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Abstract

In a recent paper, Alipranti et al. (2014, Price vs. quantity competition in a vertically related market, *Economics Letters*, 124: 122-126) show that in a vertically related market Cournot competition yields higher social welfare compared to Bertrand competition if the upstream firm subsidises the quantity setting downstream firm's production via negative wholesale input prices. However, the assumption of negative input prices is not economically viable as it would encourage the downstream firms to buy an unbounded amount of inputs knowing that the upstream firm would pay the downstream firms for each unit of input they purchase. We show that the welfare ranking may be reversed once we introduce a nonnegativity constraint on the input price.

JEL-Codes: D430, L130, L140.

Keywords: bargaining, Bertrand, Cournot, two-part tariffs, vertical pricing, welfare.

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1. Introduction

In a seminal paper, Singh and Vives (1984) show that Bertrand competition yields higher social welfare than Cournot competition if the goods are substitutes and the input markets are competitive. In a recent paper, Alipranti et al. (2014) show that when a monopoly input supplier bargains with the downstream firms over a two-part tariff vertical pricing contract, the upstream firm subsidises the quantity setting downstream firms via negative wholesale input prices. This creates higher social welfare under Cournot competition compared to Bertrand competition.

We believe that the assumption of a *negative input price* is not economically viable as it will encourage the downstream firms to buy an unbounded amount of inputs since the upstream firm would pay the downstream firms for each unit of input they purchase. We, therefore, impose a non-negativity constraint on the input prices. In contrast to Alipranti et al. (2014), we show that if the upstream firm's marginal cost of production is low, Cournot competition may yield lower output level, lower consumer surplus and lower social welfare compared to Bertrand competition, thus supporting the findings of Singh and Vives (1984) even in a vertical structure.

2. The model and the results

We consider an economy similar to Alipranti et al. (2014) where two downstream firms, denoted by D_i, produce differentiated products, $i, j = 1, 2$; $i \neq j$. The downstream firms require a critical input for production that they purchase from a monopoly input supplier, U, through two-part tariff contracts involving an up-front fixed-fee, F_i and a per-unit price, w_i, $i = 1, 2$. As in Alipranti et al. (2014), we assume that U produces the input at a constant marginal cost of production. While Alipranti et al. (2014) assumed that the upstream firm produces at zero marginal cost, we generalise it by assuming that the marginal cost of the upstream firm is c, where $c \in (0, a)$. This generalisation helps us to show that the negative input price in Alipranti et al. (2014) is not an artefact of their assumption of zero marginal cost of the upstream firm. We assume that the production technologies of the downstream firms are such that one unit of input is required to produce one unit of the output, and D_i and D_j can convert the inputs to the final goods without incurring any further cost.

We consider the following game. At stage 1, U bargains with D_1 and D_2 to determine the terms of the two-part tariff contracts. At stage 2, D_1 and D_2 compete in quantities (Cournot competition) or in prices (Bertrand competition) simultaneously and the profits are realised. We solve the game through backward induction.

First, we will offer a general analysis of our model by using the reduced form expressions for outputs, price and profits, thus ignoring any specific demand function. We will then use a demand function similar to Alipranti et al. (2014) to determine the closed form solutions for our variables.

At stage 2, given the two-part tariff contracts, D_i, $i = 1, 2$, determines q_i (under Cournot competition) and P_i (under Bertrand competition) to maximise its profit. Assume that, given the type of product-market competition $\rho = \{Cournot, Bertrand\}$, the reduced form equilibrium output, final good's price and the profits of D_i and U, $i = 1, 2$, are q_i^ρ, P_i^ρ, $D\pi_i^\rho - F_i^\rho = (P_i^\rho - w_i^\rho)q_i^\rho - F_i^\rho$ and $U\pi^\rho + \sum_i F_i^\rho = \sum_i (w_i^\rho - c)q_i^\rho + \sum_i F_i^\rho$ respectively.

At stage 1, the terms of the two-part tariff contract for D_i are determined by maximising the following generalised Nash bargaining expression:

$$\underset{F_i^\rho, w_i^\rho}{\text{Max}} \left[U\pi^\rho + \sum_i F_i^\rho - d\left(w_j^\rho, F_j^\rho, c\right) \right]^\beta \left[D\pi_i^\rho - F_i^\rho\right]^{1-\beta} \qquad (1)$$

where upstream firm's disagreement pay-off is $d\left(w_j^\rho, F_j^\rho, c\right) = \left(w_j^\rho - c\right)q_j^{mon} + F_j^\rho$ with monopoly output $q_j^{mon} = \frac{a - w_j^\rho}{2}$.

Maximising (1) with respect to F_i^{ρ} gives the following:

$$F_i^{\rho} = \beta D\pi_i^{\rho} - (1-\beta)\left[U\pi^{\rho} - \left(w_j^{\rho} - c\right)q_j^{mon}\right]. \tag{2}$$

Substituting (2) in (1), we get the maximisation problem as:

$$\underset{w_i^{\rho}}{\text{Max}}\ \left[\beta^{\beta}(1-\beta)^{1-\beta}\right]\left[U\pi^{\rho} + D\pi_i^{\rho} - \left(w_j^{\rho} - c\right)q_j^{mon}\right]. \tag{3}$$

Solving the first order condition gives the equilibrium input price

$$w_i^{\rho} = \frac{c\left(\frac{\partial q_i^{\rho}}{\partial w_j^{\rho}} + \frac{\partial q_j^{\rho}}{\partial w_j^{\rho}}\right) - P_j^{\rho}\left(\frac{\partial q_j^{\rho}}{\partial w_j^{\rho}}\right) - q_j^{\rho}\left(\frac{\partial P_j^{\rho}}{\partial w_j^{\rho}}\right)}{\left(\frac{\partial q_j^{\rho}}{\partial w_j^{\rho}}\right)}. \tag{4}$$

As in Alipranti et al. (2014) we now consider the inverse market demand function for D_i as $P_i = a - q_i - \gamma q_j$, where $i, j = 1, 2$ and $i \neq j$. $\gamma \in [0,1]$ denotes the degree of product differentiation. $\gamma = 0$ implies isolated products and $\gamma = 1$ implies homogeneous goods. We, however, restrict our assumption to $\gamma \in (0,1)$. The direct demand function for the ith firm is $q_i = \frac{a(1-\gamma) - P_i + \gamma P_j}{1-\gamma^2}$, $i, j = 1, 2$ and $i \neq j$.

2.1. Cournot competition

First, consider the case where the final goods producers compete in quantities. At stage 2, D_i, $i = 1, 2$, determines q_i to maximise $D\pi_i^C = (P_i - w_i)q_i = \left(a - q_i - \gamma q_j - w_i\right)q_i$. Note that F_i is sunk at stage 2. The equilibrium output of the ith downstream firm can be found as

$$q_i^C = \frac{a(2-\gamma) - 2w_i + \gamma w_j}{4-\gamma^2}\ . \tag{5}$$

The gross equilibrium profit of the ith downstream firm is

$$D\pi_i^C = \left(\frac{a(2-\gamma) - 2w_i + \gamma w_j}{4-\gamma^2}\right)^2\ . \tag{6}$$

Maximising (3) subject to (5) and (6) gives the equilibrium per-unit input price as

$w_i^C = \frac{c(4-\gamma^2)-a\gamma^2}{2(2-\gamma^2)}$.

The following lemma follows immediately from the equilibrium input price.

Lemma 1: *The equilibrium input price is negative, i.e.,* $w_i^C < 0$ *for* $0 < c \leq c^*$ *and it is positive, i.e.,* $w_i^C > 0$ *for* $c^* < c < a$ *where* $c^* = \frac{a\gamma^2}{4-\gamma^2}$.

As shown, the negotiated wholesale input price becomes negative when the upstream firm's marginal cost of production is low, i.e., $0 < c \leq c^*$. According to Alipranti et al. (2014), in this case, the monopoly input supplier subsidises downstream firms' production via input prices. However, as alluded earlier if the input price is negative, the downstream firms will want to buy an infinite number of inputs knowing that the upstream firm would pay the downstream firms for each unit of input they purchase. Hence, the assumption of a negative input price is not economically viable. Therefore, to make the analysis meaningful, we set $w_i^C = 0$ for $0 < c \leq c^*$, meaning that, the equilibrium two-part-tariff only consists only of a positive fixed fee, $F_i^C = \frac{2a^2\beta+ac(1-\beta)(4-\gamma^2)}{2(2+\gamma)^2}$ for $0 < c \leq c^*$. However, if the marginal cost input production is high, i.e., $c^* < c < a$, the upstream firm charges a positive fixed fee, $F_i^C = \frac{(a-c)^2(2-\gamma)^2(2\beta+\gamma^2-\beta\gamma^2)}{8(2-\gamma^2)^2}$ and per-unit input price, $w_i^C = \frac{c(4-\gamma^2)-a\gamma^2}{2(2-\gamma^2)}$.

Given the above-mentioned equilibrium input prices, we report the equilibrium outcomes in Table 1.

Table 1

	For $0 < c \leq \frac{a\gamma^2}{4-\gamma^2}$	For $\frac{a\gamma^2}{4-\gamma^2} < c < a$
Equilibrium output (q_i^C)	$\frac{a}{2+\gamma}$	$\frac{(a-c)(2-\gamma)}{2(2-\gamma^2)}$
Net downstream profit $(D\pi_i^C - F_i^C)$	$\frac{a^2(1-\beta)(2a-4c+c\gamma^2)}{2(2+\gamma)^2}$	$\frac{(a-c)^2(1-\beta)(2-\gamma)^2}{8(2-\gamma)^2}$
Net upstream profit $\left(U\pi^C + \sum_i F_i^C\right)$	$\frac{2a^2\beta + ac(1-\beta)(4-\gamma^2)}{(2+\gamma)^2}$	$\frac{(a-c)^2(2-\gamma)(4\beta - 2\beta\gamma - 2\beta\gamma^2 - \gamma^3 + \beta\gamma^3)}{4(2-\gamma^2)^2}$
Consumers surplus (CS^C)	$\frac{a^2(1+\gamma)}{(2+\gamma)^2}$	$\frac{(a-c)^2(2-\gamma)^2(1+\gamma)}{4(2-\gamma^2)^2}$
Social welfare (SW^C) $= \sum D\pi_i^C + U\pi^C + CS^C$	$\frac{a^2(3+\gamma)}{(2+\gamma)^2}$	$\frac{(a-c)^2(2-\gamma)(6-\gamma-3\gamma^2)}{4(2-\gamma^2)^2}$

2.2. Bertrand competition

We now turn our analysis to the case where the firms compete in prices. The ith downstream firm maximises $\pi_i^B = (P_i - w_i)q_i = (P_i - w_i)\left(\frac{a(1-\gamma)-P_i+\gamma P_j}{1-\gamma^2}\right)$, where $i = 1,2$. The equilibrium price and output of the *i*th downstream firm can be found as

$$P_i^B = \frac{a(1-\gamma)(2+\gamma)+2w_i+\gamma w_j}{4-\gamma^2} \quad \text{and} \quad q_i^B = \frac{a(1-\gamma)(2+\gamma)-(2-\gamma^2)w_i+\gamma w_j}{(1-\gamma^2)(4-\gamma^2)}. \qquad (7)$$

The gross equilibrium profit of the *i*th downstream firm is

$$D\pi_i^B = (1-\gamma^2)\left(\frac{a(1-\gamma)(2+\gamma)-(2-\gamma^2)w_i+\gamma w_j}{(1-\gamma^2)(4-\gamma^2)}\right)^2,\ i = 1,2. \qquad (8)$$

Maximising (3) subject to (7) and (8) gives the equilibrium per-unit input price and the fixed-fee as

$$w_i^B = \frac{a\gamma^2+c(4-\gamma^2)}{4} \quad \text{and} \quad F_i^B = \frac{(a-c)^2(2+\gamma)(4\beta-2\beta\gamma-2\gamma^2+\gamma^3-\beta\gamma^3-\gamma^4+\beta\gamma^4)}{32(1+\gamma)}$$

We also calculate the following equilibrium values:

$$q_i^B = \frac{(a-c)(2+\gamma)}{4(1+\gamma)}, \quad D\pi_i^B - F_i^B = \frac{(a-c)^2(1-\beta)(2+\gamma)(4-2\gamma-\gamma^3+\gamma^4)}{32(1+\gamma)},$$

$$U\pi^B + \sum_i F_i^B = \frac{(a-c)^2(2+\gamma)\big(2\beta(2-\gamma)+\gamma^3(1-\gamma)(1-\beta)\big)}{16(1+\gamma)},$$

$$CS^B = \frac{(a-c)^2(2+\gamma)^2}{16(1+\gamma)} \quad \text{and} \quad SW^B = \frac{(a-c)^2(2+\gamma)(6-\gamma)}{16(1+\gamma)}.$$

3. Results

Having derived the equilibrium outcomes under Cournot and Bertrand competition respectively, we now summarise our main results below. First, we take up the case where the upstream firm's marginal cost of input production is low, i.e., $0 < c \leq c^*$ (see propositions 1-3) and next, we consider the case where it is high, i.e., $c^* < c < a$ (see proposition 4).

Proposition 1: *If* $0 < c \leq c^*$, *the equilibrium input price is lower whereas the equilibrium fixed-fee is higher under Cournot competition than under Bertrand competition where* $c^* = \frac{a\gamma^2}{4-\gamma^2}$.

Proof: $w_i^C - w_i^B = -\frac{a\gamma^2+c(4-\gamma^2)}{4} < 0$ and finally, it can be checked that $F_i^C - F_i^B = \frac{2a^2\beta+ac(1-\beta)(4-\gamma^2)}{2(2+\gamma)^2} - \frac{(a-c)^2(2+\gamma)(4\beta-2\beta\gamma-2\gamma^2+\gamma^3-\beta\gamma^3-\gamma^4+\beta\gamma^4)}{32(1+\gamma)} > 0$ for $0 < c \leq c^*$. ■

In line with Alipranti et al. (2014), we affirm that the input supplier's *commitment problem* as well as the *opportunistic behaviour* are severe under Cournot competition than under Bertrand competition. While the commitment problem prohibits the upstream agent to set the input price higher than its marginal cost, the opportunistic behaviour enables the upstream agent to appropriate a higher amount of fixed-fee from the downstream firms under

Cournot competition than under Bertrand competition. Hence, Cournot competition yields lower input price and higher fixed-fee in comparison to Bertrand competition.

Proposition 2: *If* $0 < c \leq c'$, *the equilibrium output is lower but the prices of the final-goods are higher under Cournot competition than under Bertrand competition. The opposite holds for* $c' < c \leq c^*$ *where* $c' = \frac{a\gamma^2}{(2+\gamma)^2}$ *and* $c^* = \frac{a\gamma^2}{4-\gamma^2}$.

Proof: $q_i^C - q_i^B = \frac{c(2+\gamma)^2 - a\gamma^2}{4(1+\gamma)(2+\gamma)} < 0$ and $P_i^C - P_i^B = \frac{a\gamma^2 - c(2+\gamma)^2}{4(2+\gamma)} > 0$ for $0 < c \leq c'$ whereas $q_i^C - q_i^B > 0$ and $P_i^C - P_i^B < 0$ for $c' < c \leq c^*$.■

The reason for the above result is as follows. Although each downstream firm faces a lower marginal cost of production under Cournot competition compared to Bertrand competition, the lower marginal cost under Cournot competition is not large enough to outweigh the effects of fierce competition under Bertrand competition compared to Cournot competition. However, if the marginal cost of input production is moderately low, i.e., $c' < c \leq c^*$, the lower marginal cost under Cournot competition is large enough to outweigh the effects of fierce competition under Bertrand competition compared to Cournot competition. In this situation, the output is higher and the price is lower under Cournot competition compared to Bertrand competition.

Proposition 3: *(a) If* $0 < c \leq c'$, *consumer surplus is lower under Cournot competition than under Bertrand competition. The opposite holds for* $c' < c \leq c^*$.

(b) If $0 < c \leq c''$, *social welfare is lower under Cournot competition than under Bertrand competition. The opposite holds for* $c'' < c \leq c^*$,

where $c'' = a - 4\sqrt{\frac{a^2(1+\gamma)(3+\gamma)}{(6-\gamma)(2+\gamma)^3}}$, $c' = \frac{a\gamma^2}{(2+\gamma)^2}$, $c^* = \frac{a\gamma^2}{4-\gamma^2}$ *such that* $c'' < c' < c^*$.

Proof: $CS^C - CS^B = \frac{(c(2+\gamma)^2 - a\gamma^2)\left(4(2a-c)(1+\gamma)+\gamma^2(a-c)\right)}{16(1+\gamma)(2+\gamma)^2} < (>)0$ for $0 < c \leq c'$, ($c' < c \leq c^*$ resp.) and $SW^C - SW^B = \frac{a^2(3+\gamma)}{(2+\gamma)^2} - \frac{(a-c)^2(2+\gamma)(6-\gamma)}{16(1+\gamma)} < (>)0$ for $0 < c \leq c''$, ($c'' < c \leq c^*$ resp.). ■

As follows from Proposition 2, a lower (higher) output and higher (lower) prices of the final goods under Cournot competition result in lower (higher) consumer surplus under Cournot competition compared to Bertrand competition for $0 < c \leq c'$, ($c' < c \leq c^*$ resp.).

Further, we show that Bertrand competition is socially desirable than Cournot competition if $0 < c \leq c''$. In this case, the loss in consumer surplus and upstream profit under Cournot competition compared to Bertrand competition outweighs the gains in downstream profits[1], thus creating an overall welfare loss under Cournot compared to Bertrand competition. However, if c is relatively high i.e., $c'' < c \leq c^*$, Cournot competition becomes more efficient than Bertrand competition (follows from proposition 2). In this case, the gains in consumer surplus and downstream profit outweigh the loss in upstream profit under Cournot competition compared to Bertrand competition, and create higher social welfare under the former case than the latter.

Next, we return to the case where $c^* < c < a$. To save the analytical repetition, we only report the results in the following proposition that are similar to Alipranti et al. (2014).

[1] As in Alipranti et al. (2014) we also get that $U\pi^C - U\pi^B < 0$ and $D\pi_i^C - D\pi_i^B > 0$ for $0 < c \leq c^*$. The reasoning is straightforward. The input price being lower under Cournot than Bertrand competition, the upstream firm earns lower profit and downstream firms make higher profit in the former case than the latter.

Proposition 4: *Assume that* $c^* < c < a$.

(a) The equilibrium input price and price of the final goods are lower under Cournot competition than under Bertrand competition whereas the opposite holds for equilibrium output.

(b) Cournot competition yields higher consumer surplus and social welfare than Bertrand competition.

Proof: *(a) See that* $w_i^C - w_i^B = -\frac{\gamma^2(a-c)(4-\gamma^2)}{4(2-\gamma^2)} < 0$, $P_i^C - P_i^B = -\frac{\gamma^3(a-c)}{4(2-\gamma^2)} < 0$ and $q_i^C - q_i^B = \frac{\gamma^3(a-c)}{4(1+\gamma)(2-\gamma^2)} > 0$ for $c' < c \leq c^*$.

(b) $CS^C - CS^B = \frac{\gamma^3(a-c)^2(8+4\gamma-4\gamma^2-\gamma^3)}{16(1+\gamma)(2-\gamma^2)^2} > 0$ *and* $SW^C - SW^B = \frac{\gamma^3(a-c)^2(8-4\gamma-4\gamma^2+\gamma^3)}{16(1+\gamma)(2-\gamma^2)^2} > 0$. ■

These results are shown in Alipranti et al. (2014) under the condition that upstream firm produces under zero marginal cost and it subsidises the quantity setting downstream firms' production via negative wholesale price. The intuitions of the above results are similar to Alipranti et al. (2014).

4. Conclusion

Alipranti et al. (2014) show that if a monopoly input supplier bargains with the final goods producers over a two-part tariff pricing contract to negotiate the input price, the upstream firm subsidises the quantity setting downstream firms' production via negative input prices, and Cournot competition generates higher welfare level than Bertrand competition. However, the assumption of negative input prices is not justifiable as it will encourage the downstream firms to buy an unbounded amount of inputs knowing that the upstream firm pays the downstream firms for each unit of input they purchase. Once we correct this problem and consider non-negative input prices, the welfare ranking may reverse.

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