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Abstract

We analyze the optimal contract between a risk-averse manager and the initial shareholders in a two-period model where the manager.s investment effort, carried out in period 1, and her current effort, carried out in period 2, both impact the second-period profit, so that it may be difficult to disentangle the incentives for these two types of effort. The contract stipulates (a) the profit-contingent cash remuneration for each period, (b) the number of shares that will be granted to the manager at the end of the first period and (c) the restrictions (if any) on the sales of the granted stock. We show that stock grants play different roles according to whether the signal of investment effort is less noisy, or noisier, than that of current effort. In the former case, at the optimal solution, the firm gives more incentive to investment effort than to period 2 current effort, and there is no need to restrict the sales of granted stocks: the stock grants then serve as an incentive device for investment effort, and it is efficient to permit the manager to sell all her shares to eliminate her dividend risks. In the latter case, the efficient contract does not allow the manager to sell her granted stock, and both current and investment efforts are given the same incentive. In this case, stock grants play a different role: they serve as commitment device to overcome the time-inconsistency problem. We determine simultaneously the optimal stock grants and the optimal restrictions on sales of shares.

JEL-Codes: M510, M520.

Keywords: stock grants, executive compensation, incentive contracts, moral hazard, agency problems.

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1 Introduction

Stock grants are pervasive. In several OECD countries and especially in the U.S., they constitute a considerable share of the executive compensation package since the 1950s. The empirical evidence indicates the use of stock grants has increased substantially since the 1980s (Morgenson, 1998, Murphy, 1999, Core et al., 2002, Hall and Murphy, 2003). Indeed, according to Frydman and Saks (2007), stock grants represent a greater part of total executives pay, reaching more than 35% of total compensation in 2005. Moreover, Mishel and Sabadish (2013) estimated that in 2012 the stock grants to U.S. CEO's were 26.5 times greater than in 1978. In addition, the proportion of U.S. executives owning stocks have risen from 57% in 1980 to almost 90% in 1994 (Hall and Liebman,1998).

We show in this paper that stock grants, which are so pervasive, do have a role to play in the design of managerial optimal incentive schemes when firms' profits in any period are impacted by the efforts exerted by its manager(s) in earlier periods. The performance of a firm is indeed generally the outcome of of both (i) the efforts its managers undertake in the current period, to control its current operating costs, to optimize its pricing scheme and so on, and (ii) the efforts they (or their predecessors) have exerted in previous periods, e.g., to invest in new technologies, to explore new markets, to hire more efficient employees and the like. The distinction between those two types of efforts has been raised in the sharecropping literature (Banerjee and Ghatak, 2004). Indeed, Banerjee and Ghatak (2004) consider a two period tenancy model where the tenant farmer chooses an unobservable action that enhances the probability of high output in the next period. This action is termed an "investment effort" as its effect on profit is realized with one-period lag (e.g., developing a new environmental friendly technology, or acquiring certified green labels, etc.). This type of effort is distinct from a current action that raises the probability of high output in the current period (e.g., the operational efforts in farming, irrigating, etc.).

In our model, the managers' two types of efforts are not perfectly observable and, generally, the degrees of precision with which they can be inferred differ, so that different incentives should be ideally given to them. Obviously a remuneration scheme in which the managers only receive cash payments contingent on current profits cannot do the trick.

The present paper shows that stock grants are part of the optimal solution of this problem. We distinguish two types of efforts, an investment effort is carried out in period 1 and a second-period current effort, which both impact period 2 profits.[1] We assume that a signal about the effect of the former is observed at the beginning of period 2 and reflected in the stock price. We then posit that the degrees of 'precision' with which efforts can be inferred differ across the two types of effort, so that different incentives should be ideally given to investment and current efforts which both impact the second-period profit. Finally, we endogenize the firm's choice of restrictions on the manager's sales of shares, i.e. the optimal contract also specifies the proportion of shares granted to the manager in period 1 which she is allowed to sell at the beginning of period 2. We show that the optimal two-period contract depends on whether investment effort can be inferred 'more precisely' or 'less precisely' than current effort. In the former case, granting unrestricted shares to the manager allows the firm to give a greater incentive to investment effort. In the latter case, where giving less incentive to investment effort would be ideally required but is not feasible, restricted stock grants are a commitment device needed for enforcing a contract in which the same average incentive is optimally given to both types of effort.

Our paper is related to a number of theoretical papers on stock grants and stock options. Acharya et al. (2000) present a two-period model where the principal (the owner of the firm) offers the agent (the manager) an initial quantity of α call options on the firm's terminal value, at the strike price of 1. At the end of period 1, a signal is observed which indicates the likelihood of each of four possible terminal values. This signal is either H (high) or L (low). The probability that the signal is H depends on the manager's period 1 effort level. The authors investigate the desirability of resetting the stock options upon receiving the signal.

[1] We acknowledge that one could also include another possible interaction between current and investment efforts: current and investment efforts, when exerted in the same period t, both affect the aggregate manager's effort cost in period t in a non-additive way. For simplicity, however, we suppose here that effort costs are additive, thus abstracting from the (cost-side) substitutability or complementary of efforts.

For example, if the signal is L, should the owner revise the strike price downwards? They compare two benchmark strategies: the strategy of pre-commitment (i.e., the principal is committed to no resetting) and the strategy of reincentivization (resetting the strike price downward to encourage the manager to make more effort in period 2). They find that when the manager's utility function is linear in consumption and effort, it is better for the principal to agree from the outset that she will reset the strike price in the event of receiving the signal L. However, with other functional forms, the authors find that either benchmark strategy may dominate the other, depending on specific parametrization. Our model and that of Acharya et al. (2000) differ in a number of respects. First, Acharya et al. (2000) focus on the case of a risk-neutral manager, while our model assumes that the manager is risk-averse. (As pointed out by Aseff and Santos (2005,p 818), if the manager is risk neutral, the first-best solution is to sell the firm to the manager.) Second, in the former model, consumption takes place only at the end of period 2 while in our model consumption takes place in both periods. Third, Acharya et al. (2000) assume that the legally binding contract prohibits either party to terminate the relationship at the end of period 1, while we allow termination. Finally we determine endogenously both the number of shares granted to the manager, and the fraction of the granted stocks that the manager is not allowed to sell at the beginning of period 2, and we consider the granting of shares rather than stock options.

Aseff and Santos (2005) study the role of stock options as an incentive device in a purely static model: there is only one period. Unlike Acharya et al. (2000), they assume the manager is risk averse. Because of this feature, the risk-neutral principal must determine a contract that serves two functions: the provision of insurance and the incentivization of effort. Aseff and Santos (2005) compare the benchmark principal-agent contract (with fully contingent payments) with simple compensation schemes comprising only two components: a fixed payment T and a call option on the firm's stock (which specifies a strike price p and an option grant q). Using numerical calibration, they show that the loss arising from using simple compensation schemes is insignificant. They argue that "such small costs coupled with the benefits of implementing this simple contract may account for the popularity of stock options" (p.832).

Contrary to Aseff and Santos (2005) and Acharya and al. (2000) who considered the role of stock option grant to managers as an incentive device, Wang (1997) shows that if firms can fully commit to make cash payments that are contingent on past and present profits, stock grants or option grants are redundant instruments. Assuming that the manager is risk averse and cannot borrow to smooth consumption, Wang (1997) argues that "in a dynamic environment, if the agent performs well today, he can be rewarded both today and tomorrow" (p. 74). Using a discrete-time, infinite-horizon model, Wang (1997) shows numerically that for a large range of parameter values the optimal dynamic contract has the property that the pay-performance sensitivity is positive but very small. He concludes that "dynamic agency theory provides a resolution of Jensen and Murphy's puzzle" (p. 73).[2] Basically, the driver of this result is the fact that the dynamic compensation scheme must provide consumption smoothing across periods for the manager.

Clementi, Cooley and Wang (2006) (or CCW for short) point out that in Wang (1997) it was assumed that firms do not renege on deferred contingent cash payments. They argue that if firms have the ability to renege on deferred contingent cash payments, then stock grants are not redundant, because they can be used as a commitment device. Using a two-period model where effort is a binary choice (either 'low' or 'high'), CCW showed that stock grants may serve as an instrument to permit the firm to commit to deliver to the manager a second-period expected utility level that is strictly greater than her second-period reservation level that would be obtained in the absence of stock grants. Their paper demonstrates that this commitment lowers the firm's costs of inducing the 'high' effort level; this reduction in costs increases the expected payoffs to its initial shareholders.

Our framework differs from CCW (2006), in several respects. First, CCW assume that the manager's effort in each period affects only the profit in that period. In contrast, in our

[2] Jensen and Murphy (1990) found that empirically pay-performance sensitivity is very low, contradicting the predictions of formal static agency model.

model, in the first period the manager exerts an “investment effort”, which affects period 2 profit (in addition to his period 2 current effort). Second, the principal can make inferences about the levels of each type of efforts by observing noisy signals, and the levels of noise are different for the two types of efforts. Third, we assume that any contracting party (initial shareholders or incumbent manager) the reneges on the initial contract must fully compensate the other party for the loss inflicted by the termination. The objective of our study is to determine the optimal incentives for current effort and for investment effort.

2 The Model

We consider a two-period model. A firm needs a manager to plan and supervise production. The manager’s efforts are not verifiable. A novel feature of our agency model, inspired by the tenancy model of Banerjee and Ghatak (2004), is that the manager’s efforts are of two types: a current effort that has impact only on current profit, and an investment effort that, while exercised in period 1, has impact only on period 2 profit. Thus our model deals with moral hazard in an intertemporal context where output depends on a time profile of unverifiable inputs.

The market for managers is perfectly competitive, and all potential managers are identical. The firm wants to devise a compensation scheme in order to secure appropriate amounts of current and investment efforts. We assume that the firm is risk-neutral and the manager is risk-averse. This is the standard assumption in the principal-agent literature (Laffont and Martimort, 2001).

In our model, it is illegal to force an incumbent manager to stay with the firm in period 2. A manager is allowed to quit at the beginning of period 2 provided that she pays the firm a compensation equal to the expected damages caused by her departure. Similarly, the firm can fire the incumbent manager at the beginning of period 2, provided that it pays her a compensation equal to her loss of income.[3] If a manager ceases to be employed by the firm,

[3] We assume for simplicity that that these losses can be estimated objectively and costlessly. In practice, severances often lead to litigations, see Utz (2001a,b).

she earns an outside income $\omega > 0$. The two parties can agree to terminate the contract at the beginning of period 2, with suitable side payments. Thus termination can happen only if, taking into account compensation, both parties expect to be weakly better off as a result of termination. A two-period contract is said to be *self-enforcing* if in period 2 neither party finds it advantageous to terminate the contract and pay the compensation in order to pursue the outside option.[4]

The two-period contract specifies (a) the cash remuneration formula for period 1, (b) the amount of shares granted to the manager at the end of period 1, and the restrictions on the sales of these shares; and (c) the remuneration formula to be applied to the manager in period 2, provided that she remains employed by the firm. Since efforts are not verifiable, the contract cannot specify effort levels.

2.1 The profit function and the manager's utility function

In period 1, the manager has two choice variables: her current effort level in period 1 , e_1, and her investment effort level, I, which is carried out in period 1 but impacts only period 2 profit. The profit in period 1 (before subtracting the cash remuneration to the manager) is a linear function of $e_1 + \varepsilon_1$, where ε_1 is a random disturbance term,

$$\pi_1 = q(e_1 + \varepsilon_1)$$

where $q > 0$ is a parameter which is common knowledge. The profit π_1 is observed by all parties, but only the manager knows e_1. The firm can infer the sum $e_1 + \varepsilon_1$, but cannot separate the components of this sum. The contract specifies that, at the end of period 1, the manager will be granted a fraction λ of the shares of the firm, and that she cannot sell a fraction a of these granted shares.

At the beginning of period 2, the firm observes a signal about I. It is disturbed by a random term δ. We assume that the second-period profit π_2 is the sum of two components,

[4] Different from our model, CCW (2006) assume that either party can renege without having to compensate for the loss inflicted on the other party.

$k(I+\delta)$ and $q(e_2+\varepsilon_2)$, where $k>0$ is common knowledge. The first component is observed (but not verifiable) at the beginning of period 2, before the manager chooses to quit or to stay with the firm.

If she stays with the firm in period 2, she will choose her effort level e_2, yielding period 2 profit π_2:

$$\pi_2 = q(e_2+\varepsilon_2) + k(I+\delta)$$

We assume that ε_t $(t = 1,2)$ and δ are normally distributed, with zero mean, and respective variances σ^2 and θ^2. If $\theta^2 - \sigma^2$ is negative (respectively, positive) we say that the signal $I+\delta$ is less noisy (resp., noisier) than the signal $e_t + \varepsilon_t$.

In exercising efforts, the manager incurs effort costs. The cost of exercising current effort is

$$\phi(e_t) = \frac{1}{2}e_t^2$$

and that of exercising investment effort is

$$\psi(I) = \frac{1}{2}I^2.$$

In a more general formulation period 1' s cost function should be written as

$$C(e_1, I)$$

with a non-zero second-order cross-derivative allowing for substituability in period 1 between investment effort and simultaneous current effort if $\frac{\partial C(e_1,I)}{\partial e_1 \partial I} > 0$ or complementarity if $\frac{\partial C(e_1,I)}{\partial e_1 \partial I} < 0$. We implicitly assume here that $\frac{\partial C(e_1,I)}{\partial e_1 \partial I} = 0$.This is for the sake of simplicity and also because we intend to focus on the relationship on period 2' s current effort e_2 and (period 1' s) investment effort I which both affect second period's profits rather than on the interaction between I and e_1 which both affect managerial first period's cost. The former question has indeed not been investigated up to now and deserves accordingly some exclusive attention. Our results can be extended, at some computational cost, to a more general formulation.

Let Y_t be the manager's cash remuneration in period t. Let Z_2 be the sum of (i) the proceeds from the sale of her unrestricted shares, $(1-a)\lambda$, at the beginning of period 2, and (ii) the dividends she receives from her restricted shares, $a\lambda$, at the end of period 2. We define the manager's 'effective consumption levels' in period 1 and 2 as follows:

$$c_1 = Y_1 - \phi(e_1) - \psi(I)$$

$$c_2 = Z_2 + Y_2 - \phi(e_2)$$

Following Bommier and Rochet (2006), we assume that the manager's two-period utility function is

$$U(c_1, c_2) = \frac{1 - \exp\left[-r(c_1 + c_2)\right]}{r} \tag{1}$$

where $r > 0$ is the manager's coefficient of absolute risk-aversion.[5]This formulation allows us to derive tractable expressions for certainty equivalents.

If the manager does not work for the firm, her (outside option) effective consumption levels are $\underline{c}_1 = \underline{c}_2 = \omega$, and her two-period utility level is

$$\frac{1 - \exp\left[-r(\omega + \omega)\right]}{r} \equiv \underline{U}$$

The manager would enter into a two-period contract with the firm only if it would give her an expected two-period utility that is at least as great as $\underline{U}$. On the other hand, since the firm (the principal) faces infinitely many potential managers, there is no need for the contract to give the manager more than her reservation level of utility, $\underline{U}$. It follows that in equilibrium, the manager's expected utility $EU(c_1, c_2)$ is equal to $\underline{U}$.

We formulate the problem as one of optimal contract design by the principal (the initial shareholders of the firms). The manager is the agent. The contract must be self-enforcing

[5] A slightly more general version, used by Bommier and Rochet, is

$$U(c_1, c_2) = \frac{1 - \exp\left[-r(v(c_1) + \gamma v(c_2))\right]}{r}$$

where $v(.)$ is a concave and increasing function, and $\gamma > 0$ is a discount factor.

in the sense that after $k(I+\delta)$ is observed (though not verifiable) at the beginning of period 2, neither party will have an incentive to terminate it and pay the required compensation.

Given the above assumptions (CARA utility function, quadratic effort costs, and normally distributed noises), we can make use of the results proved by Holmstrom and Milgrom (1987,1991) that, in such a framework, the optimal cash remuneration is linear in profit. We therefore suppose that the manager is offered the following profit-contingent cash remuneration scheme

$$Y_t = \mu_t + \beta_t \pi_t, \; t = 1, 2, \tag{2}$$

where μ_t and β_t are parameters specified in the initial contract. In period 2, the firm cannot change μ_2 and β_2. Notice that, only for the sake of simplicity, this formulation does not allow for deferred cash compensation: the manager's cash remuneration Y_t depends only on current period profit, π_t. As shown in the Appendix, allowing period 2's remuneration to depend on period 1 profit would not affect the results.

Since π_2 depends on both e_2 and I, it is clear that if stock grants were not an available instrument, both e_2 and I would be rewarded by the same incentive, β_2. As will become clear later, when the firm can award unrestricted stock grants ($a = 0$ and $\lambda > 0$), it becomes possible under certain conditions to reward these two efforts with different incentives, e.g. to give greater incentive to investment effort I than to current effort e_2.

The contract specifies that at the end of period 1, after dividends have been distributed, a fraction λ of the shares is awarded to the manager, as *non-voting* shares,[6] and the manager is allowed to sell at most $(1-a)\lambda$, where $a \in [0, 1]$ is the fraction of granted stocks that must be kept until the end. Neither λ nor a are contingent on $k(I+\delta)$, because the latter is not verifiable. We will determine as part of the contract design the optimal fraction λ, as well as the fraction $1-a$ of her shares which the manager is allowed to sell immediately. She can sell the remaining fraction a (restricted shares) only after π_2 has been realized. If all of the granted stocks are restricted, obviously current and investment efforts are rewarded

[6] This implies that even if $\lambda > 0.5$, the initials shareholders, who have all the voting power, can still fire the manager.

identically. Unrestricted shares allow to differentiate the incentives to efforts e_2 and I, but only in one direction, namely giving an additional incentive to investment effort I. Giving less incentive to I is not feasible, because λ cannot be negative.

2.2 The second period problem: the firm's perspective

Let us consider the firm's decision at the beginning of period 2. Given the two-period contract, which commits it to pay the existing manager according to the scheme $\mu_2 + \beta_2\pi_2$ if she remains in charge, the firm must determine at the beginning of period 2 whether it is profitable to fire the manager, replacing her with a new one. This decision is taken after the random variable $k(I+\delta)$ has been realized and observed (though not verifiable). We denote by K the realized value of $k(I+\delta)$.[7]

If the firm hires a new manager, it will also offer her a linear contingent cash payment scheme:

$$Y_2^N(e_2^N, \varepsilon_2) = \mu_2^N + \beta_2^N \pi_2 = \mu_2^N + \beta_2^N \left[K + q(e_2^N + \varepsilon_2)\right],$$

where the superscript N stands for 'new'. Her period 2 effective consumption is

$$c_2^N(e_2^N, \varepsilon_2) = Y_2^N(e_2^N, \varepsilon_2) - \frac{(e_2^N)^2}{2}$$

A new manager would choose effort level e_2^N to maximize her expected utility:

$$\max_{e_2^N} EU\left[c_2^N(e_2^N, \varepsilon_2)\right]$$

Since the utility function is CARA, and ε_2 is normally distributed, maximizing expected utility is equivalent to maximizing the certainty equivalent of effective consumption, $CE_2\left[c_2^N\right]$, where

$$CE_2\left[c_2^N\right] = E_{\varepsilon_2}\left[c_2^N\right] - \frac{1}{2}r\left[\beta_2^N q\sigma\right]^2. \tag{3}$$

Therefore the new manager's optimal effort is

$$e_2^{N*} = \beta_2^N q \equiv e_2(\beta_2^N). \tag{4}$$

[7] Since K is not verifiable, it is not possible to write a contract that stipulates the amount of granted stock and the incumbent manager's second-period renumeration as functions of K.

That is, at the agent's optimum effort, a small increase in effort would generate a marginal effort cost $\phi' = e_2^{N*}$, which is just equal to the expected increase in renumeration brought about by that small increase in effort, $\beta_2^N q$. The resulting effective consumption is

$$c_2^{N*}(\varepsilon_2) = \mu_2^N + \beta_2^N \left[K + q(e_2(\beta_2^N) + \varepsilon_2)\right] - \frac{1}{2}\left[e_2(\beta_2^N)\right]^2.$$

The participation constraint of the new manager is

$$E_\varepsilon \left[c_2^{N^*}(\varepsilon_2)\right] - \frac{1}{2} r \left[\beta_2^N q\sigma\right]^2 \geq \omega \tag{5}$$

i.e.,

$$\mu_2^N + \beta_2^N \left[K + qe_2(\beta_2^N)\right] - \frac{1}{2}\left[e_2(\beta_2^N)\right]^2 - \frac{1}{2} r \left[\beta_2^N q\sigma\right]^2 \geq \omega. \tag{6}$$

The firm's optimal contract with the new manager must maximize the expected second period net profit,

$$\max_{\mu^N,\beta^N} \left[K + qe_2(\beta_2^N)\right] - \left\{\mu_2^N + \beta_2^N \left[K + qe_2(\beta_2^N)\right]\right\} \tag{7}$$

subject to the participation constraint (6).[8] The firm will always choose the value of μ_2^N such that the participation constraint holds with equality. Substitution for μ_2^N into the objective function yields the unconstrained maximization problem:

$$\max_{\beta_2^N} \left[K + q^2\beta_2^N\right] - \left\{\omega + \frac{1}{2}\left[q\beta_2^N\right]^2 + \frac{1}{2} r \left[\beta_2^N q\sigma\right]^2\right\} \equiv S_2^N(\beta_2^N) \tag{8}$$

That is, given that a new manager is hired and given her reaction function (4), the firm must choose β_2^N to maximize its second period profit $S_2^N(\beta_2^N)$. The solution is

$$\beta_2^{N*} = \frac{1}{1 + r\sigma^2} \tag{9}$$

We see that β_2^{N*} is decreasing in the manager's absolute risk aversion coefficient, r and in the variability of profits, σ^2. Condition (9) has a simple interpretation. Re-written as

[8] The incentive compatibility constraint has been taken into account by the agent's reaction function $e_2^N = e_2(\beta_2^N)$.

$1 = \beta_2^{N^*}(1+r\sigma^2)$, it says that, for the contract to optimize the principal's objective function, at the margin, a small increase in the 'marginal reward' parameter β_2^N that raises expected gross profit by one dollar also raises the expected managerial compensation cost by one dollar.[9]

Substituting (9) into (8), we obtain the optimized second period expected net profit under a new manager

$$\Pi_2^O \equiv S_2^N(\beta_2^{N*}) = \frac{q^2}{2(1+r\sigma^2)} + K - \omega. \tag{10}$$

Here, the superscript O indicates that this is the "outside option" for the firm. The value of Π_2^O will be important in determining the optimal self-enforcing two-period contract.

Let p^N denote the market value of the firm at the beginning of period 2 if it is run by a new manager. It is equal to the expected value of profit net of the remuneration to the new manager:

$$p^N = \Pi_2^O = \frac{q^2}{2(1+r\sigma^2)} + K - \omega. \tag{11}$$

The firm must compare (a) its expected second period profit if it keeps the incumbent manager (under the terms of the two-period contract) with (b) the value of the outside option Π_2^O minus any compensation that it must pay the incumbent manager if it breaks the two-period contract. This will become clear as we proceed.

2.3 The second-period problem: the incumbent manager's perspective

At the end of period 1, the manager receives a fraction λ of the firm's shares. We denote by p the market value of the firm at the beginning of period 2, given that the incumbent manager stays with the firm. Then p is equal to the expected second period profit net of the cash remuneration to the continuing manager:

$$p = (K + q\bar{e}_2)(1 - \beta_2) - \mu_2 \tag{12}$$

[9]If effort e_2 were verifiable, there would be no moral hazard, and the optimal effort would be $e_2 = q > \beta_2^{N*}q$.

where $\bar{e}_2$ is the market's expectation of the incumbent manager's *equilibrium* effort level.[10]

Due to risk aversion, the manager always sells as soon as possible all her unrestricted shares, $(1-a)\lambda$. Her effective consumption in period 2, denoted by c_2, consists of four terms: (a) the revenue from share sales, $(1-a)\lambda p$, (b) her cash remuneration, $\mu_2+\beta_2\left[K+q(e_2+\varepsilon_2)\right]$, (c) the dividends from the restricted shares, $a\lambda\left\{(1-\beta_2)\left[K+q(e_2+\varepsilon_2)\right]-\mu_2\right\}$, minus (d) her effort cost, $(1/2)(e_2)^2$:

$$\begin{aligned} c_2 &= (1-a)\lambda\left[(K+q\bar{e}_2)(1-\beta_2)-\mu_2\right]+\mu_2+\beta_2\left[K+q(e_2+\varepsilon_2)\right] \\ &\quad +a\lambda\left\{(1-\beta_2)\left[K+q(e_2+\varepsilon_2)\right]-\mu_2\right\}-(1/2)(e_2)^2. \end{aligned} \tag{13}$$

Given K, the random component of c_2 is $Aq\varepsilon_2$, where A is defined as

$$A \equiv \beta_2 + a\lambda(1-\beta_2). \tag{14}$$

Thus the certainty equivalent of c_2 as viewed in period 2, given that K has been observed, is

$$\begin{aligned} CE_2\left[c_2 \middle| \, K\right] &= (1-a)\lambda\left[(K+q\bar{e}_2)(1-\beta_2)-\mu_2\right]-\frac{r}{2}\left(Aq\sigma\right)^2+\mu_2 \qquad (15) \\ &\quad +\left\{\beta_2\left[K+qe_2\right]+a\lambda\left[(K+qe_2)(1-\beta_2)-\mu_2\right]-(1/2)(e_2)^2\right\}, \end{aligned}$$

where $\frac{r}{2}\left(Aq\sigma\right)^2$ is the risk premium associated with her cash renumeration. What is the continuing manager's optimal choice of effort level e_2? After having sold her unrestricted shares, and taking the market expectation $\bar{e}_2$ as given, the manager's optimal e_2 must maximize the expression {...} on the right-hand side of (15). Her optimal effort is[11]

$$e_2^* = (\beta_2 + a\lambda(1-\beta_2))q \equiv qA. \tag{16}$$

Equation (16) indicates that if $\beta_2 < 1$ the manager's current effort is increasing in the number of restricted shares. Substituting (16) into (15), the optimized certainty equivalent of c_2, given K, is

[10] We assume rational expectations, so that the market expectations of the manager's effort are correct.

[11] In this case, the market's expectation of e_2 is exactly equal to Aq.

$$CE_2\left[c_2^*\middle|\ K\right] = (1-\lambda)\mu_2 + B(Aq^2+K) - \frac{(Aq)^2}{2}\left(1+r\sigma^2\right) \equiv CE_2(K) \tag{17}$$

where B is defined as

$$B \equiv \lambda + (1-\lambda)\beta_2 = \beta_2 + \lambda(1-\beta_2). \tag{18}$$

We will see that A and B, as defined in equations (14) and (18) play a crucial role in our analysis of the optimal self-enforcing second period contract.

2.4 The first-period problem

In period 1, the manager's effective consumption is

$$c_1 = \mu_1 + \beta_1(q(e_1+\varepsilon_1)) - \frac{1}{2}e_1^2 - \frac{1}{2}I^2$$

and its certainty equivalent is

$$CE_1(e_1, I) = \mu_1 + \beta_1 q e_1 - \frac{1}{2}e_1^2 - \frac{1}{2}I^2 - \frac{r}{2}(\beta_1 q\sigma)^2$$

Her current effort level e_1 is chosen to maximize CE_1. This gives

$$e_1^* = q\beta_1 \equiv e_1(\beta_1). \tag{19}$$

and thus

$$CE_1(I) \equiv CE_1(e_1^*, I) = \mu_1 + q^2\beta_1^2\left(\frac{1-r\sigma^2}{2}\right) - \frac{1}{2}I^2 \tag{20}$$

To evaluate the two-period contract, the prospective manager must add to $CE_1(I)$ her (first-period) valuation of the random variable $CE_2(K)$.[12] Notice that in equation (17), $CE_2(K)$ depends on $K = k(I+\delta)$, where δ is a random variable. Therefore, at the beginning of period 1, for any given I, the manager must calculate the certainty equivalent of the random variable $CE_2(K)$:

$$CE\left[CE_2(K)\right] = E_\delta\left[CE_2(K)\right] - \frac{rB^2k^2\theta^2}{2}$$

[12]Recall that the rate of discount is zero.

where

$$E_\delta\left[CE_2(K)\right] = (1-\lambda)\mu_2 + \lambda + B(Aq^2 + kI) - \frac{(Aq)^2}{2}\left(1 + r\sigma^2\right).$$

To determine whether she should accept a two-period contract, the prospective manager must compute the certainty equivalent of $c_1 + c_2$. For any given I, the total certainty equivalent if she expects to stay in period 2 is then

$$CE_{total}^{stay}(I) = CE_1(I) + E_\delta\left[CE_2(K)\right] - \frac{rB^2k^2\theta^2}{2} \tag{21}$$

It is equal to the first-period certainty equivalent plus the expected second-period certainty equivalent minus the risk premium associated with $CE_2(K)$ caused by the randomness of δ.

Finally, we must determine her investment effort. If she expects that she will stay with the firm in period 2, she will choose I to maximize the total certainty equivalent $CE_{total}^{stay}(I)$. This yields her optimal investment effort I^*, which is increasing in λ,[13]

$$I^* = k\left[\beta_2 + \lambda(1-\beta_2)\right] \equiv kB. \tag{22}$$

This condition has the following interpretation. At I^*, a small increase in investment effort inflicts a marginal effort cost equal to I^*, and yields k dollars of revenue to the firm, of which the fraction captured by the manager is B. At the optimum, the manager's marginal effort cost, I^*, equals the marginal benefit that accrues to her, kB.

As can be seen from our definitions (14) and (18), and from the manager's response functions (16) and (22), it holds that

$$\frac{I^*}{k} - \frac{e_2^*}{q} = B - A = (1-\beta_2)(1-a)\lambda. \tag{23}$$

Thus, given the restrictions that $0 \leq a \leq 1$ and $\lambda \geq 0$, the investment effort cannot be given less incentive than the second period current effort, unless $\beta_2 > 1$. (At this stage we do not put any restriction on β_2.)

We can now rewrite $CE_{total}^{stay}(I^*)$ as

[13] Provided that $\beta_2 < 1$, which holds in equilibrium, as we shall see.

$$\begin{aligned} CE_{total}^{stay}(I^*) &= \mu_1 + (1-\lambda)\mu_2 + q^2\beta_1^2\left(\frac{1-r\sigma^2}{2}\right) \\ &+B(Aq^2+Bk^2) - \frac{(Aq)^2}{2}\left(1+r\sigma^2\right) - \frac{1}{2}(Bk)^2(1+r\theta^2) \end{aligned} \tag{24}$$

For her to accept the two-period contract, it is necessary that $CE_{total}^{stay}(I^*) \geq 2\omega$. Given any tuple $(\beta_1, \beta_2, \lambda, a)$, the principal will choose μ_1 and μ_2 to press the agent to her reservation utility level. Thus, $CE_{total}^{stay}(I^*) = 2\omega$, and this implies that

$$\begin{aligned} -\mu_1 - (1-\lambda)\mu_2 &= \frac{q^2\beta_1^2}{2}\left(1-r\sigma^2\right) + \frac{k^2B^2}{2}\left(1-r\theta^2\right) \\ + & \frac{q^2A^2}{2}\left[\left(\frac{2B-A}{A}\right) - r\sigma^2\right] - 2\omega \end{aligned} \tag{25}$$

This equation will be helpful in our analysis of the equilibrium two-period contract.

Accepting a two-period contract does not necessarily imply that the manager would not have any incentive to quit at the beginning of period 2. If she quits, her second period income, denoted by Y_2^O, will be the sum of (i) her fixed outside wage ω, (ii) the proceeds of her sales of a fraction $(1-a)$ of her shares, and (iii) the dividends from the remaining fraction:

$$Y_2^O = \omega + (1-a)\lambda p^N + a\lambda\left\{(1-\beta_2^{N*})\left[K + q(e_2^N + \varepsilon_2)\right] - \mu_2^{N*}\right\}$$

where p^N was given by (11). The certainty equivalent of this (outside option) income, given K, is

$$CE_2^O(K) = \omega + \lambda p^N - \frac{r}{2}\left[a\lambda(1-\beta_2^{N*})q\sigma\right]^2 \tag{26}$$

where $\frac{r}{2}\left[a\lambda(1-\beta_2^{N*})q\sigma\right]^2$ is the cost of bearing the dividend risk. We can see that, for given K and λ, a shift from the no-restriction regime ($a = 0$) to the total restriction regime ($a = 1$) would reduce the manager's valuation of her period 2 outside option by $\frac{r}{2}\left[\lambda(1-\beta_2^{N*})q\sigma\right]^2$because she would have to bear the dividend risk.

If the incumbent manager expects to be replaced by a new manager period 2, her total certainty equivalent is

$$CE_{total}^O = CE_1(I) + E_\delta\left[CE_2^O(K)\right] - \frac{r}{2}\left[\lambda(1-\beta_2^{N*})k\theta\right]^2 \tag{27}$$

Then she will choose in period 1 her investment effort I to maximize CE^{O}_{total}, giving

$$I^{O} = k\lambda(1-\beta_2^{N*}) = k\left[\lambda - (\lambda\beta_2^{N*})\right] < k\left[\lambda + \beta_2(1-\lambda)\right] = I^{*} \quad (28)$$

We see that $I^{O} < I^{*}$, i.e., she exercises less investment effort if she expects not to be in charge in period 2.

Let us turn to the interests of the initial shareholders. In period 1, the firm's expected profit net of remuneration to the manager is

$$\Pi_1 = qe_1 - (\mu_1 + \beta_1 qe_1)$$

In period 2, having observed the realized value of $k(I+\delta) = K$, the firm's expected profit (net of remuneration to the continuing manager) is

$$\Pi_2 = (1-\beta_2)(qe_2 + K) - \mu_2$$

Given that the contract specifies a fraction λ to be awarded as stock grants to the manager, the initial shareholders' period 2 income is $(1-\lambda)\Pi_2$. If the incumbent manager is retained (so that $e_2 = Aq$), the fraction of period 2 profit that accrues to the initial shareholders is

$$(1-\lambda)\Pi_2 = (1-\lambda)(1-\beta_2)\left(Aq^2 + K\right) - (1-\lambda)\mu_2 \quad (29)$$

It follows that if the contract is self-enforcing, the value of the firm to the initial shareholders is

$$\Pi_{initial} \equiv \Pi_1(\mu_1, \beta_1, e_1^{*}(\beta_1)) + (1-\lambda)\Pi_2(\mu_2, \beta_2, e_2^{*}(\beta_2, \lambda; a), I^{*}(\beta_2, \lambda)). \quad (30)$$

3 The Equilibrium Contract

Since at the beginning of period 2 the manager can quit, or the firm can fire her, we must find a two-period contract that is self-enforcing, in the sense that neither party has an incentive to terminate the contract (given that termination involves paying the other party a compensation). Self-enforceability requires that the sum of the period 2 outside option payoffs to the two parties is inferior to the sum of their period 2 payoffs if they stay together.

The contract design problem facing the initial shareholders is the following program: Choose $\beta_1, \beta_2, \mu_1, \mu_2$ as well as $a \in [0,1]$ and $\lambda \in [0,1]$ to solve

$$\max_{\beta_1,\beta_2,\mu_1,\mu_2\,,\lambda,a} \Pi_{initial}\left(\beta_1, \beta_2, \mu_1, \mu_2\,, \lambda, a\right) \tag{31}$$

subject to the participation constraint

$$CE_{total}^{stay}\left(\beta_1, \beta_2, \mu_1, \mu_2\,, \lambda, a\right) \geq 2\omega \tag{32}$$

and the self-enforceability constraint which will be fully specified below.

For the sake of argument, suppose for the moment that neither party is required to compensate the other party when the contract is terminated at the beginning of period 2. Then, after observing K, if the manager does not have to compensate the firm upon quitting, she will quit if her outside option is worth more than the certainty equivalent of continuing the relationship, i.e., if

$$CE_2(K, e_2^*, \mu_2, \beta_2, \lambda, a) < \omega + \lambda\left\{\frac{q^2}{2(1+r\sigma^2)} + K - \omega\right\} - \frac{r}{2}\left[a\lambda(1-\beta_2^{N*})q\sigma\right]^2 \tag{33}$$

where the left-hand side is given by equation (17) and the right-hand side is CE_2^O, see eq. (26) and eq. (11).

Similarly, if the firm does not have to compensate the manager upon firing her, it will fire her if the expected second period net profit (under a new manager) that accrues to the initial shareholders exceeds what they would get by continuing their relationship with the incumbent manager, i.e., if

$$(1-\lambda)\Pi_2(K, e_2^*, \mu_2, \beta_2, \lambda, a) < (1-\lambda)\left\{\frac{q^2}{2(1+r\sigma^2)} + K - \omega\right\} \tag{34}$$

where the left-hand side is given by equation (29), and the term inside the curly brackets on the right-hand side is Π_2^O, see equation (10).

It follows that if the sum of the right-hand sides of (33) and (34) exceeds the sum of the left-hand sides, then there exists a transfer T between the incumbent manager and the initial shareholders such that both parties would be better off by mutually agreeing to terminate their relationship; in other words, if that condition holds, then the two-period contract is not self-enforcing. To put it another way, for a two-period contract to be self-enforcing, we require that

$$\begin{aligned} & CE_2(K, e_2^*, \mu_2, \beta_2, \lambda, a) + (1-\lambda)\Pi_2(K, e_2^*, \mu_2, \beta_2, \lambda, a) \\ \geq \; & \frac{q^2}{2(1+r\sigma^2)} + K - \frac{r}{2}\left[a\lambda(1-\beta_2^{N*})q\sigma\right]^2 \end{aligned} \tag{35}$$

Substituting (17) and (29) into the LHS of (35), the constraint (35) reduces to an inequality where the random term K has been cancelled out:

$$q^2\left[A - \frac{A^2}{2}(1+r\sigma^2)\right] \geq \left[\frac{q^2}{2(1+r\sigma^2)}\right] - \frac{r}{2}\left[\frac{a\lambda r\sigma^2}{1+r\sigma^2}\right]^2 q^2\sigma^2. \tag{36}$$

The LHS equals the net aggregate benefits from second period effort if the relationship is continued, since the incumbent manager's second period effort is incentivized by A. The first term on the RHS is the maximum value of the net aggregate benefits yielded by second period effort under a new manager, as he is incentivized by $\beta_2^N = 1/(1+r\sigma^2)$. The second term on the RHS corresponds to the risk premium which must be given to the departing manager to compensate her for the dividends risks from holding restricted shares. If condition (36) is not satisfied, then one party will be able to 'bribe' the other party into agreeing to sever their relationship. We call condition (36) the "self-enforceability constraint."

We now prove an important intermediate result. It states that any two-period contract which specifies that the manager does not receive restricted shares (i.e. $a\lambda = 0$) is self-enforcing only if the continuing manager is incentivized to exercise as much second period effort as what the firm could obtain from a new manager, i.e., only if $\beta_2 = \beta_2^{N*} \equiv 1/(1+r\sigma^2)$.

Lemma 1 *If all the shares granted to the manager are unrestricted (i.e., $a\lambda = 0$), then self-enforceablity is not possible, unless the incumbent manager is offered the same incentive for second-period effort as that which would be offered to a new manager.*

Proof: The result follows from the observation that

$$\frac{q^2}{2(1+r\sigma^2)} = \max_A q^2\left[A - \frac{A^2}{2}(1+r\sigma^2)\right] \tag{37}$$

If $a\lambda = 0$, then $A = \beta_2$, and condition (36) can only be satisfied by setting $\beta_2 = A^U \equiv \arg\max_A q^2 \left[A - \frac{A^2}{2}(1 + r\sigma^2)\right] = 1/(1 + r\sigma^2)$. ■

According to Lemma 1, if the firm does not grant shares to the manager, or grants shares but does not restrict her freedom to sell them, then any two-period contract such that β_2 is different from $1/(1 + r\sigma^2)$ is non-enforceable, in the sense that at the beginning of period 2, at least one party will be in a position to bribe the other party to reach an agreement to terminate the contract. In other words, conditional on $a = 0$ (absence of sale restrictions), the self-enforceability constraint, alone, already pins down the value of β_2, i.e., the principal has no other choice of β_2. The principal is forced to set $\beta_2 = 1/(1+r\sigma^2)$, which is the effort level that a new period 2 manager would be induced to exercise in a one-period contract under moral hazard.

The intuition behind Lemma 1 is straightforward. An incumbent manager who has no shares or who has sold all her shares naturally behaves in the same way as a new manager. Thus, any contract with $\alpha\lambda = 0$ must also specify that β_2 is no different from $\beta_2^{N^*}$, for otherwise it would yield an inferior aggregate period 2 certainty equivalent.

In contrast to Lemma 1, as we shall see, given any $\lambda > 0$ and conditional on $a > 0$ (i.e., a fraction of the granted shares cannot be sold), it is possible to design self-enforcing two-period contracts with a non-degenerate range of possible values of β_2. This freedom, however, comes at a cost: although the manager's incentive for investment effort is strengthened by the stock grant (recall that $B = \beta_2 + \lambda(1 - \beta_2)$), the clause that she must keep a positive fraction a of her shares until the end of period 2 increases her cost of risk-bearing through her risky dividend income. This increased cost is passed on to the firm through the manager's participation constraint.

Substituting in equation (31) for $\mu_1 + (1 - \lambda)\mu_2$ using its value from (25), we can rewrite the initial shareholders' optimal contract design problem as

$$\max_{\beta_1,\beta_2,\lambda,a} \Pi_{initial} = q^2\left[\beta_1 - \frac{\beta_1^2}{2}(1+r\sigma^2)\right] + q^2\left[A - \frac{A^2}{2}(1+r\sigma^2)\right] + k^2\left[B - \frac{B^2}{2}(1+r\theta^2)\right] - 2\omega \tag{38}$$

subject to the enforceability constraint (36) and the stock-grant feasibility constraints that[14]

$$\lambda \in [0,1] \text{ and } a \in [0,1]\,. \tag{39}$$

The three bracketed terms in (38) correspond respectively to the net aggregate benefits from first-period current effort, second-period current effort and investment effort, i.e. the additional outputs generated by these efforts net of the corresponding effort costs and the risk premia. Equation (38) is the profit of the initial shareholders, having taken into account the manager's reduced-form participation constraint (25), which itself has embodied her effort supply reaction functions (16) and (22).

Since β_1 appears only in the objective function and not in the constraints, clearly its optimal value is $\beta_1^* = \frac{1}{1+r\sigma^2}$ and thus the objective function reduces to

$$\max_{\beta_2,\lambda,a} \Pi_{initial} = q^2\left[A - \frac{A^2}{2}(1+r\sigma^2)\right] + k^2\left[B - \frac{B^2}{2}(1+r\theta^2)\right] - 2\omega + \frac{q^2}{2(1+r\sigma^2)} \tag{40}$$

Let S^0 be the set of triples (β_2, λ, a) that satisfy the stock-grant feasibility constraints (39),

$$S^0 \equiv \{(\beta_2, \lambda, a) \in \mathbb{R}^3 | \quad 0 \le \lambda \le 1 \text{ and } 0 \le a \le 1\} \tag{41}$$

and S^E be the set of of triples (β_2, λ, a) that satisfy the self-enforceability constraint (36)

$$S^E \equiv \left\{(\beta_2, \lambda, a) \in \mathbb{R}^3 | \text{ condition (36) is satisfied}\right\} \tag{42}$$

We denote the feasible set by F :

$$F \equiv S^0 \cap S^E. \tag{43}$$

In solving problem (40), subject to (41) and (36), we will exploit the fact that the choice variables, β_2, λ and a, appear in the objective function in the form of two composite variables, A and B, which respectively provide incentives for e_2 and I. Then we can explore

[14] We do not rule out a priori the possibility that β_2 may be negative or larger than 1.

the objective function in terms A and B, while taking into account the restriction (41) indirectly, and in a final step we check that the candidate solution satisfies the self-enforceability constraint (36).

Disregarding for the moment the case where $a = 1$, let us see how the restriction $\lambda \in [0, 1]$ translates into restrictions on the values of A and B, for any given value of $a \in [0, 1)$. From the definitions of A and B, it follows that, for any given $a \in [0, 1)$, we can solve for λ in terms of A and B and obtain[15]

$$\lambda = \frac{B - A}{1 - a + aB - A} \text{ if } a \in [0, 1). \tag{44}$$

In Figure 1, where we impose the restriction $a \in [0, 1)$, the variable B is measured along the horizontal axis, and A along the vertical axis. We partition the first quadrant of Figure 1 into six regions, with the help of three lines. First, we draw the 45 degree line. At any point (B, A) on 45 degree line, we have $\lambda = 0$. Second, we draw the vertical line $B = 1$. On this line, clearly we have $\lambda = 1$. Third, draw the line NN' representing the denominator of the RHS of (44). The equation for this line is $A = (1 - a) + aB$. Above this line, the denominator of the RHS of (44) is negative. The line NN' passes through the point $(1, 1)$ and has the slope $a < 1$ and intercept $1 - a$. Any point (B, A) that lies above this line and to the left of the vertical line $B = 1$ yields a value $\lambda > 1$ and is therefore inadmissible. Also inadmissible is any point (B, A) that lies in the triangular region below the line NN' and above the 45 degree line (hence to the left of the vertical line $B = 1$), as it yields a negative λ. Similarly, to the right of the vertical line $B = 1$, any point (B, A) that lies in wedge between the line NN' and the 45 degree line is inadmissible because it yields a negative λ; and any point (B, A) that lies below the line NN' and to the right of the vertical line $B = 1$ yields an inadmissible value $\lambda > 1$. Consequently, the restriction that $\lambda \in [0, 1]$ implies that there are only two admissible regions: the triangle $\Delta \equiv \{(B, A) \in \mathbb{R}^2_+ | \quad B \geq A, B \leq 1\}$ and the cone $\Gamma \equiv \{(B, A) \in \mathbb{R}^2_+ | \quad B \leq A, B \geq 1\}$. Since $B - A = (1 - \beta_2)\lambda(1 - a)$, any

[15] Note that $B - A = (1 - \beta_2)\lambda(1 - a)$ and $A - aB = \beta_2(1 - a)$. It follows that $\beta_2 = (A - aB)/(1 - a)$ and $1 - \beta_2 = (1 - a + aB - A)/(1 - a)$. Then $\lambda = (B - A)/(1 - a + aB - A)$.

point in Δ with $\lambda \neq 0$ implies $\beta_2 < 1$, and any point in Γ with $\lambda \neq 0$ implies $\beta_2 > 1$.[16]

To summarize, the constraints $\lambda \geq 0$ and $\lambda \leq 1$ together imply that, for $a \in [0, 1)$, feasible couples (B, A) must belong either to the set Δ and/or to the set Γ. At any other point, the solution would violate the constraint $\lambda \in [0, 1]$ either because λ would have to be negative or because it would have to be larger than 1. This is depicted in Figure 1.

PLEASE PLACE FIGURE 1 HERE

To find the solution to the optimal contract design problem, it is convenient to try out a solution that disregards provisionnally the self-enforceability constraint (36) and the stock-grant feasibility constraints that $\lambda \in [0, 1]$ and $a \in [0, 1]$. The unconstrained solution of problem (40) is straightforward. Maximizing the RHS of (40) with respect to A and B, we obtain the unconstrained solution

$$\begin{aligned} A^U &= \frac{1}{1 + r\sigma^2}, \\ B^U &= \frac{1}{1 + r\theta^2}. \end{aligned} \tag{45}$$

where the superscript U stands for 'unconstrained'.

Let us find out under what conditions the unconstrained solution (B^U, A^U) actually satisfies the constraints $a \in [0, 1]$, $\lambda \in [0, 1]$ and the self-enforceability constraint (36). The following remark is very useful in what follows.

Remark 1: When $A = A^U$, the self-enforcement constraint (36) holds with equality if $a\lambda = 0$ and with strict inequality if a and λ are both strictly positive. To see this, we use equation (37) to deduce that (36) implies

$$0 = \left[A^U - \frac{(A^U)^2}{2}(1 + r\sigma^2)\right] - \frac{1}{2}\left(\frac{1}{1 + r\sigma^2}\right) \geq -\frac{r}{2}\left[\frac{a\lambda r\sigma^2}{1 + r\sigma^2}\right]^2 \sigma^2.$$

■

[16] Though there are no a priori reasons to rule out contractual values of β_2 outside the interval $[0, 1]$, they may appear rather unusual and we shall show that it is never necessary to use them at equilibrium.

It remains to determine the conditions under which the trial solution $(A, B) = (A^U, B^U)$ would not violate some feasibility constraints. We turn to this task in the next two subsections.

3.1 Optimal contract when $\theta^2 \leq \sigma^2$

Consider first the case where $\theta^2 \leq \sigma^2$, i.e., the noise associated with the investment effort is less than that associated with the current effort. Then we have $1 > B^U \geq A^U$.[17] In this case, it is simple to show that the unconstrained solution is also the solution of the constrained problem. Clearly, with $\theta^2 \leq \sigma^2$, the pair (B^U, A^U) belongs to the set Δ. It follows that the feasibility constraint $0 \leq \lambda \leq 1$ is satisfied. Then $I^*/k = B^U \geq A^U = e_2^*/q$. It is indeed optimal here to give more incentive to investment effort and this is possible by giving to the manager unrestricted shares ($a^* = 0$), which she will sell at the beginning of period 2, before the realization of π_2. Setting $a^* = 0$, we can infer that the optimal stock grants are $\lambda^* = (B^U - A^U)/(1 - A^U) = \left[1 - (\theta/\sigma)^2\right]/(1 + r\sigma^2) > 0$, and that $\beta_2^* = A^U = 1/(1 + r\sigma^2) = \beta_2^{N*}$. This optimal contract gives greater incentive to investment effort, I, than to period 2 current effort, e_2. The contract is self-enforcing because the incumbent manager is given the same second-period incentive as that which a new manager would receive. We conclude that if $\theta^2 \leq \sigma^2$, then it is optimal to induce the efforts levels (I^*, e_2^*) set at

$$\frac{I^*}{k} = B^U \geq A^U = \frac{e_2^*}{q} \tag{46}$$

and this can be achieved by the following triple[18]

$$(a^*, \lambda^*, \beta_2^*) = \left(0, \frac{1 - (\theta/\sigma)^2}{1 + r\sigma^2}, \frac{1}{1 + r\sigma^2}\right). \tag{47}$$

Note, however, that the optima efforts in equation (46) can also be achieved by other triples. This is because we have here two targets, I^* and e_2^*, and three instruments, a, λ and

[17] Note that

$$B^U - A^U = r(\sigma^2 - \theta^2)/(1 + r\sigma^2)(1 + r\theta^2).$$

[18] It is easy to verify that this solution belong to the feasible set $S^0 \cap S^E$.

β_2. By setting $a^* = 0$, we were able to find $\lambda^* \in [0,1)$ and $\beta_2^* \in (0,1)$. But we could have set a slightly higher value for a (as long as $a \leq 1$) and obtained a corresponding higher value for λ :

$$\begin{aligned} \lambda &= (B^U - A^U)/(1 - a + aB^U - A^U) \\ &= \frac{(\sigma^2 - \theta^2)}{(\sigma^2 + r\sigma^2\theta^2) - a\left(\theta^2 + r\sigma^2\theta^2\right)} > 0 \text{ if } \sigma^2 > \theta^2. \end{aligned}$$

Note that if we increase a, then λ must also be increased. Intuitively, an increase in restrictions on share sale must be compensated by an increase in stock grants.[19]

Thus we can state the following Proposition:

Proposition 1: *Assume that investment effort is less noisy than current effort, i.e., $\theta^2 \leq \sigma^2$. In this case, the optimal self-enforcing two-period contract is such that:*

(i) The incentives for current efforts are the same for the two periods:

$$\beta_1^* = \beta_2^* + a^*\lambda^*(1 - \beta_2^*) = A^U = \frac{1}{1 + r\sigma^2}, \tag{48}$$

(ii) The incentive for investment effort is greater than that for current effort:

$$\beta_2^* + \lambda^*(1 - \beta_2^*) = B^U = \frac{1}{1 + r\theta^2}, \tag{49}$$

(iii) If θ^2 is strictly smaller than σ^2, then $\lambda^* > 0$. In particular, the simplest solution is that there is no restriction on the sale of granted stocks:

$$(a^*, \lambda^*, \beta_2^*) = \left(0, \frac{\sigma^2 - \theta^2}{\sigma^2(1 + r\theta^2)}, \frac{1}{1 + r\sigma^2}\right) \tag{50}$$

(iv) The payoff of the initial shareholders is

$$\Pi_{initial}^* = \frac{q^2}{1 + r\sigma^2} + \frac{k^2}{2}\left[\frac{1}{1 + r\theta^2}\right] - 2\omega. \tag{51}$$

It is decreasing in θ and σ.

[19] From $\beta_2 + \lambda(1 - \beta_2) = B^U$, we have $(1 - \lambda)d\beta_2 + (1 - \beta_2)d\lambda = 0$.Thus a higher λ implies a lower β_2 if $\beta_2 < 1$.

3.2 Optimal contract when $\theta^2 > \sigma^2$

Let us turn to the case where *investment effort more less noisy than current effort, i.e.,* $\theta^2 > \sigma^2$. In this case, the unconstrained solution (B^U, A^U) given by (45) is not feasible. Indeed, with $\theta^2 > \sigma^2$, we have $B^U < A^U < 1$, which means (B^U, A^U) belongs neither to triangle Δ nor to the cone Γ, which we have shown to be the only feasible regions if we restrict a such that $a \in [0, 1)$. What about a solution with $a = 1$? Setting $a = 1$ implies $A = B$, therefore the unconstrained solution (B^U, A^U) is also not feasible under $a = 1$. We state this result as Lemma 2:

Lemma 2: *If* $\theta^2 > \sigma^2$*, then the unconstrained solution* (B^U, A^U) *is not feasible, given the constraints that* $0 \leq a \leq 1$ *and* $0 \leq \lambda \leq 1$*.*

To search for a feasible solution, let us suppose for the moment that $a = 1$. Then $A = B$. Ignore for the moment the self-enforceability constraint. Maximizing (40) with respect to A and B, subject to the constraint $A = B$, yields the candidate solution

$$B^C = A^C \equiv \frac{A^U B^U}{\frac{k^2}{k^2+q^2} A^U + \frac{q^2}{k^2+q^2} B^U} = \frac{1}{1 + \left[\frac{k^2}{k^2+q^2}(r\theta^2) + \frac{q^2}{k^2+q^2}(r\sigma^2)\right]} \tag{52}$$

where the superscript in B^C and A^C indicates that we have imposed the constraint $A = B$.

The bracketed term on the right-hand side of (52) is the weighted average of risk-bearing costs associated with providing investment and current efforts. Note that since $\theta^2 > \sigma^2$ implies that $B^U < A^U$, it can be verified that

$$B^U < A^C < A^U.$$

Remark 2: It is important to note that the proposed solution $(B, A) = (B^C, A^C)$ cannot be implemented with any $a < 1$.To see this, note that if $a < 1$ then the condition $A = B = A^C$ implies $\lambda = 0$ and $\beta_2 = A^C$. From Lemma 1, a contract with $a\lambda = 0$ and $A \neq \beta_2^{N*}$ is not self-enforcing.[20]

[20] Intuitively, this is because (i) $A^C < A^U$ does not maximize the second-period aggregate certainty equivalent and (ii) $a\lambda = 0$ implies that there is no intrinsic cost from severing the relationship.

It follows that implementing the incentives $A = A^C$ and $B = B^C = A^C$ can only be done by granting only restricted shares ($a = 1$) and by choosing a large enough value of λ in order to meet the self-enforceability constraint. The intuition is that since A^C does not maximize the second-period aggregate certainty equivalent[21], to make the solution $(B, A) = (A^C, A^C)$ self-enforceable, one has to induce a large enough cost of bearing dividend risks in the event that the relationship were severed. Recall that the self-enforceability constraint can be written as

$$\left\{ \left[A - \frac{A^2}{2}(1 + r\sigma^2) \right] - \frac{1}{2(1 + r\sigma^2)} \right\} + a^2\lambda^2 \left(\frac{r^3\sigma^6}{2(1 + r\sigma^2)^2} \right) \geq 0 \tag{53}$$

Since the term inside the curly brackets $\{...\}$ is negative if $A \neq A^U$, it is clear that for any $A \neq A^U$, the self-enforceability constraint (53) can be satisfied by choosing a sufficiently large λ, given any $a > 0$. In particular, if $a = 1$ then any $\lambda > \underline{\lambda}$ will satisfy the self-enforceability constraint, where

$$\underline{\lambda} \equiv \frac{\left|\theta^2 - \sigma^2\right| k^2 \sqrt{(1/r) + \sigma^2}}{\sigma^3 \left[k^2(1 + r\theta^2) + q^2(1 + r\sigma^2)\right]} \tag{54}$$

(See the Appendix.)

Then, provided that $\underline{\lambda} < 1$, any value of λ such that $\underline{\lambda} < \lambda < 1$ will ensure that the self-enforceability constraint is satisfied with strict inequality. Once we have selected any λ in the interval $(\underline{\lambda}, 1)$, we can deduce the corresponding value for β_2 from the equation $\beta_2 + \lambda(1 - \beta_2) = A^C$, i.e., $\beta_2 = (A^C - \lambda)/(1 - \lambda) < 1$.

We can now state and prove the following Proposition:

Proposition 2a: (The case where $\theta^2 > \sigma^2$ and $\theta^2 - \sigma^2$ is sufficiently small)

Assume that $\theta^2 > \sigma^2$ and that $\theta^2 - \sigma^2$ is sufficiently small, so that $\underline{\lambda} < 1$. Then any contract with the following properties are optimal among all self-enforcing contracts:

(i) The incentive for the current effort in the first period is provided by $\beta_1^* = \frac{1}{1+r\sigma^2}$;

(ii) The investment effort I^* and the second-period current effort e_2^* are motivated by the

[21] i.e. the LHS of (36).

same instrument, such that

$$\frac{I^*}{k} = A^C = \frac{e_2^*}{q} \tag{55}$$

where

$$\frac{1}{1+r\theta^2} < A^C \equiv \frac{1}{1+\left[\frac{k^2}{k^2+q^2}(r\theta^2)+\frac{q^2}{k^2+q^2}(r\sigma^2)\right]} < \frac{1}{1+r\sigma^2}$$

(iii) The manager receives only restricted shares, i.e. $a^* = 1$,

(iv) The optimal fraction of the firm's shares to be awarded as stock grants is any λ^* *such that* $1 > \lambda^* > \underline{\lambda}$, *where* $\underline{\lambda}$ *was defined by equation (54).*

(v) The corresponding β_2^* *is given by*

$$\beta_2^* = (A^C - \lambda^*)/(1-\lambda^*) < 1. \tag{56}$$

(vi) The equilibrium payoff to the initial shareholders is

$$\Pi^*_{initial} = \frac{q^2}{2(1+r\sigma^2)} + \frac{(k^2+q^2)^2}{2\left[k^2(1+r\theta^2)+q^2(1+r\sigma^2)\right]} - 2\omega \tag{57}$$

It is decreasing in θ *and* σ.[22]

Proof: See Appendix A.

Remark 3: We can show that there is no feasible self-enforcing contract that performs strictly better than the contract described in Proposition 2a. This is established in Appendix A.

Proposition 2a shows that if the riskiness associated with investment effort is greater than that of current effort (but the difference is not too big) it is optimal for the initial shareholders to prohibit the manager to sell her shares at the beginning of period 2, i.e. to grant her only restricted shares.The intuition is as follows. Given $\theta^2 > \sigma^2$, it would ideally be profitable to give more incentive to current effort than to investment effort. Granting unrestricted shares to the manager would amount on the contrary to give a greater incentive to investment effort. The best available option is thus to give the same incentive to the two types of effort.

[22] As θ falls toward σ, this payoff converges to the payoff in Proposition 1. There is no discontinuity in payoff.

It might at first appear that this could be implemented in two different ways. The first way would be to grant no share to the manager but to have her receive an additional fraction of second-period profit. However such a contract would not be self-enforcing: it would create an incentive for the initial shareholders to fire the incumbent manager, or renegotiate with her, when period 2 comes. This is a time-inconsistency problem in contracting. To avoid time inconsistency, the initial shareholders must create a commitment device. This is indeed the second solution: the initial owners commit to grant a sufficiently large amount of shares to the manager, who in turn is required not to sell them (another form of commitment on her part). Given the stock grant, the initial shareholders offer the manager a lower expected second-period cash remuneration than they would have to pay a replacement manager, who owns no shares. Therefore they have no incentive to hire a new manager and fire the incumbent (with compensation). As for the incumbent, quitting is not an attractive alternative, as her outside option income (which includes the random dividends from the shares she cannot sell) is not risk-free because she is not permitted to sell her shares. (It would be risk-free if there were no restrictions on share sales.)

Proposition 2a characterizes the optimal contract when $\underline{\lambda} < 1$. Can we find an optimal contract when $\underline{\lambda} > 1$? In this case, due to the restriction that $\lambda \in [0, 1]$, it is not possible to satisfy the self-enforceability constraint by setting $A = A^C$. We must find a different value of A. It is always possible to satisfy the self-enforceability constraint by setting $A = A^U$ together with $\lambda = 0$ or $a = 0$. Therefore the feasible set of triples (β_2, λ, a) is not empty. If we impose the restriction that β_2 belongs to some closed interval $\left[\underline{\beta}, \overline{\beta}\right]$, then the set of feasible triples (β_2, λ, a) is non-empty, closed, and bounded. The objective function (38) is continuous. It follows from Weierstrass Theorem that a maximum exists.

Proposition 2b: (The case where $\theta^2 > \sigma^2$ and $\theta^2 - \sigma^2$ is so large that $\underline{\lambda} > 1$)

In this case, the optimal contract has the following properties:

(i) $\lambda^* = 0$ *and* $a^* = 0$*, i.e., the optimal stock grant is zero,*

(ii) $\beta_1^* = 1/(1 + r\sigma^2)$

(iii) $\beta_2^* = A^U = 1/(1 + r\sigma^2)$ *(The manager is induced to choose second period effort level as if she were a newly hired manager for period 2).*

(iv) $B^* = A^U$*, implying that*

$$\frac{I^*}{k^*} = A^U = \frac{e_2^*}{q}$$

Thus the manager's investment effort is given same incentive as the current effort.

(v) The equilibrium payoff to the initial shareholders is

$$\Pi^*_{initial} = \frac{q^2}{2(1 + r\sigma^2)} + \frac{k^2}{2}\left[\frac{1}{1 + r\sigma^2} - \frac{r(\theta^2 - \sigma^2)}{(1 + r\sigma^2)^2}\right] - 2\omega$$

Proof: See Appendix B.

3.3 Discussion

How does the optimal size of the stock grant depend on the parameter θ^2? To illustrate, we assume that (a) if $\theta^2 < \sigma^2$, the firm uses the simplest optimal solution, i.e., equation (50), corresponding to $a = 0$, while (b) if $\theta^2 \geq \sigma^2$ (but the difference is not too large, so that $1 > \underline{\lambda}$) the stock grant λ is set at its smallest possible value, $\underline{\lambda}$. Then, keeping σ^2 constant, we see that as θ^2 increases in the range $(0, \sigma^2)$, the stock grant falls gradually to zero. As θ^2 crosses the threshold level σ^2, the size of the grant increases with θ^2. This is depicted in Figure 2 where we have set $\sigma = r = k = q = 1$.

PLEASE PLACE FIGURE 2 HERE

Unlike the CCW model, where stock grants only serve as a commitment device, our model, by allowing for the existence of an investment effort in addition to current effort, with different degrees of riskiness, shows that stock grants serve as a commitment device under one set of conditions, and as an incentive device under another set of conditions.[23] A second difference is that we endogenize the restrictions on shares sales.[24] When $\theta^2 < \sigma^2$,

[23] Of course, the two models are very different. In CCW, effort is a discrete choice variable, either 'low' or 'high', and period 2 profit is independent of period 1 actions.

[24] CCW assume that the manager is forbidden to sell the shares if neither party reneges, or if the firm reneges. Only in the case where the manager reneges, they consider three different exogenous scenarios, one of which allows the sales of shares.

granting unrestricted shares allows the initial shareholders to give a greater incentive to investment effort than to current effort. In that case, restricted shares play no specific role which could not be played equivalently by a contractual profit-contingent cash remuneration scheme.[25] When $\theta^2 > \sigma^2$, ideally less incentive should be given to investment effort, but this is not feasible. In order for the optimal common incentive to fulfill the enforceability condition, the initial shareholders distribute only restricted shares to the manager. These restricted shares act as a commitment device.[26]

4 Concluding Remarks

Using a two-period agency model, we show that granting stocks to a manager may be either a commitment device or an incentive device. Neither investment effort nor current effort are verifiable. We investigate the nature of the optimal two-period contract under the requirement that it is self-enforcing, i.e., it ensures that there are no gains to mutually agree to terminate the contract when period 2 comes. The contract stipulates the number of shares to be allocated to the manager at the end of period one, and which fraction of her shares the manager is allowed to sell before the end of the second period.

We show that stock grants play different roles, depending on the difference in the degrees of precision with which one can infer current effort and investment effort. Indeed, when the investment effort can be 'more precisely' estimated, more incentive should be and is given to this effort. In that case, unrestricted stock grants are the appropriate incentive device[27] which allows to give an additional incentive to investment effort.

When the current effort can be more precisely estimated, the ideal solution which would be to give a smaller incentive to investment effort is not feasible and accordingly the optimal contract gives an average incentive that applies equally to both investment effort and second-

[25] Indeed, there is an equilibrium in which the initial shareholders distributed only unrestricted shares.

[26] These results are in sharp constrast with CCW who noticed that, in their model, the profits to the shareholders are increasing in the degree of restriction of share sales.

[27] That is, there is an optimal contract which does not impose any restriction on the sales of shares.

period current effort. The time-inconsistency problem is resolved by using stock grants with sales restrictions as a commitment device. By granting a fraction of the stock at the end of period 1, the contract can specify a lower expected contingent cash payment in period 2 to the continuing manager, and thus the initial shareholders no longer find it profitable to replace her with a new manager. As for the incumbent manager, being unable to sell the shares implies that her outside option income (which includes the risky dividends) is not as attractive as it would be otherwise. Notice that the ideal solution could become feasible, would the initial shareholders be able to commit to hire a new manager in period 2, i.e. not to rehire the period 1- manager. The latter would receive an incentive for investment effort exclusively thanks to a grant of unrestricted shares while the former would receive contractually a greater incentive for current effort, in the form of a cash contingent remuneration scheme. However such a commitment would not be credible, especially if one introduces an (even infinitesimal) efficiency advantage for the incumbent manager over a newly hired one.

Our results show that stock grants may serve two different functions, depending on whether the current effort can be more precisely estimated than investment effort. Our paper may thus be considered on one hand, as an attempt to show that both views (stock grants as an incentive device versus stock grants as a commitment device) are correct under appropriately specified conditions while, on the other hand, it tries to characterize the optimal two-period contract of a manager when, profits being determined by manager's investment and current efforts, it is sometimes possible, and sometimes not, to differentiate the incentives for these two types of effort.

APPENDICES

A) Proof of Proposition 2a

Step 1: Let us first suppose that $a = 1$. Then $A = B$. Maximizing $\Pi_{initial}$ as defined by (38) with respect to A and B, accounting for the constraint $A = B$, yields the FOC

$$(q^2 + k^2) = \left[k^2(1 + r\theta^2) + q^2(1 + r\sigma^2)\right] A,$$

i.e.,

$$A = B = \frac{k^2 + q^2}{k^2(1 + r\theta^2) + q^2(1 + r\sigma^2)} \equiv A^C.$$

This solution satisfies the self-enforceability constraint if $\underline{\lambda}$ (which is defined in eq. (54)) is smaller than 1. Let us show this fact.

Derivation of $\underline{\lambda}$

With the proposed solution $A = B = A^C$, the self-enforceability constraint becomes

$$\frac{q^2(k^2 + q^2)(k^2(1 + 2r\theta^2 - r\sigma^2) + q^2(1 + r\sigma^2))}{2(k^2(1 + r\theta^2) + q^2(1 + r\sigma^2))^2} \geq \frac{q^2}{2(1 + r\sigma^2)} - \frac{1}{2}r(a\lambda)^2(\frac{r\sigma^2}{1 + r\sigma^2})^2 q^2\sigma^2$$

For $a = 1$, this constraint is satisfied if λ is big enough, i.e., larger than the value $\underline{\lambda}$ such that the above constraint is satisfied with equality:

$$\begin{aligned}
&\frac{(k^2(1 + r\theta^2) + q^2(1 + r\sigma^2))^2 - (1 + r\sigma^2)(k^2 + q^2)(k^2(1 + 2r\theta^2 - r\sigma^2) + q^2(1 + r\sigma^2))}{(k^2(1 + r\theta^2) + q^2(1 + r\sigma^2))^2(1 + r\sigma^2)} \\
= \; & (\underline{\lambda})^2(\frac{r\sigma^2}{1 + r\sigma^2})^2 r\sigma^2
\end{aligned}$$

Simplification yields

$$(\underline{\lambda})^2(\frac{1}{1 + r\sigma^2})(r\sigma^2)^3 = \frac{k^4 r^2 \left(\theta^2 - \sigma^2\right)^2}{(k^2(1 + r\theta^2) + q^2(1 + r\sigma^2))^2}$$

Thus

$$\begin{aligned}
(\underline{\lambda})^2 &= \frac{k^4 \left(\theta^2 - \sigma^2\right)^2 (1 + r\sigma^2)}{r\sigma^6(k^2(1 + r\theta^2) + q^2(1 + r\sigma^2))^2} \\
&= \frac{k^4 \left(\theta^2 - \sigma^2\right)^2 \left(\frac{1}{r} + \sigma^2\right)}{\sigma^6(k^2(1 + r\theta^2) + q^2(1 + r\sigma^2))^2}
\end{aligned}$$

Clearly, $\underline{\lambda} < 1$ iff

$$\left|\theta^2 - \sigma^2\right| k^2 \sqrt{(1/r) + \sigma^2} < \sigma^3 \left[k^2(1 + r\theta^2) + q^2(1 + r\sigma^2)\right]$$

Step 2: We must now show that if $a < 1$, there does not exist any pair $(B, A) \in \Gamma \cup \Delta$ that is self-enforceable and yields a payoff that is strictly greater than (57). To show this, our argument proceeds as follows.

Suppose there is some $(B^*, A^*) \in \Gamma \cup \Delta$ that is self-enforceable and yields a strictly higher payoff. Then this payoff is not greater than the payoff that is obtained from a corresponding optimization problem that disregards self-enforceability, i.e., choose (β_2, λ, a) where $a \in [0, 1)$ to maximize

$$\max_{\beta_2, \lambda, a} \Pi_{initial} = q^2 \left[A - \frac{A^2}{2}(1 + r\sigma^2)\right] + k^2 \left[B - \frac{B^2}{2}(1 + r\theta^2)\right] - 2\omega + \frac{q^2}{2(1 + r\sigma^2)} \qquad \text{(A.1)}$$

subject to $0 < \lambda < 1$. That is, choose $(B, A) \in \Gamma \cup \Delta$ to maximize (A.1), without regard to self-enforceability.

Next, we will show that the resulting maximum value $\Pi_{initial}$ is exactly the same as maximum value obtained when $a = 1$ under both self-enforceability and feasibility, thus contradicting to the claim that (B^*, A^*) yields a payoff that is strictly greater than (57).

With $a < 1$, since $\theta^2 > \sigma^2$, we know that the proposed unconstrained solution $\left(B^U, A^U\right)$ does not belong to the feasible set $\Gamma \cup \Delta$. Thus, the maximum subject to $(B, A) \in \Gamma \cup \Delta$ must occur where at least one of the constraints holds with equality, i.e., either $A = B$, or $B = 1$, or both. Consider the case where $B = 1$. Then the equilibrium candidate is $\left(1, A^U\right)$, This yields the payoff

$$\Pi_{initial}\left(B = 1, A = A^U \middle| \, a < 1\right) = \frac{q^2}{1 + r\sigma^2} + k^2 \left[1 + \frac{1}{2}(1 + r\theta^2)\right] - 2\omega$$

Next, consider the case where $A = B$, with $a < 1$. Then from step 1 above, $(A, B) = \left(A^C, B^C\right)$, yielding the payoff

$$\Pi_{initial}\left(B = B^C, A = A^C \middle| \, a < 1\right) = \frac{q^2}{2(1 + r\sigma^2)} + \frac{(k^2 + q^2)^2}{2\left[k^2(1 + r\theta^2) + q^2(1 + r\sigma^2)\right]} - 2\omega$$

This payoff is equal to (57) which was obtained by setting $a = 1$.

It is easy to verify that $\Pi_{initial}\left(B = B^C, A = A^C \mid a < 1\right) > \Pi_{initial}\left(B = 1, A = A^U \mid a < 1\right)$.

It follows that

$$(A^C, B^C) = \arg \max_{(B,A)\in\Gamma\cup\Delta} \Pi_{initial}(B, A)$$

B) Proof of Proposition 2b

Assume that $\theta^2 > \sigma^2$ and that $\underline{\lambda} > 1$. In this case, we find the solution by first re-writing the self-enforceability constraint (36) in the form

$$\underline{A} \leq A \leq \overline{A} \tag{A.2}$$

where $\underline{A}$ and $\overline{A}$ are defined as

$$\underline{A} = \frac{1 - a\lambda\sqrt{\frac{r^3\sigma^6}{(1+r\sigma^2)^2}}}{1 + r\sigma^2} = \frac{1}{1 + r\sigma^2} - a\lambda\frac{\sigma^3 r^{3/2}}{(1 + r\sigma^2)^2} = \underline{A}(\lambda, a) \tag{A.3}$$

$$\overline{A} = \frac{1 + a\lambda\sqrt{\frac{r^3\sigma^6}{(1+r\sigma^2)^2}}}{1 + r\sigma^2} = \frac{1}{1 + r\sigma^2} + a\lambda\frac{\sigma^3 r^{3/2}}{(1 + r\sigma^2)^2} = \overline{A}(\lambda, a) \tag{A.4}$$

The objective function is to maximize

$$\begin{aligned}\Pi_{initial}(\beta_2, a, \lambda) \quad = \quad & -2\omega + q^2\left[\frac{1}{2(1 + r\sigma^2)}\right] \\ & + q^2\left[A - \frac{A^2}{2}(1 + r\sigma^2)\right] + k^2\left[B - \frac{B^2}{2}(1 + r\theta^2)\right]\end{aligned}$$

subject to the following inequality constraints:

$$\lambda \geq 0, 1 - \lambda \geq 0, a \geq 0, 1 - a \geq 0$$

$$A - \underline{A} \geq 0$$

$$\overline{A} - A \geq 0$$

The first four inequality constraints ensure that (β_2, λ, a) belongs to the set S^0 defined by (41) while the last two ensure that (β_2, λ, a) belongs to the set S^E defined by (42)

Let us write the last two constraints in full:

$$\beta_2 + a\lambda(1 - \beta_2) - \frac{1}{1 + r\sigma^2} + a\lambda\frac{\sigma^3 r^{3/2}}{(1 + r\sigma^2)^2} \geq 0 \tag{A.5}$$

and

$$\frac{1}{1+r\sigma^2}+a\lambda\frac{\sigma^3 r^{3/2}}{\left(1+r\sigma^2\right)^2}-\beta_2-a\lambda(1-\beta_2)\geq 0. \tag{A.6}$$

Let $\delta_1, \delta_2, \psi_1, \psi_2, \phi_1$ and ϕ_2 be the corresponding Lagrange multipliers.

To make sure there exists a maximum, we should ensure that the feasible set is closed and bounded. Thus we add the constraints $\overline{\beta} \geq \beta_2 \geq \underline{\beta}$ where the bounds are arbitrary. The associated non-negative multipliers are η_1 and η_2)

The Lagrangian is

$$\begin{aligned}
L &= \Pi_{initial}(\beta_2, a, \lambda) \\
&\quad +\delta_1\lambda+\delta_2(1-\lambda)+\psi_1 a+\psi_2(1-a)+\eta_1(\beta_2-\underline{\beta})+\eta_t(\overline{\beta}-\beta_2) \\
&\quad +\phi_1\left[\beta_2+a\lambda(1-\beta_2)-\frac{1}{1+r\sigma^2}+a\lambda\frac{\sigma^3 r^{3/2}}{\left(1+r\sigma^2\right)^2}\right] \\
&\quad +\phi_2\left[\frac{1}{1+r\sigma^2}+a\lambda\frac{\sigma^3 r^{3/2}}{\left(1+r\sigma^2\right)^2}-\beta_2-a\lambda(1-\beta_2)\right]
\end{aligned}$$

The necessary conditions are

$$\begin{aligned}
\frac{\partial L}{\partial \beta_2} &= q^2(1-A(1+r\sigma^2))(1-a\lambda)+k^2(1-B(1+r\theta^2))(1-\lambda) \\
&\quad +\eta_1-\eta_2+(\phi_1-\phi_2)(1-a\lambda) \\
&= 0
\end{aligned} \tag{A.7}$$

$$\begin{aligned}
\frac{\partial L}{\partial \lambda} &= q^2(1-A(1+r\sigma^2))a(1-\beta_2)+k^2(1-B(1+r\theta^2))(1-\beta_2) \\
&\quad +\phi_1\left[a\left(1-\beta_2+\frac{\sigma^3 r^{3/2}}{\left(1+r\sigma^2\right)^2}\right)\right] \\
&\quad +\phi_2\left[a\left(\frac{\sigma^3 r^{3/2}}{\left(1+r\sigma^2\right)^2}-(1-\beta)\right)\right]+\delta_1-\delta_2 \\
&= 0
\end{aligned} \tag{A.8}$$

$$\begin{aligned}
\frac{\partial L}{\partial a} &= q^2(1-A(1+r\sigma^2))\lambda(1-\beta_2)+\psi_1-\psi_2 \\
&\quad +\phi_1\left[\lambda\left(1-\beta_2+\frac{\sigma^3 r^{3/2}}{\left(1+r\sigma^2\right)^2}\right)\right]+\phi_2\left[\lambda\left(\frac{\sigma^3 r^{3/2}}{\left(1+r\sigma^2\right)^2}-(1-\beta_2)\right)\right] \\
&= 0
\end{aligned} \tag{A.9}$$

and

$$\frac{\partial L}{\partial \phi_1} = \beta_2 + a\lambda(1-\beta_2) - \frac{1}{1+r\sigma^2} + a\lambda\frac{\sigma^3 r^{3/2}}{(1+r\sigma^2)^2} \geq 0 \tag{A.10}$$

$$\frac{\partial L}{\partial \phi_2} = \frac{1}{1+r\sigma^2} + a\lambda\frac{\sigma^3 r^{3/2}}{(1+r\sigma^2)^2} - \beta_2 - a\lambda(1-\beta_2) \geq 0 \tag{A.11}$$

with

$$\phi_1 \geq 0,\ \phi_1\frac{\partial L}{\partial \phi_1} = 0,\ \phi_2 \geq 0,\ \phi_2\frac{\partial L}{\partial \phi_2} = 0$$

Similarly,

$$\delta_1 \geq 0, \delta_1\lambda = 0; \delta_2 \geq 0, \delta_2(1-\lambda) = 0; \psi_1 \geq 0, \psi_1 a = 0; \psi_2 \geq 0,\ \psi_2(1-a) = 0. \tag{A.12}$$

$$\eta_1 \geq 0, \eta_1(\beta_2 - \underline{\beta}) = 0, \eta_2 \geq 0, \eta_2(\overline{\beta} - \beta_2) = 0$$

Assume $\theta^2 > \sigma^2$. Let us try the solution $(\beta_2, \lambda, a) = \left(\frac{1}{1+r\sigma^2}, 0, 0\right)$. Then $A = B = \frac{1}{1+r\sigma^2}$. Substituting these values into the necessary condition (A.7)

$$k^2(1 - \frac{1+r\theta^2}{1+r\sigma^2})(1-0) + (\phi_1 - \phi_2) = 0$$

Since $\theta^2 > \sigma^2$, $k^2(1-\frac{1+r\theta^2}{1+r\sigma^2}) < 0$, and thus this FOC is satified by setting $\phi_1 = k^2\left[\frac{1+r\theta^2}{1+r\sigma^2} - 1\right] > 0$ and $\phi_2 = 0$.

Next consider the necessary condition (A.8). We get

$$k^2\left(1 - \frac{1+r\theta^2}{1+r\sigma^2}\right)\left(1 - \frac{1}{1+r\sigma^2}\right) + \delta_1$$

so $\delta_1 > 0$ because $\theta^2 > \sigma^2$.

The necessary condition (A.9) is satisfied.

Remark A1: A solution with $a \neq 0$ (while $\lambda = 0$ and $\beta_2 = 1/(1+r\sigma^2)$) would also work if a is not too large, because then (A.8) would give

$$k^2\left(1 - \frac{1+r\theta^2}{1+r\sigma^2}\right)\left(1 - \frac{1}{1+r\sigma^2}\right) + \phi_1\left[a\left(1 - \beta_2 + \frac{\sigma^3 r^{3/2}}{(1+r\sigma^2)^2}\right)\right] + \delta_1 = 0$$

i.e

$$k^2\left[1-\frac{1+r\theta^2}{1+r\sigma^2}\right]\left[(1-a)\left(1-\frac{1}{1+r\sigma^2}\right)-a\left(\frac{\sigma^3r^{3/2}}{(1+r\sigma^2)^2}\right)\right]+\delta_1=0$$

which is consistent with $\delta_1>0$ provided that a is not too large.

ONLINE APPENDIX

Proof that a more general renumeration scheme does not affect the results

Let us modify the cash remuneration scheme by allowing the initial shareholders to commit to make payments in period 2 that are contingent on period 1 profit, i.e.

$$Y_2=\mu_2+\beta_2\pi_2+\gamma_2\pi_1. \tag{A.13}$$

Now, in Subsection 2.3., the second period market value of the firm becomes

$$p=(K+q\bar{e}_2)(1-\beta_2)-\mu_2-\gamma_2q(e_1+\varepsilon_1).$$

Accordingly, her effective consumption in period 2 obtains as

$$\begin{aligned}c_2(\beta_2,\mu_2,K,\varepsilon_2,\lambda,a,\varepsilon_1) &= (1-a)\lambda\left[(K+q\bar{e}_2)(1-\beta_2)-\mu_2-\gamma_2q(e_1+\varepsilon_1)\right]\\&+\mu_2+\beta_2\left[K+q(e_2+\varepsilon_2)\right]+\gamma_2q(e_1+\varepsilon_1)\\&+a\lambda\left\{(1-\beta_2)\left[K+q(e_2+\varepsilon_2)\right]-\mu_2-\gamma_2q(e_1+\varepsilon_1)\right\}-(1/2)(e_2)^2,\end{aligned}$$

and its certaintly equivalent is

$$\begin{aligned}CE_2(K,\beta_2,\lambda,a,\varepsilon_1) &= (1-\lambda)[\mu_2++\gamma_2q(e_1+\varepsilon_1)]+[\lambda+(1-\lambda)\beta_2]\left(Aq^2+K\right)\\&-\frac{r}{2}A^2q^2\sigma^2-\frac{(Aq)^2}{2}.\end{aligned}$$

Let us now consider how the first-period problem (Subsection 2.4.) in now modified. Notice that in the above equation, there are two random variables which are δ (since $K=$

$k(I+\delta))$ and ε_1. Let us calculate the certainty equivalent of $CE_2(K)$ for a given realization of ε_1:

$$CE\left[CE_2(K)\right] = E_\delta\left[CE_2(K)\right] - \frac{r}{2}\left[\lambda + (1-\lambda)\beta_2\right]^2 k^2\theta^2$$

where

$$\begin{aligned} E_\delta\left[CE_2(K)\right] &= (1-\lambda)[\mu_2 + \gamma_2 q(e_1+\varepsilon_1)] + \left[\lambda + (1-\lambda)\beta_2\right](Aq^2 + kI) \\ &\quad -\frac{r}{2}A^2q^2\sigma^2 - \frac{(Aq)^2}{2}. \end{aligned}$$

In order to compute the certainty equivalent of $c_1 + c_2$, it remains to determine the certainty equivalent of $c_1 + E_\delta\left[CE_2(K)\right] - \frac{r}{2}\left[\lambda + (1-\lambda)\beta_2\right]^2 k^2\theta^2$, which is itself a random variable, since it depends on ε_1. Denoting $B = \lambda + (1-\lambda)\beta_2$ and $C = \beta_1 + (1-\lambda)\gamma_2$, one obtains

$$\begin{aligned} CE_{total}^{stay}(I, e_1) &= \mu_1 + Cqe_1 - \frac{1}{2}e_1^2 - \frac{1}{2}I^2 - \frac{r}{2}(Cq\sigma)^2 + (1-\lambda)\mu_2 \\ &\quad +B(Aq^2 + kI) - \frac{r}{2}A^2q^2\sigma^2 - \frac{(Aq)^2}{2} - \frac{r}{2}B^2k^2\theta^2. \end{aligned}$$

The current effort e_1 and the investment effort I are chosen by the manager to maximize $CE_{total}^{stay}(I, e_1)$, yielding:

$$\begin{aligned} e_1^* &= qC, \\ I^* &= kB. \end{aligned}$$

We now obtain:

$$\begin{aligned} CE_{total}^{stay}(I^*, e_1^*) &= \mu_1 + (1-\lambda)\mu_2 + q^2C^2\left[\frac{1-r\sigma^2}{2}\right] - \frac{1}{2}(Bk)^2 \\ &\quad +B(Aq^2 + Bk^2) - \frac{r}{2}A^2q^2\sigma^2 - \frac{(Aq)^2}{2} - \frac{r}{2}B^2k^2\theta^2. \end{aligned}$$

In Section 2, the initial shareholders problem becomes

$$\max_{\beta_1,\beta_2,\gamma_2,\lambda,a} \Pi_{initial}(A,B,C) = q^2\left[C - \frac{C^2}{2} - \frac{r}{2}(C\sigma)^2\right] + q^2\left[A - \frac{A^2}{2} - \frac{r}{2}(A\sigma)^2\right] + k^2\left[B - \frac{B^2}{2} - \frac{r}{2}(B\theta)^2\right] - 2\omega$$

subject to the same constraints as before[28]. Since $C = \beta_1 + (1-\lambda)\gamma_2$, the constraints $(a,\lambda) \in [0,1]^2$ do not imply any constraint on C. It follows that the optimal contract is always such that $C^* = \beta_1^* + (1-\lambda^*)\gamma_2^* = \frac{1}{1+r\sigma^2}$. All other results, including the initial shareholders' equilibrium payoff remain unaffected.

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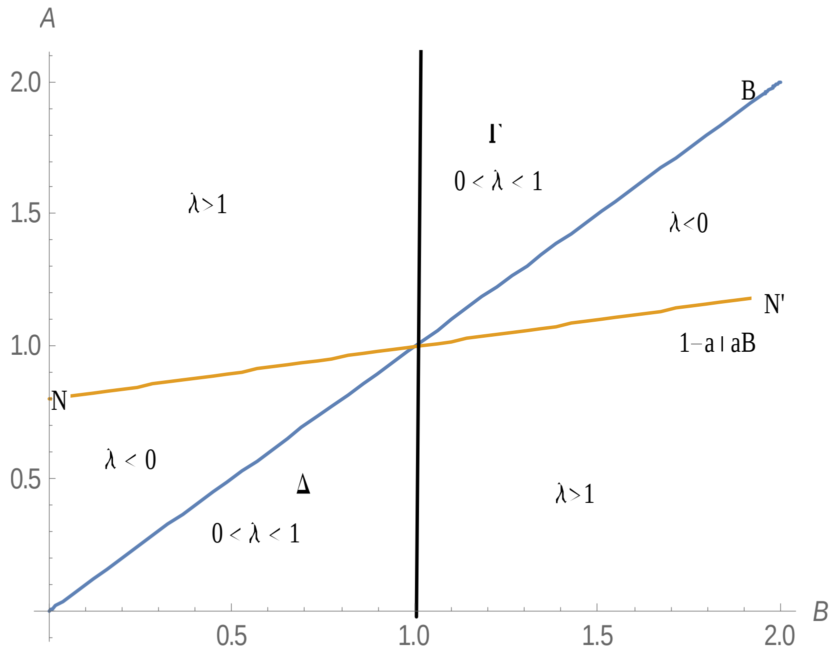

Figure 1: Feasible values of incentives A and B

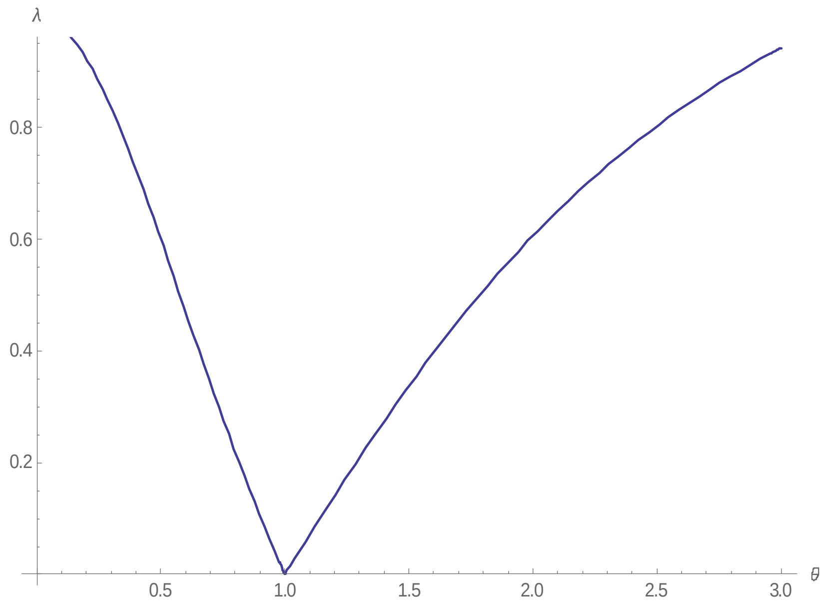

Figure 2: Optimal Stock Grant / relative precision of effort signals