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Pulling up the Tarnished Anchor: The End of Silver as a Global Unit of Account

Abstract

We use the demise of silver-based standards in the 19th century to explore price dynamics when a commodity-based money ceases to function as a global unit of account. We develop a general equilibrium model of the global economy with gold and silver money. Calibration of the model shows that silver ceased functioning as a global price anchor in the mid-1890s - the price of silver is positively correlated with agricultural commodities through the mid-1890s, but not thereafter. In contrast to Fisher (1911) and Friedman (1990), both of whom predict greater price stability under bimetallism, our model suggests that a global bimetallic system, in which the gold price of silver uctuates, has higher price volatility than a global monometallic system. We confirm this result using agricultural commodity price data for 1870-1913.

JEL-Codes: E420, F330, N100, N200.

Keywords: bimetallism, classical gold standard, silver, unit of account, fixed exchange rates.

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1 Introduction

As global trade and capital flows boomed in the latter half of the 19th century, policymakers sought an international monetary system to accommodate these flows while simultaneously anchoring prices. In modern parlance, they solved the international economic policy trilemma by opting for pegged rates and capital mobility, and in turn surrendered control over domestic monetary decision making (Obstfeld et al., 2005). Scholars have argued that this choice was perhaps more tenable in the 19th century because policymakers neither possessed the macroeconomic models nor the conceptualization of unemployment to understand the potential tradeoffs between exchange-rate stability and autonomous monetary policy. This meant that policymakers could maintain a singular focus on price stability, especially given the limited extent of the franchise in most countries (Eichengreen, 1996).

However, the decision about which commodity to use to anchor global prices was an open question—debated at international monetary conferences and ultimately only settled in a global sense by the end of the century, at which point all the largest economies had transitioned to the gold standard (Meissner, 2005). Indeed, the decision to transition from silver and bimetallism met with considerable political resistance in many parts of the world. Famously, for example, a powerful populist trend in the final quarter of the 19th century in the United States was the Free Silver Movement. Advocates blamed the steady decline in the prices of agricultural commodities on the demonetization of silver and its consequent declining price. In an obvious sense they were right: Figure 1 shows that there was in fact a very close correlation between the price of silver and the prices of agricultural commodities between 1879 and 1896. The figure, however, also points to an interesting historical irony: the prior strong correlation between the price of silver and prices of agricultural commodities dissolved abruptly in 1896, precisely at the time when the controversy over Free Silver, predicated on the existence of such a correlation, reached its peak, in the Bryan-McKinley presidential campaign fought largely on the issue.

In this paper, we provide a new explanation for and a simple model of this correlation, and why it then disappears. We connect a well-studied feature of metallic standards—their use as global price anchors—with a key historical change in the global monetary system—the abandonment of the use of silver as a global unit of account—in order to shed light on the relationship between price dynamics and global monetary transitions. Silver's historical role as a price anchor ended as countries overwhelmingly left silver and bimetallic standards in the latter half of the century in favor of gold; however, the shift to gold was not instantaneous or coordinated (Figure 5). This gradual transition away from silver as a unit of account differs from the abrupt collapses of other global monetary systems, such as Bretton Woods (with the closing of the gold window by the U.S. in 1971) or the classical gold standard (with gold embargoes and the outbreak of war in 1914), and allows us to examine the more general question of when global monetary systems cease to function as price anchors. The answer to this question is not obvious. Individual countries can jettison an existing arrangement by changing the formal rules that bind them to the existing system, but that does not necessarily imply that the unit of account, serving as an anchor for prices, ceases to function globally. Even the exit of economically or systemically important countries from a commodity-based, fixed-exchange rate regime may not end a commodity's role as a numeraire and unit of account.

We develop a general equilibrium model of the global economy that can be used to consider the transition away from a commodity-based monetary standard. Our model is in the spirit of general models of commodity money such as Barro (1979) and Sargent and Wallace (1983). Because both gold and silver were used to back currencies in different countries, we model a global two-commodity monetary system, with both precious metals having monetary and non-monetary roles in the economy. In this sense, our model is in the spirit of Flandreau (1996) and Velde and Weber (2000), but extends these by placing greater emphasis on the intra-period price formation process and the impact of different supply shocks on the prices of various goods in the economy as well as allowing for a floating global price of silver relative to gold. In order to provide predictions about changing price dynamics during the transition away from silver, the model is calibrated using new data on global commodity prices and monetary stocks of precious metals, which illustrate the gradual abandonment of silver as a unit of account.

The model delivers two main predictions that are then verified in the data. First, as countries gradually move away from the use of monetary silver, the correlation between the price of silver and the price of agricultural commodities disappears. The basic mechanism behind this gradual decline in correlation is the quantity theory of money (Friedman, 1987). In our two-commodity monetary model, a rise in the price of the non-numeraire commodity implies, ceteris paribus, a rise in the quantity of money and hence in the general price level. Although these effects become more complex in a general equilibrium setting, our model shows that the basic quantity theory intuition nonetheless provides a valid interpretation for changing global price dynamics during the gradual demonetization of silver at the end of the 19th century.

According to a new higher-frequency data set on global commodity prices we have collected, silver ceased functioning as a global price anchor in the mid-1890s, nearly two decades after many important countries abandoned bimetallism. That is, the price of silver was highly correlated with agricultural commodity prices through the mid-1890s, but not so thereafter. The timing of the end of silver's role as a price anchor is consistent with the gradual global transition from silver and bimetallic standards to gold. Many countries continued to use silver as a unit of account through the 1870s and 1880s (including Japan, Russia, the United States, Brazil, Mexico, Peru, Spain, and China). Thereafter, global political currents continued to shift in favor of gold standards, as India's suspension of the free coinage of silver in 1893, the end of Sherman Act silver purchases, and the defeat of William Jennings Bryan and the "silverites" in the U.S. presidential election of 1896 left few global economies committed to silver, further weakening silver's role as a price anchor.

A second prediction of the model is that, as countries gradually move away from the use of monetary silver, the volatility of agricultural commodity prices declines. This result contrasts with Friedman (1990), who argues that bimetallism lowers price volatility, and Velde and Weber (2000), who generate such lower price volatility in a dynamic model of bimetallism. The distinguishing feature of our setup, however, is that we consider a global two-commodity monetary system in which the gold price of silver fluctuates with global supply and demand. This assumption is realistic for the time period we consider, but contrasts with much of the existing literature on bimetallism. Of course, this contrast is essential since we show that the lower price volatility predicted for bimetallism by Fisher (1911) and Friedman (1990) reverses to higher price volatility in the case of a floating gold price of silver. The volatility result can also be understood in terms of the quantity theory of money. In a two-commodity monetary system, a shock to the price of the non-numeraire commodity is also a shock to the money supply which, in turn, affects the overall price level. This monetary effect is in addition to standard demand-side effects, and hence adds to price volatility.

This prediction for price volatility also has implications for exchange-rate regimes that peg the domestic currency price of an important export commodity, as proposed by Frankel (2003). Such an exchange-rate regime functions like commodity moneys based on silver and gold; hence our result about higher agricultural commodity price volatility suggests that if two similar-sized regions of the world peg the prices of two different commodities, then this could raise the volatility of the prices of other commodities. Even though such a commodity peg could potentially smooth terms-of-trade shocks by removing the volatility of the domestic price of the pegged export commodity in each respective region, it may also have the unintended effect of raising the volatility of other, non-pegged commodity prices.

2 Demonetization of Silver and Global Price Dynamics

The use of silver as a global price anchor gradually ended in the late 19th century as countries switched from silver and bimetallic standards to monometallic gold standards, either de jure or de facto. Until its demise, countries used silver, often in conjunction with gold, to fix exchange rates. Under bimetallism, both gold and silver served as numeraires, with all other goods priced relative to the mint par ratio of gold to silver. A country that legally permitted bimetallism was often effectively on either a gold or silver monometallic standard, depending on how the mint price of gold to the mint price of silver compared to the world price ratio of the two metals (Officer, 2008).[1]

Previous studies have attempted to model or estimate the demise of bimetallism by focusing on changes that began to occur in the 1870s in the longstanding global mint ratio of 15.5 silver ounces to one gold ounce.[2] The standard story postulates that Germany's decision to demonetize silver led many countries to drop silver and switch over to gold (Friedman and Schwartz, 1963; Gallarotti, 1994). Flandreau (1996), however, argues that France, being the largest bimetallic country in the world, was the marginal player in the bimetallic system that kept the silver-gold price ratio at 15.5:1 and that Germany's decision did not spell the end of bimetallism.[3] Rather, it is suggested that France's 1873 decision to limit the coinage of silver

[1]However, gold and silver appear to have circulated simultaneously in France from 1852-72 (Flandreau, 1996).

[2]The ratio was stabilized up through the end of the 1860s by absorbing the metal that was in oversupply while releasing the other metal that was scarce (Friedman, 1990; Flandreau, 2004). Beginning in the 1870s, this ratio fluctuated (Rockoff, 1990).

[3]Friedman (1990) argues that the United States made a big mistake when it demonetized silver in 1873, referring to this episode as Crime of 1873. He argues that this legislation destabilized prices. Velde (2002) employs a general equilibrium model to test Friedman's hypothesis that the United States could have maintained the silver-gold price ratio at 15.5:1. He finds that the United States could have kept the silver-gold price ratio fixed up until the mid-1890s. Oppers (1996), on the other hand, finds that much of the deflation of the 1870s and 1880s could have been avoided if France and Belgium continued to freely coin silver.

violated bimetallism, and led to the eventual rise of the international gold standard and to a floating silver-gold price ratio. Meissner (2013) draws on this vein of research to argue that bimetallism would have been unsustainable after 1875. On the other hand, Morys (2007) suggests that bimetallism may have unofficially ended even earlier, suggesting that the large gold discoveries in California and Australia in the 1850s increased the supply of gold and made the emergence of the classical gold standard imminent by the end of the 1860s.

Even though these bimetallic countries began to "scramble for gold" in the early 1870s, scholars have noted that that many others, including Austria-Hungary, Brazil, China, India, Japan, Mexico, Peru, Russia and Spain, continued to back money with silver well after that date (Flandreau, 2004; Bordo and Kydland, 1995). China remained on a silver standard for the entire period, while other countries gradually transitioned to gold (Figures 5 and 6).[4] Hence, silver's monetary importance persisted well after bimetallism's demise.

In comparison to analyzing bimetallism's demise, comparatively less is known about how the gradual demonetization of silver and the transition to a new global monetary system based on gold influenced price dynamics. For an individual country, once legal backing of money with silver ended (either de jure or de facto), prices were free to fluctuate relative to that metal i.e., silver. It no longer served as a numeraire. Commodity prices, however, were largely set in internationally competitive markets, suggesting that global correlations between silver and commodity prices could persist even after legal backing based on that precious metal ended for a single country, even for important ones such as France. However, widespread demonetization of silver ended silver's role as a global price anchor.

To explore further the timing of the breakdown of silver's role as a global unit of account, we assembled new monthly data on agricultural commodity prices. Volatility measures and other summary statistics for agricultural commodities were hand-collected from the "Monthly Trade Supplement" of the *Economist*. The monthly gold price of silver (expressed in pound sterling) is also from the *Economist*. The prices for agricultural commodities and precious metals were closely followed by leading financial publications during the classical gold standard, 1870-1913. For example, the *Economist*, which was published every Saturday, commented on weekly fluctuations in agricultural commodity prices. A similar story is true for silver prices which were also closely monitored by investors. The *Economist* reported the daily sterling price of silver in its weekly summary of the precious metals market. In

[4]Austria-Hungary became a de facto member of the gold standard in 1892, Brazil and Mexico joined in 1905 and 1906, and Japan and Russia became members of the gold club in 1897.

the early to mid-1870s, the *Economist* frequently discussed the depreciation of silver. In July 1876 (pp. 821-824), they attributed the depreciation of silver to three events: (1) German demonetization of silver; (2) actions by the Latin Monetary Union led by France; and (3) an increase in the tribute that India pays England has lowered the demand for silver (also see Bordo et al., 2009). Two decades later, market participants continued to track the silver market and trading in global agricultural commodities. For example, the *Economist* commented on the importance of silver's use as a money for global trade, discussing the potential effects of the U.S. government passing legislation that increased the supply of silver money.

> And coming nearer home, we have bi-metallists predicting an improvement in our cotton trade with the East as the result of a rise in silver, and forgetting, apparently, that any advance in the prices of cotton goods, especially when it comes at a time when Eastern producers are experiencing a fall in the exchange value of their products must tend to restrict the demand for them (*The Economist*, "Silver in the United States," June 14, 1890, p. 2-3).

If silver functioned as a global unit of account, then this should be observable in the co-movements of agricultural prices with silver. That is, silver and commodities should co-move in periods when silver served as a unit of account; however, as more countries abandoned silver and bimetallic standards for gold, silver's positive relationship with commodity prices should break down, a relationship we show more formally when we model the global economy. To examine this prediction, we compare the monthly gold price of silver (SILVER) with an agricultural commodity price index (AGINDX) consisting of three commodities whose prices appear monthly without breaks or changes in definitions between 1870 and 1913 and were traded and consumed globally: cotton, wheat, and tea. Each commodity was indexed relative to its value in January 1870, and then the simple average across the three was computed to create the agricultural commodity index. The correlation between silver prices and the agricultural commodity index was 0.91 between 1870-1896 and then fell to 0.01 for the period 1897-1913.

Figure 1 plots ten-year rolling correlations between silver and the agricultural commodity index starting with the ten-year period 1/1870-12/1879 and ending with the ten-year period 1/1904-12/1913. According to the figure, commodity prices are highly correlated with silver until the mid-1890s, at which point the correlation breaks down and even falls below zero for a while before briefly recovering in the mid-1900s. As we explain later, after introducing our model, this brief rise in correlation reflects the large increase in global gold production that occurred in the 1890s. By 1908, the correlation between agricultural commodity prices and silver prices begins to fall again, becoming negative by 1911. Figure 2 shows, as might be expected for a metallic commodity acting as a unit of account, that the ten-year rolling correlations exhibit similar patterns if we compare silver with individual commodity prices rather than an index. The top panel displays the three commodities used in our agricultural index and the bottom panel extends the coverage to include other commodities such as oats, barley, and rice for a shorter sample period.[5] The correlations between commodities and silver are high during the 1870s and 1880s and then dropped during the 1890s; this is followed by a brief recovery, before declining again in the last 5-7 years before the outbreak of World War I in 1914.

We can further test our co-movement hypothesis by examining the cointegration between agricultural commodity prices and silver. That is, one would expect that agricultural commodity prices and the silver price of gold are cointegrated as long as silver is widely used to purchase goods and services. As the importance of silver as a unit of account starts to wane as more and more countries leave the silver standard and adopt gold, the long-run cointegrating relationship between agricultural commodity prices and silver prices should weaken.[6] Statistically, the declining importance of silver as a money should be shown by either a failure to reject the null hypothesis of no cointegration or a decline in the value of the test statistic, λ-max, that is used to test for cointegration.

We first tested the monthly AGINDX and SILVER for a unit root using the DF-GLS unit root test over the full sample period, January 1870 - December 1913. The unit root tests yield t-statistics of -2.393 for silver and -1.606 for the agricultural commodity index. The null hypothesis of a unit root cannot be rejected at the five-percent level of significance for the agricultural commodity index or the silver price index. Thus, we then tested for the presence of a common stochastic trend between AGINDX and SILVER using the Johansen Maximum Likelihood Procedure.[7] The λ-max statistic of 5.728 suggests that the null hypothesis of no

[5] The commodities in the bottom panel of Figure 2 are not included in additional time-series tests or the agricultural commodity index due to some gaps in their coverage as well as their shorter sample period.

[6] Our main predictions from the model in Section 3 are for the correlation between silver and agricultural commodity prices, and not for cointegration. In this sense, then, these results on cointegration are suggestive and complementary to our main results.

[7] A lag length of three is employed for the cointegration analysis using the Akaike Information Criteria.

cointegration cannot be rejected at the five- or even ten-percent level of significance.

Although the baseline empirical results do not suggest the presence of cointegration, it is possible that silver was cointegrated with commodity prices during some sub-periods of the the last three decades of the 19th century. This would most likely be true at the beginning of our sample period when more countries were on silver-based standards. We investigate this possibility by testing for cointegration from 1870 through 1890 under the assumption that there may have been a long-run relationship between agricultural commodities and silver before the early to mid-1890s, when several polities abandoned the use of silver as a backing metal. The empirical evidence suggests that this may be the case. The λ-max statistic of 16 indicates that the null hypothesis of no cointegration can be rejected at the five-percent level of significance for the period 1870-1890, indicating a relationship existed in the first part of our sample period.

To further investigate when the breakdown of silver as a global unit of account occurred, we estimate a series of iterative cointegration tests to see if the λ-max test statistics declined during the 1890s. We first test for a long-run relationship between agricultural prices and silver from 1870-1887. We then add another year at the end of this sample period and test for cointegration again. We continue this procedure until we have ten λ-max test statistics—so that the last test for cointegration covers 1870-1896. Figure 3 shows that the λ-max test statistics decline in the early 1890s. Furthermore, the null hypothesis of no cointegration can be rejected at the five-percent level by incorporating the years 1887 through 1891 into the empirical analysis. For the period 1870-1892, the null hypothesis of no cointegration can only be rejected at the ten-percent level of significance. For cointegration tests that include years after 1893, we do not find evidence of a long-run cointegrating relationship between agricultural commodities and the silver price of gold.[8]

The breakdown of the cointegrating relationship and declining correlations between prices and commodities in the 1890s is consistent with several key historical events, pointing to silver's waning use as a metal to back currencies around the world. The U.S. repealed the Sherman Silver Purchase Act in 1893, which ended the U.S. Treasury's purchases of silver on the open market. Williams Jennings Bryan lost the U.S. Presidential Election in 1896

[8] We also tested the period 1896-1913 and 1900-1913 for a long-run cointegrating relationship between agricultural commodity prices and the silver price of gold. The Johansen Procedure did not find evidence of a cointegrating relationship between agricultural commodity prices and silver over either of these sample periods.

with an economic platform that called for monetizing silver to increase the price level and appeal to agricultural interests who had seen the prices of their products decline in the 1880s (Rockoff, 1990; Frieden, 1997). Developments in Asia also weakened silver's role. In June 1893, India closed its mints to the free coinage of silver, a move that de facto ended silver's role as a price anchor in that colony and preceded formal adoption of the gold standard in 1899. Further, Japan severed its formal link to silver in 1897 and adopted gold, leaving China and Straits Settlement as the only two prominent Asian polities using silver. As Figures 1 and 2 show, all of these events occur at or before the time at which the correlation between silver and agricultural commodity prices begins to drop rapidly in the mid-1890s.

Further, Figure 4, which displays ten-year rolling window average coefficients of variation for wheat, tea, and cotton over our 1870-1913 sample period, shows that there was a decline in the volatility of agricultural commodity prices during this period of silver demonetization.[9] This gradual decline in price volatility contrasts with Fisher (1911) and Friedman (1990), both of whom predict greater price stability under bimetallism. Crucially, however, this decline occurs for global commodity prices under a global bimetallic regime, in which the gold price of silver is not fixed but instead fluctuates according to the market forces of supply and demand. We now construct a micro-founded model to explain both this gradual decline in price volatility from 1870-1913 and the breakdown of the correlation between silver and agricultural commodity prices in the 1890s.

3 A Two-Commodity Monetary Model

To understand a monetary metal's role as a price anchor in the global economy, consider a simple model of an international two-commodity money monetary system.[10] Throughout, we refer to the two commodities as gold and silver, and divide the world economy into a gold region and a silver region. A representative household uses monetary gold to purchase

[9]As a robustness check, we constructed an alternative measure of volatility using the six commodities shown in Figure 2. We first calculated the growth rate of each commodity using the natural log of the price relative. Then, we estimated standard GARCH(1, 1) models for each growth rate and generated one-step ahead estimates of the conditional variance for each series. The conditional variance of the six time series is averaged to produce an equally weighted index of commodity volatility. Appendix Figure A1 shows that this alternative volatility index also exhibits declining volatility in commodity prices over our sample period. The decline in volatility is also seen when examining individual series, such as barley, oats, rice, tea, and wheat.

[10]We consider a static model only for expositional simplicity. In fact, this static model is equivalent to a dynamic model in which overlapping generations of households each live for one period.

goods in the gold region and monetary silver to purchase goods in the silver region.[11]

3.1 Consumption, Production, and Money

In addition to silver and gold, we include an agricultural consumption good in the global economy. The two metals can be held in the form of either jewelry or money, and conversion between jewelry and money is immediate and costless through minting or melting.

The representative household derives utility from consumption of the agricultural good and from its holdings of gold and silver jewelry. It has a nested constant-elasticity-of-substitution (CES) utility, so that the utility of the household is given by

$$u(C_a, \gamma_s, \gamma_g) = \left(\mu_a^{\frac{1}{\theta}} C_a^{\frac{\theta-1}{\theta}} + (1-\mu_a)^{\frac{1}{\theta}} \Gamma^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \tag{1}$$

where

$$\Gamma = \left(\mu_s^{\frac{1}{\delta}} \gamma_s^{\frac{\delta-1}{\delta}} + (1-\mu_s)^{\frac{1}{\delta}} \gamma_g^{\frac{\delta-1}{\delta}} \right)^{\frac{\delta}{\delta-1}}. \tag{2}$$

In equations (1) and (2), C_a is consumption of the agricultural consumption good, γ_s is holdings of silver jewelry, γ_g is holdings of gold jewelry, $\theta > 0$ is a constant measuring the elasticity of substitution between the agricultural consumption good and metallic jewelry, $\delta > 0$ is a constant measuring the elasticity of substitution between gold and silver jewelry, and $\mu_a > 0$ and $\mu_s > 0$ are constants measuring the household's preferences for each good relative to the others. A nested-CES function of this kind was first introduced in the context of production by Sato (1967), and has since become relatively common in other forms for both production and utility (see, for example, Atkeson and Burstein, 2008). For our purposes, the nested CES utility structure of equations (1) and (2) is particularly appropriate since it is natural to view gold and silver jewelry as more substitutable than agricultural goods and metallic jewelry. This basic observation is incorporated into our analysis when we choose $\delta > \theta$ when parameterizing the model.

In each region of the world, there is a separate competitive firm that produces the perishable agricultural consumption good. The representative household owns both of these firms, which together make up the total global production of the agricultural consumption good.

[11] In Appendix B, we consider an extension of this model that includes a bimetallic region of the world that uses both gold and silver money as in Flandreau (1996), and then show that the basic qualitative results of both models are the same.

In order to purchase the good from the silver-region firm, the household must use monetary silver, and in order to purchase the good from the gold-region firm, the household must use monetary gold. These cash-in-advance constraints force the household to use part of its gold and silver holdings to purchase the agricultural consumption good. We set gold to be the numeraire throughout, so all prices are quoted in terms of gold. It follows, then, that the representative household must satisfy the constraints:

$$M_s p_s \geq C_{s,a} p_a, \tag{3}$$

$$M_g \geq C_{g,a} p_a, \tag{4}$$

where M_s is the quantity of monetary silver, M_g is the quantity of monetary gold, p_s is the common gold price of silver, p_a is the common gold price of the agricultural consumption good, and $C_{i,a}$ is purchases of the agricultural consumption good from the firm in region $i \in \{s, g\}$.[12] As we explain below, the cash-in-advance constraints (3)-(4) are binding in equilibrium. Note that

$$C_a = C_{s,a} + C_{g,a}, \tag{5}$$

or total consumption of the agricultural consumption good is made up of the total purchases of this good from the two firms.

3.2 Equilibrium

Having described the basic structure of the economy, we can now characterize equilibrium. Let A_s and A_g denote the supplies of the agricultural consumption good in the silver and gold regions of the world, respectively, and S and G denote the world supplies of silver and gold, respectively. In the global economy, equilibrium requires that the representative household optimally chooses how much of each good to consume or hold, and that prices adjust so that all markets clear.

Definition 3.1. An equilibrium is a collection $\{C_{s,a}, C_{g,a}, \gamma_s, \gamma_g, M_s, M_g, p_a, p_s\}$ such that, (a) the representative household maximizes (1) subject to (3), (4), and (5), and (b) $C_{s,a} = A_s$, $C_{g,a} = A_g$, $\gamma_s + M_s = S$, and $\gamma_g + M_g = G$.

[12] Because the representative household can purchase goods in either region, the world economy is integrated and hence the law of one price holds for all goods.

The household cash-in-advance constraints (3)-(4) must bind in equilibrium since there is no uncertainty in this setup and there is no benefit from holding extra monetary gold or silver.[13] Using this observation, if we substitute the market-clearing conditions from above and then derive the first-order conditions from the nested CES utility-maximization problem that the household faces, then we obtain the following system of equations:

$$p_s(S - \gamma_s) = p_a A_s, \tag{6}$$

$$G - \gamma_g = p_a A_g, \tag{7}$$

$$p_s = \frac{u_s}{u_g}, \tag{8}$$

$$p_a = \frac{u_c}{u_g}. \tag{9}$$

In equations (6)-(9), u_x denotes the partial derivative of the utility function u with respect to the variable x. The solution to this system, which can be further simplified as

$$S - \gamma_s = A_s \frac{u_c}{u_s}, \tag{10}$$

$$G - \gamma_g = A_g \frac{u_c}{u_g}, \tag{11}$$

yields the equilibrium prices and quantities in this setup. As equations (10)-(11) show, exogenous fluctuations in the supplies of the agricultural consumption good, silver, and gold, A_s, A_g, S, and G, lead to changes in the household's equilibrium holdings of silver and gold jewelry, γ_s and γ_g, which also affects the equilibrium prices of the agricultural consumption good and silver, p_a and p_s.[14]

The system of equilibrium conditions described in our model is related to Flandreau (1996) and Velde and Weber (2000). In particular, these bimetallic models establish a relationship between the quantity of monetary gold and silver and nominal output that is similar to equations (6) and (7), which as discussed further below, is crucial to characterizing

[13] Another possibility is to model an uncertain cash-in-advance constraint as in Svensson (1985), where the household does not know in advance how much money it will need for purchases of goods. This alternative approach is unlikely to alter the model's basic results since it generates a similar positive relationship between money and nominal output.

[14] Alternatively, the model could have the supplies of the agricultural consumption good, silver, and gold respond to price changes. Indeed, such a positive supply response has been documented for gold during the late 1800s and early 1900s (Rockoff, 1984). Such a supply response would dampen the effects of exogenous supply fluctuations in the model, but would not alter the basic qualitative predictions.

the changing volatility of the price of the agricultural consumption good as well as the changing relationship between the prices of silver and the agricultural consumption good. However, our setup differs from these models in that we place greater emphasis on the price-formation process and the interaction between changing supplies on the prices of the various goods in the economy, as characterized by equations (8) and (9). In general, the system of equations (6)-(9) cannot be solved analytically in closed form. However, for certain values of the parameters θ and δ, we are able to obtain numerical solutions.

A modified version of the standard Friedman (1969) rule that equates the opportunity cost of holding money with the social cost of creating additional money holds true in our model. Equations (8)-(9) imply that, in equilibrium, the benefit to the representative household from reducing its holdings of either gold or silver money (and thus increasing its corresponding holdings of gold or silver jewelry) is exactly equal to the cost to the household from reducing its purchases and consumption of the agricultural consumption good. The fact that the household must reduce its consumption of the agricultural consumption good if it holds less gold or silver money is a direct consequence of the cash-in-advance constraints (3)-(4). Similarly, in equilibrium, the benefit to the representative household from increasing its holdings of either gold or silver money is exactly equal to the cost to the household from reducing its corresponding holdings of gold or silver jewelry.

3.3 Parameterization of the Model

In order to examine how shocks to the supplies of the agricultural consumption good, silver, and gold affect the prices of these goods, it is necessary to parameterize the model. Table 1 reports the parameter values used for our numerical solutions and simulations. In Appendix A, we consider alternate parameterizations of the model and show that our central qualitative results are unchanged.

As shown in the second column of Table 1, for the baseline parameterization of the model, we set the substitutability parameters θ and δ equal to 0.5 and 2, respectively. These values reflect the fact that agricultural commodities and metallic goods are less substitutable than non-monetary gold and silver holdings. We also set the parameter μ_a equal to 0.9, a value that reflects the substantially larger share of total spending on agricultural commodities relative to non-monetary gold and silver, and the parameter μ_s equal to 0.5, a value that reflects equal expenditure shares on gold and silver jewelry.

The ratio of monetary silver to monetary gold, $M_s p_s / M_g$, is particularly important in the model. Indeed, one key fact that the model uncovers and that we also observe in the data is that, as this ratio declines, there is less correlation between the price of silver and agricultural commodities—consistent with the notion that silver is ceasing its role as a unit of account. The model also shows that as the ratio of monetary silver to monetary gold declines, there is less volatility in the price of the agricultural consumption good. According to equations (6) and (7) above (together with the fact that $M_s = S - \gamma_s$ and $M_g = G - \gamma_g$), this monetary ratio is given by

$$\frac{M_s p_s}{M_g} = \frac{A_s}{A_g}. \tag{12}$$

This equation demonstrates that the ratio of monetary silver to monetary gold is equal to the ratio of production of the agricultural consumption good in the silver and gold regions of the world.

The ratio of monetary silver to gold was constructed using new data on global monetary stocks of silver and gold compiled from the U.S. Bureau of the Mint *Annual Report* (various years). The decadal estimates are based on a consistent sample of more than 30 countries or colonies for which monetary stock data are reported. For each country, we sum the reported figures for the stock of metals in circulation and in banks and public treasuries.[15]

Motivated by the relationship in equation (12), we consider several different values for the ratio of agricultural good production, A_s/A_g, which reflect the changing values of the ratio of monetary silver to monetary gold observed in our data set. Figure 6 shows that the silver-gold monetary ratio declines from around 0.9 during the 1870s through the early 1890s to just over one third by 1913. In our simulations, then, we consider values of the agricultural production ratio—which is equal to the ratio of monetary silver to monetary gold in the model—that range from zero to two, with a particular emphasis on those values between zero and one.

The results in this section show that an increase in the ratio of monetary silver to mone-

[15] Comprehensive data begin in 1873 so we used figures for each decade thereafter. Since not all country and colony data are updated for every year in this publication, the ten-year figures avoid arbitrary changes in the ratios due to reporting problems. For this reason (missing data for a few countries in the sample), we report 1894 data for the year 1893. Our sample includes the United States of America, Great Britain, France, Germany, Russia, Italy, Belgium, Netherlands, Austria-Hungary, Australasia (aggregated), Denmark, Sweden, Norway, Switzerland, Greece, Spain, Portugal, Romania, Turkey, Egypt, Mexico, Central American States (aggregated) South American States (aggregated), Japan, India, Straits Settlements, Canada, Cuba, and Haiti.

tary gold (as measured by the agricultural production ratio) produces two interesting effects: (1) it raises the correlation between the price of silver and the price of the agricultural consumption good and (2) it increases the volatility of the price of the agricultural consumption good. The first result matches the historical relationship between price correlations and monetary ratios shown in Figures 1-2 and 5-6. The second result matches the historical relationship between price volatility and monetary ratios shown in Figures 4-6.

In the numerical solutions and simulations we present, we consider a range of values for the supplies of the agricultural consumption good, silver, and gold. The goal is to examine how the prices of the agricultural consumption good and silver vary across this range of supply values and to look at the correlation between these prices when the supply values are randomly generated.

Let A denote the total world production of the agricultural consumption good, so that

$$A = A_s + A_g. \tag{13}$$

We consider only shocks to the world supply of the agricultural consumption good A, and assume that the ratio of production in the silver and gold regions of the world A_s/A_g is fixed. This choice is motivated by equation (12), which equates this silver-gold agricultural production ratio with the ratio of monetary silver to monetary gold $M_s p_s/M_g$. The positive relationship between the monetary ratio and both the correlation between silver and agricultural commodity prices and agricultural commodity price volatility is the central focus of this paper. Thus, it is natural to fix the monetary ratio at several different values and then examine how these different values affect silver and agricultural consumption good prices in equilibrium.[16] This, of course, requires fixing the silver-gold agricultural production ratio A_s/A_g.

For simplicity and symmetry, we assume that shocks to the log supplies of gold, silver, and the agricultural consumption good, G, S, and A, all follow the same distribution. In particular, we assume that these supply shocks are normally distributed with mean zero and standard deviations σ_g, σ_s, and σ_a equal to 0.096 (see the second column of Table 1). These

[16] An alternative to this approach is to not explicitly fix the silver-gold production ratio A_s/A_g but rather allow independent random fluctuations in both A_s and A_g while setting the expected values of these two supplies in a way that matches the desired ratio A_s/A_g. By equation (12), this approach introduces random fluctuations in the ratio of monetary silver to monetary gold $M_s p_s/M_g$. This does not alter the basic qualitative results of the numerical solutions and simulations.

values are equivalent to the stationary distribution of an AR-1 process with autocorrelation of 0.95 that is subject to normally distributed shocks with a mean of zero and a standard deviation of 0.03. This parameterization is consistent with the range of values chosen by Velde and Weber (2000) in their calibration of a related dynamic model of bimetallism.

With this assumption, our static model is equivalent to a dynamic model in which overlapping generations of households each live for one period, and in which the log supplies of gold, silver, and the agricultural consumption good follow AR-1 processes. In this dynamic setting, gold and silver are durable and each generation of representative households must hold these stocks as either jewelry or money in equilibrium. In contrast, the agricultural consumption good is perishable and is thus consumed by the representative household in each time period. In our static representation of such a dynamic model, fluctuations in the static supplies of gold, silver, and the agricultural consumption good are equivalent to fluctuations around a stationary distribution for dynamic supplies.

3.4 Numerical Solutions

We first examine how changes in the supplies of the agricultural consumption good, silver, and gold each affect the prices of the agricultural consumption good and silver. Table 2 reports the log responses of the prices of silver, p_s, and the agricultural consumption good, p_a, to two-standard-deviation changes in the log supplies of the agricultural consumption good, silver, and gold for values of the ratio of monetary silver to monetary gold ranging from 0 to 2. These changing equilibrium prices are for the baseline parameterization reported in column two of Table 1. Note that these price responses are standardized, so that a value of 1 implies that a 1% supply shock leads to a 1% change in price.[17]

The results in Table 2 show that an increase in the supply of silver raises the price of the agricultural consumption good and lowers the price of silver, both of which are standard results given the CES utility we assume for the representative household in the model. The table also shows that an increase in the supply of gold raises the prices of both the agricultural consumption good and silver. This latter result obtains because a shock to the supply of gold is always a monetary shock in the model and hence will affect the prices of silver and the agricultural consumption good in the same way. According to the model, then, gold supply

[17] By looking at log price changes relative to log supply changes, Table 2 implicitly log-linearizes the model. In fact, this log-linearization is highly accurate over a six-standard-deviation range for log supplies.

volatility raises the correlation between the price of the agricultural consumption good p_a and the price of silver p_s. The table also shows that these qualitative results are valid across the entire range of monetary gold to monetary silver ratios.

An implication of Table 2 is that shocks to the world supply of gold will move the prices of silver and the agricultural consumption good in the same direction, thus raising their correlation. Indeed, global supply shocks to gold help explain the rise in the correlation between silver and agricultural commodity prices in the early 1900s shown in Figures 1 and 2. The 1890s were marked by technological innovation that revolutionized the metallurgy of gold (Rockoff, 1984) as well as by the development of gold mining in Western Australia and Southern Africa. The combined effect of these led to a dramatic increase in the global production of gold (Noyes, 1905; Cagan, 1965, p.60). The widespread adoption of extraction techniques involving chlorine gas and cyanide from ore made previously abandoned mines in locations such as Colorado, Mexico, and British Columbia economically viable, and underpinned the annual average growth in gold production of 8 percent over this decade. These rates accelerated further in the latter half of the 1890s with the discovery and exploitation of lower-grade ore in the Transvaal region of Africa, which by 1900 constituted roughly a quarter of total world gold output. According to our model, this type of gold supply shock would produce the observed re-anchoring of silver and agricultural commodity prices shown in the ten-year rolling windows (i.e., a gold shock beginning in the early 1890s and gaining steam as the decade progressed will show up in the rolling average over the subsequent 10 years).

The effect of changes in the supply of the agricultural consumption good A are more subtle. According to the second and third columns of Table 2, an increase in A always causes the price of the agricultural consumption good to fall, but lowers the price of silver only when the ratio of monetary silver to monetary gold is greater than 1 and actually causes the price of silver to rise when this monetary ratio is closer to zero. In other words, the effect of volatility in the supply of the agricultural consumption good on the correlation between the prices of the agricultural consumption good and silver depends on the ratio of monetary silver to monetary gold. For high values of this ratio, this supply volatility raises the correlation, and for low values of this ratio, this supply volatility lowers the correlation. This result implies that as the silver-gold monetary ratio declines, the model predicts a de-anchoring of silver in which the correlation between silver and agricultural commodity prices also declines.[18]

The result that the correlation between the prices of the agricultural consumption good and silver falls as the ratio of monetary silver to monetary gold falls merits further discussion. It relies on essentially the same basic intuition as the quantity theory of money (Friedman, 1987). This can be seen if we combine the two binding cash-in-advance constraints from the gold and silver regions of the world. Equations (6), (7), and (13) imply that,

$$M_g + M_s p_s = A p_a. \tag{14}$$

According to the combined cash-in-advance constraint, equation (14), if M_s is greater than zero, then any change in the price of the agricultural consumption good, p_a, will lead to a corresponding change in the price of silver, p_s, holding all other quantities constant. Furthermore, as the value of monetary silver, $M_s p_s$, increases relative to the value of monetary gold, M_g, the effect of a change in p_a on p_s should increase. The quantity theory as represented by equation (14) establishes a direct relationship between the quantity of money and nominal output, so any shock that raises the price of the agricultural consumption good (and hence raises nominal output) will also raise the price of silver, thus creating a positive correlation between these different prices. This logic does not, however, apply to a global economy with a monometallic monetary regime based on gold since an increase in the price of silver no longer leads to an increase in the quantity of money in such an economy.

In general equilibrium, the full impact of a shock that alters the price of the agricultural consumption good under any monetary regime is of course more complex. Nevertheless, as long as the substitutability of the agricultural consumption good and metallic jewelry is low, this basic intuition from the quantity theory is, in fact, correct. In other words, numerical solutions and simulations both confirm that there is a higher correlation between the price of silver and the agricultural good for low values of the parameter θ.

To understand why the elasticity of substitution between the agricultural consumption good and metallic jewelry must be low, consider the effect of a shock to the supply of the agricultural consumption good A, which is a key source of fluctuations in the equilibrium price p_a. A shock that increases the supply of this good will decrease p_a, with the size of that decrease highly dependent on the elasticity of substitution θ (a standard result for

[18] In Appendix A, we demonstrate that the qualitative results from Table 2 are unchanged for different values of the utility parameters μ_a and μ_s.

CES utility). As long as θ is low, the decrease in p_a will be large enough so that Ap_a also decreases. Hence, by equation (14), the price of silver p_s decreases as well. This mechanism, which generates the positive correlation between the prices of the agricultural consumption good and silver as discussed above, is reversed when the elasticity of substitution θ is high.[19]

What does Table 2 imply about the volatility of the price of the agricultural consumption good for different silver-gold monetary ratios? The third and fifth columns of Table 2 show that the price of the agricultural consumption good p_a is least sensitive to changes in the supplies of the agricultural consumption good and silver for low ratios of monetary silver to monetary gold. In contrast, the last column of Table 2 does not indicate much difference in the sensitivity of p_a to changes in the supply of gold for different silver-gold monetary ratios. The results in Table 2 thus imply that agricultural consumption good price volatility is rising in the ratio of monetary silver to monetary gold; this is the result of a rising sensitivity to agricultural consumption good and silver supply shocks.

To understand this result, let us consider the multiple effects of a rise in the supply of silver, S. Most directly, a rise in silver supply raises the representative household's holdings of silver jewelry, γ_s, which lowers the marginal utility of silver jewelry, u_s, and raises the marginal utility of the agricultural consumption good, u_c. According to equations (8) and (9), these changes lower the price of silver and raise the price of the agricultural consumption good. However, the combined cash-in-advance constraint, equation (14), implies that this rise in the supply of silver must also lead to increases in monetary silver, M_s, and monetary gold, M_g, so that the household is able to purchase the more expensive agricultural consumption good with money. This latter effect reduces the household's holdings of gold jewelry, γ_g, which thus lowers the marginal utility of the agricultural consumption good u_c and hence reverses some of the original rise in p_a. Of course, the magnitude of the reversal depends on the relative size of the monetary roles of gold versus silver. In particular, the higher is the ratio of monetary silver to monetary gold, the weaker is this reversal (because monetary gold increases by less), and hence the greater the rise in the price of the agricultural consumption good p_a in response to a rise in the supply of silver S.

A similar logic explains why the price of the agricultural consumption good p_a is less sensitive to changes in the supply of the agricultural consumption good for low silver-gold monetary ratios. In particular, a rise in agricultural consumption good supply lowers the

[19]We confirm this reversal in Appendix A.

marginal utility of the agricultural consumption good, u_c, and thus lowers p_a as well. As in the case of a silver supply shock, the combined cash-in-advance constraint equation (14) requires that a lower price of the agricultural consumption good is matched by a decline in monetary silver and gold, the latter of which raises the representative household's holdings of gold jewelry, γ_g. The increase in gold jewelry holdings raises the marginal utility of the agricultural consumption good u_c and reverses some of the original fall in p_a, with the magnitude of this reversal declining in the ratio of monetary silver to monetary gold.

3.5 Simulations

The numerical results above suggest that the relationship between the price of silver and the non-numeraire agricultural consumption good changes as silver ceases to serve as a global unit of account. These results also indicate that the volatility of the price of the agricultural consumption good declines as silver's importance declines in the global monetary system. We now turn to simulations to examine these predictions. In particular, we wish to show that as silver's role as a unit of account fades over the period 1870-1913, there is a corresponding change in the relationship between silver and the agricultural consumption good as well as a decline in agricultural consumption good price volatility.

The results from 70,000 simulations of the baseline parameterization of the model (see column two of Table 1) for various different monetary ratios are shown in Table 3. The simulations show that an increase in the ratio of monetary silver to monetary gold leads to an increase in the correlation between the price of silver and the price of the agricultural consumption good. This is the central feature of the model and of this section's simulations, and it is consistent with the historical relationship between silver-agricultural commodity price correlations and the monetary ratio shown in Figures 1-2 and 5-6. Table 3 also shows that an increase in the ratio of monetary silver to monetary gold leads to an increase in the volatility of the price of the agricultural consumption good, consistent with the historical relationship between agricultural commodity price volatility and the monetary ratio presented in Figures 4-6.

According to Table 3, our simulations generate a decline in the correlation between the prices of silver and the agricultural consumption good from 0.455 to just below zero, as the ratio of monetary silver to monetary gold drops from one to zero. Figure 2 shows that the decline in correlations with silver prices is of a similar magnitude to the decline observed starting in the 1890s for agricultural commodities such as tea, barley, rice, and oats. In the case of wheat and cotton prices, the drop in correlation with silver prices that occurred in the 1890s is of an even larger magnitude than those generated by our simulations. Nonetheless, given the simplicity of our model and its calibration, the fact that our simulations are able to quantitatively approximate what we observe in the historical data is notable. Moreover, the key qualitative result of a simultaneously decreasing silver-gold monetary ratio and silver-agricultural commodity price correlation is clearly replicated by our simulations.

The results of the simulations in Table 3 show that there is a dramatic and nonlinear drop in the silver-agricultural consumption good price correlation as the ratio of monetary silver to monetary gold approaches zero. This nonlinearity is similar to what we observe in our historical data for the agricultural commodity index (Figure 1) as well as the prices of individual agricultural commodities (Figure 2), all of which exhibit rapidly declining correlations with silver prices toward the end of the 1890s. According to Figure 6, the silver-gold monetary ratio declines significantly during the 1890s, but remains above 0.2. Our simulation results imply that this ratio is too high to produce a dramatic, nonlinear decline in the silver-agricultural consumption good price correlation in the model. It is important to note, however, that the monetary ratios reported in Figure 6 include the silver stocks of gold-standard countries in which the free minting of silver was not allowed. The model assumes that gold and silver money can be freely minted from jewelry, therefore such silver stocks should not be included in the monetary ratio that corresponds to the model. In this sense, then, the monetary ratios reported in Figure 6 are upper bounds for the monetary ratios that are appropriate for our model, meaning that the accurate monetary ratio for the model in the late 1890s was below that in the figure and potentially below those levels that generate a rapid, nonlinear drop in the silver-agricultural consumption good price correlation.

Finally, the last two columns of Table 3 show that the coefficients of variation for the price of the agricultural consumption good are qualitatively consistent with the changing coefficients of variation reported in Figure 4 for the U.S. during the 1870-1913 period. Indeed, the simulations generate a slight decline in the coefficient of variation of the price of the agricultural consumption good from 0.139 to 0.134 as the silver-gold monetary ratio drops from one to zero. Furthermore, the coefficients of variation for the price of silver are almost all within a range of 0.05 to just over 0.2, which is what is observed for ten-year rolling window coefficients of variation for silver prices during the 1870-1913 period.

This result about the relationship between the ratio of monetary silver to monetary gold and the volatility of the price of the agricultural consumption good has implications for some recent policy proposals for small open economies that rely heavily on commodity exports and often experience disruptive terms-of-trade shocks as a result. Frankel (2003), for example, has proposed that economies whose overall production is heavily reliant on one or a few chief export commodities peg their exchange rate to the price of that commodity export(s). Such proposals are functionally equivalent to commodity-money standards. So, in the context of our model, consider two similar-sized regions pegging to two different commodities (i.e., gold and silver regions). Our model implies that this can raise the volatility of the prices of other commodities.[20] Even though pegging to a commodity price could smooth terms-of-trade shocks by removing the volatility of the domestic price of the pegged export commodity in each respective region, the model suggests it may also have the unintended effect of raising the volatility of other, non-pegged commodity prices. Ultimately, this downside could potentially mitigate any perceived advantage of adopting a commodity peg.

The central purpose of this simulation exercise is to illustrate the mechanism by which the declining role of silver as a nominal price anchor during the 1870-1913 period changed the relationship between the price of silver and the price of agricultural commodities as well as the volatility of those agricultural commodity prices. The simulations confirm that this mechanism can be understood in terms of the quantity theory of money.

4 Conclusion

We use the demise of silver as a metallic standard in the late 19th century to examine how global monetary standards anchor prices. Using a new high-frequency data set on global commodity prices, we show that silver continued to exert a strong influence over agricultural commodity prices and serve as a global unit of account into the early 1890s. Its role as a price anchor waned thereafter: silver's high positive correlation with commodity prices began to break down in the mid-1890s. We also show that this monetary de-anchoring coincided with a gradual decline in the volatility of the global prices of agricultural commodities.

We develop a general equilibrium model to illustrate why silver's role changed. The

[20]By symmetry, this result is true from the perspective of both the gold and silver regions of the world. In other words, the price of the agricultural consumption good in terms of both gold and silver rises as the silver-gold monetary ratio rises towards one.

model is derived from a variant of the quantity theory of money and is the first to provide a micro-founded framework to examine a global bimetallic monetary standard in which the gold price of silver is not fixed but instead fluctuates according to the market forces of supply and demand. This model delivers simple predictions that fit the observed patterns in the data: the correlation between agricultural commodity prices and the price of silver and the volatility of agricultural commodity prices are both positively related to silver's importance in the global monetary stock.

Our finding that the volatility of agricultural commodity prices declines as silver's monetary role wanes contrasts with Fisher (1911) and Friedman (1990), both of who predict that bimetallism should generate lower price volatility. The distinguishing feature of our setup is that we consider a global two-commodity monetary system in which the gold price of silver fluctuates with global supply and demand, a realistic assumption for the time period we consider. This distinction explains our contrasting prediction for commodity price volatility and global bimetallism, and suggests that the potential benefits from greater price stability under bimetallism depend crucially on how the relative prices of gold and silver are determined.

Our results regarding price volatility and commodity monetary standards have implications for exchange-rate regimes that peg the domestic currency price of important export commodities, as proposed by Frankel (2003). Because such an exchange-rate regime functions like a commodity monetary standard, our results suggest that if two similar-sized regions of the world peg the prices of two different commodities, then this could raise the volatility of the prices of other, non-pegged commodities. This undesirable side effect of a commodity peg is supported by both the predictions of our model and the behavior of agricultural commodity prices in the late 19th and early 20th centuries.

A Alternate Parameterizations

This section examines the robustness of our qualitative results in Section 3 for alternate parameterizations of the model. It also demonstrates that the results regarding the de-anchoring of silver prices (as silver ceases its monetary role in the global economy) depend on a low elasticity of substitution between metallic jewelry and the agricultural consumption good, as discussed in Section 3.

Table 1 lists four structural parameters—θ, δ, μ_a, and μ_s—for the model, in addition to the three standard deviation parameters for shocks to the log supplies of silver, gold, and the agricultural consumption good. The importance of the values of the standard deviations σ_s, σ_g, and σ_a for our simulation results can be seen in Table 2, and was discussed in Section 3. We wish instead to examine the importance of the values of the four structural parameters for our qualitative results. We begin with the parameters $\mu_a > 0$ and $\mu_s > 0$, which are constants measuring the representative household's preferences for each good relative to the others. In the baseline parameterization, we set $\mu_a = 0.9$ and $\mu_s = 0.5$, values that imply, realistically, a substantially larger share of total spending on agricultural commodities relative to non-monetary gold and silver, and equal expenditure shares on gold and silver jewelry.

In Appendix Table A1, we report the log responses of the prices of silver p_s and the agricultural consumption good p_a to two-standard-deviation changes in the log supplies of the agricultural consumption good, silver, and gold for an alternate parameterization of the model in which $\mu_a = 0.2$ and $\mu_s = 0.3$ (see column three of Table 1). This table shows the price responses of these two goods for a range of values of the ratio of monetary silver to monetary gold. This alternate parameterization of the model reverses both the larger share of total spending on agricultural commodities and the equal expenditures on gold and silver jewelry from the baseline parameterization. Nonetheless, the effects of these various supply shocks on the prices of the agricultural consumption good and silver are qualitatively identical to those for the baseline parameterization shown in Table 2. Namely, this alternate parameterization preserves the result that p_s de-anchors from p_a as silver ceases its monetary role. Furthermore, the lower sensitivity of p_a to changes in A and S for low silver-gold monetary ratios reported in the third and fifth columns of Table A1 confirm that this alternate parameterization also preserves the result that the volatility of p_a declines as silver ceases its monetary role.

Next, we consider the implications of different values for the parameters $\delta > 0$ and $\theta > 0$. Recall that δ is a constant measuring the elasticity of substitution between gold and silver jewelry, and θ is a constant measuring the elasticity of substitution between metallic jewelry and the agricultural consumption good. In the discussion of Section 3, we stated that a low elasticity of substitution between metallic jewelry and the agricultural consumption good was crucial to our results about the de-anchoring of silver as silver ceases its monetary role. In Appendix Table A2, we report the response of the prices of silver p_s and the agricultural consumption good p_a to supply shocks for an alternate parameterization of the model in which $\delta = 0.5$ and $\theta = 2$ (see column four of Table 1). This alternate parameterization reverses the substitutability properties of gold versus silver and the agricultural consumption good versus metallic jewelry, and as a result, should not reproduce the qualitative result that the correlation between silver and agricultural consumption good prices falls as the ratio of monetary silver to monetary gold falls. Table A2 confirms this prediction, with the second and third columns of Table A2 showing that shocks to the supply of the agricultural consumption good affect p_s and p_a in the same way only for low ratios of the silver-gold monetary ratio. Note the contrast between the second column of Table A2 and the second columns of Tables 2 and A1.

B A Bimetallic Region

In this section, we consider the implications of extending our basic model from Section 3 to include a bimetallic region in addition to the gold and silver regions of the world. In this extended model, the representative household can use either monetary gold or monetary silver to purchase goods in the bimetallic region of the world. All other assumptions are the same as in the two-region model; thus, the representative household must use monetary gold and silver to purchase goods in, respectively, the gold and silver regions of the world.

With a bimetallic region of the world, the cash-in-advance constraints (3)-(4) become

$$M_{s,s}p_s \geq C_{s,a}p_a, \tag{15}$$

$$M_{b,s}p_s + M_{b,g} \geq C_{b,a}p_a, \tag{16}$$

$$M_{g,g} \geq C_{g,a}p_a, \tag{17}$$

where $C_{b,a}$ is purchases of the agricultural consumption good from the bimetallic region, $M_{i,s}$ is the quantity of monetary silver used by the representative household in region $i \in \{s, b\}$, and $M_{j,g}$ is the quantity of monetary gold used by the representative household in region $j \in \{b, g\}$. Total consumption of the agricultural consumption good is now given by $C_a = C_{s,a} + C_{b,a} + C_{g,a}$.

As in the two-region model, the cash-in-advance constraints (15)-(17) must bind in equilibrium. We can use this observation together with market clearing and the first-order conditions from the nested CES utility-maximization problem that the representative household faces to solve this richer model. This yields a slightly more complicated system of equations than those from Section 3:

$$p_s M_{s,s} = p_a A_s, \tag{18}$$

$$p_s M_{b,s} + M_{b,g} = p_a A_b, \tag{19}$$

$$M_{g,g} = p_a A_g, \tag{20}$$

$$M_{s,s} + M_{b,s} + \gamma_s = S, \tag{21}$$

$$M_{g,g} + M_{b,g} + \gamma_g = G, \tag{22}$$

$$p_s = \frac{u_s}{u_g}, \tag{23}$$

$$p_a = \frac{u_c}{u_g}. \tag{24}$$

Note that A_b in equation (19) denotes the supply of the agricultural consumption good in the bimetallic region of the world.

Let $A_{b,s}$ and $A_{b,g}$ denote the quantity of the agricultural consumption good that is purchased in the bimetallic region of the world using, respectively, monetary silver and monetary gold. Note that this definition implies that $A_{b,s}+A_{b,g} = A_b$. We can combine the three equations (18)-(20) into the two equations

$$p_s \left(M_{s,s} + M_{b,s}\right) = p_a \left(A_s + A_{b,s}\right),$$
$$M_{g,g} + M_{b,g} = p_a \left(A_g + A_{b,g}\right),$$

which then allows us to simplify the system (18)-(24):

$$S - \gamma_s = \left(A_s + A_{b,s}\right) \frac{u_c}{u_s}, \tag{25}$$

$$G - \gamma_g = \left(A_g + A_{b,g}\right) \frac{u_c}{u_g}. \tag{26}$$

The equilibrium equations (25)-(26) for this bimetallic model are essentially the same as the equilibrium equations (10)-(11) for the basic model of Section 3. The only difference is the purchases of the agricultural consumption good in the bimetallic region, which are represented by $A_{b,s}$ and $A_{b,g}$.

This bimetallic model preserves the key qualitative and quantitative results from the original two-region model. This follows from equations (25)-(26), which together imply that the ratio of monetary silver to monetary gold in this bimetallic model is given by

$$\frac{p_s \left(M_{s,s} + M_{b,s}\right)}{M_{g,g} + M_{b,g}} = \frac{A_s + A_{b,s}}{A_g + A_{b,g}}. \tag{27}$$

Other than the inclusion of the exogenous terms $A_{b,s}$ and $A_{b,g}$ in equation (27), the monetary ratio in this bimetallic model is identical to the monetary ratio (12) from the original two-region model of Section 3.

In the original model, the correlation between the prices of silver and the agricultural consumption good and the volatility of the agricultural consumption good are both increasing in the ratio of monetary silver to monetary gold. The same results obtain in this extended model, except that now, the silver-gold monetary ratio depends on the size of the bimetallic region of the world in addition to the gold and silver regions. In our empirical analysis, we use historical monetary stocks data to determine the ratio of monetary silver to monetary gold (Figure 6). These data do not depend on the classification of different countries' monetary regimes, and hence they are unaffected by the presence of a bimetallic region in the model. According to Tables 2 and 3, the silver-gold monetary ratio is the key ratio underlying our analysis. It follows, then, that the same silver-gold monetary ratios apply to both the basic and extended models, and hence that all results are the same in the basic and extended models.

C Manufactured Goods Price Dynamics

It is illustrative to also consider the changing relationship between the prices of silver and manufactured goods. Because our two-commodity monetary model from Section 3 does not model the manufacturing process and hence does not directly apply to such manufactured goods, we must be cautious in interpreting the results. Nonetheless, it is still useful to observe the changing relationship between silver and manufactured goods prices. After all, as long as the manufactured goods are not close substitutes to metallic jewelry, then our theoretical results should still roughly apply and we should expect that the correlation between the prices of these manufactured goods and silver should de-anchor as silver's monetary role wanes in the 1890s.

To this end, we examine the changing relationship between the prices of silver and manufactured textile goods. Silverman (1930) constructs a monthly index of British textile export prices from 1880-1913 which we employ as a proxy for manufacturing prices in the 19th century. This index includes cotton yard, cotton cloths, woolen and worsted yarn, carpets, and linen and jute piece goods. Textiles were a major manufactured good of the first era of globalization and Silverman (1930)'s comprehensive textile index allows us to maintain consistency with our earlier results that employed monthly data.

Appendix Figure A2 plots ten-year rolling correlations between silver prices and this British textile export index. The figure shows a similar pattern to Figure 1, except that the de-anchoring of silver and manufactured textile goods prices is less complete than for silver and the agricultural commodity prices included in our index from Figure 1. These results are consistent with the theoretical predictions of Section 3. Because manufactured textile goods are not close substitutes to metallic jewelry, the similarity between silver-agricultural commodity price dynamics and silver-manufactured textile price dynamics shown in Figures 1 and A2 is not surprising. Furthermore, the fact that the de-anchoring of silver and manufactured textile goods prices is less complete than for raw agricultural commodities is consistent with the fact that our model does not account for the manufacturing process. Indeed, we should expect that this process would dampen the connection between silver prices and manufactured goods prices, exactly as we observe in Figure A2 versus Figure 1.

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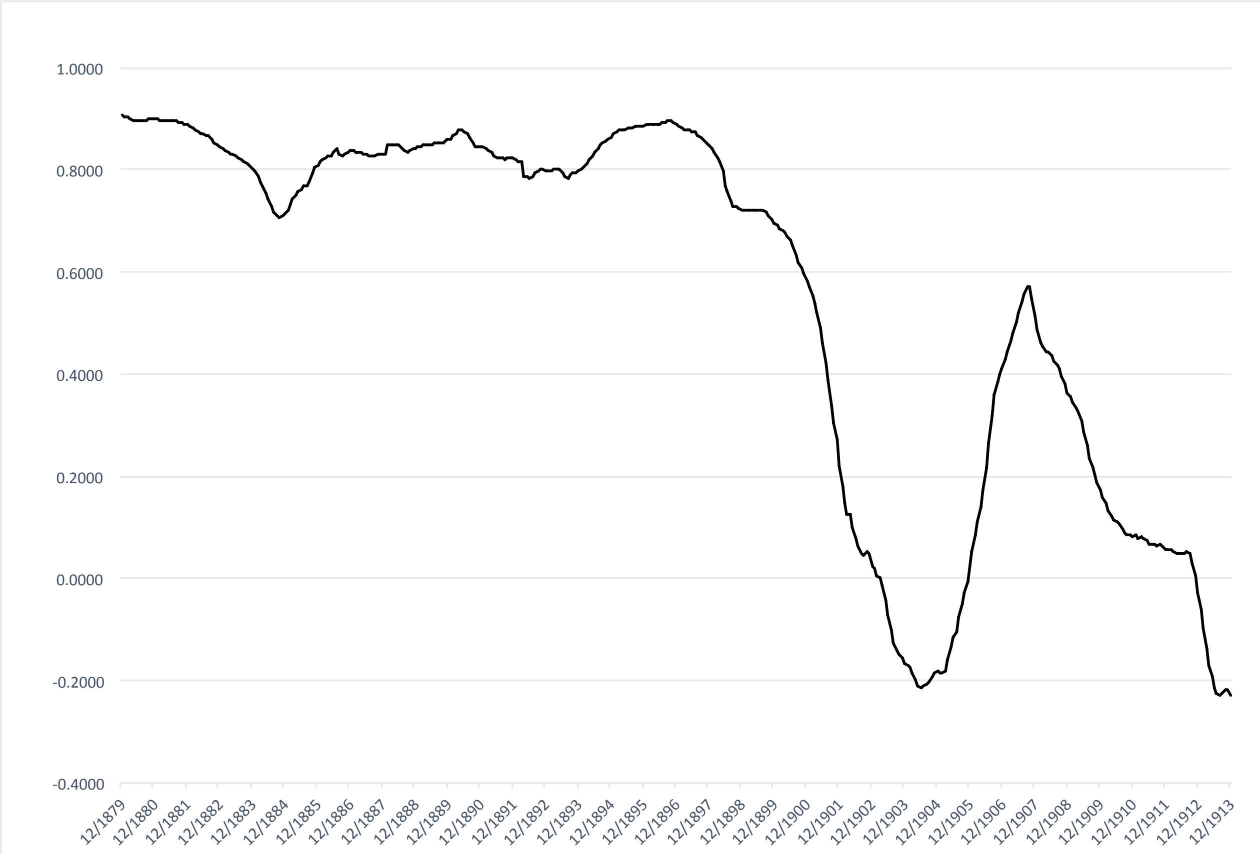
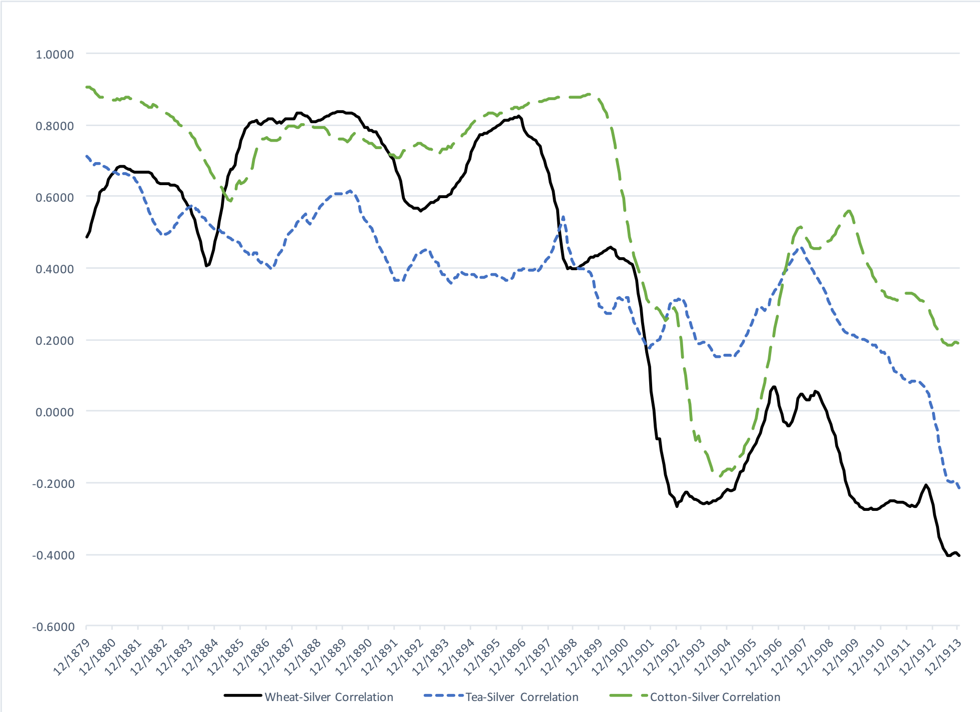


Figure 1: Correlations between the agricultural commodity index and silver prices, 1870-1913

Sources and Notes: Authors' calculations based on data from the *Economist* (various issues). These correlations are generated using monthly data over ten-year rolling windows, so the first observation covers the 1/1870-12/1879 period and the last observation covers the 1/1904-12/1913 period. The agricultural commodity index consists of wheat, cotton, and tea.



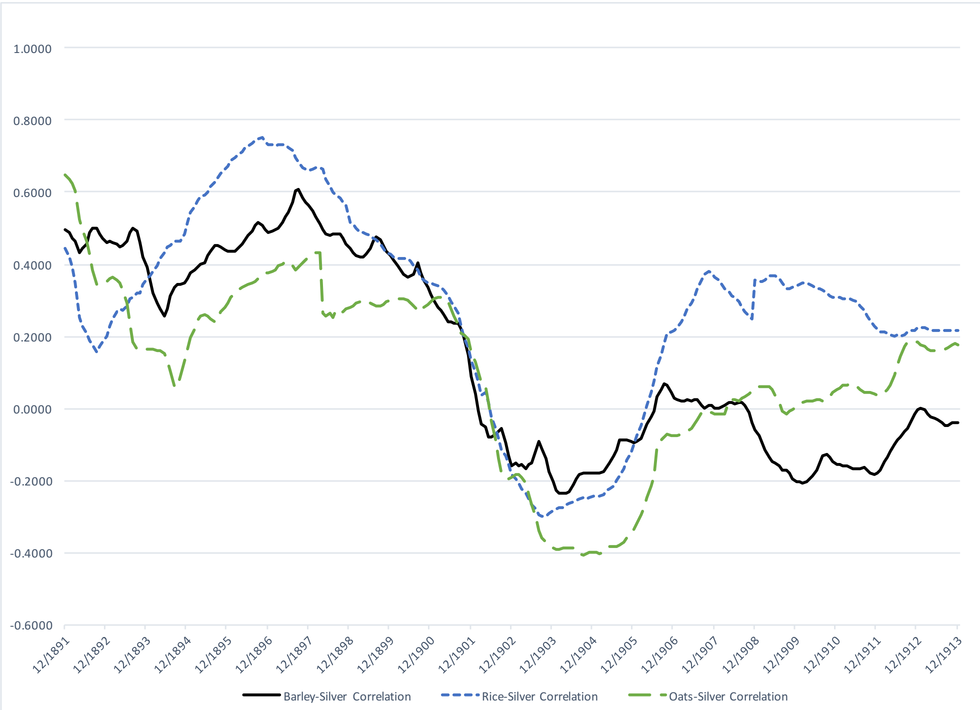


Figure 2: Top Panel: Correlations between silver prices and wheat, tea, and cotton prices, 1870-1913. Bottom Panel: Correlations between silver prices and barley, rice, and oats prices, 1882-1913

Sources and Notes: Authors' calculations based on data from the *Economist* (various issues). These correlations are generated using monthly data over ten-year rolling windows, so the last observation in both panels covers the 1/1904-12/1913 period.

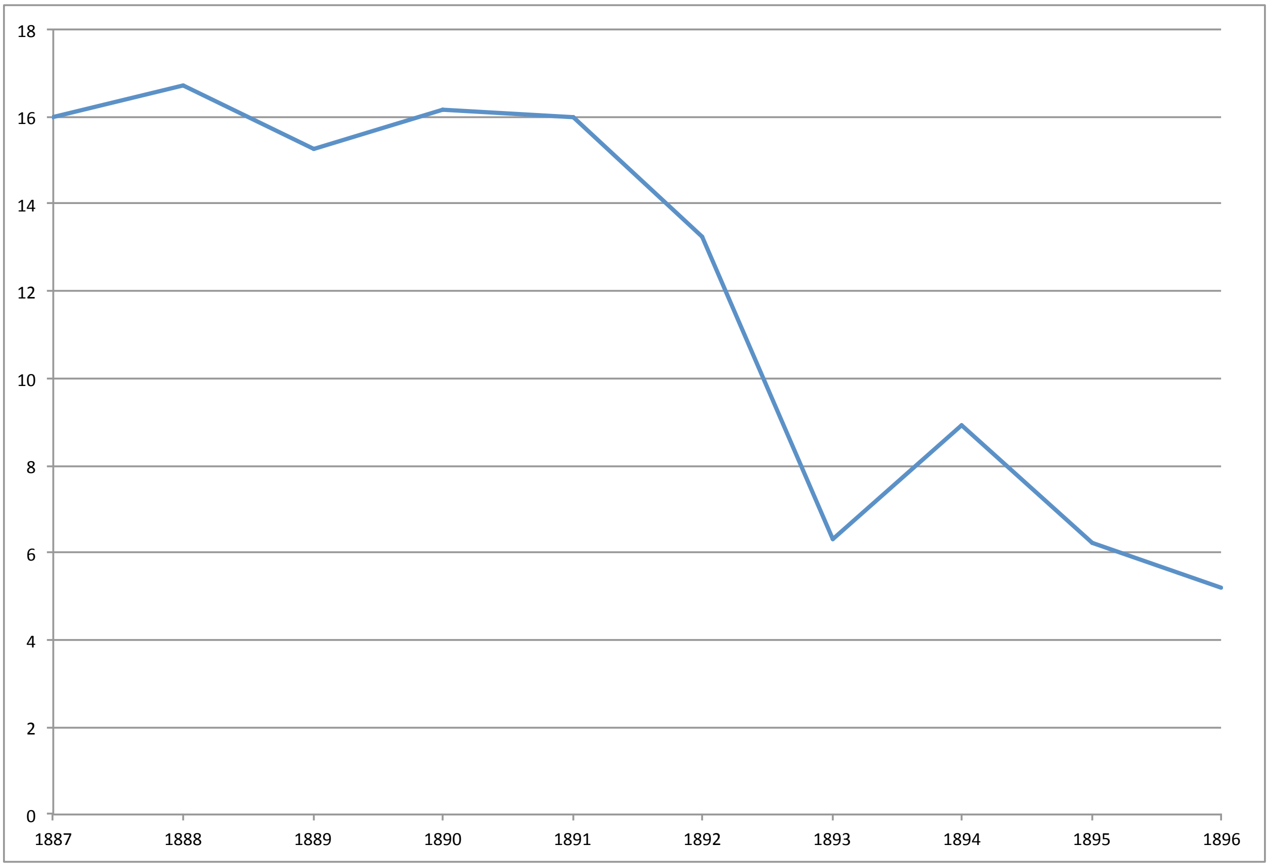


Figure 3: The breakdown of the cointegrating relationship between silver and agricultural commodity prices, 1870-1896

Notes: The figure plots ten λ-max test statistics for the cointegrating relationship between silver prices and the agricultural commodity price index, using the same starting date, 1870, but then varying the sample's ending date as displayed on the x-axis. Hence, the first test statistic is for 1870-87 and the last is for 1870-1896. The agricultural commodity index consists of wheat, cotton, and tea.

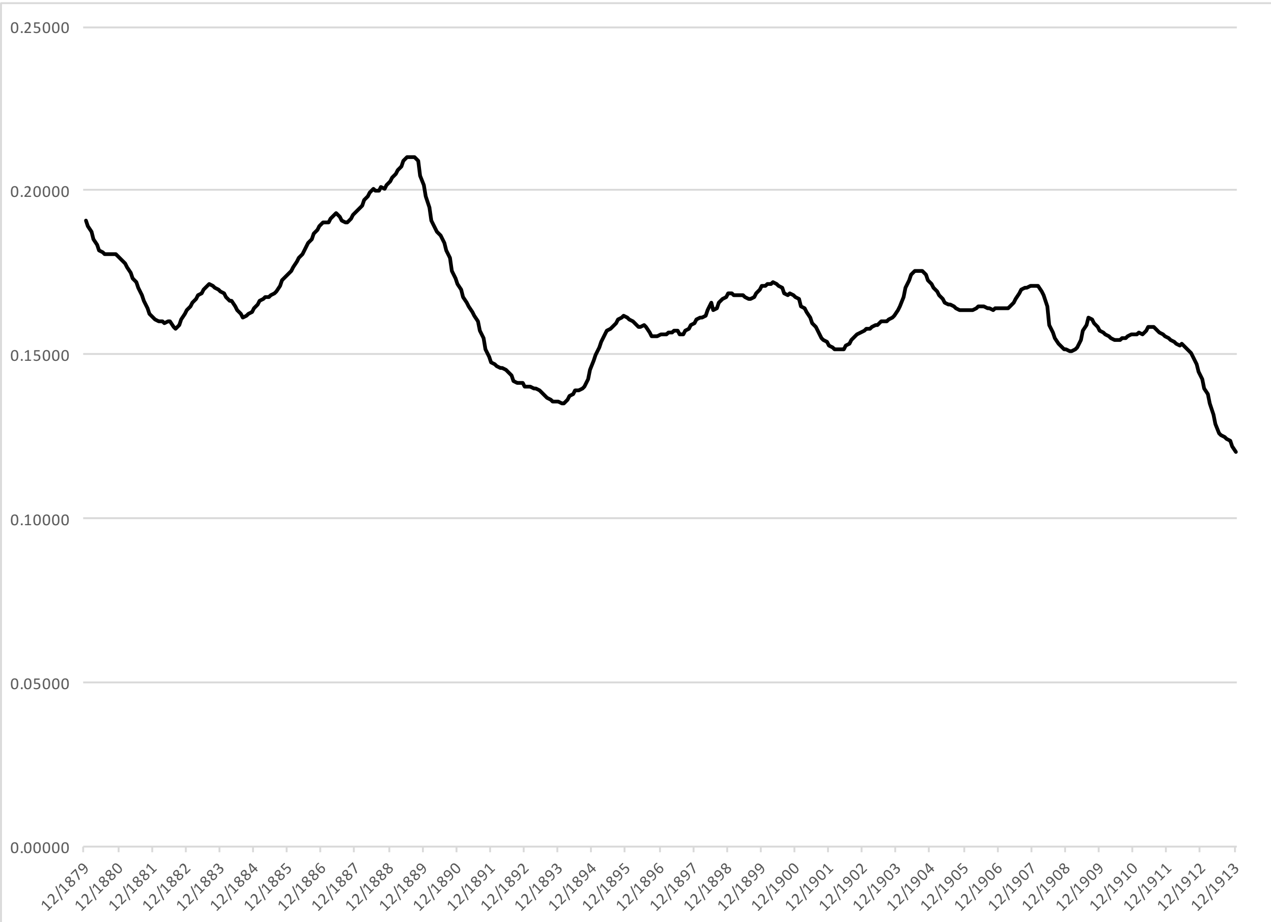


Figure 4: Average coefficient of variation for wheat, cotton, and tea, 1870-1913

Sources and Notes: Authors' calculations based on data from the *Economist* (various issues). These coefficients of variation are generated using monthly data over ten-year rolling windows, so the first observation covers the 1/1870-12/1879 period and the last observation covers the 1/1904-12/1913 period.

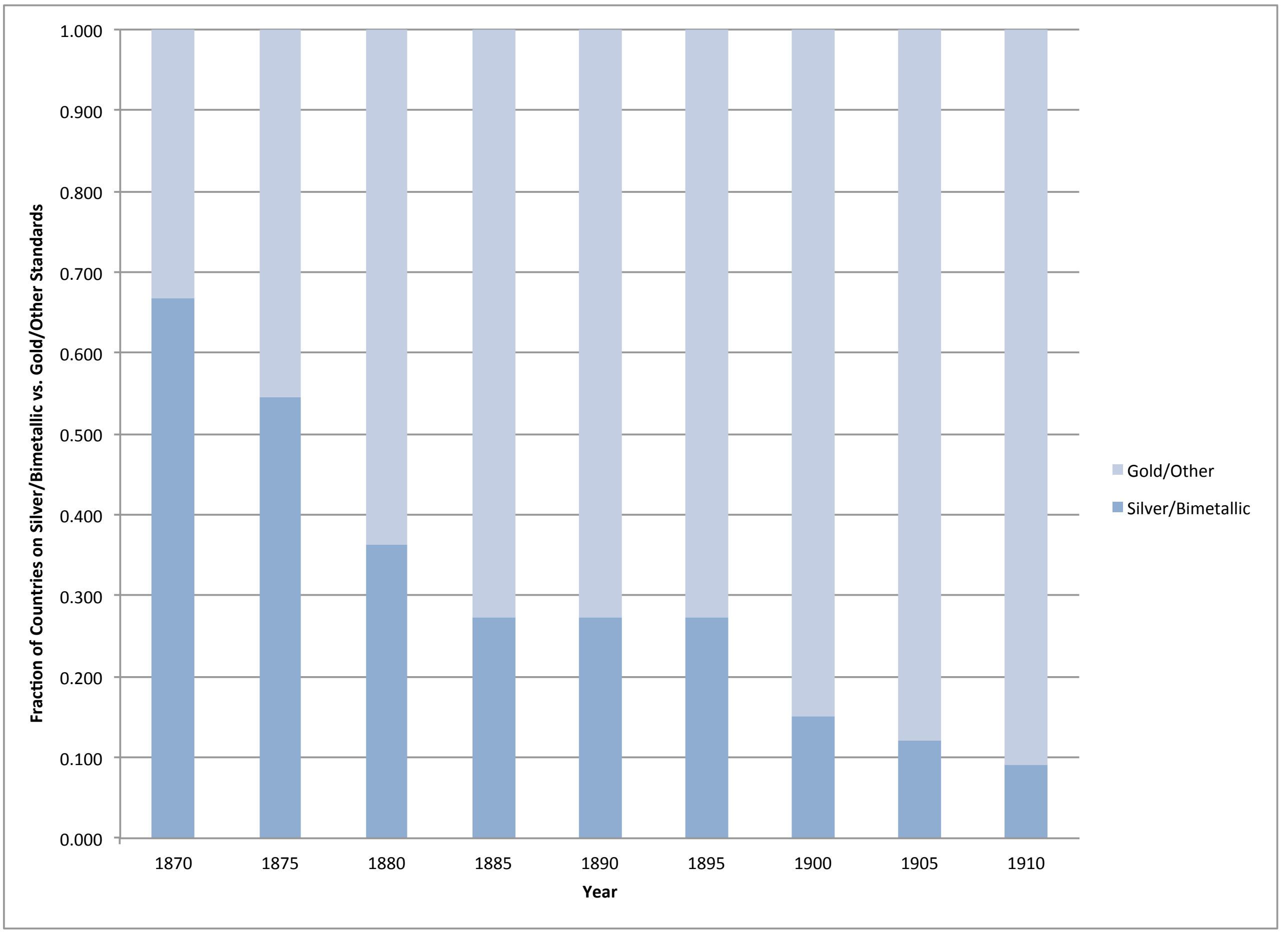


Figure 5: Monetary standards, 1870-1910

Source: Mitchener and Weidenmier (2008)

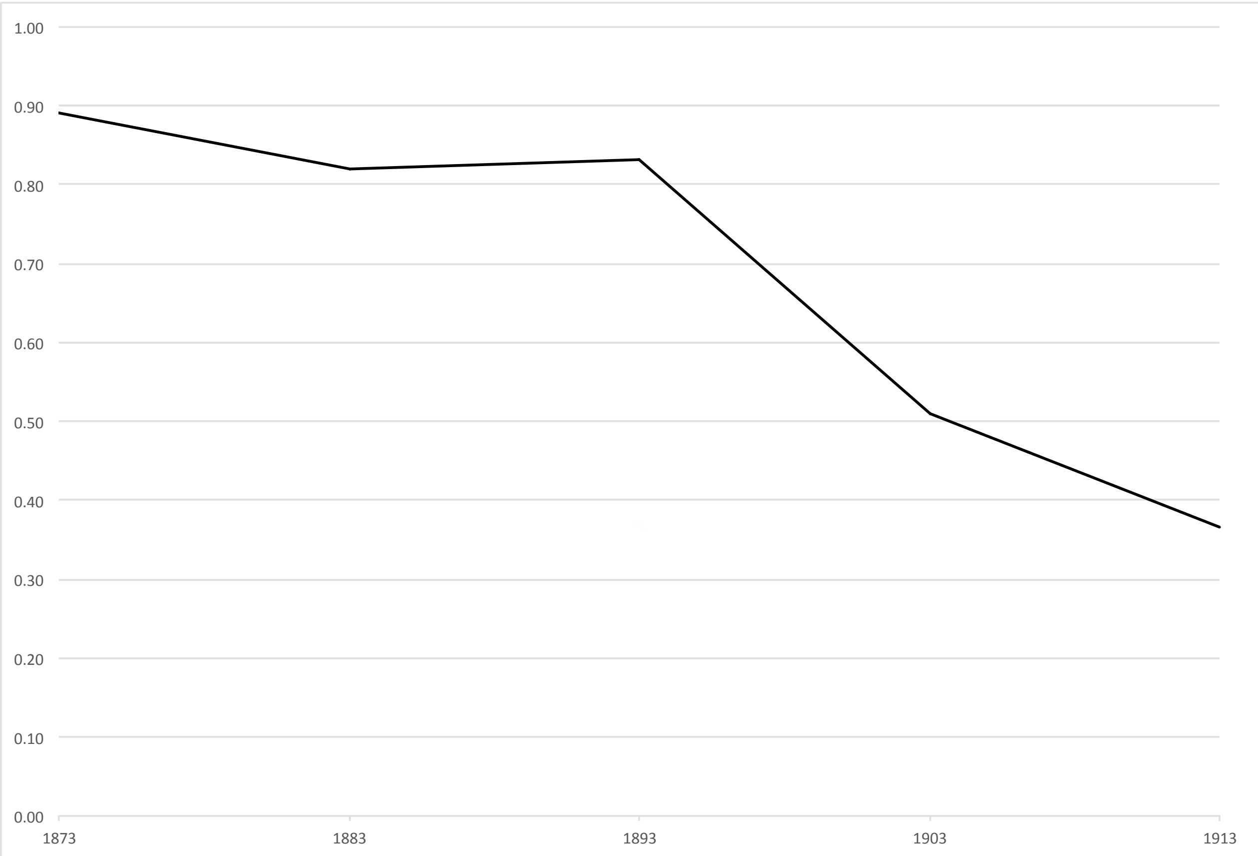


Figure 6: Global ratio of monetary silver to monetary gold, 1873-1913

Source: Authors' calculations based on data from the U.S. Bureau of the Mint (various years).

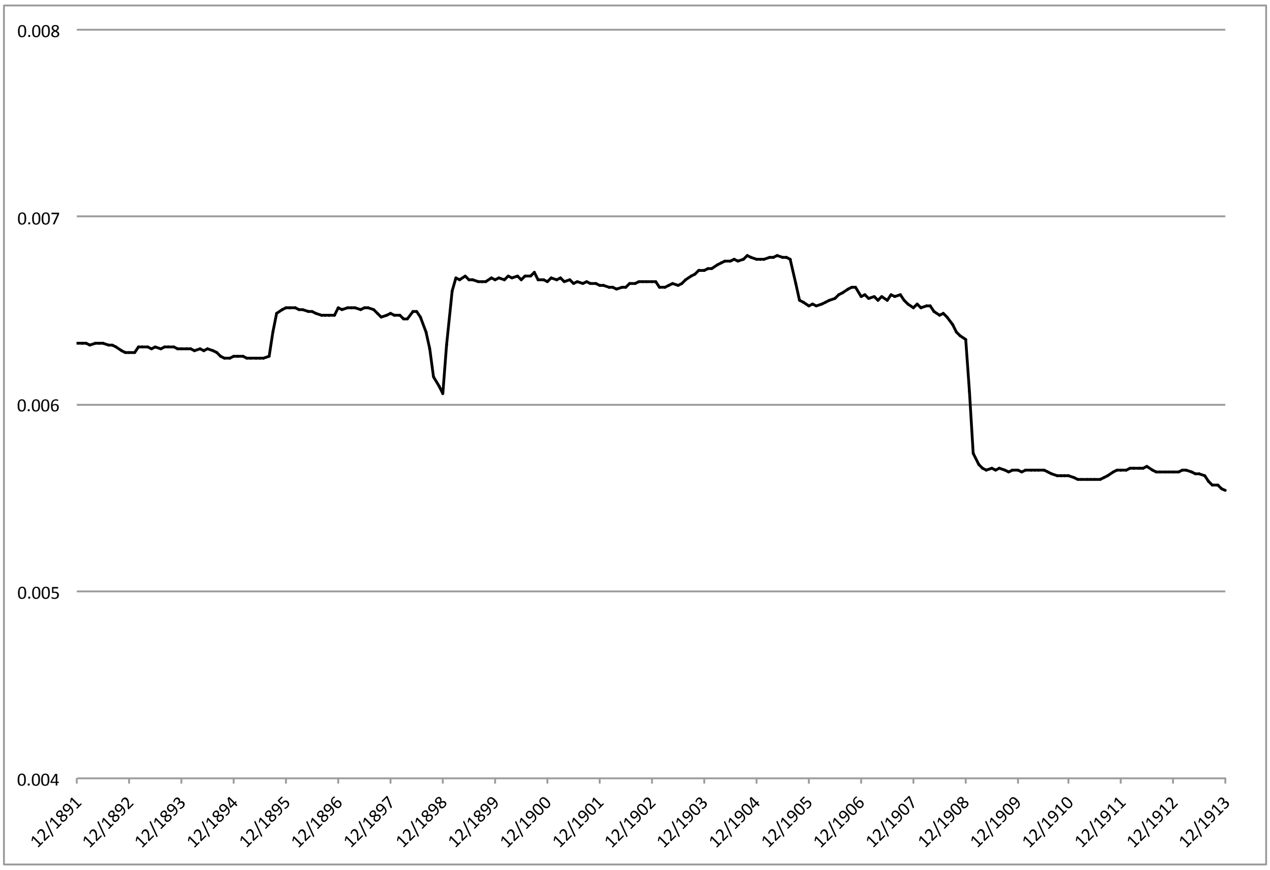


Figure A1: Average conditional variance for wheat, cotton, tea, barley, oats, and rice, 1882-1913

Sources and Notes: Authors' calculations based on data from the *Economist* (various issues). These conditional variances are estimated using a GARCH(1, 1) model and are reported as ten-year moving averages that are then averaged across all six commodities. The first observation covers the 1/1882-12/1891 period and the last observation covers the 1/1904-12/1913 period.

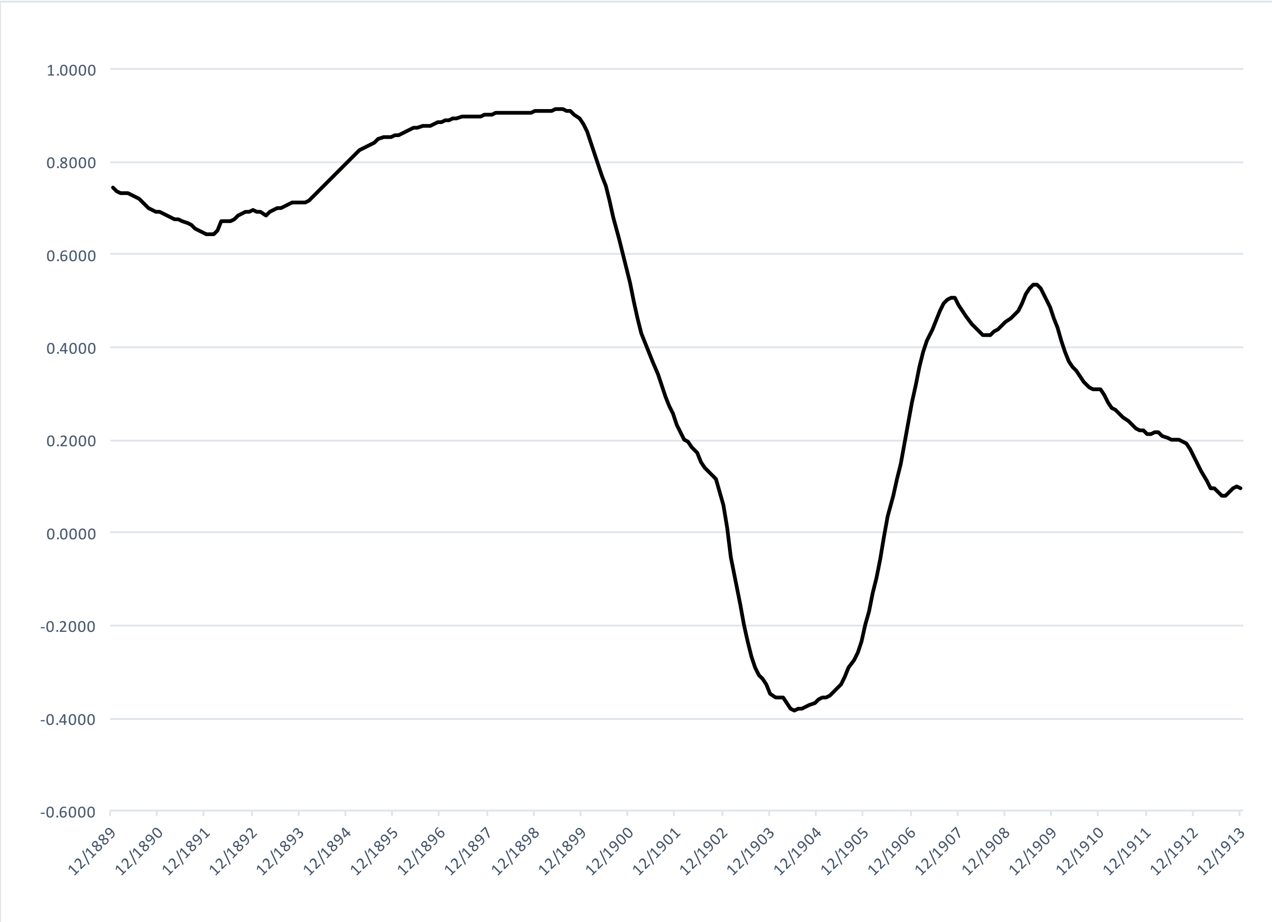


Figure A2: Correlations between the British textile export index and silver prices, 1880-1913

Sources and Notes: Authors' calculations based on data from the *Economist* (various issues) and Silverman (1930). These correlations are generated using monthly data over ten-year rolling windows, so the first observation covers the 1/1882-12/1891 period and the last observation covers the 1/1904-12/1913 period. The British textile export index consists of cotton yard, cotton cloths, woolen and worsted yarn, carpets, and linen and jute piece goods (Silverman, 1930).

Parameter	Baseline	Alternate 1	Alternate 2
θ	0.5	0.5	2
δ	2	2	0.5
μ_a	0.9	0.2	0.9
μ_s	0.5	0.3	0.5
σ_a	0.096	0.096	0.096
σ_s	0.096	0.096	0.096
σ_g	0.096	0.096	0.096

Table 1: Parameter values for baseline and alternate parameterizations of the model

	Change in A		Change in S		Change in G	
Ratio of Monetary Silver to Monetary Gold	p_s	p_a	p_s	p_a	p_s	p_a
0	0.936	-1.001	-1.872	0.002	0.936	1.000
0.1	0.186	-1.006	-1.105	0.011	0.920	0.995
0.2	0.115	-1.010	-1.030	0.020	0.914	0.991
0.5	0.045	-1.022	-0.960	0.038	0.916	0.984
1.0	0.000	-1.039	-0.930	0.055	0.930	0.985
2.0	-0.045	-1.067	-0.916	0.069	0.960	0.998

Table 2: Responses of the prices of silver and the agricultural consumption good to supply shocks in the baseline parameterization of the model

Notes: The table reports the log changes in the prices of silver (p_s) and the agricultural consumption good (p_a) in response to two-standard-deviation changes in the log supply of the agricultural consumption good (A), silver (S), and gold (G) for the model given different assumptions about the value of the ratio of monetary silver to monetary gold. The reported log price changes are standardized by dividing by the corresponding log supply changes.

Ratio of Monetary Silver to Monetary Gold	Price Correlation	Agricultural Consumption Good Coefficient of Variation	Silver Coefficient of Variation
0	-0.009	0.134	0.224
0.1	0.333	0.137	0.139
0.2	0.400	0.137	0.133
0.5	0.431	0.136	0.128
1.0	0.455	0.139	0.128
2.0	0.490	0.141	0.130

Table 3: Simulation results for baseline parameterization of the model

Notes: The table shows the correlation between the price of silver and the price of the agricultural consumption good and the coefficient of variations for the price of the agricultural consumption good and the price of silver for the model given different assumptions about the value of the ratio of monetary silver to monetary gold. Each of these results is generated by averaging a set of 10,000 model simulation results.

	Change in A		Change in S		Change in G	
Ratio of Monetary Silver to Monetary Gold	p_s	p_a	p_s	p_a	p_s	p_a
0	0.074	-1.797	-0.546	0.497	0.472	1.299
0.1	0.057	-1.802	-0.541	0.501	0.485	1.301
0.2	0.043	-1.807	-0.537	0.504	0.495	1.303
0.5	0.012	-1.818	-0.529	0.510	0.517	1.307
1.0	-0.019	-1.830	-0.520	0.517	0.540	1.313
2.0	-0.050	-1.844	-0.512	0.525	0.562	1.319

Table A1: Responses of the prices of silver and the agricultural consumption good to supply shocks in alternate parameterization 1 of the model

Notes: The table reports the log changes in the prices of silver (p_s) and the agricultural consumption good (p_a) in response to two-standard-deviation changes in the log supply of the agricultural consumption good (A), silver (S), and gold (G) for the model given different assumptions about the value of the ratio of monetary silver to monetary gold. The reported log price changes are standardized by dividing by the corresponding log supply changes.

	Change in A		Change in S		Change in G	
Ratio of Monetary Silver to Monetary Gold	p_s	p_a	p_s	p_a	p_s	p_a
0	-1.052	-0.884	-1.516	-0.053	2.568	0.937
0.1	-0.243	-0.883	-1.124	-0.060	1.367	0.943
0.2	-0.140	-0.884	-1.088	-0.067	1.228	0.951
0.5	-0.049	-0.879	-1.073	-0.085	1.123	0.964
1.0	0.000	-0.864	-1.086	-0.111	1.086	0.975
2.0	0.049	-0.830	-1.123	-0.159	1.073	0.989

Table A2: Responses of the prices of silver and the agricultural consumption good to supply shocks in alternate parameterization 2 of the model

Notes: The table reports the log changes in the prices of silver (p_s) and the agricultural consumption good (p_a) in response to two-standard-deviation changes in the log supply of the agricultural consumption good (A), silver (S), and gold (G) for the model given different assumptions about the value of the ratio of monetary silver to monetary gold. The reported log price changes are standardized by dividing by the corresponding log supply changes.