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Abstract

We analyze the impact of expansion of higher education on student outcomes in the context of competition among colleges which differentiate themselves horizontally by setting curricular standards. When public or economic pressures compel less selective colleges to lower their curricular demands, low-ability students benefit at the expense of medium-ability students. This reduces competitive pressure faced by more selective colleges, which therefore adopt more demanding curricula to better serve their most able students. This stylized model of curricular product differentiation in higher education offers an explanation for the diverging selectivity trends of American colleges. It also appears consistent with the U-shaped earnings growth profile we observe among college-educated workers in the U.S.

JEL-Codes: I210, I230, I240, J240, H440.

Keywords: curricular standard, higher education, college selectivity, enrollment expansion, income distribution.

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1 Introduction

During the last several decades, the landscape of postsecondary education in the United States has changed significantly. College education, once a gateway to the elite, has become increasingly accessible to the general public. As shown in Table 1, between 1959 and 2008, enrollment in postsecondary education has increased from 3.64 million to 19.10 million, or 525%. This growth was mainly driven by enrollment in public colleges, which has increased from 2.18 million to 13.97 million (641%). During the same period, enrollment in not-for-profit private colleges has increased from 1.46 million to 3.66 million (251%).[1]

Table 1: Enrollment in postsecondary degree-granting institutions (in thousands)

Year	Total	Public	Private		
			All	Not-for-profit	For-profit
1959	3,640	2,181	1,459	n/a	n/a
1969	8,005	5,897	2,108	2,088	20
1979	11,570	9,037	2,533	2,461	71
1989	13,539	10,578	2,961	2,731	229
1999	14,791	11,309	3,482	3,052	430
2008	19,103	13,972	5,131	3,662	1,469

Furthermore, the steady increase in college enrollment far outpaced the growth of population. As shown in Table 2, after controlling for population size, the enrollment rates within given age cohorts have shown similar patterns of multi-fold increases.[2] Part of the increase in the enrollment rate reflects better "access" to higher education, driven by public policies such as the G.I. bill and the Higher Education Act of 1965.

How did the dramatic expansion of college-bound population affect the landscape of higher education on the supply side? Empirical evidence suggests that the growth of demand over the last four decades was accompanied by a pattern of diverging selectivity among the American colleges (Hoxby, 2009), whereby "selective" colleges have been shown to have become more selective, and vice versa. This implied a progressively

[1]Source: National Center for Education Statistics, Digest of Education Statistics 2009, Tables 003 and 189. (http://nces.ed.gov/programs/digest/d09. Accessed April 13th, 2011.)

[2]Source: National Center for Education Statistics, Digest of Education Statistics 2009, Table 007.

Table 2: Enrollment in postsecondary education by age group (in %)

Year	18–19 years old			20–24 years old			25–29 years old	30–34 years old
	Total	In basic edu.	In higher edu.	All	20–21	22–24		
1959	36.8	n/a	n/a	12.7	n/a	n/a	4.9*	2.4*
1969	50.2	n/a	n/a	23.0	34.1	15.4	7.9	4.8
1979	45.0	10.3	34.6	21.7	30.2	15.8	9.6	6.4
1989	56.0	14.4	41.6	27.0	38.5	19.9	9.3	5.7
1999	60.6	16.5	44.1	32.8	45.3	24.5	11.1	6.2
2008	66.0	17.4	48.6	36.9	50.1	28.2	13.2	7.3

*Data for 1959 unavailable; reported for 1960.

"better sorting" of students across colleges, i.e., strengthening of de facto specialization of colleges, especially at the high end of selectivity, in their chosen segments in the distribution of students in terms of performance on standardized pre-college tests.

This paper aims to analyze the phenomenon of diverging selectivity of colleges that accompanied the expansion of higher education, and to examine its implications for student outcomes. Our theory is based on an innovative model of human capital production whose main novel feature is *curricular standard*, a discretionary characteristic of education technology, which is strategically chosen (as an instrument in the competition for students) and therefore potentially differs across educational institutions. In our model, the curricular standard chosen by a college is the expression of its selectivity. We argue that a student's outcome of studies at a college depends on the match between the student's aptitude (pre-college preparation) and the college's curricular standard. Each student chooses his/her best match among the available options, whereby lower ability students are better served by a less challenging curriculum and vice versa. Thus, each curriculum has a comparative advantage among certain segments of student population, and different curricular choices by colleges can be viewed as horizontally differentiated product offerings in a framework similar to Hotelling's spatial competition.

In our model, colleges differ in their exogenously (historically) established relative priorities over the quality, in terms of human capital outcomes, vs. the quantity of their graduates. These exogenous priorities determine relative positions of colleges in the selectivity rankings. A high priority that less selective colleges give to the number of students implies that such colleges can also be more exposed to additional incentives to expand enrollments further, stemming for instance from political pressure from state governments to ensure greater access to higher education, expressed directly or through financial incentives.[3]

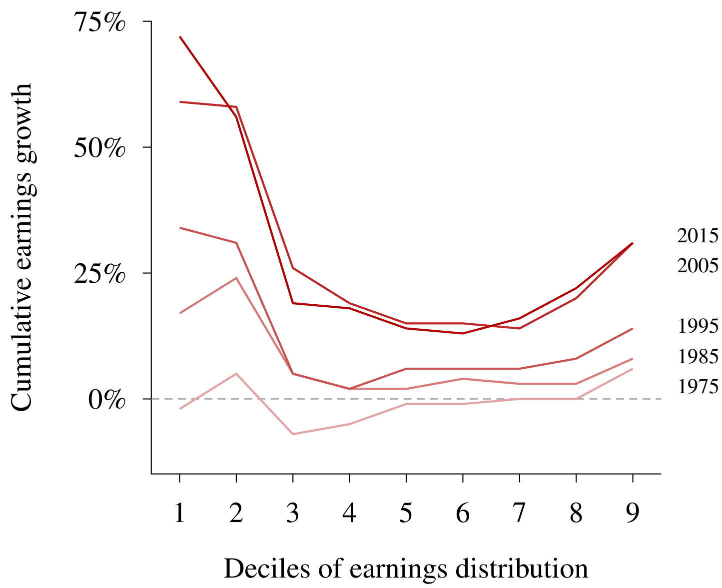
Our analysis shows that curricular standards of colleges diverge in response to the expansion of college-bound population. The diverging selectivity in terms of curricular standards in the higher education market will obviously affect the quality of optimal matches available to individual students. It will improve for some and worsen for others. For example, the downward adjustment in curricular standards of less selective colleges can manifest itself in more remedial course work offered, fewer challenging topics, and a slower pace of learning in general. This will benefit lower ability students, who would otherwise struggle to keep up, at the expense of medium ability students who are ready to learn but are not sufficiently challenged. On the other side of the spectrum, more selective colleges will respond by elevating their curricula due to reduced pressure in competition for medium ability students with the less selective colleges, whose "watered down" curricula make them a less appealing alternative for those students. As a result, high-ability students will benefit, again at the expense of their medium ability peers. Thus, our results suggest overall that students in the mid-section of the aptitude distribution of college-bound population will face deteriorating quality of their best matches, as selective colleges become too challenging for them, while the less selective colleges will offer an insufficient educational challenge.

It is then natural to ask how this stylized result describing the distributional changes in the dynamics of human capital attainment translates into labor market outcomes for college-educated workers, and in particular, whether they are consistent with earnings growth profile of such workers during the period of enrollment expansion. We address this question in Section 5, by using March Current Population Survey (IPUMS-CPS) data. Specifically, we use the repeated cross-sectional data to calculate the growth rates

[3] This interpretation would well fit the case of less selective public colleges most subject to such government policies. However, there are of course other realistic interpretations of this assumption, equally suitable for motivating our model. A bias of less selective colleges, public or private, in favor of quantity of their students has much to do with the colleges' increased reliance on tuition revenues (which are not, however, explicitly featured in our model). Indeed, the public policies to expand access to higher education are often expressed in the U.S. through tuition subsidies, either through direct appropriation for public colleges, or through financial aid to students. On the other hand, the business model of elite private as well as some top public colleges (whose lucid formalization is offered by Hoxby, 2012) is based in part on operating a private endowment which allows a college to balance its budgets intertemporally while banking on future contributions by graduates commensurate with their career earnings, whose expected levels can be deemed proportionate to the attained human capital.

of real earnings for each decile of the annual earnings distributions relative to the corresponding decile in a base year. As can be seen in Figure 1, growth rates in the top and bottom deciles are markedly higher than those in the middle of the distribution. In other words, there is a "sagging middle" in the earnings growth distribution among the college-educated workers. As a stylized pattern, this "sagging middle" phenomenon also shows up if we narrow the sample of college-educated workers to specific age cohorts, or use alternative base years to calculate the earnings growth profile. It also persists when we apply the Recentered Influence Function method (RIF) to address the fact of changing ability composition of college-bound population during the period of its expansion.

Figure 1: Earnings growth profile for college-educated workers, 1965 base year



While the fact of stagnating mid-section, relative to the ends, has been observed in the dynamics of the overall wage distribution, and theories (most notably, the "routinization" hypothesis: see, e.g., Autor et al., 2008, Acemoglu and Autor, 2011) have been advanced to explain it, the "sagging middle" phenomenon in the dynamics of wage distribution within the group of college-educated workers has not been, to our knowledge, specifically addressed by the existing literature. The results we obtain where strategic curricular adjustments by colleges in response to the demand for expanded access have a non-monotone impact on students' college outcomes appear consistent with the U-shaped earnings growth profile among college-educated workers, which we observe in the data. Although our stylized model is not equipped to serve as a predictor of quantitative patterns, our theoretical results do suggest that in addition to the explanations of such changes in the earnings distribution offered in the existing literature based on the demand side of the labor market, there may be factors also on the supply side contributing to the phenomenon. Specifically, our analysis indicates that the endogenous divergence of educational standards, and hence of the quality of educational products accessible by segments of student population, can contribute to the evolution of income inequality.

The remainder of the paper is organized as follows. Section 2 relates this paper to the existing literature on the college education market and labor market outcomes. Section 3 develops a theoretical model of college education technologies characterized by college-specific curricula, and derives students' optimal college choices given the curricula of the colleges. Section 4 endogenizes the colleges' curriculum choice strategies, defines their Nash equilibrium, and obtains our main comparative statics results, which characterizes how equilibrium college curricula and the economy's human capital distribution respond to a policy of increased college access. Section 5 presents the evidence of the "sagging middle" phenomenon in the earnings growth profile for college-educated workers. Section 6 concludes. All proofs are in the Appendix.

2 Literature Review

This paper's main focus is on the heterogeneous human capital gains in college. It builds on a growing literature that emphasizes the hierarchical structure of the education process one of whose important new insights is that the benefits from investing in superior quality of education at a given stage may critically depend, and even be contingent upon sufficient preparation at its prior stage. Driskill and Horowitz (2002), Su (2004, 2006), Blankenau (2005), Blankenau, Cassou, and Ingram (2007), Cunha and Heckman (2007), and Gilpin and Kaganovich (2012) model education as a sequence of stages, where human capital output from lower stages acts as an input in the education technology at higher stages. In particular, the models of Su (2004, 2006) and Gilpin and Kaganovich (2012) feature a curricular threshold standard at the higher education stage, which sets the minimum pre-college preparation level necessary for making educational gains in college. The present paper takes student outcomes at the basic education stage as given, and focuses instead on curricular choices at different colleges as discussed in the Introduction. We underscore an important distinction between our concept of curricular standard, which is intrinsic to education production technology and affects students' human capital gains depending on prior aptitude, and the concept of educational standards in the literature pioneered by Costrell (1994, 1997) and Betts (1998). According to the latter, college standards are sorting devices having no effect on human capital gains in college, the idea which builds on Spence (1973) concept of college education's role as purely a signal of a graduate's aptitude.[4]

This paper also contributes to the literature on inter-school competition. Rothschild and White (1995) (see also a review by Winston, 1999), Epple and Romano (1998), and Epple et al. (2006) model segmentation of the higher education market based on students' ability to study and to pay. This literature assumes that all schools use the same curriculum. That is, schools may differ in the levels of their educational inputs, including the peer effects, but not in their education production technologies. When peer effects are present, students benefit from attending school with high-ability peers, and therefore are willing to pay higher tuition fees for such a benefit. Heterogeneity in both student ability and their family income then generates a stratification of school quality in equilibrium and the sorting of students across schools according to learning ability and the ability to pay.[5] De Fraja and Iossa (2002) add a geographic dimension to intercollegiate duopoly competition where students incur mobility as well as tuition costs. They demonstrate the emergence of two types of equilibria where colleges are either stratified in quality or offer identical quality and serve as unique regional providers, with the level of mobility costs determining which of the outcomes will obtain. In their model, school quality is determined by its admission standard. Furthermore, any student admitted to a school will benefit from a higher standard, as long as he continues to pass it. Thus a common feature of the above literature is that the educational gain of an individual student, if admitted to a school, will grow if the school's quality characteristics are increased.

[4]MacLeod and Urquiola (2015) advance this approach to explain the phenomenon of diverging selectivity of colleges as intensifying endogenous sorting, similar to, but opposite in direction, to Akerlof's "Lemons Principle."

[5]An interesting extension to the line of work on quality differentiation among colleges is offered by Brezis and Hellier (2013) who analyze the long-term intergenerational implications, in terms of social mobility, of the higher education market segmentation into elite and non-elite institutions. Our paper can offer additional insights for this research as it demonstrates that a policy of expanded access to lower ranked schools enhances polarization, as this evinces a strategy of increased selectivity among the higher ranked schools.

In our paper, each school adopts its optimal curriculum, its defining quality characteristic, which is thus endogenous and school-specific. Furthermore, our model differs from the above literature in the following important respect: not all students in a school would gain in terms of human capital if its curricular standard were raised. Instead, there will be winners and losers from such a change. This is an essential, and arguably realistic feature of our model. Therefore, students self-segregate, across different colleges, by ability based on the best match between it and a college's curriculum, rather than due to peer effects. Indeed, here, if a lower ability student were to attend a school with predominantly high-ability peers, then instead of benefiting from a peer-group effect, he would find the school's curriculum geared toward them too challenging for him in terms of maximizing his academic achievement. It is worth noting that this feature of our analysis points to an additional channel through which expanding access to education, even when universal and cost-free, can magnify rather than shrink human capital and income gaps. Indeed, we show that endogenous divergence of curricular standards offered by educational institutions, that is, expanding product differentiation resulting from the optimizing behavior of their providers, can exacerbate unequal student outcomes.

An extensive literature links students' academic achievements to their labor market outcomes. The main focus of this literature is on the college premium, i.e., the wage differential between the groups of college educated workers (with an adjustment for workers with "some college" education) and those with high school education at most. Changes in the college premium over the recent decades have been linked, in the literature focusing on the demand side of the labor market, to skill-biased technological improvements. Among others, Katz and Murphy (1992), Autor et al. (1998), and Autor et al. (2008) show that such technological changes account for several salient changes in the U.S. wage distribution over time. Davis (1992), Katz et al. (1995), Murphy et al. (1998), Card and Lemieux (2001), and Atkinson (2008) demonstrate that this explanation is consistent with cross-country differences among developed economies. An extensive survey of this literature is provided by Acemoglu and Autor (2011).

As discussed in the Introduction, our model and results can offer theoretical insights relevant for understanding the pattern of earnings growth profile *within* the group of college-educated workers. There is a substantial body of literature analyzing the evolution and recent growth of variance of earnings within this group attributable to its observed or unobserved heterogeneity. Some results point to growth, over recent decades, of this within-group residual inequality due to the variation in learning ability in particular (see, e.g., Taber, 2001, and Lochner and Shin, 2014). Some theoretical models (Galor and Moav, 2000, Gould et al., 2002) offer an explanation for these results within the directed technological change paradigm as they argue that the change is biased toward innate ability, including the ability to adjust to change.[6] According to this "ability-bias" concept, however, the magnitude of wage growth should exhibit monotone rise along the ability distribution. One might expect, therefore, that it will be the highest in the right tail of the wage distribution of college graduates, and the lowest in its left tail.

In contrast, our model makes an argument for a non-monotone pattern in the distribution of educational gains across the college-bound population, particularly with the least gains in the mid-section of student ability distribution. In Section 5, we present evidence that the insights from our model are broadly consistent with the data, despite the model being too stylized to serve as a basis for quantitative predictions. Indeed, our analysis in Section 5 of the evolution of the distribution of wage earnings of college educated workers reveals a U-shaped pattern of growth across deciles of the earnings distribution of college educated workers. We argue that this pattern cannot be easily reconciled with existing labor demand-side theories of skill premium changes. Our model thus contributes to the literature by illuminating non-monotone changes in human capital gains within the group of college educated workers during the period of expansion of higher education.

3 The Model

In this section, we introduce a model of a higher education system with a continuum of potential students and a finite number of colleges. The education production technology of colleges is characterized by college-specific parameters which we define as *curriculum*. The key feature of our model is that the human capital a particular student obtains at a particular college is a function of the relationship between this student's pre-college aptitude and the curriculum offered by the college. Thus each student's choice is about finding the best match of a college (if at all) in terms of maximizing his human capital value added. In this section, we analyze students' decisions about choosing a college given curricula of the colleges and each student's pre-college aptitude. Section 4 will

[6] Laitner (2000) analyzes a model where individual return to investment in education is enhanced by a higher individual ability as well as exogenous unbiased technological change. He notes, however, that the overall variance of income inequality within the higher education group is lowered due to composition effect, as this group expands being joined by less able agents.

characterize each college's strategic choice of a curriculum in competition with other colleges and equilibrium outcomes of the competition.

3.1 Education technology

A ***curriculum*** of a college's education technology is defined by two parameters: curricular standard c, which sets the threshold of prerequisite level of preparation to the course of study in this college, and the progress rate $A(c)$, which determines students' learning gains while in college. Thus, under curriculum $(A(c),c)$, a student's (value-added) human capital h is produced according to

$$h(q) = \begin{cases} 0 & \text{if } q \leq c, \\ A(c)(q-c) & \text{if } q > c, \end{cases} \tag{1}$$

where $q \geq 0$ denotes the student's pre-college ability. Student's ability is the only input in his human capital production. According to (1), a student will benefit from learning under curriculum $(A(c),c)$, if and only if his pre-college ability level exceeds the curricular threshold c.[7]

The curricular threshold c represents the prerequisite knowledge or skills required to study at a college under this curriculum. For example, if a course in intermediate microeconomics has algebra as prerequisite, a student not possessing such background will not benefit from learning in this course for lack of required skills, even if he attends classes. On the other hand, if a part of the course is devoted to studying the necessary math, we interpret this as lowering the curricular threshold. The student in question will then derive benefit from such a course albeit to a lesser extent than a student with superior prior preparation.[8]

The progress rate $A(c)$ represents the rate at which students can advance their knowl-

[7]A student's pre-college ability can be interpreted as his human capital level reached prior to college. This in turn can be modeled as the output of the basic education stage, where inputs may include the student's innate ability, learning effort, family inputs, as well as school inputs such as funding, teacher quality, and class size. More importantly, the production technology at the basic education stage may also be subject to different curricular choices. In this paper, we abstract from intertemporal decisions across different education stages, and treat a student's pre-college ability as exogenous.

[8]Note that the presence of a threshold c implies that there are increasing marginal returns to a student's pre-college ability level: For any $q' > q > c$, we have $(q'-c)/(q-c) > q'/q > 1$. In other words, high-ability students benefit disproportionately more from a challenging curriculum, compared to low-ability students, i.e., they enjoy a "talent premium" as also discussed by Gilpin and Kaganovich (2012) whose model features a similar education production function.

edge in the course of study. As the notation suggests, we posit that the progress rate depends on curricular standard c. It is realistic to assume, specifically, that $A(c)$ is an increasing function, which means that the higher level of presumed prior preparation allows those who possess it to make a more significant progress with this curriculum than with one requiring less from a student. Put differently, this assumption implies a trade-off: high progress rate of education requires one to meet higher curricular standards. This also means that a curriculum that is accessible to a very broad population of students can only yield modest progress rate. A curriculum with high progress rate can benefit only a smaller group of highly prepared students. The trade-off is transparent in our expression (1) for the human capital production function. Specifically, we express it through the following simplifying assumption:

Assumption 1. Curriculum progress rate $A(c) = Ac$, where $A > 0$ is a given constant.

As described further in this section, students choose a college while taking curricula of the colleges as given. In Section 4, we will model the colleges' choices of their curricula as endogenous outcomes of inter-collegiate competition for students.

3.2 Colleges

There are N colleges, denoted $s \in \{1, 2, ..., N\}$. The curriculum of college s is (Ac_s, c_s) according to Assumption 1. For now, we assume that these curricula are fixed, and ordered as follows:

$$c_1 > c_2 > ... > c_N \tag{2}$$

That is, college 1 has the most challenging curriculum with the highest threshold and fastest progress rate, and can be thought of as a highly selective, elite college. The selectivity decreases as one moves from college 1 to college N, with college N being the least selective, i.e., offering the least challenging curriculum. We take the number of colleges as exogenously given and fixed.

Of course, many factors can affect the educational progress rate of students at any given college. For example, more experienced teachers can better motivate students and allow them to learn faster than other teachers. Similarly, a small class size may allow the instructor to provide more individual feedback to students. Colleges may differ in these aspects of education quality, and therefore require different levels of funding to provide them. An improvement in such quality characteristics will benefit all students studying under the same curriculum. We, however, do not explicitly incorporate the financial aspect of education quality, including tuition and other sources of college funding in the model, noting only that these variables tend to correlate with curricular standards of colleges. We assume that all differences between colleges are captured by the differences in the parameters of their curricula.

3.3 Students

There is a continuum of students of measure 1. Pre-college aptitude or ability level of student ω is denoted by $q(\omega)$. Students are heterogeneous in their pre-college ability levels; specifically, we assume that student ability q follows a triangular distribution on $[0,Q]$ with density $f(q) = 2/Q - 2q/Q^2$. Note that the triangular distribution implies that there are few high-ability students, more middle-ability students, and even more low-ability college-bound students, i.e., there is a quality-quantity tradeoff between the student ability and the number of students.[9] Students know their own ability and observe the curriculum offered at each college.

A student faces only one choice: which school to attend. We assume that there is no capacity constraint in any of the colleges, and that attending a college is free.[10]

A student's objective is to maximize his (value-added) human capital from college education. We denote student ω's enrollment decision by $s^*(\omega) \in \{1,2,...N,N+1\}$, where $s^*(\omega) = N+1$ means that the student does not attend college. The following result proven in the Appendix characterizes this choice:

Lemma 1. *Given the curricula (Ac_s, c_s) offered by college $s \in \{1,2,...N\}$, student ω's*

[9] As a convenient shortcut, we exclude the rising portion of the density function of a standard triangular distribution which would correspond to the population with the ability below that we are explicitly considering. For example, one could consider a standard symmetric triangular density where the function $f(q) = 1/Q - q/Q^2$ represents just the declining portion of the triangle corresponding to the above-median ability population. This would correspond to the above setting under the realistic assumption, implicit in the above, that college education can serve (be meaningful for) only the individuals with higher than the median ability.

[10] Introducing (potentially different) tuition payments at the colleges will not change our main results qualitatively and will only affect the identities of the marginal students who are indifferent between two neighboring colleges (i.e., their net benefits are the same from attending two different colleges). If tuition levels are given and fixed, one can show that the model will yield qualitatively similar results obtained here for the free tuition model.

optimal enrollment decision is given by:

$$s^*(\omega) = \begin{cases} 1 & \text{if} \quad q(\omega) \geq a_2 = c_1 + c_2, \\ s & \text{if} \quad q(\omega) \in [a_{s+1}, a_s] \quad \text{for} \quad 1 < s < N, \quad \text{where} \quad a_s = c_{s-1} + c_s \\ N & \text{if} \quad q(\omega) \in [a_{N+1}, a_N], \quad \text{where} \quad a_{N+1} = c_N \\ N+1 & \text{if} \quad q(\omega) \leq a_{N+1}. \end{cases}$$

So $a_s = c_{s-1} + c_s$ for $1 < s \leq N$ is the cutoff ability level of a student indifferent between attending college $s-1$ or s (where we define $a_1 = Q$), while $a_{N+1} = c_N$ is the cutoff ability of a student indifferent between attending the least selective college or none at all.

Thus, students are stratified across colleges by their ability levels,[11] with the highest ability students attending the most selective college 1, the next highest ability segment of students going to college 2 and so forth, while the lowest ability segment of the population does not pursue higher education. The cutoffs between these subgroups are determined by the curricular standards offered by two neighboring colleges.

We conclude this section with offering a flavor of the results to come. Note that, because there is a finite number of colleges, curricula cannot be individually designed to best serve each student's needs. Instead, each college enrolls students of different pre-college ability levels pooled together to be educated using the same curriculum. For all but a measure zero of students, this will not be an ideal learning technology. For student ω, his most preferred curriculum, the one that maximizes his human capital output, is $c(\omega) = \frac{q(\omega)}{2}$. Thus, high-ability students prefer curricula with higher thresholds (curricular standards) and, accordingly, faster progress rates, while low-ability students prefer curricula with lower thresholds and slower progress rates.

Now consider a student whose pre-college ability is such that

$$c_{s+1} < c(\omega) < c_s.$$

Relative to this student's individually optimal curriculum, college $s+1$ is too easy and college s is too demanding. Of course, if either c_{s+1} or c_s is not far from $c(\omega)$, stu-

[11] Here we assume that each student is perfectly informed about his ability. When such information is imperfect, derived for instance from imprecise signals such as standardized test scores, it can over- or under-estimate one's true ability and lead students to make ex post suboptimal college choices— too "hard" or too "easy" for a particular student, and result in some deviation from perfect sorting of students across colleges in terms of true ability. However, if the informational frictions are unbiased, this will not affect the validity of our analysis qualitatively.

dent ω will be able to study in an "almost ideal" learning environment, and the fact that no college offers exactly ω's ideal curriculum will not affect this student's learning outcomes much. If, however, c_{s+1} is located substantially below $c(\omega)$, and c_s is substantially above $c(\omega)$, then student ω will find himself "stuck in the middle", i.e., placed in a suboptimal learning environment regardless of which college he chooses.

4 Equilibrium Curricula

In the previous section, the curricula of colleges were taken as given and used to derive students' enrollment choices and human capital outcomes. In this section, we focus on the choices of curricula by the colleges. We model the colleges' curricular choices as a Nash equilibrium outcome of a game played among the schools. To this end, we first need to introduce objective functions of the colleges.

4.1 Objectives of the colleges

According to Lemma 1, given $c_1 > c_2 > ... > c_N$, students with ability $q(\omega) \in [a_{s+1}, a_s]$ will enroll in college s, where $a_s = c_{s-1} + c_s$ for $1 < s \leq N$, $a_1 = Q$, and $a_{N+1} = c_N$. Therefore the aggregate human capital of college s' student body is given by

$$H_s = \int_{a_{s+1}}^{a_s} Ac_s(q - c_s) f(q)\, dq, \tag{3}$$

and the number of students enrolled at college s is

$$N_s = \int_{a_{s+1}}^{a_s} f(q)\, dq.$$

We interpret the previous expression as measuring the aggregate quality (i.e., the aggregate human capital value added by the college's students), while the last expression as measuring the quantity of students body at college s. We posit that college s chooses its curriculum c_s to maximize a linear combination of the quality and quantity measures as follows:

$$O_s = H_s + \gamma_s N_s, \tag{4}$$

where γ_s are school-specific weights assigned to quantity relative to quality.

Assumption 2. Colleges are ranked in the order of declining selectivity, such that

$$\gamma_1 < \gamma_2 < ... < \gamma_N \tag{5}$$

When choosing its curricular standard c_s to maximize (4), college s takes all other colleges' curricula as given, in particular the curricula of its neighboring colleges c_{s-1} and c_{s+1}.

We argue that the objective to maximize (4) is a meaningful proxy for the goals of both the more and the less selective colleges. Although we do not explicitly model the financial side of a college operation, for small s, i.e., relatively highly selective colleges, the argument is based on the fact that the aggregate human capital of the cohort of college s' graduates H_s correlates with their aggregate lifetime income, which, for a typical selective American college, serves as a basis for expected future alumni contributions. If the college were able to charge a full tuition payment from each student, commensurate with his expected lifetime return, the aggregate tuition revenue would be an increasing function of H_s. If the college is unable to charge thus differentiated tuition, or charges none at all as assumed in this model, the college will be arguably motivated by the future contributions of its alumni, which would tend to be proportionate to their human capital value added while in college.[12]

We note that the value of γ_s is likely to be small and in fact even negative for the most selective colleges. Indeed, a negative γ_s is meaningful because the aggregate human capital H_s defined in (3) already combines the quantity of students with their quality, so $\gamma_s = 0$ in formula (4) will merely signify a parity between quality and quantity in the college's objectives, while a negative γ_s shifts the priorities in favor of quality. It will soon become clear that in equilibrium, colleges with larger γ_s will choose lower curricular standard c_s, therefore college ranking stated in Assumption 2 leads to inequalities (2) in equilibrium.

For a large s and hence a less selective college, besides the aggregate human capital value added of its student body, the college is directly concerned about the size of enrollment, hence its likely large positive value of γ_s. This assumption reflects, without being explicitly modeled, the realities of a combination of direct pressures and financial incentives from state legislatures as well as the greater budgetary reliance of less selec-

[12]This understanding, already mentioned in the Introduction, is well aligned with Hoxby's (2012) analysis of the business model of elite private colleges and arguably to some extent applies to more selective public colleges as well.

tive, public as well as private, colleges on tuition revenues. The less selective college's main tool to pursue enrollment expansion is lowering the curricular threshold c_s. This gives a larger fraction of the population access to this college, but it comes at a cost of also lowering the curriculum's progress rate Ac_s, i.e., at the expense of the college's education quality goal.

4.2 Nash equilibrium

We now examine equilibrium curricular choices by the colleges, given their objectives described above. Each college s chooses a feasible curricular standard c_s, and accordingly, the progress rate Ac_s, to maximize (4), taking curricula chosen by other colleges as given. Thus, curricular choices at equilibrium in this game played by N colleges constitute a (pure strategy) Nash equilibrium.

For the least selective college N, the differentiation of (4) yields first order condition as

$$-\frac{c_{N-1}}{3Q^2}\left(Ac_{N-1}(6c_N+2c_{N-1}-3Q)+6\gamma_N\right)=0,$$

so the solution for c_N is given by

$$c_N = Q/2 - c_{N-1}/3 - \frac{\gamma_N}{AC_{N-1}}. \tag{6}$$

One can directly verify that the second order sufficient condition is always satisfied since $-\frac{2Ac_{N-1}^2}{Q^2}<0$.

For college s with $1<s<N$, the first-order condition is

$$-\frac{c_{s-1}-c_{s+1}}{3Q^2}\left(A(6c_s(c_{s-1}+c_{s+1})+2(c_{s-1}^2+c_{s+1}^2+c_{s-1}c_{s+1})-3Q(c_{s-1}+c_{s+1}))+6\gamma_s\right) = 0.$$

The solution to this equation, given $c_{s-1}>c_{s+1}$, is thus

$$c_s = Q/2 - \frac{c_{s-1}^2+c_{s+1}^2+c_{s-1}c_{s+1}}{3(c_{s-1}+c_{s+1})} - \frac{\gamma_s}{A(c_{s-1}+c_{s+1})}. \tag{7}$$

The second order sufficient condition is satisfied, $-\frac{2A(c_{s-1}^2-c_{s+1}^2)}{Q^2}<0$ because $c_{s-1}>c_{s+1}$.

Lastly, for the most selective college 1, the first order condition is

$$\frac{Q-c_1-c_2}{3Q^2}\left(A(-4c_1^2+4c_1c_2+2c_2^2+5Qc_1-Qc_2-Q^2)+6\gamma_1\right)=0.$$

Since $c_1+c_2<Q$ is always true, the solution to this equation is

$$c_1=\frac{5Q+4c_2-\sqrt{48c_2^2+24Qc_2+9Q^2+96\gamma_1/A}}{8}. \tag{8}$$

The second order sufficient condition for college 1 is given by

$$-\frac{2}{Q^2}\left(A(2c_1^2-c_2^2-3Qc_1+Q^2)-\gamma_1\right)<0,$$

which is satisfied if γ_1 is negative and sufficiently large by absolute value.

The Nash equilibrium is given by the solution of the N first order conditions as defined in (6), (7), and (8). The existence, uniqueness, and interiority of such a solution hinges on parametric conditions on these equations, which are not explicitly tractable in the general case of an arbitrary number of colleges N.[13] In the next sub-section, we will start by establishing the existence of interior equilibrium for the case $N=2$. This will motivate the extension of our analysis to the general multi-college case in sub-section 4.4.

4.3 The case of two colleges

When $N=2$, the curricular response c_1 of the selective college 1, given the curricular standard c_2 of the less selective college 2, is the solution of the equation

$$4c_1^2-4c_1c_2-2c_2^2-5Qc_1+Qc_2+Q^2-6\gamma_1/A=0. \tag{9}$$

The curricular response c_2 of college 2, given c_1, is

$$c_2=Q/2-c_1/3-\gamma_2/(Ac_1). \tag{10}$$

[13]The above first order conditions presume that solutions are, in fact, interior. As will be seen in the next sub-section, under some parameter values corner solutions can occur where the most selective college will set $c_1=Q$, i.e., will admit measure zero of students, and/or the least selective college will choose $c_N=0$, making it accessible to everyone.

A Nash equilibrium is then a pair of curricular thresholds (c_1^*, c_2^*) which are mutual best responses—that is, $c_1^* = c_1(c_2^*)$ and $c_2^* = c_2(c_1^*)$. The equilibrium is *locally stable*, if a small perturbation of the equilibrium results in best response dynamics converging back to it (Moulin, 1986). The following result provides a sufficient condition for the existence, uniqueness, and stability of an interior Nash equilibrium in the curricular choices in the game of two colleges.

Proposition 1. (Existence and Stability) *There are bounds $\underline{\gamma_1} < \overline{\gamma_1} < 0$ and $0 < \underline{\gamma_2} < \overline{\gamma_2}$ such that when $\gamma_1 \in (\underline{\gamma_1}, \overline{\gamma_1})$ and $\gamma_2 \in (\underline{\gamma_2}, \overline{\gamma_2})$, a unique locally stable Nash equilibrium exists with $c_1^* > c_2^* > 0$ and $c_1^* + c_2^* < Q$.*

Proposition 1 shows that when the selective college (college 1) is moderately biased toward quality, and likewise, when the less selective college (college 2) is moderately biased toward quantity, they will optimally choose to differentiate their curricular offerings to students. More specifically, since the selective college cares more about quality than quantity, it chooses a more challenging curriculum to attract the high-ability students. On the other hand, the less selective college cares more about quantity than quality, so it chooses a less challenging curriculum to attract a large number of less able students. The bounds on the parameter values ensure that both colleges' curricular choices are interior solutions.[14]

Remark. The proof of Proposition 1, placed in the Appendix, additionally provides a localization for the colleges' strategies. Specifically, it pins down the locations of c_1^* and c_2^* within the intervals $(Q/4, Q/2)$ and $(0, Q/6)$ respectively. Recall our assumption (accompanied by a discussion in footnote 9) that ability distribution of potential college bound young agents is triangular, and that the distribution of ability q on $[0, Q]$ with density function $f(q) = 2/Q - 2q/Q^2$ that we analyze represents the upper half of the distribution of ability in the population of youths at large, i.e., just the declining portion of the overall triangular distribution. According to this understanding, the ability variable is recentered such that $q = 0$ corresponds to the median ability in the population of all youths. Therefore, the above localization implies, according to Lemma 1, that this model puts the overall share of young population attending a college between one third and one half, which corresponds well with the facts, especially given that our

[14]Specifically, condition $\gamma_1 > \underline{\gamma_1}$ ensures that college 1 will want to admit a positive measure of students. Likewise, condition $\gamma_2 < \overline{\gamma_2}$ prevents the situation where college 2 gives extreme priority to the quantity of students over their quality and as a result will completely eliminate curricular standard, i.e., choose the corner solution $c_2 = 0$.

model does not factor in financial constraints faced by students. Further, these estimates also imply that the share of students attending the more selective of the two colleges is between 34% and 55% of those attending a college, which is consistent with Hoxby's (2009) estimates.

We now proceed to the comparative statics analysis of the equilibrium responses of both colleges to a positive shock to the less selective college's priority to admit more students. We obtain the following

Proposition 2. (Diverging Selectivity) *At the Nash equilibrium,* $\frac{\partial c_2^*}{\partial \gamma_2} < 0$*,* $\frac{\partial c_1^*}{\partial \gamma_2} > 0$*, and* $\frac{\partial (c_1^* + c_2^*)}{\partial \gamma_2} < 0$*. In other words, while college 2 will lower its curricular threshold, college 1 will do the opposite; the absolute enrollments will expand in both colleges.*

Proposition 2 shows that when the less selective college 2 experiences a shock compelling it to increase the weight it places on the quantity of its students (an increase in γ_2), it lowers its curricular threshold to pursue enrollment expansion. Such exogenous change in college 2 priorities may stem from budgetary or political pressures. Recall that under the triangular distribution, the population is denser at lower ability levels. Thus lowering the curricular threshold makes college education attractive for an additional densely populated segment of less prepared students for whom this was not the case before. However, such a lower curriculum threshold comes at the expense of human capital attainment of the top segment of the students originally bound for college 2. For these medium-ability students, the downward adjustment of curriculum makes college 2 less appealing an option, so some may shift their college choice toward college 1. In other words, the competitive pressure faced by the more selective college 1 will somewhat weaken, and as a strategic response it will be able to afford giving less attention to the human capital gains of its lower-end students. Thus college 1 will find it optimal to raise its curricular standard to the benefit of its better prepared students, because the human capital loss of its lower-end students is more than offset by the increase in human capital of its high-ability students, due to the increased marginal returns to ability (see Footnote 8). In sum, the pursuit of enrollment expansion therefore causes the less selective college 2 to make its curriculum less demanding, eliciting the optimal response from the selective college to further elevate its curriculum.

Figure 2, which depicts the appropriately sloped (as can be seen in the proof of Proposition 1 in the Appendix) best response curves of the selective college $c_s(c_2)$ and its less selective counterpart $c_2(c_1)$, also illustrates the comparative statics adjustments discussed above. As γ_2 increases, college 2 best response curve shifts downward (the

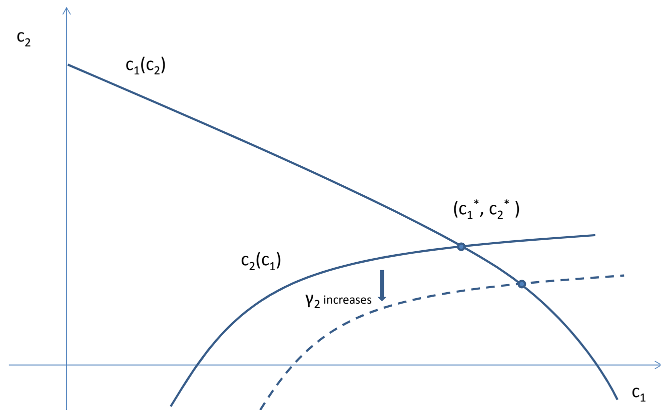


Figure 2: Best responses and Nash equilibrium

dotted line). College 1's best response curve, on the other hand, is independent of γ_2 and stays fixed. Thus, as γ_2 increases, the equilibrium slides down along college 1's best response curve. This implies that c_1^* increases and c_2^* decreases, as stated in Proposition 2. Furthermore, the stability of the Nash equilibrium implies that the best response curve for college 1 is steeper than the best response curve for college 2, so an increase in γ_2 leads to a smaller increase in c_1^* and a larger decrease in c_2^*, with the overall effect on the cut-off $a_2 = c_1^* + c_2^*$ to be a net decrease. Thus, student enrollment in both the less selective *and* the more selective college increases when γ_2 increases. This happens despite the fact that the selective college becomes even more challenging (c_1^* increases). The reason is that students of relatively high ability who previously would have enrolled in the less selective college will switch to the selective college when the former's curriculum becomes less demanding, because the relative learning gain of attending college 1 now becomes more significant.

We shall now examine how increased access to higher education affects the welfare and human capital outcomes of students. Consider an increase in γ_2, and let $(c_1^{\text{old}}, c_2^{\text{old}})$ and $(c_1^{\text{new}}, c_2^{\text{new}})$ denote the equilibrium curricula before and after the change in γ_2. According to Proposition 2,

$$c_2^{\text{new}} < c_2^{\text{old}} < c_1^{\text{old}} < c_1^{\text{new}}.$$

That is, the "wedge" between the less selective and the more selective colleges' curricular thresholds widens. The following result describes how students' human capital is affected by this.

Corollary 1. (Distributional Effects) *At the Nash equilibrium, a positive shock to γ_2 will have the following effects on students' human capital:*

(a) Students with ability levels $q \in \left(c_2^{new}, c_2^{old} + c_2^{new}\right)$ and $q \in \left(c_1^{old} + c_1^{new}, Q\right]$ accumulate more human capital.

(b) Students with ability levels $q \in \left(c_2^{old} + c_2^{new}, c_1^{old} + c_1^{new}\right)$ accumulate less human capital.

(c) Students with ability levels $q \in \left[0, c_2^{new}\right]$ do not attend college before or after the change in γ_2. They accumulate the same amounts of human capital, before and after the change.

Thus, the changes in equilibrium college curricula affect students differently, depending on their initial ability. Corollary 1 characterizes the distributional impacts of curricular adjustments. If γ_2 increases, medium ability students lose out, while high ability and low ability college enrollees are made better off. As the gap between the curricular standards of the less selective and the more selective colleges widens, the curriculum of the more selective college moves closer to the ideal curriculum of the most able students. Similarly, the less selective college's curriculum moves closer to the ideal curriculum of the least able students. Both curricula, however, move away from the ideal curriculum of medium ability students.

Overall, the endogenous strategic curricular responses of both colleges resulting from an exogenous shock to the less selective college's priority to expand admission has a non-monotone impact on students with respect to their ability. It is important to emphasize that the presence of strategic interaction between colleges plays an essential role in this non-monotonicity phenomenon. If the selective college did not have an incentive to adjust its curriculum in response to the less selective college's move, enrollment expansion at the latter would have increased the human capital of low ability students and negatively affected medium ability students (some of whom would have switched to the selective college as a result), without affecting the high-ability students who are already enrolled in the selective college. It is the selective college's strategic "beefing up" of its curriculum—in response to the less selective college's "watering down" of its curriculum—that enhances the gains of the most able students. But now, the selective college's curriculum adjustment adds to the losses suffered by the medium ability students by making the education at the selective college also a less adequate match for them.

Remark. In light of Corollary 1, parametric localizations provided in the Remark following Proposition 1 now allow us to estimate the range of student abilities where human capital losses would occur resulting from the diverging selectivity established in Proposition 2. Specifically, some losses will be experienced by students with abilities in the interval between $2c_2^*$ and $2c_1^*$, with the most pronounced "sagging middle" centering around $c_2^* + c_1^*$. Thus, according to our estimates discussed in the previous Remark, this "sagging middle" tends to occur between the 4-th and 6-th decile of the ability distribution of college attendees. This is largely consistent with our estimates of the earnings growth profiles among US college-educated workers over the period of college expansion: see Figure 1, as well as some more refined analysis of the data in Section 5.

4.4 The general case: multiple colleges

In the previous subsection, we considered the special two-college case. We first proved that there are meaningful parametric conditions under which locally stable interior Nash equilibrium of curricular choices exists (Proposition 1), and then went on to obtain the main comparative statics result demonstrating the diverging selectivity of the colleges when the overall college access expands (Proposition 2). We first note that the task of closed form characterization of the parametric conditions under which a *locally stable interior Nash equilibrium* of curricular choices exists, as provided in Proposition 1, is generally not tractable in the general case of multiple colleges. We shall therefore proceed with our theoretical analysis under the assumption that a locally stable interior Nash equilibrium exists under given parameter values for a given $N > 2$. We shall then provide numerical examples, for the cases $N = 3$ and $N = 4$, featuring locally stable interior Nash equilibria, along with their comparative statics analysis, under meaningful parameter values.

We also make an additional simplifying assumption that in a Nash equilibrium, for $j \in \{1, 2, ..., N-1\}$, if one or several of the least selective colleges $s = N - j + 1, ..., N$ were to fix their curricular standards, i.e., stopped responding to external changes, then the Nash equilibrium for the subgame among the rest of the colleges, $s = 1, ..., N - j$, remained locally stable. We assume the same for the subgame among colleges $s = j + 1, ..., N$ when the j most selective colleges, $s = 1, ..., j$, fix their curricular positions.

Thus, we take the above existence and local stability conditions (i.e., standard sufficient conditions for comparative statics analysis) in the multi-college case for granted, having established them under appropriate parametric conditions for the two-college case, and focus, in the remainder of this section, on carrying out the comparative statics analysis. Specifically, we focus on extending the diverging selectivity result of Proposition 2 to the multi-college case, based on the above assumptions.

Recall that if Nash equilibrium is interior, it satisfies the system of first-order conditions given by (6), (7), and (8). We begin by making two observations: (1) parameter γ_s only shows up in the first order condition for college s; (2) the best response of college s, namely c_s, depends only on the curricular choices of its immediate neighboring colleges, namely c_{s-1} and c_{s+1}. Therefore, when there is a shock to γ_m, the priority college m gives to the quantity of its students, one can separate the analysis of its impact on the Nash equilibrium into the direct impact of this exogenous change on the curriculum of college m itself, c_m, the sequential indirect impact on the lower-ranked colleges $c_{m+1}, c_{m+2}, ..., c_N$, and the sequential indirect impact on the higher-ranked colleges $c_{m-1}, c_{m-2}, ..., c_1$.

According to Assumption 2 colleges are ranked according to their selectivity, with the more selective colleges characterized by smaller weight γ they put on the quantity of students. As we explained in the discussion following Assumption 2, a bias in college's priorities in favor of quality is appropriately characterized by a negative value of γ_s. One can therefore consider college s to be "selective" if $\gamma_s < 0$, and "less selective" otherwise. We will assume that our list of N colleges contains both types. We also note that since triangular distribution of student ability is denser at lower ability levels, strategic interaction of colleges in equilibrium implies that a college with larger γ (giving more weight to the quantity of students) will choose lower curricular standard c than its counterpart with the next smaller value of γ. This implies that college selectivity ranking according to Assumption 2 yields, in a Nash equilibrium, a corresponding their ranking in terms of curricular standards $c_1 > c_2 > ... > c_N$.

The following Proposition establishes the comparative statics result for the general multi-college case, a counterpart to Proposition 2 for the two-college case.

Proposition 3. (Diverging Selectivity) *Assume that N colleges with $\gamma_1 < ... < 0 < ... < \gamma_N$ are in a Nash equilibrium and thereby $c_1^* > c_2^* > ... > c_N^*$. There exist two cutoffs K and M, where $1 \leq K \leq N-1$ and $M \geq K+1$, such that for any m satisfying $m \geq M$, when college m experiences a positive shock to γ_m, then*

(a) $\frac{\partial c_s^*}{\partial \gamma_m} < 0$ for all $s \geq K+1$, i.e., college $K+1$ and all colleges ranked below it

become less selective by reducing their curricular thresholds;

(b) $\frac{\partial c_K^*}{\partial \gamma_m} > 0$*, i.e., college K becomes more selective by means of raising its curricular threshold, i.e., the group of colleges ranked at K or above becomes collectively more selective. Within this group, a pairwise clustering pattern emerges: colleges K,* $K-2$*, etc. raise their selectivity, while colleges* $K-1$*,* $K-3$*, etc. lower theirs.*

Proposition 3 divides the set of colleges into two subgroups, the less selective colleges and the more selective ones. It shows that when one of the less selective colleges (specifically, colleges ranked at M or below) experiences an exogenous shock compelling it to increase the priority it gives to the size of its enrollment, the entire group of less selective colleges (namely all colleges ranked at $K+1$ or below) becomes more accessible to students, i.e., all of these colleges lower their curricular standards. On the other hand, the group of more selective colleges, namely those ranked K and above, *collectively* becomes less accessible. Interestingly, the impact is not monotonic within the group of more selective colleges: As college K raises its curricular standard c_K^*, college $K-1$ lowers its curriculum c_{K-1}^* (albeit still remaining above c_K^*), college $K-2$ raises c_{K-2}^*, and college $K-3$ lowers c_{K-3}^*, and so forth. In other words, there is a pairwise clustering pattern of changing selectivity within the group of more selective colleges. Nonetheless, despite this pairwise clustering effect, the entire group becomes more selective overall, as evidenced by the increased gap between the curricular choices of college $K+1$ and college K. It is also worth noting that the above result remains true if several of the less selective colleges, i.e., not just college m, experience simultaneous positive shocks to the weights they assign to the quantity of their students.

Recall that Corollary to Proposition 2 in the previous sub-section derived the taxonomy of the human capital gains across student population resulting from the phenomenon of the "diverging selectivity" of colleges, i.e., the comparative statics effects established in the Proposition for the case of two colleges. Namely, it demonstrated the U-shaped effect on human capital value added with relative gains at its ends and relative losses in the middle. Since Proposition 3 establishes the phenomenon for the multi-college case whereby the two groups of colleges (the more and the less selective) diverge, a similar analysis applies.

Indeed, since the curricular standard of college $K+1$ and all the colleges below it will be adjusted downward, the overall quality of the student-college matches in terms of educational gains will improve for the lower ability fraction of the student population and worsen for those in the medium ability range. In particular, the relatively high ability students among those attending college $K+1$ will suffer. Likewise, since as we have shown, the curricular standard of college K will be adjusted upward, the quality of the student-college match will suffer for the relatively lower ability fraction of its student body, unlike their higher ability peers who will benefit. Together, two segments of student population in the medium ability range, namely those with relatively high ability among the attendees of the less selective colleges and the relatively low ability ones among students attending the more selective colleges, form the "sagging middle" in the distribution of the effects of diverging selectivity on the educational gains across the population, as compared to the very low and the very high ability students who benefit from the improved student-college matches.[15]

The following numerical examples illustrate the diverging selectivity results of Proposition 3 along with their implications discussed above. Nash equilibrium we obtain in each of the examples satisfy all the provisions of Proposition 3. Below, we use the same simplifying notation helping to streamline the set of relevant parameters that we apply in the proofs of Propositions 1-3 in the Appendix: $r_s = \gamma_s/(AQ^2)$ and $x_s = c_s/Q$ for $s = 1,2,...,N$.

Numerical Example 1: The number of colleges $N = 3$.

Case A: $r_1 = -0.1$, $r_2 = 0.02$, and $r_3 = 0.08$.

Equilibrium curricula: $x_1 = 0.3475$, $x_2 = 0.3294$, and $x_3 = 0.1471$.

Comparative statics (positive shock to r_3): x_3^* decreases, while x_2^* increases, and x_1^* decreases. Here $K = 2$ and $M = 3$, so colleges 1 and 2 form the selective group while college 3 is less selective. Furthermore, the above implies that the total enrollment in the selective colleges increases, and so does enrollment in the less selective college.

The "sagging middle" centers at $x_2 + x_3 = 0.4765$, that is in the 6-th decile of the ability distribution among the college attendees.

[15] We note that such effects will have intervals of non-monotonicity, where the curricular change at each of the less selective colleges benefits its lower ability fraction of the students while hurting the higher ability fraction of its students. Likewise, while the quality of student-college matches improves overall at the higher end of the ability distribution (since such students benefit from higher standards), such effects will have intervals of non-monotonicity according to our result on the pairwise clustering of selective colleges. Indeed, the curricular change at the colleges ranked at K or above by an even number (K, $K-2$, $K-4$, etc.) benefits its higher-ability fraction of the students while hurting the lower-ability fraction, and the opposite is true for the colleges above K by an odd number ($K-1$, $K-3$, etc.). We conjecture that the aforementioned intervals of non-monotonicity amount to minor departures from the overall U-shaped pattern.

Case B: $r_1 = -0.05$, $r_2 = 0.05$, and $r_3 = 0.08$.
Equilibrium curricula: $x_1 = 0.2898$, $x_2 = 0.2686$, and $x_3 = 0.1126$.
Comparative statics (positive shock to r_2 or r_3 or both): x_3^* and x_2^* decrease, while x_1^* increases (here $K = 1$ and $M = 2$). Furthermore, enrollments at both groups, selective college (1) and the less selective colleges (2 and 3), increase.
The "sagging middle" centers at $x_1 + x_2 = 0.5584$, that is in the 7-th deciles of the ability distribution among the college attendees.

Numerical Example 2**:** The number of colleges $N = 4$.
Case A: $r_1 = -0.1$, $r_2 = 0.01$, $r_3 = 0.03$, and $r_4 = 0.07$.
Equilibrium curricula: $x_1 = 0.3529$, $x_2 = 0.3184$, $x_3 = 0.3126$, and $x_4 = 0.1719$.
comparative statics (positive shock to r_4): x_4^* decreases, while x_3^* increases, x_2^* decreases, and x_1^* increase (here $K = 3$ and $M = 4$). Furthermore, enrollments in both groups, selective colleges (1, 2, and 3) and the less selective one (college 4), increase.
The "sagging middle" centers at $x_3 + x_4 = 0.4845$, that is in the 6-th decile of the ability distribution among the college attendees.
Case B: $r_1 = -0.08$, $r_2 = 0.03$, $r_3 = 0.05$, and $r_4 = 0.09$.
Equilibrium curricula: $x_1 = 0.3264$, $x_2 = 0.3017$, $x_3 = 0.2597$, and $x_4 = 0.0669$.
Comparative statics (positive shock to r_3 or r_4 or both): x_4^* and x_3^* decrease, while x_2^* increases, and x_1^* decreases (here $K = 2$ and $M = 3$). Furthermore, enrollments in both groups, selective colleges (1 and 2) and the less selective ones (3 and 4), increase.
The "sagging middle" centers at $x_2 + x_3 = 0.5614$, that is in the 7-th decile of the ability distribution among the college attendees.

We observe that similar to the estimates discussed in the Remarks in subsection 4.3, the results in the above examples can be seen as numerically meaningful scenarios for the aggregate college attendance statistics as well as the earnings growth profiles among college-educated workers over the period of college expansion.

In the next section we explore the changes in the earnings profile of the population of workers who attended college, over the 1965-2015 period, during which the higher education underwent the transformations studied in this paper, namely the sustained expansion of access accompanied by the pattern of diverging selectivity among the groups of more and less selective colleges. We find evidence that the changes in the distribution of earnings consistently exhibited a U-shaped pattern, with larger gains in the tails and relative stagnation in the middle. While some of the literature we discussed in Section 2 offers demand-side explanations of this phenomenon, it appears to also be consistent with the U-shaped pattern we have found to characterize educational gains across the population of workers who attended college. This allows us to conjecture that the changing distribution of educational gains across segments of student population could be among the supply side factors contributing to the earnings distribution phenomenon.

5 U.S. Earnings Growth Profile, 1965–2015

5.1 Data

We shall now examine the changes in the earnings profile of college-educated workers by using the March Current Population Survey (IPUMS-CPS) data, a household survey conducted jointly by the U.S. Census Bureau and the Bureau of Labor Statistics and covering the period from 1962 to 2015. We use the information on respondents' age, highest educational level attained, and annual wage income for the previous year. The education variable is missing for the year 1963, so we use data from 1964 onward. To avoid over-crowding the graphs, we only report results in ten-year intervals (the general pattern holds for results for the interim years). Table 1 in the Introduction shows that this period featured drastic enrollment expansion in U.S. higher education, both in absolute terms and as a fraction of the college-age population.

With respect to the educational outcome, we categorize as "college-educated" all workers with at least some college education, ranging from workers with "some college" to those with graduate or professional degrees. In contrast, non-college-educated workers are defined as those with at most a high school diploma. With respect to the labor market outcomes, we focus on workers' earnings—namely, annual wage incomes whether fully or partly employed—rather than on weekly or hourly wage rates.[16] Besides the wage rate, a worker's earnings may also depend on factors such as the nature of the job (part-time or full-time), unemployment risks (likelihood and duration), compensation schemes (regular wage versus bonus), etc. These factors are known to correlate with a worker's educational attainment and human capital level, so the earnings variable provides a broader measurement of a worker's labor market outcome. Our sample includes all workers whose annual earnings are not missing or equal zero.

[16] Recent studies, including Autor et al. (2008), focus on hourly wage rates of full-time full-year workers, i.e., a sub-group whose share in the workforce has exhibited considerable change.

We are interested in documenting the differences in the earnings growth at the high and the low end of the earnings distribution of college-educated workers as compared to that in its mid-section. To do so, we first deflate the reported nominal earnings by Consumer Price Index (CPI) to obtain real earnings. Next, taking earnings distribution in a given year $t(0)$ as the base year, for each decile of the distribution ($\tau = 0.1, 0.2, ..., 0.9$), we find the natural logarithm of the real earnings level $q_{\tau}^{t(0)}$. Similarly, for a subsequent year $t > t(0)$, we find the natural logarithm of the real earnings for all deciles of the earnings distribution q_{τ}^{t} in that year. We then calculate the cumulative growth rate in each decile relative to the base year, $g_{\tau}^{t} = q_{\tau}^{t} - q_{\tau}^{t(0)}$.

As can be seen in Figure 1 presented in the Introduction, the growth rates in the top and bottom deciles are markedly higher than those in the middle of the distribution. In other words, there is a "sagging middle" in the earnings growth distribution among the college-educated workers. Below, we offer a more detailed analysis of this empirical phenomenon while addressing various changes in the composition of the population relevant to its interpretation.

5.2 Empirical patterns

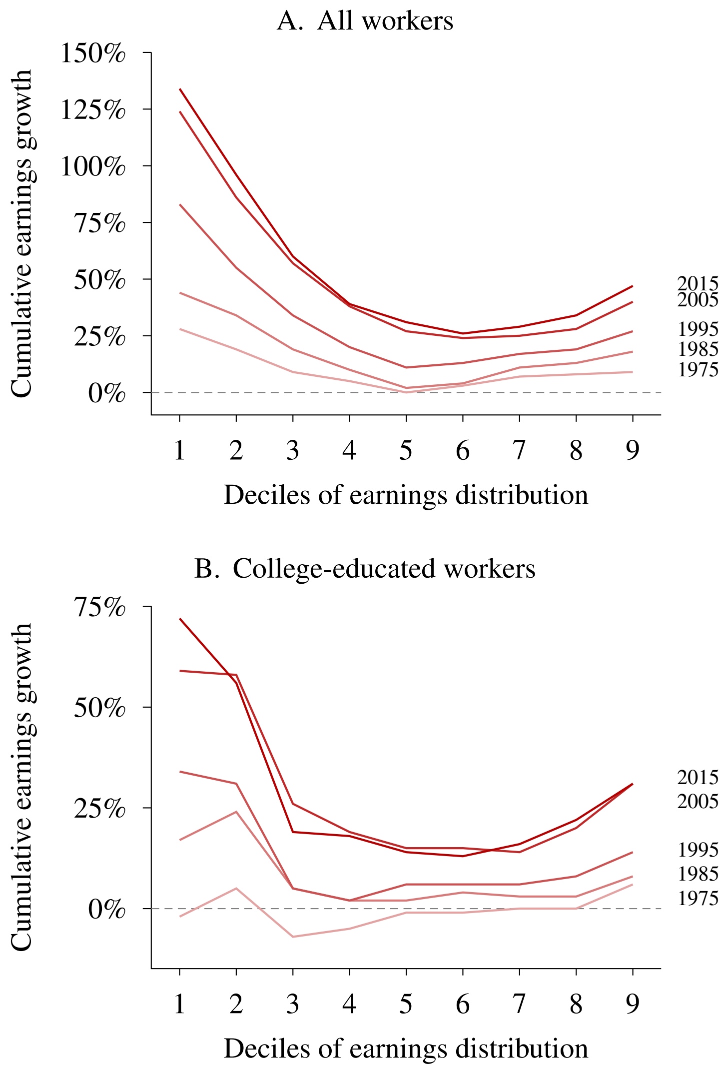
Although our focus is on the relative differences in the earnings growth rates across deciles within the group of college-educated workers, we start with a look at the entire population to see whether the pattern for the wage income dynamics of all workers is similar to that documented in the literature, which tends to focus on wage rates of workers who are employed full time for a full year. Figure 3A shows the earnings growth profile for all workers, regardless of age or educational attainment. Similarly to what has been documented in the literature (e.g., Acemoglu and Autor 2011), Figure 3A exhibits the "polarization" phenomenon, i.e., the fact that earnings at both ends of the distributions grow faster than those in the middle, producing U-shaped graphs of the earnings growth profiles.[17]

Figure 3B, which replicates the graph in Figure 1 in the Introduction, shows the earnings growth profile we obtain applying the same methodology to the group of college-

[17] See Figures 1 and 11 in Autor et al. (2008) and Figure 9 in Acemoglu and Autor (2011). Unlike these authors, who do employ similar percentile-wise growth measures but only look at full-time, full-year workers, we include all workers with positive earnings. This may explain why we obtain a relatively steeper declining segment of the "U-shape" and a somewhat earlier onset of this phenomenon: mid- to late 1970's as opposed to mid- to late 1980's observed in the aforementioned literature which focuses on wage rate dynamics of full-time workers.

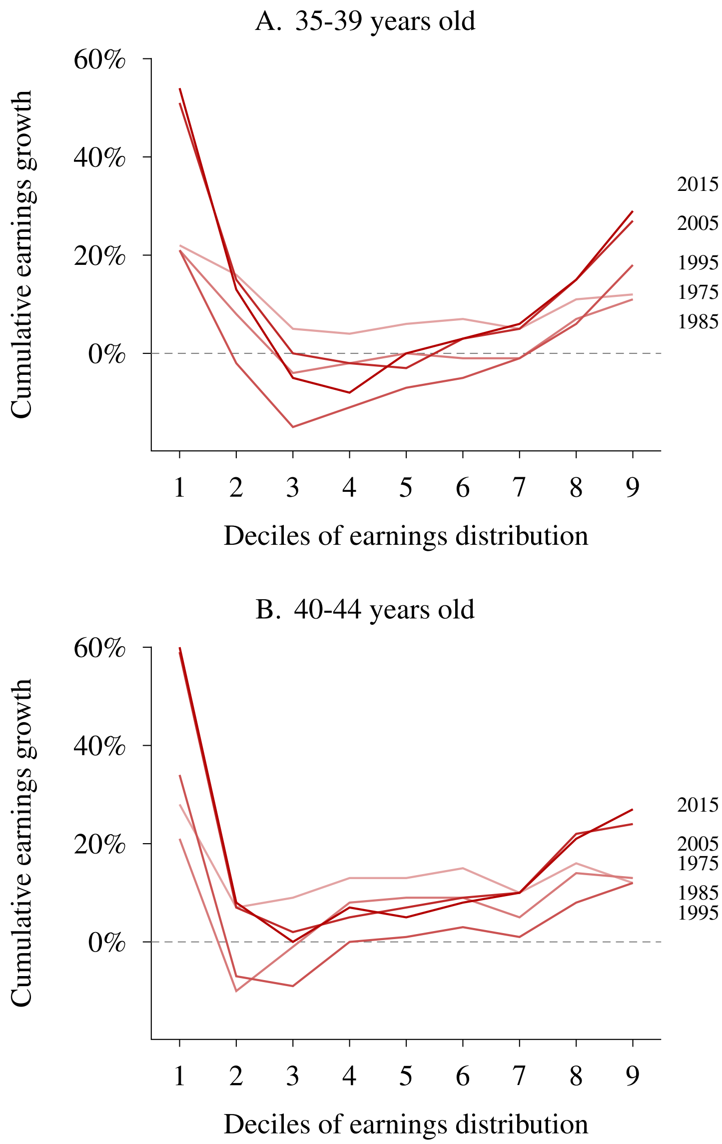
educated workers only.Remarkably, the sagging middle phenomenon observed in Figure 3A for the entire working population persists after excluding workers with at most high school education.

Figure 3: Earnings growth profile, all age groups, 1965 base year



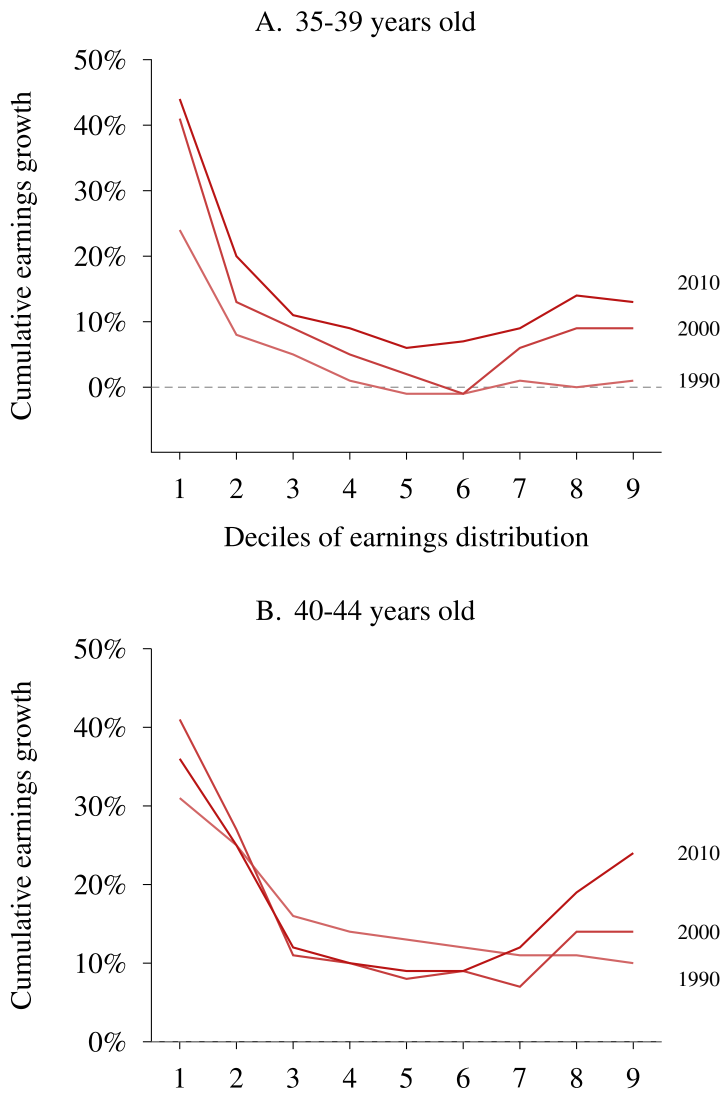
It is well known that the time period under consideration has seen substantial changes in the composition of the labor force, which can cause concern about the impact of these structural changes on the result above. To address the issue of the change in age composition of the labor force, we restrict our sample to specific age-cohorts of the college-educated workers. Figures 4A and 4B represent, respectively, the cohorts of 35 to 39 year olds and 40 to 44 year olds. The U-shaped earnings growth profile persists in both age cohorts, which shows that the sagging middle phenomenon is not an artifact of major changes in the age composition of the workforce over the time period.

Figure 4: Earnings growth for college-educated workers, by age, 1965 base year



Furthermore, we use 1980 instead of 1965 as the base year to examine the robustness of the sagging middle phenomenon in the later part of the overall period, so as to reduce a possible effect of structural changes during this period. Among such structural changes, one of particular note is the enrollment expansion itself. More specifically, for a given year in the later part of the period, say 2015, the enrollment expansion is obviously less pronounced relative to 1980 than relative to 1965, so the change in the underlying ability distribution of college students would be smaller over the period from 1980 to 2015. Figure 5 reproduces the earnings growth profiles for the two aforementioned age cohorts of college-educated workers relative to the 1980 base year. Again, the sagging middle phenomenon persists.

Figure 5: Earnings growth for college-educated workers, by age, 1980 base year



In the next sub-section, we explicitly address the issues associated with a rise in the share of population with some college education, a significant compositional change that occurred during the time period under consideration.

5.3 Changing ability composition

Over the time period under consideration, the share of workers with at least some college education has expanded significantly. As a result, the underlying distribution of abilities (an unobserved variable) has also changed over time, which could have contributed to the distribution of wage growth. To address this issue, we apply the method of Recentered Influence Functions (RIF) which accounts for changes in the underlying distribution of the unobserved variable.

Note that one way to interpret the earnings growth curves depicted in the graphs above is that they represent the quantile treatment effects (QTEs) due to enrollment expansion along with the curricular changes it effected. In this sense, every point on a given curve of earnings growth distribution is an estimate of a simple non-parametric quantile regression on the real earnings for college-educated workers in the base year $t(0)$ and a subsequent year t. A well-known limitation of the quantile regression is the assumption that the distribution of the unobserved variable (in this case, student ability) remains unchanged in both years, so that a n-th decile individual in year t has the same (unobserved) ability as the n-th decile individual in year $t(0)$. However, in our model, enrollment expansion and the changes in curricular standards of colleges associated with it have obviously affected the ability distribution of college-bound population, so that common distribution assumption cannot be expected to hold in general. When this is the case, the observed earnings difference can be attributed to either ability difference and/or curricular changes, making the U-shaped pattern harder to interpret.

To address this concern, we compare the quantile regression results (as seen in the Figures above) to those obtained by applying the Recentered Influence Function (RIF) method. The RIF method is a recently developed approach (Firpo, Fortin, and Lemieux 2009) explicitly relaxing the common distribution assumption, which is required for the quantile regression.[18] More specifically, when the observed outcomes (in this case, earnings) vary monotonically with the unobserved variable (in this case, ability), instead of comparing two individuals belonging to the same quantile (as in the quantile regression case), RIF compares, instead, two individuals with the same earnings. For example,

[18] Given its flexibility, the RIF method has recently been applied to analyze a range of issues such as cigarette taxes (Maclean et al 2014) and child care (Havnes and Mogstad 2015).

given a real earnings level q_τ which corresponds to the quantile $\tau_{t(0)}$ in the base year, RIF determines the corresponding quantile τ_t of this real earnings level in the subsequent year t. The impact of enrollment expansion and the associated curricular changes, measured as the difference in the population shares, is then given by $-(\tau_t - \tau_{t(0)})$. This difference is then divided by a kernel estimate of the joint density of earnings at the level q_τ to arrive at the associated QTE. In other words, instead of using the quantile outcome q_τ directly, the RIF method adds the Influence Function as an additional term, so we have

$$RIF(y;q_\tau) = q_\tau + \frac{\tau - I[Y \leq q_\tau]}{f_Y(q_\tau)}, \tag{11}$$

where $f_Y(q_\tau)$ is the probability density function of outcome Y evaluated at q_τ, and $I[Y \leq q_\tau]$ is an indicator variable that takes a value of 1 when the outcome value, Y, is less than q_τ, and 0 otherwise.

Again, using 1980 as the base year, focusing on the 35-39 years old college-educated workers, we compare the RIF estimates with the quantile regression results across deciles of the earnings distribution.[19] Table 3 reports these estimates, where the upper panel containing results from quantile regressions, and the lower panel containing results from RIF regressions. Robust standard error is reported in parenthesis, and the stars indicate significance at the 10% (*), 5% (**), and 1% (***) level respectively. As can be seen, after accounting for possible changes in the underlying ability distribution over the years, the application of the RIF method imparts only quantitative but not qualitative changes on the overall pattern of the earnings dynamics. In particular, the "sagging middle" phenomenon persists.

6 Concluding Remarks

This paper introduces the concept of an educational institution's curriculum as a characteristic of its human capital production function, which posits that the value added attained by an individual student depends on the relationship between his aptitude (prior preparation) and the curricular standard set by the institution. Our model also suggests that colleges' curricula are set endogenously and moreover strategically, as an instrument in their competition over the relevant populations of students. This paper makes a first step toward understanding colleges' strategies to determine their curricular choices

[19] Results from other years and age cohorts are qualitatively similar and not reported here.

Table 3: Earnings growth for college-educated workers, 35-39 years old, 1980 base year

Year	Deciles								
	1	2	3	4	5	6	7	8	9
Quantile regression									
1990	0.235**	0.081*	0.050*	0.010	-0.005	-0.010	0.006	0.000	0.010
	(0.092)	(0.048)	(0.027)	(0.024)	(0.022)	(0.020)	(0.017)	(0.024)	(0.026)
2000	0.412***	0.128***	0.089***	0.050**	0.022	-0.008	0.065***	0.087***	0.094***
	(0.087)	(0.048)	(0.027)	(0.024)	(0.023)	(0.020)	(0.021)	(0.026)	(0.026)
2010	0.438***	0.198***	0.107***	0.089***	0.059***	0.074***	0.091***	0.138***	0.131***
	(0.090)	(0.042)	(0.027)	(0.024)	(0.020)	(0.021)	(0.019)	(0.023)	(0.022)
RIF regression									
1990	0.278***	0.121***	0.080***	0.062***	0.013	-0.003	0.002	-0.007	0.005
	(0.082)	(0.045)	(0.027)	(0.023)	(0.020)	(0.019)	(0.018)	(0.020)	(0.025)
2000	0.420***	0.158***	0.080***	0.053**	0.059***	0.052***	0.052**	0.088***	0.128***
	(0.076)	(0.037)	(0.027)	(0.023)	(0.021)	(0.020)	(0.021)	(0.022)	(0.026)
2010	0.445***	0.193***	0.143***	0.084***	0.099***	0.090***	0.098***	0.106***	0.193***
	(0.076)	(0.038)	(0.025)	(0.022)	(0.020)	(0.019)	(0.019)	(0.020)	(0.023)

and the effects of these choices on the distribution of human capital attainment by heterogeneous students looking for the best curricular match for their aptitude. Our model offers an explanation for the evidence of diverging selectivity of American colleges that accompanied the expansion of access to higher education in the U.S., particularly a downward adjustment in the curricular standards of historically less selective colleges, and an upward adjustment of the curricula in the more selective ones.

We argue that the distributional impact of these changes is non-monotone: While low and high-ability students gain in terms of human capital value added, medium ability students lose out. We further find that these results are consistent with the U-shaped earnings growth profile among college-educated workers that we observe in the wage income data over the period which exhibited the aggregate enrollment expansion of higher education and the pattern of diverging selectivity among the American colleges, and suggest a connection between the phenomena. To be sure, the model we use is highly stylized, which is necessitated by keeping the focus on the key insights associated with the impact of curricula changes while preserving analytical tractability. While we make a step in advancing the concept of product (curricula) differentiation across colleges, it is clear that each college in fact offers multiple curricular products such as majors, which differ vastly in their selectivity (both through strategic discretion of their providers and as may be dictated by labor market requirements in the corresponding professions), and even curricular differentiation within majors (e.g., their Honors versions). Thus on the one hand, in reality, students' choices are multidimensional, such as between college-major combinations. On the other hand, students continue making choices upon entering college as more information is revealed, potentially changing majors or dropping out altogether.

Another substantial limitation of our model is the exclusion of an explicit account for the financial side of college economics. An extension of our analysis to incorporate education expenditure in the education production technology could help capture school quality aspects besides their curricula, such as teacher quality, class size, classroom and lab technology, and so forth. While different students prefer different curricula, all students can benefit from higher levels of the aforementioned characteristics of quality. Allowing for interactions across different school quality characteristics can help produce novel insights into their roles in educational attainment in different segments of student population, the trade-offs involved, and to explore the question of optimal costs associated with different curricula standards. These questions are especially important when examining policies which allocate public funds to achieve college access objectives.

Finally, a framework in which schools choose their curricula could also be useful for reexamining the role of peer effects. In the existing literature on peer effects (e.g., Epple and Romano (1998)), students directly benefit from interacting with better qualified peers. When school curricula are endogenous, as proposed here, a new dimension of peer interactions emerges: If a school's curriculum is geared toward the majority of its student body, a less prepared student who attends a school with significantly better qualified peers may indirectly suffer from being exposed to too challenging a learning environment. The interaction between these two effects suggests another non-monotone relationship: While having slightly better peers has a positive effect on human capital, facing a more challenging curriculum geared toward these better peers can have a detrimental impact on one's learning outcomes. These effects are relevant for evaluating the efficiency and equity of outcomes under different education systems, such as *mixing* (i.e., pooling students of different ability in one class) or *tracking* (i.e., separating students by ability).

Appendix

Proof of Lemma 1. Note that staying out of college is equivalent to learning under curriculum with $c = 0$. A student of ability q prefers curricular standard c to c' whenever $c(q-c) > c'(q-c')$. Now suppose $c_1 > c_2 > ... > c_N > 0$. A student with ability q prefers staying out of college to attending college N if $q < c_N$. He prefers college n over college $n+1$ for $n = 1,2,...,N-1$ if $c_n(q-c_n) > c_{n+1}(q-c_{n+1})$. This implies the Lemma's result. □

Proof of Proposition 1. To streamline notation, we denote $x = c_1/Q$, $y = c_2/Q$, $r = \gamma_1/(AQ^2)$, and $t = \gamma_2/(AQ^2)$. Then the first order condition for college 1 becomes

$$4x^2 - 4xy - 2y^2 - 5x + y + 1 - 6r = 0.$$

Solving for y in terms of x, we have

$$y = \phi(x) = \frac{1 - 4x + \sqrt{48x^2 - 48x + 9 - 48r}}{4}.$$

Similarly the first order condition for college 2 becomes

$$y = g(x) = 1/2 - x/3 - t/x.$$

Below we show that when $\underline{r} \equiv -\frac{1}{16} < r < \bar{r} \equiv -\frac{1}{27\times 16} < 0$ and $0 < \underline{t} \equiv \frac{1}{12} < t < \bar{t} \equiv \frac{5}{48}$, a unique stable Nash equilibrium exists with $x^* \in (1/4, 1/2)$ and $0 < y^* < x^*$.

First, focusing on the function $g(\cdot)$, we make the following observations. To ensure $g(x) < x$, it is sufficient to require that $t > \frac{3}{64}$. To ensure $g(x) > 0$, it is both necessary and sufficient that $t < \frac{3}{16}$ and $x \in (\frac{3-\sqrt{9-48t}}{4}, \frac{3+\sqrt{9-48t}}{4})$. So when $0 < \underline{t} < t < \bar{t}$ and $x \in (1/4, 1/2)$, it is guaranteed that $0 < g(x) < x$. Furthermore, when $0 < \underline{t} < t < \bar{t}$ and $x \in (0, 1/2)$, we have $g'(x) = -1/3 + \frac{p}{x^2} > 0$ and $g''(x) = -\frac{2p}{x^3} < 0$. In other words, $g(x)$ is upward sloping and concave.

Next, focusing on the function $\phi(\cdot)$, we make the following observations. When $r > -\frac{1}{16}$, $\phi(x)$ is not defined for $x > 1/2 - \sqrt{1/16 + r}$, so the relevant domain for $\phi(x)$ is $x \in (0, 1/2 - \sqrt{1/16 + r}]$. Furthermore, when $\underline{r} < r < \bar{r} < 0$ and $x \in (0, 1/2 - \sqrt{1/16 + r}]$, we have $\phi'(x) = -1 + \frac{12x-6}{\sqrt{48x^2-48x+9-48r}} < 0$ and $\phi''(x) = -\frac{3+48r}{(48x^2-48x+9-48r)^{3/2}} < 0$. In other words, $\phi(x)$ is downward sloping and concave.

Now we establish the single crossing between $\phi(x)$ and $g(x)$. When $x = 1/4$, for the inequality $\phi(1/4) > g(1/4)$ to hold, it is sufficient to have $\sqrt{-48r} > \frac{1}{3}$, or equivalently $r < -\frac{1}{27\times 16}$. Next, when $x = 1/2 - \sqrt{1/16+r}$, for the inequality $\phi(1/2 - \sqrt{1/16+r}) < g(1/2 - \sqrt{1/16+r})$ to hold, it is sufficient to have $\phi(1/2 - \sqrt{1/16+r}) < 0$, or equivalently $r < 0$.

Put together, when $\underline{r} < r < \overline{r} < 0$ and $0 < \underline{t} < t < \overline{t}$, the single-crossing property ensures that a unique Nash equilibrium exists with $1/4 < x^* < 1/2 - \sqrt{1/16+r} < 1/2$, $x^* > y^* > 0$, and hence $x^* + y^* < 1$.

To establish local stability of the Nash equilibrium, it is sufficient to show that $-\phi'(x) > g'(x)$, so that the mutual best response dynamics upon a small perturbation of the equilibrium (x^*, y^*) converges back to the equilibrium. Recall that $\phi''(x) < 0$ and $g''(x) < 0$ for $x \in [1/4, 1/2 - \sqrt{1/16+r}]$, so it is sufficient to show that $-\phi'(1/4) > g'(1/4)$. This inequality is indeed satisfied since $-\phi'(1/4) = 1 + \frac{3}{\sqrt{-48r}} > 2 > g'(1/4) = -1/3 + 16t$ for the parameter values $r \in (\underline{r}, \overline{r})$ and $t \in (\underline{t}, \overline{t})$.

The above concludes the proof, i.e., the establishment of the bounds on the values of parameters $r = \gamma_1/(AQ^2)$ and $t = \gamma_2/(AQ^2)$ ensuring the existence of stable interior Nash equilibrium. In the process, we were also able to estimate the localizations of the colleges' strategies represented here by $x = c_1/Q$ and $y = c_2/Q$. As stated above, x^* must belong to the interval $(1/4, 1/2)$. Combining this with the bounds established for r and t, it is straightforward to derive that $y^* \in (0, 1/6)$. □

Proof of Proposition 2. Keeping the above notation, it is easy to see that at a given equilibrium (x^*, y^*), when t is marginally increased, the reaction curve $\phi(x)$ is unaffected but $g(x)$ shifts down. So the change in equilibrium results in a movement along the $\phi(x)$ curve. This implies that $dy^* < 0$ and since, as established in the proof of Proposition 1, $y = \phi(x)$ is downward sloping, it follows that $dx^* > 0$. Furthermore, since $-\phi'(x^*) > -\phi'(1/4) = 1 + \frac{3}{\sqrt{-48r}} > 1$, we also know $dx^* < |dy^*|$, and therefore, $dx^* + dy^* < 0$. It then follows that $sign(\frac{\partial c_1^*}{\partial \gamma_2}) = sign(\frac{\partial x^*}{\partial t}) > 0$, $sign(\frac{\partial c_2^*}{\partial \gamma_2}) = sign(\frac{\partial y^*}{\partial t}) < 0$, and $sign(\frac{\partial (c_1^* + c_2^*)}{\partial \gamma_2}) = sign(\frac{\partial (x^* + y^*)}{\partial t}) < 0$. □

Proof of Corollary 1. According to Proposition 2, when γ_2 increases, curricular thresholds adjust from $(c_1^{\text{old}}, c_2^{\text{old}})$ to $(c_1^{\text{new}}, c_2^{\text{new}})$ such that

$$c_2^{\text{new}} < c_2^{\text{old}} < c_1^{\text{old}} < c_1^{\text{new}},$$

and

$$c_1^{\text{new}} + c_2^{\text{new}} < c_1^{\text{old}} + c_2^{\text{old}}.$$

According to Lemma 1, student population can be partitioned into the following five subgroups:

1. Students with $q \in [0, c_2^{\text{new}}]$ do not attend college after the policy change.

2. Students with $q \in (c_2^{\text{new}}, c_2^{\text{old}}]$ do not attend college before the policy change, but attend college 2 after the change.

3. Students with $q \in (c_2^{\text{old}}, c_2^{\text{new}} + c_1^{\text{new}}]$ attend college 2 before and after the change.

4. Students with $q \in (c_2^{\text{new}} + c_1^{\text{new}}, c_2^{\text{old}} + c_1^{\text{old}}]$ attend college 2 before the policy change, but switch to college 1 after the change.

5. Students with $q \in (c_2^{\text{old}} + c_1^{\text{old}}, Q]$ attend college 1 before and after the change.

By Assumption 1, a student accumulates more human capital and hence prefers curriculum c over $c' > c$, iff

$$Ac(q-c) > Ac'(q-c') \text{ or equivalently, } q < c + c'.$$

Students in the first subgroup are obviously unaffected by the policy change. Students in the second subgroup gain in human capital after the policy change.

A student in the third group (who attends college 2 before and after the policy change) prefers c_2^{new} to c_2^{old}, if $q < c_2^{\text{old}} + c_2^{\text{new}}$; otherwise he prefers c_2^{old} to c_2^{new}.

All students in the fourth group (who attend college 2 before, and college 1 after the change) prefer c_2^{old} to c_1^{new} because for these students $q < c_2^{\text{old}} + c_1^{\text{old}} < c_2^{\text{old}} + c_1^{\text{new}}$.

Any student in the fifth group (who attends college 1 before, and after the change) prefers c_1^{old} to c_1^{new} if $q < c_1^{\text{old}} + c_1^{\text{new}}$; otherwise he prefers c_1^{new} to c_1^{old}.

Putting these results together, after the policy change, students with $q \in (c_2^{\text{new}}, c_2^{\text{old}} + c_2^{\text{new}})$ or $q \in (c_1^{\text{old}} + c_1^{\text{new}}, Q]$ acquire more human capital, but students with $q \in (c_2^{\text{old}} + c_2^{\text{new}}, c_1^{\text{old}} + c_1^{\text{new}})$ acquire less human capital. □

Proof of Proposition 3. To streamline notation, similarly to that used in the proof of Proposition 1, we denote $x_i = c_i/Q$, and $r_i = \gamma_i/(AQ^2)$ for $i = 1, ..., N$. Then a Nash equilibrium is a solution $(x_1^*, x_2^*, ..., x_N^*)$ of the system of equations below, given parameter values $r_1, r_2, ..., r_N$:

$$x_1 \equiv f_1(x_2, r_1) = \frac{5 + 4x_2 - \sqrt{48x_2^2 + 24x_2 + 9 + 96r_1}}{8},$$
$$x_s \equiv f_s(x_{s-1}, x_{s+1}, r_s) = 1/2 - \frac{x_{s-1}^2 + x_{s+1}^2 + x_{s-1}x_{s+1}}{3(x_{s-1} + x_{s+1})} - \frac{r_s}{x_{s-1} + x_{s+1}}, \quad 1 < s < N$$
$$x_N \equiv f_N(x_{N-1}, r_N) = 1/2 - x_{N-1}/3 - r_N/x_{N-1}. \tag{a.1}$$

Now consider a small change in the parameter values. The resulting new equilibrium can be written as $(x_1^* + dx_1, x_2^* + dx_2, ...x_N^* + dx_N)$. Linearizing system (a.1) around the original equilibrium, we can write

$$dx_1 = f_{12}dx_2 + D_1dr_1,$$
$$dx_s = f_{s,s-1}dx_{s-1} + f_{s,s+1}dx_{s+1} + D_sdr_s, \quad 1 < s < N$$
$$dx_N = f_{N,N-1}dx_{N-1} + D_Ndr_N, \tag{a.2}$$

where $f_{s,s\pm1} = \frac{\partial f_s}{\partial x_{s\pm1}}$, and $D_s = \frac{\partial f_s}{\partial r_s}$ for $s = 1, ..., N$ are direct derivatives with respect to the parameters. According to the equations in (a.1), $D_s < 0$ for all s.

For tractability, we break down the rest of the proof into several steps as follows.

Step 1.

Consider the signs of $f_{s,s+1}$ for $s = 1, ..., N-1$. Note that $f_{12} = \frac{1}{2} - \frac{12x_2+3}{2\sqrt{48x_2^2+24x_2+9+96r_1}}$, so $f_{12} < \frac{1}{2}(1 - \sqrt{\frac{144x_2^2+72x_2+9}{48x_2^2+24x_2+9}}) < 0$ whenever $r_1 < 0$. Likewise, for any $1 < s < N$, $f_{s,s+1} = \frac{-2x_{s-1}x_{s+1}-x_{s+1}^2+3r_s}{3(x_{s-1}+x_{s+1})^2}$, so $sign(f_{s,s+1}) = sign(-2x_{s-1}x_{s+1} - x_{s+1}^2 + 3r_s)$. Since $x_1 > x_2 > ... > x_N$ and $r_1 < r_2 < ... < r_N$, the numerator $-2x_{s-1}x_{s+1} - x_{s+1}^2 + 3r_s$ is strictly increasing. Thus there exists $1 \le K \le N-1$ such that $f_{s,s+1} < 0$ for all $s \le K$, and $f_{s,s+1} > 0$ for all $s > K$. Note that $K = N-1$ would imply that $f_{s,s+1} < 0$ for all possible $s = 1, ..., N-1$.

Similarly, consider the signs of $f_{s,s-1}$ for $s = 2, ..., N$. Note that for any $1 < s < N$, $f_{s,s-1} = \frac{-2x_{s-1}x_{s+1}-x_{s-1}^2+3r_s}{3(x_{s-1}+x_{s+1})^2}$, and $f_{N,N-1} = \frac{-x_{N-1}^2+3r_N}{3x_{N-1}^2}$. So overall, if we denote $x_{N+1} = 0$, $sign(f_{s,s-1}) = sign(-2x_{s-1}x_{s+1} - x_{s-1}^2 + 3r_s)$ for all $s = 2, ..., N$. Since $x_1 > x_2 > ... > x_N > x_{N+1} = 0$ and $r_1 < r_2 < ... < r_N$, the numerator $-2x_{s-1}x_{s+1} - x_{s-1}^2 + 3r_s$ is again strictly increasing. Thus there exists $1 \le L \le N$ such that $f_{s,s-1} < 0$ for all $s \le L$, and $f_{s,s-1} > 0$ for all $s > L$. Note that $L = 1$ would imply that $f_{s,s-1} > 0$ for all possible

$s = 2, ..., N$, while $L = N$ would imply that $f_{s,s-1} < 0$ for all possible $s = 2, ..., N$.

Finally, we observe that for any $1 < s < N$, we have $f_{s,s+1} > f_{s,s-1}$ since $c_{s+1} < c_{s-1}$. It then follows that $K \leq L$.

Step 2.

Rewrite the local linearization equations (a.2) in the matrix format:

$$\mathbf{A} \cdot \mathbf{dx} = \mathbf{b}, \tag{a.3}$$

where $\mathbf{A}$ is the $N \times N$ tridiagonal matrix with non-zero elements $a_{s,s} = 1$, $a_{s,s-1} = -f_{s,s-1}$, and $a_{s,s+1} = -f_{s,s+1}$, $\mathbf{dx}$ is an $N \times 1$ vector with elements dx_s, and $\mathbf{b}$ is also an $N \times 1$ vector with elements $D_s dr_s$.

Further, the solution of the linearized system (a.3) is the equilibrium of the following dynamic system, with the superscript as the time period index,

$$\mathbf{dx}^{t+1} = \mathbf{B}\mathbf{dx}^t + \mathbf{b}, \tag{a.4}$$

where $\mathbf{B} = \mathbf{A} - \mathbf{I}$, i.e., the tridiagonal matrix which differs from $\mathbf{A}$ in that $\mathbf{B}$ has all zeros on its main diagonal. It is easy to see that system (a.4) represents a linearization of the system of best responses of colleges for which the original non-linear system (a.1) defines the Nash equilibrium. To ensure local stability of the Nash equilibrium, we follow Moulin (1986) and impose a sufficient condition that all eigenvalues of matrix $\mathbf{B}$ lie strictly within the unit circle. As we will show immediately below, this implies that matrix $\mathbf{A}$ is non-singular, so the solution of the linearized system (a.3) is uniquely defined and comparative statics analysis is applicable, similar to the case under $N = 2$ in Proposition 2. (As mentioned above, having established the sufficient conditions for comparative statics under feasible parametrization for the two-college case, we assume that appropriate feasible ranges for parameters exist for $N > 2$ and focus on applying comparative statics analysis.)

Lemma 2 (i) Matrix $\mathbf{A}$ is non-singular. Moreover, its determinant $|\mathbf{A}| > 0$. (ii) The same is true for all the leading and trailing principal minors of matrix $\mathbf{A}$, namely $|\mathbf{A^j}| > 0$ and $|\mathbf{A_j}| > 0$ for the leading and trailing principal minors of order $j = 1, ..., N-1$ respectively.

Proof Consider all eigenvalues of the matrix $\mathbf{B}$: λ_k for $k = 1, ..., m$. Since they lie strictly within the unit circle, function $h(\lambda_k) = (1 - t\lambda_k)^{-1}$ is well defined for all real numbers $t \in [0, 1]$. Then, according to Gantmacher (1959), the matrix function

$h(\mathbf{B}) = (\mathbf{I} - t\mathbf{B})^{-1}$ is also well defined by a matrix all of whose eigenvalues are given by $h(\lambda_k) = (1 - t\lambda_k)^{-1}$, $k = 1, \ldots, m$ with their respective multiplicities. This means that matrix $\mathbf{I} - t\mathbf{B}$ is invertible so $|\mathbf{I} - t\mathbf{B}| \neq 0$ for all $t \in [0,1]$. Note that the determinant $|\mathbf{I} - t\mathbf{B}|$ is a continuous function of t and that it equals 1, and hence positive when $t = 0$. Since, as we have just shown, $|\mathbf{I} - t\mathbf{B}| \neq 0$ for all $t \in [0,1]$, it must also be positive for $t = 1$. We can thus conclude that $|\mathbf{A}| = |\mathbf{I} - \mathbf{B}| > 0$, which completes the proof of part (i) of the Lemma.[20]

The proof of part (ii) is completely analogous to the above. Note that matrices $\mathbf{A^j}$ and $\mathbf{B^j}$, the leading principal submatrices of order j of matrices $\mathbf{A}$ and $\mathbf{B}$ respectively, correspond to the system of linearized equations similar to (a.3) and the dynamical system of mutual best responses similar to (a.4), which arise in the subgame of colleges $1, 2, \ldots, j$ when the positions of the $N - j$ least selective colleges ($s = j+1, \ldots, N$) are fixed at the equilibrium levels. As assumed above, the sufficient condition for the stability of Nash equilibrium in this subgame is satisfied, i.e., all eigenvalues of $\mathbf{B^j}$ lie strictly within the unit circle. Thus, following the footsteps of part (i) proof, we obtain $|\mathbf{A^j}| > 0$. The result for the trailing principal minors $|\mathbf{A_j}| > 0$ obtains similarly, arising from the analysis of the subgame played by colleges $N - j + 1, \ldots, N$ when positions of the $N - j$ most selective colleges ($s = 1, \ldots, N - j$) are fixed at the equilibrium levels.

Step 3.

Denote $M \equiv \max\{L, K+1\}$. Now we derive the comparative statics when there is a positive shock to γ_m (hence r_m), the weight college m attaches to the quantity of its students, where $m \geq M$.

First, since $L \leq N$ and $K \leq N - 1$, we know $M = \max\{L, K+1\} \leq N$, so we can always consider a positive shock to r_N, i.e., vector $\mathbf{b}$ has only one non-zero element $b_N < 0$. Using Cramer's Rule, we have $dx_N = \frac{b_N |\mathbf{A}^{N-1}|}{|\mathbf{A}|} < 0$. Similarly, $dx_{N-1} = \frac{f_{N-1,N} b_N |\mathbf{A}^{N-2}|}{|\mathbf{A}|}$, so $dx_{N-1} < 0$ if $f_{N-1,N} > 0$, and $dx_{N-1} > 0$ if $f_{N-1,N} < 0$. In general, $dx_s = \frac{f_{s,s+1} \cdots f_{N-1,N} b_N |\mathbf{A}^{s-1}|}{|\mathbf{A}|}$, so $dx_s < 0$ if $s > K$ (see Step 1), $dx_s > 0$ if $s = K$, $dx_s < 0$ if $s = K - 1$, $dx_s > 0$ if $s = K - 2$, and so forth.

Next, if $L < N$ and $K < N - 1$, we have $M \leq N - 1$, so we can also consider a positive shock to γ_{N-1} (hence r_{N-1}), i.e., vector $\mathbf{b}$ has only one non-zero element $b_{N-1} < 0$. Using Cramer's Rule, we have $dx_{N-1} = \frac{b_{N-1} |\mathbf{A}^{N-2}|}{|\mathbf{A}|} < 0$. For college N, i.e., the only college ranked below $N - 1$, since $L < N$ implies $f_{N,N-1} > 0$, we have $dx_N = \frac{f_{N,N-1} b_{N-1} |\mathbf{A}^{N-2}|}{|\mathbf{A}|} < 0$. For colleges ranked above $N - 1$, we have $dx_{N-2} = \frac{f_{N-2,N-1} b_N |\mathbf{A}^{N-3}|}{|\mathbf{A}|}$,

[20] The idea for this last step of the proof is borrowed from Bellman (1960).

so $dx_{N-2} < 0$ if $f_{N-2,N-1} > 0$, and $dx_{N-2} > 0$ if $f_{N-2,N-1} < 0$. In general, $dx_s = \frac{f_{s,s+1} \cdots f_{N-2,N-1} b_{N-1} |\mathbf{A}^{s-1}|}{|\mathbf{A}|}$, so again $dx_s < 0$ if $s > K$, $dx_s > 0$ if $s = K$, $dx_s < 0$ if $s = K-1$, $dx_s > 0$ if $s = K-2$, and so forth.

Lastly, if $L < N-1$ and $K < N-2$, we have $M \le N-2$, we can consider in general a positive shock to r_m for any $m \ge M$. Using Cramer's Rule, we have $dx_m = \frac{b_m |\mathbf{A}^{m-1}||\mathbf{A}_{N-m}|}{|\mathbf{A}|} < 0$. For colleges ranked below m, we have $dx_N = \frac{f_{m+1,m} \cdots f_{N,N-1} b_m |\mathbf{A}^{m-1}|}{|\mathbf{A}|} < 0$, and $dx_{N-1} = \frac{f_{m+1,m} \cdots f_{N-1,N-2} b_m |\mathbf{A}^{m-1}|}{|\mathbf{A}|} < 0$. More generally, for $m < s \le N-2$ (when $m < N-2$), we have $dx_s = \frac{f_{m+1,m} \cdots f_{s,s-1} b_m |\mathbf{A}_{N-s}||\mathbf{A}^{m-1}|}{|\mathbf{A}|} < 0$. For colleges ranked above m, we have $dx_1 = \frac{f_{12} \cdots f_{m-1,m} b_m |\mathbf{A}_{N-m}|}{|\mathbf{A}|}$, so $dx_1 > 0$ if K is odd (so $K-1$ is even), and $dx_1 < 0$ if K is even. Next, if $m > 2$, we have $dx_2 = \frac{f_{23} \cdots f_{m-1,m} b_m |\mathbf{A}_{N-m}|}{|\mathbf{A}|}$, so $dx_2 < 0$ if $K = 1$, $dx_2 > 0$ if $K = 2$, $dx_2 < 0$ if $K = 3$, and so forth. More generally, for $3 \le s < m$ (when $m > 3$), we have $dx_s = \frac{f_{s,s+1} \cdots f_{m-1,m} b_m |\mathbf{A}^{s-1}||\mathbf{A}_{N-m}|}{|\mathbf{A}|}$, so $dx_s < 0$ if $s > K$, $dx_s > 0$ if $s \le K$ and $K-s$ is even, and $dx_s < 0$ if $s < K$ and $K-s$ is odd.

Overall, we have shown that when there is a positive shock to the relative weight γ_m that a relatively low-ranked college $m \ge M$ attaches to the quantity of its students, the equilibrium curriculum choices of all colleges change in a divergent pattern: all less selective colleges $s \ge K+1$ lower their curricular thresholds and become less selective; colleges $s \le K$ become collectively more selective, while a pair-wise clustering pattern occurs within, namely college K increases its curricular threshold, college $K-1$ lower its curricular threshold, and so forth. □

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