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## Decentralized Leadership

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# Decentralized Leadership

## Abstract

This paper studies the efficiency of decentralized leadership in federations where selfish regional governments provide regional and federal public goods and the benevolent central government implements interregional earmarked and income transfers. Without residential mobility, unlimited decentralized leadership is efficient only if the center implements redistributive interregional income and earmarked transfers to equate consumption of private and regional public goods across regions. Such policies perfectly align the incentives of the selfish regional governments. With imperfect residential mobility, decentralized leadership is efficient if the center adopts redistributive interregional income and earmarked policies and there is a common labor market in the federation.

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# 1. Introduction

In a family context, in which the well-known “rotten kid theorem” holds, benevolent parents are unable to commit to incentive schemes (carrots and sticks) to induce their selfish children to be well behaved (see, e.g., Becker (1981), Bergstrom (1989), Cornes and Silva (1999)). Children may choose actions to promote their self-interests in lieu of their family’s common good. However, parents are the family workers and control the allocation of bequests, which occurs after parents observe their children’s actions. Becker (1981) demonstrates that selfish children are well behaved (i.e., they maximize their family’s welfare) if they anticipate that they are personally better off by taking actions that maximize family income. Bergstrom (1989) shows that Becker’s rotten kid theorem is not general, but it holds if the children have quasilinear preferences and their wellbeing are normal goods for their parents. Cornes and Silva (1999) show that the rotten kid theorem holds when the kids’ preferences can be represented by general but identical continuous and concave utility functions for two normal goods, a private good (numeraire) and a standard pure public good.

The interactions between selfish children and a loving, benevolent, parent in a family is similar in many respects to those between self-interested regional and benevolent central governments in federations where regional governments have (some) policy autonomy. In many federations, regional governments are able to implement some policies without seeking approval or support from federal authorities. The literature refers to this phenomenon as “decentralized leadership.” In Canada, the provinces of British Columbia and Alberta have been leaders in global environmental policy – they unilaterally moved forward with the levying of carbon taxes even though a nationwide policy on carbon emissions is still lacking. In the European Union, the nation states have considerable power vis-à-vis the central government to adopt several types of policies, ranging from policies that govern the provision of local public goods (e.g., policing, health care, national security) to policies that determine their contributions to public goods that generate transnational benefits (e.g., abatement of carbon emissions). In such federations, characterized by decentralized leadership, one also observes substantial interregional income and fiscal transfers.<sup>1</sup> These reflect policies that central authorities control and which intend to reduce income and fiscal disparities across regions.

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<sup>1</sup> See, e.g., Silva (2015) and Silva (2016). These papers provide ample evidence of the importance of interregional earmarked and income transfers in several federations.

Caplan, Cornes and Silva (2000), motivated by the parallels between families and federations, demonstrate that (two) self-interested regional governments provide efficient contributions to a federal, pure public good if they make these contributions knowing that the benevolent central government will redistribute income across regions after it observes the regional governments' actions. The authors show that this rotten kid theorem continues to hold when one extends the model to allow imperfectly mobile residents to choose their region of residence after they observe the policies implemented by regional and central authorities. Residents are imperfectly mobile due to idiosyncratic regional attachment benefits (e.g., language, culture, customs, family ties).

This paper revisits the efficiency of decentralized leadership, the main issue studied by Caplan, Cornes and Silva (2000), but in situations in which regional governments provide multiple public goods and the central government controls multiple interregional-transfer instruments.<sup>2</sup> Each regional government contributes to the provision of two public goods, one of which is regional and the other is federal. The quantity of federal public good is an aggregation of regional contributions where the aggregation consumption technology is represented by a concave function. Particular cases of this concave function are summation (pure public good) and Cobb-Douglas (weaker link). Cornes (1993) and Cornes and Hartley (2007) advanced the study of weaker-link public goods. At the federal level, good examples are control of infectious diseases and counterterrorism effort.

The central government is responsible for interregional income and earmarked fiscal transfers. To facilitate comparisons and illustrate the social desirability of earmarked fiscal transfers, the center's policy arsenal is initially restricted to contain an instrument to implement interregional income transfers only. The initial setting is further restricted with the assumption that individuals are immobile.

The results of this paper demonstrate that there are efficiency-enhancing incentives promoted by centralized income and earmarked transfers when used together. In the simpler model without residential mobility, interregional income transfers equalize marginal utilities of income. Interregional earmarked transfers, implemented to reduce fiscal disparities in the provision of regional public goods, equalize marginal utilities of consumption of regional public goods. With

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<sup>2</sup> See Cornes and Itaya (2010) for an interesting study of voluntary contributions to multiple pure public goods. The authors show that the provision levels in equilibrium are too low (relative to efficient levels), among other things.

separable (or homothetic) utility functions, the centralized choices imply equalization of consumption of private and regional public goods across regions, which lead forwarding looking regional governments to realize that they wish to maximize the same objective function. In the presence of such perfect incentive equivalence, regional governments provide efficient contributions to regional and federal public goods. In the absence of earmarked transfers, regional governments provide efficient contributions to the federal public good, but overprovide the regional public goods when they are able to commit to provision of both types of public goods. As in Caplan, Cornes and Silva (2000), the interregional income transfers promote incentives for efficient decentralized behavior on the provision of a federal public good. However, they also essentially make the regional governments to treat regional public goods as a federal public good, since the interregional-income-transfer mechanism creates a “universal” subsidy rate for the provision of all public goods that are subject to decentralized leadership. By including the earmarked fiscal transfer in the arsenal of instruments controlled by the center, one perfectly cures the anomaly caused by interregional income transfers, since the implicit subsidy rate disappears in the provision of regional public goods.

The efficiency of decentralized leadership remains in the extended model with imperfect residential mobility provided there is a common labor market in the federal economy. The existence of a common labor market implies that individual choices of region to work and to reside are independent. In such circumstances, interregional income and fiscal transfers lead to equalization of consumption of private and regional public goods across regions, and hence perfect incentive equivalence.

This paper contributes to multiple branches of the fiscal federalism literature.<sup>3</sup> It is the first paper, to my knowledge, to examine the efficiency of decentralized leadership in a setting in which regional governments commit to the provision of regional and federal public goods and the center is endowed with instruments to implement income and earmarked interregional transfers. Silva (2014) considers a decentralized leadership setting in which regional governments provide regional and federal public goods and the center promotes interregional transfers. However, the center does not have an instrument to implement earmarked transfers. Silva (2015) examines the

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<sup>3</sup> Please see Silva (2014), Silva (2015) and Silva and Lucas (2016) for important contributions to fiscal federalism in the areas of decentralized leadership, earmarking and soft budgets, and imperfect residential mobility due to regional attachment, respectively.

efficiency of interregional earmarked fiscal transfers when used together with interregional income transfers in a federation where regional governments provide multiple regional public goods only. This paper is also the first to combine the policy ingredients described above with a common labor market in the extended model with imperfectly mobile residents. Silva and Lucas (2016) show that decentralized leadership is efficient in the presence of a common labor market and imperfectly mobile residents when regional governments provide multiple types of federal public goods, but no regional public goods, and the center implements interregional income transfers. In the absence of regional public goods, the center does not need to have an instrument to implement earmarked transfers in order to promote incentives for efficient behavior at the regional level.

In what follows, section 2 introduces the basic model, and section 3 examines the socially optimal allocation and the subgame perfect equilibria for games in decentralized leadership settings. Individuals are assumed to be immobile in section 3. In section 4, the model is extended to allow residential mobility. In this section, one first considers the socially optimal allocation and later examines the subgame perfect equilibrium for a game with decentralized leadership where the regional governments provide contributions to both regional and federal public goods prior to the center's choices of interregional income and earmarked transfers. Section 5 offers conclusion remarks and suggestions for future research.

## 2. Basic Model

Consider an economy with two regions. There are  $n_i$  residents in region  $i$ ,  $i = 1, 2$ , and  $N = n_1 + n_2$  in the economy. There are two regional governments and one central government. Each region contributes to the provision of two types of public goods, regional and federal. Region  $i$  provides  $y_i$  units of the regional public good and  $g_i$  units of the federal public good. The contributions of both regions to the federal public good are aggregated according to an aggregation consumption technology, which is formally described by the function  $f(g_1, g_2)$ , where  $f(\cdot)$  is increasing in each argument and concave. The regional contributions to the federal public good produce  $Q$  units of this good; that is,  $Q = f(g_1, g_2)$ . Common examples of the federal public good are the pure public good,  $Q = g_1 + g_2$  (e.g., national defense and abatement of greenhouse gases), and the weaker-link public good,  $Q = g_1^\alpha g_2^{1-\alpha}$ ,  $0 < \alpha < 1$  (e.g., control of contagious diseases and national border management).

The representative resident of region  $i$  derives utility  $u(x_i, y_i, Q)$  from consumption of  $x_i$  units of a numeraire good,  $y_i$  units of the regional public good and  $Q$  units of the federal public good. For simplicity, the utility function is assumed to be strongly separable:  $u(x_i, y_i, Q) = b(x_i) + r(y_i) + v(Q)$ , where  $b' > 0$ ,  $b'' < 0$ ,  $r' > 0$ ,  $r'' < 0$ ,  $v' > 0$ ,  $v'' < 0$ .

The provision of each type of public good has a unitary cost equal to one unit of the numeraire good. In the setting with immobile residents, the representative resident of region  $i$  is endowed with a fixed amount of income,  $w_i$ . In this case, region  $i$ 's budget constraint is  $n_i x_i + g_i + y_i = n_i w_i$  in the absence of transfers controlled by the central government. In the setting in which residents are mobile, the representative resident of region  $i$  earns market income from supplying labor and from profit shareholding.

The governments are utilitarian. In the setting with immobile residents, the payoffs for regional and central governments are  $U^i = n_i [b(x_i) + r(y_i) + v(Q)]$ ,  $i = 1, 2$ , and  $U = U^1 + U^2$ , respectively. In the setting with mobile residents, the payoffs for regional and central governments also include psychic attachment benefits, to be described in detail in section 4.

### 3. Economy with Immobile Residents

In this section, assume that  $n_1 = n_2 = n = N/2$  and  $w_1 > w_2$ . These assumptions guarantee that region 1 is richer than region 2, which then provides the impetus for the central government to implement interregional income transfers. The assumption that the regions are equally populated is also consistent with all equilibria examined in section 4. Therefore, it facilitates comparisons and helps the reader to understand the key rationale for the results.

#### 3.1. Social Optimum

The social planner chooses non-negative  $\{x_1, x_2, y_1, y_2, g_1, g_2\}$  to maximize

$$n[b(x_1) + b(x_2) + r(y_1) + r(y_2)] + Nv(Q) \quad (1)$$

subject to the national resource constraint:

$$n(x_1 + x_2) + G + Y = W, \quad (2)$$

where  $G = g_1 + g_2$ ,  $Y = y_1 + y_2$  and  $W = n(w_1 + w_2)$ . Letting  $\lambda \geq 0$  denote the Lagrangian multiplier associated constraint (2), the first order conditions are as follows (for  $i = 1, 2$ ):

$$nb'(x_i) - n\lambda = 0, \quad (3)$$

$$nr'(y_i) - \lambda = 0, \quad (4)$$

$$Nf_i v'(Q) - \lambda = 0. \quad (5)$$

Combining equations (3) and (4) yields

$$\frac{nr'(y_i)}{b'(x_i)} = 1. \quad (6)$$

Combining equations (3) and (5) yields

$$\frac{Nf_i v'(Q)}{b'(x_i)} = 1. \quad (7)$$

Equations (6) state that the regional public goods are provided at levels that equate each region's sum of marginal rates of substitution between regional and numeraire goods to the marginal cost of provision. Equations (7) show that the socially optimal contributions to the federal public good equate the nation's sum of marginal rates of substitution between each region's contribution to their marginal contribution costs. Equations (3) and (4) imply that  $x_1 = x_2$  and  $y_1 = y_2$ , respectively, because  $b'' < 0$  and  $r'' < 0$ .

## 3.2. Decentralized Leadership

### 3.2.1. Unlimited Regional Commitments

The regional governments are able to commit to contributions to regional and federal public goods. The center observes these contributions and then implements interregional income transfers. Let  $\tau_i$  denote the amount of income transfer that region  $i$  receives (if positive) or pays (if negative).

Region  $i$ 's budget constraint is

$$nx_i + g_i + y_i = nw_i + \tau_i, \quad i = 1, 2. \quad (8)$$

The center's income transfers are redistributive. Hence,

$$\tau_1 + \tau_2 = 0. \quad (9)$$

The center chooses  $\{\tau_1, \tau_2\}$  to maximize social welfare (1) subject to constraints (8) and (9). The first order conditions yield constraints (8), (9) and

$$b'(x_1) = b'(x_2). \quad (10)$$



Since  $b'' < 0$ , equation (10) implies that  $x_1 = x_2 = x$ . Combining this result with equations (8) and (9) yields  $Nx + G + Y = W$ . For simplicity, one can express the center's response function in terms of the numeraire good rather than in terms of the income transfer instruments. Hence, let  $x(g_1, g_2, y_1, y_2)$  denote the center's best response function. The national resource constraint enables one to write

$$x(g_1, g_2, y_1, y_2) = \frac{W - G - Y}{N}. \quad (11)$$

In the first stage of the game, regional government  $i$  chooses non-negative  $\{g_i, y_i\}$  to maximize  $n[b(x(g_1, g_2, y_1, y_2)) + r(y_i) + v(f(g_1, g_2))]$ , taking the choices of the other region as given. The first order conditions yield (for  $i = 1, 2$ )

$$\frac{Nf_i v'(Q)}{b'(x_i)} = 1, \quad (12)$$

$$\frac{nr'(y_i)}{b'(x_i)} = \frac{1}{2}. \quad (13)$$

Conditions (12) are the modified Samuelson conditions for optimal contributions to the federal public good. Conditions (13) are the conditions that determine the contributions to the regional public goods. They equate each region's sum of the marginal rates of substitution between consumption of regional public good and numeraire good to the perceived marginal rate of transformation between regional and numeraire good. The latter is distorted by the income transfer mechanism. There is an implicit subsidy, which reduces the perceived marginal cost of provision of the regional public good by  $\frac{1}{2}$ . Consequently, each region oversupplies the regional public good. Intuitively, each regional government anticipates that the center will redistribute consumption of the private good, equating individual marginal utilities of income, and thus has an incentive to overspend resources in consumption of the regional public good.

### 3.2.2. Selective Centralized Earmarking

The distortion created by the income transfer mechanism can be eliminated if the center introduces earmarking transfers to equalize fiscal capacities. Suppose now that the center earmarks the provision of regional public goods. As before,  $y_i$  denotes the amount of regional public good that region  $i$  provides. Let  $e_i + s_i$  denote the total tax revenue that region  $i$  has available to provide

the regional public good, where  $e_i$  represents the portion of tax revenue that is collected in the region and  $s_i$  is the amount of a (positive or negative) fiscal transfer promoted by the center. Since the fiscal transfer is earmarked, the regional government must balance its budget with respect to provision of the regional public good:  $y_i = e_i + s_i$ ,  $i = 1, 2$ . This implies that region  $i$ 's budget constraint becomes

$$nx_i + g_i + e_i + s_i = nw_i + \tau_i, \quad i = 1, 2. \quad (14)$$

Assume that the fiscal transfers are redistributive, so that

$$s_1 + s_2 = 0. \quad (15)$$

In this setting, regional government  $i$  chooses  $\{e_i, g_i\}$  in the first stage. Having observed  $\{g_1, g_2, e_1, e_2\}$ , the center chooses  $\{s_1, s_2, \tau_1, \tau_2\}$  to maximize social welfare (1) subject to constraints (9), (14), (15) and  $y_i = e_i + s_i$ ,  $i = 1, 2$ . The conditions that maximize social welfare are the constraints, equation (10) and the following:

$$r'(y_1) = r'(y_2). \quad (16)$$

As before, equation (10) implies that  $x_1 = x_2 = x$ . Equation (16) informs us that the fiscal transfers equalize the marginal social utilities of consumption of regional public goods. Since  $r'' < 0$ , this equation implies that  $y_1 = y_2 = y$ . Let  $x(e_1, e_2, g_1, g_2)$  and  $s^i(e_1, e_2)$ ,  $i = 1, 2$ , denote the center's best response functions. These functions satisfy the following system of equations:

$$x(e_1, e_2, g_1, g_2) = \frac{W - E - G}{N}, \quad (17)$$

$$e_1 + s^1(e_1, e_2) = e_2 + s^2(e_1, e_2), \quad (18)$$

$$s^1(e_1, e_2) + s^2(e_1, e_2) = 0, \quad (19)$$

where  $E = e_1 + e_2$  is the national expenditure incurred in the provision of regional public goods.

Equations (18) and (19) imply that

$$\frac{\partial s_1}{\partial e_1} = \frac{\partial s_2}{\partial e_2} = -\frac{1}{2}. \quad (20)$$

In the first stage, regional government  $i$  chooses non-negative  $\{e_i, g_i\}$  to maximize  $n[b(x(e_1, e_2, g_1, g_2)) + r(e_i + s^i(e_1, e_2)) + v(f(g_1, g_2))]$ , taking the choices of the other regional government as given. Assuming interior solutions, the first order conditions are (for  $i = 1, 2$ ):

$$b'(x_i) \frac{\partial x}{\partial e_i} + r'(y_i) \left(1 + \frac{\partial s^i}{\partial e_i}\right) = 0 \Rightarrow \frac{N r'(y_i)}{2 b'(x_i)} = 1 \Rightarrow \frac{nr'(y_i)}{b'(x_i)} = 1, \quad (21)$$

$$b'(x_i) \frac{\partial x}{\partial g_i} + f_i v'(Q) = 0 \Rightarrow \frac{N f_i v'(Q)}{b'(x_i)} = 1. \quad (22)$$

As revealed by equations (20) and (21), the fiscal transfer mechanism neutralizes the distortion created by the income transfer mechanism on the provision of regional public goods. As before, the regions face correct incentives for the contributions to the federal public good, as shown by equation (22).

### 3.2.3. Selective Decentralized Leadership

In order to formally demonstrate that the distortion created by the income transfer mechanism arises only if the regional governments are able to commit to their contributions to regional public goods, suppose now that regional governments are unable to commit to provision of regional public goods. In the first stage of the game, they choose their contributions to the federal public good. In the second stage of the game, the regional governments choose the amounts of regional public goods and the center chooses the amounts of private goods.

Consider the second stage. Given equations (8), one can say that regional government  $i$  chooses non-negative  $y_i$  to maximize  $n \left[ b \left( w_i - \frac{g_i + y_i - \tau_i}{n} \right) + r(y_i) + v(Q) \right]$ , taking the choices of the other governments as given. In addition, the central government chooses  $\{\tau_1, \tau_2\}$  to maximize social welfare,  $n \left[ b \left( w_1 - \frac{g_1 + y_1 - \tau_1}{n} \right) + b \left( w_2 - \frac{g_2 + y_2 - \tau_2}{n} \right) + r(y_1) + r(y_2) + 2v(Q) \right]$ , subject to the income redistribution constraint (9), taking the choices of the other governments as given. Assuming interior solution, the first order conditions yield equations (6), (9) and (10). Combining equations (8) and (10) yields the national resource constraint (2).

Let  $y^i(g_1, g_2)$  and  $\tau^i(g_1, g_2)$  denote the best response functions for the regional and central governments, respectively. Let

$$x^i(g_1, g_2) = w_i + \frac{\tau^i(g_1, g_2) - g_i - y^i(g_1, g_2)}{n}. \quad (23)$$

Equation (10) implies that  $x^1(g_1, g_2) = x^2(g_1, g_2)$ . Hence, this result and equations (23) imply

$$w_1 + \frac{\tau^1(g_1, g_2) - g_1 - y^1(g_1, g_2)}{n} = w_2 + \frac{\tau^2(g_1, g_2) - g_2 - y^2(g_1, g_2)}{n}. \quad (24)$$

Equation (9) yields

$$\tau^1(g_1, g_2) + \tau^2(g_1, g_2) = 0. \quad (25)$$

Equations (24) and (25) yield

$$\tau^1(g_1, g_2) = \frac{n(w_2 - w_1) + (g_1 + y^1(g_1, g_2) - g_2 - y^2(g_1, g_2))}{2} = -\tau^2(g_1, g_2). \quad (26)$$

Letting  $x^i(g_1, g_2) = x(g_1, g_2)$  and combining equations (23) and (26) implies

$$x(g_1, g_2) = \frac{W - G - Y(g_1, g_2)}{N} = \frac{W - G - 2y(g_1, g_2)}{N}. \quad (27)$$

Equation (27) uses the fact that  $Y(g_1, g_2) = y^1(g_1, g_2) + y^2(g_1, g_2) = 2y(g_1, g_2)$ , where the last equality follows from the combination of equations (6) and (10), which yields  $y^1(g_1, g_2) = y^2(g_1, g_2) = y(g_1, g_2)$ .

Consider now the first stage. Regional government  $i$  chooses non-negative  $g_i$  to maximize  $n[b(x(g_1, g_2)) + r(y(g_1, g_2)) + v(f(g_1, g_2))]$ , taking the choice of the other regional government as given. Assuming interior solutions, the first order conditions in the first stage are (for  $i = 1, 2$ ):

$$\begin{aligned} b'(x) \frac{\partial x}{\partial g_i} + r'(y) \frac{\partial y}{\partial g_i} + f_i v'(Q) = 0 &\Rightarrow \frac{\partial y}{\partial g_i} \left( r'(y) - \frac{2b'(x)}{N} \right) - \frac{b'(x)}{N} + f_i v'(Q) = 0 \\ \Rightarrow \frac{2b'(x)}{N} \frac{\partial y}{\partial g_i} \left( \frac{nr'(y)}{b'(x)} - 1 \right) + \frac{b'(x)}{N} \left( \frac{Nf_i v'(Q)}{b'(x)} - 1 \right) = 0 &\Rightarrow \frac{Nf_i v'(Q)}{b'(x)} = 1. \end{aligned}$$

In sum, the subgame perfect equilibrium for the selective decentralized leadership game is socially optimal.

## 4. Economy with Mobile Residents

Having considered the special case in which residents are immobile and the regions are equally populated, suppose now that individuals are free to choose their region of residence. Every individual is endowed with one unit of labor, which he/she supplies to competitive firms that produce the numeraire good. Assume that labor is the only variable input. There are  $J_i \geq 2$  firms in region  $i$ . Let  $j$  index the firms, with  $j = 1, \dots, J_i$  in region  $i$ . The firms in both regions use a standard, constant-returns-to-scale technology. Let  $\Phi(\cdot)$  denote the concave production function that represents this technology. Assume that  $\Phi(\cdot)$  increases in all arguments at decreasing rates and assume that all inputs are essential (i.e.,  $\Phi(\cdot)$  satisfies the standard Inada conditions).<sup>4</sup> Let  $\phi^i(l_{ji}) \equiv \Phi(l_{ji}; \mathbf{z}_{ji})$  be the reduced form for the production function utilized by the representative firm in region  $i$ , where  $l_{ji}$  is the amount of labor that the firm hires and  $\mathbf{z}_{ji}$  is the vector of fixed inputs. Assume that all firms in region  $i$  use the same quantities of fixed inputs; that is,  $\mathbf{z}_{ji} = \mathbf{z}_i$ , for all  $j = 1, \dots, J_i$ . In addition, assume that  $\mathbf{z}_1 \gg \mathbf{z}_2$  and  $J_1 \geq J_2$ . Region 1 is more abundant in the fixed resources than region 2. This is the sole source of asymmetry in the model.

All firms operate in a common labor market. Let  $\omega$  denote the market wage. The representative firm in region  $i$  chooses  $l_{ji} \geq 0$  to maximize  $\phi^i(l_{ji}) - \omega l_{ji}$ , taking the choices of all other firms as given. The first order conditions yield  $\phi_{l_{ij}}^i = \omega$ ,  $i = 1, 2$ , where  $\phi_{l_{ij}}^i \equiv d\phi^i/dl_{ij}$ . The amount of labor hired by the representative firm satisfies the equalization of the marginal product of labor to the marginal cost of labor. Let  $l_j^i(\omega)$  denote the labor demand function for the representative firm in region  $i$ . Note that  $l_j^i(\omega) = l^i(\omega)$ , for all  $j = 1, \dots, J_i$ . In words, all firms in region  $i$  hire the same amount of labor.

The labor market clears if and only if

$$L^1(\omega) + L^2(\omega) = N, \quad (28)$$

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<sup>4</sup> The Inada conditions guarantee that both regions are populated in equilibrium.

where  $l^i(\omega) \equiv J_i l^i(\omega)$ ,  $i=1,2$ . The market-clearing condition (28) can be used to define the market wage as an implicit function of the labor market characteristics,  $\omega = \omega(J_1, J_2, N)$ .

The representative consumer in region  $i$  earns an amount  $w_i$  of market income from supplying labor in the market and from being a shareholder in all firms located in the region.<sup>5</sup> Then, we can write the per capita market income function in region  $i$  as follows:

$$w^i(n_i, \omega) = \omega + J_i \pi^i(\omega) / n_i, \quad i = 1, 2, \quad (29)$$

where  $\pi^i(\omega) \equiv f(l^i(\omega)) - \omega l^i(\omega)$  is the profit obtained by the representative firm in region  $i$ . In addition to market income, the representative resident of region  $i$  also receives (if positive) or pays (if negative) a transfer of  $\tau_i / n_i$  units of income from the central government. This consumer spends  $x_i + t_i$  units of income to pay for his/her private and public consumption levels, where  $t_i$  is the amount of tax that he/she pays to the regional government. Hence, the consumer's budget constraint yields:

$$x^i(n_i, t_i, \tau_i) = w^i(n_i, \omega) - t_i + \tau_i / n_i, \quad i = 1, 2. \quad (30)$$

Regional government  $i$  must balance the regional public budget:

$$t^i(g_i, n_i, y_i) = (g_i + y_i) / n_i, \quad i = 1, 2, \quad (31)$$

where  $t^i(g_i, n_i, y_i)$  is region  $i$ 's per capita tax function. The central government's income transfers are redistributive. Hence, constraint (9) holds.

Due to idiosyncratic regional benefits (e.g., family ties, culture, language, etc.), consumers are attached to regions. Let  $n \in [0, N]$  denote a consumer in the economy. This individual gets an attachment benefit equal to  $a(N - n)$  if he/she resides in region 1 and an attachment benefit equal to  $an$  if he/she resides in region 2, where  $a > 0$  is the attachment intensity. The total utility individual  $n$  derives from residing in region 1 is  $u(x_1, y_1, Q) + a(N - n)$ , while the total utility this

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<sup>5</sup> For simplicity, I omit rental income sources that residents obtain from supplying fixed inputs (say, land and capital) in the market. If each resident of region  $i$  is endowed with equal supplies of region  $i$ 's fixed resources, equation (28) would remain the same: the rental incomes would exactly cancel out with the amounts spent by the firms to hire such resources. Thus, the analysis in the text is consistent with the assumption that regional residents are equally endowed with all regional resources.

individual derives from residing in region 2 is  $u(x_2, y_2, Q) + an$ . In the migration equilibrium, there is an individual,  $n_1$ , who is indifferent between residing in region 1 and residing in region 2:

$$\begin{aligned} u(x_1, y_1, Q) + a(N - n_1) &= u(x_2, y_2, Q) + an_1 \\ \Rightarrow b(x_1) + r(y_1) + a(N - n_1) &= b(x_2) + r(y_2) + an_1. \end{aligned} \quad (32)$$

#### 4.1. Social optimum

Let  $U^1 = \int_0^{n_1} [b(x_1) + r(y_1) + v(Q) + a(N - n)] dn$  and  $U^2 = \int_{n_1}^N [b(x_2) + r(y_2) + v(Q) + an] dn$  be the welfare levels enjoyed by regions 1 and 2, respectively. We assume that the social planner is utilitarian,  $U = U^1 + U^2$ . Hence,

$$U = n_1 \left[ b(x_1) + r(y_1) + v(Q) + a \left( N - \frac{n_1}{2} \right) \right] + n_2 \left[ b(x_2) + r(y_2) + v(Q) + a \left( N - \frac{n_2}{2} \right) \right]. \quad (33)$$

Assume that consumers/migrants do not observe the social planner's policy choices prior to making their migration decisions and the social planner does not observe the outcome of the migration decisions (i.e., the population distribution) prior to making its policy choices.<sup>6</sup> These assumptions imply that consumers/migrants take the planner's choices as given while the planner takes the population distribution as given. The migration equilibrium is determined by equation (33). Taking  $\{n_1, n_2\}$  as given, the planner chooses  $\{g_1, g_2, y_1, y_2, \tau_1, \tau_2\}$  to maximize

$$n_1 \left[ b \left( w^1(n_1, \omega) + \frac{\tau_1 - g_1 - y_1}{n_1} \right) + r(y_1) \right] + n_2 \left[ b \left( w^2(n_2, \omega) + \frac{\tau_2 - g_2 - y_2}{n_2} \right) + r(y_2) \right] + Nv(Q), \quad (34)$$

subject to constraint (9), where the objective function (34) neglects the attachment benefits present in the social welfare function (33) because the planner takes the population distribution as given. In writing (34), equations (29) and (30) are taken into account. Remember that the per capita market income functions are given by equations (28). Straightforward calculations yield conditions (7), (10) and the following:

$$\frac{n_i r'(y_i)}{b'(x_i)} = 1, \quad i = 1, 2, \quad (35)$$

$$n_1 x_1 + n_2 x_2 + Y + G = W, \quad (36)$$

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<sup>6</sup> Silva and Lucas (2016) show that the subgame perfect equilibrium for the decentralized leadership game is the same whether government authorities take migration responses into account or consider the population distribution as given. To simplify exposition, the government authorities take the population distribution as given in the current setting.

where, in the national resource constraint (36),  $W = n_1 w^1(n_1, \omega) + n_2 w^2(n_2, \omega)$ . Since the objective function is strictly concave and the constraint (9) is linear, the solution to the planner's problem satisfies the sufficient second order conditions and it is unique.

Combining equations with the migration equilibrium equation (32) and  $N = n_1 + n_2$  yields the socially optimal allocation subject to free mobility of residents. Equation (10) implies that  $x_1 = x_2 = x$ . Combining equations (10) and (35), one obtains

$$n_1 r'(y_1) = n_2 r'(y_2). \quad (37)$$

The fact that  $x_1 = x_2 = x$  implies that the migration equilibrium equation (32) simplifies to

$$r(y_1) + a n_2 = r(y_2) + a n_1. \quad (38)$$

For arbitrary  $a$  and  $N$  values, equations (37), (38) and  $N = n_1 + n_2$  hold simultaneously if and only if  $n_1 = n_2 = n = N/2$  and  $y_1 = y_2 = y$ .

## 4.2. Unlimited Decentralized Leadership with Selective Earmarking

Suppose that the regional governments are able to commit to their contributions to regional and federal public goods. The center, however, is able to make income and fiscal transfers. The fiscal transfer that a region receives or pays is earmarked. As in section 3.2.2, let  $y_i = e_i + s_i$ ,  $i = 1, 2$ . Assume also that  $s_1 + s_2 = 0$ . The budget constraint for the representative resident of region  $i$  yields

$$x^i(e_i, g_i, n_i, s_i, \tau_i) = w^i(n_i, \omega) + \frac{\tau_i - g_i - e_i - s_i}{n_i}, \quad i = 1, 2. \quad (39)$$

Having observed  $\{e_1, e_2, g_1, g_2\}$ , the center chooses  $\{s_1, \tau_1\}$  to maximize

$$n_1 \left[ b(x^1(e_1, g_1, n_1, s_1, \tau_1)) + r(e_1 + s_1) \right] + n_2 \left[ b(x^2(e_2, g_2, n_2, -s_1, -\tau_1)) + r(e_2 - s_1) \right],$$

in the second stage, taking  $\{n_1, n_2\}$  as given. The objective function takes  $s_2 = -s_1$  and  $\tau_2 = -\tau_1$  into account and ignores the national benefit from consumption of the federal public good, since  $Q$  has already been determined in the first stage. The first order conditions yield equations (10) and (37). Let  $s^i(e_1, e_2, g_1, g_2)$  and  $\tau^i(e_1, e_2, g_1, g_2)$  denote the center's best response functions,  $i = 1, 2$ . Equation (10) yields



$$w^1(n_1, \omega) + \frac{\tau^1(\cdot) - g_1 - e_1 - s^1(\cdot)}{n_1} = w^2(n_2, \omega) + \frac{\tau^2(\cdot) - g_2 - e_2 - s^2(\cdot)}{n_2}. \quad (40)$$

Plugging  $\tau^2(\cdot) = -\tau^1(\cdot)$  into equation (40) and solving the resulting expression yields

$$\tau^1(\cdot) = \frac{n_1 n_2 [w^2(n_2, \omega) - w^1(n_1, \omega)] + n_2 [e_1 + g_1 + s^1(\cdot)] - n_1 [e_2 + g_2 + s^2(\cdot)]}{N} = -\tau^2(\cdot). \quad (41)$$

Combining equations (40) and (41) one obtains

$$\hat{x}(e_1, e_2, g_1, g_2) = \frac{W - E - G}{N}. \quad (42)$$

Equation (37) yields

$$n_1 r'(e_1 + s^1(\cdot)) = n_2 r'(e_2 + s^2(\cdot)). \quad (43)$$

Plugging  $s^2(\cdot) = -s^1(\cdot)$  into equation (43) and differentiating with respect to each policy variable controlled by the regional governments leads to ( $i = 1, 2$ ):

$$\frac{\partial s^i}{\partial e_i} = -\frac{n_i r''(y_i)}{n_1 r''(y_1) + n_2 r''(y_2)} < 0, \quad (44)$$

$$\frac{\partial s^i}{\partial e_{-i}} = \frac{n_{-i} r''(y_{-i})}{n_1 r''(y_1) + n_2 r''(y_2)} > 0, \quad -i = 2 \text{ if } i = 1 \text{ and vice versa,} \quad (45)$$

$$\frac{\partial s^i}{\partial g_i} = \frac{\partial s^i}{\partial g_{-i}} = 0, \quad -i = 2 \text{ if } i = 1 \text{ and vice versa.} \quad (46)$$

In the first stage, regional government  $i$  chooses non-negative  $\{e_i, g_i\}$  to maximize

$$n_i \left[ b(\hat{x}(e_1, e_2, g_1, g_2)) + r(e_i + s^i(\cdot)) + v(f(g_1, g_2)) \right]$$

taking  $\{n_1, n_2\}$  and the choices of the other regional government as given. Assuming interior solutions, the first order conditions yield equations (7) and the following (for  $i = 1, 2$ )

$$\frac{N r'(y_i)}{b'(x)} \left( \frac{n_{-i} r''(y_{-i})}{n_1 r''(y_1) + n_2 r''(y_2)} \right) = 1. \quad (47)$$

Since the equilibrium also satisfies equation (38) and  $N = n_1 + n_2$ , one obtains  $n_1 = n_2 = n = N/2$  and  $y_1 = y_2 = y$ . Combining these results with equations (47) yields conditions (7).

## 5. Conclusion

In most nations, regional governments provide multiple public goods. Some of these goods yield regional consumption benefits while some others benefit residents and non-residents. In federal regimes where regional governments have some policy autonomy, the provision of regional and federal public goods may be efficient. This paper demonstrates that there are common circumstances under which regional and central governments interact in efficient manners in federations. In particular, regional and central governments may implement socially optimal policies if regional governments commit to provision of regional and federal public goods prior to the center's decisions concerning interregional income and earmarked transfers. The efficiency of decentralized leadership is robust to imperfect residential mobility if there is a common labor market in the federation.

Caplan, Cornes and Silva (2000) applied the rotten kid theorem obtained by Cornes and Silva (1999) to a federal regime context due to the similarities there exist between families with rotten kids and loving and benevolent parents and federations with self-interested regional governments and benevolent central governments. Caplan, Cornes and Silva (2000) was also motivated by the fact that individuals seem to be attached to regions and hence not perfectly mobile in many federations. In the current paper, the positive message of Caplan, Cornes and Silva (2000) is shown to hold to more general situations, which include multiple types of public goods, but under conditions that they did not anticipate. Without residential mobility and unlimited regional commitments, the efficiency result goes through only if the center possesses instruments to implement interregional income and earmarked transfers. With residential mobility subject to attachment to regions and unlimited regional commitments, the efficiency result goes through only if the center is endowed with the previously mentioned instruments and there is a common labor market in the federation.

Since the application to federal regimens has produced new insights, a natural question is to ask to what extent such insights are applicable to family contexts. Rotten kids consume a large and diverse basket of goods. Some of such goods are private goods which yield private benefits, others are private goods that produce (positive or negative) externalities while others still are family (i.e., public) goods. The insights from this paper generate the following behavioral hypotheses: rotten kids overconsume private goods other than the numeraire and provide efficient contributions to family goods in the presence of family income redistribution (e.g., inter-vivo

transfers and bequests). As data about family expenditures become more abundant and available, such behavioral hypotheses should be tested. Another interesting avenue for future research in family contexts is to study interfamily interactions and the provision of multiples public goods, some of which are commonly shared (see, e.g., Cornes, Itaya and Tanaka (2012)). In family networks, there may be special arrangements of income and earmarked transfers that motivate family members to be well behaved.

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