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CESIFO WORKING PAPER NO. 6381
CATEGORY 11: INDUSTRIAL ORGANISATION
FEBRUARY 2017



***ISSN 2364-1428***

CESifo
Center for Economic Studies & Ifo Institute

CESifo *Working Paper No. 6381*

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## Abstract

We model leadership selection, competition, and decision making in teams with heterogeneous membership composition. We show that if the choice of leadership in a team is imprecise or noisy—which may arguably be the case if appointment decisions are made by non-expert administrators—then it is not necessarily the case that the best individuals should be selected as team members. On the contrary, and in line with what has been called the "Apollo effect," a "dream team" consisting of unambiguously higher performing individuals may perform worse in terms of team output than a group composed of lower performers. We characterize the properties of the leadership selection and production processes which lead to the Apollo effect and clarify when the opposite effect occurs in which supertalent performs better than comparatively less qualified groups.


JEL-Codes: C700, D700, J800.

Keywords: team composition, leadership, mistakes.

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Thanks for helpful comments and discussion to Philipp Hungerländer, Alberto Versperoni, and seminar participants at the University of Vienna. (27-Feb-2017).

# 1 Introduction

The "Apollo Syndrome" is a phenomenon first described and popularized in the management literature by Belbin (1981). It describes situations in which teams of highly capable individuals, collectively, perform badly. The phenomenon is named after the mission teams in NASA's Apollo space program and refers to situations in which one team is composed of unambiguously more capable individuals than the comparison teams. Contrary to intuition, in the experiments Belbin conducted in the 60ies at what is now Henley Business School, the Apollo teams often finished near the bottom among the competing teams.[1] One of the reasons Belbin gives for the Apollo teams' failure is that Apollo team members "spent a large part of their time engaged in abortive debate, trying to persuade the other members of the team to adopt their own particular, well-stated point of view. No one seemed to convert another or be converted. However, each seemed to have a flair for spotting the weak points of the other's argument. [...] Altogether, the Apollo company of supposed supertalent proved an astonishing disappointment." (Belbin, 1981, p. 15)[2]

For our main result, we model a team production problem in which an executive or administrator (either a principal or the team itself) appoints a single leader and subsequently all team members produce joint output by exerting individual efforts. We assume that the administrator is more likely to select a "wrong" or suboptimal leader if the skills of the candidates are similar. The model represents the administrator's selection capabilities through a symmetric black-box function (for which we supply micro justifications) that selects individuals with some probability for leadership positions on the basis of their innate leadership skills which are unknown to the executive. The higher the skill differences, the easier it is to find the better team leader. We show that in this environment the Apollo effect—which we define as a team of highly skilled individuals being outperformed by a team consisting entirely of lower-qualified members—is generally inescapable and arises for any noisy selection process.

In terms of the selection of leadership roles we think of the following process. The

[1] "Of 25 companies that we constructed according to our Apollo design, only three became the winning team. The favourite finishing position out of eight was sixth (six times), followed by fourth (four times)." (Belbin, 1981, p. 20) The performance data of the remaining Apollo teams is not available. If we allocate the remaining 12 teams with equal probability to each remaining rank, the resulting hypothetical expected Apollo rank is 4.6.

[2] The general observation itself is not necessarily novel, as "it chanced unto this gentleman, as the common proverb is, — *the more cooks the worse potage*, he had in his ship a hundred marines, the worst of them being able to be a master in the best ship within the realm; and these so maligned and disdained one the other, that refusing to do that which they should do, were careless to do that which was most needful and necessary, and so contending in envy, perished in forwardness." Hooker, J., *The Life of Sir Peter Carew*, 1575.

(human resources) executive or administrator charged with assigning tasks to workers and managers is not an expert on the production processes for which the appointments under consideration are made. She collects information on the performance of the individuals according to some standardized management selection protocol. Although she may perform her job admirably, she occasionally makes the wrong leadership assignment.

The narrative offered in this introduction explains the Apollo effect based on competition for leadership. This does not need to be taken literally. Any potential for conflicting opinions, differential styles of conducting business, management philosophies, etc, can be similarly thought of as the basis for the frictions that are modeled through our black-box assignment function. Among the paper's extensions we define and describe the properties of a task-matching model in which the single leadership interpretation is replaced by a matching interpretation of workers to differentially productive tasks. There, the assignment function models the potential for mistakenly assigning the wrong worker to a given task. Although the Apollo effect is less ubiquitous in this environment than for the leadership game, we show that there are always skill profiles of workers for which the Apollo effect can arise for suitably noisy task-selection technologies. While we assume in the main body of our analysis that workers know each other's skills, we show that the Apollo effect persists under incomplete skill information among workers. Finally, we show that the Apollo effect exists regardless of the introduction of a profit-maximizing principal into the pure team environment.

The plan for the remainder of the paper is as follows. After a short overview of the applicable literature we define our model in Section 2. Section 3 presents and illustrates our main result, the ubiquity of the Apollo effect. Section 4 discusses several extensions, alternative interpretations, and the robustness of the main model. In the concluding section we discuss a further set of potential applications and extensions. Proofs of all results and details of some derivations can be found in the appendix.

## Literature

Belbin (1981) introduces a "team role" theory designed to enhance team composition based on a series of business (school) training games.[3] The described Apollo syndrome is an effect of team composition and is as such distinct from the "Ringelmann-type" free-riding (or social loafing) due to moral hazard in teams (Gershkov et al., 2016).

[3] For recent management surveys on team composition and pointers to empirical work see, for example, Aritzeta et al. (2007) or Mathieu et al. (2013). There is a topical link to the literatures on collective intelligence in organizations (Woolley et al., 2015) and to swarm intelligence/stupidity (Kremer et al., 2014).

Following and building upon Cyert & March (1963), Marschak & Radner (1972), and Holmström (1977), a rich literature developed on the economics of organizations. We are unaware, however, of an attempt of introducing systematic errors into (team) decision making processes and an analysis of their effect on team performance and team composition in the theoretical literature. There are accounts of cognitive biases and heuristics in the management literature (e.g., Schwenk, 1984; Gary, 1998), in psychology (e.g., Kahneman, 2003; Gigerenzer & Gaissmaier, 2011), in sports (e.g., Lombardi et al., 2014), and administrative science (e.g., Tetlock, 2000), but we know of no directly related explorations in economics.

The existing economic literature on team composition problems consists only of a handful of papers. Chade & Eeckhout (2014) analyze problems of team composition when teams compete subsequent to the matching stage.[4] Their matching setup results in a model in which externalities affect the obtained sorting patterns that substantially differ from the standard case.[5] Palomino & Sákovics (2004) discuss a model of revenue sharing when sports teams competitively bid to attract talent. They find that the organization of the league(s) is key to the optimal design of remuneration schemes and the resulting availability of talent. In a paper on board composition, Hermalin & Weisbach (1988) discuss how firm performance and CEO turnover determine the choice of directors. None of these papers develops the core of our paper, namely, leadership selection under assignment errors.

The endogenous emergence of team leadership is modeled explicitly in several papers in the recent literature. In Kobayashi & Suehiro (2005), each of two players gets imperfect, private signals on team productivity. The individual incentives to lead by example (as in Hermalin, 1998) give rise to a coordination problem. Andreoni (2006) analyzes a public goods provision game in which a team can learn the project type by individually expending some small cost. The investing "leader" faces free-riding incentives. Huck & Rey-Biel (2006) analyze teams of asymmetrically productive and conformism-biased agents. They find that the less-productive among two equally biased agents should lead. In contrast, our paper does not model a particular leadership game but employs a black-box assignment function yielding selection probabilities based on idiosyncratic skills which should, in principle, be compatible with a large set of selection

[4] In their motivation, Chade & Eeckhout (2014) ask whether or not a single "superstar" team would have been able to confirm the existence of the Higgs Boson quicker than the competing ATLAS and CMS teams at CERN's Large Hadron Collider.

[5] In the settings we analyze, the optimal allocation is usually given by assortative matching, that is, the more talented team member should be assigned a leadership position or the higher productivity task. However, as the administrator (or organization) assigns leadership based on imprecise skill information, this results in a noisy allocation (for bounds on efficiency in case of coarse matching see McAfee, 2002). The main difference to this literature is that, in our analysis, the matching procedure is taken into account in the specified compensation scheme.

procedures.

There are many further papers analyzing organizational design issues that are touched upon by this paper such as, for instance, the concept of leadership (Hermalin, 1998; Lazear, 2012), battles for control (Rajan & Zingales, 2000), sequentiality of production (Winter, 2006), transparency of efforts (Bag & Pepito, 2012), and repetition (Che & Yoo, 2001). For other aspects of organizational theory see the excellent, recent overviews (e.g., Bolton et al., 2010; Hermalin, 2012; Waldman, 2012; Garicano & Van Zandt, 2012).

## 2 The model

There is a team consisting of two members $\{1, 2\}$. Each team member is supposed to exert unobservable effort that contributes to joint output. In addition, each team member $i \in \{1, 2\}$ is attributed with managerial or leadership skill $\theta_i \in \mathbb{R}_+$. The team's output depends on the assigned leader and on the efforts of all team members.[6] Denote by $y(\theta_i, e_1, e_2)$ the team output when agent $i \in \{1, 2\}$ is assigned to lead the team, agent 1 exerts effort of $e_1$ and agent 2 exerts effort of $e_2$. The cost of exerting effort $e_i$ is the same for both agents, $c(e_i)$, with $c' > 0$ and $c'' > 0$. The effect of the agents' effort exertion on output is symmetric, that is, for any $\theta_i$, $e_1$ and $e_2$ the team generates

$$y(\theta_i, e_1, e_2) = y\left(\theta_i, e_2, e_1\right). \tag{1}$$

We assume that $y$ is differentiable with

$$y_1 \ = \frac{\partial y}{\partial \theta_i} > 0,\ y_{j+1} = \frac{\partial y}{\partial e_j} > 0,\ y_{j+1,j+1} = \frac{\partial^2 y}{\partial e_j^2} < 0,\ y_{j+1,\mathbf{1}} = \frac{\partial^2 y}{\partial e_j \partial \theta_i} > 0 \tag{2}$$

for any $j \in \{1, 2\}$. The time structure of the modelled events is the following: at the first stage of the interaction, one of the agents is appointed the team leader. At the second stage, the agents exert uncontractible efforts after observing the chosen leader and his leadership skill.[7] The resulting output is divided equally between the team members.[8] Monotonicity of output $y$ with respect to the leader's skill attribute

[6] The leadership position creates a (sufficiently high) private and non-monetary benefit to the appointed leader which renders the trivial (and potentially first-best) solution of "selling the project to the manager" infeasible. For empirical justifications of such benefits including "self-dealing" see, for instance, Tirole (2006, p. 17).

[7] Similarly to the sequential game outlined above, the Apollo effect can be shown to exist in a simultaneous production version of the model in which all players choose their respective strategies at the same time.

[8] The paper's results hold regardless of the chosen output division rule. In particular, it is unimportant for the occurrence of the Apollo effect whether incentives are provided to exert (constrained) efficient efforts or not (Gershkov et al., 2016).

implies that it is optimal to choose the agent with the highest leadership skills as a team-leader.

The main premise of the paper is, however, that selecting a team leader (or decision making in general) is a complex process that sometimes involve mistakes. More precisely, we denote by $f(\theta_i, \theta_j)$ the probability that agent $i$ is appointed to the leadership position when $i$'s leadership skills are represented by parameter $\theta_i$, while the other team member's skill is $\theta_j$. With probability $1 - f(\theta_i, \theta_j)$ player $j$ is assigned the leadership. We assume that the assignment function is symmetric

$$f(\theta_i, \theta_j) = 1 - f(\theta_j, \theta_i), \tag{3}$$

responsive

$$\frac{\partial f(\theta_i, \theta_j)}{\partial \theta_i} > 0, \tag{4}$$

and satisfies appropriate probability limit behavior, in particular $f(0, \hat{\theta}) = 0$ for $\hat{\theta} > 0$.[9]

In the introduction we motivate informally how this function $f$ may arise from some management selection processes. We now give two more formal micro-justifications for the main properties of the black-box function we use throughout the paper. In the first formalization, we think of the appointing executive having access to a test which is potentially capable of ranking the candidates: if either one candidate is below and the other candidate is above the test location, then the test returns the ranking. If both candidates are below or above the test location, then one candidate is picked at random. Being less than perfectly well informed, however, the executive can choose the location of the test only probabilistically. Assume that the test realizes at threshold $\hat{\theta}$ with positive density $t(\hat{\theta})$. Then the probability of player 1 with skill $\theta_1$ being chosen under this test is

$$\frac{1}{2}\left[\int_0^{\min(\theta_1,\theta_2)} t(\hat{\theta})d\hat{\theta} + \int_{\max(\theta_1,\theta_2)}^1 t(\hat{\theta})d\hat{\theta}\right] + \mathbf{1}_{\{\theta_1 \geq \theta_2\}} \int_{\min(\theta_1,\theta_2)}^{\max(\theta_1,\theta_2)} t(\hat{\theta})d\hat{\theta}. \tag{5}$$

The derivatives for any realization of $\theta_1 > \theta_2$ are as required by our assumptions.

Our second micro-foundation is based on the idea that the administrator can make noisy observations of the two agents' types $\theta_i + \varepsilon_i$ and only knows that $\varepsilon_i$ is distributed independently and identically according to any continuous distribution $H$ (for a complete model development, see Lazear & Rosen, 1981). The administrator then bases a decision on her noisy observation of leadership abilities. In this environment, the probability that agent 1 will be appointed is

$$\Pr(\theta_1 + \varepsilon_1 > \theta_2 + \varepsilon_2) \ = \Pr(\varepsilon_2 - \varepsilon_1 < \theta_1 - \theta_2) \equiv P \tag{6}$$

[9] The implied discontinuity at $f(0, 0)$ does not play a role in our analysis.

in which the difference between the two independently distributed random variables is itself a continuously distributed random variable. The derivatives of this assignment probability $P$ satisfy the required properties of $f(\theta_1, \theta_2)$.

# 3 The main result

This section presents the principal finding of this paper, the ubiquity of the Apollo effect. Before we start the formal analysis we would like to point out that the first-best efficient selection in which the better qualified player is always appointed the team leader by an uniformed administrator is generally unattainable in the specified game based on selection capabilities $f$. We start the discussion by means of a simple, illustrative example of the main idea.

Example 1: In the following comparative static arguments we distinguish between two teams $j \in \{\mathrm{A}, \mathrm{B}\}$ and typically assume that team members' abilities are ranked $\theta_1^A \geq \theta_1^B$ and $\theta_2^A \geq \theta_2^B$, so team A consists of unambiguously higher ability players than team B. For leadership selection, an administrator employs a black-box function based on ability ratios which gives the probability of player $i \in \{1,2\}$ being selected as leader as[10]

$$f(\theta_i, \theta_j) \ = \frac{\theta_i^r}{\theta_1^r + \theta_2^r}, \ r > 0. \tag{7}$$

If player $i \in \{1,2\}$ is selected as team $j$'s leader ($j \in \{A, B\}$), then $\hat{\theta} = \theta_i^j$ and the team generates simple linear output

$$y(\hat{\theta}, e_1, e_2) = \hat{\theta}(e_1 + e_2). \tag{8}$$

As either player 1's or player 2's ability is employed exclusively for leadership we refer to this case as "exclusive" management or production.[11] Following the time structure outlined above, workers know whether or not they are assigned leadership roles before exerting efforts, i.e., any mistakes are made during a first leadership assignment stage while unobservable efforts are exerted by perfectly informed agents at a second stage. More specifically, player $i$'s stage-2 objective, given that the player with type $\hat{\theta}$ is chosen as leader and output is shared equally, is

$$\max_{e_i} \ u_i(\hat{\theta}) = \frac{y(\hat{\theta}, e_i, e_j)}{2} - c(e_i). \tag{9}$$

[10] In different environments similar functions have been called "logistic" or "sigmoid" functions. The contest literature refers to a variant of (7) as "ratio," "power," or "Tullock" contest success function (Jia et al., 2013). Note that—as there are no strategies involved at this stage—our use of this function for leadership selection is purely descriptive and constitutes no game.

[11] In order to capture also shared production aspects in teams of complementary skills where individuals are matched to tasks, we will later allow for a "task matching" model extension.

Assuming quadratic effort costs $c(e) = e^2$, symmetric equilibrium efforts are simply

$$e_1(\hat{\theta}) \ = e_2(\hat{\theta}) = \hat{\theta}/2. \tag{10}$$

At the leadership selection stage, the administrator selects either player 1 with probability $f(\theta_1, \theta_2)$ or player 2 with probability $1 - f(\theta_1, \theta_2)$ as the team leader. Hence, first stage expected equilibrium team output is

$$\begin{aligned} Y(\theta_1, \theta_2) &= f(\theta_1, \theta_2) y(\theta_1, e(\theta_1), e(\theta_1)) + (1 - f(\theta_1, \theta_2)) y(\theta_2, e(\theta_2), e(\theta_2)) \\ &= \theta_2^2 + f(\theta_1, \theta_2)(\theta_1^2 - \theta_2^2) \\ &= \frac{\theta_1^{r+2} + \theta_2^{r+2}}{\theta_1^r + \theta_2^r}. \end{aligned} \tag{11}$$

We now implicitly define an "isoquant" function $\theta_2(\bar{y}, \theta_1)$ which determines the type $\theta_2$ that achieves the constant output level $\bar{y}$ for some type $\theta_1$. An example is shown in Figure 1: low precision $r = .25$ is shown on the left, moderate precision $r = 2$ in the middle, and high precision $r = 15$ on the right.[12] We restrict attention (without loss of generality) to $\theta_1 \geq \theta_2$, so only the subset under the diagonal is relevant in the figure. Team compositions "under the isoquant," that is, to the left of the isoquant $\theta_2(\bar{y}, \theta_1)$, produce lower output than $\bar{y}$. Skill pairs "above the isoquant" to the right of isoquant $\theta_2(\bar{y}, \theta_1)$ produce higher output than $\bar{y}$. The Apollo effect arises here because, for any point $(\hat{\theta}_1, \hat{\theta}_2)$ on a positively sloped part of an isoquant, we can find a point $(\theta_1 > \hat{\theta}_1, \theta_2 > \hat{\theta}_2)$ *under this isoquant* (close to where it is vertical), such that $y(\hat{\theta}_1, \hat{\theta}_2) > y(\theta_1, \theta_2)$. Note that one would not expect a positive slope of the isoquants in Figure 1 without the possibility of making mistakes in leadership assignment. In this example, the Apollo effect shows for all selection precisions, provided that the type spread $\theta_1 - \theta_2$ is sufficiently high. $\triangleleft$

One may be curious, however, how pervasive the occurrence of the Apollo effect arising in the above example is. In order to answer this question, we start the formal argument by defining the Apollo effect in a general production environment with two teams.

**Definition 1.** *The environment expresses the Apollo effect, if there exist two teams* $\{A, B\}$ *with leadership skills* $\left(\theta_1^A, \theta_2^A\right) >> \left(\theta_1^B, \theta_2^B\right)$ *with* $y^A < y^B$*, where* $y^A$ *is the equilibrium output of team* $A$ *and* $y^B$ *is that of team* $B$*.*

[12] We refer to the exponent $r$ in (7) as "selection precision" of player 1 because it parameterizes the derivative of the assignment function with respect to $\theta_1$. The comparison case of no mistakes is obtained for $r \to \infty$, i.e., $f(\theta_1, \theta_2) = 1$ iff $\theta_1 \geq \theta_2$. In this case, the level sets in the right panel of Figure 1 become a perfectly rectangular map. In contrast, if $r = 0$, we have $f(\theta_1, \theta_2) = 1/2$ for any $\theta_1$ and $\theta_2$.

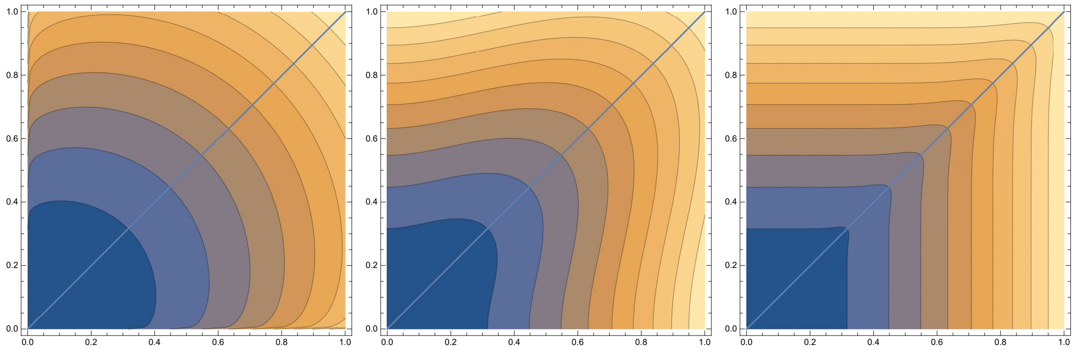


Figure 1: Shown are "isoquant" expected team output level sets with $\theta_1$ on the horizontal and $\theta_2$ on the vertical axis for $r = .25$ on the left, $r = 2$ in the middle, and $r = 15$ on the right.

Without loss of generality, we assume that $\theta_1 \geq \theta_2$. Observing the appointed leader of type $\hat{\theta}$ and assuming equal sharing of output, team member $i$ maximizes effort stage utility

$$\max_{e_i} \; u_i = \frac{y(\hat{\theta}, e_1, e_2)}{2} - c(e_i). \tag{12}$$

Taking the derivative with respect to $e_i$ defines symmetric equilibrium effort $e^* = e_1 = e_2$ as

$$y_{i+1}(\hat{\theta}, e_i, e_j) - 2c'(e_i) = 0 \tag{13}$$

in which subscripts on functions denote derivatives. The assumed curvature of the output and cost functions guarantee that $e^*(\hat{\theta})$ is non-decreasing. We substitute these equilibrium efforts into output which determines equilibrium team output as

$$Y(\theta_1, \theta_2) = f(\theta_1, \theta_2) y(\theta_1, e^*(\theta_1), e^*(\theta_1)) + (1 - f(\theta_1, \theta_2))\, y(\theta_2, e^*(\theta_2), e^*(\theta_2)). \tag{14}$$

It turns out that the following is an analytically convenient way to demonstrate that the Apollo effect exists: we show that there exists a skill combination $(\theta_1, \theta_2)$ such that $y(\theta_1, \theta_2)$ has a positive gradient, i.e., there exist $(\eta_1, \eta_2) >> 0$ such that

$$\frac{\partial Y(\theta_1, \theta_2)}{\partial \theta_1}\eta_1 + \frac{\partial Y(\theta_1, \theta_2)}{\partial \theta_2}\eta_2 < 0. \tag{15}$$

Our main claim is that there exists a team endowed with skills $\theta^A$ for which equilibrium team output shrinks if either or both types are increased infinitesimally.

**Lemma 1.** *The Apollo effect arises if and only if*

$$-\frac{f_2(\theta_1, \theta_2)}{1 - f(\theta_1, \theta_2)} > \frac{e'^*(\theta_2) 2 y_e(\theta_2, e^*(\theta_2), e^*(\theta_2)) + y_1(\theta_2, e^*(\theta_2), e^*(\theta_2))}{y(\theta_1, e^*(\theta_1), e^*(\theta_1)) - y(\theta_2, e^*(\theta_2), e^*(\theta_2))}. \tag{16}$$

The Lemma allows us to specify when the Apollo effect is plausible. It shows that increasing the difference between the team members increases the chance of observing the Apollo effect (if $y(\theta_1, e^*(\theta_1), e^*(\theta_1)) - y(\theta_2, e^*(\theta_2), e^*(\theta_2))$ is high, $f(\theta_1, \theta_2)$ is high and hence it is easier to satisfy the condition of the last Lemma). Moreover, the probability of misallocation must be responsive to the skills, that is, $f_2(\theta_1, \theta_2)$ is substantially low (and negative).

An immediate implication of the Lemma and its proof is that while improving the leadership skill of the best team member is always beneficial, this is certainly not the case for the lower-qualified team member. We proceed to state a general property of exclusive production.

**Lemma 2.** *For exclusive production $y(\theta, e(\theta), e(\theta))$ and $\hat{\theta} > 0$, we have*

$$\begin{aligned} &f(\hat{\theta},\hat{\theta})y(\hat{\theta},e(\hat{\theta}),e(\hat{\theta})) + (1 - f(\hat{\theta},\hat{\theta}))y(\hat{\theta},e(\hat{\theta}),e(\hat{\theta})) \\ &\quad = f(\hat{\theta},0)y(\hat{\theta},e(\hat{\theta}),e(\hat{\theta})) + (1 - f(\hat{\theta},0))y(0,e(0),e_2(0)). \end{aligned} \tag{17}$$

Therefore, for any $\hat{\theta}$, the points $(\hat{\theta}, \hat{\theta})$ and $(\hat{\theta}, 0)$ belong to the same isoquant. Note that for symmetric functions $f$, the isoquants' slope at $\theta_1 = \theta_2$ must be $-1$ at the diagonal of our level sets. Together, these observations imply the following general result.

**Proposition 1.** *The Apollo effect arises under exclusive leadership assignment for every feasible continuous function $f$.*

This results shows that the only case in which the Apollo effect cannot arise is if the possibility for leadership selection mistakes is entirely absent. For concave production technology and any conceivable, not infinitely accurate continuous leadership selection technology $f$, there will be skill profiles which give rise to the Apollo effect, i.e., where unambiguously better qualified teams must be expected to produce lower output than a set of "underdogs." We now illustrate our main result through a series of applications and direct extensions in the form of remarks.

**Remark 1** (Labor market)**.** *This environment can be used to study the effect of imprecise leadership selection on the optimal assignment of agents to several teams. We keep the same informational assumptions as in the rest of this section but are here only interested in characterizing the optimal team composition, not a game capable of bringing it about. In particular, we ask which agent types from the ordered set $\theta_1 > \theta_2 > \cdots > \theta_n$, $n \geq 3$ should optimally self-select into what team structure?*

*We assume that the firm wishes to create $k < n/2$ teams of two agents each. Subsequent to the creation of the teams, a leader will be chosen in each team following* *the procedure we introduced. How should the hiring and team creation process take the later leader selection process into account? To answer this question, we have to identify the optimal hiring and coupling assuming that the types are observable at this stage. This illustrates which types should be targeted and the information that needs to be collected on candidates.*

*Absent a possibility for subsequent leadership selection mistakes, an optimal matching is to form $k$ teams with team $j$ led by agent $\theta_{2j-1}$, i.e., one of the $k$ agents with the highest leadership ability with any second agent chosen from the lower half of types. If the lower-skill partners' types have an arbitrarily small output contribution, then the lowest type(s) ($\theta_{2k+1}, \ldots, \theta_n$) will never be employed.*[13] *Therefore, it is important to identify and exclude the lowest ability types. Yet, if leadership assignment is imprecise, an implication of the Apollo effect is that a set of workers strictly better qualified than these "worst" types should be optimally excluded.*

*Consider, for example, the ordered set of $n = 5$ agent types $\theta_i = (n-i)/(n-1)$ with identical, linear production $y(\theta, e_1, e_2) = \theta(e_1 + e_2)$, quadratic costs $c(e) = e^2/2$, and ratio assignment $f(\theta) = \theta_i^r/(\theta_i^r + \theta_j^r)$, $r > 0$. Assume that the organization needs two teams and, hence, seeks to exclude one agent. For $r \geq 1$ it is optimal to exclude the agent with median ability $\theta_3$. The example intuition of "dropping the middle" types for sufficiently precise assignment $f$ can be generalized and has implications for the labor market: firms demand the right types, not necessarily the highest available types. In the example, given sufficient precision of $f$, the middle types are left unemployed but the lowest type $\theta_n$ is employed in all optimal matchings!*

**Remark 2** (Project selection)**.** *Consider a manager's choice between two projects of unknown quality $\theta_1, \theta_2$ guided by the imperfect selection technology $f(\theta_1, \theta_2)$. In this application, project output $y(\theta_i, K, L)$ is increasing in $\theta_i$, satisfies the equivalents of assumptions* (1) *&* (2)*, and the symmetric factors $K$ and $L$ are chosen by strategic project employees who privately observe quality $\theta_i$. Proposition 1 shows that there are situations in which improving both individual projects to $\theta_1' > \theta_1$ and $\theta_2' > \theta_2$ actually decreases the firm's expected revenue relative to the original, unambiguously worse project environment.*

**Remark 3** (Larger teams)**.** *While our other results are stated for assignment functions defining selection probabilities for just two players we now analyze the consequences*

[13] This positive influence can be made precise and formalized by an infinitesimally small, positive multiplier $t_l$ in the task-matching environment of section 4.1 which, in general, gives qualitatively similar results to exclusive production only for intermediate precision of assignment function $f$.

*of increasing the team size.*[14]

*For example, consider an $n$-player version of our model governed by the usual linear production $y(\theta, e_1, \ldots, e_n) = \theta(e_1 + \cdots + e_n)$ and quadratic efforts cost $c(e) = e^2/2$. We adopt a ratio-assignment function which gives the probability of (the highest-type) player 1 being selected as*

$$f(\theta_1, \ldots, \theta_n) \;= \frac{\theta_1^r}{\theta_1^r + \cdots + \theta_n^r},\; r > 0. \tag{18}$$

*Provided that all team members share output equally, this results in type-contingent equilibrium efforts of $e = \theta/n$ while a benevolent planner would dictate the efficient $e^* = \theta$. As in the two agent case, the Apollo effect arises in this example.*

# 4 Further results

## 4.1 Task matching

The main result of this paper rests on a conflict (for leadership) interpretation to explain the Apollo effect since either team member's management skills enter the production process exclusively. Only one of the team members is appointed the leader and the other player's leadership skill is completely discarded. Deviating from this leadership interpretation, we now assume that the production technology requires that all workers are matched to their "correct" tasks and therefore both individual skills enter production.[15] That is, we consider an environment in which the organization or its executives must assign team members to different tasks and, after the assignment, the agents apply their skills and exert effort on the allocated tasks. This assignment, however, may involve mistakes or misallocations of agents to tasks. We employ the following output function

$$y(\theta_i, \theta_j, e_i, e_j) = y^h(\theta_i, e_i) + y^l(\theta_j, e_j) \tag{19}$$

in which both $y^h(\theta, e)$ and $y^l(\theta, e)$ are weakly concave and increasing in both arguments. That is, each worker is matched either with task $h$ or with task $l$. Each worker

[14] Amazon's Jeff Bezos is reported to employ a "two pizza rule:" if a team cannot be fed by two pizzas, then that team is too large. The idea is that having more people work together is less efficient, i.e., team output decreases beyond the optimal size. This is the case in Shellenbarger (2016) who argues that participants tend to feel less accountable in crowded meetings, therefore doubt that any contribution they make will be rewarded, and hence reduce effort.

[15] Referring back to our motivational example of the NASA Apollo missions, the Apollo teams were composed of distinct roles. The Apollo 11 team, for instance, consisted of mission commander Neil Armstrong, command module pilot Michael Collins, and lunar module pilot Edwin Aldrin. Hence, team performance depended on each member of the crew being selected into and performing a very specific task.

uses "leadership" skills and efforts on the allocated task. Function $f$ chooses the assignment of the workers to the tasks. Otherwise the model is the same as in the previous section. Without loss of generality and as before we assume that $\theta_1 \geq \theta_2$. We assume that for any $e \geq 0$

$$y_1^h(\theta, e) \; > y_1^l(\theta, e) > 0. \tag{20}$$

Therefore, the efficient assignment is that the higher ability agent 1 is assigned task $h$, while agent 2 is assigned task $l$. Given an allocation, the agents will exert efforts dictated by the first-order conditions $(e_i^h(\theta_i), e_j^l(\theta_j))$:

$$y_2^h(\theta_i, e_i) \; = 2c'(e_i), \; y_2^l(\theta_j, e_j) = 2c'(e_j). \tag{21}$$

Assuming, in addition to (20), that

$$y_2^h(\theta, e) \; > y_2^l(\theta, e) > 0, \; y_{12}^h(\theta, e) > y_{12}^l(\theta, e) > 0 \text{ and } 0 > y_{22}^h(\theta, e) > y_{22}^l(\theta, e) \tag{22}$$

implies that equilibrium effort on both tasks is increasing in type and that both $e^h(\theta) > e^l(\theta) > 0$ and $e^{h\prime}(\theta) > e^{l\prime}(\theta) > 0$. At the selection stage, expected team output under task matching is

$$Y(\theta_i, \theta_j) \; = f(\theta_i, \theta_j) z(\theta_i, \theta_j) + (1 - f(\theta_i, \theta_j)) \, z(\theta_j, \theta_i), \tag{23}$$

in which we assume that $z(\theta_i, \theta_j)$ is the equilibrium output if agent $i$ is assigned to task $h$ and agent $j$ is assigned to task $l$, i.e.,

$$z(\theta_i, \theta_j) = y(\theta_i, \theta_j, e_i^h(\theta_i), e_j^l(\theta_j)). \tag{24}$$

Our assumptions above imply that $z(\theta_1, \theta_2) > z(\theta_2, \theta_1)$. As for our main result, we fix ideas by starting with a motivating example.

Example 2: We assume that team output is created by the simple production function

$$y\left(\theta_i, \theta_j, e_i, e_j\right) = t_h \theta_i e_i + t_l \theta_j e_j, \text{ with } t_h \geq t_l. \tag{25}$$

Similarly to the previous example, we assume that costs are quadratic, $c(e) = e^2$, and that the allocation technology is

$$f(\theta_i, \theta_j) \; = \frac{\theta_i^r}{\theta_1^r + \theta_2^r}, \; r > 0 \tag{26}$$

specifying the probability that agent $i$ is assigned task $h$. Then task-specific equilibrium efforts are $e^x(\theta) = t_x \theta$, $x \in \{h, l\}$, and expected equilibrium team output is

$$\begin{aligned} Y(\theta_i, \theta_j) &= f(\theta_i, \theta_j) z(\theta_i, \theta_j, e_i^h(\theta_i), e_j^l(\theta_j)) + (1 - f(\theta_i, \theta_j)) z(\theta_j, \theta_i, e_j^h(\theta_j), e_i^l(\theta_i)) \\ &= \frac{f(\theta_i, \theta_j)(\theta_i^2 - \theta_j^2)(t_h^2 - t_l^2) + \theta_j^2 t_h^2 + \theta_i^2 t_l^2}{2} \\ &= \frac{t_h^2\left(\theta_i^{r+2} + \theta_j^{r+2}\right) + t_l^2\left(\theta_j^2 \theta_i^r + \theta_i^2 \theta_j^r\right)}{2\left(\theta_i^r + \theta_j^r\right)}. \end{aligned}$$

The arising isoquants under different selection precisions $r$ are shown in Figure 2: as in Figure 1 for the exclusive leadership case, low precision $r = .25$ is shown on the left, moderate precision $r = 2$ in the middle, and high precision $r = 15$ on the right. The example task values are $t_h = 2/3$, $t_l = 1/3$.

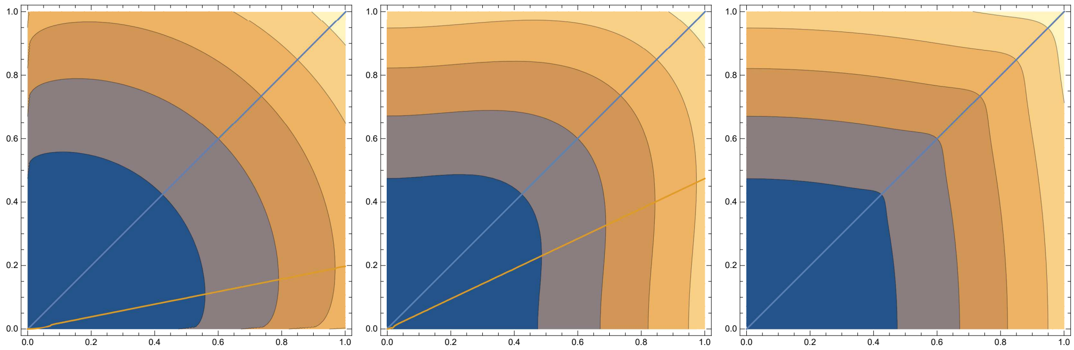

Figure 2: Task matching level sets showing expected output for $t_h = 2/3$ and $t_l = 1/3$. The level sets are drawn for $r = .25$ on the left, $r = 2$ in the middle, and $r = 15$ on the right. The solid golden line represents condition (27).

The Figure illustrates that under task matching and for a given pair $(t_h, t_l)$, the Apollo effect only shows in cases where the subsequent selection precision $r$ is below the minimal threshold which in the present example is implicitly given by

$$\frac{3\theta_2^2(\theta_1^r + \theta_2^r)}{(\theta_1^2 - \theta_2^2)(\theta_1\theta_2)^r} = r\frac{t_h^2 - t_l^2}{\theta_1^r t_h^2 + \theta_2^r t_l^2} \tag{27}$$

or, plugging in example values, $r \leq 2.52$. This threshold condition expresses that the less it matters who is assigned to which task, i.e., the closer $t^h$ and $t^l$ are, the more likely assignment mistakes must be in order for the Apollo effect to arise. $\lhd$

Intuitively, we can decompose the second player's marginal output contribution into two components: productive and disruptive. For the moment, consider the (efficient) case of an infinitely precise allocation function $f$ so the disruptive effect does not arise. Starting at any interior point $\hat{\theta} = \theta_1 = \theta_2$ on the diagonal in figures 2, 3 and 4, a decrease in $\theta_2$ goes along with lower output that must be compensated by an increase in $\theta_1$ in order to stay on the same isoquant $I(\cdot)$. Hence, the isoquants in the middle panel of the top row of Figure 3 now become "triangular" in the sense that the point on the diagonal where $\theta_1 = \theta_2 = \hat{\theta}$ is connected by a negatively sloped curve with the point on the horizontal axis where $(\tilde{\theta}_1 > \hat{\theta}_1, \theta_2 = 0)$. This point is to the right of the point $(\hat{\theta}_1, \theta_2 = 0)$ directly under the diagonal from which we started. The horizontal shift of the isoquant depicts the marginal productive influence of player 2 which we call the productive effect (which includes, more generally speaking, the "synergies" created by teamwork).

The discussed isoquant maps are illustrated in Figure 3, the detailed decomposition into productive and disruptive marginal effect is shown in Figure 4. The latter displays the productive marginal effect (the negative vertical slope of the blue isoquant $I(a', b')$) and the total marginal effect (the vertical slope of the red isoquant $I(a'', b'')$) for the task-matching case of $t_h > t_l$ and intermediate selection precision $f$.

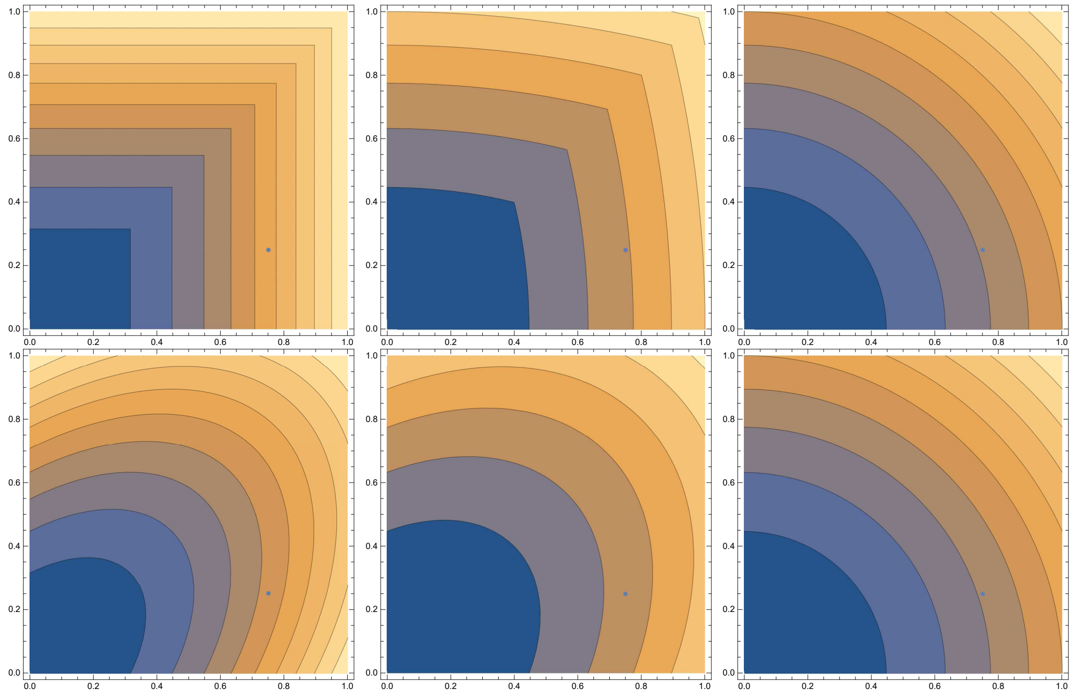

Figure 3: Isoquants for infinitely precise $f$ in the first row (illustrating the pure productive effect) and $r = 1$ in the second row (illustrating both productive and disruptive effects). Plotted are the cases of $t_h = 1$ and $t_l = 0$ (left), $t_l = 1/2$ (middle), and $t_l = 1$ (right).

Any assignment function $f$ which satisfies our assumptions introduces allocative inefficiency, thereby shifting all points of the efficient-assignment blue isoquant—except for the two points on the diagonal and horizontal axis just pinned down—further to the right, resulting in the red isoquant of Figure 4. This is what we call the disruptive effect. The disruptive effect tends to shift points $(\theta_1, \theta_2)$ close to the diagonal (where the chance of mistakes is highest) further to the right than those with lower $\theta_2$. But only in the extreme case in which the disruptive effect causes an isoquant to become positively sloped the Apollo effect arises. This happens precisely if the (negative) marginal disruptive effect—described by $f_2(\theta_1, \theta_2)$—more than outweighs the (positive) marginal productive effect of a marginal increase of $\theta_2$.

Compare this to the exclusive leadership case considered in the previous section (illustrated in the two left-hand panels of Figure 3 and the black isoquant of Figure 4): there, the efficient isoquant map was perfectly rectangular and any imprecision

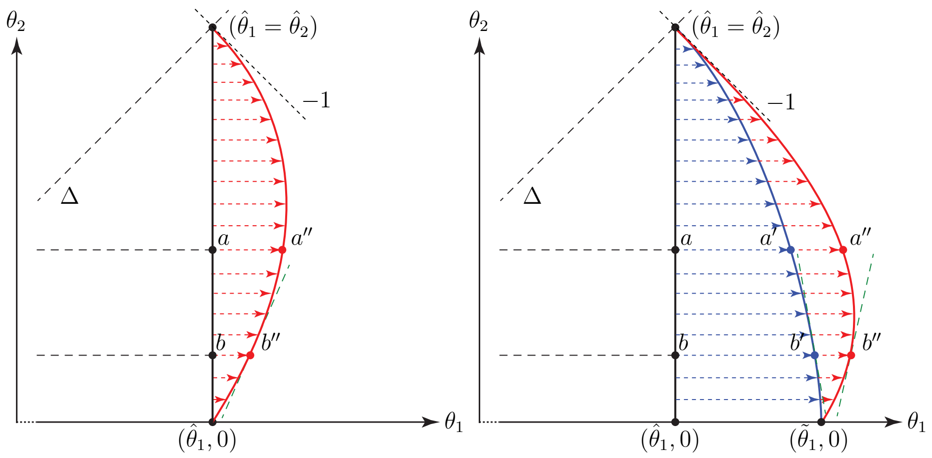


Figure 4: Three isoquants are shown on the right: infinitely precise $f$: $I(a, b)$ under exclusive leadership (black), infinitely precise $f$: $I(a', b')$ for task-matching (blue), and finite $f$: $I(a'', b'')$ under task-matching with $t_h > t_l$ (red). The marginal, positively sloped (total) Apollo effect is shown as dashed tangent through $b''$. The necessity of the Apollo effect under exclusive leadership for finite $f$ is illustrated on the left.

of $f$ leads to disruption shifting all points of the isoquant (expect for those on the diagonal and horizontal axis) to the right. Hence, the Apollo effect is always present in the simpler exclusive leadership environment of proposition 1. The existence of the Apollo effect in the task matching environment of this section, however, depends on the marginal output of player 2 (1)—her productive contribution—being smaller than the disruptive effect introduced through the possibility of wrongly assigning her to the more (less) productive task $h$ ($l$). Our next result summarizes this intuition and generalizes the previous example by identifying a condition on the assignment function $f$ that guarantees the Apollo effect to arise also in the task-matching environment.

**Proposition 2.** *For equilibrium task-matching production $z(\theta_i, \theta_j)$, a sufficient condition for the Apollo effect to arise for some type profile $\theta_1 > \theta_2$ is that the selection technology $f(\theta_1, \theta_2)$ satisfies*

$$f_2(\theta_1, \theta_2) < \frac{z_2(\theta_1, \theta_2) + z_1(\theta_2, \theta_1)}{2z(\theta_2, \theta_1) - 2z(\theta_1, \theta_2)}. \tag{28}$$

Notice that the condition of this proposition holds if $f_2(\theta_1, \theta_2)$ is sufficiently low (and negative). To get a better understanding of the last condition, observe that for $f$ infinitely precise, we have $f_2(\theta_1, \theta_2) = 0$ for any $\theta_1 > \theta_2$. Therefore, indeed as we write in the intuition before the proposition, the Apollo effect arises for sufficiently imprecise assignment functions.

Example 3: We continue in the setup of the previous example with

$$z(\theta_i, \theta_j) = t_h y^p(\theta_i, e^h) + t_l y^q(\theta_j, e^l).$$

For quadratic costs and task-specific, linear production (25), the equilibrium production is $y^x(\theta, e(\theta)) = t_x\theta^2$, $x \in \{h, l\}$. The condition for an isoquant to have positive slope (expression (43) in the proof of proposition 2) is

$$\frac{t_h^2}{t_h^2 - t_l^2} < f(\theta_1, \theta_2) - f_2(\theta_1, \theta_2)\frac{\theta_1^2 - \theta_2^2}{2\theta_2}. \tag{29}$$

For the general ratio assignment function (7), this condition (29) equals

$$\frac{t_h^2}{t_h^2 - t_l^2} < \frac{\theta_1^r}{\theta_1^r + \theta_2^r} - (r\theta_2^{r-2})\frac{\theta_1^r\left(\theta_2^2 - \theta_1^2\right)}{2\left(\theta_1^r + \theta_2^r\right)^2} \tag{30}$$

in which the term $r\theta_2^{r-2}$ goes to infinity as $\theta_2 \to 0$ for $0 < r < 2$, irrespective of $\theta_1 > \theta_2$. Hence the claimed inequality holds for some spread of types. This is confirmed by the sufficient condition (28) which specifies in this case

$$-\frac{r\theta_1^r\theta_2^{r-1}}{\left(\theta_1^r + \theta_2^r\right)^2} < -\frac{\theta_2\left(t_h^2 + t_l^2\right)}{\left(\theta_1^2 - \theta_2^2\right)\left(t_h^2 - t_l^2\right)} \tag{31}$$

which equals at arbitrary point $\theta_1 = 3/4, \theta_2 = 1/4$ (indicated in the below figure) and task multipliers $t_h = 1, t_l = 1/4$

$$-\frac{r}{\cosh(r\log(3)/2)^2} < -\frac{17}{30} \Leftrightarrow r \in [0.64, 2.5]. \tag{32}$$

Figure 5 shows examples of the corresponding output contour sets for different task multipliers and selection precisions. ◁

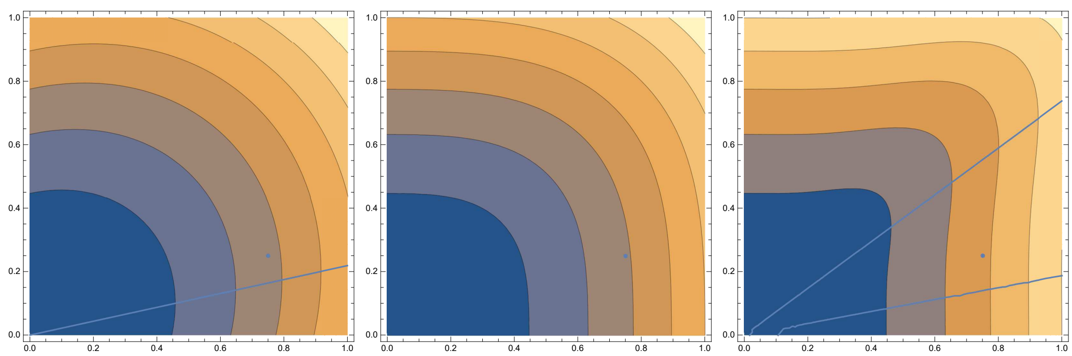


Figure 5: Left and center: type combinations for $t_h = 1, t_l = 3/4$ producing the same output $Y(\theta_1, \theta_2)$. Left is the case of $r = 1$, center are the same isoquants for $r = 2$. The blue lines shows type-pairs for which the isoquants are vertical. The right panel illustrates a case of multiple critical locations ($t_h = 1$, $t_l = 1/4$, and $r = 4$).

## 4.2 Incomplete information among agents

In this section we illustrate the robustness of our previous results by relaxing the assumption that agents know each others' types. For this purpose, we assume the types distribute independently and identically according to distribution function $G$ with density $g$ on support $[a,b]$. When exerting effort, each agent knows only his own type and whether or not (s)he was assigned as a leader. Therefore, a symmetric equilibrium is characterized by two functions: $e^L(\theta)$, the effort function of the agent who was selected to be the team-leader and $e^F(\theta)$, the effort function of the agent who was not selected as team-leader.

**Proposition 3.** *A pair of necessary conditions for equilibrium efforts under incomplete information among agents' skills is*

$$\int_a^b y_2\left(\theta, e^L(\theta), e^F(\theta')\right) f(\theta,\theta')\, g(\theta')\, d\theta' = 2c'(e^L(\theta)) \int_a^b f(\theta,\theta')\, g(\theta')\, d\theta' \quad (33)$$

*and*

$$\int_a^b y_3\left(\theta', e^L(\theta'), e^F(\theta)\right) f(\theta',\theta)\, g(\theta')\, d\theta' = 2c'(e^F(\theta)) \int_a^b f(\theta',\theta)\, g(\theta')\, d\theta'. \quad (34)$$

We illustrate this result in the same environment as for the previous examples. Assume that $c(e) = e^2/2$ and $y(\theta, e_1, e_2) = \theta\,(e_1 + e_2)$, then the above first-order conditions (33) and (34) become

$$\begin{aligned} e^L(\theta) &= \theta/2, \\ e^F(\theta) &= \frac{\int_a^b \theta' f(\theta',\theta)\, g(\theta')\, d\theta'}{2\int_a^b f(\theta',\theta)\, g(\theta')\, d\theta'} = \mathbb{E}_{\theta'|\text{follower has type }\theta}\left[\theta'\right] \\ &= \frac{r+1}{2(r+2)} \frac{{}_2F_1\left(1, \frac{r+2}{r}; 2+\frac{2}{r}; -\theta^{-r}\right)}{{}_2F_1\left(1, 1+\frac{1}{r}; 2+\frac{1}{r}; -\theta^{-r}\right)} \end{aligned} \quad (35)$$

in which ${}_2F_1(x)$ is the ordinary hypergeometric function (representing the hypergeometric series).[16] An example for the Uniform distribution and the case of $\theta = 2/3$ is shown in Figure 6. These equilibrium efforts yield expected team output

$$Y(\theta_1,\theta_2) = f(\theta_1,\theta_2) y(\theta_1, e^L(\theta_1), e^F(\theta_2)) + (1 - f(\theta_1,\theta_2))\, y(\theta_2, e^F(\theta_1), e^L(\theta_2)). \quad (37)$$

Isoquants as in the previous examples are shown for precisions $r \in \{.25, 2, 8\}$ in Figure 6. As the positively sloped parts of the isoquants illustrate, the Apollo effect is present in this example with incomplete information as well.

[16] The ordinary hypergeometric function is defined as

$${}_2F_1(a,b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}. \quad (36)$$

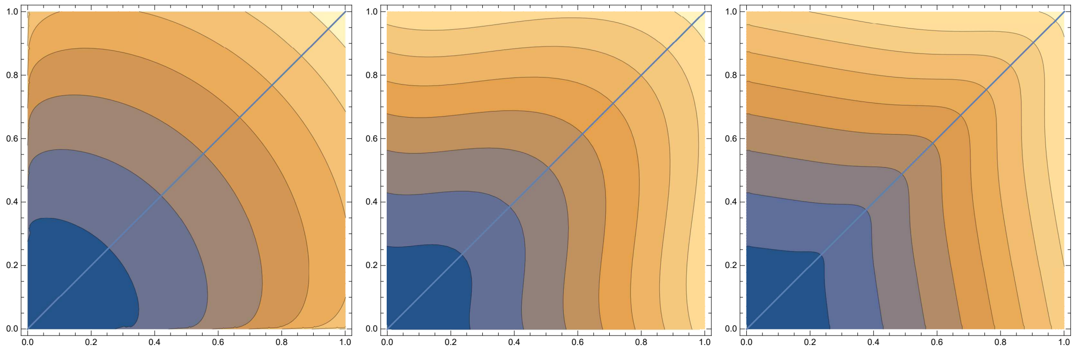


Figure 6: Expected team output level sets for uniformly distributed partner types for $r = .25$ on the left, $r = 2$ in the middle, and $r = 8$ on the right.

## 4.3 Principal-agent model

Contrary to the team production environment used for all other results of this paper, this section explores the robustness of our findings with respect to the presence of a profit-maximizing principal who may act as a budget breaker and can therefore discipline the team members engaged in production. While we assume that the agents' efforts remain unobservable, we assume that final output is observe- and contractible. Moreover, we assume that the principal—although (s)he does not observe the attributes of the chosen leader—knows the composition of skills in the team. Therefore, the contract that the principal specifies may depend on the produced output and the composition of the leadership skills in the team (but not of the skill of the assigned leader).

We analyze the same production setup as before in an environment in which a board (the principal) appoints a manager among a team of heterogeneous agents. We model the situation in which this principal may make mistakes in assigning the "correct" leader to the team by assuming that the principal only observes ranking information on agent types' $\theta$, summarized by function $f$ in (7).

<u>Example 4</u>: In the exclusive production environment, assume that the principal pays a fixed wage $w$.[17] Assume further, as a first step, that agent efforts are observable to the principal (but types are not) and wages can condition on these efforts. Moreover, we assume the same linear production function (8) as in the previous examples. Then the principal and agents solve the problem

$$\begin{aligned} \max_{w(e)} \quad & y = f\left[\theta_1\left(2e_i^1\right) - 2w(e_i^1)\right] + (1-f)\left[\theta_2\left(2e_i^2\right) - w(e_i^2)\right] \\ \text{s.t.} \quad & u_i^j = w(e_i^j) - c(e_i^j) \geq 0. \end{aligned} \tag{38}$$

[17] A similar example for the principal-agent model under task-matching exhibits qualitatively comparable effects and is available from the authors.

Under the standard quadratic costs, this is solved by

$$e_i^j(\theta_j) \ = \theta_j, \ w(e_i^j) = \frac{(\theta^j)^2}{2}. \tag{39}$$

Since efforts can be observed by the principal, she can ex-post invert the observed efforts to learn the agents' types. This information, however, is not available to her at the ex-ante stage when she makes the leadership assignment. Under the assumed ratio assignment mistakes (7), expected equilibrium team output is

$$2\frac{\theta_1^{r+2} + \theta_1^{r+2}}{\theta_1^r + \theta_2^r}. \tag{40}$$

Our usual example confirms the possibility of the Apollo effect in this environment: Figure 7 shows the principal's equilibrium profit exhibiting the Apollo effect in all cases (team output would show the same effect).

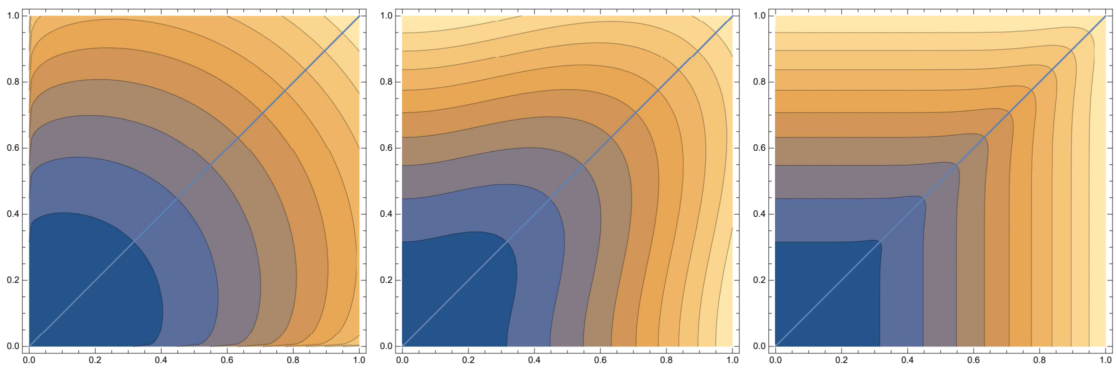

Figure 7: Expected profits in the principal-agent environment with observable efforts. The panels show selection precisions $r \in \{1/4, 2, 15\}$ from left to right.

We proceed to the case of unobservable efforts. We stay in the linear production environment with $y = \theta(e_1 + e_2)$, quadratic costs $c(e) = e^2/2$, and only two possible assignments: $\theta_1 > \theta_2$. The principal sets the wage $w$ based on the observed output $y$. Without loss of generality we can assume that the principal pays equally to both agents. The principal wants to induce effort of $e(\theta_1)$ when the assignment is $\theta_1$, and $e(\theta_2)$ when the assignment is $\theta_2$. Therefore, along the equilibrium path, the principal expects to see either $y(\theta_1) = 2\theta_1 e(\theta_1)$ or $y(\theta_2) = 2\theta_2 e(\theta_2)$. Without loss of generality we can assume that there are two wage levels: $w(y(\theta_1))$ and $w(y(\theta_2))$, for any other output, the principal pays a wage of zero.

Hence, the combined problem of the principal and the two agents we consider is

$$\begin{aligned}
\underset{e(\theta),w(y(\theta))}{max} \quad & f\left[y(\theta_1) - 2w(y(\theta_1))\right] + (1-f)\left[y(\theta_2) - 2w(y(\theta_2))\right] \\
\text{s.t.} \quad & (\text{IR}_1): \; w(y(\theta_1)) - c(e(\theta_1)) \geq 0, \\
& (\text{IR}_2): \; w(y(\theta_2)) - c(e(\theta_2)) \geq 0, \\
& (\text{IC}_1): \; w(y(\theta_1)) - c(e(\theta_1)) \geq w(y(\theta_2)) - c\left(\frac{y(\theta_2)}{\theta_1} - \frac{y(\theta_1)}{2\theta_1}\right), \\
& (\text{IC}_2): \; w(y(\theta_2)) - c(e(\theta_2)) \geq w(y(\theta_1)) - c\left(\frac{y(\theta_1)}{\theta_2} - \frac{y(\theta_2)}{2\theta_2}\right).
\end{aligned} \tag{41}$$

The wages (58) and efforts (59) which solve this problem are derived in the appendix. We insert them into the principal's problem and plot level sets of the principal's expected profit in Figure 8 for different precisions of the assignment function $f$. As isoprofit curves have positive slopes for some type profiles in all cases, we confirm the Apollo effect also in the principal-agent environment.$\triangleleft$

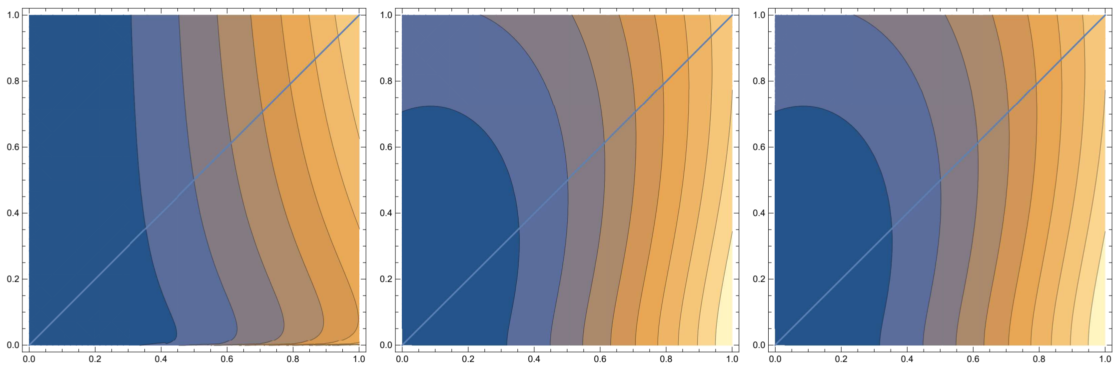

Figure 8: Expected PAM-profits for unobservable efforts exhibiting the Apollo effect. The level sets are drawn for $r \in \{.25, 2, 15\}$; only region $\theta_1 > \theta_2$ below the diagonal is relevant.

## 5 Concluding remarks

Successful law firms, medical or accounting partnerships, etc, strive to hire the brightest graduates for their organizations. By definition, these firms are Apollo teams, consisting of competitive individuals whose professional training may not always have emphasized lateral relationship skills. This paper provides a model for systematically thinking about the implications of this observation.

At its core, the present paper analyzes the influence of potential appointment mistakes on team production. For this purpose we model team member skills as exogenous and let an official who has only statistical information on the workers' skills match the team members to tasks or positions. The baseline analysis shows that mistakes of this kind lead inevitably to what has been called the Apollo effect: the property that teams composed of weaker individuals may outperform teams of unambiguously higher qualified individuals in terms of team output. Our model extensions allow for more complex task assignment or production modes, private information on skills among the workers, and the presence of a profit-maximizing principal. We show that in all cases, to some extent, the Apollo effect cannot be avoided.

Many other economically interesting situations could be modeled with the methodology developed in this paper. For instance, a standard electoral competition model could be enriched through politicians choosing platforms (their "types" in our model) and voters who are unable to perfectly discriminate between these platforms may make mistakes in choosing their candidates. This would presumably counteract the tendency of candidates to move towards the median as such a convergence would maximize the probability of mistakes by the electorate. Another application of a similar idea is the possibility of making mistakes when identifying the "best" bid in general auction environments when (potentially multi-dimensional) bids are close.

This paper presents an analytically rigorous way of generating the Apollo effect in a variety of production environments. The resulting way of thinking about organizations has, in our view, important implications. Similar effects to those we report for leadership selection are at work for imperfect project selection with unobserved quality and training investments in human capital. Outside of the production environment, selecting a speaker among competing party officials, choosing the most promising among several architectural designs, or selecting a replacement goalie among sets of alternatives in a soccer team may all give rise to similarly negative effects in terms of expected overall performance.[18]

Among potentially fruitful model extensions are a temporal alternative to our probabilistic selection mistakes in which one could model the time cost of decision making as an increasing function of the proximity of alternatives, exploring the possibilities for efficient incentive provision in the presence of selection errors, and the study of the precise characteristics of the possible functions governing selection errors.

# Proofs

**Proof of Lemma 1**. We show that while $\partial Y(\theta_1, \theta_2)/\partial\theta_1 > 0$ always holds, $\partial Y(\theta_1, \theta_2)/\partial\theta_2 < 0$ if and only if the condition of the Lemma holds. In the latter case, there exist $(\eta_1, \eta_2) >> 0$ such that (15) holds. Taking the derivative of (14) with respect to $\theta_2$

[18] The motivation of Woolley et al. (2015) contains a particularly nice example of the performance of the Russian (Apollo) ice hockey team at the 2014 Sochi olympics. For an account of other recent dream team failures, see Martinez (2013).

gives the change in output for an increase in type $\theta_2$ as

$$\begin{aligned}
&f_2(\theta_1,\theta_2)(y(\theta_1,e^*(\theta_1),e^*(\theta_1))-y(\theta_2,e^*(\theta_2),e^*(\theta_2)))\\
&\quad+(1-f(\theta_1,\theta_2))\left(e'^*(\theta_2)\left(y_3(\theta_2,e^*(\theta_2),e^*(\theta_2))+y_2(\theta_2,e^*(\theta_2),e^*(\theta_2))\right)\right.\\
&\quad\left.+\,y_1(\theta_2,e^*(\theta_2),e^*(\theta_2))\right)
\end{aligned} \tag{42}$$

in which $y_e(\theta_2,e^*(\theta_2),e^*(\theta_2)) = y_2(\theta_2,e^*(\theta_2),e^*(\theta_2)) = y_3(\theta_2,e^*(\theta_2),e^*(\theta_2))$. This change is negative if (16) holds. Moreover and as claimed, the derivative of $y(\theta_1,\theta_2)$ with respect to $\theta_1$ is

$$\begin{aligned}
&f_1(\theta_1,\theta_2)\left[y(\theta_1,e^*(\theta_1),e^*(\theta_1))-y(\theta_2,e^*(\theta_2),e^*(\theta_2))\right]\\
&\quad+f(\theta_1,\theta_2)\left[e'^*(\theta_1)2y_e(\theta_1,e^*(\theta_1),e^*(\theta_1))+y_1(\theta_1,e^*(\theta_1),e^*(\theta_1))\right]>0. \quad\square
\end{aligned}$$

**Proof of Lemma 2**. By assumption of symmetry and $f(0,\hat{\theta})=0$. □

**Proof of Proposition 1**. From lemmata 1 & 2 and the intermediate value theorem, every feasible continuous function $f$ has a range in which the slope of the isoquant is positive. □

**Proof of Proposition 2**. The condition for the isoquant to have positive slope, i.e., the derivative of output $Y(\theta_1,\theta_2)$ from (23) with respect to $\theta_2$ to be negative, is

$$\frac{z_1(\theta_2,\theta_1)}{z_1(\theta_2,\theta_1)-z_2(\theta_1,\theta_2)} < f(\theta_1,\theta_2)-f_2(\theta_1,\theta_2)\frac{z(\theta_1,\theta_2)-z(\theta_2,\theta_1)}{z_1(\theta_2,\theta_1)-z_2(\theta_1,\theta_2)}. \tag{43}$$

Assumptions (20) and (22) imply single-crossing of $z_1$ and $z_2$ since

$$\begin{aligned}
z_1(\theta_2,\theta_1)-z_2(\theta_1,\theta_2) &= e_1^h(\theta_2)y_2^h(\theta_2,e^h(\theta_2))-e_1^l(\theta_2)y_2^l(\theta_2,e^l(\theta_2))\\
&\quad+y_1^h(\theta_2,e^h(\theta_2))-y_1^l(\theta_2,e^l(\theta_2))>0
\end{aligned} \tag{44}$$

where the second line of the last expression is positive due to the assumption that $y_1^h(\theta,e)>y_1^l(\theta,e)>0$ and $e^h(\theta_2)>e^l(\theta_2)$. The first line is positive since $e^{h\prime}(\theta_2)>e^{l\prime}(\theta_2)$ and $y_2^h(\theta_2,e^h(\theta_2))>y_2^l(\theta_2,e^l(\theta_2))$, which, in turn follows from

$$y_2^h(\theta_2,e^h(\theta_2)) = 2c'\left(e^h(\theta_2)\right) \text{ and } y_2^h(\theta_2,e^l(\theta_2)) = 2c'\left(e^l(\theta_2)\right), \tag{45}$$

$e^h(\theta_2)>e^l(\theta_2)$, and $c''>0$. Thus, the left-hand side of (43) exceeds 1 while the term multiplied with $f_2(\theta_1,\theta_2)$ on the right-hand side of (43) is positive. Hence, as $f(\theta_1,\theta_2)\in[1/2,1]$, a sufficient condition for the Apollo effect to arise for some type profile $\theta_1>\theta_2$ is (28). □

**Proof of Proposition 3**. Equilibrium effort functions must satisfy

$$\begin{aligned} e^L(\theta) &\in \arg\max_e \mathbb{E}_{\theta'|\text{ leader has type }\theta}\left[\frac{y\left(\theta, e, e^F\left(\theta'\right)\right)}{2}\right] - c\left(e\right), \\ e^F(\theta) &\in \arg\max_e \mathbb{E}_{\theta'|\text{ follower has type }\theta}\left[\frac{y\left(\theta', e, e^L\left(\theta'\right)\right)}{2}\right] - c\left(e\right). \end{aligned} \tag{46}$$

We calculate the conditional expectations as

$$\begin{aligned} \Pr\left(\Theta \leq \theta' | \text{leader has type } \theta\right) &= \frac{\Pr\left(\Theta \leq \theta' \text{ \& leader has type } \theta\right)}{\Pr\left(\text{leader has type } \theta\right)} \\ &= \frac{\int_a^{\theta'} f\left(\theta, \theta"\right) g\left(\theta"\right) d\theta"}{\int_a^b f\left(\theta, \theta"\right) g\left(\theta"\right) d\theta"}. \end{aligned} \tag{47}$$

Therefore, the density of $(\theta'|\text{leader has type } \theta)$ is

$$\frac{f\left(\theta, \theta'\right) g\left(\theta'\right)}{\int_a^b f\left(\theta, \theta"\right) g\left(\theta"\right) d\theta"}. \tag{48}$$

Therefore,

$$E_{\theta'|\text{leader has type }\theta} \frac{y\left(\theta, e, e^F\left(\theta'\right)\right)}{2} = \frac{\int_a^b y\left(\theta, e, e^F\left(\theta'\right)\right) f\left(\theta, \theta'\right) g\left(\theta'\right) d\theta'}{2\int_a^b f\left(\theta, \theta"\right) g\left(\theta"\right) d\theta"} \tag{49}$$

The first-order condition is given by

$$\frac{\int_a^b y_2\left(\theta, e, e^F\left(\theta'\right)\right) f\left(\theta, \theta'\right) g\left(\theta'\right) d\theta'}{2\int_a^b f\left(\theta, \theta"\right) g\left(\theta"\right) d\theta"} - c'(e) = 0 \tag{50}$$

Therefore, $e^L(\theta)$ must satisfy (33).

Calculating the conditional expectations for the second case gives

$$\begin{aligned} \Pr\left(\Theta \leq \theta' | \text{follower has type } \theta\right) &= \frac{\Pr\left(\Theta \leq \theta' \text{ \& follower has type } \theta\right)}{\Pr\left(\text{follower has type } \theta\right)} \\ &= \frac{\int_a^{\theta'} f\left(\theta", \theta\right) g\left(\theta"\right) d\theta"}{\int_a^b f\left(\theta", \theta\right) g\left(\theta"\right) d\theta"} \\ &= \frac{\int_a^{\theta'} \left[1 - f\left(\theta, \theta"\right)\right] g\left(\theta"\right) d\theta"}{\int_a^b \left[1 - f\left(\theta, \theta"\right)\right] g\left(\theta"\right) d\theta"}. \end{aligned} \tag{51}$$

Therefore, the density of $\theta'|\text{follower has type } \theta$ is

$$\frac{f\left(\theta', \theta\right) g\left(\theta'\right) d\theta'}{\int_a^b f\left(\theta", \theta\right) g\left(\theta"\right) d\theta"}. \tag{52}$$

Therefore,

$$E_{\theta'|\text{follower has type }\theta} \frac{y\left(\theta', e, e^L\left(\theta'\right)\right)}{2} = \frac{\int_a^b y\left(\theta', e^L\left(\theta'\right), e\right) f\left(\theta', \theta\right) g\left(\theta'\right) d\theta'}{2\int_a^b f\left(\theta", \theta\right) g\left(\theta"\right) d\theta"}. \tag{53}$$

The first-order condition is given by

$$\frac{\int_a^b y_3\left(\theta', e^L\left(\theta'\right), e\right) f\left(\theta', \theta\right) g\left(\theta'\right) d\theta'}{2\int_a^b f\left(\theta', \theta\right) g\left(\theta'\right) d\theta'} - c'(e) = 0. \tag{54}$$

Therefore, we know that $e^F(\theta)$ must satisfy (34). ☐

**Derivation of equilibrium efforts and wages in Example 5.**
Assume that both ($\mathrm{IR}_2$) and ($\mathrm{IC}_1$) bind. Combining the binding ($\mathrm{IR}_2$) and ($\mathrm{IC}_1$) gives

$$e(\theta_2) \ = \sqrt{2}\sqrt{w(y(\theta_2))},\ e(\theta_1) = \frac{\theta_1^2(w(y(\theta_1)) - w(y(\theta_2))) + 4\theta_2^2 w(y(\theta_2))}{2\sqrt{2}\theta_1\theta_2\sqrt{w(y(\theta_2))}}. \tag{55}$$

Inserting these into the binding ($\mathrm{IR}_1$) gives

$$w(y(\theta_1)) = w(y(\theta_2))\frac{(\theta_1 + 2\theta_2)^2}{\theta_1^2}. \tag{56}$$

Inserting this back into the principal's problem in (41) gives her unconstrained objective

$$2\sqrt{w(y(\theta_2))}\left(\sqrt{2}\theta_2 + \frac{(\theta_1+\theta_2)\theta_1^{r-2}\left(\sqrt{2}\theta_1^2 - 4\theta_2\sqrt{w(y(\theta_2))}\right)}{\theta_1^r + \theta_2^r} - \sqrt{w(y(\theta_2))}\right). \tag{57}$$

Taking the derivative with respect to $w_2 = w(y(\theta_2))$ and solving results in the pair of wages

$$w(y(\theta_1)) = \frac{\theta_1^2(\theta_1+2\theta_2)^2\left((\theta_1+2\theta_2)\theta_1^r + \theta_2^{r+1}\right)^2}{2\left((\theta_1+2\theta_2)^2\theta_1^r + \theta_1^2\theta_2^r\right)^2}, \tag{58a}$$

$$w(y(\theta_2)) = \frac{\theta_1^4\left((\theta_1+2\theta_2)\theta_1^r + \theta_2^{r+1}\right)^2}{2\left((\theta_1+2\theta_2)^2\theta_1^r + \theta_1^2\theta_2^r\right)^2} \tag{58b}$$

implying efforts of

$$e(\theta_1) = \frac{\theta_1(\theta_1+2\theta_2)\left((\theta_1+2\theta_2)\theta_1^r + \theta_2^{r+1}\right)}{(\theta_1+2\theta_2)^2\theta_1^r + \theta_1^2\theta_2^r}, \tag{59a}$$

$$e(\theta_2) = \frac{\theta_1^2\left((\theta_1+2\theta_2)\theta_1^r + \theta_2^{r+1}\right)}{(\theta_1+2\theta_2)^2\theta_1^r + \theta_1^2\theta_2^r}. \tag{59b}$$

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