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Abstract

It has been noted that the search and matching model cannot account for the observed unemployment fluctuations. Gertler and Trigari (2009) show this weakness of the model disappears when wage stickiness is introduced to the model. Pissarides (2009) disagrees with this modification, arguing that new hires' wages are not sticky. We argue that there is heterogeneity in wage setting: while some wages are sticky, the others are not. We generalise the model to account for this heterogeneity. We find that the new model with even only a small fraction of sticky wage contracts comes closer to matching the data.

JEL-Codes: E240, E320, J640.

Keywords: search and matching, heterogeneity in staggered wage bargaining, unemployment volatility puzzle.

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1. Introduction

The search and matching model proposed by Diamond (1982a,b), Mortensen (1982), and Pissarides (1985) (see also Mortensen and Pissarides (1994)) has been a standard tool in analysing unemployment dynamics. The model is rich enough to account for several important features of labour market dynamics and, yet, it is analytically tractable. In the model, in each period, a certain fraction of workers lose their job and search for a new job. Aiming to capture the difference between workers' productivity and workers' wage, firms create new jobs. The model can account for the fact that in a tight market, firms find it difficult to hire new workers. As a result, unemployed workers find it easy to find jobs. Another important feature of the model is that workers and firms bargain to split the surplus generated by the new job. The bargaining is done according to the Nash Bargaining approach.

However, Shimer (2005) criticises the model on the grounds that it cannot generate sufficient volatility in labour market variables in response to an increase in productivity, calling into question the empirical relevance of the model. Since then many papers emerged addressing the puzzle. The reason for this result is as follows. As there is nothing in the model that prevents wages to adjust fully to an increase in productivity, all the increase in productivity is absorbed by wages, leaving no incentive to firms to create jobs. As a result, employment does not change much in the model.

One suggestion that is put forward by Shimer (2005), Hall (2005) and Gertler and Trigari (2009)(GT) to solve the puzzle is to introduce wage stickiness to the model. The idea that wage stickiness plays an important role in understanding aggregate fluctuations is an old one in macroeconomics. It dates back at least to Keynes (1936). The model in GT differs from those in Shimer (2005) and Hall (2005) in that the GT model assumes general equilibrium Real Business Cycle (RBC) framework, while Shimer (2005) and Hall (2005) assume partial equilibrium. GT show that adding wage stickiness to the model leads to spillover effects, resulting in larger fluctuations in unemployment and in other labour market variables. Since wages are sticky, not all wages are adjusted to reflect the increased productivity. As a consequence, relative to the increased productivity, wages paid by some employers remain low, lowering workers' opportunity costs. For this reason, workers who negotiate their wages ask for lower wages than they otherwise would. Increased productivity and low wages induce firms to create jobs, reducing unemployment further.

Despite the intuitive appeal of the GT model, Pissarides (2009) disagrees with this suggestion. He argues that the relevant wage for job creation is new hires' wages and the micro-data on wages suggest that these wages are highly volatile. Based on this evidence, he concludes new hires' wages cannot be sticky. Therefore, wage stickiness cannot be the answer to the puzzle.

It may well be the case that wages of new hires, on average, respond more to shocks than wages of existing workers and the majority of new hires' wages are flexible. However, this observation does not completely rule out the fact that some wages are sticky. In fact, as the quote from Taylor (1999) indicates, there is heterogeneity in wage setting. Specifically, John Taylor points out that:

> Wages in some industries change once per year on average, while others change per quarter and others once every two years. One

> might hope that a model with homogenous representative ...wage setting would be a good approximation to this more complex world, but most likely some degree of heterogeneity will be required to describe reality accurately.

In the New Keynesian literature, accounting for the heterogeneity in prices has proved to be helpful in addressing the criticisms directed at New Keynesian models. For example, Kara (2015) shows that two disturbing problems of the standard New Keynesian model disappear when heterogeneity in price stickiness is introduced. First, the model requires large price shocks to explain inflation dynamics (see Chari et al. (2009)) and, second, firm level pricing in the model is inconsistent with that in reality (see Bils et al. (2012)). Findings like this one lead Taylor (2016) to conclude that "...heterogeneity is not simply a nuisance; it has major implications for aggregate dynamics, and it has been offered as a response to criticism of staggered wage and price setting models. Often that criticism applies to a particular simple staggered contract model ... and that criticism disappears when heterogeneity is taken into account as Kara (2010) and Knell (2013) have emphasized..".

To test our argument that heterogeneity in wage stickiness may solve the Shimer (2005) puzzle, we extend the standard search and matching model to include many sectors, each with a different degree of wage stickiness. To be more specific, we assume that there is a large number of firms and of households. Households have many members, who can be both unemployed and employed. We first group firms according to the degree of wage stickiness they face. There are N groups (or sectors). The household members are also divided into N sub-groups, one for each sector. Within each sector, there is more or less the standard search and matching process. The rest of our assumptions are standard RBC assumptions, as in GT. Our model has the standard search and matching model and the model by GT, as special cases. When all sectors have flexible wages, we have the standard search and matching model. When all sectors face the same degree of wage stickiness, the model becomes the same as that in GT.

Our assumption that search and matching is done at the sectoral level is consistent with recent micro-level evidence provided by Du Caju et al. (2008). They report that in most of European countries, wage negotiations are done at the sectoral level. Findings reported in Alvarez and Shimer (2011) seem to suggest that high labour mobility is inconsistent with the U.S. wage data. Woodford (2003) also emphasizes that the quasi-fixed feature of factor inputs is one of the main reasons for larger aggregate fluctuations in macroeconomic variables over the business cycle. We also present results from the version of the model with common labour market.

To make our point, we focus on the simplest version of our model with two sectors only. In one of the sectors, wages are fully flexible and in the other they are sticky. The reason for this choice is that while there is evidence for heterogeneity in wage stickiness, the evidence on precise distribution of wage contract lengths for new hires is scarce. Therefore, we consider cases in which the majority of contracts are flexible, just as suggested by Pissarides (2009).

Our main finding is that, despite the fact that only a small fraction of wages are sticky in the economy, the new model comes closer in matching the data. This is true even when the share of sticky wage sector is very small.

There are two reasons why allowing heterogeneity in wages in the model significantly improves the empirical performance of the model. First, spillover effects are stronger in the two sector model. This is true since an increase in mean-preserving spread means that in one of the sectors wage is stickier than the mean. Therefore, when the shock hits the economy, wages in this sector do not change much, resulting in a stronger and more persistent spillover effects in that sector. Stronger spillover effects increase incentives for firms in this sector to create more jobs. Although the volatility in the flexible sector is low, due to the fact that spillover effects are absent in this sector, the increase in volatility in the sticky sector more than offsets low volatility in the flexible sector. The second reason is our assumption that workers are only allowed to work for a specific sector. If we were to assume a common labour market then the volatilities in labour market variables would be lower. To understand this result, first note that in the sticky sector, there are more vacancies due to lower wages. In the common labour market, workers can move from the flexible sector to the sticky sector, where it is easier to find jobs. This increases labour market tightness and wages in the flexible sector and lowers those in the sticky sector. Increased wages in the flexible sector lead to lower employment in the flexible sector, while reduced wages in the sticky sector increase employment further. Aggregate employment increases less in the common labour case, as the share of flexible sector is larger.

The remainder of the paper is organised as follows. Section 2 presents the model. Section 3 presents the log-linearised model and discusses the calibration of model parameters. Section 4 evaluates whether the model can match the data on the volatility of labour market variables and shows that it comes closer in matching the data. Section 5 explains why a model with the heterogeneity in wage stickiness matches the data better than the existing models. The role of assumptions about the labour market structure on the results is also discussed. Section 6 concludes the paper.

2. The model

There is a continuum of competitive firms with index $f \in [0,1]$. The firms are divided into sectors with index i and the sector shares are given by α_i. Corresponding to the continuum of firms, there is a continuum of identical households of measure unity. Each household has a continuum of members. Household members can either be workers or unemployed. Each household is also divided into N sub-groups, one for each sector i. To put it differently, each sector i is twinned with a subgroup of households, meaning that households in that subgroup can work only for firms in sector i. Firm f in sector i employs n_{fit} workers.

A firm's workforce consists of workers that are employed in the past and new hires. It is assumed that firms lose a fraction λ of workers in each period. Therefore, the number of existing workers in firm f in sector i in the current period is given by $(1-\lambda)n_{fit-1}$. In each period, firm f posts vacancies v_{fit} to hire new workers $q(\bar{\theta}_{it})v_{fit}$ where $q(\bar{\theta}_{it})$ denotes the vacancy-filling rate. The hiring rate of the firm i is defined as $x_{fit} \equiv q(\bar{\theta}_{it})v_{fit}/n_{fit-1}$. Therefore, firm f's employment evolves according to

$$n_{fit} = (1 - \lambda + x_{fit})n_{fit-1} \tag{1}$$

This equation is based on the assumption that new hires participate in production immediately. The same assumption is made in Blanchard and Gali (2010) and Gertler et al. (2008). The total number of workers searching for a job in sector i is given by

$$\bar{u}_{it} = \alpha_i - \bar{n}_{it-1} \tag{2}$$

Within each sector, there is a Calvo (1983) process. A randomly chosen $1 - \delta_i$ fraction of firms negotiate a new wage contract with both the existing workers and new matches (or new hires). The degree of wage stickiness increases with i. The wage rate is determined according to a Nash bargaining approach. Due to the assumption of constant returns in matching, all workers are the same. They all set the same wage. The firms that are not chosen to renegotiate their wage contracts, all existing workers and new hires get the same wage that is set in the past. Reset wages differ across sectors, since when firms and workers set their wages, they set them for different horizons.

In our model, when $\alpha_i = 1$, the model reduces to the standard search and matching model. When all the sectors face the same degree of wage stickiness, the model is the same as that in GT. In the rest of this section, we will outline the main building blocks of the model.

2.1. The matching function

Instead of an economy-wide job search and matching process, we assume that job matching is done at the sectoral level. Therefore, workers who search for jobs in sector i only search for jobs in that sector and firms in that sector can only hire workers in sector i. Given these assumptions, the total number of successful matches in period t in sector i is given by the following matching function

$$m(\bar{v}_{it}, \bar{u}_{it}) \equiv \mu_m \bar{u}_{it}^{\mu} \bar{v}_{it}^{1-\mu}, \qquad 0 < \mu < 1$$

where $\bar{v}_{it}$ is the total number of vacancies posted by firms in sector i and $\bar{u}_{it}$ is the total number of job seekers (or unemployed) workers in that sector. The parameter μ_m denotes the scale parameter that measures the efficiency of matching. As noted earlier, $\bar{u}_{it} = \alpha_i - \bar{n}_{it-1}$, since it is assumed that all unemployed workers search for jobs and the newly separated workers do not participate in searching in the same period. The unemployed workers's job finding rate, $p(\bar{\theta}_{it})$, and the firms' vacancy filling rate, $p(\bar{\theta}_{it})$, are given by

$$p\left(\bar{\theta}_{it}\right) \equiv \frac{m(\bar{v}_{it}, \bar{u}_{it})}{\bar{u}_{it}} = \mu_m \bar{\theta}_{it}^{1-\mu}$$
$$q\left(\bar{\theta}_{it}\right) \equiv \frac{m(\bar{v}_{it}, \bar{u}_{it})}{\bar{v}_{it}} = \mu_m \bar{\theta}_{it}^{-\mu}$$

where $\bar{\theta}_{it} \equiv \bar{v}_{it}/\bar{u}_{it}$ denotes the labour market tightness in sector i.

2.2. Firms

There is a continuum of competitive firms. A firm produces a homogeneous consumption good. In each period, firm f employs n_{fit} workers to produce output y_{fit}. Each worker receives a wage w_{fit}. The production function with a constant returns to scale technology is given by

$$y_{fit} = A_t n_{fit} \tag{3}$$

A_t denotes productivity which is assumed to follow an AR(1) process

$$a_t = \rho_a a_{t-1} + \epsilon_t^a \tag{4}$$

where $a_t \equiv \log A_t$ and ϵ_t^a is an *iid* productivity shock with mean zero. The firm posts v_{fit} vacancies and hires $q(\bar{\theta}_{it})v_{fit}$ workers in period t. The hiring process is costly. Following GT, we assume that hiring costs take the following form

$$\frac{\kappa}{2} x_{fit}^2 n_{fit-1}$$

Taking into account exogenous job separations and newly created matches the law of motion of the employment stock in firm f is given by

$$n_{fit} = (1 - \lambda + x_{fit}) n_{fit-1} \tag{5}$$

In each period firm f chooses x_{fit} to maximise its value by taking the total number of its employees at the beginning of period t (n_{fit-1}) and the current and expected path of wages as given. Specifically firms solve the following problem

$$F_{fit}(n_{fit-1}) = \max_{x_{fit}} \left[A_t n_{fit} - w_{fit} n_{fit} - \frac{\kappa}{2} x_{fit}^2 n_{fit-1} + E_t \beta_{t,t+1} F_{fit+1}(n_{fit}) \right] \tag{6}$$

subject to $n_{fit} = (1 - \lambda + x_{fit}) n_{fit-1}$. The solution to this maximisation problem results in the job creation condition of the firm, which is given by

$$\kappa x_{fit} = A_t - w_{fit} + E_t \beta_{t,t+1} \left[\frac{\kappa}{2} x_{fit+1}^2 + (1 - \lambda) \kappa x_{fit+1} \right] \tag{7}$$

This equation shows that the hiring rate depends on the net marginal product of labour ($A_t - w_{fit}$), savings on hiring costs in the next period and the continuation value of the match.

We define $J_{fit}^F(w_{fit})$ as the firm's marginal surplus when it hires an additional worker at the wage rate w_{fit}. This is given by

$$J_{fit}^F(w_{fit}) = A_t - w_{fit} + E_t \beta_{t,t+1} \left[-\frac{\kappa}{2} x_{fit+1}^2 + (1 - \lambda + x_{fit+1}) J_{fit+1}^F(w_{fit+1}) \right] \tag{8}$$

Comparing the last two equations yields

$$J^F_{fit}(w_{fit}) = \kappa x_{fit} \tag{9}$$

This equation requires that in equilibrium the value of an additional worker to be equalized with the costs of adding one more worker.

2.3. Households

As it is standard in this literature (see, for example, GT), we use the representative family setup proposed by Merz (1995). As noted earlier, there is a continuum of identical households. Each household has a continuum of members, which can either be workers or unemployed. While household members work in different sectors, they pool income together and get full risk sharing within the household. As a consequence of these assumptions, all household members consume the same amount. An unemployed member of the representative household receives unemployment benefit. The representative household holds bonds and is a shareholder of firms and receives dividends. Given these assumptions, the representative household's life-time utility and the corresponding budget constraint are given by

$$U_t = \max_{c,B} \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \beta U_{t+1}$$

where σ is the constant relative risk aversion parameter and β is the subjective discount rate. The household's budget constraint is

$$c_t + B_t \leq (1 + r_{t-1})B_{t-1} + \int_0^1 n_{fit} w_{fit} \, \mathrm{d}f + b(1 - n_t) + \Pi_t - T_t$$

where r_{t-1} is the (real) interest rate between period $t-1$ and t, B_{t-1} are holdings of one-period real bonds, w_{fit} is the wage rate in firm f in sector i, and

Π_t are aggregate dividends from the firms. In addition, b is unemployment benefit measured in consumption units and T denotes taxes. The representative household maximises its utility subject to the budget constaint. The first order condition of this problem is given by

$$1 = \beta E_t \left[(1 + r_t) \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \right] \tag{10}$$

Since the probability to find a job is $p(\bar{\theta}_{it})$, the household's employment in firm f in sector i evolves according to

$$n_{fit} = (1 - \lambda) n_{fit-1} + p(\bar{\theta}_{it}) \frac{v_{fit}}{\bar{v}_{it}} \bar{u}_{it} \tag{11}$$

where $p(\bar{\theta}_{it}) v_{fit} / \bar{v}_{it}$ is the probability of finding a job at firm f in sector i.

We define $J_{fit}^W(w_{fit})$ as the workers' surplus from a job, i.e., the marginal value of additional employment in firm f in sector i to the household in consumption units. By making use of the envelope condition we obtain the following expression.

$$J_{fit}^W(w_{fit}) = w_{fit} - b - E_t \beta_{t,t+1} \left[p(\bar{\theta}_{it+1}) J_{xit+1}^W - (1 - \lambda) J_{fit+1}^W(w_{fit+1}) \right] \tag{12}$$

where

$$J_{xit+1}^W \equiv \int_{\alpha_i} \frac{v_{jit+1}}{\bar{v}_{it+1}} J_{jit+1}^W(w_{jit+1}) \, \mathrm{d}j$$

denotes the average surplus of a worker who is newly hired in time $t + 1$. $\beta_{t,t+s} \equiv \beta c_{t+s}^{-\sigma} / c_t^{-\sigma}$ is the stochastic discount factor between periods t and $t + s$. The last two terms in Equation (12) come from Equation (11) and reflect the fact that an additional unit of employment at firm f results in $1 - \lambda$ unit of surviving match at the firm in the next period but this comes at a cost for workers. Workers lose opportunities to find jobs elsewhere in that sector in the next period, as workers who are employed cannot search for jobs in the next period.

2.4. Staggered wage bargaining in sector i

In each period, only a fraction $1-\delta_i$ of firms in sector i negotiate their wages with their workers. Newly hired workers are assumed to receive the same wage with the existing workers if they enter the firm between the contracts. When negotiating their wages, workers and firms take into account of the fact that the wage set is going to be valid for some time. If we denote w^*_{it} as the renegotiated wage in sector i[1], the firm's surplus from the marginal worker, which is given by Equation (8), can be rewritten as

$$J^F_{it}(w^*_{it}) = A_t - w^*_{it} + E_t\beta_{t,t+1}\left[\delta_i\frac{\kappa}{2}x^2_{it+1}(w^*_{it}) + (1-\delta_i)\frac{\kappa}{2}x^2_{it+1}(w^*_{it+1})\right]$$
$$+(1-\lambda)E_t\beta_{t,t+1}\left[\delta_i J^F_{it+1}(w^*_{it}) + (1-\delta_i)J^F_{it+1}(w^*_{it+1})\right] \qquad (13)$$

Similarly the worker's surplus from the match, Equation (12), can be expressed as

$$J^W_{it}(w^*_{it}) = w^*_{it} - [b + E_t p(\bar{\theta}_{it+1})\beta_{t,t+1}J^W_{ixt+1}]$$
$$+(1-\lambda)E_t\beta_{t,t+1}\left[\delta_i J^W_{it+1}(w^*_{it}) + (1-\delta_i)J^W_{it+1}(w^*_{it+1})\right] \qquad (14)$$

The Nash bargaining involves choosing the wage rate w^*_{it} that maximises the product of the worker's and the firm's surpluses. The resulting sharing rule is given by

$$J^W_{it}(w^*_{it}) = \eta\left[J^W_{it}(w^*_{it}) + J^F_{it}(w^*_{it})\right] \qquad (15)$$

where $\eta \in (0,1)$ denotes workers bargaining power[2]. Finally, the average wage in sector i is given by

$$\bar{w}_{it} = \delta_i\bar{w}_{it-1} + (1-\delta_i)w^*_{it} \qquad (16)$$

[1]Since all renegotiating firms in a given sector set the same wage, we drop the subscript f from now on.

[2]Following Thomas (2008), we exclude the horizon effect, which results from the fact

where w_{it}^* is the reset wage in sector i and $\bar{w}_{it} \equiv \int_{\alpha_i} w_{fit}\,\mathrm{d}f$ is the average wage in sector i.

2.5. Government and market clearing

The resource constraint is given by

$$\bar{y}_t = \bar{c}_t + \frac{\kappa}{2}\int_0^1 x_{fit}^2 n_{fit-1}\,\mathrm{d}f \tag{17}$$

The government budget constraint is

$$b(1 - \bar{n}_t) = T_t \tag{18}$$

The equation is based on the assumption that the government finances unemployment benefits with taxes.

3. The log-linearised economy

In this section we present the complete set of log-linearised equilibrium conditions. The steady-state of the model economy is presented in Appendix B. Variables with a hat are log deviations from the steady-state value. We begin by presenting the key equations describing wage dynamics and job creation. By log-linearising firm and worker surpluses (Equations (13) and (14)) and substituting the resulting expressions into the log-linearised version

that workers and firms have different horizons when negotiating wages. A firm takes into account of the fact that new hires will receive the same wage too. On the other hand, for workers, the current wage rate is only relevant during the time they work for the firm. GT report that, this effect is not significant. Therefore, for simplicity but without loss of significant generality, we ignore this effect.

of sharing rule (Equation (15)), we obtain the following expressions for the reset wage in period t (w_{it}^*) and the target wage ($\hat{w}_{it}^o(w_{it}^*)$).

$$\hat{w}_{it}^* = (1-\tau_i)\hat{w}_{it}^o(w_{it}^*) + \tau_i E_t \hat{w}_{it+1}^* \tag{19}$$

$$\begin{aligned} \hat{w}_{it}^o(w_{it}^*) &= \varphi_a \eta \hat{a}_t + \frac{\varphi_x}{2}\eta E_t \left[\hat{\Lambda}_{t,t+1} + 2\hat{x}_{it+1}(w_{it+1}^*)\right] \\ &+ \varphi_\theta (1-\eta) E_t \left[(1-\mu)\hat{\bar{\theta}}_{it+1} + \hat{\Lambda}_{t,t+1} + \hat{J}_{xit+1}^W\right] \end{aligned} \tag{20}$$

where $\tau_i \equiv \frac{(1-\eta)\xi_i + \eta\chi_i}{1+(1-\eta)\xi_i+\eta\chi_i}$, $\chi_i \equiv \frac{\beta\delta_i}{1-\beta\delta_i}$, $\xi_i \equiv \frac{\beta(1-\lambda)\delta_i}{1-\beta(1-\lambda)\delta_i}$, $\varphi_a \equiv \tilde{A}/\tilde{w}$, $\varphi_x \equiv \beta\kappa\tilde{x}^2/(\tilde{w})$, $\varphi_\theta \equiv \tilde{p}(\theta)\beta\tilde{J}^W/\tilde{w}$, and $E_t\hat{\Lambda}_{t,t+1} = \sigma\left[\hat{c}_t - E_t\hat{c}_{t+1}\right]$. The first equation implies that the reset wage is a sum of discounted current and future target wages. The target wage is the period-by-period Nash bargaining solution for wages, taking the other firms' wages as given. Since the other firms in the economy set their wages in a staggered way, the target wage ($\hat{w}_{it}^o(w_{it}^*)$) is different from the target wage when all firms negotiate wages every period. The difference between the two target wages reflects the spillovers of economy-wide average wages on the individual firm's wage negotiation. Therefore, following GT, we call the latter as spillover-free target wage and denote it by $\hat{\bar{w}}_{it}^o$. To obtain $\hat{\bar{w}}_{it}^o$, we first derive expressions for the new hire's average surplus ($\hat{J}_{xit+1}^W$) and the resetting firm's hiring rate ($\hat{x}_{it+1}(w_{it+1}^*)$) in terms of sectoral hiring rate ($\hat{\bar{x}}_{it+1}$) and the difference between the average wage and reset wage ($\hat{\bar{w}}_{it+1} - \hat{w}_{it+1}^*$). Substituting the resulting expressions into (20) yields the following relationships between $\hat{w}_{it}^o(w_{it}^*)$ and $\hat{\bar{w}}_{it}^o$, along with the expression for $\hat{\tilde{w}}^o_{it}$[3]

$$\hat{w}^o_{it}(w^*_{it}) = \hat{\tilde{w}}^o_{it} + \tau_{1i}(\hat{\tilde{w}}_{it+1} - \hat{w}^*_{it+1}) + \tau_{2i}(\hat{\tilde{w}}_{it+1} - \hat{w}^*_{it+1}) \tag{21}$$

$$\begin{aligned}\hat{\tilde{w}}^o_{it} &= \varphi_a \eta \hat{a}_t + \frac{\varphi_x}{2}\eta \left[\hat{\Lambda}_{t,t+1} + 2\hat{\tilde{x}}_{it+1}\right] \\ &\quad + \varphi_\theta (1-\eta) E_t \left[(1-\mu)\hat{\tilde{\theta}}_{it+1} + \hat{\Lambda}_{t,t+1} + \hat{\tilde{x}}_{it+1}\right]\end{aligned} \tag{22}$$

with $\tau_{1i} \equiv \frac{p(\bar{\theta})\beta}{1-\tau_i}$ and $\tau_{2i} \equiv \beta\tilde{x}\eta(1+\chi_i)$. The last brackets in Equation (20) and Equation (22) capture the worker's opportunity cost. The only difference is that the last term in the bracket is $\hat{J}^W_{xit+1}$ in Equation (20) while it is $\hat{\tilde{x}}_{it+1}$ in Equation (22). Equation (9) dictates that $\hat{\tilde{x}}_{it+1} = \hat{\tilde{J}}^F_{it+1}$. Moreover, the firm surplus always reflects productivity, regardless of bargaining process. As a results, we can conclude that the spillover-free target wage depends more on the current economic conditions than the target wage does[4]. The log-linearised average wage in sector i (Equation (16)) is given by

$$\hat{\tilde{w}}_{it} = (1-\delta_i)\hat{\tilde{w}}_{it-1} + \delta_i \hat{w}^*_{it} \tag{23}$$

Next, when we log-linearise the job creation condition (Equation (7)), we obtain the hiring rate in sector i.

$$\hat{\tilde{x}}_{it} = \varkappa_a \hat{a}_t - \varkappa_w \hat{\tilde{w}}_{it} + \varkappa_\Lambda E_t \hat{\Lambda}_{t,t+1} + \beta E_t \hat{\tilde{x}}_{it+1} \tag{24}$$

where $\varkappa_a \equiv \tilde{A}/\tilde{J}^F$, $\varkappa_w \equiv \tilde{w}/\tilde{J}^F$ and $\varkappa_\Lambda \equiv \beta(1-\lambda/2)$. Iterating this equation forward suggests that the hiring rate depends on the current and future productivity net of wage.

[3] The detailed derivations are available in Appendix A.

[4] The spillover-free target wage and the Nash wage are different. The two wages are the same in the way they split the total surplus. However, the total surpluses in the two cases are different since when wages are sticky, wages are different from the target wage.

Log-linearsing unemployment ($\bar{u}_{it} = \alpha_i - \bar{n}_{it-1}$), labour market tightness ($\bar{\theta}_{it} = \bar{v}_{it}/\bar{u}_{it}$) and the hiring rate ($\bar{x}_{it} = q(\bar{\theta}_{it})\bar{v}_{it}/\bar{n}_{it-1}$) gives

$$\hat{\hat{u}}_{it} = -\frac{\tilde{p}(\theta)}{\lambda}\hat{\hat{n}}_{it-1} \tag{25}$$

$$\hat{\hat{\theta}}_{it} = \hat{\hat{v}}_{it} - \hat{\hat{u}}_{it} \tag{26}$$

$$\hat{\hat{x}}_{it} = -\mu\hat{\hat{\theta}}_{it} + \hat{\hat{v}}_{it} - \hat{\hat{n}}_{it-1} \tag{27}$$

The employment evolution in sector i (Equation (5)) is log-linearised as

$$\hat{\hat{n}}_{it} = \hat{\hat{n}}_{it-1} + \lambda\hat{\hat{x}}_{it} \tag{28}$$

Aggregating individual firm's production function (Equation (3)) across firms in sector i and then log-linearising the resulting equation yield the sectoral output.

$$\hat{\hat{y}}_{it} = \hat{a}_t + \hat{\hat{n}}_{it} \tag{29}$$

The log-linearised version of the Euler equation (Equation (10)) is given by

$$\sigma(\hat{\hat{c}}_{it} - E_t\hat{\hat{c}}_{it+1}) + E_t r_{t+1} = 0 \tag{30}$$

As noted above (Equation (4)), the productivity shock follows an AR(1) process.

$$\hat{a}_t = \rho_a\hat{a}_{t-1} + \epsilon_t^a \tag{31}$$

Finally, we aggregate sectoral output, wage, employment, unemployment, vacancy, tightness, and hiring rate to obtain economy-wide output ($\hat{\hat{y}}_t$), wage ($\hat{\hat{w}}_t$), employment ($\hat{\hat{n}}_t$), unemployment ($\hat{\hat{u}}_t$), vacancy ($\hat{\hat{v}}_t$), tightness ($\hat{\hat{\theta}}_t$), and hiring rate ($\hat{\hat{x}}_t$). For a variable z_t, the economy-wide aggregate $\hat{\hat{z}}_t$ is given by a weighted average of sectoral aggregate $\hat{\hat{z}}_{it}$ with the sector share α_i as the weights as shown by the following equation.

$$\hat{\tilde{z}}_t = \sum_{i=1}^{n} \alpha_i \hat{\tilde{z}}_{it} \tag{32}$$

The resource constraint closes the model.

$$\hat{\tilde{y}}_t = \tilde{y}_c \hat{c}_t + (1 - \tilde{y}_c) \left(2\hat{\tilde{x}}_t + \hat{\tilde{n}}_{t-1}\right) \tag{33}$$

where $\tilde{y}_c \equiv \tilde{c}/\tilde{y}$ denotes consumption share in output.

3.1. Calibration

Since we build on GT and since we want to compare our results with theirs, when calibrating our model, we slavishly follow GT. Except for parameters that are specific to our model, all parameters are taken from GT. The calibration of the model is monthly, since, as noted by GT, monthly calibration captures the high job finding rate in the US better. The discount factor(β) is set to $\beta = 0.99^{1/3}$, the persistence parameter of the productivity shock (ρ_z) is set to $\rho_z = 0.95^{1/3}$ and the constant relative risk aversion parameter (σ) is assumed to be $\sigma = 1$. These are all standard values in the RBC literature.

Turning to the parameter values that are specific to the search and matching model, the separation rate is calibrated at $\lambda = 0.035$, which is based on the evidence that jobs last about two years and a half. The job finding rate is assumed to be $\tilde{p}(\theta) = 0.45$, as in Shimer (2005). These two assumptions lead to steady-state unemployment rate of 0.072. The matching elasticity is calibrated at $\mu = 0.5$, while workers' bargaining power is set to $\eta = 0.5$. The unemployment benefit ratio $\tilde{b}$ is the ratio of the unemployment flow value (b) to the steady-state flow contribution of the worker to the match ($\tilde{A} + \frac{\kappa}{2}\tilde{\tilde{x}}^2$).

κ is the hiring cost parameter. κ and b are chosen in a way so that $\tilde{b}$ is equal to 0.4. This requires setting $\kappa = 33.32$ and $b = 0.4081$. The implied replacement ratio is $b/\tilde{w} = 0.42$ and the steady-state hiring costs to output ratio is $\frac{\kappa}{2}\tilde{\tilde{x}}^2\frac{\tilde{\tilde{n}}}{\tilde{\tilde{y}}} = 0.02$[5].

Finally, we focus on parameters that are specific to our model. These parameters are sector shares (α_i) and the degree of wage stickiness in each sector (δ_i). Unfortunately, as noted above, the data on these parameters are scarce. Our approach when calibrating these parameter are as follows. Given the lack of evidence on the distribution of wage contracts, we consider a special case of our model with only two sectors. In one of the sectors, wages are fully flexible, as is the case in the standard search and matching model and in the other sector they are sticky. We calibrate the share of each sector in a way so that the majority of wage contracts in the economy are flexible. We consider two cases. In case 1, which is our benchmark case, we assume that the share of firms in the flexible sector (α_1) is $\alpha_1 = 0.65$ and the share of the sticky sector (α_2) is $\alpha_2 = 0.35$. In the sticky sector we assume that the average age of contracts (δ_2) is $\delta_2 = 1 - 1/12$. These assumptions imply that the average age of wage contracts in the economy is only 5 months, which is much lower than that assumed by GT (i.e. 9 months and 12 months). Our second calibration is based on the evidence provided by Du Caju et al. (2008). Du Caju et al. (2008) provide evidence for 23 European countries on

[5]This value is higher that assumed in GT. The is because in their model production function consists of both capital and labour. In our model labour is the only input. We check the robustness of our findings when $\frac{\kappa}{2}\tilde{\tilde{x}}^2\frac{\tilde{\tilde{n}}}{\tilde{\tilde{y}}} = 0.01$, as in GT. Our main conclusions appear robust.

wage bargaining. They find that the average length of collective bargaining agreements is between one and three years, suggesting that average duration of wage contracts is longer than 12 months. Given this finding, in another case, holding all the other factors constant, we assume that renegotiated wages lasts on average 18 months in the sticky sector.

4. Unemployment volatility puzzle

In this section, we evaluate the performance of our model in matching the volatility of labour market variables, especially of unemployment, vacancies and labour market tightness. Panel A of Table 2 reports the empirical moments of labour market variables along with those of output in the US during 1964:Q1-2005:Q1, which are taken from GT. The standard deviations of variables are expressed relative to the standard deviation of output. We will first consider the performance of the standard search and matching model, next the sticky wage model suggested by GT and, finally, our model with heterogeneity in wage stickiness.

Panel B of Table 2 reports the statistics for the standard search and matching model. Consistent with the findings reported in Shimer (2005), the search model cannot generate enough (relative) volatility in labour market dynamics to match the data. The volatility of unemployment relative to output in the model is only 1.26, while it is 5.15 in the data. The relative volatility of vacancy implied by the model is much lower than that in the data (1.62 in the model vs 6.30 in the data). This is also true for the labour market tightness measure (2.78 in the model vs. 11.28 in the data).

Panel C of Table 2 reports the results for the GT model. As noted by

GT, the results show that adding wage stickiness to the model significantly increases the volatility of labour market variables. The volatilities in unemployment, vacancy, and tightness come closer to the data at 2.75, 3.82, and 6.19, respectively. These numbers are lower than those reported in GT, since the degree of wage stickiness we assume is lower than that assumed in the GT. GT consider cases with average age of contracts 9 and 12 months, while we assume an average age of 5 months.

We now turn to examine the performance of our model with heterogeneity. We first consider our benchmark case. As discussed in the calibration section, in this case, we assume that $\alpha_1 = 0.65$, $\alpha_2 = 0.35$, $\delta_1 = 0$ and $\delta_2 = 1 - 1/12$. The results from this model are reported in Panel D of Table 2. The results suggest that the two-sector model generates more volatility in labour market variables than the one sector model with the same mean. This is true despite the fact that in the two sector model almost 70% of wage contracts are flexible and the average duration of wage contracts are the same. The volatilities in unemployment, vacancy, and tightness come closer to the data at 3.75, 4.85, and 8.29, respectively, compared to 5.15, 6.30, and 11.28 in the data.

In addition, the two-sector model appears to capture the persistence of the variables, as measured by first autocorrelations, of labour market variables better. The autocorrelations of labour market variables, reported in the second row of each panel, in our model are closer to the data than those in the GT model. The autocorrelations of unemployment, vacancy, and tightness are 0.90, 0.85, and 0.89, while they are 0.91, 0.91, and 0.91 in the data. In the GT model, they are 0.83, 0.71, and 0.80.

Finally, we consider our second case. The results from this experiment are reported in Panel E of Table 2. The only difference between this case and the previous one is that in this case the average duration of wage contracts in the sticky sector is slightly longer at 18 months. As the table clearly shows, in this case, our model matches the data on the standard deviations and persistence of labour market variables almost perfectly.

5. How does the two-sector model generate more volatility?

There are two reasons why our model generates larger fluctuations in labour market variables than both the standard search and matching model and the GT model. The first and main reason is the presence of heterogeneity in wages stickiness. The second reason is our assumption of sector-specific labour market. We now explain each reason in turn.

5.1. Heterogeneity in wage stickiness and wage dynamics

To understand how heterogeneity in wage stickiness increases volatility of labour market variables, it is helpful to study the aggregate wage equation in our model. As noted earlier, the aggregate wage in the two sector version of our model is given by

$$\hat{\bar{w}}_t = \alpha_1 \hat{\bar{w}}_{1t} + \alpha_2 \hat{\bar{w}}_{2t} \tag{34}$$

where $\bar{w}_{1t}$ is the average wage in sector 1 with flexible wages and $\bar{w}_{2t}$ is the average wage in sector 2 with sticky wages. As we show in Appendix A, the average wage in sector i is

$$\hat{\bar{w}}_{it} = \gamma_{bi} \hat{\bar{w}}_{it-1} + \gamma_{oi} \hat{\bar{w}}_{it}^{o} + \gamma_{fi} E_t \hat{\bar{w}}_{it+1} \tag{35}$$

where $\gamma_{bi} \equiv \delta_i/\phi_i$, $\gamma_{oi} \equiv (1-\delta_i)(1-\tau_i)/\phi_i$, $\gamma_{fi} \equiv [\tau_i - \delta_i(1-\tau_i)(\tau_{1i}+\tau_{2i})]/\phi_i$, and $\phi_i \equiv 1 + \delta_i[\tau_i - (1-\tau_i)(\tau_{1i}+\tau_{2i})]$.

This equation implies that the average wage in sector i depends not only on current spillover-free target wage but also on expected future and past wages. The $\gamma-$coefficients on the variables on the RHS of the equation depend mainly on the degree of wage stickiness (δ_i) in that sector. In the flexible sector, when $\delta_1 = 0$, $\gamma_{b1} = \gamma_{f1} = 0$ and $\gamma_{o1} = 1$. Therefore, the wage rate in that sector is simply given by[6]

$$\hat{\bar{w}}_{1t} = \gamma_{o1}\hat{\bar{w}}^o_{1t} \tag{36}$$

An increase in wage stickiness in the sticky sector leads to more sluggish adjustment in average wage in this sector, as, with increased wage stickiness, a smaller fraction of wage agreements are renewed in each period. This can easily be seen by considering Equation (35). The coefficient on the lagged wage (γ_{bi}) increases with wage stickiness and the weight on the current economic conditions (γ_{oi}) decreases. Moreover the wage rate in the previous period $\hat{\bar{w}}_{it-1}$ becomes more sluggish as wage stickiness increases, increasing the persistence of average wage further.

This sluggish adjustment in wages leads to a spillover effect. The fact that wages respond less to productivity shocks affects workers opportunity costs. Since wages are sticky, there are firms that have not renegotiated wages after the productivity shock. Therefore, not all wages in that sector reflect the increased productivity and some of the wages are lower than they should be.

[6]When $\delta_i = 0$, since wages are flexible, there is no spillover effect. So, the spillover-free target wage $\hat{\bar{w}}^o_{it}$ is equal to the Nash wage.

These lower wages decrease employed workers' opportunity costs. Reduced opportunity costs lead to lower target wage and, consequently, lower reset wages. Moreover, since wages adjust persistently, the spillover effects are persistent too. The lower and persistent wages induce firms in this sector to post more vacancies and hire more workers, resulting in a decrease in unemployment. The labour market variables are more volatile than in the case of the standard search and matching model with fully flexible wages.

This intuition can also be seen by considering the aggregate wage in the economy. Substituting sectoral wages into Equation (34) gives

$$\hat{\hat{w}}_t = \alpha_2 \gamma_{b2} \hat{\hat{w}}_{2t-1} + \alpha_1 \gamma_{o1} \hat{\hat{w}}^o_{1t} + \alpha_2 \gamma_{o2} \hat{\hat{w}}^o_{2t} + \alpha_2 \gamma_{f2} E_t \hat{\hat{w}}_{2t+1} \tag{37}$$

The aggregate wage depends on the lagged wage rate in sticky sector. Since this wage rate adjusts sluggishly, the aggregate wage in the economy adjusts sluggishly. The channel becomes more important as the share of the sticky sector increases (α_2). When $\alpha_2 = 0$, as in the standard search model, the adjustment in wages happens very quickly. As noted above, the sluggish adjustment in wages leads to persistent spillovers effects.

Figure 1 confirms these suggestions. There we plot the irfs of several variables in the two sector model, in the GT model, and in the flexible-wage version of the model to a productivity shock. Lets first consider the irfs in our model with two sectors. Increased productivity directly affects firm surplus, leading firms to post more vacancies. Vacancies increase. Given the fact that wages respond sluggishly in the sticky sector and the presence of persistent spillover effects, the average wages in this sector remain low for some time, increasing the number of vacancies further. As a result, as the figure shows, labour market tightness increases and unemployment falls. As productivity fades away, all variables go back to their steady-state values.

Next, we consider the irfs in the GT model. It is useful to compare the irfs in the GT model with those of the aggregate variables in the two-sector model. If we look at the irfs in the GT model, when the shock first hits the economy, the wage rate is lower than that in the two-sector model. This is because of the fact that in the two-sector model, there are lots of flexible wages. However, in the later part, wages in the two-sector model adjust more sluggishly, due to the presence of the sticky wage sector. Consequently, spillover effects are weaker and wages change more in response to the shock and unemployment changes less. In the version of the model with flexible wages, as is known, and as the irfs show, most of the increase in productivty is absorbed by increasing wages. As a result, unemployment and other labour market variables do not change much.

To show the significant role that spillover effects play in our model, in Table 3, we calculate the volatilities of labour market variables removing spillover effects from the model[7]. These results suggest that without spillover effects, the volatilities are around 40% lower. In the one sector model, at around 30%, the decrease in volatilities is large but not as large as in the two sector models, indicating that spillover effects are stronger in the two-sector

[7] To remove spillover effects from the model, following GT, we set the coefficients τ_{1i} and τ_{2i}, which determine the size of the spillover effects, equal to zero. The two coefficients are functions of parameters of the model, but we do not change those parameters. Therefore, this simulation should be viewed as a rather informal way to quantify the significance of the spillover effects.

model.

5.2. Sector-specific labour market vs. common labour market model

In the common labour market model, the model economy is similar as in the sector specific labour market model, except for two assumptions. First, all household members are not divided into sub-groups and, therefore, can work for all firms in the economy. Second, the search and matching is done at the economy-wide labour market, instead of at sectoral level. This assumption implies that all workers in the economy compete for all vacancies posted by all firms in the economy. As a result the labour market tightness θ and unemployment u are defined only at economy-wide level, and so are the job finding rate $p(\theta)$ and vacancy-filling rate $q(\theta)$, while vacancy v_{it} and hiring rate x_{it} are defined at sectoral level. The matching function is given by

$$m(\bar{v}_t, \bar{u}_t) \equiv \mu_m \bar{u}_t^{\mu} \bar{v}_t^{1-\mu} \tag{38}$$

where $\bar{v}_t \equiv \sum_{i=1}^{n} \bar{v}_{it}$ is the total vacancies posted by firms in all sectors and $\bar{u}_t$ is the total number of job seekers in the economy. The common labour market assumption changes the household's optimization problem in two ways. First, the household's employment in firm f in sector i evolves according to

$$n_{fit} = (1-\lambda) n_{fit-1} + p(\bar{\theta}_t) \frac{v_{fit}}{\bar{v}_t} \bar{u}_t \tag{39}$$

The worker surplus in firm f in sector i is given by

$$J_{fit}^{W}(w_{fit}) = w_{fit} - b - E_t \beta_{t,t+1} \left[p(\bar{\theta}_{t+1}) J_{xt+1}^{W} - (1-\lambda) J_{fit+1}^{W}(w_{fit+1}) \right] \tag{40}$$

where the opportunity cost is consisted of the economy-wide job finding rate $p(\bar{\theta}_{t+1})$, and the worker surplus conditional on being a new hire in the next period. The new hire's surplus takes into account the probability to find a job in all sectors, instead of in any specific sector as in the sector-specific labour market model. Therefore the economy-wide new hire's surplus is an weighted average of the sectoral new hire's surplus, with the weights given by the number of vacancies in each sector relative to total vacancies.

$$J^W_{xt+1} \equiv \sum_{i=1}^{n} \int_{\alpha_i} \frac{v_{jit+1}}{\bar{v}_{t+1}} J^W_{jit+1}(w_{jit+1})\,\mathrm{d}j = \sum_{i=1}^{n} \frac{\bar{v}_{it+1}}{\bar{v}_{t+1}} J^W_{xit+1} \tag{41}$$

As a consequence of these changes, the firm optimization problem and the sharing rule are unaffected. The log-linearised economy-wide labour market tightness, total unemployment and the sectoral hiring rate are given by

$$\hat{\bar{\theta}}_t = \hat{\bar{v}}_t - \hat{\bar{u}}_t \tag{42}$$

$$\hat{\bar{u}}_t = -\frac{\tilde{p}(\theta)}{\lambda}\hat{\bar{n}}_{t-1} \tag{43}$$

$$\hat{\bar{x}}_{it} = -\mu\hat{\bar{\theta}}_t + \hat{\bar{v}}_{it} - \hat{\bar{n}}_{it-1} \tag{44}$$

In this specification, the target wage of resetting firms in sector i is

$$\hat{w}^o_{it}(w^*_{it}) = \eta\varphi_a\hat{a}_t + \eta\varphi_x/2E_t\left[\hat{\Lambda}_{t,t+1} + 2\hat{\bar{x}}_{it+1}\right] \tag{45}$$

$$+(1-\eta)\varphi_\theta E_t\left[(1-\mu)\hat{\bar{\theta}}_{t+1} + \hat{\Lambda}_{t,t+1} + \hat{J}^W_{xt+1}\right] \tag{46}$$

$$+\tau_{2i}\left(\hat{\bar{w}}_{it+1} - \hat{w}^*_{it+1}\right) \tag{47}$$

Finally, the economy-wide new worker's surplus is

$$\hat{J}^W_{xt+1} = \sum_{i=1}^{n} \alpha_i \hat{J}^W_{xit+1} \tag{48}$$

To understand the implications of labour market structure, it is useful to compare the irfs in response to the productivity shocks in the two specifications. Figure 2 reports the irfs for several key variables. Lets first focus on the flexible-sector. In the initial part of the wage adjustment process, opportunity costs in the common labour market case is lower than in the sector-specific labour market case. The reason for this is that in the sticky sector wages are lower, resulting in a lower average wage in the economy and, consequently, lower opportunity costs. As a result, initially, the target wage in this sector is lower. In the sticky sector, since wages adjust sluggishly to shocks, there are more jobs available. In the common market case, since workers can also work for the sticky sector, this leads to an increase in labour market tightness in the flexible sector. As tightness increases, target wage in the flexible sector increases and, consequently, wages in this sector increase. Increased wage rate lowers employment in this sector.

Turning to the sticky sector, in the common labour market case, at the beginning of the wage adjustment process, opportunity costs are higher than the sector-specific labour market case, since wages in the flexible sector are higher. In the later part, labour market becomes less tight, as workers from the flexible sector moves to the sticky sector, lowering opportunity costs and target wage in the sticky sector. Reduced wages in this sector leads to higher employment than in the sector-specific case. Note that average wage in the sticky sector does not fall as much as the fall in target wage, since wages are sticky.

Taken together, the assumptions about labour market structure affect sectoral target wages, sectoral wages and employment levels. In the flexible sector, employment is lower, while it is higher in the sticky sector. Aggregate employment is lower in the common labour market than in the sector-specific labour market simply because of the larger share of the flexible sector in the economy.

Table 4 reports the results for the version of the model with common labour market. Consistent with the above discussion, the model generates smaller volatilities in the case of common labour market.

These findings have an important implication. They suggest that the existence of a little bit of wage stickiness and the sector-specific labour market can generate a significant degree of volatility in the labour market variables. To make this point clearer, we repeat the same calculations as in Table 4 but vary the share of flexible-sector between 0.1 and 0.9. Figure 3 reports the volatility of unemployment from this experiment. In sharp contrast to what one would expect, increasing the share of flexible wages in the economy significantly increases the volatility in unemployment, relative to the common labour market case. In the case in which the share of flexible sector is 0.9 is especially interesting. Even if the share of sticky sector is very small, the model is able to generate volatility in unemployment closer to that seen in the data.

5.3. Bargaining set in a model with heterogeneity

In this section we show that our model is not subject to the criticism by Barro (1977). If a wage is set for a very long time, then, after some time, the wage may fall out of the bargaining set. The lower bound of the bargaining set is given by the reservation wage of workers (r^W_{fit}), while the upper bound is determined by the maximum wage the firm is willing to pay (r^F_{fit}). In particular, the bargaining set is given by

$$B_{fit} = [r^W_{fit}, r^F_{fit}]$$

where r^W_{fit} is the wage at which the worker surplus from the job equals to zero,

$$J^W_{fit} = r^W_{fit} - b - E_t\beta_{t,t+1}\left[p(\bar{\theta}_{it+1})J^W_{xit+1} - (1-\lambda)J^W_{fit+1}\right] = 0$$

and r^F_{fit} is the wage that makes the firm surplus from the job equal to zero.

$$J^F_{fit} = A_t - r^F_{fit} + E_t\beta_{t,t+1}\left[\frac{\kappa}{2}x^2_{fit+1} + (1-\lambda)J^F_{fit+1}\right] = 0$$

We now test if contract wages stay within the bargaining set over the life of the contract. To check this, 1,000 observations of the model economy are simulated. Productivity shocks are assumed to be normally distributed with zero mean and standard deviation of 0.0075 as in GT and in Thomas (2008). For each period, we compute the wage rate for the contract that has survived for 25 months and the corresponding bargaining set. The share of such contracts in the economy is only 4%.

Figure 4 reports the results for this experiment. Although the wage rate draws near to the boundary a few times in 1,000 months (83 years), it stays within the bargaining set. Although we do not report here, we also do the same experiment for a wage contract that has been in place for 27 months. The share of such contract is only 3%. The main result still holds. In any case, even if we drop the contracts that are older than 25 months, our main conclusions do not change significantly.

6. Summary and Conclusions

We have extended the staggered mutli-period wage contracting model of Gertler and Trigari (2009), which is based on the Diamond-Mortensen-Pissarides (DMP) framework to include many sectors, each with different degree of wage stickiness. We assume that search and matching is done at the sectoral level. Within each sector, there is a more or less standard search and matching process. When all sectors have the same degree of wage stickiness, the model reduces to the Gertler and Trigari (2009). Assuming in all sectors wages adjust every period gives the Diamond-Mortensen-Pissarides model.

We have then used our model to see if it can provide an explanation for the Shimer (2005) puzzle. Our main finding is that allowing for even a small degree of wage stickiness significantly improves the model's performance in matching the volatility of labour market variables. The presence of even a few longer term contracts within a sector holds the average wage in that sector, reducing opportunity costs for workers and, consequently, wages. Reduced wages create incentives for firms to create new jobs, lowering unemployment in the economy. Our assumption that search and matching is done at the sectoral level plays a role in our results. If we were to assume a common labour market, our model generates a little lower volatility. The reason is that in common labour market, since there are more job opportunities in the sticky wage sector, workers from the flexible wage sectors would move to the sticky sector, reducing employment in the flexible-wage sectors and increasing employment in the sticky wage sectors. Since the share of flexible wage sector is large in the economy, aggregate employment increases less in the common labour case.

Our findings have two important implications. First, although the model with heterogeneity generates larger volatilities than the corresponding one sector model, as in Gertler and Trigari (2009), one sector model is not a bad approximation of the model with heterogeneity. However, optimal monetary policy implications of the two models may be different. Second, even when majority of contracts in an economy is flexible, labour market can be very volatile.

Appendices

Appendix A. Derivation of wage equations

Appendix A.1. New hire surplus and resetting firm's hiring rate

We find the expressions for new hire's surplus ($\hat{J}^W_{xit+1}$) and the resetting firm's hiring rate ($\hat{x}_{it+1}(w^*_{it+1})$) in terms of sectoral hiring rate ($\hat{\bar{x}}_{it+1}$) and the difference between the avearge and the reset wages ($\hat{\bar{w}}_{it+1} - \hat{w}^*_{it+1}$). Log-linearising the firm surplus (Equation (13)), obtaining $\hat{J}^F_{it+1}(w^*_{it+1})$ and $\hat{J}^F_{it+1}(\bar{w}_{it+1})$, and then taking the difference between the two of them yield

$$\hat{J}^F_{it+1}(w^*_{it+1}) - \hat{J}^F_{it+1}(\bar{w}_{it+1}) = -\frac{\tilde{w}}{\tilde{J}^F}(1+\chi_i)(\hat{w}^*_{it+1} - \hat{\bar{w}}_{it+1}) \tag{A.1}$$

By taking into account $\hat{x}_{it+1}(w_{it+1}) = \hat{J}^F_{it+1}(w_{it+1})$, we obtain the expression for resetting firm's hiring rate ($\hat{x}_{it+1}(w^*_{it+1})$).

$$\hat{x}_{it+1}(w^*_{it+1}) = \hat{x}_{it+1}(\bar{w}_{it+1}) - \frac{\tilde{w}}{\tilde{J}^F}(1+\chi_i)(\hat{w}^*_{it+1} - \hat{\bar{w}}_{it+1}) \tag{A.2}$$

Similarly, log-linearising the worker surplus (Equation (14)), obtaining $\hat{J}^W_{it+1}(w^*_{it+1})$ and $\hat{J}^W_{it+1}(\bar{w}_{it+1})$, and then taking the difference yield

$$\hat{J}^W_{it+1}(w^*_{it+1}) - \hat{J}^W_{it+1}(\bar{w}_{it+1}) = \frac{\tilde{w}}{\tilde{J}^W}(1+\xi_i)(\hat{w}^*_{it+1} - \hat{\bar{w}}_{it+1}) \tag{A.3}$$

By using Equation (A.1) and Equation (A.3) together with the log-linearised version of the sharing rule (Equation (15)), we obtain the expression for the new hire's surplus $\hat{J}^W_{xit+1}$.

$$\begin{aligned}\hat{J}^W_{it+1}(\bar{w}_{it+1})=&\hat{J}^W_{it+1}(w^*_{it+1}) - \frac{\tilde{w}}{\tilde{J}^W}(1+\xi_i)(\hat{w}^*_{it+1} - \hat{\bar{w}}_{it+1})\\ =&\hat{J}^F_{it+1}(w^*_{it+1}) - \frac{\tilde{w}}{\tilde{J}^W}(1+\xi_i)(\hat{w}^*_{it+1} - \hat{\bar{w}}_{it+1})\\ =&\hat{J}^F_{it+1}(\bar{w}_{it+1}) - \frac{\tilde{w}}{\tilde{J}^W}\frac{1}{(1-\tau)(1-\eta)}(\hat{w}^*_{it+1} - \hat{\bar{w}}_{it+1})\\ =&\hat{x}_{it+1}(\bar{w}_{it+1}) - \frac{\tilde{w}}{\tilde{J}^W}\frac{1}{(1-\tau)(1-\eta)}(\hat{w}^*_{it+1} - \hat{\bar{w}}_{it+1}) \quad \text{(A.4)}\end{aligned}$$

Note that $\hat{J}^W_{xit+1} = \hat{J}^W_{it+1}(\bar{w}_{it+1})$ up to a first order approximation. Finally, by subsituting Equation (A.2) and Equation (A.4) into the target wage (Equation (20)), we obtain Equation (21) and Equation (22) in the text.

Appendix A.2. Average wage in sector i

Next, we show the derivation of the average wage equation (Equation (35)). Combining Equation (19), Equation (21), and Equation (23), and then rearranging terms yield

$$\begin{aligned}\hat{\bar{w}}_{it}=&\delta_i\hat{\bar{w}}_{it-1} + (1-\delta_i)\hat{w}^*_{it}\\ =&\delta_i\hat{\bar{w}}_{it-1} + (1-\delta_i)\left[(1-\tau_i)\left\{\hat{\bar{w}}^o_{it} + (\tau_{1i}+\tau_{2i})E_t(\hat{\bar{w}}_{it+1} - \hat{w}^*_{it+1})\right\} + \tau_i E_t\hat{w}^*_{it+1}\right]\end{aligned}$$

Substituting $(1-\delta_i)E_t\hat{w}^*_{it+1} = E_t\hat{\bar{w}}_{it+1} - \delta_i\hat{\bar{w}}_{it}$ (from Equation (23)) into the latter equation, and then collecting terms give Equation (35) in the main text.

Appendix B. Steady State

Consumption and savings:

$$1 = \beta(1+\tilde{r})$$

Production in sector i:

$$\tilde{\bar{y}}_i = \tilde{A}\tilde{\bar{n}}_i$$

Separation and hiring rate in sector i:

$$\tilde{\bar{x}}_i = \lambda$$

Flows in and out of unemployment in sector i:

$$\tilde{\bar{x}}_i(\alpha_i - \tilde{\bar{u}}_i) = \tilde{p}(\theta)\tilde{\bar{u}}_i$$

Matching in sector i:

$$\tilde{p}(\theta)\tilde{\bar{u}}_i = \sigma_m \tilde{\bar{u}}_i^{\mu}\tilde{\bar{v}}_i^{1-\mu}$$

Job creation in sector i:

$$\kappa\tilde{\bar{x}}_i = \tilde{A} - \tilde{\bar{w}}_i + \beta\left[\frac{\kappa}{2}\tilde{\bar{x}}_i^2 + (1-\lambda)\kappa\tilde{\bar{x}}_i\right]$$

Wage in sector i:

$$\tilde{\bar{w}}_i = \eta\left[\tilde{A} + \beta\frac{\kappa}{2}\tilde{\bar{x}}_i^2 + \beta\tilde{p}(\theta)\kappa\tilde{\bar{x}}_i\right] + (1-\eta)b$$

Economy-wide output:

$$\tilde{\bar{y}} = \sum_{i=1}^{n}\tilde{\bar{y}}_i$$

Economy-wide hiring:

$$\tilde{\bar{x}} = \sum_{i=1}^{n}\alpha_i\tilde{\bar{x}}_i$$

Resource constraint:

$$1 = \frac{\tilde{\bar{c}}}{\tilde{\bar{y}}} + \frac{\kappa}{2}\tilde{\bar{x}}^2\frac{\tilde{\bar{n}}}{\tilde{\bar{y}}}$$

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Table 1: Calibration

Subjective discount factor	β	0.997
Productivity autoregressive parameter	ρ_a	0.983
Productivity standard deviaiton	σ_a	0.0075
Separation rate	λ	0.035
Job-finding rate	$\tilde{p}(\theta)$	0.45
Elasticity of matching to unemployment	μ	0.5
Worker's bargaining power	η	0.5
Labour adjustment cost parameter	κ	33.32
Unemployment flow value	b	0.4081
Unemployment rate	$\tilde{u}$	0.07

Note: Parameters in the rows 1-7 are fix. The rest of the parameters, i.e. the labour adjustment cost parameter (κ), the unemployment flow value (b), and steady-state unemployment rate ($\tilde{u}$) are implied.

Table 2: Main statistics

	y	w	n	u	v	θ
	A. U.S. Economy, 1964:1-2005:1					
Relative s.d.	1.00	0.52	0.60	5.15	6.30	11.28
Autocorrelation	0.87	0.91	0.94	0.91	0.91	0.91
	B. Model w/o heterogeneity (Flexible wages)					
Relative s.d.	1.00	0.89	0.10	1.26	1.62	2.78
Autocorrelation	0.78	0.79	0.91	0.91	0.84	0.89
	C. Model w/o heterogeneity (Sticky wages), $1-\delta = 0.21$					
Relative s.d.	1.00	0.69	0.21	2.75	3.82	6.19
Autocorrelation	0.79	0.90	0.83	0.83	0.71	0.80
	D. Model with heterogeneity, $1-\delta = 0.21$ ($\alpha_2 = 0.35$)					
Relative s.d.	1.00	0.60	0.29	3.75	4.85	8.29
Autocorrelation	0.82	0.86	0.90	0.90	0.85	0.89
	E. Model with heterogeneity, $1-\delta = 0.14$ ($\alpha_2 = 0.35$)					
Relative s.d.	1.00	0.48	0.40	5.17	6.52	11.36
Autocorrelation	0.85	0.85	0.92	0.92	0.89	0.92

Note: Statistics for the U.S. economy are for the periods during 1964:Q1-2005:Q1, which are taken from GT. Statistics for the model economies are computed by simulating the model 500 times for 300 periods conditional on productivity shock with zero mean and standard deviation 0.75%. Changes in the number of simulations do not change the results. The statistics are averages over the HP-filtered simulations with smoothing parameter 10^5. The standard deviations (s.d.) of all variables are relative to output.

Table 3: The spillover effects

	y	w	n	u	v	θ
	A. U.S. Economy, 1964:1-2005:1					
	1.00	0.52	0.60	5.15	6.30	11.28
	B. Model w/o heterogeneity (Flexible wages)					
With spillover	1.00	0.89	0.10	1.26	1.62	2.78
W/o spillover	1.00	0.89	0.10	1.26	1.62	2.78
	C. Model w/o heterogeneity (Sticky wages), $1-\delta=0.21$					
With spillover	1.00	0.69	0.21	2.75	3.82	6.19
W/o spillover	1.00	0.77	0.15	1.96	2.72	4.40
	D. Model with heterogeneity, $1-\delta=0.21$ ($\alpha_2=0.35$)					
With spillover	1.00	0.60	0.29	3.75	4.85	8.29
W/o spillover	1.00	0.74	0.17	2.21	2.95	4.92
	E. Model with heterogeneity, $1-\delta=0.14$ ($\alpha_2=0.35$)					
With spillover	1.00	0.48	0.40	5.17	6.52	11.36
W/o spillover	1.00	0.68	0.22	2.81	3.68	6.22

Note: The panel A reports the relative standard deviation of variables in the U.S. data during 1964:Q1-2005:Q1. In the panels B-E, the first rows show the relative standard deviations of variables with spilover-effect. The second rows of the panels report the relative standard deviation when we exclude the spillover effects by setting the coefficients for the spillover effects, i.e. τ_{1i} and τ_{2i}, at zero. For the details of the simulation, see the note in Table 2.

Table 4: The sector specificity

	y	w	n	u	v	θ
	A. U.S. Economy, 1964:1-2005:1					
	1.00	0.52	0.60	5.15	6.30	11.28
	B. Model with heterogeneity, $1-\delta=0.21$ ($\alpha_2=0.35$)					
Sector-specific	1.00	0.60	0.29	3.75	4.85	8.29
Common labour	1.00	0.63	0.26	3.31	4.35	7.34
	C. Model with heterogeneity, $1-\delta=0.14$ ($\alpha_2=0.35$)					
Sector-specific	1.00	0.48	0.40	5.17	6.52	11.36
Common labour	1.00	0.54	0.33	4.31	5.57	9.52

Note: The panel A reports the relative standard deviation of variables in the U.S. data during 1964:Q1-2005:Q1. In the panels B and C, the first rows report the relative standard deviation for the sector-specific labour market model, while the second rows show the results for the common labour market model. Heterogeneity in wage stickiness is assumed in both models. For the details of the simulation, see the note in Table 2.

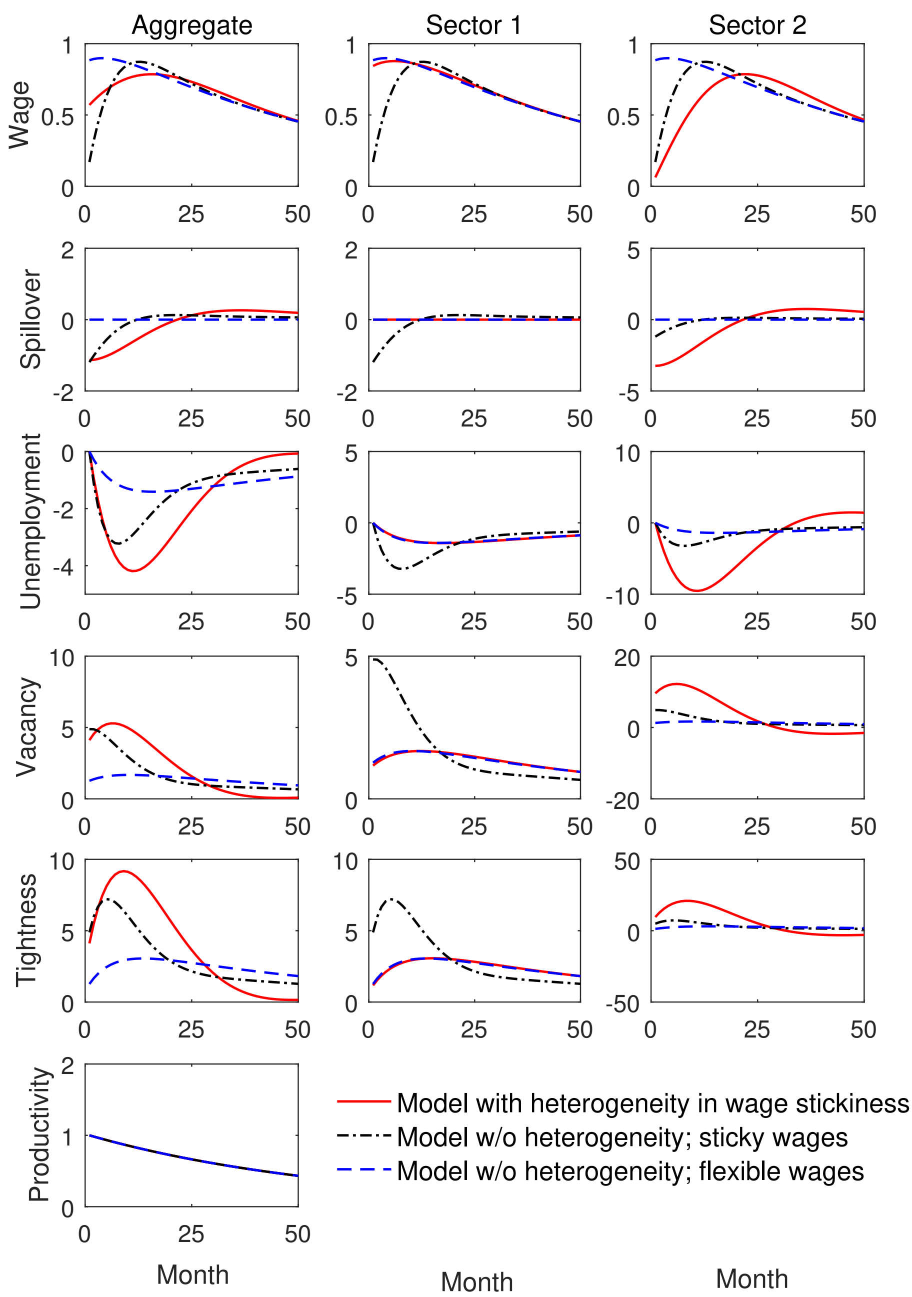

Figure 1: Impulse Response Functions (IRF) to Productivity Shock in Alternative Models
Note: The red lines denote the IRFs in our model with heterogeneity in wage stickiness. The black dash-dotted lines show the IRFs in the one-sector model with sticky wages (the GT model). The blue dashed lines denote the IRFs in the model with flexible wages.

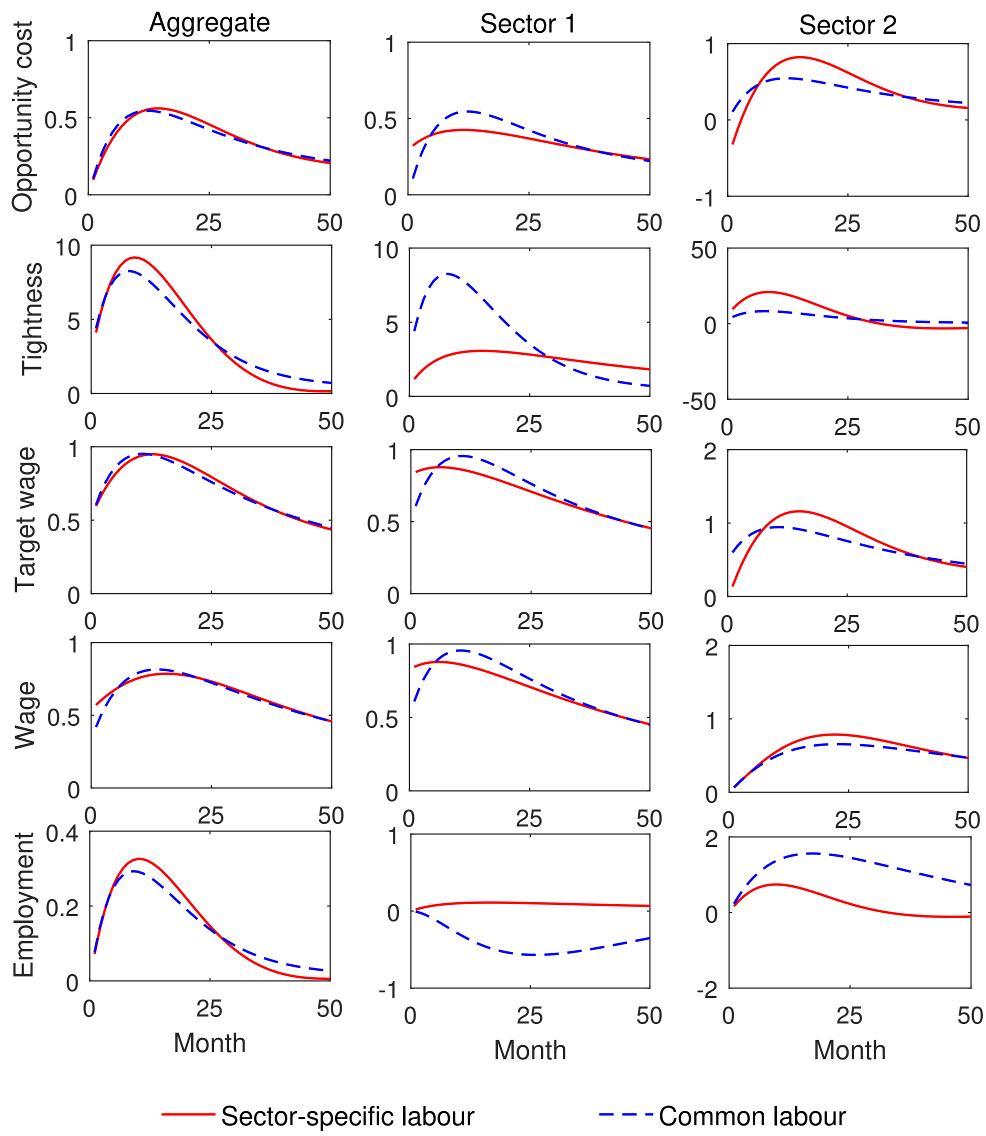

Figure 2: Impulse Response Functions (IRF) to Productivity Shock in Sector-specific Labour Market Model vs. Common Labour Market Model

Note: The red lines denote the IRFs in the benchmark model with sector-specific labour market, while the blue dashed lines show those with the common labour market.

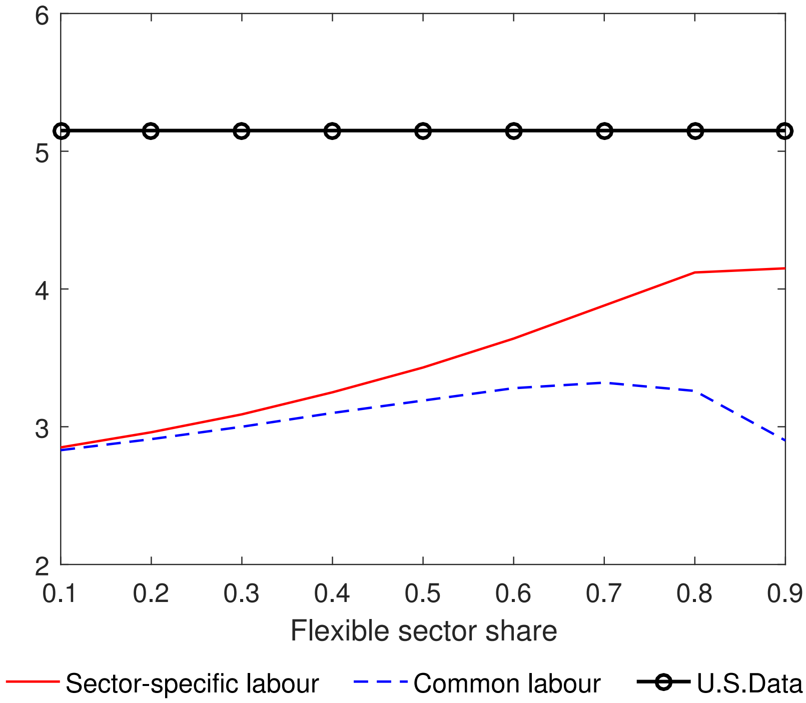

Figure 3: Unemployment Volatility with Different Flexible Sector Share in Sector-specific Labour Market Model vs. Common Labour Market Model

Note: The red line denotes the unemployment volatility in the sector-specific labour market model for each flexible sector share. The blue dashed line shows the volatility in the common labour market model while the black circled line indicates the unemployment volatility in the U.S. data during 1964:Q1-2005:Q1. Unemployment volatility is relative to output.

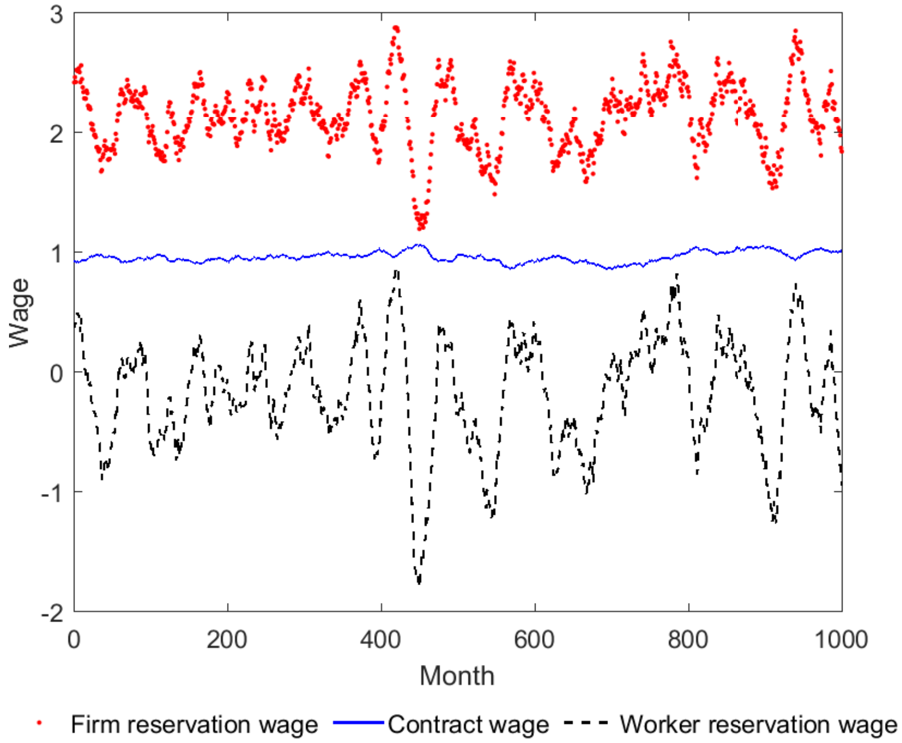

Figure 4: Simulated Bargaining Set for 25-Month-old contract

Note: The red dotted line shows the firm reservation wage, while the black dashed line denotes the worker reservation wage. The blue line denotes the wage rate that has survived 25 months. All observations are obtained by a simulation conditional on productivity shocks with standard deviation 0.75%.