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Abstract

This paper studies general equilibrium when workers in the economy are also consumers of final goods. Once a firm and a worker are matched, there is a standard moral hazard problem. However, the firm's profit depends on the price of the good the worker produces, and the price is determined by the total supply and demand in the economy. The worker's expected utility also depends on the number of units they consume and therefore depends on the price of the good. I characterize the set of equilibria and show that there is a unique equilibrium level of worker's outside option to price ratio. When the government changes minimum wage, the outside option for workers change through limited liability. In any equilibrium, the price responds proportionally to the change in minimum wage; the incentivized effort, the expected outcome, consumption and the expected utility of workers all remain exactly the same, and only the prices change as a result.

Keywords: general equilibrium, contracts, moral hazard, minimum wage.

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1 Introduction

Standard general equilibrium models don't allow for moral hazard in production; firms take capital and labor and turn them into outcomes. I study what happens if each firm hires a worker and there is moral hazard between the firm and the worker. I assume workers are consumers of the final good, and firms maximize their net profit, i.e., total revenue subtracted by the payment to the worker. In this model, each worker cares about their consumption utility and the cost of working. The wage gives them the budget constraint, and the worker cares about his cost (therefore the effort), the wage and the price of the good. Each firm cares about the price of the good, the outcome the worker produces and the payment to the worker.

I characterize the set of equilibria of this model and show that there is a unique equilibrium ratio of the worker's outside option to the price of the good. Within each firm-worker pair, the optimal contract and the effort it incentivizes are pinned down by any heterogeneous characteristics of the firm and the worker and this ratio of outside option to price. Since the ratio of outside option to price is uniquely pinned down, the optimal contract and the effort are determined by the firm characteristic and the worker characteristic; the only difference across equilibria with different outside options is the price, but the ratio has to be constant.

I explicitly characterize the optimal contract provided by each firm in an equilibrium. This allows me to do comparative statics of the optimal contract and effort with respect to the firm characteristic, the worker characteristic, the worker's outside option and the price of the good. The firm characteristic and the worker characteristic affect the outcome distribution and the cost.

One of the main applications of my results is the comparative statics with respect to the worker's outside option. Minimum wage set by the government works as the worker's outside option, and it imposes limited liability to the firm's optimization problem. In any equilibrium, the ratio of the worker's outside option to the price of the good is uniquely pinned down. When the minimum wage changes, the price changes in proportion so that the ratio remains constant. What this implies is that for a given firm-worker pair, the firm profit is scaled up, but the incentive problem is identical as before. The incentivized effort and the optimal contract offered by the firm don't change. The worker's expected utility also doesn't change. Higher minimum wage translates into proportional inflation of the price, and there is no change to the workers' welfare.

(related literature and comment on minimum wage)

The rest of the paper is organized as follows. Section 2 describes the model, and section 3 presents the results. Section 4 concludes.

2 Model

There are a unit mass of firms and a unit mass of workers. The joint distribution of firm-worker characteristic is given by $G(\theta)$ where $\theta \in [0,1]^2$, and I do not explicitly model the match process in this paper. When a firm and a worker are matched, the firm offers a contract that consists of outcome-contingent payments. If the worker accepts the contract, the worker exerts effort, but the effort is unobservable to the firm which leads to moral hazard. An outcome is realized and observed by both the firm and the worker. The firm makes the payment specified by the contract. I assume everything except for workers' efforts is common knowledge; there is no adverse selection.

Specifically, the worker chooses $e \in [0,\infty)$, and the disutility (cost) of working is $c(e|\theta)$ where $c_e > 0,\ c_{ee} \geq 0,\ c(0|\theta) = 0$ for all θ. Outcome is in $[\underline{y}, \bar{y}]$ and the pdf is $f(y|\theta, e)$ with no atoms and positive density everywhere. f is twice differentiable, and the cdf is denoted by F. I assume $\int y dF(y|\theta, e)$ strictly increases with e for all θ and $f(y|\theta, e)$ is weakly concave in e. Workers are subject to limited liability, i.e., $w(y|\theta) \geq \underline{w} \geq 0$. I assume that if the firm doesn't hire the worker, they both get 0.

The firm's profit depends both on the outcome y and the price of the good p. The firm maximizes its net profit which is the revenue subtracted by the payment to the worker: $\Pi(\theta, e) = \mathbb{E}_{y,p}[py - w(y|\theta)|\theta, e]$. The worker maximizes his expected utility which is $\mathbb{E}_{y,p}[u(\frac{w(y|\theta)}{p}) - c(e|\theta)|\theta, e]$. u is strictly increasing, concave and $u(0) = 0$.

There are three main differences from the standard model, and the first difference is that there is moral hazard between the firm and the worker. Instead of maximizing the revenue subtracted by the production cost, the firm maximizes the revenue subtracted by the payment to the worker.

The second difference is that the profit of the firm not only depends on the outcome the worker produces but also on the price in the market. The price is determined by the market clearing condition. In standard moral hazard models, firms only care about the level of outcome, which can be considered as p being constant.

Third, each worker also cares about the total number of unit consumption which depends on his wage and the price of the good. In models where the worker only supplies labor, their utilities are pinned down by wage, and in models where the consumer buys the good, they only care about the price of the good and their wealth. This model captures the simplest case where the workers are also the consumers of the good they produce.

The utility of the worker, $u(\frac{w(y|\theta)}{p}) - c(e|\theta)$, implicitly assumes that the worker doesn't value having extra money and spends it all on consumption. But this still endogenizes the total demand because the total demand is $\mathbb{E}[w(y|\theta)]$ in the economy.

An equilibrium satisfies the following conditions: (i) each firm offers an optimal contract, (ii) each worker optimally chooses his effort, (iii) the price satisfies the market clearing condition.

3 Results

There are two main results of the paper. I first characterize the set of equilibria of the model and show that the ratio of the worker's outside option to the price of the good is uniquely pinned down. The contract offered by the firm and the effort the agent chooses are pinned down by heterogeneous characteristics of the firm and the worker and the ratio of outside option to price. The second result is comparative statics with respect to the worker's outside option. I show that when the worker's outside option increases, the price increases proportionally so that the firm's profit scales up but the incentive provision and the worker's expected utility remain exactly the same.

Throughout the paper, $\int_+ h(y)dF(y|\theta,e) = \int_{f_e(y|\theta,e)>0} h(y)f(y|\theta,e)dy$.

Theorem 1. *Suppose the outside option for workers is a binding constraint for a positive measure of firms. Given the distributions F, G and the worker's cost and utility functions c, u, there exists a unique ratio of $\frac{\underline{w}}{p^*}$ across all equilibria. Let p^* be the equilibrium price. The optimal effort for firm θ is pinned down by*

$$\arg\max_e \int ydF(y|\theta,e) - \frac{\underline{w}}{p^*} - \int_+ \max\{0, (u')^{-1}(\frac{f(y|\theta,e)}{\lambda(\frac{\underline{w}}{p^*},\theta,e)f_e(y|\theta,e)}) - \underline{w}\}dF(y|\theta,e),$$

and the firm offers

$$w(y|\theta) = \begin{cases} \max\{\underline{w}, p^*(u')^{-1}(\frac{f(y|\theta,e)}{\lambda f_e(y|\theta,e)})\} & \text{if } f_e(y|\theta,e) > 0 \\ \underline{w} & \text{if } f_e(y|\theta,e) \leq 0 \end{cases}$$

where the Lagrange multiplier $\lambda(\frac{\underline{w}}{p^*}, \theta, e)$ *is uniquely pinned down by*

$$\int u(\frac{w(y|\theta)}{p^*}) f_e(y|\theta, e) dy = c'(e|\theta).$$

Sharper results with additional assumptions are stated after the proof of Theorem 1. Theorem 1 shows that there exists a unique equilibrium ratio $\frac{\underline{w}}{p^*}$ for given primitives of the model, and furthermore, when the worker's outside option $\underline{w}$ changes, the equilibrium price p^* changes in proportion, so that $\frac{\underline{w}}{p^*}$ is constant across all equilibria. The incentivized effort depends on $\underline{w}, p^*$ only through the ratio of the two. The optimal contract provides payments as a function of likelihood ratio.

Proof of Theorem 1. Suppose the characteristic of the firm-worker pair is θ and the equilibrium price is p^*.[1] The firm's problem is

$$\begin{aligned} &\max_{w(\cdot),e} \int p^* y - w(y|\theta) dF(y|\theta, e) \\ s.t. \quad &\int u(\frac{w(y|\theta)}{p^*}) dF(y|\theta, e) - c(e|\theta) \geq 0 \\ &\int u(\frac{w(y|\theta)}{p^*}) dF(y|\theta, e) - c(e|\theta) \geq \int u(\frac{w(y|\theta)}{p^*}) dF(y|\theta, e') - c(e'|\theta), \ \forall e' \\ &w(y|\theta) \geq \underline{w} \end{aligned}$$

From $\underline{w} \geq 0$ and the IC for deviating to $e' = 0$, we don't need to worry about IR. IC becomes

$$\begin{aligned} &\int u(\frac{w(y|\theta)}{p^*}) dF(y|\theta, e) - c(e|\theta) \geq \int u(\frac{w(y|\theta)}{p^*}) dF(y|\theta, e') - c(e'|\theta), \ \forall e' \\ \Leftrightarrow &\int u(\frac{w(y|\theta)}{p^*})(f(y|\theta, e) - f(y|\theta, e')) dy \geq c(e|\theta) - c(e'|\theta), \ \forall e'. \end{aligned}$$

[1] If there is a distribution of prices in an equilibrium, we need to be more careful with taking the worker's expected utility. However, from the market clearing condition, when there is a continuum of firms and workers, the price is a degenerate distribution.

From the concavity of f, local IC is sufficient for all IC, and first-order approach is valid: IC becomes

$$\int u(\frac{w(y|\theta)}{p^*})f_e(y|\theta,e)dy = c'(e|\theta)\ \forall e \neq 0.$$

If the firm wants to induce $e=0$, it can provide $w(y|\theta) = \underline{w}$ for all y.

I will first characterize the minimum expected payment for the given level of effort. Since the first-order approach is valid for all payments, we just need to minimize the payment for the principal subject to IC and limited liability:

$$\begin{aligned} &\min \int w(y|\theta)dF(y|\theta,e) \\ s.t. \quad &\int u(\frac{w(y|\theta)}{p^*})f_e(y|\theta,e)dy = c'(e|\theta) \\ &w(y|\theta) \geq \underline{w} \end{aligned}$$

Rewriting $v_y = u(\frac{w(y|\theta)}{p^*})$, we can express the principal's problem as

$$\begin{aligned} &\min \int u^{-1}(v_y)dF(y|\theta,e) \\ s.t. \quad &\int v_y f_e(y|\theta,e)dy = c'(e|\theta) \\ &v_y \geq u(\frac{\underline{w}}{p^*}). \end{aligned}$$

First, note that the optimal solution v_y only depends on the ratio $\frac{\underline{w}}{p^*}$ and not on individual level of $\underline{w}, p^*$. The first-order condition becomes

$$\frac{1}{u'(u^{-1}(v_y))}f(y|\theta,e) = \lambda f_e(y|\theta,e) + \mu_y$$

where $\lambda, \mu_y \geq 0$, and $\mu_y = 0$ if $w(y|\theta,e) > \underline{w}$. Essentially, we need

$$\begin{aligned} &\frac{1}{u'(u^{-1}(v_y))}f(y|\theta,e) = \lambda f_e(y|\theta,e) \text{ if } w(y|\theta) > \underline{w} \\ &\frac{1}{u'(u^{-1}(v_y))}f(y|\theta,e) \geq \lambda f_e(y|\theta,e) \text{ if } w(y|\theta) = \underline{w} \end{aligned}$$

and

$$w(y|\theta) = \begin{cases} \max\{\underline{w}, p^*(u')^{-1}(\frac{f(y|\theta,e)}{\lambda f_e(y|\theta,e)})\} & \text{if } f_e(y|\theta,e) > 0 \\ \underline{w} & \text{if } f_e(y|\theta,e) \leq 0. \end{cases} \tag{1}$$

λ is pinned down by the first-order condition

$$\int u(\frac{w(y|\theta)}{p^*})f_e(y|\theta,e)dy = c'(e|\theta)$$

and has to be greater than 0 for all $e > 0$. When u is strictly concave, the left-hand side strictly increases with λ, there is a unique λ for given p^*, e and the primitives of the model. The solution always exists if $\lim_{c\to\infty} u(c) \to \infty$, and if there is no solution, the principal cannot implement the particular e.

The firm profit for implementing e is

$$\Pi(\theta,e) = \int p^* y dF(y|\theta,e) - \underline{w} - \int_+ \max\{0, p^*(u')^{-1}(\frac{f(y|\theta,e)}{\lambda(\frac{\underline{w}}{p^*},\theta,e)f_e(y|\theta,e)}) - \underline{w}\}dF(y|\theta,e)$$

and the optimal effort is

$$\arg\max_e \int ydF(y|\theta,e) - \frac{\underline{w}}{p^*} - \int_+ \max\{0,(u')^{-1}(\frac{f(y|\theta,e)}{\lambda(\frac{\underline{w}}{p^*},\theta,e)f_e(y|\theta,e)}) - \frac{\underline{w}}{p^*}\}dF(y|\theta,e) \tag{2}$$

From the market clearing condition, we have

$$p^*\int ydF(y|\theta,e)dG(\theta) = \underline{w} + \int\int_+ \max\{0,p^*(u')^{-1}(\frac{f(y|\theta,e)}{\lambda(\frac{\underline{w}}{p^*},\theta,e)f_e(y|\theta,e)}) - \underline{w}\}dF(y|\theta,e)dG(\theta)$$

$$\Leftrightarrow \quad \int ydF(y|\theta,e)dG(\theta) - \frac{\underline{w}}{p^*} \tag{3}$$

$$-\int\int_+ \max\{0,(u')^{-1}(\frac{f(y|\theta,e)}{\lambda(\frac{\underline{w}}{p^*},\theta,e)f_e(y|\theta,e)}) - \frac{\underline{w}}{p^*}\}dF(y|\theta,e)dG(\theta) = 0. \tag{4}$$

Comparative statics of optimal $\int u^{-1}(v_y)dF(y|\theta,e)$ for given e,θ can be obtained by the envelope theorem:

$$\frac{d}{d\frac{\underline{w}}{p^*}}: \int(\frac{1}{u'(u^{-1}(v_y))}f(y|\theta,e) - \lambda f_e(y|\theta,e))u'(\frac{\underline{w}}{p^*})dy \geq 0.$$

It is 0 if and only if limited liability is not binding. If the outside option for workers is a binding constraint for a positive measure of firms, then the left-hand side of (4) is a strictly decreasing function of $\frac{\underline{w}}{p^*}$, and there is a unique equilibrium level of $\frac{\underline{w}}{p^*}$.

The optimal contract offered by each firm is given in (1), and the optimal effort to implement is given by (2). □

I will first state the main implication of Theorem 1 then discuss the theorem in more detail. Theorem 1 implies the following comparative statics with respect to the worker's outside option.

Theorem 2. *Suppose the government changes the minimum wage from $\underline{w}$ to $\underline{w}'$. The unique equilibrium price changes from p^* to $p^{*\prime}$, but*

$$\frac{\underline{w}}{p^*} = \frac{\underline{w}'}{p^{*\prime}}.$$

The incentivized effort for each firm, expected outcome, consumption and the expected utility for each worker are the same in both equilibria. The price, payments and the firm profit change proportionally to the minimum wage.

Proof. From Theorem 1, the ratio $\frac{\underline{w}}{p^*}$ has to be constant. (2) only depends on the ratio, and (1) shows that the payments are scaled up in proportion. When the incentivized effort doesn't change, the expected outcome also doesn't change. Since the price and wages increase in proportion to the minimum wage, the consumption level for each worker remains the same. Each worker's expected utility only depends on his consumption and his cost of effort, and therefore, the expected utility of workers are the same. Plugging everything back into the firm profit, one can see that the firm profit increases in proportion. □

Theorem 2 shows that if (i) there is moral hazard between each firm and a worker, (ii) workers are consumers of the good, (iii) firm revenue depends on the equilibrium price, then any change in minimum wage is absorbed by proportional changes in prices and wages, and nothing else in the economy changes. Incentive provision, production and the workers' welfare are exactly the same. My model provides an alternative explanation as to why the scope of minimum wage has been limited. One can also draw the same conclusion from Melitz (2003). Compared to Melitz (2003), my result shows that the intuition is robust to moral hazard on the production side and linking the labor market with consumption.

Theorem 1 doesn't tell us whether there is a unique optimal effort to incentivize for each firm, whether the firm's optimization problem over effort is concave and so forth. With further assumptions, one can state sharper results.

Theorem 3. *Suppose*

$$\int y f_{ee}(y|\theta,e)dy + \lambda(\frac{w}{p^*},\theta,e)(\int u(\frac{w(y|\theta)}{p^*})f_{eee}(y|\theta,e)dy - c_{eee}(e|\theta))$$
$$+\frac{\partial\lambda}{\partial e}(\frac{w}{p^*},\theta,e)(\int u(\frac{w(y|\theta)}{p^*})f_{ee}(y|\theta,e)dy - c_{ee}(e|\theta)) \leq 0 \ \forall e.$$

where $w(y|\theta), \lambda(\frac{w}{p^*},\theta,e)$ correspond to the optimal contract in Theorem 1. The firm's problem over effort is concave, and each firm incentivizes either the critical point or $e=0$. In particular, if

$$\frac{\partial}{\partial e}\Big(\frac{f(y|\theta,e)}{f_e(y|\theta,e)}\Big) \geq 0, \ \ f_{eee} \leq 0, \ \ c_{eee} \geq 0,$$

the firm's problem is concave and there is a unique optimal effort for each θ.

Proof. From the envelope theorem, we have the following comparative statics of the minimum expected payment with respect to e:

$$\frac{d}{de} : -p^*\lambda(\frac{w}{p^*},\theta,e)(\int u(\frac{w(y|\theta)}{p^*})f_{ee}(y|\theta,e)dy - c_{ee}(e|\theta)),$$

and the theorem follows from differentiating it with respect to e once more. When

$$\frac{\partial}{\partial e}\Big(\frac{f(y|\theta,e)}{f_e(y|\theta,e)}\Big) \geq 0,$$

we have $\partial\lambda/\partial e \geq 0$, and together with the third derivatives, the condition is satisfied. □

Theorem 3 is a condition on third derivatives, but this is because the optimal payments depend on the first derivatives of the distribution and the cost. In order to verify whether the firm's problem over effort is concave, we need to know how payments vary with the effort, and it has to involve the third derivatives.

Theorem 4. *Suppose $F(y|\theta,e)$ satisfies the monotone likelihood ratio property for* *each θ, i.e.,*

$$\frac{\partial}{\partial y}\Big(\frac{f_e(y|\theta,e)}{f(y|\theta,e)}\Big) \geq 0.$$

There exists $y(\theta, e, p^*)$ such that the optimal contract provides

$$w(y|\theta,e) = \begin{cases} \underline{w} & \text{if } y \leq y(\theta,e,p^*) \\ p^*(u')^{-1}(\frac{f(y|\theta,e)}{\lambda f_e(y|\theta,e)}) & \text{if } y > y(\theta,e,p^*). \end{cases}$$

Proof. Since u is strictly concave, we know u' is decreasing and $(u')^{-1}$ is also decreasing. Together with $\lambda > 0$, we get the theorem. □

If the outcome distribution satisfies the monotone likelihood ratio property, then the optimal contract has a cutoff on outcomes below which the worker gets paid his outside option and above which the payment is a function of the likelihood ratio.

4 Conclusion

I study general equilibrium with three properties in this paper. Within each firm, there is moral hazard between the firm and the worker, and the firm revenue depends on the equilibrium price and the payment to the worker; the worker's utility depends on his wage, equilibrium price and the cost of effort. I characterize the optimal contract and the incentivized effort for each firm which depends on the firm-worker characteristic and the ratio of worker's outside option to equilibrium price. Then I show that there exists a unique ratio of the worker's outside option to equilibrium price across all equilibria.

The main implication of my model is that variations in minimum wage, which works as the worker's outside option within each firm-worker pair, translates into proportional changes in prices and wages. Once the price effect is taken into account, the production side and the consumption side remain exactly the same when minimum wage changes. My model is in a static setting, but since the same insight can be also drawn from Melitz (2003) without moral hazard but in a dynamic setting, the insight is robust to frictions in the market.

There are sufficient conditions for the firm's problem to be concave, and if the outcome distribution satisfies the monotone likelihood ratio property, we restore familiar conditions that provides the outside option for low outcomes and higher outcomes are paid as a function of the likelihood ratio. Other comparative statics with respect to the firm-worker characteristic is also possible.

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