0 < \alpha < 1$ is a technology parameter.

In addition, following Calvo (1983), firms choose their prices facing a nominal fixity. In particular, in each period, each firm h faces an exogenous probability θ of not being able to reset its price. A firm h, which is able to reset its price at time t, chooses its price $P_t^{\#}(h)$ to maximize the sum of discounted expected nominal profits for the next k periods in which it may have to keep its price fixed. This objective is given by:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \widetilde{\Omega}_{t+k}(h) = E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left\{ P_t^{\#}(h)\, y_{t+k}^H(h) - \Psi_{t+k}\left(y_{t+k}^H(h)\right) \right\}$$

where $\Xi_{t,t+k}$ is a discount factor taken as given by the firm (but, in equilibrium, it equals the household's intertemporal marginal rate of substitution in consumption), $y_{t+k}^H(h) = \left[\frac{P_t^{\#}(h)}{P_{t+k}^H}\right]^{-\phi} y_{t+k}^H$ is the demand function in future periods and $\Psi_t(.)$ denotes the minimum nominal cost function for producing $y_t^H(h)$ at t so that $\Psi_t'(.)$ is the associated nominal marginal cost.

Further details on the firm's problem and its first-order conditions are in Appendix 2.

2.4 Government budget constraint

This subsection presents the government budget constraint in the domestic economy (details are in Appendix 3). The period budget constraint of the consolidated government sector expressed in real terms and aggregate quantities is:

$$\begin{array}{c} b_t + \frac{S_t P_t^*}{P_t} f_t^g + m_t = R_{t-1} \frac{P_{t-1}}{P_t} b_{t-1} + Q_{t-1} \frac{S_t P_t^*}{P_t} \frac{P_{t-1}^*}{P_t^*} f_{t-1}^g + \frac{P_{t-1}}{P_t} m_{t-1} + \\ + \frac{P_t^H}{P_t} g_t - \tau_t^c \left(\frac{P_t^H}{P_t} c_t^H + \frac{P_t^F}{P_t} c_t^F\right) - \tau_t^k \left(r_t^k \frac{P_t^H}{P_t} k_{t-1} + \widetilde{\omega}_{i,t}\right) - \tau_t^n w_t n_t - \tau_t^l \end{array} \quad (14)$$

where b_t is the end-of-period domestic real public debt held by domestic households, f_t^g is the end-of-period domestic real public debt held by foreing households and expressed in foreign prices,[15] and m_t is the end-of-period stock of real money balances. As said, small letters denote real variables, $b_t \equiv \frac{B_t}{P_t}$, $m_t \equiv \frac{M_t}{P_t}$ and $f_t^g \equiv \frac{F_t^g}{P_t^*}$. [16] Also, the government allocates its total expenditure among product varieties h by solving an identical problem with household i, so that $g_t(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} g_t$.

If we define total nominal public debt in the domestic country as $D_t \equiv B_t + S_t F_t^g$, so that in real terms $d_t \equiv b_t + \frac{S_t P_t^*}{P_t} f_t^g$, we have $b_t \equiv \lambda_t d_t$ and $\frac{S_t P_t^*}{P_t} f_t^g \equiv (1-\lambda_t) d_t$, where $0 \leq \lambda_t \leq 1$ is the fraction of domestic public debt held by domestic private agents and $0 \leq 1-\lambda_t \leq 1$ is the fraction of domestic public debt held by foreign private agents.

In each period, one of the fiscal instruments (τ_t^c, τ_t^k, τ_t^n, g_t, τ_t^l, λ_t, d_t) follows residually to satisfy the government budget constraint.[17] We assume, except otherwise said, that this role

[15] That is, since the returns to bonds held by domestic agents and to the same bonds held by foreign agents can differ, our modelling implies that the government bond market can be segmented.

[16] Note that we also use the definitions $c_t^H \equiv \sum_{i=1}^N c_{i,t}^H$, $c_t^F \equiv \sum_{i=1}^N c_{i,t}^F$, $k_{t-1} \equiv \sum_{i=1}^N k_{i,t-1}$, $\widetilde{\Omega}_t \equiv \sum_{i=1}^N \widetilde{\Omega}_{i,t}$, $n_t \equiv \sum_{i=1}^N n_{i,t}$, $F_{t-1}^h \equiv \sum_{i=1}^N F_{i,t-1}^h$, $B_{t-1} \equiv \sum_{i=1}^N B_{i,t-1}$ and $T_t^l \equiv \sum_{i=1}^N T_{i,t}^l$.

[17] That is, we treat the share of public debt held by foreign private agents, $(1-\lambda_t)$, as a policy instrument rather than as an endogenous variable. In general, an assumption like this is unavoidable sooner or later. To understand this, recall that, in our model, there is a single internationall asset subject to a single transaction cost. Thus, all international borrowing (by private and by public agents) takes place via this market. Since we do not allow for separare international asset markets (one for private and one for public assets), we need an extra

is played by the end-of-period total public debt, d_t.

2.5 World financial intermediary

We use a simple and popular model of financial frictions (see e.g. Uribe and Yue, 2006, Curdia and Woodford, 2009 and 2010, Benigno et al., 2014). International borrowing, or lending, takes place through a financial intermediary or a bank. This bank is located in the home country. The bank plays a traditional role only, which consists in collecting deposits from lenders and lending the funds to borrowers.

In particular, the bank raises funds from domestic private agents, $\left(f_t^h - f_t^g\right)$, at the rate Q_t and lends to foreign agents, $(f_t^{*g} - f_t^{*h})$, at the rate Q_t^*.[18] In addition, the bank faces operational costs, which are increasing and convex in the volume of the loan, $(f_t^{*g} - f_t^{*h})$. The profit of the bank is revenue minus cost where revenue is net of transcation or monitoring costs. Thus, the profit written in real terms in the domestic country is given by (details are in Appendix 4):

$$\pi_t = Q_{t-1}^* \left[\frac{P_{t-1}}{P_t}(f_{t-1}^{*g} - f_{t-1}^{*h}) - \frac{P_t^H}{P_t}\frac{P_{t-1}^H}{P_t^H}\frac{\psi}{2}(f_{t-1}^{*g} - f_{t-1}^{*h})^2 \right] - Q_{t-1}\frac{S_t}{S_{t-1}}\frac{P_{t-1}}{P_t}(f_{t-1}^{*g} - f_{t-1}^{*h}) \quad (15)$$

where $\frac{\psi}{2}(f_{t-1}^{*g} - f_{t-1}^{*h})^2$ is the real cost function and $\psi \geq 0$ is a parameter (see subsection 3.1 below for its value). The first term in the brackets on the RHS is the bank's return on the loan net of monitoring cost, while the last term is payments to the savers.[19]

At each t, the bank chooses the volume of its loan taking Q_t and Q_t^* as given. The optimality condition is (details are again in Appendix 4):

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assumption to get a solution and this is provided by treating λ_t as an exogenous variable in each country (it will be set as in the data average). Alternatively, we could assume that private agents in each country can separately invest in foreign private assets and foreign governemnt bonds (rather than in a single international asset). But, as is known, this modelling would lead to a non-well specified system (a kind of portfolio indeterminacy), except if one is willing to assume different transaction costs in different asset markets. In the latter case, portfolio shares could be determimed but their solution would depend on the parameterization of the associated transcation cost function (see e.g. Economides et al. (2013)). This would not be different from treating λ_t exogenously in the first place.

[18] Recall that f_t^h denotes private foreign assets and f_t^g denotes public foreign debt (i.e. public debt held by foreign agents) in the home country. Similarly in the foreign country. Thus, if it so happens that $\left(f_t^h - f_t^g\right)$ is positive, it denotes net foreign assets in the home country and if it so happens that $(f_t^{*g} - f_t^{*h})$ is positive, it denotes net foreign liabilities in the foreign country. In equilibrium, $(f^{*g} - f^{*h}) + \frac{S_t P_t^*}{P_t}(f^g - f^h) = 0$.

[19] Note that, as in Curdia and Woodford (2009 and 2010), any resources consumed by the bank for the monitoring of its financial operations are part of the aggregate demand for the Dixit-Stiglitz composite good (details are in Appendices 2 and 5). Also note that the bank is located in the home country so that its profits are distributed to private agents in the domestic country in a lump-sum fashion, where $\pi_t = \sum_{i=1}^N \pi_{i,t}$ in equilibrium.

$$Q^*_{t-1} = \frac{Q_{t-1}\frac{S_t}{S_{t-1}}}{1 - \frac{P^H_{t-1}}{P_{t-1}}\psi(f^{*g}_{t-1} - f^{*h}_{t-1})} \tag{16}$$

where, in a currency union, $S_t \equiv 1$; thus, $Q^*_t > Q_t$ which means that borrowers pay a sovereign premium.

It needs to be stressed that the implied property in equation (16) - namely, that the interest rate, at which the country borrows from the rest of the world, is increasing in the nation's total foreign debt - is supported by a number of empirical studies (see e.g. EMU-Public Finances (2012) by the European Commission). It should also be stressed that a similar type of endogeneity of the country premium can be produced by several other models, including models of default risk.[20]

2.6 Monetary and fiscal policy

We now specify monetary and fiscal policy. As in most of the related literature, we follow a rule-like approach to policy.

2.6.1 Single monetary policy rule in a monetary union

If we had flexible exchange rates, the exchange rate would be an endogenous variable and the two countries' nominal interest rates, R_t and R^*_t, could be free to be set independently by the national monetary authoritis, say, to follow national Taylor-type rules (see section 6 for flexible exchange rates). Here, by contrast, to mimic the eurozone regime, we assume that only one of the interest rates, R_t, can follow a Taylor-type rule, while R^*_t is an endogenous variable replacing the exchange rate which becomes an exogenous policy variable (this modelling, where the union's central bank uses one of national governments' interest rates as its policy instrument, is similar to that in e.g. Galí and Monacelli (2008) and Benigno and Benigno (2008)).[21]

In particular, we assume a single monetary policy rule of the form:

[20] Default risk reflects the fear of repudiation of debt obligations but also the fear of new wealth taxes with retroactive effect on debt repayments (see Alesina et al., 1992, for an early study). As Corsetti et al. (2013) point out, there are two approaches to sovereign default. The first approach models it as a strategic choice of the government (see e.g. Eaton and Gersovitz, 1981, Arellano, 2008, and many others). The second approach assumes that default occurs when debt exceeds an endogenous fiscal limit (see Bi, 2012, and many others). In terms of modelling, a simple way of allowing for sovereign default risk is to replace R^* with $R^* \equiv (1-\Delta^*)R^{**}$, where R^{**} is the gross interest rate on government bonds and $0 \leq \Delta^* < 1$ is the sovereign default rate in the foreign country. The latter, namely Δ^*, can be either exogenoulsy set or endogenously determined as, say, a function of the level of public debt (see the papers mentioned above). Then, our solution for the net-of-default interest rate, R^*, would in turn give a solution for the gross interest rate, R^{**}. Obviously, the higher the default rate, the higher the gross interest rate.

[21] For various ways of modelling monetary policy in a monetary union, see e.g. Dellas and Tavlas (2005) and Collard and Dellas (2006).

$$\log\left(\frac{R_t}{R}\right) = \phi_\pi\left(\eta\log\left(\frac{\Pi_t}{\Pi}\right)+(1-\eta)\log\left(\frac{\Pi_t^*}{\Pi^*}\right)\right)+$$
$$+\phi_y\left(\eta\log\left(\frac{y_t^H}{y^H}\right)+(1-\eta)\log\left(\frac{y_t^{*H}}{y^{*H}}\right)\right) \tag{17}$$

where $\phi_\pi \geq 0$ and $\phi_y \geq 0$ are respectively feedback monetary policy coefficients on inflation and the output gap, $0 \leq \eta \leq 1$ is the political weight given to the domestic country relative to the foreign country (see subsection 3.1 below for the value of this parameter) and variables without time subscripts denote policy targets (in the case of monetary policy, the policy targets are simply the steady state values of the corresponding variables).

2.6.2 National fiscal policy rules

Countries can follow independent fiscal policies. As in the case of monetary policy above, we focus on simple rules meaning that national fiscal authorities react to a small number of easily observable macroeconomic indicators. In particular, in each country, we allow the main spending-tax policy instruments, namely, government spending as share of output, defined as s_t^g, and the tax rates on consumption, capital income and labor income, τ_t^c, τ_t^k and τ_t^n, to react to the public debt-to-output ratio as deviation from a target, as well as to the output gap, according to simple linear rules:[22]

$$s_t^g - s^g = -\gamma_l^g\left(l_{t-1}-l\right)-\gamma_y^g\left(y_t^H-y^H\right) \tag{18}$$

$$\tau_t^c - \tau^c = \gamma_l^c\left(l_{t-1}-l\right)+\gamma_y^c\left(y_t^H-y^H\right) \tag{19}$$

$$\tau_t^k - \tau^k = \gamma_l^k\left(l_{t-1}-l\right)+\gamma_y^k\left(y_t^H-y^H\right) \tag{20}$$

$$\tau_t^n - \tau^n = \gamma_l^n\left(l_{t-1}-l\right)+\gamma_y^n\left(y_t^H-y^H\right) \tag{21}$$

where public liabilities are defined as:

$$l_t \equiv \frac{R_t\lambda_t D_t + Q_t\frac{S_{t+1}}{S_t}\left(1-\lambda_t\right)D_t}{P_t^H y_t^H} \tag{22}$$

[22] For similar rules, see e.g Schmitt-Grohé and Uribe (2007) and Cantore et al. (2015). See also EMU-Public Finances (2011) by the European Commission for fiscal reaction functions used in practice.

where $\gamma_l^q \geq 0$ and $\gamma_y^q \geq 0$, for $q \equiv (g,\ c,\ k,\ n)$, are respectively feedback fiscal policy coefficients on inherited public liabilities and the output gap, and variables without time subscripts denote policy targets (see subsection 4.1 below for definition of fiscal policy targets). Notice that the rest of fiscal policy instruments (namely, lump-sum tranfers, τ^l, and the fraction of public debt held by domestic agents, λ) are set at their data average values in all time periods.

Fiscal policy in the foreign country is modelled similarly.

2.7 Exogenous variables and shocks

We now specify the exogenous variables, $A_t, A_t^*, \tau_t^l, \lambda_t, \tau_t^{l*}, \lambda_t^*$ and $\frac{S_{t+1}}{S_t}$. Starting with TFP in the two countries, A_t and A_t^*, we assume stochastic $AR(1)$ processes of the form:

$$\log(A_t) = (1-\rho^a)\log(A) + \rho^a \log(A_{t-1}) + \varepsilon_t^\alpha \tag{23}$$

$$\log(A_t^*) = (1-\rho^{*a})\log(A^*) + \rho^{*a} \log\left(A_{t-1}^*\right) + \varepsilon_t^{*\alpha} \tag{24}$$

where $0 < \rho^a, \rho^{*a} < 1$ are persistence parameters, variables without time subscript denote long-run values and $\varepsilon_t^a \sim N\left(0,\ \sigma_a^2\right), \varepsilon_t^{*a} \sim N\left(0,\ \sigma_a^{*2}\right)$.

The exogenously set fiscal policy instruments, $\{\tau_t^l, \lambda_t, \tau_t^{l*}, \lambda_t^*\}_{t=0}^\infty$, or equivalently, if we express lump-sum transfers as share of output, $\{s_t^l, \lambda_t, s_t^{l*}, \lambda_t^*\}_{t=0}^\infty$,[23] are assumed to be constant and equal to their data average values in all time preiods. Finally, as said, in a regime of a currency union, we set $S_t \equiv 1$ at any t.

In other words, we assume that stochasticity comes from shocks to TFP only (we report however that our main results do not depend on this).

2.8 Equilibrium system in the status quo economy

We now combine all the above to get the equilibrium system for any feasible policy. This is defined to be a sequence of allocations, prices and policies such that: (i) households maximize utility; (ii) a fraction $(1-\theta)$ of firms maximize profits by choosing an identical price $P_t^{\#}$, while a fraction θ just set prices at their previous period level; (iii) the international bank maximizes its profit (iv) all constraints, including the government budget constraint and the balance of payments, are satisfied; (v) all markets clear, including the international asset market; (vi) policy instruments are set by rules.

[23] Thus, $s_t^l \equiv \frac{\tau_t^l}{y_t^H TT_t^{\nu-1}}$ and $s_t^{l*} \equiv \frac{\tau_t^{l*}}{y_t^{H*} TT_t^{1-\nu^*}}$.

This equilibrium system is presented in detail Appendix 5. It consists of 59 equations in 59 variables, $\{V_t,\, y_t^H, c_t,\, c_t^H,\, c_t^F,\, n_t,\, x_t,\, k_t,\, f_t^h,\, m_t,\, TT_t,\, \Pi_t,\, \Pi_t^H,\, \Theta_t,\, \Delta_t,\, w_t,\, mc_t,\, \widetilde{\omega}_t,\, r_t^k,\, d_t,\, \Pi_t^*,\, z_t^1,\, z_t^2,\, \pi_t,\, q_t,\, Q_t,\, l_t,\, V_t^*,\, y_t^{*H},\, c_t^*,\, c_t^{H*},\, c_t^{F*},\, n_t^*,\, x_t^*,\, k_t^*,\, f_t^{h*},\, m_t^*,\, \Pi_t^{H*},\, \Theta_t^*,\, \Delta_t^*,\, w_t^*,\, mc_t^*,\, \widetilde{\omega}^*,\, r_t^{*k},\, d_t^*,\, z_t^{1*},\, z_t^{2*},\, Q_t^*,\, l_t^*,\, R_t,\, s_t^g,\, \tau_t^c,\, \tau_t^k,\, \tau_t^n,\, R_t^*\, ,\, s_t^{g*},\, \tau_t^{c*},\, \tau_t^{k*},\, \tau_t^{n*}\}_{t=0}^{\infty}$. This is for given the exogenous variables, $\{A_t, A_t^*, s_t^l, \lambda_t, s_t^{l*}, \lambda_t^*, \frac{S_{t+1}}{S_t}\}_{t=0}^{\infty}$, as defined in subsection 2.7, the values of feedback policy coefficients as defined in subsection 2.6 and initial conditions for the state variables.

2.9 Plan of the rest of the paper

Our main goal in this paper is to evaluate the implications of various hypothetical and actual debt consolidation policies. We will therefore work as follows. First, using commonly employed parameter values and fiscal data from Germany and Italy, we numerically solve the above model. This is in the next section (section 3). In turn, to the extent that the steady state solution of this model is empirically relevant (meaning that it can mimic the data averages over the euro area period of study), we will use this regime - defined as the status quo - as a point of reference in order to evaluate the implications of various debt consolidation policies. Details on policy experiments and the solution strategy are in section 4, while numerical solutions are in sections 5 and 6.

3 Data, parameteres and solution of the status quo model

This section solves numerically the above model by using data from Germany and Italy over the period 2001-2011. We start in 2001 because this year marked the introduction of the euro and we stop at 2011 because 2012 marked the beginning of fiscal consolidation efforts in Italy (see e.g. EMU-Public Finances (2015) by the European Commission). The data sources are OECD and Eurostat. As it turns out, the model's steady state solution mimics the main empirical characteristics of the two countries over 2001-2011.

3.1 Parameter values and fiscal policy variables

The baseline parameter values and the data averages of fiscal policy variables, used in the numerical solution of the above model, are listed in Tables 1a and 1b respectively. The time unit is meant to be a year. The two countries can differ only in their discount factors (see β and β^* in Table 1a) and fiscal policy variables (see the fiscal policy instruments in Table 1b). In all other respects, the two countries are assumed to be symmetric. Interestingly, as said above, these two differences will prove to be enough to give a steady state solution close to the data averages during 2001-2011.

Regarding parameter values, the model's key parameters are the discount factors in the two countries, β and β^*, and the cost coefficient driving the wedge between the borrowing and the lending interest rate, ψ. The values of these parameters are calibrated to match the real interest rates and the net foreign asset position of the two countries in the time period under consideration. In particular, the values of β and β^* follow from the Euler equations in the two countries which, at the steady state, are reduced to:

$$\beta Q/\Pi = 1 \tag{25}$$

$$\beta^* Q^*/\Pi^* = 1 \tag{26}$$

where Q/Π and Q^*/Π^* are the real interest rates in the two countries.[24] Since $Q/\Pi < Q^*/\Pi^*$ in the data over the period under consideration, it follows $\beta = 0.9833 > \beta^* = 0.9780$. That is, the Germans are more patient than the Italians.

In turn, the optimality condition of the bank, (16), written at the steady state, is (as said, $S \equiv 1$ in a currency union):

$$Q^* = \frac{Q}{1 - \frac{P^H}{P}\psi(f^{*g} - f^{*h})} \tag{27}$$

from which the value of the parameter ψ is calibrated.

All other papameter values, as listed in Table 1a, are the same across countries and are set at values commonly used in related studies. We start by setting the value of the political weight, η, at the "neutral" value of 0.5 (a sensitivity analysis regarding this parameter is in section 6 below). We report that our main results are robust to changes in these values (see section 6 below for details). Thus, although our numerical simulations below are not meant to provide a rigorous quantitative study, they illustrate the qualitative dynamic features of the model in a robust way.

[24] Here, $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ and $\Pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$ (see Appendix 5 for detailed definitions of all variables).

Table 1a: Baseline parameter values

Parameter	Home	Foreign	Description
a, a^*	0.3	0.3	share of physical capital in production
ν, ν^*	0.5	0.5	home goods bias in consumption
μ, μ^*	3.42	3.42	money demand elasticity in utility
δ, δ^*	0.1	0.1	capital depreciation rate
ϕ, ϕ^*	6	6	price elasticity of demand
φ, φ^*	1	1	inverse of Frisch labour elasticity
σ, σ^*	1	1	inverse of elasticity of substitution in consumption
θ, θ^*	0.2	0.2	price rigidity parameter
χ_m, χ_m^*	0.001	0.001	preference parameter related to money balances
χ_n, χ_n^*	5	5	preference parameter related to work effort
χ_g, χ_g^*	0.1	0.1	preference parameter related to public spending
ξ, ξ^*	0.01	0.01	adjustment cost parameter of physical capital
ζ, ζ^*	1	1	public spending elasticity in utility
η	0.5	0.5	political weight in union-wide policies
β, β^*	0.9833	0.9780	time discount factor
ψ	0.072	-	cost parameter in international borrowing
$\sigma_\alpha, \sigma_{\alpha^*}$	0.01	0.01	standard deviation of TFP
$\rho^\alpha, \rho^{\alpha^*}$	0.92	0.92	persistence of TFP

Regarding fiscal policy instruments in the two countries as defined in subsection 2.6.2 above, the steady state tax rates and government spending-to-output ratios are all set equal to their average values in the data in Germany and Italy over 2001-2011 (see Table 1b).

Table 1b: Fiscal policy variables (2001-2011 data averages)

Variable	Home	Foreign	Description
τ^c, τ^{*c}	0.1934	0.1756	consumption tax rate
τ^k, τ^{*k}	0.2041	0.3118	capital income tax rate
τ^n, τ^{*n}	0.3833	0.421	labour income tax rate
s^g, s^{*g}	0.2131	0.2423	government spending on goods/services as share of GDP
$-s^l$, $-s^{*l}$	-0.2039	-0.2163	government transfers as share of GDP
λ, λ^*	0.52	0.61	share of public debt held by domestic agents

3.2 Steady state solution in the status quo model

The equilibrium system was defined in subsection 2.8 and the associated steady state follows simply if we assume that variables do not change over time (details are at the very end of Appendix 5). Table 2 presents the steady state solution when parameters and policy instruments are set at the values in Tables 1a-b. It is worth pointing out that, since policy instruments react to deviations of macroeconomic indicators from their steady state values, feedback policy coefficients do not play any role in steady state solutions. In this steady state solution, the residually determined public financing variable is public debt in both countries. Table 2 also presents some key ratios in the German and Italian data and, as can be seen, the respective ratios implied by the steady state solution are close to their values in the data. In particular, the solution can mimic rather well the data averages of public debt-to-GDP ratios and foreign debt-to-GDP ratios in the two countries over 2001-2011.

This solution will serve as a point of departure. That is, in what follows, we will depart from this solution to study various policy experiments. We report (and this is confirmed below) that an exogenous reduction is public debt stimulates output and improves welfare in both countries; this can provide a justification for our fiscal consolidation experiments.

Table 2: Status quo steady state solution

Variables	Description	Home	Data	Foreign	Data
$u,\ u^*$	utility	0.0376	-	0.0315	-
y^H, y^{H*}	output	0.3912	-	0.3543	-
c, c^*	consumption	0.2314	-	0.2278	-
$n,\ n^*$	hours worked	0.3116	-	0.3063	-
k, k^*	capital	0.6655	-	0.4976	-
$w,\ w^*$	real wage rate	0.6976	-	0.7085	-
$r^k,\ r^{k*}$	real return to capital	0.1470	-	0.1780	-
$Q^* - Q$	interest rate premium	-	-	0.0055	0.0055
$\frac{c}{y^H TT^{1-\nu}}, \frac{c^*}{y^{H*} TT_t^{\nu^*-1}}$	consumption as share of GDP	0.5633	-	0.6752	-
$\frac{k}{y^H}, \frac{k^*}{y^{H*}}$	capital as share of GDP	1.7009	-	1.4045	-
$\frac{d}{TT^{\nu-1} y^H}, \frac{d^*}{TT^{1-\nu^*} y^{*H}}$	total public debt as share of GDP	0.6907	0.6861	1.0871	1.08
$\frac{\left(\frac{(1-\lambda)d}{TT^{\nu-1}} - TT_t^{\nu^*} f^h\right)}{y^H}, \frac{\frac{(1-\lambda^*)d^*}{TT^{1-\nu-\nu^*}} - f^{*h}}{TT_t^{\nu} y^{*H}}$	total foreign debt as share of GDP*	-0.2109	-0.2501	0.2114	0.2109

Notes: Parameters and policy variables as in Tables 1a-b.

3.3 Transition dynamics in the status quo model and determinacy

It is well recognized that the interaction between fiscal and monetary policy, and, in particular, the magnitude of the associated feedback policy coefficients in the policy rules, are crucial to determinany (see e.g. Leith and Wren-Lewis, 2008). This is also the case in our paper. In particular, when we assume that fiscal policy instruments remain constant at their data average values in Table 1b without any reaction to public debt and there is no interest rate policy reaction to inflation, the model, when approximated around the status quo steady state solution, exhibits dynamic instability meaning that there is no convergence to the steady state solution reported above. In other words, economic policy can guarantee a unique transition path when fiscal policy instruments (s_t^g, τ_t^c, τ_t^k, τ_t^n and s_t^{g*}, τ_t^{c*}, τ_t^{k*}, τ_t^{n*}) react to public liabilities between critical minimum and maximum non-zero values, where these critical values differ across different fiscal policy instruments, and when monetary policy satisfies the so-called Taylor principle, meaning that the single nominal interest rate reacts aggressively to inflation. This will also be confirmed by the results for optimized policy rules below. By contrast, fiscal and monetary policy reaction to the output gap has not been found to be crucial for determinacy. Further details regarding ranges of feedback policy coefficients guaranteeing local determinacy are available upon request. In sum, determinacy requires stabilizing fiscal reaction to inherited public debt and monetary reaction to inflation. This holds in all cases studied below.

4 Description of policy experiments and solution strategy

This section defines in some detail our policy experiments and explains the solution strategy. Numerical results will be presented in sections 5 and 6.

In our main thought experiment, we will depart from the status quo steady state solution (in other words, the initial values of the predetermined variables will be those found by the steady state solution in Table 2) and compute the equilibrium transition path as we travel towards a new reformed steady state (policy reforms are defined in subsection 4.1 below). Alternative policy scenarios are defined in subsection 4.2. Transition dynamics from the status quo steady state to a new steady state will be driven by extrinsic shocks and by debt consolidation policies in the high-debt country. Along this transition, we experiment with fiscal policy mixes, which means that we allow all national fiscal policy instruments at the same time to react to their policy targets in the policy rules.[25] The feedback monetary and fiscal policy coefficients of the instrument(s) used along the transition path - by the single monetary authority as well as by the two national fiscal authorities - are chosen optimally (this is explained in subsection 4.3).

Regarding transition results, we will compute second-order approximate solutions, around the associated steady state, by following the methodology of Schmitt-Grohé and Uribe (2004). Details are below in subsection 4.3. At this point, we just mention that we focus on second-order accurate approximate solutions because the model is stochastic and, as is known, first-order approximations can give spurious results when used to compare the welfare under alternative policies (see e.g. the review in Galí, 2008, pp. 110-111).[26]

4.1 National fiscal policies and reforms in the main policy experiment

In our main thought experiment, motivated by the facts discussed in the opening paragraph of the Introduction, we focus on two types of fiscal action, one for each country.

[25] To understand the logic of our results, and following usual practice in related studies, we have also experimented with one fiscal instrument at a time. This means that, along the early costly phase, we have allowed only one of the fiscal policy instruments to react to public debt imbalances and, at the same time, it is the same fiscal policy instrument that adjusts residually in the long-run to close the government budget. Thus, the same policy instrument bears the cost of, and reaps the benefit from, debt consolidation. We directly report results with policy mixes only to save on space.

[26] We have also experimented with non-approximate (or non-linear) solutions. Our results are similar.

4.1.1 Fiscal policy scenario in the domestic country with solid public finances

The domestic country (defined to be Germany) is assumed to follow a neutral fiscal policy. In other words, we assume that the domestic country does not take any active fiscal consolidation measures but it just stabilizes the public debt-to-GDP ratio at its average level, where the latter, namely the public debt target in the country's feedback policy rules, is defined to be the steady state value of the public debt-to-GDP ratio as determined residually by the within-period government budget constraint. That is, in this country, we depart from, and end up at, the same tax-spending position, which is as in the average data in Germany (however, as explained below, the new steady state solution will differ from the status quo solution because of fiscal consolidation in the foreign country). This is usually called "debt accomodation" in the related literature (see Wren-Lewis, 2010).

Specifically, fiscal policy in the domestic country is defined as follows: (a) All exogenously set tax-spending national policy instruments remain at the same value (data average value) in both the status quo steady state and the new steady state. (b) Along the transition to the new steady state, all tax-spending national policy instruments are allowed to react to deviations from policy targets in a optimized way (see below), where the policy targets are the endogenously determined new steady state values. (c) All the time, namely, both during the transition and in the steady state, the public debt serves as the residually determined public financing instrument closing the within-period government budget constraint.

To understand this scenario, imagine that the economy is hit by a temporary adverse shock to TFP as modelled in equations (23)-(24). This, as the impulse response functions can show, leads at impact to a contraction in output and a rise in the public debt-to-output ratio. Then, the policy questions are which tax-spending policy instrument to use over time, and how strong the reaction of those policy instruments to deviations from targets should be, in order to minimize cyclical volatility.

4.1.2 Fiscal policy scenario in the foreign country with weak public finances

In our main thought experiment, the role of fiscal policy is twofold in the foreign country (defined to be Italy): to stabilize the economy against the same shock(s) as in the home country and, at the same time, to improve resource allocation by bringing down its public debt-to-GDP ratio over time. In other words, we depart from the status quo steady state solution, but we end up at a new reformed steady state with a new fiscal position in the high-debt country. This is typically called "debt consolidation" in the related literature (see Wren-Lewis, 2010).

Specifically, in our main thought experiment, fiscal policy in the foreign country (i.e. Italy) is defined as follows: (a) In the new reformed steady state, the country's output share of public debt is exogenously set at the target value of 90% (recall that it was around 110% of GDP in the status quo steady state solution in subsection 3.2).[27] Actually, we will study two subcases here: one in which sovereign premia may remain in this reformed steady state, as determined endogenously by equation (27), similarly to the status quo model; and one in which, not only public debt is reduced to 90%, but also sovereign premia are eliminated in the new reformed steady state, meaning that now we also impose $Q = Q^*$ in equation (27). Obviously, the second case, the one without premia, is more ambitious. Modelling details are provided in the next subsection right below. (b) In this new reformed steady state, since the country's public debt has been reduced and thus fiscal space has been created relative to the status quo, fiscal spending can be increased and/or tax rates can be cut, depending on which fiscal policy instrument is assumed to follow residually to close the government budget constraint. This is known as the long-term fiscal gain from debt consolidation. Here, we will report results only for the case in which the fiscal space created by debt reduction is used to reduce the capital tax rate; as our solutions show, this is the most efficient way of making use of the fiscal space created and is consistent with the Chamley-Judd well known normative results in the long run. (c) Along the transition to the new reformed steady state, the national tax-spending policy instruments are allowed to react to deviations from policy targets in a optimized way (see below). Given that the new debt policy target is set at a value lower than in the status quo (i.e. we depart from 110% but the policy target in Italy's feedback fiscal policy rules is 90%), this requires lower public spending, and/or higher tax rates, during the early phase of the transition period. This is known as the short-term fiscal pain of debt consolidation.[28] The reaction to policy targets is again chosen optimally.

[27] We choose the target value of 90% simply because this is consistent with evidence provided by e.g. Reinhart and Rogoff (2010) and Checherita-Westphal and Rother (2012) that, in most advanced economies, the adverse effects of public debt arise when it is around 90-100% of GDP. We report that our main results are not sensitive to this value. For instance, we have experimented with a debt target value of 70% or 60% and the results are qualitatively the same.

[28] It is well recognized that debt consolidation implies a tradeoff between short-term pain and medium-term gain (see e.g. Coenen et al., 2008, for a discussion). During the early phase of the transition, debt consolidation comes at the cost of higher taxes and/or lower public spending. In the medium- and long-run, a reduction in the debt burden allows, other things equal, a cut in tax rates, and/or a rise in public spending. Thus, one has to value the early costs of stabilization vis-a-vis the medium- and long-term benefits from the fiscal space created. It is also recognized that the implications of fiscal reforms, like debt consolidation, depend heavily on the public financing policy instrument used, namely, which policy instrument adjusts endogenously to accommodate the exogenous changes in fiscal policy (see e.g. Leeper et al., 2009). In the case of debt consolidation, such implications are expected to depend both on which policy instrument bears the cost of adjustment in the early period of adjustment and on which policy instrument is expected to reap the benefit, once consolidation has been achieved. Notice that if lump-sum policy instruments were available, the costs of adjustment, as well as the benefits after adjustment has been achieved, would be trivial.

4.1.3 Equilibrium system in the reformed economy: modelling

As said above, in the reformed economy, the debt policy target in Italy is set at 90%. Also as said above, we will distinguish two subcases in the steady state of this reformed economy: one with premia and one without premia. The equilibrium system and modelling details are in Appendix 6. As explained in that Appendix, the case in which premia are allowed in the new steady state is similar to the status quo regime in terms of modelling except that in the reformed economy the debt policy target is 90%. On the other hand, the case in which we also set $Q = Q^*$ in the new steady state is more demanding. In particular, elimination of premia, or equivalently equalization of interest rates, $Q = Q^*$, means that the international capital market becomes perfect so that agents can borrow and lend at the same interest rate internationally. For this to happen, however, and as already discussed in the beginning of section 2, the discount factors need also to be equalized across countries. Namely, without financial frictions, the agents should become equally patient eventually. Thus, β^* needs to become equal to β at the new steady state (although this is not required along the transition). To model this, in a relatively unbiased way, we assume that the discount factor in Italy, β^*, follows over time the AR(1) process:

$$\beta_t^* = \rho^{\beta^*} \beta_{t-1}^* + \left(1 - \rho^{\beta^*}\right) \beta \tag{28}$$

where the initial value is the value used in the status quo solution (see Table 1a) while the value in the new reformed steady state is set as in Germany (see Table 1a again).[29] It is important to stress that, in case premia are eliminated in the new steady state so that $\beta^* = \beta$, we will choose the autoregressive parameter, ρ^{β^*}, optimally, alongside all other feedback policy parameters, so as not to force results in one direction or another. In general, ρ^{β^*} can be thought of as capturing some form of cultural, or social norm, change relative to the status quo, as studied by e.g. Becker and Mulligan (1997) and Doepke and Zilibotti (2008); see our discussion in subsection 2.1 above.

4.2 Other fiscal policy scenarios studied

In addition to the above defined main experiment, and for reasons of comparison, we will also study two other policy scenarios:

First, the case in which, other things equal, Italy does not take any active fiscal consoli-

[29] The exact numerical value we use for steady state β^* is not important to our main results. But we do need $\beta^* = \beta$ to get a well-defined steady state solution to the extent that we do not have premia in this new steady state.

dation measures. That is, acting like Germany, it just departs from, and returns to, the same tax-spending position (which is the status quo steady state). This case of debt accomodation typically serves as a benchmark to evaluate the possible merits of fiscal consolidation. Notice that again feedback policy coefficients will be chosen optimally.

Second, we study an ad hoc case in which fiscal policy variables in Italy and Germany mimic their values in the actual data since 2012 (see e.g. EMU-Public Finances (2015, p. 15) by the European Commission). This means that, in Italy, any debt consolidation is achieved by an increase in total tax revenues as share of GDP by around 2 percentage points, while the public spending share remains practically unchanged.[30] In Germany, fiscal policy is kept neutral meaning no changes relative to the status quo steady state solution. We shall call this the "ad hoc" fiscal policy scenario. Note that all this will be achieved by appropriately adjusting the feedback policy coefficients on public debt in the fiscal policy rules in the two countries. Thus, under this scenario, policy reaction is not chosen optimally but the fiscal feedback policy coefficients are adjusted so as to give values for fiscal policy variables (public spending and tax revenues as shares of GDP in the two countries) as in the data since 2012. The monetary authority's reaction to weighted inflation in the two countries, ϕ_π, is also set exogenously at, say, 2 (we report that our results are not sensitive to this value to the extent that $\phi_\pi > 1$, which is the so-called Taylor principle - see below). Further details and results of this ad hoc scenario are in subsection 5.2. It should be said that we do this experiment in order to see how inferior is this ad hoc case relative to the case in which the authorities choose their policies in the best possible way.

4.3 Optimized policy rules, solution methodology and welfare comparison

To make the comparison of different policies meaningful, we compute optimized feedback policy rules (except in the case of the ad hoc scenario discussed in subsection 4.2 above), so that our results do not depend on arbitrary differences in feedback policy coefficients across different regimes. Recall that here the single monetary authority can choose the feedback policy coefficients on inflation and output in the two countries in its single rule for the nominal interest rate (see equation 17 above), while each national fiscal authority can choose the feedback policy coefficients on national public debt and output in its rules for public spending and tax rates (see equations 18-21 above for each country).

We start with defining the welfare objective of policymakers.

[30] In Italy, tax revenues as share of GDP were 45.6% in 2011 and this increased to 47.8% in 2012. See European Commission (2015, p. 15).

4.3.1 Welfare objective of policymakers

There can be many institutional scenarios regarding the degree of cooperation between the single monetary authority and the two national fiscal authorities, ranging from full cooperation to zero cooperation.[31] Here we will focus on a scenario of full cooperation at policy level. Appart from computational simplicity, we focus on this scenario because, these days, most macroeconomic measures, and especially fiscal consolidation measures, are taken under the advice, or coordination, of the European Union and the ECB (see EMU-Public Finances of the European Commission, 2013).

In particular, we assume that all monetary and fiscal feedback policy coefficients are chosen jointly and simultaneously so as to maximize a weighted average of households' expected discounted lifetime utility in the two countries:

$$W_t = \eta V_t + (1-\eta) V_t^* \tag{29}$$

where $0 \leq \eta \leq 1$ is the political weight of the domestic country vis-a-vis the foreign country, i.e. the higher is η, the higher the say of Germany in policy-making (see also equation (17) above), and V_t and V_t^* are as defined in equation (7) above. As said above, we will start with the neutral case $\eta = 0.5$, but we will then experiment with various values of η.

4.3.2 Computation of optimized feedback policy rules

Except from the case of the ad hoc scenario in subsection 4.2, we compute the welfare-maximizing values of feedback policy coefficients in the policy rules (this is what Schmitt-Grohé and Uribe, 2005 and 2007, call optimized policy rules). The welfare criterion is to maximize the conditional welfare of the two households as defined in (29) above, where conditionality refers to the initial conditions chosen; the latter are given by the status quo solution in Table 2 above, which is close to data averages over 2001-2011. To this end, following Schmitt-Grohé and Uribe (2004), we take a second-order approximation to both the equilibrium conditions and the welfare criterion. Specifically, we first compute a second-order approximation of both the conditional welfare and the decentralized equilbrium, as functions of feedback policy coefficients using Dynare and in, turn, we use a matlab function (such as fminsearch.m) to compute the values of the feedback policy coefficients that maximize this approximate system (Dynare

[31] Without any cooperation, the single monetary authority chooses its reaction to inflation and output in the two countries in its Taylor rule for the single nominal interest rate, while the two national fiscal auhorities choose their own reaction to domestic debt and output in their own fiscal rules, all three of them playing Nash vis-a-vis each other. See e.g. Mendoza and Tesar (2005) who have computed Nash equilibria in a two-country model.

and matlab routines are available upon request). In this exercise, if necessary, the feedback policy coefficients are restricted to be within some prespecified ranges so as to deliver determinacy and satisfy the Taylor principle meaning $\phi_\pi > 1$. All this is with, and without, debt consolidation, where the case without consolidation will serve as a benchmark.

4.3.3 Welfare comparison of alternative regimes

Comparisons of alternative policy regimes will be in terms of expected lifetime discounted utility (or "welfare"). Welfare differences will then be expressed in terms of consumption equivalences, as is the tradition in the related literature (see e.g. Lucas, 1990). As said, the case without debt consolidation will serve as the benchmark in all welfare comparisons.

5 Main results

We work as explained in the previous section. Namely, we depart from the status quo steady state solution in Table 2 and travel towards a new reformed steady state. In this new reformed steady state, in Italy, under debt consolidation, public debt has been cut at 90% and the resulting fiscal space is used to finance a decrease of the capital tax rate. In Germany, by contrast, tax-spending policy instruments remain as in the status quo steady state. Along the transition to this new reformed steady state, the debt policy target in Italy's feedback fiscal policy rules is set at 90%. Also, along the transition, except in the case of the ad hoc policy scenario, all feedback policy coefficients are optimally chosen as explained in the previous section.

Ideally, we would like to study the case in which all feedback policy coefficients are chosen optimally and at the same time. Namely, when the single monetary authority reacts to both inflation and output (see the rule in subsection 2.6.1) and each national fiscal authority reacts to both public debt and output (see the rules in subsection 2.6.2). However, when all feedback policy coefficients are chosen optimally simultaneously, the model becomes too heavy computationally. Therefore, we will start with the case in which all national fiscal policy instruments are allowed to react to the public debt gap only and the single monetary policy instrument is allowed to react to the weighted sum of national inflation rates only, while policy reaction to the output gap will be exogenously added in the robustness section below.

In all cases studied, as said, the reference will be the case without debt consolidation, other things equal.

5.1 Debt consolidation with optimized policy

In the reformed economy with debt consolidation in Italy, the debt policy target is 0.9. Thus, in the steady state of this economy, we set the Italian public debt to GDP ratio, $\frac{d^*}{TT^{1-\nu^*}y^{*H}}$, at this target value, 0.9, and allow the capital tax rate, τ^{k*}, to follow residually. As said in subsection 4.1, we distinguish two cases of this scenario: one in which sovereign premia are allowed in the reformed steady state (the results are in subsection 5.1.1) and a more ambitious one where such premia are eliminated (the results are in subsection 5.1.2).

5.1.1 Allowing for sovereign premia in the steady state of the reformed economy

In this case, the endogenously determined steady state capital tax rate, τ^{k*}, falls from 0.31 (in the data) to 0.292.[32] Table 3 reports the values of the optimized feedback policy coefficients (see notes of Table 3) and hence the associated policy mix, as well as the resulting level of expected discounted lifetime utility (see the last column in Table 3). In addition, we report results over shorter time horizons (see the first three columns).[33] This is in both countries. Numbers in parentheses report welfare levels in the benchmark case without debt consolidation in Italy, other things equal. Welfare gains/losses of debt consolidation vis-a-vis no debt consolidation are in terms of percentage consumption equivalences; a positive number means a gain vis-a-vis the case without debt consolidation and vice versa for a negative number.

Table 3: Welfare over different time horizons with, and without, debt consolidation in Italy (when premia remain in the reformed steady state)

	2 periods	4 periods	20 periods	lifetime
Home (Germany)	0.1304	0.2373	0.7681	2.4843
	(0.0811)	(0.1589)	(0.6802)	(2.3889)
welfare gain/loss	0.0169	0.0163	0.0049	0.0016
Foreign (Italy)	0.0418	0.126	0.6221	1.5846
	(0.068)	(0.1339)	(0.5732)	(1.5340)
welfare gain/loss	-0.0089	-0.0016	0.0029	0.0011

Notes: (i) Values of optimized policy coefficients $\phi_\pi = 1.1$, $\gamma_l^g = 0.1188$, $\gamma_l^c = 0.244$, $\gamma_l^{*g} = 0.587$, $\gamma_l^{*c} = \gamma_l^k = \gamma_l^{*k} = \gamma_l^n = \gamma_l^{*n} = 0$.(ii) We set $\eta \equiv 0.5$. (iii) Results without debt consolidation in parentheses. (iv) Welfare gains/losses in terms of consumption equivalences.

[32] Modelling details and the full steady state solution are in Appendix 6.

[33] The welfare criterion for the choice of feedback policy coefficients is the maximization of expected discounted lifetime utilities, as defined in equation (29) above. Thus, when we report welfare results for shorter time horizons, we just use these lifetime optimal feedbacks into the discounted utility of different time periods.

In terms of policy reaction, the values of the optimized feedback policy coefficients, as reported in the notes of Table 3, imply that the single interest rate should react aggressively to weighted inflation, $\phi_\pi = 1.1 > 1$, which is according to the Taylor principle, while national fiscal reactions to public debt should be achieved by government spending and consumption taxes in Germany and by government spending only in Italy. Obviously, government spending reaction to public debt should be stronger in Italy (the debt consolidating country) than in Germany (the debt accomodating country); that is, $0.1188 = \gamma_l^g < \gamma_l^{*g} = 0.587$. Interestingly, both countries should not use any income taxes for debt accomodation (in Germany) or debt consolidation (in Italy); in other words, the optimal values of the associated feedback policy coefficients in the rules for the capital and labor income tax rates are all zero in both countries, $\gamma_l^k = \gamma_l^{*k} = \gamma_l^n = \gamma_l^{*n} = 0$. Intuitively, these results imply that, since Italy goes for debt consolidation, which requires relatively big cuts in public spending and/or big tax rises, it is better to avoid the use of fiscal instruments that are particularly distorting like tax rates. Germany, on the other hand, just stabilizes cyclical debt fluctuations, which does not require big changes in fiscal instruments, so this can be achieved by the use of consumption taxes too. But, in both countries, it is a bad idea to use income taxes, the most distorting fiscal instruments, to address public debt problems.

In terms of welfare implications of debt consolidation, expressed as said above in terms of consumption equivalences, the respective signs in Table 3 imply that debt consolidation in Italy hurts Italians in the short term, but there are welfare gains in the medium and long term. By contrast, Germany gains all the time from debt consolidation in Italy. More specifically, our numerical solutions reveal that Italy gains from undertaking debt consolidation mainly because its capital, real wages and output rise. On the other hand, in an integrated world, such developments in the high-debt country, in combination with a fall in interest rates in the whole currency area, have positive spillover effects on other countries.

Notice, however, that the size of welfare effects from debt consolidation is relatively small in Table 3 and this is especially the case in Italy which is the debt consolidating country (for instance, a value of 0.0011 for lifetime gain means that consumption rises by 0.11% in each period in Italy). Such a welfare effect looks to be "small" at least when it is compared to the welfare effects of e.g. Lucas (1990), who has found a lifetime welfare gain of around 0.027 or 2.7%, when capital taxes are eliminated in the USA. This is why we will also consider a more ambitious consolidation scenario right below.

Before we move on, Table 4 reports the implications for fiscal policy variables as a result of the above policy. Numbers are expressed as absolute changes relative to the corresponding values in the status quo steady state solution. In other words, a value of -0.0665 means that public spending, as share of GDP, should fall by around 6.65 percentage points relative to its value in the status quo steady state solution. As can be seen, in the country that undertakes debt consolidation, public spending to GDP should fall but tax revenues to GDP should not rise practically; the non-increase in tax revenues mitigates the short-term recessionary implications of debt consolidation policies achieved by spending cuts.

Recall that here we computed optimized rules so all these are normative results.

Table 4: Resulting government spending and tax revenues as shares of GDP

	2 periods average	4 periods average	10 periods average	20 periods average
Gov. spending to GDP in Germany	0.0078	0.0059	0.0013	0
Gov. spending to GDP in Italy	-0.0665	-0.0367	-0.0172	-0.0093
Tax revenues to GDP in Germany	-0.0092	-0.0069	-0.0014	0.0004
Tax revenues to GDP in Italy	0.0014	-0.0009	-0.0025	-0.0033

Notes: See notes in Table 3.

5.1.2 Eliminating sovereign premia in the steady state of the reformed economy

We now consider the more ambitious case in which, not only public debt is reduced to 90% but also sovereign premia are eliminated in the new reformed steady state.[34] Results are reported in Tables 5 and 6. Tables 5 and 6 are the analogues of Tables 3 and 4 respectively. That is, the difference between Tables 5 and 6 on one hand, and Tables 3 and 4 on the other hand, is that in 5 and 6 we also eliminate sovereign premia in the steady state of the refomed economy. Recall that elimination of premia implies a perfect international capital market so that, in turn, discount factors have to be equalized across countries in the new steady state. By contrast, this was not necessary in Tables 3 and 4 where we allowed for premia so, in that case, the values of the discount factors remained as in the status quo economy (see Table 2).

Comparison of Tables 5 and 3 reveals that the qualitative effects from fiscal consolidation in Italy do not change. Namely, Germany gains all the time, while Italy gains in the longer term

[34] Modelling details and the full steady state solution are in Appendix 6.

only. But there are some new results too. Welfare effects are much bigger now. In particular, Germany is better off over all time windows when, eventually, premia are also eliminated and discount factors are equalized (i.e. Germany's discounted utilities are all higher in Table 5 than in Table 3). Second, the elimination of premia also benefits Italy in terms of discounted lifetime utility (i.e. in Table 3, it was 1.5846, while it is 2.765 in Table 5) but, on the other hand, the elimination of premia in the new steady state comes at a higher cost in the short term (i.e. Italy's discounted utilities over the first 2, 4 and 20 periods are lower in Table 5 than in Table 3). Intuitively, a more ambitious policy leads to higher payoffs in the medium and long term but it also means bigger sacrifices in the short term.

Table 5: Welfare over different time horizons with, and without, debt consolidation in Italy (when we elinimate premia in the reformed steady state)

	2 periods	4 periods	20 periods	lifetime
Domestic (Germany)	0.1424	0.2751	1.0543	2.8979
	(0.0811)	(0.1589)	(0.6802)	(2.3889)
welfare gain/loss	0.0210	0.0243	0.0212	0.0085
Foreign (Italy)	-0.0421	-0.0099	0.4585	2.765
	(0.068)	(0.1339)	(0.5732)	(1.534)
welfare gain/loss	-0.0345	-0.0296	-0.0067	0.0208

Notes: (i) Values of optimized policy coefficients $\phi_\pi = 1.103$, $\gamma_l^g = 0.014$, $\gamma_l^{*g} = 0.5619$, $\gamma_l^c = \gamma_l^{*c} = \gamma_l^k = \gamma_l^{*k} = \gamma_l^n = \gamma_l^{*n} = 0$, $\rho^{\beta^*} = 0$. (ii) We set $\eta \equiv 0.5$. (iii) Results without debt consolidation in parentheses. (iv) Welfare gains/losses in terms of consumption equivalences.

In terms of policy reaction, the values of the optimized feedback policy coefficients, as reported in the notes of Table 5, are similar to those in Table 3. The only difference is that now both countries should react to public debt by using government spending only. In other words, the optimal values of the feedback policy coefficients in the rules for the tax rates are all zero in both countries, $\gamma_l^c = \gamma_l^{*c} = \gamma_l^k = \gamma_l^{*k} = \gamma_l^n = \gamma_l^{*n} = 0$. Obviously, as before, government spending reaction to public debt should be stronger in Italy (the debt consolidating country) than in Germany (the debt accomodating country); that is, $0.014 = \gamma_l^g < \gamma_l^{*g} = 0.5619$ in Table 5.

Besides, it is important to notice, as reported in the notes of Table 5, that the optimally chosen value of the persistence parameter in the AR(1) process for Italy's discount factor, ρ^{β^*}, is practically zero meaning that it would be optimal for Italians to adopt the patience of Germans and this should be done as soon as possible. Although we realize that cultural characteristics, like the degree of patience, can change very slowly, we believe that this is a useful normative result.

Table 6: Resulting government spending and tax revenues as shares of GDP

	2 periods average	4 periods average	10 periods average	20 periods average
Gov. spending to GDP in Germany	0.0005	0.0009	0.001	0.0006
Gov. spending to GDP in Italy	-0.0538	-0.0252	-0.014	0.0101
Tax revenues to GDP in Germany	0.0028	0.0023	0.0016	0.001
Tax revenues to GDP in Italy	-0.0087	-0.0104	-0.0104	-0.01

Notes: See notes in Table 5.

5.2 Debt consolidation with ad hoc policy

Here, we study the ad hoc policy scenario described in subsection 4.2. Namely, we repeat the same policy experiment as in subsection 5.1.1 above, except that now debt reduction to 0.9 in Italy is achieved by ad hoc changes in fiscal policy variables, whish are similar to those actually implemented in the post-2011 period. Namely, in Italy, the tax revenue to GDP ratio rises by around 2 percentage points, while the spending ratio remains practically unchanged. In Germany, fiscal policy is kept neutral, meaning no real changes. Recall that, now, the feedback policy coefficients are not chosen optimally. They are just adjusted so as to give this ad hoc scenario and, of course, to guarantee dynamic determinacy. We also assume, in the new reformed steady state, that there can be risk premia, which is as in subsection 5.1.1, and that the fiscal space created by debt reduction in Italy is used to finance an increase in transfer payments, rather than a decrease in capital tax rates as assumed so far (although this is not crucial to our qualitative results).

Results are reported in Tables 7 and 8. These two tables are respectively like Tables 3 and 4 (which were with optimized policy and with premia in the steady state) or like Tables 5 and 6 (which were with optimized policy and without premia in the steady state). Inspection of the results in the new Tables 7 and 8, and comparison with the previous ones, reveals that consolidation in Italy is now harmful for both countries and across all time intervals, even in terms of lifetime utility. Along the same lines, welfare in the medium and long term is lower in Table 7 than in Table 3 and even lower than in Table 5.

Therefore, the way public debt is brought down is important. Bringing public debt down in an ad hoc way, similar to the one actualy followed in Italy (i.e. an increase in tax rates and tax revenues) proves to be welfare-deteriorating all the time and for both countries vis-a-vis the case without debt consolidation and, naturally, is welfare inferior relative to the normative case where fiscal policy is chosen optimally (meaning a cut in public spending). In other words, there is room for considerable improvement in European policies in view of the recent debt crisis.

Table 7: Welfare over different time horizons with, and without, debt consolidation in Italy (policy as in the actual data)

	2 periods	4 periods	20 periods	lifetime
Domestic (Germany)	0.0745	0.1411	0.5688	2.1645
	(0.0811)	(0.1589)	(0.6802)	(2.3889)
welfare gain/loss	-0.0022	-0.0037	-0.0062	-0.0062
Foreign (Italy)	0.0677	0.1276	0.5083	1.4092
	(0.068)	(0.1339)	(0.5732)	(1.534)
welfare gain/loss	-0.0004	-0.0013	-0.0038	-0.0027

Notes: (i) Ad-hoc policy coefficients $\phi_\pi = 2$, $\gamma_l^g = 0.05$, $\gamma_l^{*g} = 0$, $\gamma_l^c = \gamma_l^k = \gamma_l^n = 0$, $\gamma_l^{*c} = \gamma_l^{*k} = \gamma_l^{*n} = 0.08$, (ii) We set $\eta \equiv 0.5$. (iii) Results without debt consolidation in parentheses. (iv) Welfare gains/losses in terms of consumption equivalences.

Table 8: Resulting government spending and tax revenues as shares of GDP (policy as in the actual data)

	2 periods average	4 periods average	10 periods average	20 periods average
Gov. spending to GDP in Germany	0.001	0	0	0
Gov. spending to GDP in Italy	0	0	0	0
Tax revenues to GDP in Germany	0	0	0	0
Tax revenues to GDP in Italy	0.023	0.0205	0.0155	0.01

Notes: See notes in Table 7.

6 Sensitivity analysis

We now check the robustness of our results. We will focus on changes in countries' political power (subsection 6.1), reaction to the output gap (subsection 6.2) and shocks to initial debt when consolidation efforts start (subsection 6.3). But, before we present results for these cases, we report that the above results are robust to changes in all other parameter values at least within reasonable ranges.

Since the welfare benefits of debt consolidation have been found to be stronger in the case in which premia are also eliminated in the steady state of the reformed economy (see the results in Tables 5 and 6), in what follows, we work with this case when we refer to consolidation. We report, however, that the qualitative results are the same when the comparison is relative to the less ambitious policy experiment in Tables 3 and 4.

6.1 Does political power matter?

So far, we have restricted ourselves to the "politically correct" case in which the two countries shared equal political power in policy decision making. That is, so far, we have set the weight η in equation (29) at the neutral value of 0.5**.** The higher is η, the more Germany matters to policy decision-making in this equation. New results in the range $0.5 \leq \eta < 1$ are reported in Tables 9 and 10. To save on space, we focus again on the best policy mix found, namely, when both Italy and Germany use public spending in the transition phase, while Italy cuts capital taxes once its fiscal consolidation has been implemented.

First of all, observe that welfare differences are small quantitatively as η changes. This should be expected since here we compare results under optimized rules. Keeping this in mind, the main messages are as follows. Table 9 implies that the higher the say of Germany in policy decision making, the better off Germany becomes and the worse off Italy becomes. This is as expected. Table 10 implies that the higher the say of Germany, the stronger the fiscal consolidation in Italy. This is shown by the monotonic positive effect of η on the magnitude of the feedback fiscal policy coefficient on public debt in Italy. In particular, the optimized value of γ_l^{g*} rises monotonically, as η rises (the other optimized feedback policy coefficients remain practically zero, as in the previous section).

Table 9: Effect of political weight on lifetime utility

weight	world E_0W_0	home E_0V_0	foreign $E_0V_0^*$
$\eta = 0.5$	2.8315	2.8979	2.765
$\eta = 0.6$	2.8460	2.9009	2.7635
$\eta = 0.7$	2.8598	2.9012	2.7634
$\eta = 0.8$	2.8735	2.9027	2.7565
$\eta = 0.9$	2.8887	2.9042	2.7488

Notes: The weight in the Taylor rule is kept at 0.5 (not important).

Table 10: Effect of political weight on feedback policy coefficients

political weight	monetary reaction to inflation	home fiscal reaction to debt	foreign fiscal reaction to debt
$\eta = 0.5$	$\phi_\pi = 1.1$	$\gamma_l^g = 0.014$	$\gamma_l^{*g} = 0.5619$
$\eta = 0.6$	$\phi_\pi = 1.1$	$\gamma_l^g = 0.015$	$\gamma_l^{*g} = 0.6389$
$\eta = 0.7$	$\phi_\pi = 1.1$	$\gamma_l^g = 0.014$	$\gamma_l^{*g} = 0.6454$
$\eta = 0.8$	$\phi_\pi = 1.1$	$\gamma_l^g = 0.016$	$\gamma_l^{*g} = 0.7636$
$\eta = 0.9$	$\phi_\pi = 1.1$	$\gamma_l^g = 0.014$	$\gamma_l^{*g} = 0.8607$

Notes: See notes in Table 7.

6.2 Allowing also for reaction to the output gap

So far we have allowed for optimal reaction to inflation only (on the part of the single monetary authority) and to public debt only (on the part of national fiscal authorities). That is, we have not allowed monetary and/or fiscal policy instruments to react optimally to the output gap too. As explained above, this has been for computational reasons only. Nevertheless, although we cannot allow all feedback policy coefficients in subsections 2.6.1 and 2.6.2 to be chosen optimally and at the same time, we can at least experiment with various exogenously set values of national feeback fiscal reaction to the output gap. Monetary policy reaction to inflation and national fiscal policy reactions to public debt are optimally chosen as above (while again we set $\phi_y = 0$ for the central bank).[35]

For instance, Table 11 reports the implications for expected discounted lifetime utility in the two countries, when Italy uses its income (capital and labor) tax rates in a counter-cyclical way, meaning that these two distorting tax rates decrease when the output gap is negative and the opposite when the output gap is positive, while the rest of the experiment remains as above. In particular, in Table 11, we set $\gamma_l^{*k} = \gamma_l^{*n} \equiv 0.5$ in the rules for the capital and labor tax rates in Italy (we report that our qualitative results do not depend on the particular values assumed for these two feedback coefficients).

Then, comparison of Table 11 to, for instance, Table 5 implies two results: First, welfare rises in Table 11 relative to Table 5. Second, Table 11 implies a clear assigment of policy instruments to policy targets during the transition phase: public spending should be cut to address the public debt gap and, at the same time, capital and labor tax rates should also be reduced to mitigate the recessionary effects of debt consolidation.

Table 11: Welfare over different time horizonswith, and without, debt consolidation in Italy (Plus ad-hoc reaction to output gap via income taxes in Italy)

	2 periods	4 periods	20 periods	lifetime
Home (Germany)	0.1522	0.2947	1.1395	3.0220
	(0.0811)	(0.1589)	(0.6802)	(2.3889)
welfare gain/loss	0.0244	0.0285	0.0261	0.0106
Foreign (Italy)	−0.0397	−0.0084	0.4936	2.7848
	(0.068)	(0.1339)	(0.5732)	(1.534)
welfare gain/loss	-0.0358	-0.029	-0.0045	0.0211

Notes: (i) Values of optimized policy coefficients $\phi_\pi = 1.1$, $\gamma_l^g = 0.014$, $\gamma_l^{*g} = 0.4942$, $\gamma_l^c = \gamma_l^k = \gamma_l^n = 0$, $\gamma_l^{*c} = \gamma_l^{*k} = \gamma_l^{*n} = 0$, $\rho^{\beta^*} = 0$ (ii) We set $\gamma_l^{*k} = \gamma_l^{*n} \equiv 0.5$ and $\eta \equiv 0.5$ (iii) Results without debt consolidation in parentheses. (iv) Welfare gains/losses are in terms of consumption equivalences.

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[35] Monetary policy reaction to output, $\phi_y > 0$, is bad for welfare. That is, as ϕ_y rises, welfare deteriorates in both countries. See also e.g. Schmitt-Grohé and Uribe (2007) in a closed economy.

6.3 Shocks to initial debt

Now we shock the initial public debt in Italy so as to rise from 110% to, say, 130%. All the rest remains as in section 5 above. The new results are reported in Table 12. Qualitatively, the results are as in Tables 3 or 5. Notice however that, with a higher public debt initially, the fiscal pain is bigger than in the previous tables.

Table 12: Welfare over different time horizons
with, and without, debt consolidation in Italy

	2 periods	4 periods	20 periods	lifetime
Home (Germany)	0.1775	0.3449	1.2575	3.1538
	(0.0811)	(0.1589)	(0.6802)	(2.3889)
welfare gain/loss	0.0332	0.0392	0.0329	0.0129
Foreign (Italy)	-0.0839	-0.0605	0.3765	2.6945
	(0.068)	(0.1339)	(0.5732)	(1.534)
welfare gain/loss	-0.0502	-0.0394	-0.011	0.0196

Notes: (i) Values of optimized policy coefficients $\phi_\pi = 1.1$, $\gamma_l^g = 0.014$, $\gamma_l^{*g} = 0.443$, $\gamma_l^c = \gamma_l^k = \gamma_l^n = 0$, $\gamma_l^{*c} = \gamma_l^{*k} = \gamma_l^{*n} = 0$, $\rho^{\beta^*} = 0$.(ii) We set $\eta \equiv 0.5$. (iii) Results without debt consolidation in parentheses. (iv) Welfare gains/losses are in terms of consumption equivalences.

7 Concluding remarks and possible extensions

This paper has studied fiscal and monetary policy in a New Keynesian model consisting of two heterogeneous countries being part of a monetary union. We have used optimized, simple and implementable feedback policy rules for various categories of taxes and public spending, as well as of the union-wide nominal interest rate, in order to study the general equilibrium implications of fiscal consolidation in a high-debt country. A main result is that, although there is a conflict of national interests in shorter horizons, there is a common interest in the medium and long term. This is with optimized policy rules. By contrast, debt consolidation is welfare inferior to non-consolidation for both countries and all the time, if it is implemented in an ad hoc way, like an increase in income taxes. Thus, the policy mix is important.

The paper can be extended in various ways. For instance, we could distinguish different sub-categories of public spending used for fiscal consolidation. Or we could add heterogeneity across agents within each country and so study distributional implications too. Or we could add more and asymmetric distortions (at market and policy level) in the two countries. We leave these extensions for future work.

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8 Appendix 1: Households

This Appendix presents the solution of the household's problem in the domestic country (the problem of the household in the foreign country is analogous except otherwise said). There are $i = 1, 2, ..., .N$ identical domestic households who act competitively.

8.1 Household's optimality conditions

Each domestic household i maximizes (7)-(8) subject to (1)-(6), (9) and (10) in the main text. We work in two steps. We first suppose that the household determines its desired consumption of composite goods, $c_{i,t}^{H}$ and $c_{i,t}^{F}$, and, in turn, chooses how to distribute its purchases of individual varieties, $c_{i,t}^{H}(h)$ and $c_{i,t}^{F}(f)$. The first-order conditions of each i include its constraints, as listed in the main text, plus:

$$\begin{aligned} &\frac{\partial u_{i,t}}{\partial c_{i,t}}\frac{\partial c_{i,t}}{\partial c_{i,t}^{H}}\frac{P_t}{P_t^H(1+\tau_t^c)} = \\ &= \beta E_t \frac{\partial u_{i,t+1}}{\partial c_{i,t+1}}\frac{\partial c_{i,t+1}}{\partial c_{i,t+1}^{H}}\frac{P_{t+1}}{P_{t+1}^H\left(1+\tau_{t+1}^c\right)}R_t\frac{P_t}{P_{t+1}} \end{aligned} \tag{30}$$

$$\begin{aligned} &\frac{\partial u_{i,t}}{\partial c_{i,t}}\frac{\partial c_{i,t}}{\partial c_{i,t}^{H}}\frac{P_t}{P_t^H(1+\tau_t^c)}\frac{S_t P_t^*}{P_t} = \\ &= \beta E_t \frac{\partial u_{i,t+1}}{\partial c_{i,t+1}}\frac{\partial c_{i,t+1}}{\partial c_{t+1}^{H}}\frac{P_{t+1}}{P_{t+1}^H\left(1+\tau_{t+1}^c\right)}Q_t\frac{S_{t+1}P_{t+1}^*}{P_{t+1}}\frac{P_t^*}{P_{t+1}^*} \end{aligned} \tag{31}$$

$$\begin{aligned} &\frac{\partial u_{i,t}}{\partial c_{i,t}}\frac{\partial c_{i,t}}{\partial c_{i,t}^{H}}\frac{1}{(1+\tau_t^c)}\left\{1-\xi\left(\frac{k_{i,t}}{k_{i,t-1}}-1\right)\right\} = \\ &= \beta E_t \frac{\partial u_{i,t+1}}{\partial c_{i,t+1}}\frac{\partial c_{i,t+1}}{\partial c_{i,t+1}^{H}}\frac{1}{\left(1+\tau_{t+1}^c\right)}\left\{\begin{array}{c} (1-\delta)-\frac{\xi}{2}\left(\frac{k_{i,t+1}}{k_{i,t}}-1\right)^2 + \\ \xi\left(\frac{k_{i,t+1}}{k_{i,t}}-1\right)\frac{k_{i,t+1}}{k_{i,t}}+\left(1-\tau_{t+1}^k\right)r_{t+1}^k \end{array}\right\} \end{aligned} \tag{32}$$

$$\chi_m \frac{\partial u_{i,t}}{\partial m_{i.t}} = \frac{\partial u_{i,t}}{\partial c_{i,t}}\frac{\partial c_{i,t}}{\partial c_{i,t}^{H}}\frac{P_t}{P_t^H\left(1+\tau_t^c\right)} - \beta E_t \frac{\partial u_{i,t+1}}{\partial c_{i,t+1}}\frac{\partial c_{i,t+1}}{\partial c_{i,t+1}^{H}}\frac{P_{t+1}}{P_{t+1}^H\left(1+\tau_{t+1}^c\right)}\frac{P_t}{P_{t+1}} \tag{33}$$

$$-\chi_n \frac{\partial u_{i,t}}{\partial n_{i,t}} = \frac{(1-\tau_t^n)}{(1+\tau_t^c)} w_t \frac{\partial u_{i,t}}{\partial c_{i,t}}\frac{\partial c_{i,t}}{\partial c_{i,t}^{H}}\frac{P_t}{P_t^H} \tag{34}$$

$$\frac{c_{i,t}^H}{c_{i,t}^F} = \frac{\nu}{1-\nu}\frac{P_t^F}{P_t^H} \tag{35}$$

$$c_{i,t}^H(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} c_{i,t}^H \tag{36}$$

$$c_{i,t}^F(f) = \left[\frac{P_t^F(f)}{P_t^F}\right]^{-\phi} c_{i,t}^F \tag{37}$$

Equations (30)-(32) are respectively the Euler equations for domestic bonds, foreign assets and domestic capital, (33) is the optimality condition for money balances and (34) is the optimality condition for work hours. Finally, (35) shows the optimal allocation between domestic and foreign goods, while (36) and (37) show the optimal demand for each variety of domestic and foreign goods respectively.

8.2 Implications for price bundles

Equations (35), (36) and (37), combined with the household's budget constraints, imply that the three price indexes are:

$$P_t = (P_t^H)^{\nu}(P_t^F)^{1-\nu} \tag{38}$$

$$P_t^H = \left[\sum_{h=1}^{N} \kappa [P_t^H(h)]^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{39}$$

$$P_t^F = \left[\sum_{f=1}^{N} \kappa [P_t^F(f)]^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{40}$$

where $\kappa = 1/N$.

9 Appendix 2: Firms

This Appendix presents the solution of the firm's problem in the domestic country (the problem of the firm in the foreign country is analogous except otherwise said). There are $h = 1, 2, ..., .N$ differentiated domestic firms. Each firm h produces a differentiated good of variety h under monopolistic competition facing Calvo-type nominal fixities.

9.1 Demand for the firm's product

Each domestic firm h faces demand for its product, $y_t^H(h)$. The latter comes from domestic households' consumption and investment, $c_t^H(h)$ and $x_t(h)$, where $c_t^H(h) \equiv \sum_{i=1}^{N} c_{i,t}^H(h)$ and $x_t(h) \equiv \sum_{i=1}^{N} x_{i,t}(h)$, from the domestic government, $g_t(h)$, from the financial intermediary located in the domestic country, $q_t(h)$,[36] and from foreign households' consumption, $c_t^{F*}(h) \equiv$

[36] See also Curdia and Woodford (2009) for a similar modelling of resources consumed by banks.That is, the

$\sum_{i=1}^{N*} c_{i,t}^{F*}(h)$. Thus, the demand for the Dixit-Stiglitz good produced by each firm h is written as:

$$y_t^H(h) = c_t^H(h) + x_t(h) + g_t(h) + q_t(h) + c_t^{F*}(h) \tag{41}$$

where, for each component:

$$c_t^H(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} c_t^H \tag{42}$$

$$x_t(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} x_t \tag{43}$$

$$g_t(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} g_t \tag{44}$$

$$q_t(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} q_t \tag{45}$$

$$c_t^{F*}(h) = \left[\frac{P_t^{F*}(h)}{P_t^{F*}}\right]^{-\phi} c_t^{F*} \tag{46}$$

where, using the law of one price discussed above, we have in (46):

$$\frac{P_t^{F*}(h)}{P_t^{F*}} = \frac{\frac{P_t^H(h)}{S_t}}{\frac{P_t^H}{S_t}} = \frac{P_t^H(h)}{P_t^H} \tag{47}$$

Since, at the economy level, agreggate demand for the domestically produced good is:

$$y_t^H = c_t^H + x_t + g_t + q_t + c_t^{F*} \tag{48}$$

the above equations imply that the demand for each domestic firm's product is:

$$y_t^H(h) = c_t^H(h) + x_t(h) + g_t(h) + q_t(h) + c_t^{F*}(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} y_t^H \tag{49}$$

model requires the bank to use real resources in the period in which the loan is originated.

9.2 Firm's problem

Each domestic firm h maximizes nominal profits, $\widetilde{\Omega}_t(h)$:

$$\widetilde{\Omega}_t(h) = P_t^H(h)y_t^H(h) - r_t^k P_t^H(h)k_{t-1}(h) - W_t n_t(h) \tag{50}$$

This is subject to the production function:

$$y_t^H(h) = A_t[k_{t-1}(h)]^\alpha [n_t(h)]^{1-\alpha} \tag{51}$$

and, since the firm operates under imperfect competition, the product demand function:

$$y_t^H(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} y_t^H \tag{52}$$

In addition, following Calvo (1983), firms choose their prices facing a nominal fixity. In each period, firm h faces an exogenous probability θ of not being able to reset its price. A firm h, which is able to reset its price, chooses its price $P_t^{\#}(h)$ to maximize the sum of discounted expected nominal profits for the next k periods in which it may have to keep its price fixed.

9.3 Firm's optimality conditions

To solve the firm's problem, we work in two steps. We first solve a cost minimization problem, where each firm h minimizes its cost by choosing factor inputs given technology and prices. The solution will give a minimum nominal cost function, which is a function of factor prices and output produced by the firm. In turn, given this cost function, each firm, which is able to reset its price, solves a maximization problem by choosing its price.

The solution to the cost minimization problem gives the input demand functions:

$$w_t = mc_t(1-a)\frac{y_t\,(h)}{n_t\,(h)} \tag{53}$$

$$\frac{P_t^H}{P_t} r_t^k = mc_t a \frac{y_t\,(h)}{k_{t-1}\,(h)} \tag{54}$$

where mc_t is real marginal cost.

Then, the firm chooses its price to maximize nominal profits written as:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \widetilde{\Omega}_{t+k}(h) = E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left\{ P_t^{\#}(h)\, y_{t+k}^H(h) - \Psi_{t+k}\left(y_{t+k}^H(h)\right) \right\}$$

where $\Xi_{t,t+k}$ is a discount factor taken as given by the firm, $y_{t+k}^H(h) = \left[\frac{P_t^{\#}(h)}{P_{t+k}^H}\right]^{-\phi} y_{t+k}^H$ and $\Psi_t(.)$ denotes the minimum nominal cost function for producing $y_t^H(h)$ at t so that $\Psi_t'(.)$ is the associated marginal cost (namely, $\Psi_t'(.) = mc_t P_t$).

The first-order condition gives:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[\frac{P_t^{\#}(h)}{P_{t+k}^H}\right]^{-\phi} y_{t+k}^H \left\{ P_t^{\#}(h) - \frac{\phi}{\phi-1} \Psi_{t+k}' \right\} = 0 \tag{55}$$

Dividing by the aggregate price index, P_t^H, we have:

$$E_t \sum_{k=0}^{\infty} \theta^k [\Xi_{t,t+k} \left[\frac{P_t^{\#}(h)}{P_{t+k}^H}\right]^{-\phi} y_{t+k}^H \left\{ \frac{P_t^{\#}(h)}{P_t^H} - \frac{\phi}{\phi-1} mc_{t+k} \frac{P_{t+k}}{P_t^H} \right\}] = 0 \tag{56}$$

Therefore, the behaviour of each firm h is summarized by (53), (54) and (56). A recursive expression of this problem is presented below.

Note that each firm h, which can reset its price in period t, solves an identical problem, so $P_t^{\#}(h) = P_t^{\#}$ is independent of h, and each firm h, which cannot reset its price, just sets its previous period price $P_t^H(h) = P_{t-1}^H(h)$. Thus, the evolution of the aggregate price level is given by:

$$\left(P_t^H\right)^{1-\phi} = \theta \left(P_{t-1}^H\right)^{1-\phi} + (1-\theta)\left(P_t^{\#}\right)^{1-\phi} \tag{57}$$

10 Appendix 3: Government budget constraint

This Appendix presents the government budget constraint in some detail. In the domestic economy, the government budget constraint in nominal terms is:

$$\begin{gathered} B_t + M_t + S_t F_t^g = R_{t-1} B_{t-1} + M_{t-1} + \\ + Q_{t-1} S_t F_{t-1}^g + P_t^H g_t - \tau_t^c (P_t^H c_t^H + P_t^F c_t^F) - \tau_t^k (r_t^k P_t^H k_{t-1} + \widetilde{\Omega}_t) - \tau_t^n W_t n_t - T_t^l \end{gathered} \tag{58}$$

Now let $D_t \equiv B_t + S_t F_t^g$ denote total nominal public debt. This debt can be held both by domestic private agents, $\lambda_t D_t$, where in equilibrium $B_t = \lambda_t D_t$, and by foreign private agents, $S_t F_t^g = (1-\lambda_t) D_t$. Then, the government budget constraint can be written as:

$$\begin{gathered} D_t + M_t = R_{t-1}\lambda_{t-1}D_{t-1} + M_{t-1} + \\ +Q_{t-1}\tfrac{S_t}{S_{t-1}}\left(1-\lambda_{t-1}\right)D_{t-1} + P_t^H g_t - \tau_t^c(P_t^H c_t^H + P_t^F c_t^F) \\ -\tau_t^k(r_t^k P_t^H k_{t-1} + \widetilde{\Omega}_t) - \tau_t^n W_t n_t - T_t^l \end{gathered} \tag{59}$$

and in real terms as:

$$\begin{gathered} d_t + m_t = R_{t-1}\lambda_{t-1}\tfrac{P_{t-1}}{P_t}d_{t-1} + \tfrac{P_{t-1}}{P_t}m_{t-1} + \\ +Q_{t-1}\tfrac{S_t P_t^*}{P_t}\tfrac{P_{t-1}^*}{P_t^*}\tfrac{P_{t-1}}{P_{t-1}^* S_{t-1}}\left(1-\lambda_{t-1}\right)d_{t-1} + \tfrac{P_t^H}{P_t}g_t - \tau_t^c(\tfrac{P_t^H}{P_t}c_t^H + \tfrac{P_t^F}{P_t}c_t^F) \\ -\tau_t^k(r_t^k\tfrac{P_t^H}{P_t}k_{t-1} + \widetilde{\omega}_t) - \tau_t^n w_t n_t - \tau_t^l \end{gathered} \tag{60}$$

Thus, the liabilities of the domestic government as a share of output are:

$$l_t \equiv \frac{R_t\lambda_t D_t + Q_t\frac{S_{t+1}}{S_t}\left(1-\lambda_t\right)D_t}{P_t^H y_t^H} \tag{61}$$

Similarly, the government budget constraint in nominal terms in the foreign country is:

$$\begin{gathered} B_t^* + M_t^* + \tfrac{F_t^{*g}}{S_t} = \tfrac{\phi^{*g}}{2}\left(\tfrac{F_t^{*g}}{P_t^* S_t} - \tfrac{F^{*g}}{SP^*}\right)^2 + R_{t-1}^* B_{t-1}^* + M_{t-1}^* + \\ +Q_{t-1}^*\tfrac{1}{S_t}F_{t-1}^{*g} + P_t^{*H}g_t^* - \tau_t^{*c}(P_t^{*H}c_t^{*H} + P_t^{*F}c_t^{*F}) - \tau_t^{*k}(r_t^{*k}P_t^{*H}k_{t-1}^* + \widetilde{\Omega}_t^*) - \tau_t^{*n}W_t^* n_t^* - T_t^{*l} \end{gathered} \tag{62}$$

Let denote D_t^* to be the total foreign public debt in foreign currency. This can be held by foreign private agents, $B_t^* = \lambda_t^* D_t^*$, and by domestic private agents, $\frac{F_t^{*g}}{S_t} = (1-\lambda_t^*) D_t^*$. Then, we have in nominal terms:

$$\begin{gathered} D_t^* + M_t^* = R_{t-1}^*\lambda_{t-1}^* D_{t-1}^* + M_{t-1}^* + \\ +Q_{t-1}^*\tfrac{S_{t-1}}{S_t}\left(1-\lambda_{t-1}^*\right)D_{t-1}^* + P_t^{*H}g_t^* - \tau_t^{*c}(P_t^{*H}c_t^{*H} + P_t^{*F}c_t^{*F}) \\ -\tau_t^{*k}(r_t^{*k}P_t^{*H}k_{t-1}^* + \widetilde{\Omega}_t^*) - \tau_t^{*n}W_t^* n_t^* - T_t^{*l} \end{gathered} \tag{63}$$

Thus, the liabilities of the foreign government as a share of output are:

$$l_t^* \equiv \frac{R_t^*\lambda_t^* D_t^* + Q_t^*\frac{S_t}{S_{t+1}}\left(1-\lambda_t^*\right)D_t^*}{P_t^{*H}y_t^{*H}} \tag{64}$$

11 Appendix 4: Financial intermediary or bank

The profit of the international bank from loans between $t-1$ and t is distributed at time t. In nominal terms, this profit is defined as:[37]

$$Q^*_{t-1}\left[(F^{*g}_{t-1}-F^{*h}_{t-1})-\frac{\psi}{2}P^H_{t-1}(f^{*g}_{t-1}-f^{*h}_{t-1})^2\right]-Q_{t-1}S_t\left(F^h_{t-1}-F^g_{t-1}\right) \tag{65}$$

where the real resources used by the bank are assumed to be consumed at the same time the interest payments/income are repaid/received, namely at time t, rather then when the loan contract was originated, namely at time $t-1$.

Dividing by P_t, the real profit, π_t, is:

$$\pi_t = Q^*_{t-1}\left[\frac{P_{t-1}}{P_t}(f^{*g}_{t-1}-f^{*h}_{t-1})-\frac{P^H_t}{P_t}\frac{\psi}{2}\frac{P^H_{t-1}}{P^H_t}(f^{*g}_{t-1}-f^{*h}_{t-1})^2\right]-Q_{t-1}\frac{S_tP^*_t}{P_t}\frac{P^*_{t-1}}{P^*_t}\left(f^h_{t-1}-f^g_{t-1}\right) \tag{66}$$

Since, in equilibrium, borrowing equals lending, namely, $F^{*g}_t - F^{*h}_t = S_t\left(F^h_t - F^g_t\right)$ or $f^{*g}_t - f^{*h}_t = \frac{S_tP^*_t}{P_t}\left(f^h_t - f^g_t\right)$ or in turn $f^{*g}_{t-1} - f^{*h}_{t-1} = \frac{S_{t-1}P^*_{t-1}}{P_{t-1}}\left(f^h_{t-1} - f^g_{t-1}\right)$, this is rewritten as:

$$\pi_t = Q^*_{t-1}\left[\frac{P_{t-1}}{P_t}(f^{*g}_{t-1}-f^{*h}_{t-1})-\frac{P^H_t}{P_t}\frac{\psi}{2}\frac{P^H_{t-1}}{P^H_t}(f^{*g}_{t-1}-f^{*h}_{t-1})^2\right]-Q_{t-1}\frac{S_t}{S_{t-1}}\frac{P_{t-1}}{P_t}(f^{*g}_{t-1}-f^{*h}_{t-1}) \tag{67}$$

If the volume of the loan, $(f^{*g}_{t-1} - f^{*h}_{t-1})$, is chosen optimally, the first-order condition is:

$$Q^*_{t-1} = \frac{Q_{t-1}\frac{S_t}{S_{t-1}}}{1-\frac{P^H_{t-1}}{P_{t-1}}\psi(f^{*g}_{t-1}-f^{*h}_{t-1})} \tag{68}$$

Finally, as said, the profit is distributed in lump-sum fashion at the start of period t to domestic households so that $\pi_t = \sum_{i=1}^{N}\pi_{i,t}$ in equilibrium.

[37] Thus, at the beginning of period t, agents carry over assets and liabilities from period $t-1$. Borrowers honor their preexisting obligations to lenders. In particular, in the international capital market, where transactions take place via the bank, the bank receives interest income from borrowers and pays off the lenders. The latter is the interest payments that the bank promised at $t-1$ to pay at t. The bank also pays the monitoring cost associated with these transactions.

12 Appendix 5: Equilibrium in the status quo economy

This Appendix presents the status quo equilibrium system, given feedback policy coefficients. We work in steps.

12.1 Equilibrium equations

The home country is summarized by the following equations:

$$\frac{\partial u_t}{\partial c_t}\frac{\partial c_t}{\partial c_t^H}\frac{P_t}{P_t^H\left(1+\tau_t^c\right)}=\beta\frac{\partial u_{t+1}}{\partial c_{t+1}}\frac{\partial c_{t+1}}{\partial c_{t+1}^H}\frac{P_{t+1}}{P_{t+1}^H\left(1+\tau_{t+1}^c\right)}R_t\frac{P_t}{P_{t+1}} \tag{69}$$

$$\begin{gathered}\tfrac{\partial u_t}{\partial c_t}\tfrac{\partial c_t}{\partial c_t^H}\tfrac{1}{(1+\tau_t^c)}\tfrac{P_t}{P_t^H}\tfrac{S_tP_t^*}{P_t}=\\=\beta\tfrac{\partial u_{t+1}}{\partial c_{t+1}}\tfrac{\partial c_{t+1}}{\partial c_{t+1}^H}\tfrac{1}{\left(1+\tau_{t+1}^c\right)}\tfrac{P_{t+1}}{P_{t+1}^H}Q_t\tfrac{S_{t+1}P_{t+1}^*}{P_{t+1}}\tfrac{P_t^*}{P_{t+1}^*}\end{gathered} \tag{70}$$

$$\begin{gathered}\tfrac{\partial u_t}{\partial c_t}\tfrac{\partial c_t}{\partial c_t^H}\tfrac{1}{(1+\tau_t^c)}\left\{1+\xi\left(\tfrac{k_t}{k_{t-1}}-1\right)\right\}=\\=\beta E_t\tfrac{\partial u_{t+1}}{\partial c_{t+1}}\tfrac{\partial c_{t+1}}{\partial c_{t+1}^H}\tfrac{1}{\left(1+\tau_{t+1}^c\right)}\left\{(1-\delta)-\tfrac{\xi}{2}\left(\tfrac{k_{t+1}}{k_t}-1\right)^2+\xi\left(\tfrac{k_{t+1}}{k_t}-1\right)\tfrac{k_{t+1}}{k_t}+\left(1-\tau_{t+1}^k\right)r_{t+1}^k\right\}\end{gathered} \tag{71}$$

$$\frac{\partial u_t}{\partial m_t}=\frac{\partial u_t}{\partial c_t}\frac{\partial c_t}{\partial c_t^H}\frac{P_t}{P_t^H\left(1+\tau_t^c\right)}-\beta\frac{\partial u_{t+1}}{\partial c_{t+1}}\frac{\partial c_{t+1}}{\partial c_{t+1}^H}\frac{P_{t+1}}{P_{t+1}^H\left(1+\tau_{t+1}^c\right)}\frac{P_t}{P_{t+1}} \tag{72}$$

$$\frac{\partial u_t}{\partial n_t}=\left(1-\tau_t^n\right)w_t\frac{\partial u_t}{\partial c_t}\frac{\partial c_t}{\partial c_t^H}\frac{P_t}{P_t^H\left(1+\tau_t^c\right)} \tag{73}$$

$$\frac{c_t^H}{c_t^F}=\frac{\nu}{1-\nu}\frac{P_t^F}{P_t^H} \tag{74}$$

$$k_t=(1-\delta)k_{t-1}+x_t-\frac{\xi}{2}\left(\frac{k_t}{k_{t-1}}-1\right)^2k_{t-1} \tag{75}$$

$$c_t=\frac{\left(c_t^H\right)^\nu\left(c_t^F\right)^{1-\nu}}{\nu^\nu(1-\nu)^{1-\nu}} \tag{76}$$

$$w_t=mc_t(1-a)A_tk_{t-1}^an_t^{-a} \tag{77}$$

$$\frac{P_t^H}{P_t}r_t^k=mc_taA_tk_{t-1}^{a-1}n_t^{1-a} \tag{78}$$

$$\widetilde{\omega}_t=\frac{P_t^H}{P_t}y_t^H-\frac{P_t^H}{P_t}r_t^kk_{t-1}-w_tn_t \tag{79}$$

$$\sum_{k=0}^{\infty}\theta^k E_t\ \Xi_{t,t+k}\left[\frac{P_t^{\#}}{P_{t+k}^H}\right]^{-\phi} y_{t+k}^H\left\{\frac{P_t^{\#}}{P_t^H}\frac{P_t^H}{P_t}\frac{P_t}{P_{t-1}}-\frac{\phi}{(\phi-1)}mc_{t+k}\frac{P_t}{P_{t-1}}...\frac{P_{t+k}}{P_{t+k-1}}\right\}=0 \tag{80}$$

$$y_t^H=\frac{1}{\left(\frac{\widetilde{P}_t^H}{P_t^H}\right)^{-\phi}}A_t k_{t-1}^a n_t^{1-a} \tag{81}$$

$$\begin{array}{c} b_t+m_t+\frac{S_tP_t^*}{P_t}f_t^g=\frac{R_{t-1}b_{t-1}}{\Pi_t}+\frac{m_{t-1}}{\Pi_t}+Q_{t-1}\frac{S_tP_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}f_{t-1}^g+ \\ +\frac{P_t^H}{P_t}g_t-\tau_t^c(\frac{P_t^H}{P_t}c_t^H+\frac{P_t^F}{P_t}c_t^F)-\tau_t^k(r_t^k\frac{P_t^H}{P_t}k_{t-1}+\widetilde{\omega}_t)-\tau_t^n w_t n_t-\tau_t^l \end{array} \tag{82}$$

$$y_t^H=c_t^H+x_t+g_t+q_t+c_t^{F*} \tag{83}$$

$$\begin{array}{l} \frac{P_t^H}{P_t}(c_t^H+x_t+g_t-y_t^H)+\frac{P_t^F}{P_t}c_t^F+Q_{t-1}\frac{S_tP_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}\left(f_{t-1}^g-f_{t-1}^h\right)= \\ =\frac{S_tP_t^*}{P_t}\left(f_t^g-f_t^h\right)+\pi_t \end{array} \tag{84}$$

$$\left(P_t^H\right)^{1-\phi}=\left[\theta\left(P_{t-1}^H\right)^{1-\phi}+(1-\theta)\left(P_t^{\#}\right)^{1-\phi}\right] \tag{85}$$

$$P_t=(P_t^H)^{\nu}(P_t^F)^{1-\nu} \tag{86}$$

$$P_t^F=S_tP_t^{H*} \tag{87}$$

$$P_t^*=(P_t^{*H})^{\nu^*}(P_t^H/S_t)^{1-\nu^*} \tag{88}$$

$$\left(\widetilde{P}_t^H\right)^{-\phi}=\left[\theta\left(\widetilde{P}_{t-1}^H\right)^{-\phi}+(1-\theta)\left(P_t^{\#}\right)^{-\phi}\right] \tag{89}$$

$$q_t=Q_{t-1}^*\frac{\psi}{2}\frac{P_{t-1}^H}{P_t^H}(f_{t-1}^{*g}-f_{t-1}^{*h})^2 \tag{90}$$

$$\pi_t=Q_{t-1}^*\left[\frac{P_{t-1}}{P_t}(f_{t-1}^{*g}-f_{t-1}^{*h})-\frac{P_t^H}{P_t}\frac{\psi}{2}\frac{P_{t-1}^H}{P_t^H}(f_{t-1}^{*g}-f_{t-1}^{*h})^2\right]-Q_{t-1}\frac{S_t}{S_{t-1}}\frac{P_{t-1}}{P_t}(f_{t-1}^{*g}-f_{t-1}^{*h}) \tag{91}$$

$$Q_{t-1}^{*}=\frac{Q_{t-1}\frac{S_t}{S_{t-1}}}{1-\frac{P_{t-1}^{H}}{P_{t-1}}\psi(f_{t-1}^{*g}-f_{t-1}^{*h})} \tag{92}$$

where $\Xi_{t,t+k}\equiv\beta^{k}\frac{c_{t+k}^{-\sigma}}{c_{t}^{-\sigma}}\frac{P_t}{P_{t+k}}\frac{\tau_t^c}{\tau_{t+k}^c}$, $\frac{S_{t+1}}{S_t}=1$ in a currency union model and recall that $\frac{P_t^H}{P_t}=\left(\frac{P_t^H}{P_t^F}\right)^{1-\nu}$.

The foreign country is summarized by the following equations:

$$\frac{\partial u_t^*}{\partial c_t^*}\frac{\partial c_t^*}{\partial c_t^{*H}}\frac{P_t^*}{P_t^{*H}\left(1+\tau_t^{*c}\right)}=\beta\frac{\partial u_{t+1}^*}{\partial c_{t+1}^*}\frac{\partial c_{t+1}^*}{\partial c_{t+1}^{*H}}\frac{P_{t+1}^*}{P_{t+1}^{*H}\left(1+\tau_{t+1}^{*c}\right)}R_t^*\frac{P_t^*}{P_{t+1}^*} \tag{93}$$

$$\begin{gathered}\frac{\partial u_t^*}{\partial c_t^*}\frac{\partial c_t^*}{\partial c_t^{*H}}\frac{P_t^*}{P_t^{*H}(1+\tau_t^{*c})}\frac{P_t}{S_tP_t^*}=\\=\beta\frac{\partial u_{t+1}^*}{\partial c_{t+1}^*}\frac{\partial c_{t+1}^*}{\partial c_{t+1}^{*H}}\frac{P_{t+1}^*}{P_{t+1}^{*H}(1+\tau_{t+1}^{*c})}Q_t^*\frac{P_{t+1}}{S_{t+1}P_{t+1}^*}\frac{P_t}{P_{t+1}}\end{gathered} \tag{94}$$

$$\begin{gathered}\frac{\partial u_t^*}{\partial c_t^*}\frac{\partial c_t^*}{\partial c_t^{*H}}\frac{1}{(1+\tau_t^{*c})}\left\{1+\xi^*\left(\frac{k_t^*}{k_{t-1}^*}-1\right)\right\}=\\=\beta\frac{\partial u_{t+1}^*}{\partial c_{t+1}^*}\frac{\partial c_{t+1}^*}{\partial c_{t+1}^{*H}}\frac{1}{\left(1+\tau_{t+1}^{*c}\right)}\left\{(1-\delta^*)-\frac{\xi^*}{2}\left(\frac{k_{t+1}^*}{k_t^*}-1\right)^2+\xi^*\left(\frac{k_{t+1}^*}{k_t^*}-1\right)\frac{k_{t+1}^*}{k_t^*}+\left(1-\tau_{t+1}^{*k}\right)r_{t+1}^{*k}\right\}\end{gathered} \tag{95}$$

$$\frac{\partial u_t^*}{\partial m_t^*}=\frac{\partial u_t^*}{\partial c_t^*}\frac{\partial c_t^*}{\partial c_t^{*H}}\frac{P_t^*}{P_t^{*H}\left(1+\tau_t^{*c}\right)}-\beta\frac{\partial u_{t+1}^*}{\partial c_{t+1}^*}\frac{\partial c_{t+1}^*}{\partial c_{t+1}^{*H}}\frac{P_{t+1}^*}{P_{t+1}^{*H}\left(1+\tau_{t+1}^{*c}\right)}\frac{P_t^*}{P_{t+1}^*} \tag{96}$$

$$-\frac{\partial u_t^*}{\partial n_t^*}=\left(1-\tau_t^{*n}\right)w_t\frac{\partial u_t^*}{\partial c_t^*}\frac{\partial c_t^*}{\partial c_t^{H*}}\frac{P_t^*}{P_t^{*H}\left(1+\tau_t^{*c}\right)} \tag{97}$$

$$\frac{c_t^{*H}}{c_t^{*F}}=\frac{\nu^*}{1-\nu^*}\frac{P_t^{*F}}{P_t^{*H}} \tag{98}$$

$$k_t^*=(1-\delta^*)k_{t-1}^*+x_t^*-\frac{\xi^*}{2}\left(\frac{k_t^*}{k_{t-1}^*}-1\right)^2k_{t-1}^* \tag{99}$$

$$c_t^*=\frac{\left(c_t^{*H}\right)^{\nu^*}\left(c_t^{*F}\right)^{1-\nu^*}}{\nu^{*\nu^*}(1-\nu^*)^{1-\nu^*}} \tag{100}$$

$$w_t = mc_t^*(1-a^*)A_t^* k_{t-1}^{*a^*} n_t^{*-a^*} \tag{101}$$

$$\frac{P_t^{*H}}{P_t^*} r_t^{*k} = mc_t^* a^* A_t^* k_{t-1}^{*a-1} n_t^{*1-a} \tag{102}$$

$$\widetilde{\omega}_t^* = \frac{P_t^{*H}}{P_t^*} y_t^{*H} - \frac{P_t^{*H}}{P_t^*} r_t^{*k} k_{t-1}^* - \frac{W_t^*}{P_t^*} n_t^* \tag{103}$$

$$\sum_{k=0}^{\infty} (\theta^*)^k E_t\ \Xi_{t,t+k}^* \left[\frac{P_t^{*\#}}{P_{t+k}^{*H}}\right]^{-\phi} y_{t+k}^{*H} \left\{ \frac{P_t^{*\#}}{P_t^{*H}} \frac{P_t^{*H}}{P_t^*} \frac{P_t^*}{P_{t-1}^*} - \frac{\phi}{(\phi-1)} mc_{t+k}^* \frac{P_t^*}{P_{t-1}^*} \cdots \frac{P_{t+k}^*}{P_{t+k-1}^*} \right\} = 0 \tag{104}$$

$$y_t^{H*} = \frac{1}{\left(\frac{\widetilde{P}_t^{H*}}{P_t^{H*}}\right)^{-\phi}} A_t^* k_{t-1}^{*a^*} n_t^{*1-a^*} \tag{105}$$

$$\begin{gathered} b_t^* + m_t^* + \tfrac{P_t}{S_t P_t^*} f_t^{*g} = R_{t-1}^* b_{t-1}^* \tfrac{P_{t-1}^*}{P_t^*} + m_{t-1}^* \tfrac{P_{t-1}^*}{P_t^*} + Q_{t-1}^* \tfrac{P_t}{S_t P_t^*} \tfrac{P_{t-1}}{P_t} f_{t-1}^{*g} + \\ + \tfrac{P_t^{H*}}{P_t^*} g_t^* - \tau_t^{*c}(P_t^{*H} c_t^{*H} + P_t^{*F} c_t^{*F}) - \tau_t^{*k}(r_t^{*k} P_t^{*H} k_{t-1}^* + \widetilde{\omega}_t^*) - \tau_t^{*n} w_t^* n_t^* - \tau_t^{*l} \end{gathered} \tag{106}$$

$$y_t^{*H} = c_t^{*H} + x_t^* + g_t^* + c_t^F \tag{107}$$

$$\begin{gathered} \tfrac{P_t^{*H}}{P_t^*}(c_t^{*H} + x_t^* + g_t^* - y_t^{*H}) + \tfrac{P_t^{*F}}{P_t^*} c_t^{*F} + Q_{t-1}^* \tfrac{P_t}{S_t P_t^*} \tfrac{P_{t-1}}{P_t} \left(f_{t-1}^{*g} - f_{t-1}^{*h}\right) = \\ = \tfrac{P_t}{S_t P_t^*} \left(f_t^{*g} - f_t^{*h}\right) \end{gathered} \tag{108}$$

$$\left(P_t^{*H}\right)^{1-\phi^*} = \left[\theta^* \left(P_{t-1}^{*H}\right)^{1-\phi^*} + (1-\theta^*)\left(P_t^{*\#}\right)^{1-\phi^*}\right] \tag{109}$$

$$\left(\widetilde{P}_t^{*H}\right)^{-\phi^*} = \left[\theta^* \left(\widetilde{P}_{t-1}^{*H}\right)^{-\phi^*} + (1-\theta^*)\left(P_t^{*\#}\right)^{-\phi^*}\right] \tag{110}$$

where see below for number of equations and variables in this system.

12.2 Transformed variables

As in the related literature (see e.g. Schmitt-Grohé and Uribe (2005, 2007)), we transform some variables and introduce some new ones.

First, instead of price levels, we work with inflation rates and relative prices. Thus, we define $\Pi_t \equiv \frac{P_t}{P_{t-1}}, \Pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}, \Pi_t^H \equiv \frac{P_t^H}{P_{t-1}^H}, \Theta_t \equiv \frac{P_t^\#}{P_t^H}, \Delta_t \equiv \left(\frac{\widetilde{P}_t^H}{P_t^H}\right)^{-\phi}, \epsilon_t \equiv \frac{S_t}{S_{t-1}}$ and $TT_t \equiv \frac{P_t^F}{P_t^H}$. We also express some policy variables as shares of output. In particular, we define $b_t \equiv s_t^b y_t^H TT_t^{\nu-1}$, $s_t^l \equiv s_t^l y_t^H TT_t^{\nu-1}$ and $g_t \equiv s_t^g y_t^H$. So, in what follows, we use Π_t, Π_t^*, Π_t^H, Θ_t, Δ_t, ϵ_t, TT_t, s_t^g, s_t^l instead of P_t, P_t^*, P_t^H, $P_t^\#$, $\widetilde{P}_t$, S_t, P_t^F, g_t, τ_t^l respectively. Note that we also use (in the steady state only), $f_t^g \equiv s_t^f y_t^H \frac{1}{TT_t^{\nu^*}}$.

Second, working as in Schmitt-Grohé and Uribe (2007), we rewrite the firm's optimality conditions in recursive form. In particular, instead of equation (80), we now use:

$$z_t^1 = \frac{\phi}{(\phi - 1)} z_t^2 \tag{111}$$

where

$$z_t^1 = \Theta_t^{1-\phi} y_t TT_t^{\nu-1} + \beta\theta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \left(\frac{\Theta_t}{\Theta_{t+1}}\right)^{1-\phi} \left(\frac{1}{\Pi_{t+1}^H}\right)^{1-\phi} z_{t+1}^1 \tag{112}$$

$$z_t^2 = \Theta_t^{-\phi} y_t mc_t + \beta\theta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \left(\frac{\Theta_t}{\Theta_{t+1}}\right)^{-\phi} \left(\frac{1}{\Pi_{t+1}^H}\right)^{-\phi} z_{t+1}^2 \tag{113}$$

thus, we add two more equations and two new endogenous variables, z_t^1 and z_t^2.

Third, again as in Schmitt-Grohé and Uribe (2007), in order to compute expected discounted lifetime utility, denoted as V_t, we add a new equation and a new endogenous variable, V_t:

$$V_t = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_t^{1+\varphi}}{1+\varphi} + \chi_m \frac{m_t^{1-\mu}}{1-\mu} + \chi_g \frac{\left(s_t^g y_t^H\right)^{1-\zeta}}{1-\zeta} + \beta E_t V_{t+1} \tag{114}$$

We work similarly for the foreign country. That is, first, we use Π_t^{*H}, Θ_t^*, Δ_t^*, s_t^{*g}, s_t^{*l} instead of P_t^{*H}, $P_t^{*\#}$, $\widetilde{P}_t^*$, g_t, τ_t^{*l} respectively, second, we have for the foreign firm:

$$z_t^{*1} = \frac{\phi}{(\phi - 1)} z_t^{*2} \tag{115}$$

$$z_t^{*1} = \Theta_t^{*1-\phi^*} y_t^{*H} TT_t^{1-\nu^*} + \beta^*\theta^* E_t \frac{c_{t+1}^{*-\sigma^*}}{c_t^{*-\sigma^*}} \frac{1+\tau_t^{*c}}{1+\tau_{t+1}^{*c}} \left(\frac{\Theta_t^*}{\Theta_{t+1}^*}\right)^{1-\phi^*} \left(\frac{1}{\Pi_{t+1}^{*H}}\right)^{1-\phi^*} z_{t+1}^{*1} \tag{116}$$

$$z_t^{*2} = \Theta_t^{*-\phi^*} y_t^{*H} mc_t^* + \beta^*\theta^* E_t \frac{c_{t+1}^{*-\sigma^*}}{c_t^{*-\sigma^*}} \frac{1+\tau_t^{*c}}{1+\tau_{t+1}^{*c}} \left(\frac{\Theta_t^*}{\Theta_{t+1}^*}\right)^{-\phi^*} \left(\frac{1}{\Pi_{t+1}^{*H}}\right)^{-\phi^*} z_{t+1}^{*2} \tag{117}$$

and, thirdly, we have the new value function:

$$V_t^* = \frac{c_t^{*1-\sigma^*}}{1-\sigma^*} - \chi_n^* \frac{n_t^{*1+\varphi^*}}{1+\varphi^*} + \chi_m^* \frac{m_t^{*1-\mu^*}}{1-\mu^*} + \chi_g^* \frac{\left(s_t^{*g} y_t^{*H}\right)^{1-\zeta^*}}{1-\zeta^*} + \beta E_t V_{t+1}^* \tag{118}$$

Finally, given the above, notice that we make use of the following equations:

$$\frac{P_t}{S_t P_t^*} = TT_t^{1-\nu-\nu^*}$$

$$TT_t = \frac{P_t^F}{P_t^H} = \frac{\frac{P_t^F}{S_t}}{\frac{P_t^H}{S_t}} = \frac{P_t^{*H}}{P_t^{*F}}$$

$$\frac{P_t^{*H}}{P_t^*} = \frac{P_t^{*H}}{\left(P_t^{*H}\right)^{\nu^*}\left(P_t^{*F}\right)^{1-\nu^*}} = \left(\frac{P_t^{*H}}{P_t^{*F}}\right)^{1-\nu^*} = TT_t^{1-\nu^*}$$

$$\frac{P_t^{*F}}{P_t^*} = \frac{P_t^{*F}}{\left(P_t^{*H}\right)^{\nu^*}\left(P_t^{*F}\right)^{1-\nu^*}} = \left(\frac{P_t^{*F}}{P_t^{*H}}\right)^{\nu^*} = \left(\frac{1}{TT_t}\right)^{\nu^*}$$

12.3 Final equilibrium system in the status quo economy

Using the above, we now present the final equilibrium system (given feedback policy coefficients).

The domestic country is summarized by the following equations:

$$V_t = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_t^{1+\varphi}}{1+\varphi} + \chi_m \frac{m_t^{1-\mu}}{1-\mu} + \chi_g \frac{\left(s_t^g y_t^H\right)^{1-\zeta}}{1-\zeta} + \beta E_t V_{t+1} \tag{119}$$

$$\beta E_t \frac{c_{t+1}^{-\sigma}}{\left(1+\tau_{t+1}^c\right)} \frac{R_t}{\Pi_{t+1}} = \frac{c_t^{-\sigma}}{\left(1+\tau_t^c\right)} \tag{120}$$

$$\beta E_t \frac{c_{t+1}^{-\sigma}}{\left(1+\tau_{t+1}^c\right)} \frac{Q_t TT_{t+1}^{v^*+\nu-1}}{\Pi_{t+1}^*} = \frac{c_t^{-\sigma}}{\left(1+\tau_t^c\right)} TT_t^{v^*+\nu-1} \tag{121}$$

$$\beta E_t \frac{c_{t+1}^{-\sigma}}{\left(1+\tau_{t+1}^c\right)} TT_{t+1}^{\nu-1} \left\{1-\delta-\frac{\xi}{2}\left(\frac{k_{t+1}}{k_t}-1\right)^2+\xi\left(\frac{k_{t+1}}{k_t}-1\right)\frac{k_{t+1}}{k_t}+\left(1-\tau_{t+1}^k\right)r_{t+1}^k\right\} = \\ = \frac{c_t^{-\sigma}}{\left(1+\tau_t^c\right)} TT_t^{\nu-1}\left[1+\xi\left(\frac{k_t}{k_{t-1}}-1\right)\right] \tag{122}$$

$$\chi_m m_t^{-\mu} = \frac{c_t^{-\sigma}}{\left(1+\tau_t^c\right)} - \beta E_t \frac{c_{t+1}^{-\sigma}}{\left(1+\tau_{t+1}^c\right)} \frac{1}{\Pi_{t+1}} \tag{123}$$

$$\chi_n n_t^{\varphi} = \left(1-\tau_t^n\right) w_t \frac{c_t^{-\sigma}}{\left(1+\tau_t^c\right)} \tag{124}$$

$$\frac{c_t^H}{c_t^F} = \frac{\nu}{1-\nu} TT_t \tag{125}$$

$$k_t = (1-\delta)k_{t-1} + x_t - \frac{\xi}{2}\left(\frac{k_t}{k_{t-1}}-1\right)^2 k_{t-1} \tag{126}$$

$$c_t = \frac{\left(c_t^H\right)^{\nu}\left(c_t^F\right)^{1-\nu}}{(\nu)^{\nu}(1-\nu)^{1-\nu}} \tag{127}$$

$$w_t = mc_t(1-a)A_t k_{t-1}^a n_t^{-a} \tag{128}$$

$$\frac{1}{TT_t^{1-v}} r_t^k = mc_t a A_t k_{t-1}^{a-1} n_t^{1-a} \tag{129}$$

$$\widetilde{\omega}_t = \frac{1}{TT_t^{1-v}} y_t^H - \frac{1}{TT_t^{1-v}} r_t^k k_{t-1} - w_t n_t \tag{130}$$

$$z_t^1 = \frac{\phi}{(\phi-1)} z_t^2 \tag{131}$$

$$y_t^H = \frac{1}{\Delta_t} A_t k_{t-1}^a n_t^{1-a} \tag{132}$$

$$d_t + m_t = \frac{R_{t-1}}{\Pi_t}\lambda_{t-1}d_{t-1} + \frac{Q_{t-1}TT_t^{v+v^*-1}}{\Pi_t^*}\frac{1}{TT_{t-1}^{v+v^*-1}}(1-\lambda_{t-1})d_{t-1} + \\ + \frac{1}{\Pi_t}m_{t-1} + TT_t^{\nu-1}s_t^g y_t^H - \tau_t^c\left(\frac{1}{TT_t^{1-v}}c_t^H + TT_t^v c_t^F\right) - \\ - \tau_t^k\left(r_{t-1}^k \frac{1}{TT_t^{1-v}} k_{t-1} + \widetilde{\omega}_t\right) - \tau_t^n w_t n_t - TT_t^{\nu-1} s_t^l y_t^H \tag{133}$$

$$(1-\lambda_t)d_t - TT_t^{\nu^*+\nu-1}f_t^h + \pi_t + TT_t^{\nu-1}q_t = -TT_t^{\nu-1}c_t^{F*} + TT_t^{\nu}c_t^F + \\ + \tfrac{Q_{t-1}TT_t^{\nu^*+\nu-1}}{\Pi_t^*}\left(\tfrac{1}{TT_{t-1}^{v+v^*-1}}(1-\lambda_{t-1})d_{t-1} - f_{t-1}^h\right) \tag{134}$$

$$y_t^H = c_t^H + x_t + s_t^g y_t^H + q_t + c_t^{F*} \tag{135}$$

$$\left(\Pi_t^H\right)^{1-\phi} = \theta + (1-\theta)\left(\Theta_t \Pi_t^H\right)^{1-\phi} \tag{136}$$

$$\frac{\Pi_t}{\Pi_t^H} = \left(\frac{TT_t}{TT_{t-1}}\right)^{1-\nu} \tag{137}$$

$$\frac{TT_t}{TT_{t-1}} = \frac{\epsilon_t \Pi_t^{*H}}{\Pi_t^H} \tag{138}$$

$$\frac{\Pi_t^*}{\Pi_t^{*H}} = \left(\frac{TT_{t-1}}{TT_t}\right)^{1-\nu^*} \tag{139}$$

$$\Delta_t = \theta\Delta_{t-1}\left(\Pi_t^H\right)^{\phi} + (1-\theta)\left(\Theta_t\right)^{-\phi} \tag{140}$$

$$z_t^1 = \Theta_t^{1-\phi} y_t TT_t^{\nu-1} + \beta\theta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c}\left(\frac{\Theta_t}{\Theta_{t+1}}\right)^{1-\phi}\left(\frac{1}{\Pi_{t+1}^H}\right)^{1-\phi} z_{t+1}^1 \tag{141}$$

$$z_t^2 = \Theta_t^{-\phi} y_t mc_t + \beta\theta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c}\left(\frac{\Theta_t}{\Theta_{t+1}}\right)^{-\phi}\left(\frac{1}{\Pi_{t+1}^H}\right)^{-\phi} z_{t+1}^2 \tag{142}$$

$$q_t = Q_{t-1}^* \frac{\psi}{2} \frac{P_{t-1}^H}{P_t^H}(f_{t-1}^{*g} - f_{t-1}^{*h})^2 \tag{143}$$

$$\pi_t = Q_{t-1}^*\left[\frac{P_{t-1}}{P_t}(f_{t-1}^{*g} - f_{t-1}^{*h}) - \frac{P_t^H}{P_t}\frac{\psi}{2}\frac{P_{t-1}^H}{P_t^H}(f_{t-1}^{*g} - f_{t-1}^{*h})^2\right] - Q_{t-1}\frac{S_t}{S_{t-1}}\frac{P_{t-1}}{P_t}(f_{t-1}^{*g} - f_{t-1}^{*h}) \tag{144}$$

$$Q^*_{t-1} = \frac{Q_{t-1}\frac{S_t}{S_{t-1}}}{1 - \frac{P^H_{t-1}}{P_{t-1}}\psi(f^{*g}_{t-1} - f^{*h}_{t-1})} \tag{145}$$

Next, the foreign country is summarized by the following equations:

$$V^*_t = \frac{c_t^{*1-\sigma^*}}{1-\sigma^*} - \chi^*_n \frac{n_t^{*1+\varphi^*}}{1+\varphi^*} + \chi^*_m \frac{m_t^{*1-\mu^*}}{1-\mu^*} + \chi^*_g \frac{\left(s^{*g}_t y^{*H}_t\right)^{1-\zeta^*}}{1-\zeta^*} + \beta^* E_t V^*_{t+1} \tag{146}$$

$$\beta^* E_t \frac{c^{*-\sigma}_{t+1}}{\left(1+\tau^{*c}_{t+1}\right)} \frac{R^*_t}{\Pi^*_{t+1}} = \frac{c^{*-\sigma}_t}{\left(1+\tau^{*c}_t\right)} \tag{147}$$

$$\beta^* E_t \frac{c^{*-\sigma}_{t+1}}{\left(1+\tau^{*c}_{t+1}\right)} \frac{Q^*_t TT^{1-\nu-v^*}_{t+1}}{\Pi_{t+1}} = \frac{c^{*-\sigma}_t}{\left(1+\tau^{*c}_t\right)} TT_t^{1-\nu-v^*} \tag{148}$$

$$\begin{aligned} \beta E_t TT^{1-\nu^*}_{t+1} \frac{c^{*-\sigma}_{t+1}}{\left(1+\tau^{*c}_{t+1}\right)} \left\{1 - \delta^* - \frac{\xi^*}{2}\left(\frac{k^*_{t+1}}{k^*_t} - 1\right)^2 + \xi^*\left(\frac{k^*_{t+1}}{k^*_t} - 1\right)\frac{k^*_{t+1}}{k^*_t} + \left(1 - \tau^{*k}_{t+1}\right) r^{*k}_{t+1}\right\} = \\ = TT^{1-\nu^*}_t \frac{c^{*-\sigma}_t}{\left(1+\tau^{*c}_t\right)} \left[1 + \xi^*\left(\frac{k^*_t}{k^*_{t-1}} - 1\right)\right] \end{aligned} \tag{149}$$

$$\chi^*_m m_t^{*-\mu^*} = \frac{c^{*-\sigma}_t}{\left(1+\tau^{*c}_t\right)} - \beta^* E_t \frac{c^{*-\sigma}_{t+1}}{\left(1+\tau^{*c}_{t+1}\right)} \frac{1}{\Pi^*_{t+1}} \tag{150}$$

$$\chi^*_n n_t^{*\varphi^*} = \left(1 - \tau^{*n^*}_t\right) w^*_t \frac{c^{*-\sigma}_t}{\left(1+\tau^{*c}_t\right)} \tag{151}$$

$$\frac{c^{*H}_t}{c^{*F}_t} = \frac{\nu^*}{1-\nu^*}\frac{1}{TT_t} \tag{152}$$

$$k^*_t = (1-\delta^*)k^*_{t-1} + x^*_t - \frac{\xi^*}{2}\left(\frac{k^*_t}{k^*_{t-1}} - 1\right)^2 k^*_{t-1} \tag{153}$$

$$c^*_t = \frac{\left(c^{*H}_t\right)^{\nu^*}\left(c^{*F}_t\right)^{1-\nu^*}}{\left(\nu^*\right)^{\nu^*}\left(1-\nu^*\right)^{1-\nu^*}} \tag{154}$$

$$w^*_t = mc^*_t(1-a^*)A^*_t k^{*a^*}_{t-1} n^{*-a^*}_t \tag{155}$$

$$TT^{1-v^*}_t r^{*k}_t = mc^*_t a^* A^*_t k^{*a^*-1}_{t-1} n^{*1-a^*}_t \tag{156}$$

$$\widetilde{\omega}^*_t = TT^{1-v^*}_t y^{*H}_t - TT^{1-v^*}_t r^{*k}_t k^*_{t-1} - w^*_t n^*_t \tag{157}$$

$$z_t^{*1} = \frac{\phi^*}{(\phi^* - 1)} z_t^{*2} \tag{158}$$

$$y_t^{*H} = \frac{1}{\Delta_t^*} A_t^* k_{t-1}^{*a^*} n_t^{*1-a^*} \tag{159}$$

$$\begin{aligned} d_t^* + m_t^* = {} & \tfrac{R_{t-1}^*}{\Pi_t^*} \lambda_{t-1}^* d_{t-1}^* + \tfrac{Q_{t-1}^* TT_t^{1-v-v^*}}{\Pi_t} \tfrac{1}{TT_{t-1}^{1-v-v^*}} (1 - \lambda_{t-1}^*) d_{t-1}^* + \\ & + \tfrac{1}{\Pi_t^*} m_{t-1}^* + TT_t^{1-\nu^*} s_t^{*g} y_t^{*H} - \tau_t^{*c} (TT_t^{1-v^*} c_t^{*H} + \tfrac{1}{TT_t^{v^*}} c_t^{*F}) - \\ & - \tau_t^{*k} (r_{t-1}^{*k} TT_t^{1-v^*} k_{t-1}^* + \widetilde{\omega}_t^*) - \tau_t^{*n} w_t^* n_t^* - s_t^{*l} y_t^{*H} TT_t^{1-\nu^*} \end{aligned} \tag{160}$$

$$y_t^{*H} = c_t^{*H} + x_t^* + s_t^{*g} y_t^{*H} + c_t^F \tag{161}$$

$$\begin{aligned} (1 - \lambda_t^*) d_t^* - TT_t^{1-\nu^*-\nu} f_t^{*h} = {} & -TT_t^{1-\nu^*} c_t^F + TT_t^{-\nu^*} c_t^{F^*} \\ & + \tfrac{Q_{t-1}^* TT_t^{1-\nu^*-\nu}}{\Pi_t} \left(\tfrac{1}{TT_{t-1}^{1-v-v^*}} (1 - \lambda_{t-1}^*) d_{t-1}^* - f_{t-1}^{h^*} \right) \end{aligned}$$

$$\left(\Pi_t^{*H}\right)^{1-\phi^*} = \theta^* + (1 - \theta^*) \left(\Theta_t^* \Pi_t^{*H}\right)^{1-\phi^*} \tag{162}$$

$$\Delta_t^* = \theta^* \Delta_{t-1}^* \left(\Pi_t^{*H}\right)^{\phi^*} + (1 - \theta^*) \left(\Theta_t^*\right)^{-\phi^*} \tag{163}$$

$$z_t^{*1} = \Theta_t^{*1-\phi^*} y_t^{*H} TT_t^{1-\nu^*} + \beta^* \theta^* E_t \frac{c_{t+1}^{*-\sigma^*}}{c_t^{*-\sigma^*}} \frac{1 + \tau_t^{*c}}{1 + \tau_{t+1}^{*c}} \left(\frac{\Theta_t^*}{\Theta_{t+1}^*} \right)^{1-\phi^*} \left(\frac{1}{\Pi_{t+1}^{*H}} \right)^{1-\phi^*} z_{t+1}^{*1} \tag{164}$$

$$z_t^{*2} = \Theta_t^{*-\phi^*} y_t^{*H} mc_t^* + \beta^* \theta^* E_t \frac{c_{t+1}^{*-\sigma^*}}{c_t^{*-\sigma^*}} \frac{1 + \tau_t^{*c}}{1 + \tau_{t+1}^{*c}} \left(\frac{\Theta_t^*}{\Theta_{t+1}^*} \right)^{-\phi^*} \left(\frac{1}{\Pi_{t+1}^{*H}} \right)^{-\phi^*} z_{t+1}^{*2} \tag{165}$$

where $S_t F^g = (1 - \lambda_t) D_t$, $F^g = \frac{(1-\lambda_t) D_t}{S_t}$, $\frac{F^g}{P_t^*} = \frac{(1-\lambda_t) D_t}{P_t^* S_t}$, $f_t^g = (1 - \lambda_t) d_t \frac{P_t}{P_t^* S_t} = (1 - \lambda_t) d_t \frac{1}{TT_t^{\nu^*+\nu-1}}$,

$\frac{F^{*g}}{S_t} = (1 - \lambda_t^*) D_t^*$, $F^{*g} = (1 - \lambda_t) D_t^* S_t$, $\frac{F^{*g}}{P_t} = \frac{(1-\lambda_t^*) D_t^* S_t}{P_t}$, $f_t^{*g} = (1 - \lambda_t^*) d_t^* \frac{S_t P_t^*}{P_t} = (1 - \lambda_t^*) d_t^* TT_t^{\nu^*+\nu-1}$,

$\frac{P_{t-1}^H}{P_{t-1}} = TT_{t-1}^{\nu-1}$, $\frac{S_t P_t^*}{P_t} = TT_t^{\nu^*+\nu-1}$, $\frac{P_t}{S_t P_t^*} = TT_{t+1}^{1-\nu-v^*}$, $\frac{P_t^{*H}}{P_t^*} = \left(\frac{P_t^{*H}}{P_t^{*F}} \right)^{1-\nu^*}$, $TT_t = \frac{P_t^F}{P_t^H} =$

$\frac{\frac{P_t^F}{S_t}}{\frac{P_t^H}{S_t}} = \frac{P_t^{*H}}{P_t^{*F}}$, $\frac{P_t^{*F}}{P_t^{*H}} = \frac{1}{TT_t}$, $\epsilon_t = \frac{S_t}{S_{t-1}}$.

We finally have the feedback monetary and fiscal policy rules:

$$\begin{aligned} \log\left(\frac{R_t}{R}\right) &= \phi_\pi\left(\eta\log\left(\frac{\Pi_t}{\Pi}\right) + (1-\eta)\log\left(\frac{\Pi_t^*}{\Pi^*}\right)\right) + \\ &+\phi_y\left(\eta\log\left(\frac{y_t^H}{y^H}\right) + (1-\eta)\log\left(\frac{y_t^{*H}}{y^{*H}}\right)\right) \end{aligned} \tag{166}$$

$$s_t^g - s^g = -\gamma_l^g\left(l_{t-1} - l\right) - \gamma_y^g\left(y_t^H - y^H\right) \tag{167}$$

$$\tau_t^c - \tau^c = \gamma_l^c\left(l_{t-1} - l\right) + \gamma_y^c\left(y_t^H - y^H\right) \tag{168}$$

$$\tau_t^k - \tau^k = \gamma_l^k\left(l_{t-1} - l\right) + \gamma_y^k\left(y_t^H - y^H\right) \tag{169}$$

$$\tau_t^n - \tau^n = \gamma_l^n\left(l_{t-1} - l\right) + \gamma_y^n\left(y_t^H - y^H\right) \tag{170}$$

$$s_t^{*g} - s^{*g} = -\gamma_l^{*g}\left(l_{t-1}^* - l^*\right) - \gamma_y^{*g}\left(y_t^{*H} - y^{*H}\right) \tag{171}$$

$$\tau_t^{*c} - \tau^{*c} = \gamma_l^{*c}\left(l_{t-1}^* - l^*\right) + \gamma_y^{*c}\left(y_t^{*H} - y^{*H}\right) \tag{172}$$

$$\tau_t^{*k} - \tau^{*k} = \gamma_l^{*k}\left(l_{t-1}^* - l^*\right) + \gamma_y^{*k}\left(y_t^{*H} - y^{*H}\right) \tag{173}$$

$$\tau_t^{*n} - \tau^{*n} = \gamma_l^{*n}\left(l_{t-1}^* - l^*\right) + \gamma_y^{*n}\left(y_t^{*H} - y^{*H}\right) \tag{174}$$

$$l_t = \frac{R_t \lambda_t d_t + Q_t \epsilon_{t+1} (1 - \lambda_t) d_t}{TT_t^{\nu-1} y_t^H} \tag{175}$$

$$l_t^* = \frac{R_t^* \lambda_t^* d_t^* + Q_t^* \frac{1}{\epsilon_{t+1}} (1 - \lambda_t^*) d_t^*}{TT_t^{1-\nu^*} y_t^{*H}} \tag{176}$$

Therefore, we have 27 equations for the home country, 21 equations for the foreign country and 11 equations for the policy reules. This is 59 equations in total. We also have 59 endogenous variables, which are $\{V, y^H, c, c^H, c^F, n, x, k, f^h, m, TT, \Pi, \Pi^H, \Theta, \Delta, w, mc, \widetilde{\omega}, r^k, d, \Pi^*, z^1, z^2, Q, \pi, q\}$ and $\{R, s^g, \tau^c, \tau^k, \tau^n, l\}$ for the home country, and $\{V^*, y^{*H}, c^*, c^{*H}, c^{*F}, n^*, x^*, k^*, f^{*h}, m^*, \Pi^{*H}, \Theta^*, \Delta^*, w^*, mc^*, \widetilde{\omega}^*, r^{*k}, d^*, z^{*1}, z^{*2}, Q^*, R^*\}$ and $\{s^{*g}, \tau^{*c}, \tau^{*k}, \tau^{*n}, l^*\}$ for the foreign country. This is given given the exogenous variables, $\{\epsilon, \lambda, s^l, \lambda^*, s^{*l}, A, A^*\}$, initial conditions for the state variables and the values of the feedback (monetary and fiscal) policy coefficients in the policy rules.

Notice that, since all market-clearing conditions have been already included, the above system also satisfies the international asset market-clearing condition, $(f^{*g} - f^{*h}) + \frac{S_t P_t^*}{P_t}(f^g - f^h) = 0$. This can be seen if we add up the two balance of payments above; this will give $(f^{*g} - f^{*h}) + \frac{S_t P_t^*}{P_t}(f^g - f^h) = 0$ residually.

12.4 Steady state and transition

The steady state system follows directly from the above defined system when variables do not change over time. At steady state, regarding monetary policy, we set $\Pi = \Pi^* = 1$ and let the nominal interest rates to follow residually from the Euler for bonds in each country. Regarding fiscal policy, the residual policy instrument is total public debt in each country. To get the transition path, we approximate the dynamic system around its steady state solution, as explained in the main text (see section 4).

13 Appendix 6: Equilibrium in the reformed economy

We study two cases as said in the main text.

13.1 When sovereign premia are allowed in the new reformed steady state

The equilibrium system in the reformed economy is the same as above (see the system of 59 equations above) except that now, in Italy, the debt target in the feedback policy rules in subsection 2.6.2 is set at 0.9. In the steady state of this reformed economy, with $\frac{d^*}{TT^{1-\nu^*} y^{*H}}$

set at 0.9, the capital tax rate falls to 0.302. Table A.1 reports the associated steady state solution.

Table A1: Reformed steady state with debt consolidation in Italy (with premia)

Variables	Description	Home	Foreign
$u,\ u^{*}$	utility	0.0397	0.0337
y^{H}, y^{H*}	output	0.3912	0.3569
c, c^{*}	consumption	0.2319	0.2283
$n,\ n^{*}$	hours worked	0.3116	0.3067
k, k^{*}	capital	0.6654	0.508
$w,\ w^{*}$	real wage rate	0.6904	0.7111
$r^{k},\ r^{k*}$	real return to capital	0.147	0.1756
TT	terms-of-trade	1.098	
$Q^{*}-Q$	interest rate premium		0.0055
$\frac{c}{y^{H}TT^{1-\nu}}, \frac{c^{*}}{y^{H*}TT_{t}^{\nu^{*}-1}}$	consumption as share of GDP	0.5656	0.6704
$\frac{k}{y^{H}}, \frac{k^{*}}{y^{H*}}$	capital as share of GDP	1.7	1.4236
$\frac{d}{TT^{\nu-1}y^{H}}, \frac{d^{*}}{TT^{1-\nu^{*}}y^{*H}}$	total public debt as share of GDP	0.69	0.9
$\frac{\left(\frac{(1-\lambda)d}{TT^{\nu-1}}-TT_{t}^{\nu^{*}}f^{h}\right)}{y^{H}}, \frac{\frac{(1-\lambda^{*})d^{*}}{TT^{1-\nu-\nu^{*}}}-f^{*h}}{TT_{t}^{\nu}y^{*H}}$	total foreign debt as share of GDP*	-0.21	0.209

13.2 When sovereign premia are not allowed in the new reformed steady state

The equilibrium system in the reformed economy is the same as above (see the system of 59 equations above) except that now, in Italy, not only the debt target in the feedback policy rules in subsection 2.6.2 is set at 0.9, but also the discount factor, β^{*}, follows the AR(1) rule:

$$\beta_{t}^{*}=\rho^{\beta^{*}}\beta_{t-1}^{*}+\left(1-\rho^{\beta^{*}}\right)\beta \tag{177}$$

where variables' definitions are in the main text and, as said, the value of $\rho^{\beta^{*}}$ is chosen optimally.

In the steady state of this reformed economy: (a) In Italy, since the public debt-to-GDP ratio is set at an exogenously given value, one of the other fiscal variables becomes endogenous. As said, we report results when it is the capital tax rate that plays that role. (b) Sovereign premia are eliminated by setting $Q^* = Q$. This, in turn, implies $\beta^* = \beta$ via the Euler equations for the international asset in the two countries written at the steady state. It also implies that that we lose one equation (the two Euler equations for the international asset become identical), but, at the same time, in order to have $Q^* = Q$, we also have a new equation, $f^{*g} - f^{*h} = 0$ (see equation (27)), so the steady state system remains well defined in terms of equations and unknowns. The associated steady state solution is in Table A2.

Table A2: Reformed steady state with debt consolidation in Italy (without premia)

Variables	Description	Home	Foreign
$u,\ u^*$	utility	0.0403	0.054
y^H, y^{H*}	output	0.3925	0.367
c, c^*	consumption	0.23	0.23
$n,\ n^*$	hours worked	0.31	0.307
k, k^*	capital	0.667	0.556
$w,\ w^*$	real wage rate	0.70	0.7259
$r^k,\ r^{k*}$	real return to capital	0.147	0.1653
TT	terms-of-trade	1.086	-
$Q^* - Q$	interest rate premium	-	0
$\frac{c}{y^H TT^{1-\nu}},\ \frac{c^*}{y^{H*} TT_t^{\nu^*-1}}$	consumption as share of GDP	0.5678	0.6588
$\frac{k}{y^H},\ \frac{k^*}{y^{H*}}$	capital as share of GDP	1.7009	1.5123
$\frac{d}{TT^{\nu-1} y^H},\ \frac{d^*}{TT^{1-\nu^*} y^{*H}}$	total public debt as share of GDP	0.6433	0.9
$\frac{\left(\frac{(1-\lambda)d}{TT^{\nu-1}} - TT_t^{\nu^*} f^h\right)}{y^H},\ \frac{\frac{(1-\lambda^*)d^*}{TT^{1-\nu-\nu^*}} - f^{*h}}{TT_t^{\nu} y^{*H}}$	total foreign debt as share of GDP*	0	0