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Abstract

We study the effect of technological progress on the optimal transition to a renewable energy-fueled world economy. We develop a dynamic general equilibrium model where energy is used as an input in production and can come from fossil or renewable sources. Both require the use of capital, which is also needed in the production of final goods. Renewable energy firms can invest in improving the productivity of their capital stock. The actual improvement is subject to spillovers and involves an opportunity cost. This results in underinvestment in the productivity of renewable energy capital. In the presence of environmental externalities, the optimal allocation can be implemented through a Pigouvian tax on fossil fuel, together with policy that promotes new renewable technologies. We calibrate our model using world-economy data and characterize the transition toward a low carbon economy. We find that it is optimal for renewables to "start small" and pick up their market penetration only later. In the short run, investment is needed mainly to improve productivity in the renewable energy sector. Later, renewable energy contributes by becoming a "modest" engine of economic growth. It takes approximately 150 years before fossil fuel is phased out entirely, resulting in a 2.8 degree Celsius temperature increase.

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1 Introduction

Economic growth generates a tremendous demand for energy. Historically, this need has been met largely through the use of fossil fuel. In recent decades, renewable energy sources, such as solar and wind, have been increasing their representation in many nations' energy supply. While renewables are still too costly to directly compete with fossil fuel sources in many areas, the transition toward renewable energy is expected to accelerate as concerns about climate change become more prevalent and as technological progress reduces costs.

A common belief holds that markets currently underinvest in renewable energy.[1] This argument can take many different forms. Underinvestment might refer to resources spent on innovation and R&D, or on actual installation and usage of facilities that harvest renewable energy. Similarly, the reasons for under-investment can range from the externalities associated with climate change to spillovers associated with innovation. In addition, renewable energy is often hailed as a potential engine of economic growth. To better understand the connection between innovation, renewable energy supply, and economic growth, and in order to evaluate alternative policies, we need to have a handle on the rate at which declining costs will lead to increased competitiveness for renewables. What determines the productivity improvements in renewable energy production and how does the rate of these improvements respond to policy? What are the consequences for the fossil fuel sector and for the optimal transition toward renewable energy-fueled growth? Traditionally, most economic analysis of energy and environmental issues focuses on computable general equilibrium models which as a rule, abstract from endogenous technological progress.[2] Our paper attempts to shed light on the above questions in the context of a dynamic general equilibrium model where technological progress in the renewable sector is endogenous.

We investigate the optimal transition from a mainly fossil-fueled to a mainly renewable energy-fueled world economy. This transition depends on several factors. First, although fossil fuel sources are plentiful, they constitute an exhaustible resource. A second and more binding consideration involves environmental concerns. As fossil fuel generates externalities due to an increasing stock of carbon emissions, the need for a clean substitute becomes more prevalent. We model two important considerations in the process of innovation in the renewable energy sector. First, spillovers imply that investment in renewable technology has both a direct and an indirect effect on innovation. Second, investing in new technologies involves an opportunity cost, as some resources are diverted toward innovation in order to make future renewable energy supply less costly. This, in turn, reduces the temporary production of renewable energy.

We develop a model where energy is an input in the production of a consumption good. Energy can be produced from either fossil or renewable sources. Both require capital, which is also needed for the production of the final good. At each point in time, renewable energy-producing firms can improve their productivity. The actual improvement is subject to a

[1] For example, IPCC (2012) argues that "Additional policies would be required to attract the necessary increases in investment in (renewable) technologies and infrastructure."

[2] See, for example, Nordhaus and Boyer (2000) and references therein.

spillover, as it depends on the aggregate investment in the renewable sector. The spillover effect leads to an overall under-investment in renewable energy. At the same time, the optimal mix of energy supply involves a declining use of fossil fuel. We characterize the optimal transition toward a renewable energy-fueled economy. The optimal intertemporal allocation can be implemented through a policy that promotes renewables, together with a Pigouvian tax on the environmental externality created by fossil fuel use. We show that the optimal policy for promoting renewables can be revenue-neutral. We then calibrate the model using world economy data. We find that it is optimal for renewables to "start small" and pick up their market penetration only later. In the short run, investment is needed mainly to improve productivity in renewable energy production. Later, renewable energy contributes by becoming a modest engine of economic growth. It takes approximately 150 years before fossil fuel is phased out completely, resulting in a 2.8 degree Celsius temperature increase for the planet. The optimal carbon tax is constant at about $17,000/ton of carbon equivalent.

1.1 Related Literature

Our paper contributes to the small but growing literature on energy transitions and on energy-related innovation and growth. Nordhaus (1994) was a pioneer in introducing climate factors in dynamic economic modeling, but he abstracts from technological progress. Parente (1994) studies a model in which firms choose to adopt new technologies as they gain specific expertise through learning-by-doing. He identifies conditions under which equilibria exhibit constant growth of per capita output. As in most of the literature on innovation and growth, Parente abstracts from issues related to energy and the environment, which are the focus of our study. Acemoglu et al. (2012) study a growth model that takes into consideration the environmental effect from operating "dirty" technologies. They consider policies that tax innovation and production in the dirty sectors. They find that subsidizing research in the "clean" sectors can speed up environmentally friendly innovation without the corresponding slowdown in economic growth. Consequently, optimal behavior in their model implies an immediate increase in clean energy R&D, followed by a complete switch toward the exclusive use of clean inputs in production. The implied opportunity costs of early adoption in our model lead to a different conclusion. We find that renewable energy should be adopted only gradually, as it becomes more productive. Manuelli and Seshadri (2014) study technology diffusion of tractors in American agriculture during the first part of the 20th century. They argue that part of the reason for the slow rate of adoption was that tractor quality kept improving over that period. As a result, farmers chose to postpone their purchase, rather than investing in a tractor that would soon become obsolete. Our paper has a similar message in the context of energy supply. Atkeson and Burstein (2015) study the impact of policy-induced changes in innovative investment and the implications for medium-and long-run innovation and growth. They too abstract from energy and environmental considerations.

Golosov, Hassler, Krusell, and Tsyvinski (GHKT, 2014) build a dynamic general equilibrium model that incorporates the use of energy and the resulting environmental consequences.

They derive a formula describing the optimal tax due to the externality from carbon emissions and provide numerical values for the size of the tax. They, however, abstract from the costs associated with endogenous technological progress. We will employ several elements from their work in what follows, including the modeling of the environmental externality. Acemoglu, Akcigit, Hanley, and Kerr (2012) use the structure in GHKT to study questions related to the transition to clean technologies. They employ a "ladder" model to study technological progress in both the clean and the dirty sectors, and they estimate their model using R&D and patent data. They find that, in addition to carbon taxes, quantitatively significant R&D subsidies are a necessary ingredient of optimal policy. The reason is that subsidies encourage technological progress without overtaxing short-run future output. More recently, van der Ploeg and Rezai (2016) extend the model in GHKT in several ways. They allow for general fossil fuel extraction costs, a negative impact of global warming on growth, mean reversion in climate damages, labor-augmenting and green technology progress, and a direct effect of the emissions stock on welfare. They characterize the social optimum, as well as the optimal carbon tax and the renewable energy subsidies.[3]

The paper proceeds as follows. Section 2 introduces the model. Section 3 discusses efficiency and optimal policy. Section 4 introduces our calibration and quantitative findings. A brief conclusion follows. Some of the more technical material appears in the Appendix.

2 The Economic Model

We assume discrete time and infinite horizon, $t = 0, 1, \ldots$. There is a single final consumption good per period, and all markets are competitive. The economy is populated by a representative infinite-lived household. The household discounts the future at rate $\beta \in (0, 1)$ and values period-t consumption, c_t, through a utility function $u(c_t)$. We assume that u is smooth, strictly increasing, strictly concave, and that the usual Inada conditions hold. The labor endowment is normalized to 1, and labor is supplied inelastically. There are three different kinds of firms, all owned by the household. In each period, the household chooses how much capital, k, to rent in the market at rate r and receives all profits resulting from firms' activities. All capital depreciates at rate $\delta \in (0, 1)$.

The final good-producing representative firm uses capital, k, labor, l, and energy, e, in order to produce output. In addition, we assume that environmental quality, Γ, can affect production through a damage function $D(\Gamma)$. The final good production function is, thus, given by

[3] Other related papers include Stokey (1998), who considers growth under environmental constraints; Goulder and Schneider (1999), who study endogenous innovations in abatement technologies; Van der Zwaan et al. (2002), who study the impact of environmental policies in a model with learning-by-doing; and Popp (2004), who studies innovation in the energy sector and the costs of environmental regulation. See also Hartley et al. (2012), who study technological progress and the optimal energy transition, and Van der Ploeg and Withagen (2011), who study the possibility of a *green paradox*.

$$
\begin{aligned}
y_t &\leq A_t(k_t^g)^{\theta_k}(l_t)^{\theta_l}(e_t)^{1-\theta} = (1 - D_t(\Gamma_t))\left[\widetilde{A}_t(k_t^g)^{\theta_k}(l_t)^{\theta_l}(e_t)^{1-\theta}\right] \\
&= \exp\left[-\pi_t\left(\Gamma_t - \overline{\Gamma}\right)\right]\widetilde{A}_t\left[(k_t^g)^{\theta_k}(l_t)^{\theta_l}(e_t)^{1-\theta}\right],
\end{aligned} \tag{1}
$$

where $\widetilde{A}$ is a productivity parameter, $A_t = (1 - D_t(\Gamma))\widetilde{A}_t$, while θ, θ_k, $\theta_l \in (0,1)$, and $\theta_k + \theta_l = \theta$. Following Golosov, Hassler, Krusell, and Tsyvinski (GHKT, 2014), we assume that

$$
D_t(\Gamma) = 1 - \exp\left[-\pi_t\left(\Gamma_t - \overline{\Gamma}\right)\right], \tag{2}
$$

where $\overline{\Gamma}$ is the pre-industrial greenhouse gas concentration in the atmosphere, and π is a random variable that parametrizes the effect of higher greenhouse gas concentrations on the level of damages. Environmental quality evolves according to

$$
\Gamma_t = \sum_{n=0}^{t+T}(1 - d_n)f_{t-n}, \tag{3}
$$

where $d_n \in [0,1]$, and f_n indicates the fossil fuel used in period n. Assuming that a fraction φ_L of emitted carbon stays in the atmosphere forever, while a fraction $(1 - \varphi_0)$ of the remaining emissions exits into the biosphere and the remaining part decays at geometric rate φ, we obtain

$$
\begin{aligned}
1 - d_n &= \varphi_L + (1 - \varphi_L)\varphi_0(1 - \varphi)^n \\
\Gamma_t^p &= \Gamma_{t-1}^p + \varphi_L f_t \\
\Gamma_t^d &= (1 - \varphi)\Gamma_{t-1}^d + (1 - \varphi_L)\varphi_0 f_t \\
\Gamma_t &= \Gamma_t^p + \Gamma_t^d.
\end{aligned} \tag{4}
$$

Energy can be produced by using fossil or renewable sources. We assume that the two types of energy are perfect substitutes in the production of the final good.[4] We let w_t denote the available stock of fossil fuel in period t, while, as mentioned above, f_t denotes the fossil fuel used in energy production. Thus, the law of motion for the stock of fossil fuel is $w_{t+1} \leq w_t - f_t$. The fossil-fuel-derived energy production function is given by

$$
e_t^f \leq (f_t)^{1-\alpha_f}\left(k_t^f\right)^{\alpha_f}, \tag{5}
$$

where $\alpha_f \in (0,1)$. Thus, to produce energy through fossil fuel we need to use fossil fuel and capital. This specification captures the feature that, by using additional capital, we can produce more energy from the remaining fossil fuel reserves.

[4] A high substitutability across energy seems a reasonable benchmark in a model like ours, where we concentrate on long-run effects. For similar reasons, our analysis abstracts from short-run fluctuations in the supply and demand for energy and from the corresponding short-run volatility in energy prices.

We assume a competitive sector of renewable energy-producing firms. As these firms are heterogenous, we will need to keep track of the identity of each individual firm. The renewable energy production function for firm j is given by

$$e^r_{j,t} \leq \Psi(i_{j,t}) \left(\mathcal{E}_{j,t}\right)^{1-\alpha_r} \left(k^r_{j,t}\right)^{\alpha_r}, \tag{6}$$

where $\mathcal{E}_{j,t}$ is the firm's productivity parameter, and $\alpha_r \in (0,1)$. We interpret $i_{j,t}$ as the *new technology adoption rate* by renewable firm j in period t. This specification captures the feature that the process of adopting a new technology can result in interrupting production using the old method, thus reducing output. In other words, new technology adoption has an opportunity cost in terms of a contemporaneous output loss, but it boosts future productivity in the renewable energy sector.[5] We capture the technology adoption cost by reducing firm j's current output by a factor $\Psi(i_{j,t})$, where $\Psi(\cdot)$ is such that $\Psi(0) = 1$, $\Psi'(\cdot) < 0$, $\Psi''(\cdot) < 0$, and $\Psi(i) = 0$, for $i = \bar{i}$.

We will assume that there is a *spillover effect*, as aggregate technology adoption also affects the productivity of each individual firm. Put differently, as more firms adopt new technologies, the benefits affect the entire sector. This feature creates an externality, leading to a discrepancy between equilibrium and desirable levels of new renewable energy adoption. We consider this effect especially relevant, as investments in energy tend to be capital intensive. Thus, if innovators do not expect to capture the resulting returns, under-adoption of new technologies is likely to occur.[6] We assume that the productivity of each renewable energy firm depends on its own adoption, as well as on the aggregate level of adoption by all renewable-energy firms. More precisely, the productivity of firm j evolves deterministically according to

$$\ln \mathcal{E}_{j,t+1} \leq \xi i_{j,t} + (1-\xi) \left(\int_0^1 i_{j,t} k^r_{j,t} dj \Big/ \int_0^1 k^r_{j,t} dj \right) + \ln \mathcal{E}_{j,t}, \tag{7}$$

where $0 \leq \xi \leq 1$ parametrizes the strength of the spillover effect. For example, $\xi = 1$ corresponds to the case where there are no spillovers, while $\xi = 0$ describes the case where productivity is entirely determined by spillovers. In the above expression, we normalize total technology adoption by capital stock in order to abstract from any size-dependent advantage to firms. Finally, total capital used in the economy cannot exceed the total supply; i.e., for all t,

$$k^g_t + k^f_t + \int_0^1 k^r_{j,t} dj \leq k_t. \tag{8}$$

[5] Admittedly, innovation also takes place in the fossil fuel sector. Mainly for simplicity, in this paper we will concentrate on technological progress in the renewable sector.

[6] Bosettia et al. (2008) argue that international knowledge spillovers tend to increase the incentive to free-ride, thus decreasing investments in energy R&D. Braun et al. (2009) perform an empirical study of spillovers in renewable energy. They document significant domestic and international knowledge spillovers in solar technology innovation, as well as significant international spillovers in wind.

We begin by characterizing efficient allocations. These allocations solve the following social planner's problem.

$$\max_{\{c_t,k_{t+1},k_t^g,k_t^f,w_{t+1}^f,f_t,e_t^f,\Gamma_t^p,\Gamma_t^d,\{i_{j,t+1},k_{j,t}^r,\mathcal{E}_{j,t+1},e_{j,t}^r\}_{j=0}^1\}_{t=0}^\infty} E\sum_{t=0}^{\infty}\beta^t u(c_t)$$

$$\text{s.t. } c_t + k_{t+1} \leq A_t (k_t^g)^{\theta_k}(l_t)^{\theta_l}(e_t)^{1-\theta} + (1-\delta)k_t : \mu_{R,t}$$

$$w_{t+1} \leq w_t - f_t : \mu_{W,t}$$

$$\Gamma_t^p = \Gamma_{t-1}^p + \varphi_L f_t : \mu_{\Gamma_t^p}$$

$$\Gamma_t^d = (1-\varphi)\Gamma_{t-1}^d + (1-\varphi_L)\varphi_0 f_t : \mu_{\Gamma_t^d}$$

$$e_{j,t}^r \leq \Psi(i_{j,t})\left(\mathcal{E}_{j,t}\right)^{1-\alpha_r}\left(k_{j,t}^r\right)^{\alpha_r} : \left(\mu_{r,t}^j dj\right),$$

$$e_t^f \leq (f_t)^{1-\alpha_f}\left(k_t^f\right)^{\alpha_f} : \mu_{F,t}$$

$$k_t \geq k_t^g + k_t^f + \int_0^1 k_{j,t}^r dj : \mu_{K,t}$$

$$\ln \mathcal{E}_{t+1}^j \leq \xi i_{j,t} + (1-\xi)\left(\int_0^1 i_{j,t}k_{j,t}^r dj \Big/ \int_0^1 k_{j,t}^r dj\right) + \ln \mathcal{E}_{j,t} : \left(\mu_{\mathcal{E},t}^j dj\right)$$

$$e_t \leq e_t^f + \int_0^1 e_{j,t}^r dj : \mu_{E,t}$$

$$0 \leq f_t : \mu_{f_t}$$

$$k_{t+1} \geq 0, w_{t+1} \geq 0, \text{ all } t$$

$$k_0 > 0, \mathcal{E}_{j,t} > 0, w_0 > 0, \text{ given.}$$

The μ-variables indicate the corresponding Lagrange multipliers. The first-order conditions (FOCs), which are also sufficient, for the planner's problem are:

$$\partial c_t : \beta^t u'(c_t) = \mu_{R,t} \tag{9}$$

$$\partial k_{t+1} : -\mu_{R,t} + (1-\delta)E_t\mu_{R,t+1} + E_t\mu_{K,t+1} = 0 \tag{10}$$

$$\partial w_{t+1} : \mu_{W,t} = E_t\mu_{W,t+1} \tag{11}$$

$$\partial \Gamma_t^p : \mu_{\Gamma_t^p} - E_t\mu_{\Gamma_{t+1}^p} - \pi y_t \mu_{R,t} = 0 \tag{12}$$

$$\partial \Gamma_t^d : \mu_{\Gamma_t^d} - (1-\varphi)E_t\mu_{\Gamma_{t+1}^d} - \pi y_t \mu_{R,t} = 0 \tag{13}$$

$$\partial f_t : -\mu_{W,t} + \mu_{F,t}(1-\alpha)\left(\frac{k_t^f}{f_t}\right)^{\alpha_f} - \mu_{\Gamma_t^p}\varphi_L - \mu_{\Gamma_t^d}(1-\varphi_L)\varphi_0 + \mu_{f_t} = 0 \tag{14}$$

$$\partial e^r_{jt} : \mu^j_{r,t} dj = \mu_{E,t}, \text{ or, } \int \mu^j_{r,t} dj = \mu_{E,t}. \tag{15}$$

Note that the marginal utility of having a firm producing an extra infinitesimal amount of renewable energy should be equal across firms; i.e., $\mu^j_{r,t} = \mu^m_{r,t}$ for any two firms j and m. We then have

$$\partial e^f_t : -\mu_{F,t} + \mu_{E,t} = 0 \tag{16}$$

$$\partial e_t : \mu_{R,t} (1-\theta) A_t \frac{(k^g_t)^{\theta_k} (L_t)^{\theta_l}}{(e_t)^{\theta}} = \mu_{E,t} \tag{17}$$

$$\partial k^g_t : \mu_{R,t} \theta_k A_t (k^g_t)^{\theta_k - 1} (L_t)^{\theta_l} (e_t)^{1-\theta} = \mu_{K,t} \tag{18}$$

$$\partial k^f_t : \mu_{F,t} \alpha_f \left(\frac{f_t}{k^f_t} \right)^{1-\alpha_f} = \mu_{K,t} \tag{19}$$

$$\partial k^r_{j,t} : (1-\xi) \left(\frac{i_{j,t} - \overline{i_t}}{\overline{k^r_t}} \right) \int_0^1 \mu^m_{\mathcal{E},t} dm + \Psi(i_{j,t}) \alpha_r \mu^j_{r,t} \left(\frac{\mathcal{E}_{jt}}{k^r_{jt}} \right)^{1-\alpha_r} = \mu_{K,t}, \tag{20}$$

where $\overline{k^r_t} = \int k^r_{t,m} dm$, and $\overline{i_t} = \int_0^1 i_{j,t} k^r_{j,t} dj / \int_0^1 k^r_{j,t} dj$.

As from (15) we have that $\mu^j_{r,t} = \mu_{E,t}$, (20) implies that $i_{j,t}$ is a function of $\frac{\mathcal{E}_{jt}}{k^r_{jt}}$ only. Another first order condition gives

$$\partial i_{j,t} : \Psi'(i_{j,t}) (\mathcal{E}_{j,t})^{1-\alpha_r} \left(k^r_{j,t}\right)^{\alpha_r} \mu^j_{r,t} dj + \xi \mu^j_{\mathcal{E},t} dj + (1-\xi) \frac{k^r_{j,t}}{\overline{k^r_t}} dj \int_0^1 \mu^m_{\mathcal{E},t} dm = 0, \tag{21}$$

which implies

$$\partial i_{j,t} : -\mu^j_{r,t} \Psi'(i_{j,t}) \left(\frac{\mathcal{E}_{j,t}}{k^r_{j,t}} \right)^{1-\alpha_r} = \xi \frac{\mu^j_{\mathcal{E},t}}{k^r_{j,t}} + (1-\xi) \frac{\int_0^1 \mu^m_{\mathcal{E},t} dm}{\overline{k^r_t}}. \tag{22}$$

As $\mu^j_{r,t} = \mu_{E,t} = \mu_{F,t}$, and $i_{j,t}$ is only a function of $\frac{\mathcal{E}_{jt}}{k^r_{jt}}$, we have that $\frac{\mu^j_{\mathcal{E},t}}{k^r_{j,t}}$ is also a function of $\frac{\mathcal{E}_{jt}}{k^r_{jt}}$. The following Lemma greatly simplifies our analysis. It asserts that if $\mathcal{E}_{j,t}$ and $k^r_{j,t}$ are proportional to the initial values of $\mathcal{E}_{j,0}$, then $i_{j,t} = i_t$, for all j and t. In other words, although renewable energy-producing firms are heterogeneous, efficiency is consistent with them choosing identical levels of investment in R&D.

Lemma 1. *There is an efficient allocation satisfying* $\frac{k^r_{m,t}}{\mathcal{E}_{m,t}} = \frac{k^r_t}{\mathcal{E}_t}$ *and* $i_{m,t} = i_t$*, for all* m.

Proof. For any initial values of $\mathcal{E}_{j,0}$, there is a solution such that $\mathcal{E}_{j,t}$, $k^r_{j,t}$, $\mu^j_{\mathcal{E},t}$, and $\mu^j_{r,t}$ are proportional to the initial values of $\mathcal{E}_{j,0}$. In that case, (22) implies that $i_{m,t} = i_t$, for all $j \in [0,1]$. From (20), $\frac{\mathcal{E}_{jt}}{k^r_{jt}}$ is a function of $i_{j,t}$ only. As $i_{m,t} = i_t$, we have $\frac{\mathcal{E}_{jt}}{k^r_{jt}} = \frac{\mathcal{E}_t}{k^r_t}$. □

3 Equilibrium, Optimality, and Policy

We will demonstrate that, as discussed earlier, there is a discrepancy between equilibrium and optimal allocations. The problems of the household and the firms are studied in the Appendix. First, we characterize the equilibrium choice of investment in the renewable technology. We establish that, provided that $\xi < 1$, this investment will be lower than optimal. Clearly, the magnitude of the distortion depends on the level of the externality, ξ.

Proposition 2. *In competitive equilibrium, i is lower than optimal when $\xi < 1$.*

Proof. The social planner's FOCs for $i_{j,t}$ gives

$$\partial i_{j,t} : \Psi'(i_{j,t}) \left(\mathcal{E}_{j,t}\right)^{1-\alpha_r} \left(k^r_{j,t}\right)^{\alpha_r} \mu^j_{r,t} + \xi \mu^j_{\mathcal{E},t} + (1-\xi) \frac{k^r_{j,t}}{\overline{k^r_t}} \int_0^1 \mu^l_{\mathcal{E},t} dl = 0 \tag{23}$$

or

$$\partial i_{j,t} : -\Psi'(i_{j,t}) \left(\mathcal{E}_{j,t}\right) \left(\frac{k^r_{j,t}}{\mathcal{E}_{j,t}}\right)^{\alpha_r} \mu^j_{r,t} = \xi \mu^j_{\mathcal{E},t} + (1-\xi) \frac{k^r_{j,t}}{\overline{k^r_t}} \int_0^1 \mu^l_{\mathcal{E},t} dl. \tag{24}$$

As $\beta^t u'(c_t) p^e_t = \beta^t u'(c_t) (1-\theta) A_t \frac{\left(k^g_t\right)^{\theta_k} (L_t)^{\theta_L}}{(e_t)^\theta} = \mu_{R,t} (1-\theta) A_t \frac{\left(k^g_t\right)^{\theta_k} (L_t)^{\theta_L}}{(e_t)^\theta} = \mu_{E,t} = \mu^j_{r,t}$, we can rewrite this condition as

$$\partial i_{j,t} : -\beta^t u'(c_t) p^e_t \Psi'(i_{j,t}) \left(\mathcal{E}_{j,t}\right) \left(\frac{k^r_{j,t}}{\mathcal{E}_{j,t}}\right)^{\alpha_r} = \xi \mu^j_{\mathcal{E},t} + (1-\xi) \frac{k^r_{j,t}}{\overline{k^r_t}} \int_0^1 \mu^l_{\mathcal{E},t} dl. \tag{25}$$

As the social planner chooses $i_{j,t} = i_t$ and $\frac{k^r_{j,t}}{\mathcal{E}_{j,t}} = \frac{k^r_t}{\mathcal{E}_t}$, the RHS is independent of j, or,

$$\partial i_{j,t} : -\beta^t u'(c_t) p^e_t \Psi'(i_t) \left(\mathcal{E}_{j,t}\right) \left(\frac{k^r_t}{\mathcal{E}_t}\right)^{\alpha_r} = \xi \mu^j_{\mathcal{E},t} + (1-\xi) \frac{k^r_{j,t}}{\overline{k^r_t}} \int_0^1 \mu^l_{\mathcal{E},t} dl. \tag{26}$$

The condition for $\mu^l_{\mathcal{E},t}$ implies

$$\partial \mathcal{E}_{j,t+1} : \mu^l_{\mathcal{E},t+1} \frac{1}{\mathcal{E}^j_{t+1}} + \mu^j_{r,t+1} (1-\alpha_r) \Psi(i_{j,t+1}) \left(\frac{k^r_{j,t+1}}{\mathcal{E}_{j,t+1}}\right)^{\alpha_r} = \mu^j_{\mathcal{E},t} \frac{1}{\mathcal{E}^j_{t+1}} \tag{27}$$

or

$$\partial \mathcal{E}_{j,t+1} : \mu^l_{\mathcal{E},t+1} \frac{1}{\mathcal{E}^j_{t+1}} + \beta^{t+1} u'(c_{t+1}) p^e_{t+1} (1-\alpha_r) \frac{e^r_{j,t+1}}{\mathcal{E}_{j,t+1}} = \mu^j_{\mathcal{E},t} \frac{1}{\mathcal{E}^j_{t+1}} \tag{28}$$

or

$$\partial \mathcal{E}_{j,t+1} : \mu^l_{\mathcal{E},t+1} + \beta^{t+1} u'(c_{t+1}) p^e_{t+1} (1-\alpha_r) e^r_{j,t+1} = \mu^j_{\mathcal{E},t}. \tag{29}$$

Solving for $\mu^j_{\mathcal{E},t}$, we obtain

$$\mu^j_{\mathcal{E},t} = \sum_{\tau=1}^{\infty} \beta^{t+\tau} u'(c_{t+\tau}) p^e_{t+\tau} (1-\alpha_r) e^r_{j,t+\tau}, \text{ if } \lim_{\tau\to\infty} \mu^j_{\mathcal{E},\tau} = 0. \tag{30}$$

Replacing in the above expression, we obtain

$$-\Psi'(i_t)\left(\mathcal{E}_{j,t}\right)\left(\frac{k_t^r}{\mathcal{E}_t}\right)^{\alpha_r} = \xi\sum_{\tau=1}^{\infty}\beta^{\tau}\frac{u'(c_{t+\tau})p_{t+\tau}^e}{u'(c_t)p_t^e}(1-\alpha_r)e_{j,t+\tau}^r + (1-\xi)\frac{k_{j,t}^r}{k_t^r}\sum_{\tau=1}^{\infty}\beta^{\tau}\frac{u'(c_{t+\tau})p_{t+\tau}^e}{u'(c_t)p_t^e}(1-\alpha_r)\int_0^1 e_{l,t+\tau}^r dl. \tag{31}$$

The competitive equilibrium FOCs for $i_{j,t}$ gives

$$\partial i_{j,t} : -\beta^t u'(c_t)p_t^e\Psi'(i_{j,t})\left(\mathcal{E}_{j,t}\right)\left(\frac{k_{j,t}^r}{\mathcal{E}_{j,t}}\right)^{\alpha_r} = \xi\lambda_{\mathcal{E},t}^j \tag{32}$$

The competitive equilibrium FOCs for $\mathcal{E}_{t+1}^j$ gives

$$\partial\mathcal{E}_{t+1}^j : \lambda_{\mathcal{E},t+1}^j\frac{1}{\mathcal{E}_{t+1}^j} + \beta^{t+1}u'(c_{t+1})p_{t+1}^e(1-\alpha_r)\Psi(i_{j,t+1})\left(\frac{k_{j,t+1}^r}{\mathcal{E}_{j,t+1}}\right)^{\alpha_r} = \lambda_{\mathcal{E},t}^j\frac{1}{\mathcal{E}_{t+1}^j} \tag{33}$$

or

$$\partial\mathcal{E}_{t+1}^j : \lambda_{\mathcal{E},t+1}^j\frac{1}{\mathcal{E}_{t+1}^j} + \beta^{t+1}u'(c_{t+1})p_{t+1}^e(1-\alpha_r)\frac{e_{j,t+1}^r}{\mathcal{E}_{j,t+1}} = \lambda_{\mathcal{E},t}^j\frac{1}{\mathcal{E}_{t+1}^j} \tag{34}$$

or

$$\partial\mathcal{E}_{t+1}^j : \lambda_{\mathcal{E},t+1}^j + \beta^{t+1}u'(c_{t+1})p_{t+1}^e(1-\alpha_r)e_{j,t+1}^r = \lambda_{\mathcal{E},t}^j. \tag{35}$$

Solving for $\lambda_{\mathcal{E},t}^j$, we have

$$\lambda_{\mathcal{E},t}^j = \sum_{\tau=1}^{\infty}\beta^{t+\tau}u'(c_{t+\tau})p_{t+\tau}^e(1-\alpha_r)e_{j,t+\tau}^r,\ \text{if}\ \lim_{\tau\to\infty}\lambda_{\mathcal{E},\tau}^j = 0. \tag{36}$$

Replacing in the above FOCs gives

$$\partial i_{j,t} : -\Psi'(i_{j,t})\left(\mathcal{E}_{j,t}\right)\left(\frac{k_{j,t}^r}{\mathcal{E}_{j,t}}\right)^{\alpha_r} = \xi\sum_{\tau=1}^{\infty}\beta^{\tau}\frac{u'(c_{t+\tau})p_{t+\tau}^e}{u'(c_t)p_t^e}(1-\alpha_r)e_{j,t+\tau}^r. \tag{37}$$

The RHS of (31) is larger than the RHS of (37), and $-\Psi'(i_{j,t})$ is increasing in $i_{j,t}$. Everything else equal, the value of $i_{j,t}$ that satisfies (37) is lower than the one that satisfies (31). □

Next, we discuss optimal policy, which needs to take into account two distortions. The first relates to under-investment in i, due to the spillover effects. The second involves the social costs associated with the environmental externality. The next proposition demonstrates that both distortions can be fully accommodated through the use of two policy instruments. First, a policy that taxes firms in proportion to their under-investment in i restores optimal investment by making firms indifferent when they choose between paying the tax or pursuing the optimal level of investment. Second, a Pigouvian tax internalizes the externality from carbon emissions. As in GHKT, under the special assumptions of log utility and 100% depreciation of capital, the Pigouvian tax imposed on the fossil fuel firms does not depend on the growth rate of the economy.

Proposition 3. *(1) The optimal allocation can be supported by a combination of a revenue-neutral policy,* $\Phi_t^j(i_{j,t}) = -(1-\xi)p_t^e\Psi'(i_t^*)\left(\frac{e_{j,t}^{*r}}{\Psi(i_{j,t}^*)}\right)(i_{j,t}-i_t^*)$*, imposed on renewable firms, together with a Pigouvian tax on fossil fuel use,* $\tau_t^f = \sum_{j=0}^{\infty}\beta^j\frac{u'(c_{t+j}^*)}{u'(c_t^*)}\pi_{t+j}y_{t+j}^*(1-d_j)$*, where* $\{c_t^*, y_t^*, i_t^*\}_{t=0}^{\infty}$ *is the solution to the planner's problem, and* $1-d_j = \varphi_L + (1-\varphi_L)\varphi_0(1-\varphi)^j$*; (2) If* $u(c) = \log(c)$, $\alpha_r = \alpha_f = \alpha$*, and* $\delta = 1$, τ_t^f *does not depend on the growth rate of the economy.*

Proof. (1) Renewable-energy-producing firm j's problem can be written as

$$\max_{\{i_{j,t}\}_{t=0}^{\infty}} \sum_{\tau=0}^{\infty}\beta^{\tau}\frac{u'(c_{t+\tau})}{u'(c_t)}\left[p_{t+\tau}^e\Psi(i_{j,t+\tau})\left(\mathcal{E}_{j,t+\tau}\right)^{1-\alpha_r}\left(k_{j,t}^r\right)^{\alpha_r} - r_{t+\tau}k_{j,t}^r + \Phi_{t+\tau}^j\right]$$

$$\begin{aligned} \text{s.t. } \ln\mathcal{E}_{t+1}^j &\leq \ln\mathcal{E}_t^j + \xi i_{j,t} + (1-\xi)\left(\int_0^1 i_{j,t}k_{j,t}^r dj \Big/ \int_0^1 k_{j,t}^r dj\right) : \lambda_{\mathcal{E},t}^j \\ & i_{j,t} \geq 0, \text{ and } \mathcal{E}_0 \text{ given.} \end{aligned}$$

The FOCs are

$$\partial k_{j,t+\tau}^r : p_{t+\tau}^e\alpha_r\Psi(i_{j,t+\tau})\left(\frac{\mathcal{E}_{j,t+\tau}}{k_{j,t+\tau}^r}\right)^{1-\alpha_r} = r_{t+\tau} \tag{38}$$

$$\partial i_{j,t+\tau} : \beta^{\tau}\frac{u'(c_{t+\tau})}{u'(c_t)}p_{t+\tau}^e\Psi'(i_{j,t+\tau})\left(\frac{e_{j,t+\tau}^r}{\Psi(i_{j,t+\tau})}\right) - (1-\xi)\beta^{\tau}\frac{u'(c_{t+\tau})}{u'(c_t)}p_{t+\tau}^e\Psi'(i_{j,t+\tau}^*)\left(\frac{e_{j,t+\tau}^{*r}}{\Psi(i_{j,t+\tau}^*)}\right) = \xi\lambda_{\mathcal{E},t+\tau}^j \tag{39}$$

$$\partial\mathcal{E}_{t+\tau}^j : \lambda_{\mathcal{E},t+\tau}^j\frac{1}{\mathcal{E}_{t+\tau}^j} - \lambda_{\mathcal{E},t+\tau-1}^j\frac{1}{\mathcal{E}_{t+\tau}^j} + \beta^{\tau}\frac{u'(c_{t+\tau})}{u'(c_t)}p_{t+\tau}^e(1-\alpha_r)\left(\frac{k_{j,t+\tau}^r}{\mathcal{E}_{j,t+\tau}}\right)^{\alpha_r} = 0.$$

Using (39), at the optimum we have

$$\lambda_{\mathcal{E},t+\tau}^j = \beta^{\tau}\frac{u'(c_{t+\tau})}{u'(c_t)}p_{t+\tau}^e\Psi'(i_{j,t+\tau}^*)\left(\frac{e_{j,t+\tau}^{*r}}{\Psi(i_{j,t+\tau}^*)}\right)$$

Substituting the above in (38), we obtain an expression identical to that for the optimal i.

Next, suppose sellers of fossil fuel face a linear tax rate,

$$\tau_t^f = \sum_{j=0}^{\infty}\beta^j\frac{u'(c_{t+j}^*)}{u'(c_t^*)}\pi_{t+j}y_{t+j}^*(1-d_j), \tag{40}$$

where $\{c_t^*, y_t^*\}_{t=0}^{\infty}$ solves the planner's problem, and $1-d_j = \varphi_L + (1-\varphi_L)\varphi_0(1-\varphi)^j$. Under this tax, the fossil-fuel-producers' optimal intertemporal substitution implies

$$u'(c_t)\left\{p_t^f - \tau_t^f\right\} = \beta u'(c_{t+1})\left\{p_{t+1}^f - \tau_{t+1}^f\right\}.$$

Using (54) and (57) for the price of fossil fuel and (40) for the tax, we obtain

$$\begin{aligned}
& u'(c_t)\left\{MPF_t - \pi_t y_t^* \left(\varphi_L + (1-\varphi_L)\varphi_0\right)\right\} \\
& + \sum_{j=1}^{\infty} \beta^j u'(c_{t+j}^*)\pi_{t+j} y_{t+j}^* ((1-\varphi_L)\varphi_0(1-\varphi)^{j-1}\varphi) \\
= \; & \beta u'(c_{t+1})\left\{MPF_{t+1}\right\},
\end{aligned}$$

where MPF_t is the period-t marginal productivity of fossil fuel in units of the final good. Clearly, if $\frac{y_{t+j}^*}{c_{t+1}^*} = \chi$, a constant, the claim would be true. First, observe that $\frac{c_t}{y_t} = \chi \Leftrightarrow \frac{k_{t+1}^g}{y_t} = \theta^k \beta$. This equation follows from the FOCs of the social planner, which include

$$\frac{y_t}{c_t} = \frac{y_{t+1}}{c_{t+1}} \frac{\theta^k \beta y_t}{k_{t+1}^g}.$$

It remains to be shown that

$$\frac{k_{t+1}^f}{y_t} + \frac{k_{t+1}^r}{y_t} = 1 - \chi - \theta^k \beta \equiv \varrho,$$

where $k_t^r = \int k_{t,m}^r dm$.

The social planner problem's FOCs with respect to $k_{j,t}^r$ implies

$$\alpha_r \Psi\left(i_t\right) \left(\frac{\mathcal{E}_{j,t}}{k_{j,t}^r}\right)^{1-\alpha_r} (1-\theta) \frac{y_t}{e_t} = \alpha_r \left(\frac{e_{j,t}^r}{k_{j,t}^r}\right) (1-\theta) \frac{y_t}{e_t} = \theta^k \frac{y_t}{k_t^g}$$

$$\alpha_r (1-\theta) \frac{e_{j,t}^r}{e_t} = \frac{1}{\beta} \frac{k_{j,t}^r}{y_{t-1}}.$$

The social planner problem's FOCs with respect to k_t^f implies

$$\alpha_f (1-\theta) \frac{e_t^f}{e_t} = \frac{1}{\beta} \frac{k_t^f}{y_{t-1}}.$$

It is sufficient to show that

$$\beta (1-\theta) \left(\frac{\alpha_r e_{t+1}^r + \alpha_f e_{t+1}^f}{e_{t+1}}\right) = \varrho,$$

which is true if $\alpha_r = \alpha_f = \alpha$. □

The optimal policy in our model has some interesting implications. First, the tax on renewable energy firms generates no revenue, but it reduces the household's profits from the renewable sector as a result of inducing additional innovation compared to the competitive equilibrium. Second, the Pigouvian tax reduces the household's profits from the fossil fuel sector. However, the household receives a lump-sum transfer of equal magnitude; thus, its budget constraint remains unchanged. Finally, there is no interaction between these two schemes, as the total effect on the household's budget is the same as the resource cost of innovation in the planner's problem.

For the remainder of the paper, we will impose the assumption that $u(c) = \log(c)$ and $\delta = 1$. Moreover, we will assume that the stock of fossil fuel is large enough so that consumption of fossil fuel is never constrained. In the Appendix we solve the planner's problem backward, from a state where only renewable energy is used to the state where both renewable and fossil fuel energy are used. We also show that the total consumption of fossil fuel is endogenously bounded. In other words, the transition to renewable energy takes place prior to the exhaustion of fossil fuel resources. The transition takes place because the growing productivity in the renewable sector eventually surpasses a threshold that makes using fossil fuel an inferior source of energy. While allocating additional capital to the fossil fuel sector increases the production of energy per unit of fossil fuel, the present value of the marginal environmental damages limits the overall benefit from fossil fuel use. In the next section we calibrate our model in order to study the optimal timing of the transition to a renewable energy regime, as well as the effects of the greenhouse gas (GHG) accumulation prior to this transition.

4 Calibration

Our calibration relies on the characterization of the optimal allocation across the transition, which is given in the Appendix. The coefficient of relative risk aversion is set to $\sigma = 1$. Discounting is given by $\beta = .96^{10}$, and depreciation is full; i.e., $\delta = 1$ (recall that the length of a period is 10 years). Moving to the production function for the final good, the percentage capital share is $\theta_k = (1/3) \times .95$, while the percentage labor share is set to $\theta_l = (2/3) \times .95$. The energy share is then given by $1 - \theta = 1 - (\theta_k + \theta_l)$. The percentage capital share in the energy sector is $\alpha = .5$. We set the percentage productivity growth rate in the final good sector so that the balanced growth rate is 2%, or $g = (exp(0.02)^{\theta_l})^{10}$, while the long-run percentage population growth rate is $g^l = \exp(0)$. Note that, for a balanced growth path, we require that $\beta exp(\bar{i}) < 1$.

For our calibration, we assume the following form for the technology adoption cost function, Ψ:

$$\Psi(i) = \left(1 - \left(\frac{i}{\bar{\bar{i}}}\right)^{\psi}\right)^{1/\psi}. \tag{41}$$

This form satisfies the earlier assumptions that $\Psi(0) = 1$, $\Psi'(\cdot) < 0$, $\Psi''(\cdot) < 0$, and $\Psi(i) = 0$, for $i = \bar{\bar{i}}$. Moreover, under this functional form, the elasticity of the technology adoption cost with respect to the adoption rate is given by

$$\frac{\Psi'}{\Psi} = \frac{1}{i} \times \frac{(i/\bar{\imath})^{\psi}}{1-(i/\bar{\imath})^{\psi}}. \tag{42}$$

As shown in the appendix, this elasticity plays an important role in determining the long-run technological adoption rate in the renewable sector. The parameter ψ provides us with a degree of freedom to match a long-run technological adoption rate that is consistent with the long-run growth rate of the economy.

To calibrate Ψ, we need to assign values to its two parameters, $\bar{\imath}$ and ψ. To calibrate $\bar{\imath}$, we use the fact that, when fossil fuel is used, expression (82) characterizes the "backward-asymptotic" value of i. This implies that $\beta e^{i_f} < 1$.[7] We set $\bar{\imath} = -\log(\beta) - 0.1 = 0.3082$, so $\bar{\imath}$ is 1% below the maximum value. We set the long-run technology adoption rate to $i^l = log(\widehat{g})$, where $exp(i^l)$ is the long-run growth rate of the renewable productivity. That is, the productivity of the renewable energy sector can improve at most by 3.13% annually, which is broadly consistent with historical cost-reduction averages in the sector. Given $\bar{\imath}$, we match the 5% representation of renewables in today's energy supply. In order to have a balanced growth path, the long-term i should be equal to the long term growth rate. The long-run i is, in turn, characterized by equation (76) in the Appendix. Given $\bar{\imath}$, we can use this property to back up ψ. By matching the optimal long-run i to i^l, we obtain

$$\psi = log\left(\frac{\beta/(1-\beta)(1-\alpha)i^l}{(1+\beta/(1-\beta)(1-\alpha)i^l)}\right)/log(i^l/\bar{\imath}). \tag{43}$$

Given values of ψ and $\bar{\imath}$ we characterize Ψ. We do not need to calibrate ξ, as we assume the optimal level of renewable production in each period.[8] Finally, we calibrate the environmental damages as in GHKT (2014) by setting $\pi = 2.379 \times 10^{-5} \times 10$, $\varphi = 0.0228$, $\varphi_L = 0.2$, and $\varphi_0 = 0.393$.

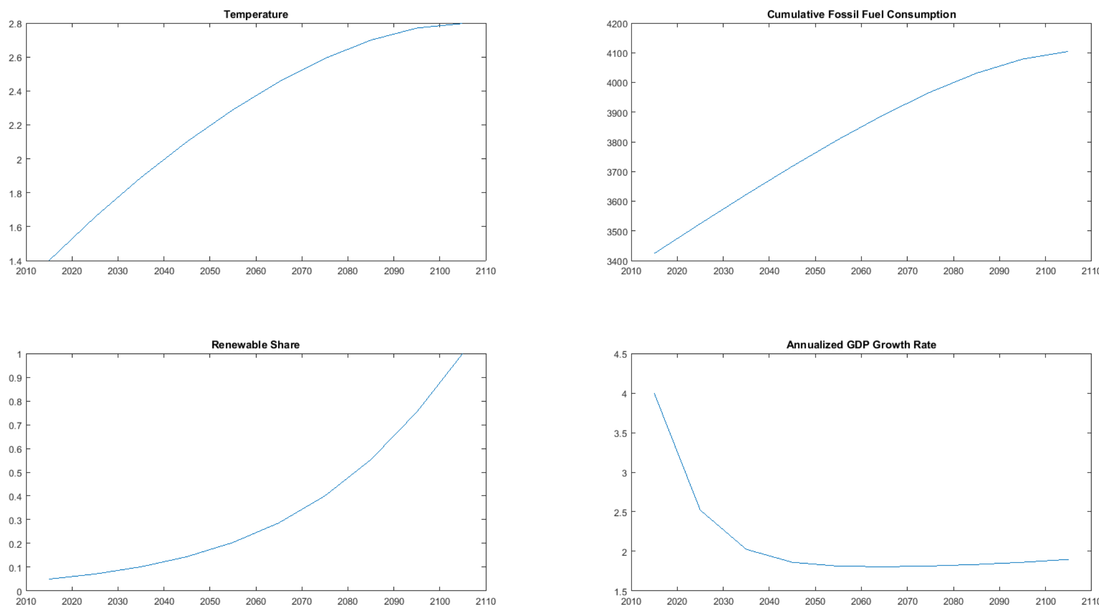
Assuming that the initial reserves are large enough so that on the optimal path the economy stops using fossil fuel before exhausting it, the upper bound for the consumption of fossil fuel is given by the right-hand side of expression (71). The interpretation is straightforward, as this implies equating the marginal benefit from using fossil fuel to the present value of the resulting future damages. These values imply that this upper bound is 105.78 GtC. Finally, the optimal carbon tax follows from the last part of (79). This tax is constant and given by

$$\mathcal{T} = \beta\left\{\varphi_L \frac{\pi}{(1-\beta)(1-\beta\Theta)} + ((1-\varphi_L)\varphi_0)\frac{\pi}{(1-\beta(1-\varphi))(1-\beta\Theta)}\right\}, \tag{44}$$

where $\Theta = \theta_k + (1-\theta_k-\theta_\ell)\alpha$. Given our calibration, this equation gives $\mathcal{T} = 3.0578e-04$, or about \$17,000/ton of carbon equivalent.

[7] If $\beta e^{\bar{\imath}} > 1$, then growth would be unbounded. In that case, one could set $i = \bar{\imath}$ and grow $\mathcal{E}$ (hence, output) at a rate faster than discounting.

[8] Thus, our calibration does not address the potential discrepancy between the optimal and the decentralized allocation.



Our *Matlab* code simulates the optimal i, $\mathcal{E}$, and fossil fuel production going backward. Moreover, we calculate the share of renewable energy from the total energy production. Assuming a current share of renewables below 5% and a current GDP growth rate of 4%, we find that it would take approximately 150 years (15 periods) to end the use of fossil fuel. The cumulative future consumption of fossil fuel would be around 680 GtC. In order to map carbon concentrations into global temperatures, we use the following expression from GHKT

$$T(S_t) = 3\ln\left(\frac{S_t}{\overline{S}}\right) / \ln(2), \tag{45}$$

where $\overline{S}$ is the pre-industrial level of atmospheric carbon concentration. This formula implies a 2.8 degree Celsius temperature increase. The following graph shows the resulting optimal paths for global temperatures, cumulative fossil fuel use, the share of renewable energy, and the annualized GDP growth rates for the world economy. As a result of renewable energy adoption, the GDP growth rate increases slightly, but only after renewables approach a 40% penetration in global energy supply around 2070.

5 Conclusion

We studied the interplay between optimal growth, policy, and the adaptation of new technologies by renewable energy-producing firms in a framework where energy is an input in the production of final goods and where fossil fuels create damages associated with an externality from carbon emissions. We assumed that, in order to make improvements in the supply of renewable energy, firms must incur an opportunity cost involving a loss in the current renewable energy production. Importantly, we took into consideration that these improvements are subjects to spillovers. Efficiency requires a policy that promotes the adaptation of new technologies by subsidizing investment in renewables, as well as a Pigouvian tax on the environmental externality.

We found that, contrary to current policy recommendations that favor a large scale of renewable energy installation, it is optimal for renewables to start "small" and pick up their market penetration only later. In the short run, investment is needed mainly to improve productivity in renewable energy production. Later, renewable energy contributes by becoming a modest engine of economic growth. Assuming a current GDP growth rate of 4%, we found that it would take approximately 150 years for fossil fuel to be completely phased out. The cumulative future consumption of fossil fuel is around 680 GtC, implying a 2.8 degree Celsius temperature increase for the planet. The optimal carbon tax is constant and about $17,000/ton of carbon equivalent.

6 Appendix

6.1 Household's and Firms' Optimization

The household's problem is given by

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} p_t \left[c_t + k_{t+1} - (1-\delta)k_t\right] \leq \sum_{t=0}^{\infty} p_t \left[r_t k_t + w_{l,t} l_t + p_t^f f_t + \pi_t^g + \pi_t^f + \int_0^1 \pi_{j,t}^r dj\right] : \lambda$$

$$w_{t+1} \leq w_t - f_t : \mu_t, \tag{46}$$

where p_t is the Arrow-Debreu price of the period-t final good, r_t is the rental price of capital in period t, p_t^f is the price of fossil fuel in period t, π stands for the respective firms' profits, and λ, μ are the corresponding Lagrange multipliers. The FOCs, which are also sufficient for a maximum, can be written as

$$\frac{p_{t+1}}{p_t} = \frac{p_t^f}{p_{t+1}^f} \tag{47}$$

$$\beta^t \frac{u'(c_t)}{p_t} = \lambda \tag{48}$$

$$1 - \delta + r_{t+1} = \frac{p_t}{p_{t+1}} \tag{49}$$

$$\beta^t u'(c_t) p_t^f = \mu_t = \mu. \tag{50}$$

The final good firm's problem is

$$\max \left[A_t \cdot \left(k_t^g\right)^{\theta_k} \left(l_t\right)^{\theta_l} \left(e_t\right)^{1-\theta} - r_t k_t^g - w_{l,t} l_t - p_t^e e_t \right]. \tag{51}$$

The first-order conditions are

$$\partial k_t^g : \theta_k A_t \left(k_t^g\right)^{\theta_k - 1} \left(l_t\right)^{\theta_l} \left(e_t\right)^{1-\theta} = r_t \tag{52}$$

$$\partial l_t : \theta_l A_t \left(k_t^g\right)^{\theta_k} \left(l_t\right)^{\theta_l - 1} \left(e_t\right)^{1-\theta} = w_{l,t} \tag{53}$$

$$\partial e_t : \left(1-\theta\right) A_t \frac{\left(k_t^g\right)^{\theta_k} \left(l_t\right)^{\theta_l}}{e_t^{\theta}} = p_t^e. \tag{54}$$

The fossil fuel firm's problem is

$$\max \left[p_t^e \left(f_t\right)^{1-\alpha_f} \left(k_t^f\right)^{\alpha_f} - r_t k_t^f - p_t^f f_t \right]. \tag{55}$$

The first-order conditions are

$$\partial k_t^f : p_t^e \alpha_f \left(\frac{f_t}{k_t^f}\right)^{1-\alpha_f} = r_t \tag{56}$$

$$\partial f_t : p_t^e \left(1-\alpha_f\right) \left(\frac{k_t^f}{f_t}\right)^{\alpha_f} = p_t^f. \tag{57}$$

Finally, the renewable firm j's problem in period t is

$$\begin{gathered} \max_{\{i_{j,t+\tau}, k_{j,t+\tau}^r\}_{\tau=0}^{\infty}} \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{u'(c_{t+\tau})}{u'(c_t)} \left[p_{t+\tau}^e \Psi(i_{j,t+\tau}) \left(\mathcal{E}_{j,t+\tau}\right)^{1-\alpha_r} \left(k_{j,t+\tau}^r\right)^{\alpha_r} - r_{t+\tau} k_{j,t+\tau}^r \right] \\ \text{s.t. } \ln \mathcal{E}_{t+1}^j \leq \ln \mathcal{E}_t^j + \xi i_{j,t} + (1-\xi) \left(\int_0^1 i_{j,t} k_{j,t}^r dj \Big/ \int_0^1 k_{j,t}^r dj \right) : \lambda_{\mathcal{E},t}^j \\ i_{j,t} \geq 0, \text{ and } \mathcal{E}_0 \text{ given.} \end{gathered} \tag{58}$$

Recall that $\Psi(\cdot)$ is a convex function with $\Psi(0) = 0$, $\Psi' > 0$, $\Psi'' > 0$, and $\lim_{x \to 0} \Psi'(x) = 0$. The FOCs are

$$\partial k_{j,t+\tau}^r : p_{t+\tau}^e \alpha_r \Psi(i_{j,t+\tau}) \left(\frac{\mathcal{E}_{j,t+\tau}}{k_{j,t+\tau}^r}\right)^{1-\alpha_r} = r_{t+\tau} \tag{59}$$

$$\partial i_{j,t+\tau} : \beta^{\tau} \frac{u'(c_{t+\tau})}{u'(c_t)} p_{t+\tau}^e \Psi'(i_{j,t+\tau}) \left(\mathcal{E}_{j,t+\tau}\right)^{1-\alpha_r} \left(k_{j,t+\tau}^r\right)^{\alpha_r} = \xi \lambda_{\mathcal{E},t+\tau}^j \tag{60}$$

$$\partial \mathcal{E}_{t+\tau}^j : \lambda_{\mathcal{E},t+\tau}^j \frac{1}{\mathcal{E}_{t+\tau}^j} - \lambda_{\mathcal{E},t+\tau-1}^j \frac{1}{\mathcal{E}_{t+\tau}^j} + \beta^{\tau} \frac{u'(c_{t+\tau})}{u'(c_t)} p_{t+\tau}^e (1-\alpha_r) \Psi(i_{j,t+\tau}) \left(\frac{k_{j,t+\tau}^r}{\mathcal{E}_{j,t+\tau}}\right)^{\alpha_r} = 0. \tag{61}$$

6.2 Optimal Transition

Here we characterize the optimal allocation across the transition, and we derive some expressions that will be used when we calibrate the model. Let $V(k; A, L, \mathcal{E}; \Gamma^p, \Gamma^d)$ denote the value given k available units of capital and given that the aggregate productivity is A, the labor supply is l, the productivity in the renewable energy sector is $\mathcal{E}$, and the stocks of permanent and depreciating emissions are Γ^p and Γ^d, respectively. We let g stand for the percentage productivity growth rate in the final good sector, while g^l is the population growth rate. The optimal consumption and saving decision under log utility and full depreciation is given by $c = (1 - \beta\Theta)y$, and $k' = \beta\Theta y$, where $\Theta = \theta_k + (1 - \theta_k - \theta_\ell)\alpha$ is the marginal product of capital. The recursive formulation for $V(\cdot)$ is given by

$$\begin{aligned} V\left(k; A, L, \mathcal{E}; \Gamma^p, \Gamma^d\right) &= \max_{i,f} \ln\left((1-\beta\Theta)y\right) \\ &\quad +\beta V(\beta\Theta y; gA, g^l l, e^i\mathcal{E}; \Gamma^p + \varphi_L f, (1-\varphi)\Gamma^d + (1-\varphi_L)\varphi_0 f) \\ &\quad \text{where} \\ &\quad y = e^{-\pi(\bar{\Gamma}-\Gamma^p-\Gamma^d)} A L^{\theta_\ell} \left(f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}\right)^{(1-\alpha)(1-\theta_k-\theta_\ell)} k^{\Theta}. \end{aligned} \tag{62}$$

Utilizing the envelope theorem, we have $V_k = \Theta\frac{1}{k} + \beta\Theta\frac{k'}{k}V'_{k'}$, which implies

$$kV_k = \Theta + \beta\Theta k' V_{k'}. \tag{63}$$

Next, we guess that kV_k is a constant and we verify that

$$V_k = \frac{\Theta}{1-\beta\Theta}\frac{1}{k}. \tag{64}$$

Using the same method, we have that $V_A = \frac{1}{A} + \beta\left\{\frac{k'}{A}V'_{k'} + gV'_{A'}\right\}$, which, in turn, implies

$$AV_A = 1 + \beta\left\{\frac{\Theta}{1-\beta\Theta} + (gA)\,V'_{A'}\right\} \tag{65}$$

Next, we guess that AV_A is a constant. As $A' = gA$, this equation allows us to verify that

$$V_A = \frac{1}{(1-\beta)(1-\beta\Theta)}\frac{1}{A}. \tag{66}$$

Similarly, we obtain

$$V_L = \frac{\theta_l}{(1-\beta)(1-\beta\Theta)}\frac{1}{L} \tag{67}$$

$$V_{\Gamma^p} = \frac{1}{(1-\beta)(1-\beta\Theta)}(-\pi) \tag{68}$$

$$V_{\Gamma^d} = \frac{1}{(1-\beta(1-\varphi))(1-\beta\Theta)}(-\pi). \tag{69}$$

The last expression reflects the depreciation rate of the temporary part of the emissions stock.

The optimal choice of f implies

$$\begin{aligned} 0 &\leq \frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\left(f+\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}\right)} + \beta\left\{\frac{(1-\alpha)(1-\theta_k-\theta_\ell)k'}{\left(f+\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}\right)}V'_{k'} + \varphi_L V'_{\Gamma^{p\prime}} + ((1-\varphi_L)\varphi_0)\, V'_{\Gamma^{d\prime}}\right\} \\ &= \frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\left(f+\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}\right)(1-\beta\Theta)} \\ &\quad -\beta\left\{\varphi_L\frac{\pi}{(1-\beta)(1-\beta\Theta)} + ((1-\varphi_L)\varphi_0)\,\frac{\pi}{(1-\beta(1-\varphi))(1-\beta\Theta)}\right\}. \end{aligned} \tag{70}$$

The first and second terms in the above expression give the marginal benefit from consumption and from future capital accumulation, respectively. The third term gives the marginal cost from next period's damages resulting from emissions (in utils per GtC). This value, in turn, implies that

$$f+\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E} \leq \frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\beta\left\{\frac{\pi\varphi_L}{(1-\beta)} + \frac{\pi(1-\varphi_L)\varphi_0}{(1-\beta(1-\varphi))}\right\}} \tag{71}$$

with equality for $f>0$.

The optimal choice for i implies

$$0 = (1-\alpha)(1-\theta_k-\theta_\ell)\frac{\frac{1}{1-\alpha}\Psi'(i)\Psi(i)^{\frac{1}{1-\alpha}-1}\mathcal{E}}{f+\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}\left\{1+\beta\frac{\Theta}{1-\beta\Theta}\right\} + \beta\underbrace{e^i\mathcal{E}}_{\mathcal{E}'}V'_{\mathcal{E}'} \tag{72}$$

or

$$\frac{-\Psi'(i)}{\Psi(i)}\frac{(1-\theta_k-\theta_\ell)}{1-\beta\Theta}\frac{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{f+\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}} = \beta\mathcal{E}'V'_{\mathcal{E}'}, \tag{73}$$

where

$$\mathcal{E}V_{\mathcal{E}} = \frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{1-\beta\Theta}\frac{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{f+\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}} + \beta\mathcal{E}'V'_{\mathcal{E}'}. \tag{74}$$

Combining, we obtain

$$\frac{-\Psi'(i)}{\Psi(i)} = \beta\frac{\frac{\Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}'}{f'+\Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}'}}{\frac{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{f+\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}}\left((1-\alpha)+\frac{-\Psi'(i')}{\Psi(i')}\right). \tag{75}$$

For a balanced growth path, we need all industries to grow at the same rate. If $f=f'=0$, we have $\frac{-\Psi'(i)}{\Psi(i)} = \beta\left((1-\alpha)+\frac{-\Psi'(i')}{\Psi(i')}\right)$, which implies $i=i'=\widehat{i}$, where $\widehat{i}$ is determined by

$$\frac{-\Psi'(\widehat{i})}{\Psi(\widehat{i})} = \frac{\beta}{1-\beta}(1-\alpha). \tag{76}$$

If $f > f' = 0$, then

$$\begin{aligned}
\frac{-\Psi'(i)}{\Psi(i)} &= \beta \frac{f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}} \left((1-\alpha) + \frac{-\Psi'(i')}{\Psi(i')} \right) \\
&= \beta \frac{f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}} \frac{1-\alpha}{1-\beta} \\
&= \frac{(1-\alpha)^2}{1-\beta} \frac{(1-\theta_k-\theta_\ell)}{\left\{ \frac{\pi\varphi_L}{(1-\beta)} + \frac{\pi(1-\varphi_L)\varphi_0}{(1-\beta(1-\varphi))} \right\}} \frac{1}{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}. \qquad (77)
\end{aligned}$$

If $f,\ f' > 0$, then

$$\begin{aligned}
f' + \Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}' &= f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E} \\
&= \frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\beta\left\{ \frac{\pi\varphi_L}{(1-\beta)} + \frac{\pi(1-\varphi_L)\varphi_0}{(1-\beta(1-\varphi))} \right\}},
\end{aligned}$$

which implies

$$\frac{-\Psi'(i)}{\Psi(i)} = \beta \frac{\Psi(i')^{\frac{1}{1-\alpha}} e^{i}}{\Psi(i)^{\frac{1}{1-\alpha}}} \left((1-\alpha) + \frac{-\Psi'(i')}{\Psi(i')} \right). \qquad (78)$$

Finally, if $f = 0$, we have

$$V_{\mathcal{E}} = \frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{(1-\beta)(1-\beta\Theta)} \frac{1}{\mathcal{E}}.$$

Hence,

$$\begin{aligned}
V\left(k; A, L, \mathcal{E}; \Gamma^p, \Gamma^d\right) &= C + \frac{\Theta}{1-\beta\Theta} \ln k \\
&+ \frac{1}{(1-\beta)(1-\beta\Theta)} \left\{ \ln A + \theta_l \ln L + (1-\alpha)(1-\theta_k-\theta_\ell) \ln \mathcal{E} \right\} \\
&- \frac{\pi}{(1-\beta)(1-\beta\Theta)} \Gamma^p - \frac{\pi}{(1-(1-\varphi)\beta)(1-\beta\Theta)} \Gamma^d, \qquad (79)
\end{aligned}$$

where C is a constant.

To determine the path of i and f, we begin by determining $\widehat{i}$. Then the minimum $\mathcal{E}$ for which $f = 0$ is determined by

$$\Psi(\widehat{i})^{\frac{1}{1-\alpha}} \underline{\mathcal{E}} = \frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\beta\left\{ \frac{\pi\varphi_L}{(1-\beta)} + \frac{\pi(1-\varphi_L)\varphi_0}{(1-\beta(1-\varphi))} \right\}}. \qquad (80)$$

For $\mathcal{E}' \in \left[\underline{\mathcal{E}}, e^{\widehat{i}}\underline{\mathcal{E}}\right)$, the previous period's consumption of fossil fuel is positive, $f > 0$, while the current period's is zero. Hence, using (77) and $e^{i}\mathcal{E} = \mathcal{E}'$, we have

$$\frac{-\Psi'(i)}{\Psi(i)}\Psi(i)^{\frac{1}{1-\alpha}}\underbrace{e^{-i}\mathcal{E}'}_{\mathcal{E}} = \frac{(1-\alpha)^2}{1-\beta}\frac{(1-\theta_k-\theta_\ell)}{\left\{\frac{\pi\varphi_L}{(1-\beta)}+\frac{\pi(1-\varphi_L)\varphi_0}{(1-\beta(1-\varphi))}\right\}}. \tag{81}$$

Denote the i and the $\mathcal{E}$ corresponding to $\mathcal{E}' = \underline{\mathcal{E}}$ by $\underline{\underline{i}}$ and $\underline{\underline{\mathcal{E}}}$, respectively. For $\mathcal{E}' \in \left[\underline{\underline{\mathcal{E}}}, \underline{\mathcal{E}}\right]$, and using the corresponding i', we can now compute the previous period's i using (78) and proceed backward. In that case, i converges to $\widehat{i^f}$ determined by

$$\frac{-\Psi'(\widehat{i^f})}{\Psi(\widehat{i^f})} = \frac{\beta\widehat{i^f}}{1-\beta\widehat{i^f}}(1-\alpha). \tag{82}$$

Finally, given i and $\mathcal{E}$, we can compute the optimal f. Proceeding recursively, we characterize the entire optimal path.

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