CESifo Working Papers
6473
2017
May 2017

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Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
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Editors: Clemens Fuest, Oliver Falck, Jasmin Gröschl
www.cesifo-group.org/wp

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CESifo *Working Paper No. 6473*
Category 4: Labour Markets

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Abstract

Correlations between parent and child earnings reflect intergenerational mobility and, more broadly, correlations between siblings' earnings reflect shared community and family background. These earnings relationships capture important aspects of relations in socio-economic status more generally. We estimate intergenerational transmission and sibling correlations of life-cycle earnings jointly within a unified framework that nests previous models. Using data on the Danish population of father/first-son/second-son triads we find that intergenerational effects account for on average 72 percent of sibling correlations. This share is higher than all previous studies because we allow for heterogeneous intergenerational transmission between families. Sibling correlations exhibit a U-shape over the working life, consistent with differences in human capital investments between families.

JEL-Codes: D310, J620.

Keywords: sibling correlations, intergenerational transmission.

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May 2017
We benefitted from discussions with Anders Björklund, Mette Ejrnæs, Bo Honorè, Markus Jäntti, Matthew Lindquist, Bash Mazumder, Costas Meghir, Claudia Olivetti, Luigi Pistaferri, Gary Solon and from the comments of audiences at Copenhagen Business School, University of Copenhagen, the University of Milan, La Sapienza University in Rome, ZEW Mannheim, LEED (Lisbon), ICEEE (Genoa), EEA (Gothenburg), ESPE (Aarhus), EALE (Turin) and SOLE (Arlington). Cappellari gratefully acknowledges the hospitality of SFI in Copenhagen. Financial support was provided by the Danish Strategic Research Council (DSF-09-065167) and the Danish Council for Independent Research – Social Sciences (DFF-6109-00226). The usual disclaimers apply.

I. Introduction

Explaining inequality of individual incomes on the basis of family background is the subject of a vast literature in economics, sociology, and other disciplines. The theoretical background in economics for the analysis of family effects dates back to the contributions of Becker and Tomes (1979). In their model, parents care about the lifetime earnings of their children and maximize utility by choosing between their own consumption and investment in child earnings capacity. Offspring outcomes also depend on other productive endowments which parents transfer to their children. As a result, lifetime earnings are transmitted between generations, through parental investments and productive endowments. Solon (1999), Björklund and Jännti (2009) and Black and Devereux (2011) document the progress of economists in this field, illustrating various angles from which one can study the importance of family background. Among these, intergenerational and sibling studies represent two prominent research approaches: the first explicitly considers parent-child transmission, while the second provides an omnibus measure of family and community influences on offspring outcomes.

How much of the correlation in sibling earnings is due to intergenerational persistence? Answering this question is key for understanding the channels through which outcomes are transmitted within the family and the importance of shared community factors. Evidence based on calibrations suggests that parent-child transmission is not the main driver of sibling correlations (Solon, 1999; Björklund et al., 2010; Björklund and Jännti, 2012), implying that to a large extent the resemblance of siblings' earnings stems from factors that the siblings share independently of their parents' outcomes. However, the share of sibling earnings correlation due to intergenerational persistence has never been estimated directly. In this paper we fill this gap by developing an econometric model of intergenerational transmission and sibling correlations. We find that other studies substantially underestimate the relevance of intergenerational persistence within sibling correlations.

We make two main contributions to the literature. Firstly we introduce a new approach for jointly analyzing sibling correlations and intergenerational transmission, and secondly we model multi-person earnings dynamics. Our first contribution is to study intergenerational transmission and sibling correlations of earnings jointly within a unified framework. We draw data from administrative registers of the Danish population and use a novel research design based on father/first-son/second-son triads. Using these triads identifies intergenerational effects separately from other factors shared by siblings within the overall sibling correlation. Our model nests the models of previous research which separately consider either intergenerational or sibling correlations – approaches that have complemented each other over the past thirty years, albeit indirectly. Assuming that the degree of intergenerational transmission is homogeneous in the population, Corcoran et al. (1990) and Solon (1999) show analytically how the sibling correlation can be decomposed into a part due to parent-child transmission and a residual sibling effect, the latter being interpreted as "[T]he combined effect of family background characteristics uncorrelated with parental income" (Solon, 1999, p. 1776). Subsequent research uses this decomposition in calibration, by combining statistics of intergenerational and sibling associations estimated for different families and from separate studies.

Calibration studies find that intergenerational correlations account for a relatively low share of sibling correlations: 40 percent in the US, 25 percent in Sweden and only 6 percent in Denmark. In our paper we provide a direct decomposition of sibling correlations allowing intergenerational effects to differ between families. Compared to previous studies we find that intergenerational transmission accounts for a much larger share of sibling correlations than previously thought – 72 percent on average. The importance of intergenerational persistence may reflect the effect of parental resources, but may also capture transmission of other important determinants of socio-economic status more generally, such as ability or preferences.

Our second main contribution is to combine insights from the sibling and intergenerational literatures with the literature on individual earnings dynamics. The seminal works of Lillard and Willis (1978), Lillard and Weiss (1979), Hause (1980) and MaCurdy (1982) initiated a long tradition of studies of individual earnings dynamics, surveyed in Meghir and Pistaferri (2011). Moffitt and Gottschalk (1995) pioneered the use of these models for analyzing trends in earnings inequality, opening up a stream of empirical research on the evolution of permanent and transitory components of earnings inequality and their impacts on earnings mobility.[1] For the first time we apply this approach to the analysis of the joint earnings dynamics of fathers, sons, and brothers.[2]

Our model allows for individual heterogeneity in earnings growth and serially correlated transitory shocks, enabling us to account for life-cycle biases and transitory shocks, which are the estimation issues plaguing the study of intra-family earnings correlations. Previous studies of sibling or intergenerational earnings correlations deal with these estimation issues either by taking averages of individual earnings over time to smooth out transitory shocks, or by limiting the analysis to a specific age range to mitigate life-cycle biases. Both approaches entail a loss of information. Furthermore, tension exists between the two approaches because the time span required for integrating out shocks of even moderate persistence is longer than the one for which

[1] The Moffitt and Gottschalk approach complements other approaches to earnings mobility, such as transition matrices and comparisons of inequality in current and lifetime earnings (see Buchinsky and Hunt, 1999, and Bönke et al., 2015).
[2] The one study of multi-person earnings dynamics is Ostrovsky (2012), who analyzes spouses' earnings in Canada. He builds on the earlier work of Hyslop (2001) who modeled the covariance structure of spouses' earnings in the U.S., but without allowing for life-cycle effects.

life-cycle bias is considered minimized. We take an entirely different route by modeling both sources of bias, enabling us to avoid informational losses and to show how sibling correlations and their intergenerational components evolve from ages 25 to 51.

Other studies estimate the sibling correlation of permanent earnings to be about 0.23 in Denmark (Björklund et al., 2002). While confirming this finding on average, our results show that sibling correlations vary considerably over the life-cycle, being about 0.5 at age 25, dropping to 0.15 by the mid-30s, and then rising again to 0.25 by age 51. The U-shaped life-cycle pattern of sibling correlations reflects Mincerian cross-overs of earnings profiles within birth cohorts: there is a negative association between starting earnings and earnings growth between individuals, so that the intra-generational distribution of permanent earnings first shrinks and then fans out over the working life (Mincer, 1958). We find that the compression/decompression occurs through the earnings component shared by siblings, generating a U-shaped pattern of sibling correlations. This finding is consistent with differences between families in human capital investments. We also find significant variation in the relative importance of intergenerational factors over the life-cycle. At age 25 intergenerational transmission accounts for approximately half of the sibling correlation, but from the early 30s intergenerational effects become predominant, and in the longer run intergenerational transmission matters more than other shared factors for explaining similarities in the earnings profiles of brothers.

Our model encompasses the other models used in the literature, and we exploit this property to reconcile our findings with those of previous studies. We rule out data differences by applying the calibrated decomposition common in the literature to our data, finding an intergenerational share of 4-5 percent within the sibling correlation, which lines up with existing results. Then, by relaxing the assumptions of the basic model we approach our more general model and measure the contribution of each assumption to the total reconciliation of 65 percentage points (from 5 to 72 percent) between estimates. We show that the key difference between the previous approaches and ours is that we allow for heterogeneous intergenerational transmission between families. We illustrate this key difference by developing a modified version of the standard calibration formula that incorporates heterogeneous intergenerational elasticities.

Our model for family triads averages intergenerational transmission for two sons. In the final part of the paper we extend the model to allow for differential intergenerational transmission to first son and second son. Differential transmission may be important in the light of the literature showing significant birth order effects on outcome levels in favor of the first born (see e.g. Black et al., 2005). Moreover, this extended model helps understanding whether sibling correlations underestimate intergenerational influences due to differential treatment of first and second sons. We find slightly stronger transmission to second sons which is consistent with birth order effects operating especially in poorer families. However, these differences are not statistically significant, suggesting that sibling correlations provide a reliable estimate of the earnings generating factors transferred from father to son.

II. Related Literatures

We draw upon two strands of literature that have a long tradition in labor economics: sibling correlations and earnings dynamics. Ours is a model of sibling correlations and intergenerational transmission in life-cycle earnings. We now review these two strands of literature, focusing on the aspects that are relevant for highlighting our contributions.

A. *Sibling Studies*

Research on sibling correlations in outcomes is well established in economics and sociology (see the reviews in Griliches, 1979; Solon, 1999; Björklund and Jännti, 2009; and Black and Devereux, 2011). Siblings are "[M]ore alike than a randomly selected pair of individuals on a variety of socioeconomic measurements" (Griliches, 1979; p. S38); sibling correlations of earnings or other outcomes have been used as a way of capturing many of the influences that siblings share. These influences may not only originate in the intergenerational transmission of outcomes, but may also stem from other factors passed from parents to children, factors (at least partly) independent of parental outcomes, such as values (Behrman et al., 1982). In addition, sibling effects capture those influences that are shared by siblings but that do not come from the parents, such as orthogonal school or community effects. However, there may be family-transmitted factors that are not shared by siblings, (e.g., because of differential treatment from parents) and which are not captured by sibling correlations.

The prototypical model of sibling earnings correlations specifies individual log earnings in deviation from the mean (w) as the sum of three orthogonal components:

$$w_{ijt} = a_{ij} + f_j + v_{ijt}, \qquad a_{ij} \sim \big(0, var(a)\big), f_j \sim \big(0, var(f)\big), v_{ijt} \sim \big(0, var(v)\big) \tag{1}$$

where i indexes individuals, j indexes families and t indexes time (see e.g. Solon, 1999, and Björklund et al., 2009). The a and f components are assumed time-invariant and measure permanent earnings, whereas v is a transitory shock typically assumed to be white noise.[3] Permanent earnings depend on an individual-specific factor a_{ij} capturing idiosyncratic components not shared by

[3] One exception is Björklund et al. (2009) who adopt a stationary AR(1) process.

siblings, and depend on a family-specific factor f_j absorbing all determinants of permanent earnings that are shared by siblings. This f_j includes both intergenerational persistence and all other sources of sibling similarities in earnings; we label all these other sources "residual sibling effects". Intergenerational earnings transmission may depend on endowments passed on at birth, educational investments or on the extent with which parents are able to transmit their skills and preferences to their children after birth. Conversely, residual sibling effects include parental influences not captured by earnings, or other community effects shared by siblings which are independent of the parents' earnings. Schools, friendship networks, or other influences operating at the community level are examples of residual sibling effects. We label the effects captured by f_j (intergenerational plus residual sibling) "overall sibling effects".

This model has been used to estimate the sibling correlation of permanent earnings (r_S) as the ratio between the variance of the overall sibling effect and the total variance of individual permanent earnings:

$$r_S = \frac{var(f)}{var(a) + var(f)} \tag{2}$$

The sibling correlation provides an omnibus measure of family and community effects, which is the share of inequality in permanent earnings accounted for by family and community background. Identification of the sibling correlation is achieved when data are available on earnings for sibling pairs over multiple years, as the multi-year requirement enables separation of permanent from transitory earnings. Previous studies report estimates of the correlation in brothers' permanent incomes ranging from 0.4 to 0.5 in the US (Solon et al., 1991; Altonji and Dunn, 1991; Solon, 1999; Mazumder, 2008) to a little higher than 0.3 in Sweden (Björklund et al., 2009) and about 0.2 for Norway and Denmark (Björklund and Jännti, 2009). Thus between one fifth and one half of the dispersion of permanent earnings is due to differences between sibling pairs in income-generating factors and the remainder is due to idiosyncratic differences within sibling pairs.

Estimating how much of the sibling correlation is due to intergenerational transmission is important both for understanding the mechanisms behind between-family differences in the distribution of outcomes and for measuring the contribution of community factors. A formal characterization of the link between sibling income correlation and the intergenerational income elasticity (IGE, the slope coefficient of a regression of sons' log incomes on fathers' log incomes) is provided by Corcoran et al. (1990) and Solon (1999). They start with the model of equation (1) and write the overall sibling effect as a linear function of father's permanent income through the IGE and an additive residual sibling effect orthogonal to father's income, capturing remaining shared factors independent of father's income:

$$f_j = \beta y_j^F + \mu_j^R \tag{3}$$

where the IGE (β) is assumed constant between families, y_j^F denotes the log of father's permanent income in deviation from the mean and μ_j^R denotes the residual sibling effect. Assuming stationarity in the distribution of permanent incomes between generations (i.e. assuming that $var(a) + var(f) = var(y_j^F)$), the resulting decomposition of the sibling correlation is:

$$r_S = \beta^2 + r_R, \tag{4}$$

where $r_R = \frac{var(\mu_j^R)}{var(a)+var(f)}$. We label r_R "residual sibling correlation", because it measures the part of r_S that does not depend on father's permanent income.

Solon (1999) reports an IGE of 0.4 for the U.S., which when matched to a sibling correlation of about the same size implies that 40 percent (=0.4^2/0.4) of the sibling correlation can be ascribed to intergenerational persistence. Subsequent research has applied this decomposition indirectly as a calibration using sibling correlations and IGEs which are sometimes estimated from different families and different samples. Intergenerational factors are generally found to have only a small effect. For Denmark, Björklund and Jännti (2009) report an IGE of 0.12 and a brother correlation of 0.23, implying that the role of parental income is negligible, explaining 6 percent of the overall brother correlation. One of our contributions is to provide a counterpart to this decomposition, directly estimated from a unified model of intergenerational and sibling associations in earnings that allows intergenerational transmission to be heterogeneous in the population.

By relaxing the assumption of stationarity, Björklund et al. (2010) obtain a decomposition analogous to (4) in terms of the intergenerational *correlation* (rather than *elasticity*) of permanent incomes:

$$r_S = \lambda^2 + r_R, \tag{4'}$$

where λ denotes the intergenerational correlation (IGC). Björklund and Jännti (2012) use Swedish register data and apply the sibling correlation model to a range of traits and outcomes such as IQ, non-cognitive skills, height, schooling, and long-term earnings. They find that sibling correlations in earnings are the lowest, and that the strongest associations are for height and IQ. They also estimate the IGC and apply the decomposition formula without assuming stationarity, finding that parental effects account for a small share of the overall sibling correlation, irrespective of the trait or outcome considered.

An alternative approach for assessing the role of parental incomes in shaping sibling correlations is provided by Mazumder (2008) who estimates the correlation before and after conditioning sibling earnings on family attributes in a mixed model framework. When family attributes are limited to fathers' permanent incomes, this approach is similar to the decomposition of equation (4'), with one difference being that mixed models are estimated via Restricted Maximum Likelihood (REML) and assume that unobserved components of log income are normally distributed. Using this method on U.S. data, Mazumder reports that approximately one third of the sibling correlation in long-run earnings is accounted for by paternal incomes. Applying this approach to Swedish data, Bjorklund et al. (2010) report a 13 percent reduction of the sibling correlation after controlling for fathers' income.

Understanding which factors determine the sibling correlation is also the aim in Page and Solon (2003a, 2003b) where the focus is on neighborhood effects rather than parental incomes. They contrast sibling correlations in earnings with correlations in earnings between neighboring boys and girls, finding that neighbors' correlations—in turn an upper bound of the underlying effect of neighborhoods—account for about half of the sibling correlation in earnings. Thus, similar to the studies that consider the effects of parental incomes, they find that a large portion of the sibling correlation remains unexplained.

B. *Estimation Issues and Models of Earnings Dynamics*

Estimating intra-family income associations is complicated by two fundamental measurement issues. First, data on annual incomes are mixtures of long-term incomes and transitory income shocks, the latter being equivalent to classical measurement error. Solon (1992) and Zimmerman (1992) show that when estimating the IGE the bias can be substantial and that averaging parental incomes over a limited number of years is sufficient for mitigating the bias. Mazumder (2005) shows that when transitory shocks are characterized by serial correlation, measurement error becomes more severe and harder to integrate out, requiring sequences of individual income data as long as 30 years.

The second measurement issue stems from life-cycle bias, whereby fathers' and sons' incomes are usually sampled at different phases of the life-cycle, typically too early for sons and too late for fathers, when current measures under- and over- estimate (respectively) long-term measures (see Jenkins, 1987; and Grawe, 2006). Haider and Solon (2006) show that if there is individual heterogeneity in life-cycle earnings growth, then the relationship between current and lifetime earnings varies over the life-cycle, and the bias incurred by using annual measures instead of lifetime measures is minimized in the 30-40 age range.[4] In the context of intergenerational analyses, Nybom and Stuhler (2016) show how life-cycle bias gives rise to non-classical measurement error and call for an explicit allowance for heterogeneous life-cycle growth between individuals in studies of intergenerational income associations.

The strategies used by previous studies for coping with transitory shocks and life-cycle biases conflict with each other. While transitory shocks are better dealt with using long strings of individual earnings, life-cycle bias is minimized over a limited age range, between 30 and 40.[5] In this paper we follow a different strategy, one that allows us to resolve this tension. We use tools from the earnings dynamics literature to model (rather than averaging out) the two sources of bias. Our approach avoids informational losses and allows for life-cycle effects in intra-family correlations of permanent earnings.

There exists a well-established literature on modeling individual earnings dynamics. In this tradition, studies typically start from a permanent-transitory characterization of the log earnings process (in deviation from some central tendency) and pay considerable attention to the dynamic properties of the permanent and transitory components.[6] The latter are usually specified as low order ARMA processes, thus allowing for serial correlation of transitory shocks. Long-term earnings are specified as either Random Growth (RG, also called Heterogeneous Income Profile-HIP) or Random Walk (RW, also called Restricted Income Profile – RIP—because there is no heterogeneity in earnings profiles) processes.

In the RG-HIP model individual earnings are assumed to evolve according to an individual-specific linear age (or experience) profile (see e.g. Lillard and Weiss, 1979; Hause, 1980; Baker, 1997; Haider, 2001; Guvenen, 2007; and Gladden and Taber, 2009). There are two sources of persistent individual earnings differences, time-invariant heterogeneity and growth rate heterogeneity. The presence of growth rate heterogeneity makes the model particularly attractive for studying interpersonal dynamics as it enables controlling for the source of life-cycle biases. Linearity in earnings levels implies a quadratic age profile of earnings variances. The RG-HIP model can be summarized as follows:

[4] For Sweden, Böhlmark and Lindquist (2006) obtain results remarkably close to the U.S. estimates of Haider and Solon (2006).
[5] See e.g. Björklund et al. (2009) for an application of this approach to sibling studies.
[6] See Moffitt and Gottschalk (2012) and the survey articles by Meghir and Pistaferri (2011) and Browning and Ejrnæs (2013). Most of these studies focus on the earnings process in isolation from other outcomes; exceptions are Abowd and Card (1989) and Altonji et al. (2013).

$$y_{it} = a_i + b_i A_{it}; (a_i\ b_i) \sim \left(0,0; var(a), var(b), cov(a,b)\right) \quad (5)$$

where y_{it} is log permanent earnings and A_{it} is age. Many studies have found a negative covariance between intercepts and slopes, implying that individuals starting with low pay will see their earnings grow faster than initially higher paid individuals (see e.g. Gladden and Taber, 2009). As pointed out by Baker and Solon (2003), these different trajectories can be interpreted in a human capital framework. According to human capital theory, differences in human capital investments between individuals would generate heterogeneity of both initial earnings and earnings growth (Mincer, 1958; Ben-Porath, 1967). Moreover, heterogeneous investments induce a negative correlation between initial earnings and earnings growth, because investors trade off lower initial earnings with higher earnings growth. In these circumstances, heterogeneous earnings profiles will converge at some point after labor market entry. Conventionally, the cross-over point of converging profiles can be computed as the age of minimum earnings variance: $A^* = -cov(a,b)/var(b)$ (Hause, 1980). Hence, within a birth cohort permanent inequality displays a U-shaped profile minimized at A^*. In principle, the covariance of intercepts and slopes may also be positive, indicating a complementarity between starting earnings and earnings growth: in that case the life-cycle profile of the variance would follow an ever increasing quadratic trend.

RW-RIP models assume earnings evolve over the life-cycle through the arrival of permanent shocks (z):[7]

$$y_{it} = y_{it-1} + z_{it}; y_{it(A_0)} \sim \left(0, var\left(y_{t(A_0)}\right)\right);\ z_{it} \sim \left(0, var(z)\right) \quad (6)$$

where A_0 is the starting age and $t(A_0)$ is the corresponding time period, so that $y_{it(A_0)}$ is the initial condition of the process. Examples of permanent shocks are promotions, displacements or chronic diseases. The accumulation of shocks and the model assumptions imply that the RW-RIP process produces a constantly growing earnings variance that follows a linear trend over the life-cycle. One virtue of the RW-RIP model is that it fits well within models of life-cycle optimization with rational expectations. Guvenen (2007) shows that a process of individual learning on the heterogeneous profile needs to be specified in order for the RG-HIP model to be used in a dynamic optimization framework.

[7] See, among others, MaCurdy (1982), Meghir and Pistaferri (2004) and Hryshko (2012).

While most studies choose either the RG-HIP or RW-RIP model, there are examples of eclectic approaches using mixtures of the two, such as Baker and Solon (2003), and Moffitt and Gottschalk (2012). We use a combination of RG-HIP and RW-RIP in this paper.

III. Data Description

We use data from administrative registers of the Danish population. The civil registration system was established in 1968 and everyone resident in Denmark then and since has been registered with a unique personal identification number which has subsequently been used in all national registers enabling accurate linkage. Links from children to legal parents originate from municipal and parish records and are complete for births from 1955 onwards (Pedersen, et al. 2006). We sample fathers born from 1935 who have two sons with the same mother, but drop fathers who were younger than 18 when their first son was born. For remaining fathers we sample first and second sons, but drop grandsons in the sense that no son is recycled as a father in the analysis. Subsequent sons are rare in the population (4 percent) and are ignored. Brothers born less than 12 months or more than 12 years apart are dropped from the sample. Boys changing legal parentage through adoption before age 18 are also dropped. In this way we derive a sample of father/first-son/second-son triads.[8] Women play no role in the main analysis after determining full brotherhood.[9]

We select fathers born 1935-1961, first sons born 1959-1985 and second sons born 1962-1985. This selection is because of completeness of registered parentage and the small number of first sons observed born before 1959. We group individuals into 3-year birth cohorts, imputing the central age to each cohort group, and hereafter refer to cohort groups by this central age. Imposing a cohort structure on the data is fundamental for separating life-cycle effects from calendar time, and this is the established practice of earnings dynamics studies (see for example Baker and Solon, 2003).

We model annual pre-tax labor earnings which are obtained from income tax returns. Each January employers report earnings for each employee for the previous year to the tax authorities, and in March the tax authorities send these returns to the employees themselves for verification. We use the sum of earnings from all employments during the year for the period 1980-2014 over which it is available in the Statistics Denmark Income Statistics Register; see Baadsgaard and Quitzau

[8] In sensitivity analyses we include families consisting of father/first-son couples as well as our main estimation sample of father/first-son/second-son triads. We find that the inclusion of single-son families reinforces our headline result of a predominant intergenerational effect within the brother correlation.

[9] Son birth order is determined irrespective of the presence of daughters: for example, we do not make any distinction for whether there is a daughter born in-between the two sons, before or after. We study men and do not consider mother/son, father/daughter or brother/sister associations. Our results are robust to the presence of sisters.

(2011) for a detailed description of Danish income registers. In common with most of the earnings dynamics literature, we exclude zero earnings observations and assume that earnings are missing at random. It is also common to drop zeros in the sibling correlation literature, see for example Björklund et al. (2009). In order to model life-cycle dynamics we need to observe individual earnings strings over time and conventionally set the start of the life-cycle (A_0) at age 25 and its final point at age 60. Consequently we observe fathers throughout this range 25-60, first sons 25-54 and second sons 25-51. Mean observed ages are 46.7, 35.1 and 33.4 respectively.

The combination of sample selection criteria generates a dataset which is described in the left panel of Table 1 for selected years in terms of first and second moments of the annual earnings distribution and average age. For this sample we apply two additional selections which are typical in the earnings dynamics literature. First, we exclude outliers by trimming half a percentile on each tail of the earnings distribution of each year; since the analysis will exploit empirical earnings moments separately by family members, we perform the trimming within the distribution of each type of member.[10] Secondly, in order to measure earnings profiles precisely we require at least five consecutive positive earnings observations. This selection rule is intermediate between the one used by Baker and Solon (2003), of continuous positive earnings strings, and the approach of Haider (2001), allowing individuals to move in and out of the sample only requiring two positive but not necessarily consecutive earnings observations.

The right panel of Table 1 describes the estimation sample after making these restrictions. Trimming outliers and imposing partially continuous earnings strings has an impact on sample size. There is also an impact on earnings dispersion, while average earnings are not much affected. In total, our sample consists of 303,231 men belonging to 101,077 families. Individuals are observed for 18.1 years on average (fathers 21.8, first sons 17.9 and second sons 14.6), giving 5,488,445 earnings observations in total. Most observations – 2,199,507 – are for fathers' earnings, with 1,810,396 for first sons and 1,478,542 for second sons.

We begin describing patterns of earnings associations within the family in Figure 1, which plots intergenerational and brother correlations of log real annual earnings, adjusted for time and age effects by taking residuals of regressions for each birth cohort and type of family member on calendar year dummies and a quadratic age trend. We discard empirical second moments that are based on fewer than 100 cases throughout the analysis.[11] The figure provides an overview of correlations in raw earnings, which reflect both permanent and transitory earnings components.

[10] Using Danish registers Bingley et al. (2013) show that estimates of earnings dynamics models are robust to alternative trimming rules.

[11] There are 62,065 empirical moments in total, of which 6,324 are dropped because they are estimated on fewer than 100 cases. Our results are essentially unchanged if we set the threshold to 50 or 150 cases.

In Panel A we consider brother correlations. The fixed-age plot represents the average brother correlation by age of the younger brother, conditional on the age of the older brother being fixed at 35; the same-age plot is calculated from the average of correlations when brothers reach the same given age. There is a contrast between the two plots. The same-age plot displays a declining age profile, consistent with an underlying RG-HIP model of permanent earnings dynamics with Mincerian cross-overs, in which brothers share the determinants of their human capital investments. However, the fixed-age correlations are very low at young ages and converge to the level of same-age correlations at 30. This fixed-age pattern is consistent with life-cycle bias: estimating brother correlations between brothers observed at different stages of the life-cycle provides an underestimate of the correlation at the same stage. That we can observe this bias suggests our data provides an adequate basis for controlling the bias.

Besides being an outcome of a RG-HIP process of permanent earnings, large same-age correlations of raw earnings while young may also reflect shared transitory shocks. It is well known that earnings are unstable while young (see e.g. Baker and Solon, 2003) and it is plausible that siblings are subject to common shocks, for example because of similar local economic conditions at labor market entry. To assess if the relatively large brother same-age correlation while young is driven by permanent earnings differences or transitory fluctuations, we compute brother correlations for brothers born at least five or eight years apart (not shown). Brothers from more distant cohorts are less likely to share transitory shocks at labor market entry. The same-age profile of correlations persists even after excluding closely spaced brothers, suggesting that the source of the declining age profile of brother correlations is in the permanent earnings component, and supporting a RG-HIP specification.[12]

In Panel B of Figure 1 we repeat the exercise for intergenerational correlations, obtained by averaging father-son correlations for both sons by sons' ages. Fixed-age correlations refer to fathers aged 35 and follow an increasing pattern similar to the fixed-age brother correlation, suggestive of life-cycle bias. Same-age correlations are relatively flat and do not display the sharp initial decline that we observe for brother correlations. While this pattern might mean that the intergenerational component of permanent earnings is not generated by a RG-HIP model, it might also reflect much greater age spacing between fathers and sons than between brothers, making life-cycle bias more severe and harder to control with same-age correlations, especially at young ages, when there is

[12] An additional reason for the declining brother correlation between age 25 and 30 could be selection into the labour market: at age 25 school-to-work transitions might still be incomplete for a non-random sample of the population and the sources of non-randomness might be correlated between brothers. In our estimating sample 8 percent of brothers enter the labour market after the age of 25. To assess whether life-cycle patterns of brother correlations are an artefact of selection into the labour market, we re-estimated raw correlations limiting the sample to brothers whose earnings profiles are observed from the age of 25, and found that the level and life-cycle evolution of the brother correlation are virtually identical to the ones depicted in Figure 1.

larger measurement error from transitory shocks. The model of the next section features both age-dependent transitory shocks and RG-HIP permanent earnings, thus enabling us to test whether the intergenerational component of permanent earnings can be characterized as a RG-HIP process with Mincerian cross-overs.

IV. A Model of Earnings Dynamics for Fathers and Sons

We study earnings dynamics within the family and set up a multi-person model which contributes to the strands of literature reviewed in Section II. We contribute to the earnings dynamics literature because ours is a model of the joint earnings dynamics of three family members. This enables us to resolve the tension faced by previous studies on estimation biases when choosing the length of the income strings to analyze, because we model both heterogeneous earnings growth and serially correlated transitory shocks. Moreover, we contribute to the sibling literature by decomposing the sibling correlation into intergenerational and residual sibling components.

For the first time, we derive the decomposition from a unified model of intergenerational and sibling correlations in earnings. We can perform a *direct* decomposition of the sibling correlation which, unlike *indirect* decompositions based on the calibration formula of equations (4) or (4'), allows for IGE heterogeneity between families. Heterogeneous transmission can be rationalized in the Becker and Tomes (1979) framework as long as parental altruism, liquidity constraints or the ability or willingness to transmit pre- or post-birth endowments are family-specific. As we will show in Section VII, allowing IGE heterogeneity is key to finding a much greater role for father-son transmission within the brother correlation.

We focus on men and distinguish three types of family members, fathers (F), first-born sons ($S1$) and second-born sons ($S2$). For each family member, we consider individual log earnings in deviation (w) from the mean, where the mean varies by year, birth cohort and type of family member.[13] Log earnings deviations from the mean consist of a permanent (long-term) component (y) and an orthogonal transitory (mean-reverting) shock (v). Orthogonality holds by definition of permanent and transitory components of earnings, and total earnings are written as the sum of the two orthogonal components:

$$w_{ijt} = y_{ijt} + v_{ijt}; E\left(y_{ijt}, v_{ijt}\right) = 0 \tag{7}$$

[13] Considering earnings in deviation from yearly means by birth cohort is a flexible way of removing average age effects that may confound the estimation of individual life-cycle profiles, see Baker and Solon (2003). Here we apply the "de-meaning" procedure distinguishing the different types of family members and taking residuals from cohort/member-specific regressions of log earnings on calendar year dummies and quadratic age trends, where the latter adjust for within-cohort age differences (recall that we work with birth cohorts defined over three-year ranges).

where the indices i, j, and t stand for individual, family and year of observation.

A. *Permanent Earnings*

We model permanent earnings by combining insights from literatures on sibling correlations and earnings dynamics and extend the model of equation (1) in two ways. First, we distinguish between an intergenerational component and an additive residual sibling component within the overall sibling correlation in an unrestricted way. This distinction allows us to attribute part of the omnibus sibling earnings correlation to intergenerational earnings persistence while allowing transmission to be heterogeneous between families. We identify the transmission of earnings from fathers to sons, and we are silent about other channels of intergenerational persistence working independently of father's earnings. In this sense our decomposition provides a lower bound to the intergenerational component of sibling earnings correlations.

Secondly, we extend the model of equation (1) by introducing life-cycle effects. We specify earnings components shared between family members using the RG-HIP parameterisation. This is motivated by the need to allow for heterogeneous earnings profiles in order to avoid life-cycle biases. Also, in the previous section we provided evidence that empirical brother correlations are U-shaped in age, consistent with a model of permanent earnings dynamics with Mincerian cross-overs in which brothers share the determinants of their human capital investments; a pattern that can be captured by a RG-HIP model and not by a RW-RIP. Note that in this multi-person context the RG-HIP model can be more easily justified from the informational viewpoint than in models of single-person earnings dynamics; for example, sons may already know the parameters of their earnings process at labor market entry by observing the earnings profiles of their fathers or of other members of their communities. We maintain the RW-RIP specification for the idiosyncratic component of permanent earnings, and we allow its distribution to be member-specific for greater generality.[14] Sons' earnings are written as:

$$y_{ijt} = \left(\left(\mu_j^I + \mu_j^R\right) + \left(\gamma_j^I + \gamma_j^R\right) A_{it} + \omega_{ijt} \right) \pi_t, \qquad (8)$$
$$\omega_{ijt} = \omega_{ijt-1} + \phi_{ijt}.$$

The earnings profile is linear in age, and intercepts (μ) and slopes (γ) of the RG-HIP model depend upon family-specific effects. Family effects have an intergenerational component indexed by superscript I, and a residual sibling component indexed by superscript R. Intergenerational and

[14] There are additional empirical considerations supporting our choice of specification, see Footnote 17.

residual sibling components split permanent earnings shared by brothers into that due to father's permanent earnings and other factors independent of father's earnings. The idiosyncratic component (ω_{ijt}) is a RW-RIP process capturing persistent individual-specific deviations from the family effect. We also introduce time effects through period-specific loading factors π_t, following Moffitt and Gottschalk (1995), in order to avoid life-cycle variation being confounded by secular trends of earnings inequality.

Fathers' earnings need to be modelled jointly with sons' earnings in order to identify an intergenerational component within the overall sibling correlation. We specify a model for fathers' earnings similar to that of sons, with the exception of residual sibling effects that are shared by siblings only and do not feature in fathers' earnings. The model for fathers' earnings is:

$$y_{ijt} = \left(\mu_j^I + \gamma_j^I A_{it} + \omega_{ijt} \right) \pi_t. \tag{9}$$

As we will show in Section VII, allowing for family-specific intergenerational factors amounts to allowing for the IGE to be heterogeneous between families and possibly correlated with the level of paternal earnings, introducing a key difference between our model and the model of sibling and intergenerational correlations of Section II.

Each individual- or family-specific parameter of the model is drawn from a zero mean unspecified distribution. RG-HIP intercepts and slopes are correlated within each dimension of family-specific heterogeneity (intergenerational and residual siblings) and are assumed to be independent between dimensions. RW-RIP parameters of idiosyncratic components are drawn from member-specific distributions. In sum, the distribution of permanent earnings is specified as follows:

$$\begin{gathered}
\left(\omega_{ijt(A_0)}, \phi_{ijt}\right) \sim \left(0,0; \sigma^2_{\omega_0 h}, \sigma^2_{\phi h}\right), \qquad h = F, S1, S2 \\
\left(\mu_j^I, \gamma_j^I\right) \sim \left(0,0;\ \sigma^2_{\mu I}, \sigma^2_{\gamma I}, \sigma_{\mu\gamma I}\right) \\
\left(\mu_j^R, \gamma_j^R\right) \sim \left(0,0;\ \sigma^2_{\mu R}, \sigma^2_{\gamma R}, \sigma_{\mu\gamma R}\right).
\end{gathered} \tag{10}$$

where $h = h(i)$ denotes the member type of individual i within the family.

Having specified a model with age related growth and idiosyncratic, intergenerational and residual sibling sources of heterogeneity in permanent earnings, we show in the Appendix how we can use parameter estimates to derive a linear additive decomposition of the overall sibling correlation (r_S) into intergenerational (r_I) and residual sibling (r_R) components over the life-cycle:

$$r_S(A) = r_I(A) + r_R(A). \tag{11}$$

This decomposition can be compared with the ones used in previous research, in particular equation (4') that is not based on the assumption of stationarity of the earnings distribution of fathers and sons. There are two main differences between the decompositions of equations (4') and (11). First, our decomposition in equation (11) is obtained from a model of life-cycle earnings and therefore it varies over the life-cycle, while there is no life-cycle variation in equation (4'). Second, in our decomposition the intergenerational parameter enters linearly, while in equation (4') the intergenerational term enters *quadratically*. The reason why Equation (4') uses the squared IGC is because it derives from a model with homogeneous IGE between families; the IGE turns out to be a multiplicative constant in the computation of the sibling covariance and is squared in the process, resulting in a squared IGC when stationarity is not assumed. In Section VII we show that introducing IGE heterogeneity within the framework used by previous research results in an extended decomposition formula that includes an additional positive term related to paternal earnings.[15]

B. *Transitory Earnings*

Studies of individual earnings dynamics use low order ARMA processes to model transitory shocks. In contrast, intergenerational or sibling studies take multi-period averages to smooth out earnings shocks and reduce measurement error biases, choosing the number of periods on the basis of the assumed degree of serial correlation. One exception is Björklund et al. (2009) who explicitly model correlated shocks as stationary AR(1) processes concentrating on the 30 – 40 age range, assuming shocks are uncorrelated between siblings.

In this paper we specify transitory earnings as member-specific AR(1) processes. We allow for age-related heteroskedasticity in the innovations of the process by using an exponential spline. Baker and Solon (2003) find that the dispersion of transitory shocks is U-shaped in age. Age-related heteroskedasticity in our model might otherwise be spuriously attributed to permanent earnings. We

[15] Another difference between approaches is that, as shown in the Appendix, the intergenerational component on the right hand side of (11) (r_I) is the ratio between the intergenerational earnings covariance and the variance sons' permanent earnings; instead equation (4') uses the IGC, that is equal to r_I divided by the ratio between the standard deviations of fathers' and sons' permanent earnings. In general this ratio will be larger than one if fathers are sampled at older ages than sons. Using life-cycle averages to measure permanent earnings, in our data the ratio is equal to 1.2, and in practice which of the two intergenerational parameters is used for the decomposition does not substantively affect the conclusions.

also allow for contemporaneous correlation of transitory shocks between family members. Our transitory earnings model is as follows:

$$v_{ijt} = \tau_t u_{ijt} = \left(\rho_h u_{ijt-1} + \varepsilon_{ijt}\right)\tau_t, \quad \varepsilon_{ijt} \sim (0, \sigma^2_{\varepsilon hA}), \sigma^2_{\varepsilon hA} = \sigma^2_{\varepsilon h} \exp\left(g_h(A_{it})\right)$$
$$u_{ijs} \sim \left(0, \eta_c^{d(s=t_0)} \sigma^2_{u_0 h}\right), \qquad s = \max(t_0, c + A_0), \tag{12}$$
$$E\left(\varepsilon_{ijt}\varepsilon_{kjt}\right) = \sigma_{hl}, \; h, l = F, S1, S2, \quad h \neq l$$

where $c = c(i)$ denotes the birth cohort of person i and t_0 the first year of data, so that s is the first year in which individuals from a given cohort are observed. We allow for non-stationarity by modeling the initial condition of the transitory process ($\sigma^2_{u_0 h}$) and introduce cohort effects in initial conditions (η_c) for cohorts starting their working life prior to the initial observation year, *d*( ) is a binary indicator function. τ_t is a period specific loading factor and g_h( ) a member-specific linear spline. Each family member draws transitory shocks from a member-specific distribution indexed by h, and shocks are (contemporaneously) correlated between members. Our model includes parameters capturing intergenerational and sibling correlations of transitory earnings shocks which have both been assumed away in previous studies. If, for example, transitory shocks are positively correlated between persons, then the between-member correlation of current earnings yields an upward biased estimate of correlations in permanent earnings.

C. *Estimation*

The model fully specifies the inter-temporal distribution of permanent and transitory earnings for each family member and between members. The second moments of this distribution are a non-linear function of a parameter vector θ that contains RW-RIP, RG-HIP and AR(1) coefficients, plus period factor loadings on permanent and transitory earnings. Details of moment restrictions are provided in the Appendix. We estimate θ by Minimum Distance (see Chamberlain, 1984; Haider, 2001).[16] In order to identify age effects separately from time effects, we derive empirical earnings moments specific to birth cohorts and stack them in a single moment vector for estimation. The model is estimated using age in deviations from 25.

[16]We use Equally Weighted Minimum Distance (EWMD) and a robust variance estimator Var(θ)=$(G'G)^{-1}G'VG(G'G)^{-1}$, where V is the fourth moments matrix and G is the gradient matrix evaluated at the solution of the minimisation problem.

V. Results

We begin the discussion of results by focusing on estimates of parameters for the permanent and transitory components, which are reported in Tables 2 and 3; estimates of period factor loadings for both components (the πs and τs) are not reported and are available upon request. Parameters are estimated by imposing the moment restrictions implied by the model on empirical second moments of earnings, after excluding moments based on fewer than 100 observations. We base the analysis on 7,140 within-person moments (of which 3,609 refer to fathers, 1,809 to first sons and 1,344 to second sons), 20,419 father/first-son moments, 17,085 father/second-son moments and 11,475 brother/brother moments. There are 55,741 empirical moments in total that are used in estimation, which we fit with 115 parameters.

A. *Permanent Earnings*

Results for the RG-HIP/RW-RIP model are reported in Table 2. The table distinguishes parameters of the distribution of the (member-specific) idiosyncratic components (in Panel A) from those of the shared components (intergenerational and residual sibling, Panels B.1 and B.2).

There are differences in idiosyncratic parameters between fathers and sons, demonstrating the importance of allowing for member-specific distributions of idiosyncratic effects. These differences are especially evident in the variance of RW-RIP innovations, which is the parameter driving the evolution of idiosyncratic earnings dispersion. Shocks are more dispersed for sons than fathers, which might reflect life-cycle variation in the variance of shocks, as fathers in our sample are observed on average at later stages of their careers than sons.[17]

The next set of estimates refers to the parameters of shared components, see Panel B of Table 2. Heterogeneity in sibling effects—intergenerational and residual sibling—is substantial. Considering initial earnings, half of their total variance comes from sibling effects. In turn, intergenerational and residual sibling effects each account for about half of the variance within the sibling-specific distribution of initial earnings.[18] Sibling effects are also evident in the distribution of earnings growth rates, but with different patterns, with the intergenerational component showing the greater dispersion of growth rates.

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[17] In preliminary analyses we experimented with a RW-RIP specification also for the shared components, obtaining negative estimated variances of the shocks for the residual sibling component, suggesting that this specification is not identified in the data. This result might be a consequence of the U-shaped pattern of empirical brother correlations illustrated in Figure 1, which cannot be captured by a RW-RIP specification. Similarly, augmenting the RG-HIP model of the intergenerational component with a RW-RIP shock yielded an insignificant estimate of the variance for slope parameters, again indicating insufficient variation in the data.

[18] Using the average of brothers' estimated variance of initial idiosyncratic earnings, the total variance of initial earnings is given by $0.5\widehat{\sigma^2_{\omega_0 S1}} + 0.5\widehat{\sigma^2_{\omega_0 S2}} + \widehat{\sigma^2_{\mu I}} + \widehat{\sigma^2_{\mu R}} = 0.1625$ of which 0.0828 ($=\widehat{\sigma^2_{\mu I}} + \widehat{\sigma^2_{\mu R}}$) come from the factors that brothers share.

The covariance between RG-HIP intercepts and slopes for both the intergenerational and residual sibling components are negative and statistically significant. The negative signs indicate the presence of Mincerian cross-overs in the distribution of permanent earnings, with cross-over age $A*$ at 31 $(=-\widehat{\sigma_{\mu\gamma I}}/\widehat{\sigma_{\gamma I}^2}+25)$ and 39 $(=-\widehat{\sigma_{\mu\gamma R}}/\widehat{\sigma_{\gamma R}^2}+25)$ years for the intergenerational and residual sibling components, respectively. The faster compression of the intergenerational component reflects the fact that there is more heterogeneity of earnings profiles in the intergenerational component.

Insofar as Mincerian cross-overs emerge from heterogeneous investments in human capital, results suggest that the determinants of these investments are shared by siblings. In families that invest more in human capital, siblings will have lower earnings at age 25, but their earnings profiles will be steeper than those of siblings from families investing less. Mincerian cross-overs imply that the predicted variance of permanent earnings explained by earnings components shared by siblings will first decrease and then fan out over the life-cycle. It is worth stressing that we obtain this result in a model that controls for age effects in transitory earnings, ruling out "omitted variable bias" induced by shared instability while young. It is also worth noting that we perform unconstrained parameter estimation, such that heterogeneity of income profiles and Mincerian cross-overs are not imposed on the data, but may in principle be rejected. [19]

B. *Sibling Correlation in Permanent Earnings and its Components*

Figure 2 shows the life-cycle evolution of the brother correlation and its decomposition into intergenerational and residual sibling effects. This decomposition is obtained by substituting estimated parameters into equation (11) using the formulae provided in the Appendix.[20] It represents the counterpart of decompositions from previous studies on the basis of equations (4) or (4').

The overall brother correlation is just above 0.5 at the start of the life-cycle and depends on both intergenerational and residual sibling effects. The value of the brother correlation at age 25 is about twice that found previously for Denmark without allowing for life-cycle variation. As individuals age, overall brother correlations diminish, and become smaller than 0.3 for ages 30 and onwards. The reason for the rapid drop in brother correlations is the shrinking of the overall intra-generational earnings distribution (Mincerian cross-overs) which is driven by brother effects. After the cross-over point, the intra-generational earnings distribution starts opening up again as an effect

[19] Table 2 also reports Newey's (1985) χ^2 statistic for a test of the model against the alternative hypothesis of an unspecified covariance structure. As noted in Baker and Solon (2003, footnote 25), with many empirical moments and relatively few parameters, the test is bound to reject the maintained specification.
[20] Figure 2 is drawn using the average of brothers' idiosyncratic parameters.

of heterogeneous earnings growth, so that the overall brother correlation increases. The U-shaped profile of brother correlation in Figure 2 results from the combination of variance estimates of intercepts and slopes of RG-HIP processes and the estimated negative intercept-slope covariance. Because in estimation we do not constrain parameters for variances to be positive, we are not imposing a particular sign on the slope or curvature of the life-cycle profile of brother correlations. The average brother correlation is 0.22 (see Panel C of Table 2), and is very close to that reported by Björklund et al. (2002) for Denmark.

The most striking result from Figure 2 is the importance of intergenerational factors in shaping the overall brother correlation. Intergenerational effects explain on average 72 percent of the total brother correlation (see Panel C of Table 2). This is a larger share than previously thought, but is in line with sibling studies concluding that community factors external to the family explain a limited share of the income correlation between brothers (see e.g. Page and Solon, 2003a, 2003b). While life-cycle variation is evident in both the residual sibling and intergenerational components, it is more pronounced for the former. In contrast to the intergenerational correlations of *raw* earnings at same age which are rather flat (Figure 1) the intergenerational correlation of *permanent* earnings in Figure 2 displays a U-shaped profile consistent with Mincerian cross-overs and heterogeneity of human capital investments between families. The residual sibling component is sizeable only at the start of the life-cycle and falls to insignificance for ages 35-48 before rising slightly at age 51.[21]

C. *Transitory Earnings*

Results for the member-specific AR(1) model for the transitory part of the earnings process are reported in Table 3. The characteristics of the process governing transitory shocks are similar for the three family members. The estimate of the parameter $\sigma^2_{\varepsilon h}$ (baseline shock volatility at age 26) is larger for sons than fathers, but differences are not statistically significant. The volatility of shocks decreases while young, and only increases late in the life-cycle. Baker and Solon (2003) find

[21] In our main estimation sample fathers are born 1935-64, first sons are born 1959-85 and second sons are born 1962-85, implying that the oldest observation for older (younger) sons in the model is 54 (51), because the data end in 2014 and we model age imputing each 3-year birth cohort its central age. One issue is that if we could observe the entire life-cycle profile, rising residual sibling correlation after age 51 would reduce the average relevance of intergenerational effects within the sibling correlation, and consequently we would exaggerate the importance of intergenerational transmission by not observing sons when they are older. To address these concerns, we extended the sample to include fathers born 1930-34. For these additional early cohorts of fathers we do not know complete parentage, and the first observed son might well not be fathers' first son. With this extension, we could add one additional cohort group of sons, such that the oldest observation for older (younger) sons becomes 57 (54). Using the extended sample we find that the brother correlation is 0.21, of which 77 percent is accounted for by intergenerational effects. Considering that early pension benefits were available from age 60 during our sample period, and that we are sufficiently close to observing the full working life, we are confident that our assessment of the intergenerational share within the brother correlation will not substantively change by further extending the data.

similar age-related heteroskedasticity. Increasing shock volatility is only evident for fathers and older sons, the only family members observed ages 52-60.

Autoregressive coefficient estimates are of moderate size comparable to those reported in other studies (e.g. in Baker and Solon, 2003) and tend to be rather stable between family members. However, the variance of the AR(1) initial conditions ($\sigma^2_{u_0 h}$) is larger for sons because we do not allow for cohort-specific shifters of the initial conditions on uncensored cohorts, and all birth cohorts of sons are first observed at age 25. Therefore, for sons the variance of initial conditions also reflects heterogeneity between cohorts.

Transitory shocks are positively correlated between family members, especially between brothers. For brothers, the correlation coefficient at age 26 implied by the estimates is 0.031 ($= \frac{\widehat{\sigma_{S1S2}}}{\widehat{\sigma_{\varepsilon S1}}\widehat{\sigma_{\varepsilon S2}}}$), which is about 7 percent of the overall sibling correlation of permanent earnings at that age. While quantitatively small, the existence of a significant correlation questions the ubiquitous assumption in sibling studies that transitory shocks are uncorrelated between siblings. There is a stronger covariance of earnings between fathers and second sons than between fathers and first sons, but the difference is insignificant.

VI. Sensitivity Analysis

Unlike previous studies, we find that most of the brother correlation in long-term earnings is accounted for by correlation with fathers' earnings. Our model is fairly complex because of considerations from the individual earnings dynamics literature, and as a consequence lacks transparency. In this section we estimate simpler versions of the model to establish the extent to which our distinctive findings are driven by modeling choices. First, we estimate nested versions of the model; we focus on life-cycle effects, transitory correlations and calendar time effects. Second, we abstract from the life-cycle aspects of the model altogether and consider time-invariant outcomes using the simpler sibling correlation model common in the literature described in equations (1) and (2).

Table 4 reports the results of these sensitivity checks. In Panel A.1 we start with a model that ignores life-cycle variation in permanent and transitory earnings. We estimate the brother correlation to be 0.25 and attribute 70 percent to the intergenerational component, which compares to 0.22 and 72 percent in the more comprehensive model with life-cycle variation. In Panel A.2 we present results from a model that excludes between- and within-individual correlation in transitory earnings and specifies these as a simple white noise process without life-cycle heteroskedasticity. We estimate the brother correlation to be 0.27 and attribute 54 percent to the intergenerational component. This downward bias of the intergenerational share when ignoring serial correlation and life-cycle heteroskedasticity of transitory shocks is consistent with the low raw same-age intergenerational correlations early in the life-cycle presented in Figure 1, which we argued could reflect larger transitory shocks at young ages exacerbating life-cycle biases. In Panel A.3 we combine the restrictions of Panels A.1 and A.2; and in Panel A.4 we further restrict to excluding calendar time shifters. Brother correlations are estimated to be 0.21 and we attribute 65-68 percent to the intergenerational component.

From Table 4 Panel A it is clear that ignoring age effects leads to underestimation of the intergenerational effect, which is the less age-sensitive component. The decomposition formula used in previous studies was derived under the assumption of life-cycle invariance of earnings and those studies all concluded that intergenerational effects are not the main driver of sibling correlations. However, comparing the 65 percent intergenerational share of overall brother effects estimated in our most restricive model with the 6 percent implied by the estimates reported by Björklund and Jännti (2009) for Denmark, suggests that imposing constancy alone is not enough to replicate the finding from previous studies that intergenerational effects are negligible.

To shed further light on whether our modeling approach is responsible for the large intergenerational share, we now move away from life-cycle earnings and consider time-invariant outcomes. We apply our research design based on family triads to the variance components model that was employed by previous studies. Because we consider time-invariant outcomes there are no transitory components and therefore $w_{ijt} = y_{ij} \; \forall \; t$, so that equation (1) becomes:

$$y_{ij} = a_{ij} + f_j, \tag{13}$$

where y_{ij} now refers to any time-invariant outcome for individual i of family j. For sons, the shared component is the sum of the intergenerational and residual sibling components:

$$f_j = \mu_j^I + \mu_j^R,\; \mu_j^I \sim \left(0, \sigma_{\mu I}^2\right),\; \mu_j^R \sim \left(0, \sigma_{\mu R}^2\right), \tag{14}$$

while for fathers there is no residual sibling component and $f_j = \mu_j^I$. Given these assumptions, the share of the sibling correlation that is accounted for by intergenerational factors is $\frac{\sigma_{\mu I}^2}{\sigma_{\mu I}^2 + \sigma_{\mu R}^2}$ and the variance components are identified by sibling ($\sigma_{\mu I}^2 + \sigma_{\mu R}^2$) or intergenerational ($\sigma_{\mu I}^2$) covariances in outcomes.

We consider four time-invariant outcomes: permanent earnings (the life-cycle average of log earnings residuals), permanent earnings between ages 30 and 40, the log of the present discounted value of life-cycle earnings, and years of completed schooling.[22] For permanent earnings, we estimate a brother correlation of 0.17 and an intergenerational share of 66 percent, which is close to the estimate in Panel A.4. When we calculate permanent earnings over the 30-40 age range (Panel B.2) our findings are in line with Figure 2: the brother correlation is lower in that range compared to the full life-cycle, and the intergenerational share is larger. Also for the present discounted value of earnings and years of completed schooling we find that a similarly large percentage of the brother correlation is accounted for by the intergenerational component.

In this section we have conducted sensitivity analyses by simplifying our main model to consider nested models used in some previous studies, and by replacing annual earnings with time-invariant outcomes. Compared to our baseline finding that 72 percent of brother earnings correlation is due to intergenerational transmission, ignoring earnings dynamics gives 65 percent and simply comparing permanent incomes gives 66 percent. Specification of the earnings process in our main model is not driving the finding that the sibling correlation is largely due to intergenerational persistence.

VII. Heterogeneous Intergenerational Transmission

In this Section we reconcile our findings with those of previous studies by considering the importance of heterogeneity of intergenerational transmission between families. The previous Section shows that our specification of earnings dynamics only partially accounts for the fact that we obtain a predominant intergenerational share in the sibling correlation. Even moving away from earnings dynamics and considering time-invariant outcomes within our unified framework we obtain similar conclusions. We now aim to understand whether our distinctive findings are specific to the data we use or depend on differences in assumptions between our model and those of other studies.

We begin by performing the decompositions of equation (4) and (4'). The calibration factors are the sibling correlation of permanent earnings and the IGE or the IGC (for the non-stationary case). As in Section VI, we define permanent earnings as the life-cycle average of log earnings residuals, and we take deviations from generational means. Following Mazumder (2008), we estimate the variance components ($var(a)$ and $var(f)$) using a mixed model and a REML estimator, while we estimate the IGE (from which we also derive the IGC) using the canonical

[22] We discount earnings with a 3 percent interest rate and divide the present discounted value by the number of individual observations to account for the unbalanced design of the earnings panel. Results are very similar when using 1 or 5 percent interest rates.

intergenerational regression of log sons' earnings on log fathers' earnings. Results are reported in Panel A.1 of Table 5. We estimate the brother correlation to be 0.17, the IGE to be 0.08 and the IGC to be 0.09.[23] Using equation (4), these estimates imply that the share of the brother correlation that is associated with fathers' earnings is 4 percent; using equation (4') to relax the assumption of stationarity, the share becomes 5 percent.[24] Panel A.2 in Table 5 reports results from the sequential conditioning approach of Mazumder (2008). Again, the share of the correlation that can be ascribed to fathers' earnings is 4 percent.

Applying the approaches of previous studies on our data yields a share of brother correlation explained by fathers' earnings that is low and in line with the findings of those studies, which rules out data differences as the driver for differences in results. It must therefore be the set of assumptions behind the approaches that makes the difference. One key restriction of previous studies that we relax is that the IGE is constant between families. Specifically, while previous studies assume that intergenerational transmission occurs through a parameter (the IGE) which is common to all families, our model allows earnings components to be shared by fathers and sons in an unrestricted way, which is equivalent to having a family-specific IGE. To see this, compare equation (3) with its analogue in the time-invariant version of our approach, equation (14). While equation (3) sets $\mu_j^I = \beta y_j^F$, thus assuming that intergenerational transmission occurs through an IGE that is constant between families, in equation (14) we leave the intergenerational factor μ_j^I unrestricted, effectively allowing the degree of intergenerational transmission to differ between families.

To understand the role that IGE heterogeneity plays in our finding a predominant intergenerational share of the sibling correlation, we allow for IGE heterogeneity in equation (3) by means of a random coefficient specification within the mixed model, such that the intergenerational component becomes:

$$\mu_j^I = \left(\bar{\beta} + \beta_j\right) y_j^F;\ \beta_j \sim \mathcal{N}(0, var(\beta)). \tag{15}$$

[23] Using the same data source as us, Hussain et al. (2009) report a set of IGE estimates for Denmark showing sensitivity to sample selection and the age range when sons are observed. They report an IGE of 0.07 for sons taken in the same age range as in our sample, 25-51. The figure reported by Björklund and Jännti (2009; IGE=0.12) is cited from Hussain et al. (2009), who obtain it when using brothers data in the 30-40 age range. Using data in that same 30-40 range we obtain an IGE of 0.12.

[24] Replacing the IGC with the corresponding intergenerational component derived from our model (r_I=0.1143) yields an intergenerational share of 7.5 percent. Applying the decomposition formula of previous studies, we obtain a similarly low share (4 percent using the IGE and 7 percent using the IGC) when we use the log present discounted value of earnings in place of permanent earnings. Using years of education instead yields a larger share of the brother correlation that is ascribed to intergenerational transmission (20 percent using the IGE and 22 percent using the IGC), which is still much lower than the intergenerational share for the brother correlation in education that we obtain using the simplified version of our model in Section VI (73 percent).

The term on the right hand side introduces heterogeneity of intergenerational persistence between families, where $\bar{\beta}$ is the population average IGE and β_j is the family-specific deviation from it, assumed normally distributed and independent of y_j^F in order to apply the REML estimator of the mixed model. Because permanent earnings are in deviations from generational means, combining equations (13)—(15) and the normality assumptions of the REML estimator, sons' permanent earnings can be written as:

$$y_{ij} = a_{ij} + \left(\bar{\beta} + \beta_j\right) y_j^F + \mu_j^R,\ a_{ij} \sim \mathcal{N}(0, var(a)),\ \mu_j^R \sim \mathcal{N}\left(0, \sigma_{\mu R}^2\right) \tag{16}$$

which yields the following decomposition of the sibling correlation:

$$r_S = \frac{\left(var(\beta) + \bar{\beta}^2\right) var(y_j^F)}{var\left(y_{ij}\right)} + r_R \tag{17}$$

where $var\left(y_{ij}\right)$ is the variance of permanent earnings in the sons' generation. The first term on the right hand side captures the component of the sibling correlation that is due to paternal earnings, and the second term is the residual sibling correlation. By following Corcoran et al. (1990) and Solon (1999) in assuming a stationary distribution of permanent earnings across generations, equation (17) becomes:

$$r_S = var(\beta) + \bar{\beta}^2 + r_R \tag{18}$$

This is the counterpart of the decomposition of equation (4) in the case of heterogeneous IGE. It is clear from the decomposition formula (18) that allowing for heterogeneous IGE adds a positive component—the variance of the IGE—to the computation of intergenerational effects within the sibling correlation.

We report results obtained from the mixed model with heterogeneous IGE in Panel B.1 of Table 5. If we assume stationarity of the earnings distribution across generations (decomposition equation 18) we find fathers' earnings accounts for 18 percent of brother correlations. Fathers' earnings account for 26 percent if we do not assume stationarity (decomposition equation 17). Thus, extending the modeling approach of previous studies, by allowing for heterogeneous rather than homogenous IGEs, increases the share of brother correlation in earnings due to fathers' earnings from 4-5 percent to 18-26 percent.

The estimated intergenerational share of 26 percent is still far from the 66 percent that we have estimated in Table 4, Panel B.1, by applying our unified framework of intergenerational and sibling correlations to a simple measure of permanent earnings. The remaining discrepancy can be explained by the two distributional assumptions made when moving from the model in equations (13)-(14) to the model of equations (15)-(16): normality and independence between the family specific IGE and fathers' earnings. Both assumptions are needed by the REML estimator of the model with IGE heterogeneity, but not by the Minimum Distance estimator of variance components, and both assumptions might be rejected by the data. Regarding independence, Björklund et al. (2012) show that intergenerational transmission is stronger at the top of the earnings distribution, while Corak and Piraino (2011) report that the probability of a father and son working in the same firm is the highest at the top of the fathers' earnings distribution. Both findings are consistent with a positive covariance between father's earnings and intergenerational transmission, implying that equations (17) or (18) still provide a lower bound for the overall effect of intergenerational persistence within the brother correlation.

The factorization of the intergenerational correlation in equation (17) is obtained using the independence assumption. As a way to measure the impact of the assumption, we estimate the variance of the intergenerational factor μ_j^I without distinguishing the IGEs from father's earnings; that is, we estimate the overall variance $\sigma_{\mu I}^2 = var\left[\left(\bar{\beta} + \beta_j\right) y_j^F\right]$ without further factorizations, which amounts to estimating our unified model under normality. We retrieve the relevant parameters from the covariance matrix of the errors in a seemingly unrelated system of three normal linear equations for the permanent earnings of fathers, first sons and second sons. Results from this exercise are in Table 5, Panel B.2, with a brother correlation of 0.17 and an intergenerational share of 60 percent. Comparing this estimated intergenerational share with the analogous share estimated under the assumption of independence (26 percent in Panel B.1 of Table 5), suggests a positive association between the level of father's earnings and the extent of intergenerational transmission.

The remaining discrepancy between intergenerational shares of Table 5 Panel B.2 and Table 4 Panel B.1 can be attributed to the normality assumption. The null hypothesis of normality is overwhelmingly rejected; the $\chi^2(6)$ test statistic for the hypothesis of joint normality of permanent earnings of the three family members is 202,449. When we consider the null hypothesis of bivariate normality, the $\chi^2(4)$ test statistic is 100,513 for the brother-brother pair and 150,662 (147,069) for the father-first (second) son pair. These test statistics show that the normality assumption is more strongly rejected for intergenerational associations than for brother associations, and there are differential deviations from normality in the distribution of permanent earnings for fathers and sons, hence reducing the share of brother correlation accounted for by intergenerational persistence when normality is assumed.

In this section and the preceding section we have reconciled the difference between our estimates of the share of sibling earnings correlation accounted for by intergenerational earnings transmission and those found in other studies. On our data we estimate a model common in the literature using permanent earnings and find an intergenerational share of 4-5 percent, which is close enough to other estimates for Denmark to rule out data differences as being relevant. We show that allowing heterogeneous IGE explains most of the difference between our estimates and those from other studies.

VIII. Intergenerational Transmission and Birth Order

In the previous section we showed that allowing for heterogeneous IGE *between* families is a key feature of our model. In this section we extend our model to allow for heterogeneous IGE *within* families by relaxing the assumption that sons' intergenerational parameters are the same. Differential transmission is relevant for two reasons. First, differential treatment of children by parents is a concern for sibling studies which aim to quantify the importance of family background, because shared family influences would provide only a lower bound for the overall family effect within the distribution of outcomes in the child generation. Second, first born children have on average better outcomes than later born siblings; a difference which is often interpreted as parents investing more in first born children (Black et al., 2005).[25] Whether the intergenerational persistence of outcomes (rather than outcome levels *per se*) varies with birth order is a different and related question which has received only little attention (Behrman and Taubman, 1986), and we consider whether intergenerational persistence in earnings varies between sons within families. Birth order effects on outcome levels may induce effects also on intergenerational persistence. For example, if differential investment by birth order is more common among poor families than among affluent ones because of limited resources, then intergenerational transmission to later born children should be the strongest.[26]

In order to make our results more informative regarding birth order effects, we restrict the analysis to families where the older son is the first born child and drop families where the first born child is a daughter, reducing the sample to 91,301 families. We model differential intergenerational transmission by allowing intergenerational components in the baseline model to enter into the

[25] Björklund and Jännti (2012) show that residualization of earnings on birth order has little impact on sibling correlations estimated with mixed models.

[26] We thank Gary Solon for suggesting this mechanism to us.

second sons' earnings process through specific factor loadings on the intercept and slope of the RG-HIP process, δ_μ and δ_γ, such that the equation for second son's permanent earnings becomes:

$$y_{ijt} = \left(\left(\delta_\mu \mu_j^I + \mu_j^R \right) + \left(\delta_\gamma \gamma_j^I + \gamma_j^R \right) A_{it} + \omega_{ijt} \right) \pi_t. \tag{19}$$

The hypothesis of equal treatment corresponds to factor loadings of unity. Estimates of factor loadings which are larger (smaller) than unity mean that intergenerational transmission matters more (less) for second sons compared with first sons.

Parameter estimates of permanent earnings for the model with differential intergenerational transmission between sons are reported in Table 6. We find some difference in intergenerational persistence of permanent earnings between sons. Specifically, the difference is more pronounced for intergenerational effects in initial earnings towards second sons, but we cannot reject the hypothesis of equal treatment, the estimates of factor loadings being not statistically different from unity ($\widehat{\delta_\mu}$=1.31, s.e.=0.16; $\widehat{\delta_\gamma}$=0.89, s.e.=0.06). Figure 4 plots predicted intergenerational components of the brother correlation for the two sons. While not statistically different from one another, at age 25 for second sons' intergenerational correlations are larger than for first sons, but the difference shrinks as age increases.

Differences in intergenerational persistence between sons, concentrated at young ages to the benefit of second sons, may reflect parents' greater economic stability or longer experience of parenting. Alternatively, as discussed at the beginning of this section, stronger transmission to second sons is consistent with differential investments in favor of first sons being more common among families in the lower part of the paternal earnings distribution. Poor families may concentrate investments on first sons (consistent with findings from birth order studies) which exhaust the resources available to second sons. This will make it more likely that second sons begin working for low earnings, thus resembling their fathers' earnings position more closely than their brothers do.

With these differential transmission estimates we can now compute for each brother the share of permanent earnings inequality that is due to family and community effects. These shares are *counterfactual* brother correlations, measuring what the brother correlation would be if each son experienced the degree of intergenerational transmission that characterizes either first or second son. The counterfactual correlation is lower for first sons reflecting lower intergenerational persistence. However, differences between these counterfactual correlations and the brother correlations of the baseline model are not significant, showing that heterogeneous intergenerational transmission within families does not limit the ability of brother correlations to encompass father-son correlations.

IX. Conclusion

Family background has important effects on child outcomes in later life through parental and community influences. We show that fathers' lifetime earnings account for 72 percent of the correlation in brothers' lifetime earnings. This share is much greater than has been found in previous research that uses indirect decomposition methods and does not allow for intergenerational transmission to be heterogeneous between families. We also find large life-cycle effects in brother correlations with a U-shape between ages 25 and 51. This variation has been masked in previous studies which estimate averages over age.

Our findings relate to the large part of brothers earnings similarity which is driven by father to son transmission of earnings. This does not leave much of a role for school or neighbourhood factors that are *uncorrelated with fathers' earnings* to affect sons' earnings. But many known determinants of child outcomes are correlated with fathers' earnings, for example household income (Dahl and Lochner, 2012), school quality (Chetty et al., 2011) and neighbourhoods (Chetty et al., 2016). Nevertheless, showing that factors unrelated to fathers earnings only explain 28 percent of brothers shared earnings, leaves brothers' lifetime earnings much less affected than previously thought by common influences orthogonal to fathers' earnings.

We have established that paternal earnings are important in determining brother correlations and the same approach could be extended to further decompose the two components of sibling correlations identified in this paper. Within the family the relative importance of endowments versus parenting, and in the community the relative importance of schools versus neighbourhoods could be established. Assessing the importance of these factors is not new to the literature, but applying multi-person models of earnings dynamics with heterogeneous intergenerational transmission similar to the ones of this paper may provide new insights on these long-standing issues.

Appendix

Minimum Distance estimation of the model is based on imposing the moment restrictions implied by the model on empirical second moments estimated from the data.

Moment restrictions for the model of permanent earnings are provided below.

Father's covariance structure

$$E(y_{ijt}y_{ijq}) = (\sigma^2_{\mu I} + \sigma^2_{\gamma I}A_{it}A_{iq} + \sigma_{\mu\gamma I}(A_{it}+A_{iq}) + \sigma^2_{\omega_0 F} + \sigma^2_{\phi F}A_{it})\pi_t\pi_q \ ; \quad t \le q. \tag{A.1}$$

Son's covariance structure

$$E(y_{ijt}y_{ijq}) = \Big((\sigma^2_{\mu I} + \sigma^2_{\mu R}) + (\sigma^2_{\gamma I} + \sigma^2_{\gamma R})A_{it}A_{iq} + (\sigma_{\mu\gamma I} + \sigma_{\mu\gamma R})(A_{it}+A_{iq}) + \sigma^2_{\omega_0 Sb} + \sigma^2_{\phi Sb}A_{it}\Big)\pi_t\pi_q;\ t \le q,\ b = 1,2. \tag{A.2}$$

Father-son covariance structure

$$E(y_{ijt}y_{kjq}) = (\sigma^2_{\mu I} + \ \sigma^2_{\gamma I}A_{it}A_{kq} + \ \sigma_{\mu\gamma I}(A_{it}+A_{kq}))\pi_t\pi_q. \tag{A.3}$$

Son-son covariance structure

$$E(y_{ijt}y_{kjq}) = \Big((\sigma^2_{\mu I} + \sigma^2_{\mu R}) + (\sigma^2_{\gamma I} + \sigma^2_{\gamma R})A_{it}A_{kq} + (\sigma_{\mu\gamma I} + \sigma_{\mu\gamma R})(A_{it}+A_{kq})\Big)\pi_t\pi_q. \tag{A.4}$$

RG-HIP and RW-RIP parameters are identified by variation in age (in deviation from age 25). RW-RIP parameters of idiosyncratic components are identified by individual earnings moments. Intergenerational RG-HIP parameters are identified by Father-son earnings moments. RG-HIP sibling parameters are identified by son-son earnings moments, so that the difference between sibling moments and intergenerational moments identifies residual sibling parameters. Using parameter estimates we can decompose the total sibling correlation of permanent earnings into its intergenerational and residual sibling components over the life-cycle, obtaining equation (11) of the main text:

$$r_S(A) = r_I(A) + r_R(A). \tag{A.5}$$

where

$$r_I(A) = \frac{\sigma_{\mu I}^2 + \sigma_{\gamma I}^2 A^2 + 2\sigma_{\mu\gamma I} A}{var\big(y(A)\big)},$$

$$r_R(A) = \frac{\sigma_{\mu R}^2 + \sigma_{\gamma R}^2 A^2 + 2\sigma_{\mu\gamma R} A}{var\big(y(A)\big)}$$

and

$$var\big(y(A)\big) = \Big(\big(\sigma_{\mu I}^2 + \sigma_{\mu R}^2\big) + \big(\sigma_{\gamma I}^2 + \sigma_{\gamma R}^2\big)A^2 + \big(\sigma_{\mu\gamma I}^2 + \sigma_{\mu\gamma R}^2\big)2A \; + \sigma_{\omega_0 Sb}^2 + \sigma_{\phi Sb}^2 A\Big),$$
$$b = 1,2.$$

Within person moment restrictions for the member-specific AR(1) model are as follows:

$$E\big(v_{ijt}v_{ijq}\big) = \Big(d(t = q = s)\eta_c^{d(s=t_0)}\sigma_{u_0 h}^2 + d(t = q > s)\big(\sigma_{\varepsilon hA}^2 + var\big(u_{ijt-1}\big)\rho_h^2\big) + d(t \neq q)\big(E\big(u_{ijt-1}u_{ijq}\big)\rho_h\big)\Big)\tau_t\tau_q. \quad \text{(A.6)}$$

Allowing for contemporaneous correlation of transitory shocks between different persons, the model yields restrictions on transitory earnings also for cross-member moments:

$$E\big(v_{ijt}v_{kjq}\big) = \sigma_{hl}\left(\frac{\Big(1 - \big(\rho_h\rho_l^{|t-q|}\big)^P\Big)}{1 - \rho_h\rho_l^{|t-q|}}\right)^{d(t\leq q)}\left(\frac{\Big(1 - \big(\rho_l\rho_h^{|t-q|}\big)^P\Big)}{1 - \rho_l\rho_h^{|t-q|}}\right)^{d(t>q)}, \quad \text{(A.7)}$$
$$h = F, S1, S2;\;\; l = S1, S2;\;\; h \neq l.$$

where P is the number of years the two family members are simultaneously observed in the data.

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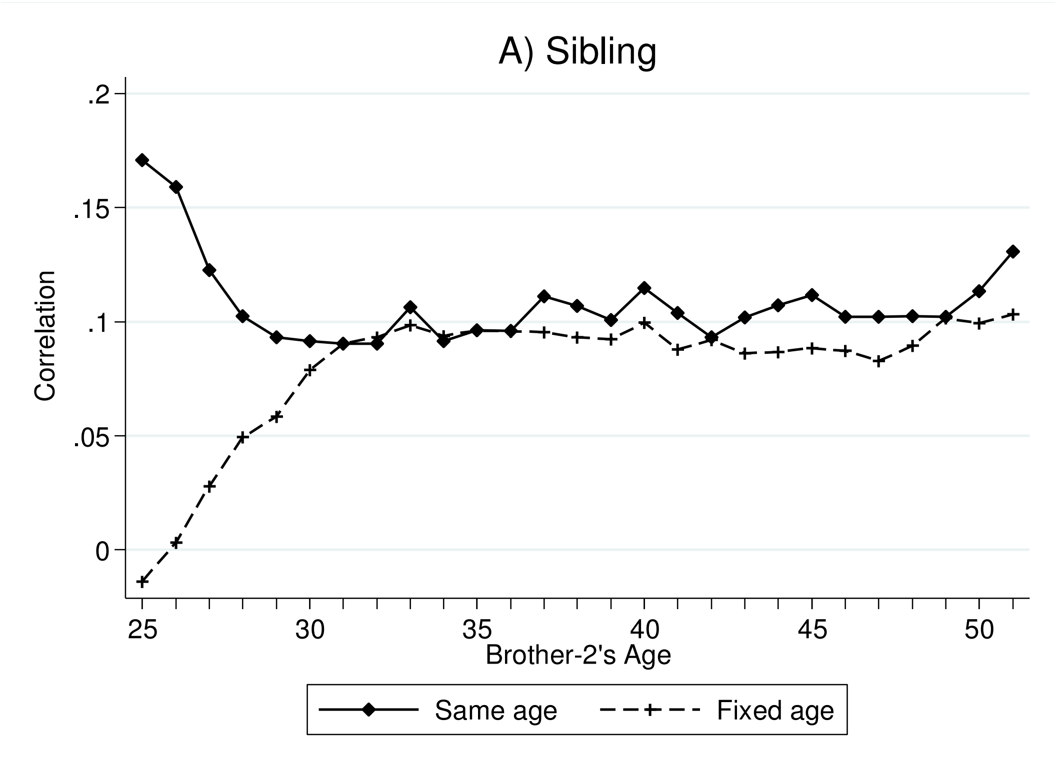
Figures and Tables

Figure 1: Intergenerational and sibling correlations of raw earnings



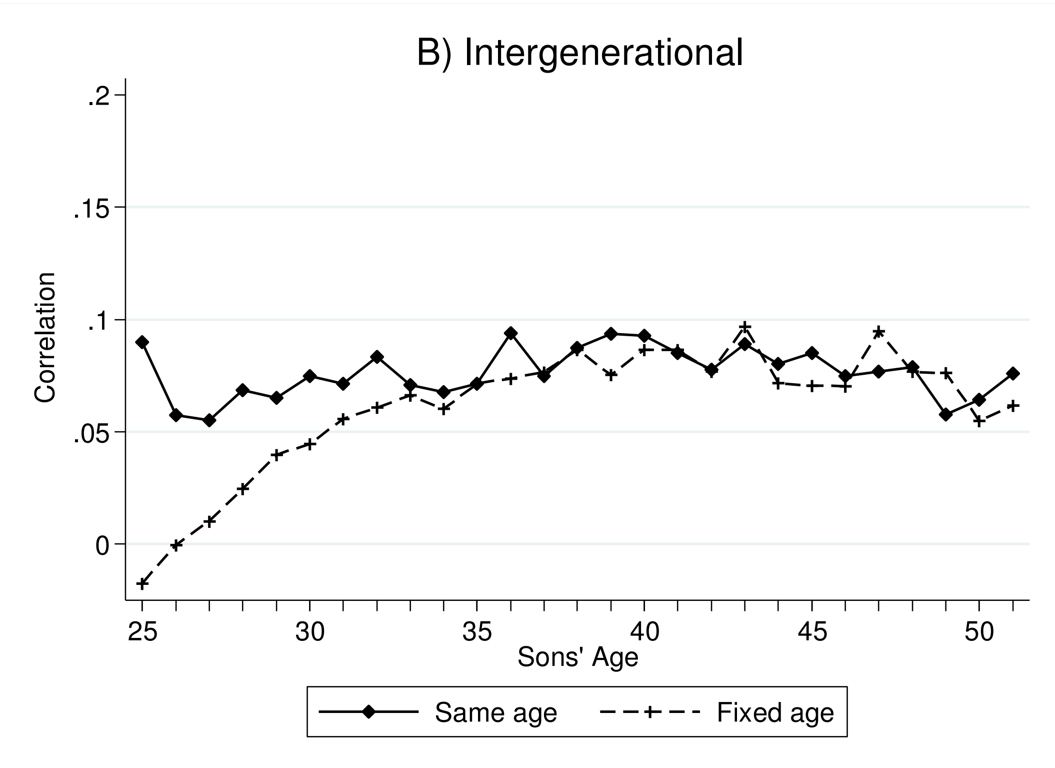


Figure 2: Predicted decomposition of sibling correlation of permanent earnings

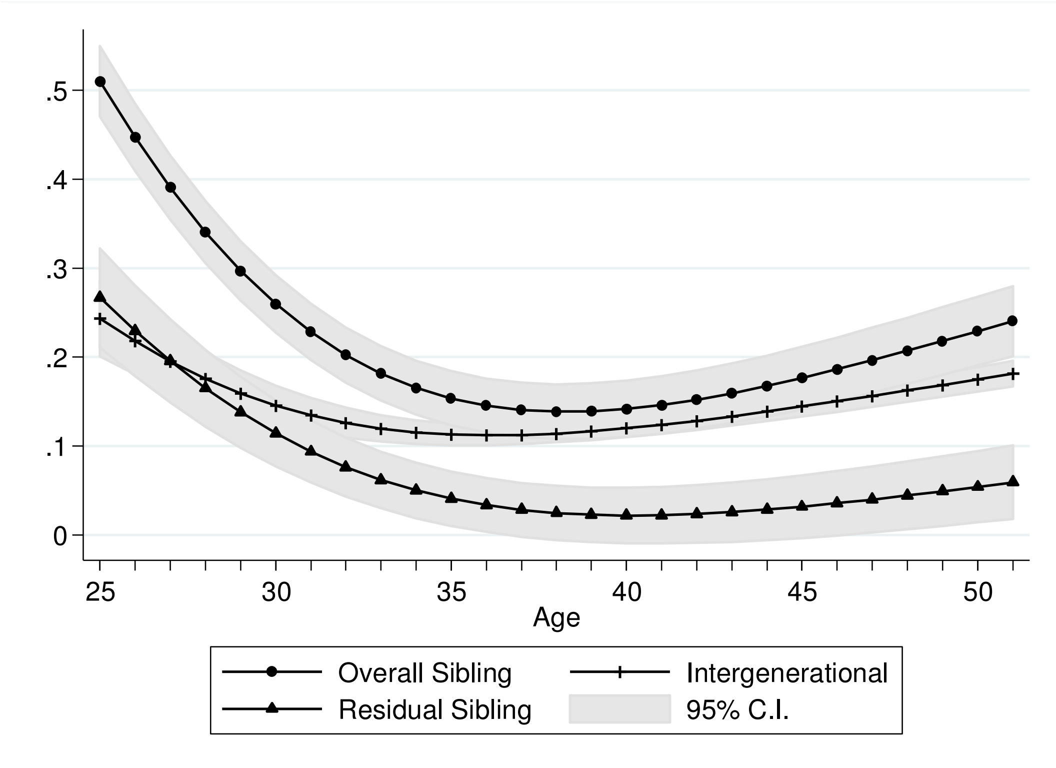


Figure 3: Predicted intergenerational components of the sibling correlation by birth order

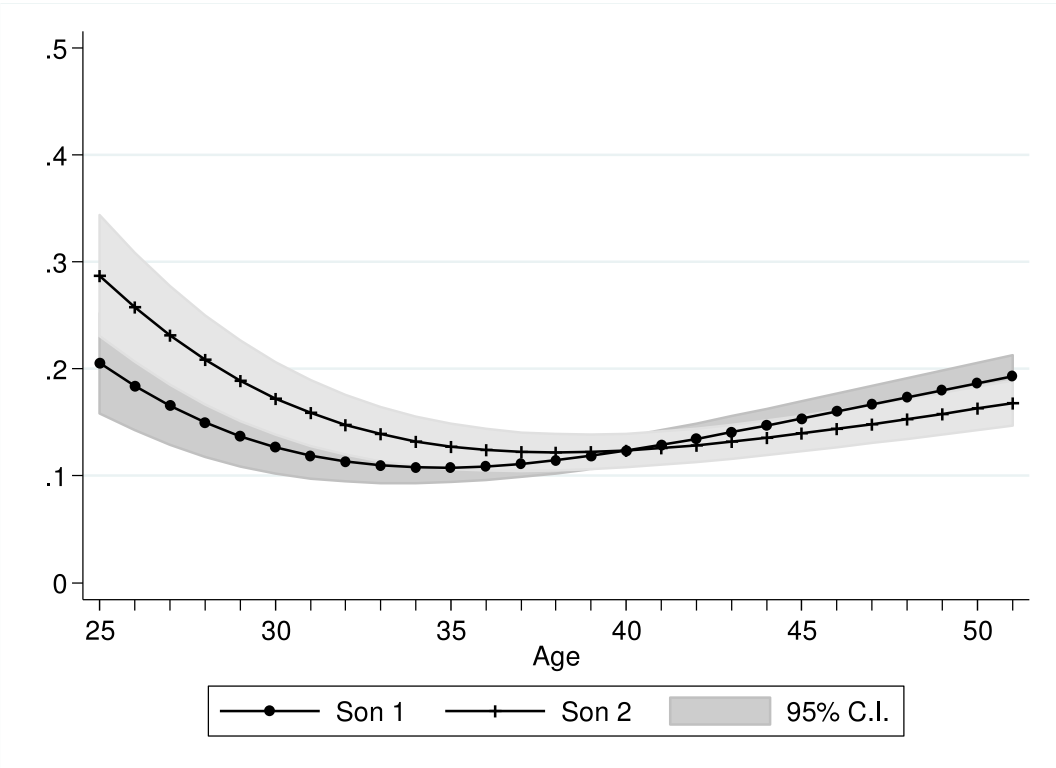


Table 1: Descriptive statistics

		(1) Sample without earnings selection			(2) Estimation sample		
		Father	Son 1	Son2	Father	Son 1	Son2
# Individuals		130179	130179	130179	101077	101077	101077
# Observations		2668550	2196964	1778117	2199508	1810396	1478542
1990	Mean Earnings	368261.83	287404.81	269512.06	367051.03	291035.22	273920.64
	SD Earnings	204049.4	136741.16	126391.34	166440.55	124137.25	118698.83
	Mean Age	45.15	27.97	27	45.42	27.96	27
1995	Mean Earnings	376523.10	302411.14	283400.41	374326.28	304465.50	286607.45
	SD Earnings	224897.48	149302.67	137780.69	181149.39	135089.14	125601.22
	Mean Age	49.93	29.54	28.20	50.19	29.52	28.19
2000	Mean Earnings	394533.93	340160.29	312005.92	390018.13	341237.57	314185.53
	SD Earnings	243685.03	192990.43	167521.24	189596.6	162519.18	146926.77
	Mean Age	52.41	31.56	29.66	52.65	31.61	29.66
2005	Mean Earnings	394381.28	386910.45	355871.94	389282.69	385764.15	356529.76
	SD Earnings	252469.38	237806.38	195480.31	189844.77	182457.51	163780.89
	Mean Age	55.71	35.39	33.18	55.91	35.46	33.17
2010	Mean Earnings	404648.26	434923.86	382454.54	397140.46	429654.23	383240.54
	SD Earnings	288662.28	325866.69	263048.95	207137.93	216977.35	194144.27
	Mean Age	56.86	39.39	35.58	56.97	39.56	35.71
2014	Mean Earnings	393356.76	459183.69	417250.16	387159.93	452759.3	418751.69
	SD Earnings	299441.99	382334.26	273269.60	195543.66	227722.10	202222.78
	Mean Age	58.71	43.37	39.55	58.77	43.58	39.75

Notes: Annual earnings are reflated to 2012 Danish Krone (1USD is worth about 7 DKK)

Table 2: Parameter estimates of permanent earnings and sibling correlation

	Coeff.	S.E.
A) Idiosyncratic component		
Variance of initial earnings		
Father ($\sigma^2_{\omega_0 F}$)	0.0452	0.0040
Son 1 ($\sigma^2_{\omega_0 S1}$)	0.0861	0.0061
Son 2 ($\sigma^2_{\omega_0 S2}$)	0.0733	0.0059
Variance of shocks		
Father ($\sigma^2_{\phi F}$)	0.0060	0.0004
Son 1 ($\sigma^2_{\phi S1}$)	0.0123	0.0009
Son 2 ($\sigma^2_{\phi S2}$)	0.0133	0.0011
B) Shared components		
B.1) Intergenerational		
Variance of initial earnings ($\sigma^2_{\mu I}$)	0.0395	0.0032
Variance of growth rates ($\sigma^2_{\gamma I}$)	0.00021	0.00001
Covariance ($\sigma_{\mu\gamma I}$)	-0.0017	0.0002
B.2) Residual Sibling		
Variance of initial earnings ($\sigma^2_{\mu R}$)	0.0433	0.0055
Variance of growth rates ($\sigma^2_{\gamma R}$)	0.00018	0.00002
Covariance ($\sigma_{\mu\gamma R}$)	-0.0026	0.0003
C) Average sibling correlation and decomposition		
Sibling correlation (r_S)	0.2205	0.0150
Intergenerational component (r_I)	0.1474	0.0064
Residual sibling component (r_R)	0.0731	0.0151
Intergenerational share (r_I/r_S)	0.7154	0.0611

Note: Reported correlations and intergenerational share are obtained as averages of their life-cycle profiles. Number of observations 5488445, number of individuals= 303231, number of moments =55741, number of parameters =115, $\chi^2(55626)$ =2.946e+08

Table 3: Estimates of member-specific AR(1) parameters of transitory earnings

	(1) Father		(2) Son 1		(3) Son 2	
	Coeff.	S.E.	Coeff.	S.E.	Coeff.	S.E.
Baseline variance ($\sigma^2_{\varepsilon h}$)	0.2397	0.0334	0.3370	0.0401	0.3253	0.0388
Age splines						
26-30	-0.0420	0.0310	-0.1504	0.0077	-0.1341	0.0059
31-35	-0.0220	0.0200	-0.0402	0.0062	-0.0649	0.0059
36-40	0.0090	0.0114	-0.0057	0.0058	-0.0013	0.0068
41-45	-0.0166	0.0143	-0.0215	0.0115	-0.0246	0.0149
46-51	-0.0213	0.0071	-0.0151	0.0134	-0.0172	0.0218
52-60	0.0669	0.0058	0.0237	0.0695		
Autocorrelation coefficient (ρ_h)	0.5251	0.0069	0.5311	0.0064	0.5429	0.0055
Baseline initial condition ($\sigma^2_{u_0 h}$)	0.3047	0.0224	0.5890	0.0695	0.5920	0.0699
Initial condition shifter for left-censored cohorts (η_c, $\eta_{1953-55}$=1)						
1935-37	0.8571	0.1108				
1938-40	1.1488	0.1259				
1941-43	1.0261	0.1077				
1944-46	0.8804	0.0885				
1947-49	0.7120	0.0758				
1950-52	0.6433	0.0712				
Between-person covariance (σ_{hl})						
Father			0.0032	0.0015	0.0034	0.0017
Son1					0.0103	0.0017

Note: See Table 2

Table 4: Sensitivity analysis

	Coeff.	S.E.
A) Nested models		
A.1: No life-cycle		
Sibling correlation	0.2453	0.0218
Intergenerational share	0.7074	0.0693
A.2: No transitory correlation		
Sibling correlation	0.2655	0.0095
Intergenerational share	0.5389	0.0273
A.3: No life-cycle, no transitory correlation		
Sibling correlation	0.2083	0.0106
Intergenerational share	0.6838	0.0424
A.4: No life-cycle, no transitory correlation, no time effects		
Sibling correlation	0.2086	0.0106
Intergenerational share	0.6532	0.0402
B) Time-invariant outcomes		
B.1: Permanent earnings		
Sibling correlation	0.1724	0.0041
Intergenerational share	0.6636	0.0221
B.2: Permanent earnings between 30 and 40		
Sibling correlation	0.1497	0.0035
Intergenerational share	0.7118	0.0305
B.3: Present discounted value of earnings		
Sibling correlation	0.1849	0.0036
Intergenerational share	0.7669	0.0247
B.4: Years of schooling		
Sibling correlation	0.3377	0.0035
Intergenerational share	0.7357	0.0112

Note: Panel A reports estimates of the sibling correlation and its intergenerational share obtained from restricted versions of our model of earnings dynamics. Panel B reports estimates of the sibling correlation and its intergenerational share for time-invariant outcomes.

Table 5: Decompositions of the sibling correlation with and without IGE heterogeneity

	Coeff.	S.E.
A) Decompositions with homogeneous IGE		
A.1: Solon (1999)'s decomposition		
IGE	0.0782	0.0023
IGC	0.0946	0.0028
Sibling correlation	0.1724	0.0030
Intergenerational share		
With stationarity	0.0355	0.0022
Without stationarity	0.0519	0.0032
A.2: Sequential conditioning		
Sibling correlation after conditioning on y_j^F	0.1649	0.0031
Intergenerational share	0.0433	0.0245
B) Decompositions with heterogeneous IGE		
B.1: With independence of β_j and y_j^F		
IGE	0.0967	0.0023
$var(IGE)$	0.0229	0.0013
Residual sibling component ($\sigma_{\mu R}^2$)	0.0366	0.0009
Sibling correlation	0.1785	0.0031
Intergenerational share		
With stationarity	0.1810	0.0078
Without stationarity	0.2626	0.0112
B.2: Without independence of β_j and y_j^F		
Sibling correlation	0.1724	0.0031
Intergenerational share	0.6037	0.0176

Note: Panel A reports estimates of the sibling correlation and its intergenerational share assuming homogeneity of intergenerational transmission between families, namely using Equation (4) or Equation (4') (Panel A.1), or sequential conditioning (Panel A.2). Panel B reports estimates of the sibling correlation and its intergenerational share without assuming homogeneity of intergenerational transmission between families, namely using Equation (18) or Equation (17) (Panel B.1), or imposing normality in our unified model of intergenerational and sibling correlations in permanent incomes (Panel B.2). The income variances of fathers and sons used to relax the stationarity assumption are estimated from the data: $var(y_j^F)$=0.4035; $var(y_{ij})$=0.2761.

Table 6: Parameter estimates of permanent earnings and sibling correlation, model with differential intergenerational transmission

	Coeff.	S.E.
A) Idiosyncratic component		
Variance of initial earnings		
Father ($\sigma^2_{\omega_0 F}$)	0.0475	0.0039
Son 1 ($\sigma^2_{\omega_0 S1}$)	0.0945	0.0074
Son 2 ($\sigma^2_{\omega_0 S2}$)	0.0594	0.0089
Variance of shocks		
Father ($\sigma^2_{\phi F}$)	0.0053	0.0004
Son 1 ($\sigma^2_{\phi S1}$)	0.0119	0.0010
Son 2 ($\sigma^2_{\phi S2}$)	0.0142	0.0011
B) Shared components		
B.1) Intergenerational		
Variance of initial earnings ($\sigma^2_{\mu I}$)	0.0341	0.0040
Brother's 2 shifter of initial earnings	1.3098	0.1623
Variance of growth rates ($\sigma^2_{\gamma I}$)	0.0002	0.00001
Brother's 2 shifter of growth rates	0.8949	0.0586
Covariance ($\sigma_{\mu\gamma I}$)	-0.0015	0.0002
B.2) Residual Sibling		
Variance of initial earnings ($\sigma^2_{\mu R}$)	0.0377	0.0068
Variance of growth rates ($\sigma^2_{\gamma R}$)	0.0002	0.00002
Covariance ($\sigma_{\mu\gamma R}$)	-0.0025	0.0003
C) Average sibling correlation and decomposition		
Sibling correlation brother 1	0.2062	0.0207
Sibling correlation brother 2	0.2220	0.0172
Intergenerational component brother 1	0.1433	0.0075
Intergenerational component brother 2	0.1578	0.0104
Intergenerational share brother 1	0.7338	0.0768
Intergenerational share brother 2	0.7558	0.0806

Note: Number of observations= 5,490,119, number of individuals=303,369, number of moments=55,732, number of parameters=117, $\chi^2(56,221) = 1.344e+08$. Correlations and the intergenerational share are obtained as averages of their respective life-cycle profiles.