CESifo WORKING PAPERS

6541
2017

Original Version: June 2017
This Version: August 2019

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*Fabrizio Germano, Francesco Sobbrio*

CESifo

## Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editors: Clemens Fuest, Oliver Falck, Jasmin Gröschl
www.cesifo-group.org/wp

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CESifo *Working Paper No. 6541*
*Category 14: Economics of Digitization*

# Opinion Dynamics via Search Engines
## (and Other Algorithmic Gatekeepers)

## Abstract

Ranking algorithms are the information gatekeepers of the Internet era. We develop a stylized model to study the nterplay between a ranking algorithm and individual clicking behavior. We consider a search engine that uses an algorithm based on popularity and on personalization. The analysis shows the presence of a feedback effect, whereby individuals clicking on websites indirectly provide information about their private signals to successive searchers through the popularity-ranking algorithm. Accordingly, when individuals provide sufficiently positive feedback to the ranking algorithm, popularity-based rankings tend to aggregate information while personalization acts in the opposite direction. Moreover, we find that, under fairly general conditions, popularity-based rankings generate an advantage of the fewer effect: fewer websites reporting a given signal attract relatively more traffic overall. This highlights a novel, ranking-driven channel that can potentially explain the diffusion of misinformation, as websites reporting incorrect information may attract an amplified amount of traffic precisely because they are few.

JEL-Codes: D830.

Keywords: ranking algorithm, information aggregation, asymptotic learning, popularity ranking, personalized ranking, misinformation, fake news.

*Fabrizio Germano*
*Department of Economics and Business*
*Universitat Pompeu Fabra & Barcelona*
*GSE / Barcelona / Spain*
*fabrizio.germano@upf.edu*

*Francesco Sobbrio*
*Department of Economics and Finance*
*LUISS "G. Carli"*
*Italy - 00197 Rome*
*fsobbrio@luiss.it*

First Version: December 2016. This Version: August 2019
We thank Larbi Alaoui, Jose Apesteguia, Emilio Calvano, Francesco Cerigioni, Stefano Colombo, Andrea Galeotti, Lisa George, Roberto Imbuzeiro Oliveira, Matthieu Manant, Andrea Mattozzi, Ignacio Monzón, Antonio Nicolò, Nicola Persico, Vaiva Petrikaite, Christian Peukert, Alessandro Riboni, Emanuele Tarantino, Greg Taylor and seminar participants in Athens (EARIE 2018), Barcelona (UPF), Florence (EUI), Geneva (EEA 2016), Hamburg (Economics of Media Bias Workshop 2015), Naples (Media Economics Workshop 2014), Padova (UP), Palma de Mallorca (JEI 2016), Paris (ICT Conference 2017), Petralia (Applied Economics Workshop 2015), Rome (IMEBESS 2016) and Toulouse (Digital Economics Conference 2018) for useful comments and conversations. We are also indebted to Riccardo Boscolo for helpful discussions on the functioning of search engines. Germano acknowledges financial support from grant ECO2017-89240-P (AEI/FEDER, UE), from Fundación BBVA (grant "Innovación e Información en la Economía Digital") and also from the Spanish Ministry of Economy and Competitiveness, through the Severo Ochoa Programme for Centres of Excellence in R&D (SEV-2015-0563).

## 1 Introduction

Search engines are among the most important information gatekeepers of the Internet era. Google alone receives over 3.5 billion search queries per day, and, according to some estimates, 80% of them are informational, dwarfing navigational and transactional searches (Jansen *et al.*, 2008). Individuals increasingly use search engines to look for information on a vast array of topics such as science (Horrigan, 2006), birth control and abortion (Kearney and Levine, 2014), or the pros and cons of alternative electoral outcomes (e.g., the Brexit referendum in the UK; see *Google Trends*). Remarkably, what determines the ranking of any website to be displayed for any given search query are automated algorithms. Such algorithms are also used by social media—such as Facebook and Twitter—to rank their posts or tweets. They are so fundamental in establishing what is relevant information or what are relevant information sources, that they necessarily convert search engines and social media platforms into *de facto* "algorithmic gatekeepers" (Introna and Nissenbaum, 2000; Rieder, 2005; Granka, 2010; Napoli, 2015; Tufekci, 2015). Despite the importance of such ranking algorithms for a wide variety of online platforms, opinion dynamics via algorithmic rankings is largely understudied.

This paper aims to fill this gap by developing a dynamic framework that studies the interaction between individual searches and a stylized ranking algorithm. Specifically, we focus on two key aspects of ranking algorithms: ($a$) rankings may be based on the *popularity* of the different websites, and ($b$) rankings may be *personalized* and may depend on individuals' characteristics. These two aspects drive the ranking of the available websites provided to individuals by a search engine.

Individuals use the search engine to look for information on the state of the world (e.g., whether or not a specific vaccine is safe). Their choices over websites are modeled by means of a stochastic choice model (Luce, 1959; Block and Marschak, 1960; Gül *et al.*, 2014). Each individual derives value from reading the content of a website, which comes from two sources or attributes, namely, ($i$) from reading news that confirms the individual's prior on the state of the world, and ($ii$) from reading majoritarian news. At the same time, individuals have an *attention bias* that translates in a propensity to choose websites that are higher-ranked (De Cornière and Taylor, 2014; Taylor, 2013; Hagiu and Jullien, 2014; Burguet *et al.*, 2015). Accordingly, individuals may trade off content and ranking as a result of their stochastic choices. This yields probabilities of reading the different websites that depend on both the website content and on the ranking.

Website rankings are endogenous, meaning that individuals' choices feed back into the search engine's ranking, thereby affecting future searches (Demange, 2014a). It is important to point out that individuals are naïve with respect to the search engine's algorithm in the sense that they do not make any inference from the websites' ranking *per se*. This last assumption—apart from making the model tractable—reflects informational and behavioral limitations of individuals in understanding the working of ranking algorithms (Granka, 2010; Eslami *et al.*, 2016).[1]

The model looks at the implications of ranking algorithms on clicking dynamics, under the implicit assumption that this also reflects individuals' opinion dynamics (that is, the contents of the websites read implicitly affect the opinions of the individuals).[2] The analysis provides three main insights deriving from the interaction of the ranking algorithm with sequential individual searches. First, it shows the presence of a feedback effect, whereby individuals clicking on websites indirectly provide information about their private signals to successive searchers through the popularity-ranking algorithm. The ranking algorithm thus acts as a mechanism for aggregating information dispersed across agents. Importantly, we provide conditions under which a popularity-based ranking can effectively aggregate information. We compare asymptotic learning under a popularity-based ranking and under uniform random ranking. We find that a popularity-based ranking does better as long as individuals generate sufficiently positive feedback through their searches (their private signals are not too noisy and the desirability of reading confirmatory news is sufficiently low (high) when majoritarian opinion is ex-ante more (less) informative than the individual prior on the state of the world).

Second, the model shows that popularity-based rankings induce what we call an *advantage of the fewer* ($AOF$) effect. It says that, all else equal, fewer websites carrying a given signal may attract more traffic overall, than if there were more of them. Popularity-based rankings amplify the static effect of simply concentrating audiences on fewer websites, since they induce relatively higher rankings for the fewer websites, which makes them more attractive for subsequent individuals, further raising their ranking and so on, generating a *"few get richer"* dynamic with a potentially sizeable effect in the limit. To the best of our knowledge, this property of popularity-based rankings has not yet been pointed out in the literature. We show that $AOF$ holds under fairly broad assumptions. Importantly, $AOF$ highlights a further, ranking-driven, channel for why "alternative-facts" websites may thrive and gain in authority in the current information environment, dominated by algorithmic gatekeepers, such as search engines and social media (Allcott and Gentzkow, 2017; Allcott *et al.*, 2018).[3]

[1]While popularity and personalization are well-established components of ranking algorithms, exact details of the algorithms are typically kept secret (Dean, 2013; MOZ, 2013; Vaughn, 2014; Kulshrestha *et al.*, 2018). As Eslami *et al.* (2016), p. 1, point out, the "operation of these algorithms is typically opaque to users."

[2] Epstein and Robertson (2015) provide empirical evidence on the ranking of webpages affecting individual choices over websites and, in turn, their voting preferences.

[3]Allcott and Gentzkow (2017) provide an economic model of fake-news and also document that, in the run-up to the 2016 US presidential election, more than 60% of traffic of fake news websites in the US came from referrals by algorithmic gatekeepers (i.e., search engines and social media). See also Azzimonti and Fernandes (2018) for a model on the diffusion of fake-news in social networks.

Third, we study whether personalized rankings can contribute to information aggregation. We compare asymptotic learning under both personalized and non-personalized rankings. We find that there is a close relationship between the conditions under which non-personalized rankings outperform personalized rankings with the ones under which popularity-based rankings outperform random rankings. For the common-value searches of our model, personalization limits the feedback among individuals in the opinion dynamic, making it better *not* to personalize the ranking when individuals' choices over websites generate a positive feedback. As a consequence, personalized rankings are often dominated in terms of asymptotic learning either by non-personalized rankings or by random rankings. Finally, we also show that personalized rankings can induce relatively similar individuals to read different websites, which can lead to *belief polarization*. At the same time, if individuals observe different (personalized) rankings but such personalization is not based on ex-ante differences across individuals (e.g., different values of reading confirmatory news), belief polarization might only be a short-run phenomenon. In the long run, asymptotic clicking dynamics would not be affected by personalized rankings.

We conclude with three caveats. The results of our model provide some first insights on opinion dynamics via an endogenous ranking used by naïve individuals. A similar model with more sophisticated individuals who can observe the evolution of the ranking sufficiently accurately may well predict that such individuals are likely to always learn the true state of the world in the limit (see Section 5.2). As emphasized above, the naïveté in our model reflects informational and behavioral limitations of individuals in assessing the working of the ranking algorithm and provides a natural benchmark for understanding online misinformation. Second, as actual algorithms used by platforms such as *Google* or *Facebook* are highly complex (Dean, 2013; MOZ, 2013; Vaughn, 2014), we do not aim to pin down the exact pattern of website traffic or opinion dynamic generated by any specific algorithm, nor do we pursue a mechanism design approach.[4] Rather, our aim is to isolate few essential components of ranking algorithms (*popularity* and *personalization*) and to study their interplay with individual clicking behavior. Finally, while our results suggest that personalization may often be sub-optimal in the context of individuals looking for information on common value issues (e.g., whether or not to vaccinate a child), when individuals care differently about the objects of their searches (e.g., where to have dinner) an appropriately personalized search algorithm might clearly outperform a non-personalized one.

**Related Literature.** To our knowledge, ours is the first paper in economics to analyze opinion dynamics via endogenous algorithmic rankings and the informational gate-keeping role of search engines.[5] The

[4] Section 2.7 provides some background information on the working of ranking algorithms.

[5] The existing economics literature on search engines has focused on the important case where search engines have an incentive to distort sponsored and organic search results in order to gain extra profits from advertising and product markets (Taylor, 2013; De Cornière and Taylor, 2014; Hagiu and Jullien, 2014; Burguet *et al.*, 2015). See also Grimmelmann (2009)

paper is broadly related to the economics literature on the aggregation of information dispersed across various agents (Bikhchandani *et al.*, 1998; Piketty, 1999; Acemoglu and Ozdaglar, 2011; Golub and Sadler, 2016; provide surveys on information aggregation, respectively, through observation of behavior of others, through voting, and through learning in social networks). We share with this literature the focus on understanding the conditions under which information that is dispersed among multiple agents might be efficiently aggregated. Our framework differs from this literature in a simple and, yet, crucial aspect: we are interested in investigating the role played by a specific (yet extensively used) "tool" of information diffusion/aggregation, namely, the ranking algorithm, which represents the backbone of many online platforms. Another feature we have in common with a subset of this literature is that we take a non-Bayesian approach and consider individuals who are naïve with respect to some key aspects of their choice situation (i.e., the ranking algorithm). In this sense, our model is closer to the papers on non-Bayesian belief formation (DeGroot, 1974; DeMarzo *et al.*, 2003; Acemoglu *et al.*, 2010; Golub and Jackson, 2010). Moreover, although our individuals perform only one search, "learning" occurs through the individuals' clicking behavior and the ranking algorithm that aggregates the information reflected in the clicking behavior and passes it on to subsequent individuals.

Our focus on search engines as information gatekeepers is close in spirit to the economic literature on news media (see DellaVigna and Gentzkow, 2010; Prat and Strömberg, 2013, for surveys). At the same time, the presence of an automated ranking algorithm makes search engines—and other algorithmic gatekeepers—fundamentally different from news media, where the choice of what information to gather and disclose is made on a discretionary, case-by-case basis. In the case of search engines the gate-keeping is unavoidably the result of automated algorithms (Granka, 2010; Tufekci, 2015).[6] Therefore, whatever *bias* might originate from search engines, its nature is intrinsically different from one arising in, say, traditional news media. As a result, studying the effects of search engines on the accuracy of individuals' beliefs, requires a different approach from the ones used so far in theoretical models of media bias (e.g., Strömberg, 2004; Mullainathan and Shleifer, 2005; Gentzkow and Shapiro, 2006).

Finally, the paper is also related to the literature outside economics discussing the possible implications of the search engines' architecture (or, more generally, of algorithmic gatekeeping) on democratic outcomes. This literature encompasses communication scholars (Hargittai, 2004; Granka, 2010), legal scholars (Goldman, 2006; Grimmelmann, 2009; Sunstein, 2009), media activists (Pariser, 2011), psychologists (Epstein and Robertson, 2015), political scientists (Putnam, 2001; Hindman, 2009; Lazer, 2015),

and Hazan (2013) for a legal perspective on the issue.

[6] Put differently, "While humans are certainly responsible for editorial decisions, these [search engine] decisions are mainly expressed in the form of software which thoroughly transforms the ways in which procedures are imagined, discussed, implemented and managed. In a sense, we are closer to *statistics* than to *journalism* when it comes to bias in Web search"; Rieder and Sire (2013), p.2.

sociologists (Tufekci, 2015) and, last but not least, computer scientists (Cho *et al.*, 2005; Menczer *et al.*, 2006; Pan *et al.*, 2007; Glick *et al.*, 2014; Flaxman *et al.*, 2013; Bakshy *et al.*, 2015).

The paper is structured as follows. Section 2 describes the framework. Section 3 presents benchmark results on opinion dynamics with rational (and naïve) individuals. Section 4 discusses our main results on opinion dynamics with behavioral individuals. Specifically, Section 4.1 presents our basic result on the the advantage of the fewer ($AOF$), Section 4.2 studies the implications of the model in terms of asymptotic learning, also providing a comparison with a fully randomized ranking, and Section 4.3 studies the effects of personalization of search results on belief polarization and asymptotic learning. Section 5 discusses some extensions, including the case of sophisticated learning. Section 6 concludes. All the proofs and some formal definitions are relegated to the Appendix.

# 2 The Model

We present a stylized model of a search environment where individuals use a search engine to look for information on a fixed issue (e.g., whether or not to vaccinate a child). At the center of the model is a search engine characterized by its ranking algorithm, which ranks and directs individuals to the different websites, using, among other things, the popularity of individuals' choices. To simplify the analysis, we assume that individuals are naïve and perform exactly one search, one after the other, without knowing who searched before them and without updating prior beliefs after observing the ranking. We also assume websites simply report their own private signal, assumed to be constant throughout. We describe the formal environment.

## 2.1 Information Structure

There is a binary state of the world $\omega$, which is a $\left(\frac{1}{2}, \frac{1}{2}\right)$ Bernoulli random variable which takes one of two values from the set $\{0, 1\}$. There are $M$ information sources (websites) and $N$ individuals, where, by slight abuse of notation, we let $M = \{1, \ldots, M\}$, $N = \{1, \ldots, N\}$ also denote the set of websites and individuals, respectively. For convenience, we assume $M$ is an odd number (unless otherwise noted). Each website $m \in M$ receives a private random signal, correlated with the true state $\omega$,

$$y_m \in \{0, 1\} \text{ with } \mathbb{P}(y_m = \omega \mid \omega = \xi) = q \in \left(\tfrac{1}{2}, 1\right), \text{ for any } \xi \in \{0, 1\}.$$

This determines the *website majority signal*, denoted $y_K \in \{0, 1\}$, which is the signal that is carried by a majority of websites; let $K = \{m \in M \mid y_m = y_K\}$ denote the set (and number) of websites carrying the signal $y_K$. Similarly, each individual $n \in N$ receives two private random signals:

$$x_n \in \{0,1\} \text{ with } \mathbb{P}(x_n = \omega \mid \omega = \xi) = p \in \left(\tfrac{1}{2}, 1\right), \text{ for any } \xi \in \{0,1\},$$

which reflects the individual's prior on the true state of the world, and

$$z_n \in \{0,1\} \text{ with } \mathbb{P}(z_n = y_K \mid y_K = \zeta) = \mu \in \left(\tfrac{1}{2}, 1\right], \text{ for any } \zeta \in \{0,1\},$$

which is independent of $x_n$, when conditioned on $(\omega, y_K)$, and reflects the individual's prior about what the majority of websites (e.g., mainstream or "authoritative" websites) are reporting. We assume all signals to be conditionally independent across individuals and websites, when conditioned on $(\omega, y_K)$.

From the onset, we emphasize that the case $\mu \leq 1$ is meant to capture the intrinsic noise faced by individuals in identifying the ex-ante more informative website majority signal ($y_K$). Accordingly, $(1-\mu)$ represents the probability that an individual incorrectly identifies the minority signal as the majority one (e.g., looking at a website reporting "alternative facts" believing that it is reporting mainstream or "authoritative" news). All the main conclusions remain unchanged if we assume $\mu = 1$, meaning that individuals can perfectly identify the majority signal (e.g., individuals know which are the mainstream websites appearing in the search result pages).

## 2.2 Information sources

Each of the $M$ websites represents an information source. A website is characterized by a signal $y_m \in \{0,1\}$ as described above, which is posted and held constant throughout. Absent any ranking effect, websites with the same signal (with $y_m = y_{m'}$) are seen as equivalent (hence, they are perfect substitutes from the individuals' perspective). This allows us to naturally partition the set of websites $M$ into two classes, namely, websites with signal 0 and ones with signal 1. Websites can be seen as articles or documents posted on the web that contain pertinent information to a given search query.

## 2.3 Individuals

Individuals in $N$ enter in a random order, sequentially, such that, at any point in time $t$, there is a unique individual $t \equiv n \in N$, who performs exactly one search, faces the ranking $r_n$, and chooses a website to read, based on his preferences and the information contained in his private signal $(x_n, z_n) \in \{0,1\}^2$ as specified above. Importantly, individuals do not update their information nor derive value from observing the headlines (or "snippets") of the ranked websites, even if they identify the websites' signals

($y_m$). The idea is that, while it easy to identify a website's signal (pro/against vaccines) from the headline provided in the search result page—as it is easy to identify the content of a newspaper article from its title—individuals need to read the posted website to learn and be informed of its content ($y_m$) and to derive utility from that. At the same time, individuals are also naïve as they do not update their beliefs from observing the ranking per se.

**Website content.** To focus on the content, suppose for a moment that individuals see the websites ranking-free. We model clicking behavior by means of a stochastic choice rule accounting for the fact that two websites with the same signal are perfect substitutes in our model.[7] We assume each individual $n$ derives value from reading the content of a website, which comes from two sources or attributes, namely, ($i$) from reading news that confirms the individual's prior on the state of the workd — this happens when $x_n = y_m$ and gives value $v_x > 0$, and gives value 0 otherwise (when $x_n \neq y_m$);[8] and ($ii$) from reading majoritarian news — this happens when $z_n = y_m$ and gives value $v_z > 0$, and gives value 0 otherwise (when $z_n \neq y_m$). We defer to Section 3 a discussion of a when a rational individual, interested in reading ex ante most informative websites, may find it optimal to read websites that confirm his prior or report the majoritarian opinion.

Letting $c \in \{0, 1\}$ denote the class type of a website, we can write individual $n$'s desirability value from reading a website from class $c$ as:

$$V_c(x_n, z_n) = \mathbb{I}_{\{x_n=c\}} \cdot v_x + \mathbb{I}_{\{z_n=c\}} \cdot v_z. \tag{1}$$

Clearly, if $c = \emptyset$, then $V_c(x_n, z_n) = 0$. Denoting the complement of class $c$ by $\neg c \equiv 1 - c$, we can write the *Luce choice rule* of choosing a website from class $c$ as:

$$\rho_c(x_n, z_n) = \frac{V_c(x_n, z_n)}{V_c(x_n, z_n) + V_{\neg c}(x_n, z_n)}. \tag{2}$$

Accordingly, consider the class of websites with signal 1 ($c = 1$). Then, an individual $n$ with signal $(x_n, z_n) = (1, 1)$ has desirability $V_1(1, 1) = v_x + v_z$ from reading any website from that class, while if the signal is $(1, 0)$, $(0, 1)$, or $(0, 0)$ the desirability is, respectively, $v_x$, $v_z$ or 0. The corresponding choice probabilities of choosing a website from that class are $\rho_1(1, 1) = 1$ for individuals with signal (1,1), $\frac{v_x}{v_x+v_z}$ for signal (1,0), $\frac{v_z}{v_x+v_z}$ for signal (0,1) and 0 for signal (0,0). Analogous choice probabilities can be computed for choosing websites from the opposite class. Let $\gamma \equiv \frac{v_x}{v_x+v_z} \in (0, 1)$ calibrate the

[7] See Luce (1959) and Block and Marschak (1960) for early papers on stochastic choice rules (or Luce rules) and Gül *et al.* (2014) on attribute rules allowing for duplicate alternatives; Agranov and Ortoleva (2017) provide empirical evidence.

[8] Mullainathan and Shleifer (2005); Gentzkow and Shapiro (2010) study individual preference for confirmatory news; Yom-Tov *et al.* (2013); Flaxman *et al.* (2013); White and Horvitz (2015) provide evidence in the context of search engines.

desirability for reading confirmatory news, and let $1-\gamma$ $(=\frac{v_z}{v_x+v_z})$ calibrate the desirability for reading the majoritarian news. Then, dividing by the number of websites in the relevant class to keep track of what in our model are equivalent or duplicate websites, we can write individual $n$'s *ranking-free choice function* for website $m$ as:

$$\rho^*_{n,m} = \begin{cases} 0 & \text{if } x_n \neq y_m \neq z_n \\ \frac{\gamma}{[m]} & \text{if } x_n = y_m \neq z_n \\ \frac{1-\gamma}{[m]} & \text{if } x_n \neq y_m = z_n \\ \frac{1}{[m]} & \text{if } x_n = y_m = z_n \end{cases}, \tag{3}$$

where $[m] = \#\{m' \in M | y_{m'} = y_m\}$ is the number of all websites with the same signal as website $m$.[9,10]

**Website rankings and stochastic choice function.** Let $r_n \in \Delta(M)$ be the *ranking* of the $M$ websites seen by individual $n$. The ranking is provided by the search engine (as discussed in the next subsection, as the ranking evolves endogenously accordingly to the stochastic individual choices over websites, it represents a distribution over websites). As mentioned, individuals ignore who searched before them and do not update their beliefs about the true state of the world $\omega$ after observing the ranking. Moreover, we assume they can process the ranked list, subject to limitations of attention, meaning they implicitly favor higher ranked websites due to attention bias.[11] Given that the ranking $(r_n)$ is modeled as a probability distribution over the websites, a natural way to incorporate the attention bias is by writing final clicking probabilities as ranking-weighted probabilities.[12] This yields the *choice function* for website $m$ defined by:

$$\rho_{n,m} = \frac{(r_{n,m})^\alpha \cdot \rho^*_{n,m}}{\sum_{m' \in M} (r_{n,m'})^\alpha \cdot \rho^*_{n,m'}}, \tag{4}$$

where the parameter $\alpha \geq 0$ calibrates the intensity of the individual's *attention bias*.[13] Here $\alpha = 1$ is a neutral benchmark in that it maintains the weight differences already present in the entries of the ranking $r_n$; $\alpha > 1$ magnifies the differences in the entries of $r_n$; $\alpha < 1$ reduces the differences in

[9] The division by $[m]$ reflects that all websites with the same signal are perfect substitutes and also avoids the usual Luce effect or "duplicate problem" when dealing with such duplicate alternatives (see, e.g., Gül *et al.*, 2014).

[10] To simplify notation, we omit the arguments $(x_n, z_n; y_m)$ of functions like $\rho^*_{n,m}$ or $\rho_{n,m}$ and keep track of them through just the subscripts $n, m$.

[11] As shown by Pan *et al.* (2007); Glick *et al.* (2014); Yom-Tov *et al.* (2013); Epstein and Robertson (2015), keeping all other things equal (e.g., the fit of a given website with respect to the individual's preferences), highest ranked websites tend to receive significantly more "attention" by users than lower ranked ones. More generally, when faced with ordinal lists, individuals often show a disproportionate tendency to select options at the top (Novarese and Wilson, 2013).

[12] Another (equivalent) alternative of deriving our choice rules is to include the ranking as a weight discounting the desirability values obtained from the content, and write the choice rule as a new Luce rule over the ranking-weighted values; see Cerigioni and Galperti (2019) for such an approach of modeling frame-order effects on desirability levels.

[13] Fortunato *et al.* (2006) and Demange (2014a) use related models of individuals' choices over ranked items.

the entries of $r_n$; so that, in the limit, as $\alpha \to 0$, all entries have the same weight, which represents the case with *no* attention bias, where all websites that provide the same signal yield the same value. Specifically, when $\alpha = 0$, $\rho_{n,m}$ implies that choices coincide with the ranking-free choices $\rho^*_{n,m}$, since all websites have the same weight $(r_{n,m})^0 = 1$.

Overall, the $\rho_{n,m}$'s summarize the following properties of website clicking behavior, namely, individuals tend to choose websites that (*i*) reflect their own prior information, (*ii*) report their perceived majoritarian opinion, (*iii*) are higher ranked and hence (*iv*) may also trade off content and ranking.

### 2.4 Search Engine and Ranking Algorithm

The *ranking algorithm* used by the search engine is at the center of our model. Whenever individual $n$ makes a search, she sees the *ranking* $r_n \in \Delta(M)$ of the $M$ websites, where an element $r_{n,m}$ is the probability that the individual $n$ is directed to website $m$ in the absence of other factors.[14]

Concretely, we assume that, given an initial ranking $r_1 \in \Delta(M)$, the *ranking* $r_n$ seen by subsequent individuals, $n = 2, 3, \ldots, N$, for website $m$ is defined by:

$$r_{n,m} = (1-\nu) r_{n-1,m} + \nu \rho_{n-1,m}, \tag{5}$$

where $\nu \in (0, 1)$ calibrates the popularity aspect of the ranking.[15] The larger $\nu$, the larger the weight placed on the individual's website search in the previous period and the less persistent is the search engine's ranking. Such ranking dynamic reflects the algorithm used by search engines to update their rankings according to how "popular" a webpage is (Dean 2013; MOZ 2013; Vaughn 2014, Section 2.7 presents background information on the working of ranking algorithms). Accordingly, we refer to such a ranking as *popularity-based* or simply *popularity ranking*.

### 2.5 Search Environments

A search environment is an *ex ante* notion that fixes the ranking algorithm, information structure and characteristics of individuals and websites, before they receive their signals and before they perform their search. More formally, we define a *search environment* $\mathcal{E}$ as a list of variables,

$$\mathcal{E} = \langle (p, q, \mu); (N, \gamma, \alpha); M; \nu \rangle, \tag{6}$$

[14] The ranking here assigns a cardinal score to each website in each period and, as in Demange (2014b, p. 918), measures the "relative strength of $[M]$ items, meaning that the values taken by the scores matter up to a multiplicative constant."

[15] See Demange (2012) for a similar specification of the popularity-ranking algorithm. Notice that, for simplicity, we let the choice probabilities rather than their realizations enter the ranking. Since by construction, the two have the same expectations, using the choice probabilities does not change the limits of the stochastic process, as shown in Appendix A, and hence does not affect any of our main results.

where $(p, q, \mu)$ describes the information structure, $(N, \gamma, \alpha)$ describes the individuals, $M$ describes the websites, and $\nu$ describes the ranking algorithm. Given a search environment $\mathcal{E}$, we refer to an *(interim) realization* of $\mathcal{E}$ as to the tuple $\langle \omega; (L, (y_m)_{m\in M}); ((x_n)_{n\in N}; (z_n)_{n\in N})\rangle$, where the true state of the world and the signals of the websites are fixed; $L$ denotes the set and the number of websites with the correct signal $y_m = \omega$; individuals with signals $(x_n; z_n)$ enter sequentially, one at a time, in a random order.

We let $r_1$ denote the *initial ranking*. Unless otherwise specified, we assume that $r_1$ is interior, that is, $r_{1,m} > 0$ for all $m \in M$. For $J \subset M$, let $r_{n,J} = \sum_{m\in J} r_{n,m}$ and $\rho_{n,J} = \sum_{m\in J} \rho_{n,m}$ denote respectively total ranking and total clicking probability at time $n$ on all websites in $J$; we will be particularly interested in the case where $J = L$. We also talk about the *expected* probability of individual $n$ accessing website $m$, $\widehat{\rho}_{n,m} = \mathbb{E}[\rho_{n,m}]$, where the expectation is taken over the private signals of agent $n$ that enters to perform the search. (These expected probabilities are discussed in more detail in Appendix A.)

### 2.6 Asymptotic Learning

To assess the effect of algoritmic gatekeepers on opinion dynamics, we consider search environments from an *interim* and an *ex-ante* perspective. Since our model endogenizes both ranking and individual clicking probabilities, it seems natural to evaluate efficiency in terms of asymptotic probability of clicking on a website carrying the correct signal. We interpret this notion of efficiency as asymptotic learning under the implicit assumption that individuals naïvely update their beliefs by only relying on the content of the website they read.[16]

Consider the probability of individual $n$ choosing a website reporting a signal corresponding to the true state of the world $(y_m = \omega)$. At the interim stage, we can write this as the probability $\rho_{n,L}$ $(= \sum_{m\in L} \rho_{n,m})$ of individual $n$ clicking on any website $m \in L$. We can also define a measure of *interim efficiency* $(\mathcal{P}_L)$, conditional on interim realizations, where the total number of websites reporting the correct signal is $L$, as:

$$\mathcal{P}_L(\alpha, \gamma, \mu, p) = \rho_{\infty,L} = \lim_{N\to\infty} \rho_{N,L}. \tag{7}$$

This implies the following measure of *ex ante efficiency* $(\mathcal{P})$:

$$\mathcal{P}(\alpha, \gamma, \mu, p, q) = \sum_{L=0}^{M} \binom{M}{L} q^L (1-q)^{M-L} \mathcal{P}_L(\alpha, \gamma, \mu, p), \tag{8}$$

which uses the accuracy of websites' signals $(q)$ to weigh the different interim levels $(\mathcal{P}_L)$.

[16] In a series of experiments on voters' opinions and preferences, Epstein and Robertson (2015) show how changing the ranking of websites may have a large impact on the website individuals read and, in turn, on political preferences over alternative candidates.

Throughout the paper, when studying asymptotic behavior, to guarantee that the stochastic dynamics is well-behaved in the limit, we set the popularity-ranking parameter $\nu = O(\frac{1}{N})$ such that $\nu \to 0$ as $N \to \infty$, ensuring that the effect of an individual's search on the ranking becomes vanishingly small in the limit, and we also restrict the ranking probabilities to always be interior, that is, $r_{t,m} \geq \epsilon$ for some arbitrarily small $\epsilon > 0$. This ensures that the dynamic process converges to its unique limit. Moreover, to better highlight the role of popularity ranking on asymptotic learning, we will use as benchmark of comparison, the case where ranking is random and uniform throughout ($r_{n,m} = \frac{1}{M}$ for all $n, m$), which we refer to simply as *random ranking.*

As explained in Appendix A, we compute the limit ranking and limit clicking probabilities using the mean dynamics approximation (Norman, 1972; Izquierdo and Izquierdo, 2013). This involves fixing an interim search environment (essentially characterized by the number of websites $L$ carrying the correct signal) and approximating the stochastic clicking probabilities $\rho_{n,m}$ in Eq. (4) for $m \in M$ by their expectations $\widehat{\rho}_{n,m} = \mathbb{E}[\rho_{n,m}]$. This leads to deterministic recursions that are easily computed in the limit by means of ordinary differential equations. To obtain the ex ante efficiency, we take expectations over all interim environments as specified in Eq. (8).

## 2.7 Discussion

Before presenting the implications of the model, we briefly discuss and provide some background information regarding the two main building blocks, namely, the individuals' stochastic choices over websites and the ranking algorithm.

**Stochastic choice over websites.** The stochastic choice rules presented in Section 2.3 are derived from certain desirability values. Specifically, we follow Gül *et al.* (2014) and consider each website has having a bundle of subjective attributes which depend on the individual private signals ($x_n$ and $z_n$) that generate the desirability values (respectively, $v_x$ and $v_z$). This leads to the Luce-type choice rules defined in Eq. (3). Block and Marschak (1960) shows that every Luce model might be derived from a random utility model (see also Becker *et al.* (1963) for a related discussion). Such a model might also be derived by an (additively) perturbed utility model with appropriately defined costs (Anderson *et al.*, 1992; Fudenberg *et al.*, 2015). Furthermore, Theorem 3 in Gül *et al.* (2014) extends the result of Block and Marschak (1960) by showing that every attribute rule (such as the one in Eq. (3)) is a random utility maximizer.

**Ranking algorithm.** There are two main challenges in constructing a formal model providing a stylized representation of a search engine's algorithm. First, search algorithms are complex: Google uses around

200 signals in determining the ranking of search results for a given query (Dean, 2013; MOZ, 2013; Vaughn, 2014). Second, the exact features of the algorithms actually used by search engines typically represent a commercial secret.[17] Nevertheless, while the exact details on these ranking algorithms are kept secret, certain key features constitute "common wisdom" among folk-tech experts (Dean, 2013; MOZ, 2013; Vaughn, 2014). Most importantly, the main components of search algorithms are well known and studied by computer scientists.

A first set of signals used by search engines to decide the ranking of a webpage (for a given query) are based on *ex-ante* parameters, such as *PageRank*, that the algorithm uses to establish the initial rank of that webpage.[18] A second set of parameters of search engines' algorithms is instead referring to the *popularity* of a websites which is captured via users-interaction (e.g., click-through-rate) and social signals (e.g., webpage likes on Facebook, tweets linking to the webpage). That is, search algorithms exploit usage data to analyze how individuals actually choose among the results displayed in the search engine's result page for a given query. Accordingly, search engines update the initial ranking of a webpage according to how "popular" the webpage is.[19] Finally, search engines' algorithms personalize search results according to individual characteristics such as, for example, her geographical location (e.g., IP address) and her past search and browsing behavior (Pariser 2011; Dean 2013; MOZ 2013; Vaughn 2014; Hannak *et al.* 2013; Xing *et al.* 2014; Kliman-Silver *et al.* 2015).[20]

In our model, to simplify and preserve tractability, we consider the initial ranking $r_1$ as reflecting the first set of *ex-ante* parameters used by algorithms to establish the initial authority or rank of a webpage. The ranking dynamic then reflects the second set of parameters, as specified in Eq. (5). That is, the ranking algorithm updates the initial ranking ($r_1$) and every subsequent ranking of webpages ($r_t$) according to how "popular" the different webpages are. In our model, this is captured by the popularity parameter ($\nu$), whereby the popularity of the webpages is determined by the clicking probabilities ($\rho_n$).

[17] As exemplified by Google "No company wants to share its secret recipes with its competitors." Search engines are secretive about their algorithm also to prevent the abusive use of it. Knowing the exact features of a search engine, websites may try to manipulate their ranking in the search results page by exploiting the components of the algorithm.

[18] *PageRank* is a recursive algorithm proposed by the founders of Google (Brin and Page, 1998). The basic idea behind it is to assign a rank to a website based on the number and quality of inbound links that it receives (i.e., each inbound link is considered as a "vote" in favor of the website by the other website). Other ex-ante parameters include domain factors; page-level factors; site-level factors; back-link factors; and brand-signals. See Dean (2013), MOZ (2013), Vaughn (2014), for a detailed description of these components.

[19] This updating algorithm via the "popularity" of a website may be interpreted both in a strict sense (e.g., direct effect of the actual clicks on the website in the search result page) and in a broad sense (e.g., a website that receives more clicks is also more likely to be more popular in other online platforms and vice versa). See Kulshrestha *et al.* (2018) for empirical evidence on the relevance of the popularity component of algorithmic rankings.

[20] For example, Hannak *et al.* (2013) run an experiment employing two hundred web users via Amazon Mechanical Turk (AMT) and then had their computers run automatic Google searches on a vast array of topics. By comparing the extent of the personalization of search results in the AMT sample with respect to the one obtained in a control one (i.e., by running an automatic script), the authors document the presence of extensive personalization of search results. See also Xing *et al.* (2014) for empirical evidence on search results personalization based on the *Booble* extension of Chrome.

In Section 4.3 we formalize and discuss personalized rankings.

Overall, given the complexity of ranking algorithms, as pointed out in the introduction, our theoretical framework cannot pin down the exact pattern of website traffic or opinion dynamic generated by any specific algorithm, nor it is meant to design an optimal mechanism.[21] Hence, the analysis simply aims to study two main components of ranking algorithms (*popularity* and *personalization*) to asses their interplay with individual clicking behavior.

# 3 Opinion Dynamics with Rational Individuals

This section provides some benchmarks for opinion dynamics with individuals who have no behavioral preference over website content or ranking, but who are nonetheless naïve in the sense that they do not update their information upon observing the ranking ($r_n$).[22] To better identify the implications of relaxing each behavioral assumption present in our model of Section 2, we discuss separately the cases, where individuals: ($i$) derive value only from reading a website carrying the ex ante most informative signal (i.e., they have no behavioral preference over website content); and ($ii$) make stochastic choices that do not depend on the website ranking (i.e., they have no attention bias). We find that when shutting down the behavioral biases, the website ranking becomes inconsequential to opinion dynamics. That is, either the ranking does not affect the class of websites read by individuals (case ($i$)) or the class of websites chosen by any individual is as if the website rankings were a uniformly random (case ($ii$)).

## 3.1 Absence of Behavioral Preferences

Suppose individuals derive value only from reading a website carrying the ex ante most informative signal. Let $w_n$ denote such signal. Rational individuals in this case will click only on websites with $y_m = w_n$. Their ranking-free choice function for website $m$ reduces to:

$$\rho^*_{n,m} = \begin{cases} 0 & \text{if } y_m \neq w_n \\ \frac{1}{[m]} & \text{if } y_m = w_n, \end{cases}$$

[21] Kremer *et al.* (2014) solve for an optimal disclosure policy in the context of a dynamic recommendation system. Palacios-Huerta and Volij (2004) and Altman and Tennenholtz (2008) study axiomatizations of static ranking systems. See Tennenholtz and Kurland (2019) for a recent survey.

[22] In Section 5.2, we briefly discuss the case where rational individuals are also sophisticated in the sense that they observe the whole ranking history and may update their beliefs accordingly. This leads us to distinguish a third source of value, namely, from reading websites carrying the ex ante most informative signal. This distinction is redundant in this section and the next, since agents are assumed to be naïve so that their ex ante most informative signal always coincides with one or both of the signals $x_n$ and $z_n$.

which implies a choice function for website $m$ of:

$$\rho_{n,m} = \begin{cases} 0 & \text{if } y_m \neq w_n \\ \frac{(r_{n,m})^\alpha}{\sum_{m' \in M} \mathbb{I}_{\{y_{m'} = w_n\}} \cdot (r_{n,m'})^\alpha} & \text{if } y_m = w_n. \end{cases}$$

It is immediate to see that, in our framework where individuals have just two signals, the ex-ante most informative signal $w_n$ will correspond to either $x_n$ and $z_n$. In particular:

$$w_n = \begin{cases} z_n & \text{if } p \leq \overline{p} \\ x_n & \text{otherwise,} \end{cases}$$

where

$$\overline{p} = \mu \sum_{K > \frac{M}{2}} \binom{M}{K} q^K (1-q)^{M-K} + (1-\mu) \sum_{K > \frac{M}{2}} \binom{M}{K} q^{M-K} (1-q)^K \tag{9}$$

is the value of $p$ (as a function of $\mu, q$ and $M$) that ensures that $x_n$ and $z_n$ are equally informative of $\omega$. It follows that, when $p < \overline{p}$, rational individuals behave as agents that derive value only from reading majoritarian news ($v_z > 0$ and $v_x = 0$, and hence $\gamma = 0$). Instead, when $p > \overline{p}$, rational individuals behave as agents that derive value only from reading confirmatory news ($v_x > 0$ and $v_z = 0$, and hence $\gamma = 1$). Accordingly, depending on the value of $p$ (in relation to (as a $\mu, q$ and $M$), two cases may arise:

1. If $p < \overline{p}$, rational individuals ignore their signal $x_n$ and follow their ex ante most informative signal $z_n$ thus clicking only on websites reporting (perceived) majoritarian news (i.e., with $y_m = z_n$). Hence, if $p < \overline{p}$ and agents are rational ($\gamma = 0$), each individual reads a website that reports the signal that coincides with that individual's majority signal, regardless of his prior information on the state of the world and regardless of the ranking. Ex ante efficiency is given by $\mathcal{P} = \mu \sum_{K > \frac{M}{2}} \binom{M}{K} q^K (1-q)^{M-K} + (1-\mu) \sum_{K > \frac{M}{2}} \binom{M}{K} q^{M-K} (1-q)^K$ while interim efficiency is given by $\mathcal{P}_L = \mu$ if the majority signal is correct ($L = K$ and hence $y_K = \omega$) and $\mathcal{P}_L = 1 - \mu$ if it is incorrect ($L \neq K$ and hence $y_K \neq \omega$). Since $\mu > 1/2$ there are more clicks on websites with $y_m = y_K$, implying that, in the limit, a website from the (majority) class $c = y_K$ will be top-ranked. In particular, there are feedback effects to the ranking, but because individuals click exclusively on websites carrying the signal $y_m = z_n$, the ranking does not affect the class of website they read, and popularity ranking is irrelevant for asymptotic learning.

2. If $p > \overline{p}$, rational individuals ignore their signal $z_n$ and follow their ex ante most informative signal $x_n$ thus clicking only on websites reporting confirmatory news (i.e., with $y_m = x_n$). Hence, if $p > \overline{p}$ and agents are rational ($\gamma = 1$), each individual reads a website that reports the signal

that coincides with that individual's signal on the true state of the world. Ex ante efficiency $\mathcal{P}$ and interim efficiency $\mathcal{P}_L$ are both equal to $p$ (excepting cases $L = 0, M$). Since $p > 1/2$ there are more clicks on websites with $y_m = \omega$, implying that, in the limit, a website from the class with the correct signal, $c = \omega$, will be top-ranked. Thus again, while there are feedback effects to the ranking, individuals click exclusively on websites carrying the signal $y_m = x_n$, so that the ranking does not affect the class of website they read, and popularity ranking is irrelevant for asymptotic learning.[23]

### 3.2 Absence of Attention Bias

Suppose now individuals have no attention bias ($\alpha = 0$) but may derive value both from reading confirmatory and majoritarian news ($v_x \geq 0$, $v_z \geq 0$, and $0 \leq \gamma \leq 1$). Then the website ranking becomes irrelevant (since all weights $(r_{n,m})^\alpha$ are equal), and it is easy to see that individuals' website choices coincide with their original ranking-free choices given in Eq. (3). As a result individuals choose websites based on their signals ($x_n$ and $z_n$) and their desirability levels ($v_x$ and $v_z$) but independently of the ranking. Again, popularity ranking does not contribute to aggregating private information, and while the intensity of the desirability for reading confirmatory news ($\gamma$) as well as the parameters $p, \mu, q$ do affect the probability with which the individuals click on websites with the correct signal, the latter probability does not change over time. Hence, it is easy to see that $\mathcal{P}$ is increasing in $p, \mu$ and $q$ and decreasing in $\gamma$ for $p$ not too large, and $\mathcal{P}_L$ is increasing in $p$ and $L$, and is increasing or decreasing in $\mu$ depending on whether the majority have the correct signal or not ($L = K$ or $L \neq K$).[24]

Such an environment with popularity ranking and no attention bias ($\alpha = 0$) is of particular interest, since it is outcome-equivalent in terms of clicking probabilities to an environment with *random ranking*—a ranking satisfying $r_{n,m} = \frac{1}{M}$, for all $n, m$. This follows because, when there is popularity ranking with zero attention bias ($\alpha = 0$), all websites receive equal weights in the choice probabilities ($\rho_{n,m}$), and when there is random ranking, then, regardless of the attention bias ($\alpha$), all websites also receive equal weights in the website choice probabilities. As a result, website choice probabilities ($\rho_{n,m}$) always coincide in the two cases and do not change as individuals enter to perform their searches. We now turn to our main results characterizing asymptotic learning in the presence of behavioral (and naïve) individuals.

[23]Clearly, in a different model where all individuals but $N$ have $\gamma = 1$, and individual $N$ naïvely clicks on the top-ranked website, asymptotic learning (by that $N$'th individual) will always be achieved with probability 1. In Section 5.2, we briefly discuss learning with sophisticated individuals.

[24]Throughout the paper, we use the term decreasing (and increasing) in the weak sense, that is, we say a function $f$ is decreasing (increasing) if $x \geq y$ implies $f(x) \leq f(y)$ ($f(x) \geq f(y)$). When $x \in \mathbb{N}$, we say $f$ is decreasing (increasing) at $x$ if $f(x+1) \leq f(x)$ ($f(x+1) \geq f(x+1)$).

# 4 Opinion Dynamics with Behavioral Individuals

The key contribution of our model is to combine endogenous ranking of websites with sequential clicking behavior of behaviorally biased and naïve individuals. We now study the general case where $v_x \geq 0$ and $v_z \geq 0$ and hence $0 \leq \gamma \leq 1$ and $\alpha \geq 0$. Before discussing the overall implications of our model in terms of asymptotic learning, we first discuss some implications on website traffic.

## 4.1 Popularity Ranking and the Advantage of the Fewer ($AOF$)

The following proposition states our first main result. It illustrates a rather general phenomenon induced by popularity ranking, which we refer to as the *advantage of the fewer* ($AOF$), whereby a set of websites with the same signal can get a *greater* total clicking probability by individuals sufficiently far up in the sequence, if the set contains fewer websites than if it contains more of them (as long as it does not switch from being a set of majority to a set of minority websites).[25]

**Proposition 1.** *Fix a search environment $\mathcal{E}$ with uniform initial ranking $r_1$, and consider interim realizations of $\mathcal{E}$ that vary in the number of websites with correct signal. Then, the limit clicking probability ($\rho_{\infty,J}$) on all websites ($J \subset M$) carrying a given signal in* $\{0, 1\}$*, is decreasing in the number of those websites ($\#J$), provided the parameters satisfy $\alpha > 0$, $0 < \gamma < 1$, and $\#J \notin \{0, \frac{M-1}{2}, M-1\}$.*

This suggests that having *fewer* websites reporting a given signal *enhances* their overall traffic.[26] The intuition behind the $AOF$ effect is straightforward. When there are fewer websites with a given signal, the flow of individuals interested in reading about that signal are concentrated on fewer websites, thus leading to relatively more clicks per website. This (trivial) static effect transforms into a dynamic, amplified one through the interaction of the popularity ranking with individuals' stochastic choices: popularity ranking induces relatively higher rankings for those fewer websites, making them more attractive for subsequent individuals, inducing more trade-offs in favor of those fewer higher ranked websites, leading to even more clicks, and so on. The process, which can be seen as embodying a *"few get richer"* dynamic, gets repeated until it stabilizes in the limit, generating a potentially sizeable amplification of total traffic on all websites with the given signal.[27]

The *amplification effect* generated by the $AOF$ hinges upon four basic assumptions underlying our framework: ($i$) the ranking is based on popularity ($\nu > 0$), ($ii$) websites are partitioned into equivalence

[25] Recall that, unless otherwise stated, our search environments always have popularity-based rankings (see Section 2.4).

[26] It is important to note that, while, for $0 < \alpha \leq 1$, the unique limit always satisfies $AOF$; for $\alpha > 1$, depending on the initial condition (i.e., the initial ranking; see also Section 5.1), there can be multiple limits, of which the asymptotically stable ones, that is, the only ones that can be reached by the process $\rho_n$, also always satisfy $AOF$.

[27] Figure A.1 in the Online Appendix, provides a graphical illustration of the *amplification effect*.

classes (there are two classes, $c \in \{0, 1\}$), (*iii*) preferences are such that some individuals only click on websites from a given class (this happens when $x_n = z_n$), while some individuals make stochastic choices trading off content as a function of the rank of the websites (this happens when $x_n \neq z_n$ and when $0 < \gamma < 1$, $\alpha > 0$, and the numerator of $\rho_{n,m}$ is homogeneous of degree $> 1$ in $r_{n,m}$ and $\rho^*_{n,m}$), (*iv*) preferences are constant on the equivalence classes and do not depend on the numbers of websites in each class ($V_c$ is constant on class $c$ and does not depend on the numbers of websites in $c$). The following result—summarizing some of the cases where $AOF$ does not apply—follows from the proof of Proposition 1.

**Corollary 1.** *Fix a search environment $\mathcal{E}$, then AOF does not apply whenever:*

1. $\nu = 0$.
2. $\alpha = 0$.
3. $\gamma = 0$ *or* $\gamma = 1$.
4. $\alpha \geq 0, \beta \geq 0$, *but* $\alpha + \beta < 1$, *for clicking probabilities defined by* $\widetilde{\rho}_{n,m} = \frac{(r_{n,m})^\alpha \cdot (\rho^*_{n,m})^\beta}{\sum_{m' \in M} (r_{n,m'})^\alpha \cdot (\rho^*_{n,m'})^\beta}$.

Points 1–3 are self-explanatory. In particular, $AOF$ does not apply when individuals are rational. As discussed in Section 3, when individuals do not have any behavioral preferences over website content ($\gamma = 0$ or $\gamma = 1$, depending on whether $p$ is below or above $\bar{p}$) they do not face any trade-off in their choice over websites, and all website traffic goes to websites of the same class. Also, when there is no attention bias ($\alpha = 0$), ranking does not matter and there is no amplification effect.

Finally, point 4 considers a slight generalization of the choice function defined in Eq. (4). In particular it allows to show that taking arbitrary positive monotonic transformations may be of consequence for $AOF$. In fact, when the effect of both the ranking ($r_{n,m}$) and the ranking-free choice probabilities ($\rho^*_{n,m}$) are sufficiently small ($0 \leq \alpha + \beta < 1$) then the trade off between content and ranking induces an opposite effect, whereby fewer outlets in a class attract *less* traffic. Given our definition of $\rho_{n,m}$ with $\rho^*_{n,m}$ representing ranking-free choice probabilities, we view our benchmark model with $\beta = 1$ as particularly natural since it allows $\rho_{n,m}$ to coincide with $\rho^*_{n,m}$ when $\alpha = 0$ (i.e., $\beta < 1$ would violate the interpretation of $\rho_{n,m}$ and $\rho^*_{n,m}$ both being clicking probabilities that moreover coincide when the ranking is irrelevant).

We conclude with two final remarks regarding $AOF$.

1. While Corollary 1 highlights some essential assumptions for $AOF$ to work, we would also like to stress that several other assumptions implicit in our framework are not necessary for $AOF$

to hold.[28] To show this, the online appendix provides examples illustrating cases, where: $(i)$ parameters are outside the assumed range, for example, when individual signals are uninformative $(p = \mu = 1/2)$, $(ii)$ individuals need not distinguish between majority and minority websites and simply choose based on independent cues ($x_n$ and $z_n$) that can be interpreted as general desirable attributes of the different websites; $(iii)$ rankings are not stochastic but rather a list of $1^{st}$ to $M^{th}$ ranked website , as a function of total traffic received up to individual $n$, $(iv)$ individual choices are pure realizations of the choice functions ($\rho_{n,m}$), or $(v)$ where the total number of websites is not necessarily fixed, thus allowing for cases, where two or more websites can merge and thereby reduce the number of overall websites $(M)$.[29] These examples further suggest that $AOF$ is a potentially robust phenomenon. Importantly, as already pointed out, in our setting $AOF$ applies only if individuals have both attention bias and behavioral preferences over website content. Yet, it is easy to see that the mechanism generating such an effect may apply to settings with rational agents as long as individuals face a trade-off in choosing alternative news sources and they have some cognitive or economic cost which is increasing in the amount of news that they read.

2. $AOF$ can contribute to the understanding of the spread of misinformation, in the sense that a signal that is carried by few websites, for example, a controversial or "fake news" report, may paradoxically receive amplified traffic precisely *because* it is carried by few websites.[30] This is consistent with various claims that the algorithms used by Google and Facebook have apparently promoted websites reporting "fake news".[31] Indeed, even in queries with a clear factual truth,

[28]In a follow-up paper, Germano *et al.* (2019) also obtain an $AOF$ effect within a related framework, where majority signals are absent, individual clicks are realizations of the clicking probabilities and where the ranking is deterministic and based on the total number of clicks received by a website. They also conduct a proof of concept experiment with participants from Amazon Mechanical Turk.

[29]One might expect that allowing for free entry would tend to weaken the $AOF$. However, while addressing free entry of websites and even modeling the strategic choices of websites is outside the scope of this paper, it is worth noting that the effect of allowing free entry on the $AOF$ may be limited, especially in those cases where the "fewer websites" are minority websites carrying "dubious" information. Indeed, it may not be in the interest of mainstream websites (which are likely to also care about their reputation) to report such information. Also, new minority websites may not necessarily be able to "steal" much traffic from the existing ones due to their lower ranking (as explained by the *rich-get-richer* dynamic, see Section 5.1). Hence, we believe the result may be particularly relevant for dubious information, carried by relatively few websites and that "resonates" with a significant fraction of individuals. More generally, the $AOF$ might also be interpreted as a mechanism ensuring that minority websites will always be present in a market equilibrium. Indeed, if too may websites reporting a majoritarian opinion enter the market, then $AOF$ ensures that at some point it might be better (i.e., more profitable in terms of expected website traffic) to enter as a website reporting a minority opinion. At the same time, if too many websites reporting the minority opinion were to enter, at some point it might be better to enter as a website reporting a majoritarian opinion.

[30]The case of $\mu = 1$ mentioned above may be such a case. Concretely, if there was a single minority "fake news" website (so that $M - L = 1$), then the probability that that website would be visited by an individual in the limit is (assuming $\mu = 1$ and $\gamma$ not too small) $\rho_{\infty,fake} = 1 - \frac{\gamma p(M-1)}{\gamma M-1}$. Moreover, such a "fake news" website will be the top ranked one if $\gamma > \frac{1}{M(1-p)}$.

[31]On Google's search algorithm prioritizing websites reporting false information, see: "Harsh truths about fake news for Facebook, Google and Twitter", *Financial Times*, November 21, 2016; "Google, democracy and the truth about internet

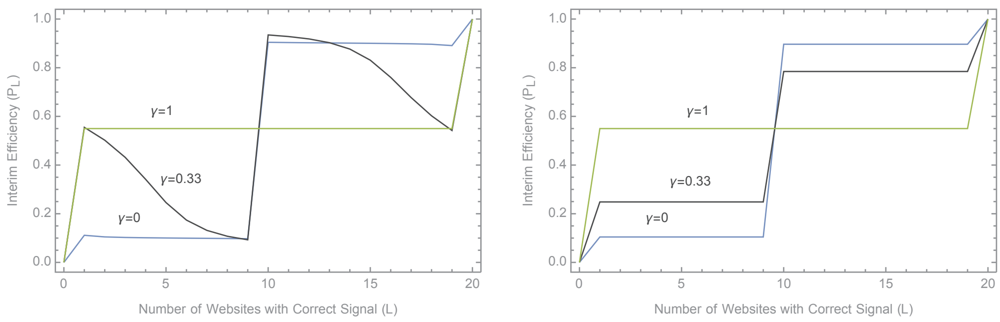


Figure 1: *Interim* efficiency ($\mathcal{P}_L$) as a function of the number of websites with correct signal ($L$) for $\gamma = 0.33$ (black line) and other values of $\gamma$ (dashed lines) for the cases of popularity ranking (left) and random ranking (right). In both panels, $M = 20$, $p = 0.55$, $\mu = 0.9$, and $r_1$ is uniform.

such as yes-no questions within the medical domain, top-ranked results of search engines provide a correct answer less than half of the time (White, 2013). Importantly, empirical evidence shows that websites reporting misinformation may acquire a large relevance in terms of online traffic and in turn may affect individuals' opinions and behavior (Carvalho *et al.*, 2011; Kata, 2012; Mocanu *et al.*, 2015; Shao *et al.*, 2016). By the same token, "authoritative" or mainstream websites that are relatively numerous will receive *less* traffic overall, the more numerous they are.

## 4.2 Popularity Ranking and Asymptotic Learning

### 4.2.1 Non-monotonicity of Interim Efficiency

Before analyzing ex-ante efficiency, we first study interim efficiency. The following result follows from Proposition 1 and illustrates how interim efficiency is a non-monotonic function of the number of websites carrying the correct signal ($L$) due to the *advantage of the fewer* effect ($AOF$).

**Corollary 2.** *Fix a search environment $\mathcal{E}$ with uniform initial ranking $r_1$, and consider interim realizations of $\mathcal{E}$ that vary only in the number of websites with correct signal ($L$). Then $\mathcal{P}_L$ is non-monotonic in $L$. In particular, when $\alpha$ and $\gamma$ are interior ($\alpha > 0$ and $0 < \gamma < 1$), then $\mathcal{P}_L$ is increasing in $L$ at $L \in \{0, \frac{M-1}{2}, M-1\}$, but is decreasing in $L$ otherwise.*

The non-monotonicity in $L$ follows from three basic facts: ($i$) small majorities (or minorities) of websites with a correct signal result in higher interim efficiency ($\mathcal{P}_L$) than large majorities (or minorities), resulting in a decreasing effect of $L$ on $\mathcal{P}_L$ induced by $AOF$, ($ii$) when $L$ increases from $\frac{M-1}{2}$ to $\frac{M+1}{2}$, then interim efficiency increases, since the correct signal ($y_L$) switches from being a minority to being

search", *The Observer*, December 4, 2016. On Facebook showing websites reporting false information in the top list of its trending topics, see: "Three days after removing human editors, Facebook is already trending fake news," *The Washington Post*, August 29, 2016.

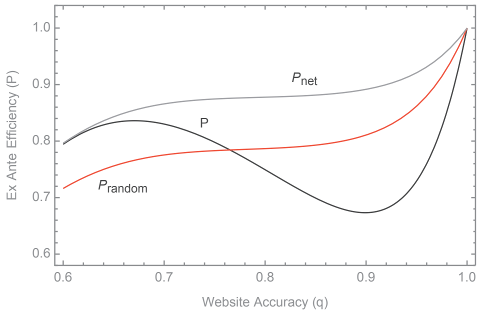


Figure 2: *Ex ante* efficiency ($\mathcal{P}$) as a function of the accuracy of websites' signals ($q$) (right) with popularity-ranking (black line) and random-ranking (red dashed). The figure also shows net *ex ante* efficiency (gray dashed) with popularity-ranking. $M = 20$, $p = 0.55$, $\mu = 0.9$, $\gamma = 0.33$ and $r_1$ is uniform.

a majority signal, $(iii)$ when $L$ increases from 0 to 1 or from $M-1$ to $M$, interim efficiency obviously increases.

Figure 1 shows the non-monotonicity of interim efficiency $\mathcal{P}_L$ in $L$, as stated in Corollary 2, as well as the $AOF$ effect as discussed in Section 4.1, in the presence of the popularity ranking when the desirability for reading confirmatory news is interior ($\gamma = 0.33$), (black line, left panel), and shows that interim efficiency is monotonic and $AOF$ is absent when $\gamma = 0, 1$ (left panel, dashed lines) or when the ranking is random (and therefore popularity ranking is switched off), (right panel).

#### 4.2.2 Ex Ante Efficiency

**Comparative statics and informational feedback effects.** Our notion of ex ante efficiency ($\mathcal{P}$) is a measure of asymptotic learning that obtains in our popularity-ranking based search environments. The following proposition describes comparative statics of $\mathcal{P}$ with respect to basic parameters of the model.

**Proposition 2.** *Let $\mathcal{E}$ be a search environment with a uniform initial ranking $r_1$. Then:*

1. *(Individual accuracy) $\mathcal{P}$ is increasing in $p$ and, if $p < \mu q$, also in $\mu$.*
2. *(Website accuracy) $\mathcal{P}$ can be both increasing or decreasing in $q$.*
3. *(Behavioral preference & attention bias) $\mathcal{P}$ can be both increasing or decreasing in $\gamma$ and in $\alpha$.*

We briefly discuss these effects.

1. (*Individual accuracy*) Higher levels of $p$ and $\mu$ always increase interim hence also ex ante efficiency. This is not just the consequence of the direct effect of more accurate private signals. The direct effect is increased by the dynamic one, since more individuals receiving a correct signal, increases

the number of clicks on websites reporting that signal, which in turn increases their ranking. This further increases the probability that subsequent individuals will also choose websites reporting a correct signal so that, even though individuals are assumed to be naïve, the higher $p$ and $\mu$ are, the more effective popularity ranking is in aggregating information and creating a positive externality that enhances asymptotic learning.

2. (*Website accuracy*) A higher $q$ may increase or decrease ex ante efficiency. This is a consequence of the non-monotonicity of the interim efficiency (Corollary 2), driven by $AOF$. Higher values of $q$ make it more likely that the number of websites with correct signal is large. Hence, due to $AOF$, a higher $q$ decreases the clicking probability on websites with a correct signal, thus reducing ex ante efficiency for intermediate values of $\gamma$. If one could switch off $AOF$, then a higher $q$ would always increase ex-ante efficiency (i.e., efficiency "net of $AOF$" is always increasing in $q$).[32] This stark difference is illustrated in Figure 2.

3. (*Behavioral preferences & attention bias*) Since a higher value of $\gamma$ leads to more weight put on signal $x_n$ relative to signal $z_n$, when the former is more informative ($p > \bar{p}$), it is not difficult to see that $\mathcal{P}$ may be increasing in $\gamma$, whereas, when it is less informative ($p < \bar{p}$), $\mathcal{P}$ may be decreasing in $\gamma$. Similarly with the attention bias. Because popularity ranking can generate an externality from individual clicking to subsequent clicking, which increases with attention bias, an increase in $\alpha$ can increase ex-ante efficiency, when externalities are positive, and decrease them otherwise. This is illustrated in Figures 2 and 3 for the case $p < \bar{p}$, where ex-ante efficiency is plotted for $\alpha = 0$ ($\mathcal{P}_{random}$) and $\alpha = 1$ ($\mathcal{P}$). Recall that, as pointed out in Section 3.2, the case of $\alpha = 0$ is outcome-equivalent to the case of uniform random ranking throughout.

**Popularity-Ranking Effect ($PoR$).** We turn to the key question of how well popularity ranking performs in terms of asymptotic learning. To simplify the analysis, we focus on the case $p < \overline{p}$, where for each individual $n$, the majority signal ($z_n$) is the ex ante most informative signal. The complementary case ($p > \overline{p}$) is analogous and leads to similar conclusions regarding asymptotic learning and popularity ranking. The following definition uses random ranking as a comparison benchmark.

[32]Specifically, one can define *ex ante efficiency "net of $AOF$"* ($\mathcal{P}_{net}$) as ex ante efficiency calculated with weighted constant "average" minority and majority traffic levels, respectively, $\mathcal{P}_{\lceil \frac{M}{4} \rceil}$ and $\mathcal{P}_{\lceil \frac{3M}{4} \rceil}$, formally:

$$\mathcal{P}_{net} = \sum_{k=1}^{\frac{M-1}{2}} \mathcal{P}_{\lceil \frac{M}{4} \rceil} \binom{M}{k} q^k (1-q)^{M-k} + \sum_{k=\frac{M+1}{2}}^{M-1} \mathcal{P}_{\lceil \frac{3M}{4} \rceil} \binom{M}{k} q^k (1-q)^{M-k} + q^M.$$

While, in principle, it would be possible to correct the ranking algorithm for the $AOF$ effect, we are unaware of any such correction undertaken in practice, nor have we seen the effect mentioned in the computer science or machine learning literature.

**Definition 1.** *Let $\mathcal{E}$ be a search environment with a uniform initial ranking $r_1$, and let $\mathcal{E}'$ be another search environment that differs from $\mathcal{E}$ only in that the ranking is always a uniform random ranking. Let $\mathcal{P}$ and $\mathcal{P}'$ denote ex ante efficiency of $\mathcal{E}$ and $\mathcal{E}'$ respectively, then we define the* popularity ranking effect *of $\mathcal{E}$ ($PoR(\mathcal{E})$) as the difference:*

$$PoR(\mathcal{E}) = \mathcal{P} - \mathcal{P}'.$$

The following result compares popularity ranking and random ranking.

**Proposition 3.** *Let $\mathcal{E}$ be a search environment with uniform initial ranking $r_1$ and $p < \overline{p}$. Then there exists $\overline{\mu} < 1$ such that, for any $\mu \in [\overline{\mu}, 1]$, there exist $0 < \overline{\gamma} < \hat{\gamma} \leq 1$, and a threshold function for $q$, $\phi : [0, 1] \to [\frac{1}{2}, 1]$, that is decreasing in $\gamma$, with $\phi(\gamma) = 1$, for $\gamma \in [0, \overline{\gamma}]$, such that:*

1. *$PoR(\mathcal{E}) \geq 0$ for $\gamma \in [0, \hat{\gamma}]$, provided $q \in [\frac{p}{\mu}, \phi(\gamma)]$.*

2. *$PoR(\mathcal{E}) \leq 0$ for $\gamma \in [\overline{\gamma}, 1]$, provided $q \in [\phi(\gamma), 1]$.*

In other words, assuming the case where the majority signal is the ex ante most informative signal ($p < \overline{p}$) and is sufficiently high ($\mu \in [\overline{\mu}, 1]$), we have that, when the desirability of reading confirmatory news is sufficiently low ($\gamma \in [0, \overline{\gamma}]$), then popularity ranking does better than random ranking. That is, when individual clicking behavior generates sufficiently positive information externalities, popularity ranking aggregates information and increases the probability of asymptotic learning relative to random ranking. Moreover, this continues to hold for higher values of the desirability of reading confirmatory news ($\gamma \in [\overline{\gamma}, \hat{\gamma}]$, for $\hat{\gamma}$ that could go up to 1), provided website accuracy is not too high ($q \leq \phi(\gamma)$) (due to $AOF$). Put differently, due to $AOF$, an increase in the ex-ante informativeness of websites ($q$), may lead to a negative popularity ranking effect.[33] By contrast, when externalities generated are not sufficiently positive ($\gamma \in [\overline{\gamma}, 1]$), then popularity ranking decreases the probability of asymptotic learning relative to random ranking, provided website accuracy is sufficiently high ($q \geq \phi(\gamma)$). Figures 2 and 3 illustrate Proposition 3: popularity ranking dominates random ranking in terms of ex-ante efficiency provided $q$ and $\gamma$ are not too large, and is dominated by random ranking otherwise. A symmetric result holds in the complementary case, where the private signal ($x_n$) is the ex-ante more informative signal ($p > \overline{p}$). In this case, when both the desirability of reading confirmatory news ($\gamma$) and the accuracy on the private signal ($p$) are sufficiently high, then popularity ranking does better than random ranking, again provided the the website accuracy parameter ($q$) is sufficiently low. This is illustrated in the Online Appendix.

[33]By contrast, ex ante efficiency "net of $AOF$" ($\mathcal{P}_{net}$) is above random ranking even for large values of $q$.

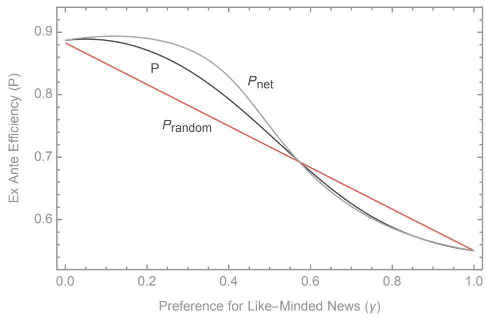


Figure 3: *Ex ante* efficiency ($\mathcal{P}$) as a function of the desirability of reading confirmatory news ($\gamma$) with popularity-ranking (black line) and random-ranking (red dashed). The figure also shows net *ex ante* efficiency (gray dashed) with popularity-ranking. The plot is drawn for $M = 20$, $p = 0.55$, $\mu = 0.9$, $q = 0.7$, and $r_1$ is uniform.

### 4.3 Personalized Ranking

Another important question for a ranking algorithm concerns whether it should keep track of, and use, information it has available concerning the individuals' identity and past searches. A search engine may want its algorithm to condition the outcomes of searches on the geographic location of the individuals (e.g., using the individual's *IP address*) or other individual characteristics (e.g., using the individual's search history). Accordingly, a personalized search algorithm may output different search results to the same query performed by individuals living in different locations and/or with different browsing histories.[34]

Suppose the set of individuals $N$ is partitioned into two nonempty groups $A, B \subset N$, such that $A \cup B = N$ and $A \cap B = \emptyset$. Suppose the two groups differ in terms of the individuals' desireability for reading confirmatory news ($\gamma_A \neq \gamma_B$). In any period an individual is randomly drawn from one of the two groups, that is, from $A$ with probability $\frac{N_A}{N}$ and from $B$ with probability $\frac{N_B}{N}$, where $N_A = \#A > 0$ and $N_B = \#B > 0$. A *personalized ranking algorithm* then consists of two parallel rankings, namely, $r_n^A$ for individuals in $A$ and $r_n^B$ for individuals in $B$. Each one is updated as in the non-personalized case, with the difference that the weight on past choices of individuals from the own group are possibly different than those from the other one. Set, for any $n$ and $m$, and for $\ell = A, B$, such that $n \in \ell$,

$$r_{n,m}^{\ell} = (1 - \nu^{\ell}) r_{n-1,m}^{\ell} + \nu^{\ell} \rho_{n-1,m}^{\ell}\,, \tag{10}$$

where $\nu^{\ell}$ now depends on whether or not the individual searching at time $n - 1$ was in the same group

[34]See Pariser (2011); Dean (2013); MOZ (2013); Vaughn (2014); Kliman-Silver *et al.* (2015). Hannak *et al.* (2013) document the presence of extensive personalization of search results. In particular, while they show that the extent of personalization varies across topics, they also point out that "politics" is the most personalized query category. See also Xing *et al.* (2014) for empirical evidence on personalization based on the *Booble* extension of Chrome.

as $n$ ($n-1 \in \ell$), and where the weight $\nu^\ell$, for $\ell = A, B$, is given by:

$$\nu^\ell = \frac{(1-\lambda_n)\nu}{1-\lambda_n\nu}, \text{ where } \lambda_n = \begin{cases} 0 & \text{if } n-1 \in \ell \\ \lambda & \text{else }, \end{cases} \tag{11}$$

with $\nu \in (0,1)$ and $\lambda \in [0,1]$ parameters of the personalized ranking algorithm. This algorithm now gives different weights to past choices over websites, depending on whether these choices were taken by individuals in the same group (weights $1-\nu, \nu$) or in the other group (weights $\frac{1-\nu}{1-\lambda\nu}, \frac{(1-\lambda)\nu}{1-\lambda\nu}$). In particular, when $\lambda = 1$, the ranking algorithm is fully personalized, whereas, when $\lambda = 0$, it coincides with the non-personalized one previously defined. We implicitly assume that the personalized algorithm partially separates individuals according to the individual characteristics (i.e., according to the different parameters $\gamma$). We will refer to a *personalized search environment* $\mathcal{E}_\lambda$ when considering the generalization of a search environment $\mathcal{E}$, defined in Section 2.5, to the case where the ranking is described by Equations (10) and (11) with $\lambda \in [0,1]$.

**Comparative statics and belief polarization.** By introducing personalization in our model, we allow the ranking of websites to be conditioned on (observable) characteristics of the individuals such that searches performed by individuals in different groups can have different weights. When $\lambda = 0$, there is no difference in the ranking of websites for the two groups, while as $\lambda$ increases the groups start observing potentially different rankings which may further trigger different website choices, thus leading to different opinions. In other words, increased personalization may lead to increased *belief polarization*.

**Definition 2.** *Fix a personalized search environment $\mathcal{E}_\lambda$ with nonempty groups of individuals $A$ and $B$. Let $K$ denote the set of websites carrying the website-majority signal, then we define the* degree of belief polarization *of $\mathcal{E}_\lambda$ as:*

$$\mathcal{BP}(\mathcal{E}_\lambda) = \lim_{N\to\infty} \left| \widehat{\rho}^A_{N,K} - \widehat{\rho}^B_{N,K} \right| .$$

*We say environment $\mathcal{E}_\lambda$ exhibits more belief polarization than $\mathcal{E}'_\lambda$, if $\mathcal{BP}(\mathcal{E}_\lambda) > \mathcal{BP}(\mathcal{E}'_\lambda)$.*

**Proposition 4.** *Let $\mathcal{E}_\lambda$ be a personalized search environment with personalization parameter $\lambda$. Suppose there are two groups of individuals $A$ and $B$ of equal size, then $\mathcal{BP}(\mathcal{E}_\lambda)$ is zero if either $\lambda = 0$ or $\gamma_A = \gamma_B$, and is otherwise increasing in $\lambda$ and $|\gamma_A - \gamma_B|$ .*

Non-trivial personalization ($\lambda > 0$) can lead to different information held by relatively similar groups of individuals and thus to polarization of opinions.[35] This suggests that individuals might end up into an

[35] If individuals in different groups were to face also different initial rankings, ($r_1^A \neq r_1^B$), then this different rankings would clearly contribute to further accentuating the evolution of rankings seen by the two groups.

algorithmically-driven echo chamber. This is in line with Flaxman *et al.* (2013), who show that search engines can lead to a relatively high level of ideological segregation, due to web search personalization embedded in the search engine's algorithm and to individuals' preference for confirmatory news. It is also in line with existing claims and empirical evidence suggesting that the Internet—together with the online platforms embedded in it—generally contributes towards increasing ideological segregation (Putnam, 2001; Sunstein, 2009; Pariser, 2011; Halberstam and Knight, 2014; Bessi *et al.*, 2015; Bar-Gill and Gandal, 2017).

The result of Proposition 4 would also apply if individuals in groups A and B were to differ in other ex-ante parameters like $p$ or $\mu$. Belief polarization would also be present if the personalized algorithm were to distinguish individuals according to the *realization* of their private signals (e.g., individuals in A and B differing in terms of $x_n$'s or $z_n$'s). At the same time, if individuals in A and B had the same ex-ante parameters and were not distinguished according to the realization of their private signals, then belief polarization, if present, would only be a short-run phenomenon driven by interim realization of individual signals. In the long run, the two groups would exhibit the same probability of clicking on the given classes of websites.

Next, we study how personalization may actually hinder asymptotic learning.

**Personalized-Ranking Effect ($PeR$).** We turn to the effect of personalization on ex-ante efficiency. Personalization here can be seen as progressively "separating" two groups of individulas by uncoupling their rankings and thereby switching off potential externalities from one group to the other. To the extent that the group with weaker behavioral bias exerts a positive externality on the groups' rankings and overall ex ante efficiency (which occurs under conditions related to the ones yielding $PoR \geq 0$ in Proposition 3), increasing the personalization may inhibit total ex ante efficiency. Again, to simplify the analysis, we focus on the case, where the majority signal is the individual's ex ante most informative signal ($p < \overline{p}$). Parallel conclusions concerning personalization and asymptotic learning can be drawn for the complementary case ($p > \overline{p}$). The following definition uses (standard) popularity ranking ($0 < \nu < 1$) with no personalization ($\lambda = 0$) as a comparison benchmark.

**Definition 3.** *Let $\mathcal{E}$ be a non-personalized and $\mathcal{E}_\lambda$ be a personalized search environment with personalization parameter $\lambda$, and both with a uniform initial ranking $r_1$. Let $\mathcal{P}$ and $\mathcal{P}_\lambda$ denote ex ante efficiency of $\mathcal{E}$ and $\mathcal{E}_\lambda$ respectively, then we can define the* personalized ranking effect *of $\mathcal{E}_\lambda$ ($PeR(\mathcal{E}_\lambda)$) as the difference:*

$$PeR(\mathcal{E}_\lambda) = \mathcal{P}_\lambda - \mathcal{P}.$$

**Proposition 5.** *Let $\mathcal{E}_\lambda$ be a personalized search environment with personalization parameter $\lambda$, and* *with a uniform initial ranking* $r_1$ *and* $p < \overline{p}$*. Suppose there are two nonempty groups of individuals* $A$ *and* $B$ *with* $0 \leq \gamma_A < \gamma_B \leq \gamma$*. Then there exists* $\overline{\mu} < 1$ *such that, for any* $\mu \in [\overline{\mu}, 1]$*, there exists* $0 < \overline{\gamma} < \hat{\gamma} \leq 1$*, and a threshold function for* $q$*,* $\phi : [0,1] \to [\frac{1}{2}, 1]$*, that is decreasing in* $\gamma$*, with* $\phi(\gamma) = 1$*, for* $\gamma \in [0, \overline{\gamma}]$*, such that:*

1. $PeR(\mathcal{E}) \leq 0$ *for* $\gamma_A, \gamma_B \leq \gamma$ *for* $\gamma \in [0, \hat{\gamma}]$*, provided* $q \in [\frac{p}{\mu}, \phi(\gamma)]$*.*

2. $PeR(\mathcal{E}) \geq 0$ *for* $\gamma_A, \gamma_B \leq \gamma$ *for* $\gamma \in [\overline{\gamma}, 1]$*, provided* $q \in [\phi(\gamma), 1]$*.*

Although the exact cutoff values for the parameters need not coincide, the parallels between the popularity ranking effect and the personalized ranking effect are stark. That is, when the desirability for reading confirmatory news is sufficiently low ($\gamma \in [0, \overline{\gamma}]$) and the accuracy on the majority signal is sufficiently high ($\mu \in [\overline{\mu}, 1]$), then personalized ranking does worse than non-personalized (popularity) ranking; moreover, this continues to hold for higher values of the desirability for reading confirmatory news ($\gamma \in [\overline{\gamma}, \hat{\gamma}]$), provided the accuracy of websites is not too high ($q \leq \phi(\gamma)$) (due to $AOF$); on the other hand for the higher values of desirability for reading confirmatory news ($\gamma \in [\overline{\gamma}, 1]$) personalized ranking will perform better than non-personalized (popularity) ranking, provided accuracy of websites is sufficiently high ($q \geq \phi(\gamma)$). In other words, the forces that lead to a positive (negative) popularity effect are similar to the ones that lead to a negative (positive) personalization effect. Again, a symmetric result holds when the private signal ($x_n$) is the ex ante more informative one ($p > \overline{p}$) and the desirability for reading confirmatory news ($\gamma$) is sufficiently high, as illustrated in the Online Appendix.

The intuition for the negative relation between the popularity ranking and the personalized ranking effects ($PoR$ and $PeR$) is due to the fact that a positive popularity ranking effect occurs when individuals' parameters generate positive feedback into the dynamics (high accuracy $\mu$ and low desirability for reading confirmatory news $\gamma$ in the case $p < \bar{p}$); the same forces favor non-personalization and hence tend to generate a negative personalized ranking effect. This is because personalization can be seen as limiting the feedback between individuals in the dynamics, and so, when individuals' signals tend to generate positive feedback, it is better not to limit the feedback and hence not to personalize the ranking, and *vice versa* when individuals' signals generate negative feedback. The similarity also with respect to the website accuracy parameter ($q$) is due to the $AOF$ effect that is present with or without personalization.

Figure 4 illustrates the negative relationship between $PoR$ and $PeR$ . Personalized ranking (green dashed) tends to be dominated (in terms of asymptotic learning) by either non-personalized ranking (black) or by random ranking (gray dashed). The right-hand panel indicates that when $AOF$ is switched off, the corresponding "net" $PeR$ effect tends to be negative for any level of ex-ante accuracy of websites.

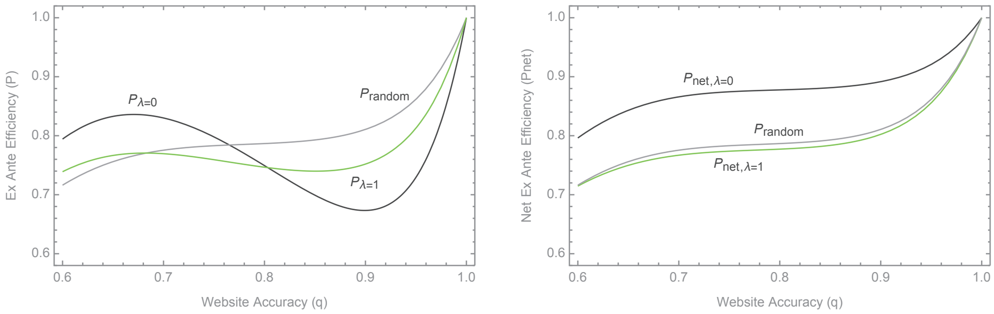


Figure 4: *Ex ante* efficiency with no personalization ($\mathcal{P}_{\lambda=0}$) (black) and with personalization w.r.t. $\gamma$ ($\mathcal{P}_{\lambda=1}$) (green dashed); and with random ranking (gray dashed). The right panel illustrates the same for ex-ante efficiency "net of $AOF$". In both panels, $M = 20$, $q = 0.7$, $p = 0.55$, $\mu = 0.9$, $\gamma = 0.33$, and $r_1$ uniform.

Finally, it is important to note that our searches are common value searches such that asymptotic learning obtains when individuals read websites that carry the (same) correct signal. When individuals have private and non-common values, then personalization may be relevant for enhancing asymptotic learning of typically multiple and distinct signals.

# 5 Robustness and Extensions

## 5.1 Non-Uniform Initial Ranking and the Rich Get Richer

Many of the propositions stated in the paper assumed a uniform initial ranking. We now study this assumption in more detail, looking at environments with general attention bias ($\alpha \geq 0$). As it turns out, the expected limit clicking probabilities $\widehat{\rho}_{\infty,m}$ only depend on the initial ranking when $\alpha > 1$. In this case, there is also a "rich-get-richer" effect.

**Proposition 6.** *Let $\mathcal{E}$ be a search environment with attention bias parameter $\alpha \geq 0$ and with interim realization $\langle \omega; (L, (y_m)) \rangle$, with interior initial ranking $r_{1,m} > 0$ for all $m$. Then, if attention bias satisfies $0 \leq \alpha \leq 1$, then the expected limit clicking probabilities $\widehat{\rho}_{\infty,m}$ do not depend on $r_1$. This is not true if $\alpha > 1$.*

The evolution of a website's ranking based on its "popularity," interacted with a sufficiently large attention bias ($\alpha > 1$) exhibits a *rich-get-richer* dynamic, whereby the ratio of the expected clicking probabilities of two websites $m, m'$, with $r_{1,m} > r_{1,m'} > 0$ increases over time as more agents perform their search. The effect is further magnified, the larger $\alpha$ is. Importantly, the differences in the ranking probabilities ($r_n$) and in the expected website choice probabilities ($\widehat{\rho}_n$), are driven by the initial ranking ($r_1$) and are amplified by the attention bias. We define this more formally.

**Definition 4.** *Fix a search environment $\mathcal{E}$ with attention bias parameter $\alpha \geq 0$ and with interim realization $\langle \omega; (L, (y_m)) \rangle$. We say that $\mathcal{E}$ exhibits the* rich-get-richer *dynamic if, for two websites $m, m' \in M$ with the same signal, $y_m = y_{m'}$, and different initial ranking, $r_{1,m} > r_{1,m'} > 0$, we also have $\frac{\widehat{\rho}_{n,m}}{\widehat{\rho}_{n,m'}} > \frac{\widehat{\rho}_{n-1,m}}{\widehat{\rho}_{n-1,m'}}$, for any $n > 0$.*

When an environment exhibits a *rich-get-richer* dynamic, then the ratio of the *expected* probability of two websites (with the same signal but different initial ranking) being visited not only persists over time (this follows from $\kappa > 0$), but actually increases. We can state the following.

**Proposition 7.** *Let $\mathcal{E}$ be a search environment with attention bias parameter $\alpha \geq 0$ and with interim realization $\langle \omega; (L, (y_m)) \rangle$. Then we have, for any two websites $m, m' \in M$ with $y_m = y_{m'}$ and $r_{1,m} > r_{1,m'} > 0$, and $n > 1$:*

$$\frac{\widehat{\rho}_{n,m}}{\widehat{\rho}_{n,m'}} \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \frac{\widehat{\rho}_{n-1,m}}{\widehat{\rho}_{n-1,m'}} \begin{cases} \text{if } \alpha < 1 \\ \text{if } \alpha = 1 \\ \text{if } \alpha > 1\,. \end{cases} \tag{12}$$

*In particular, if the attention bias is large enough ($\alpha > 1$), then $\mathcal{E}$ exhibits the rich-get-richer dynamic.*

This proposition shows that the attention bias plays a crucial role in the evolution of website traffic. When $0 \leq \alpha < 1$, initial conditions do not matter in the limit (as $N \to \infty$), in the sense that websites with the same signal will tend to be visited with the same probability in the limit. When $\alpha = 1$ the ratios of the expected clicking probabilities remain constant for websites with the same signal. When $\alpha > 1$, initial conditions matter and the evolution of website traffic follows a rich-get-richer dynamic. Traffic concentrates on the websites that are top ranked in the initial ranking. The *rich-get-richer* dynamic is in line with the "Googlearchy" suggested by Hindman (2009), who argues that the dominance of popular websites via search engines is likely to be self-perpetuating. Most importantly, the *rich get richer* pattern of website ranking (and traffic) via search engines is consistent with established empirical evidence (Cho and Roy, 2004).[36]

### 5.2 Opinion Dynamics with Sophisticated Individuals

We here briefly discuss opinion dynamics with sophisticated individuals and consider two separate cases. In Section 5.2.1, we consider sophisticated and rational individuals that can fully observe the history of the ranking and can therefore deduce the signals of the individuals entering before them. In Section 5.2.2,

[36]Indeed, even if some scholars have argued that the *overall* traffic induced by search engines is less concentrated than it might appear due to the topical content of user queries (Fortunato *et al.*, 2006), the *rich-get-richer* dynamic is still present within a specific topic.

we further allow such sophisticated and rational individuals to also derive some value from reading confirmatory or majoritarian news (besides having attention bias), so that with some probability $(1-\eta)$ they may make non-rational choices. In both cases, we consider individuals maximizing the probability of reading a website carrying the correct signal. The two cases predict rather different outcomes in terms of information aggregation. When individuals do not have any behavioral bias ($\eta = 1$) herding occurs, and relatively little information aggregation takes place. Conversely, when individuals have even a small amount of behavioral bias ($\eta < 1$ but $\eta \to 1$) full information aggregation may occur.

#### 5.2.1 Asymptotic Learning with Rational Individuals

Suppose as in Section 3 that individuals are rational in the sense that they only derive value from reading a website that carries the ex ante most informative signal. Suppose moreover that they are also sophisticated in the sense that, besides the two signals $x_n$ and $z_n$, they also know the parameters of the ex ante environment, $(\mathcal{E})$ and can observe the whole evolution of the ranking up to $n$ $(r_t, t \leq n)$. Again, their decision problem reduces to identifying which class of websites carries the ex ante most informative signal. Initially, for small $n$, the ex ante most informative signal is given by $w_n = z_n$ if $p < \overline{p}$ and by $w_n = x_n$ if $p > \overline{p}$, where $\overline{p}$ is defined by Eq. (9). Now, under a basic genericity assumption on the parameters $v_x$ and $v_z$ $(v_x \neq v_z)$, since the ranking $(r_n)$ is cardinal, as the agents enter and learn the evolution of the ranking, the first ones can also compute what signal each individual before them obtained and therefore effectively also has access to the history of signals $(x_t, z_t)$ for $t \leq n$. This happens until, after few observations, herding behavior follows and all individuals ignore their own private signals $(x_n, z_n)$ and instead click on the websites from the same class. This leads to very limited learning from clicking behavior and to an overall very low level of ex ante and interim efficiency.

#### 5.2.2 Asymptotic Learning with Behavioral Individuals

On the other hand, if we consider a slight variation of this model, and hence also of the one used throughout the paper, and assume that individuals all derive value from three sources, namely, $(i)$ from reading news that confirms their prior — again, this happens when $x_n = y_m$ and gives value $v_x > 0$, and gives value 0 otherwise (when $x_n \neq y_m$); $(ii)$ from reading majoritarian news — again, this happens when $z_n = y_m$ and gives value $v_z > 0$, and gives value 0 otherwise (when $z_n \neq y_m$), and $(iii)$ from reading news that is ex-ante most informative, that is, that has the highest probability of covering the true state of the world — this happens when $w_n = y_m$ and gives value $v_w > 0$, and gives value 0 otherwise (when $w_n \neq y_m$), where $w_n$ is determined from the individual's information at time $n$. Then, again under a generic assumption on the parameters $v_x, v_z$ and $v_w$ $(v_x \neq v_z, v_x \neq v_w, v_z \neq v_w$ as well as $v_x + v_z \neq v_w, v_x + v_w \neq v_z$ and $v_z + v_w \neq v_x$), and if individuals are sophisticated, then they will continue to learn from observing the ranking as more and more individuals enter, such that there can be asymptotic learning in the limit. We briefly sketch this.

Let $c \in \{0, 1\}$ denote again the class type of a website , then we can write individual $n$'s desirability value from reading website from class $c$ as:

$$V_c(x_n, z_n, w_n) = \mathbb{I}_{\{x_n=c\}} \cdot v_x + \mathbb{I}_{\{z_n=c\}} \cdot v_z + \mathbb{I}_{\{w_n=c\}} \cdot v_w. \tag{13}$$

and can also write the *Luce choice rule* of choosing a website from class $c$ as:

$$\rho_c(x_n, z_n, w_n) = \frac{V_c(x_n, z_n, w_n)}{V_c(x_n, z_n, w_n) + V_{\neg c}(x_n, z_n, w_n)}. \tag{14}$$

It can be shown that under the mentioned genericity assumption on $v_x, v_z$ and $v_w$, again because the ranking is cardinal and updated via clicking probabilities, as individuals enter and learn the evolution of the ranking, they can compute the individuals' signals $(x_t, z_t, w_t)$ for each individual with $t < n$. Since $p > \frac{1}{2}$, this implies that just from the signals $x_t, t < n$, individual $n$'s ex-ante most informative signal $w_n$ will in the limit converge to being an arbitrarily precise signal of the true state $\omega$. Hence, in the limit, individuals will click on websites that carry the signal $y_m = \omega$ with probability at least $\eta \equiv \frac{v_w}{v_x+v_z+v_w}$. In particular, if individuals are arbitrarily close to being rational ($\eta \to 1$), but always have some arbitrarily small desirability from reading confirmatory news ($v_x > 0$), then both ex ante and interim efficiency will be arbitrarily close to 1. Clearly, the assumption that individuals can observe the whole evolution of the ranking is a strong one. It is worth exploring the possibility of weakening this, without going to the other extreme, assumed throughout the paper, that individuals not only do not observe the evolution of the ranking but that, moreover, they do not learn anything from observing $r_n$. We leave this for future work. We note however, that under the assumption that individuals do not learn anything from observing the ranking, the case where $v_x, v_z, v_w > 0$ is equivalent to the one studied in the rest of the paper, where $v_x, v_z > 0$ and $v_w = 0$ since $w_n$ will either coincide with $x_n$ and/or $z_n$ so that individuals always ultimately only care about $x_n$ and $z_n$.

## 6 Conclusions

Several decades after the introduction of the Internet and the World Wide Web, there is still a vivid and growing popular and academic debate on the possible impact of digital platforms on public opinion (e.g., Introna and Nissenbaum, 2000; Hargittai, 2004; Rieder, 2005; Hindman, 2009; Sunstein, 2009;

Granka, 2010; Pariser, 2011; Bakshy *et al.*, 2015; Lazer, 2015; Tufekci, 2015). Even among regulators and policymakers, misinformation online ranks high as a key concern, leading some even to call for the direct regulation of online content (e.g., Germany and France have recently proposed laws to combat "fake news").[37]

Unfortunately, to understand and address these issues, it is not enough to just obtain access to the algorithm code used by digital platforms, as the interplay between ranking algorithms and individual behavior "yields patterns that are fundamentally emergent" (Lazer, 2015, p. 1090). In this sense, the theoretical framework we develop seeks to inform and provide some formal guidance to the above debate, by focusing on the interaction between users' search behavior and basic and well-established aspects of ranking algorithms (popularity and personalization). Our results point out that popularity rankings can have an overall positive effect on asymptotic learning by fostering information aggregation, as long as individuals can, on average, provide sufficiently positive feedback to the ranking algorithm. At the same time, we uncover a rather general property of popularity-based rankings, we call the *advantage of the fewer* ($AOF$) effect. Roughly speaking, it suggests that the smaller the number of websites reporting a given information, the larger the share of traffic directed to those fewer websites. Because dubious or particularly controversial information is often carried by a (relatively) small number of websites, the $AOF$ effect may help explain the spread of misinformation, since it shows how being small in number may actually boost overall traffic to such websites. The model also provides insights on a controversial component of the ranking algorithm, namely personalization. While personalized rankings can clearly be efficient for search queries on private value issues (e.g., where to have dinner), we find that for queries on common value issues (e.g., whether or not to vaccinate a child) they can hinder asymptotic learning besides also deepening *belief polarization*.

Understanding the role and effects of ranking algorithms, directly or indirectly used by billions of individuals daily, is a top priority for understanding the functioning of our information society. We view this paper as contributing a first step in this direction.

[37]See "How do you stop fake news? In Germany, with a law." *Washington Post*, April 5 2017. "Emmanuel Macron promises ban on fake news during elections." *The Guardian*, January 3, 2018.

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# APPENDIX

## A Mean Dynamics Approximation

In order to evaluate the actions taken by an agent in the limit as $n \to \infty$, we use some techniques of stochastic approximation from Norman (1972) as exposed in Izquierdo and Izquierdo (2013), which we refer to as the *mean dynamics approximation*. We here give a brief outline in order to follow our calculations and proofs, but we refer to the latter two sources for more details. The basic idea of the approach is to use the expected increments to evaluate the long run behavior of a dynamic process with stochastic increments. The key result we use allows us to relate our discrete stochastic process ($r_n$ and hence $\rho_n$ for large enough $n$) with points on a trajectory converging to an asymptotically stable point that solves an ordinary differential equation ($\dot{x} = g(x)$) to be defined below. To make the connection, we begin by rewriting the ranking probabilities using Equation (4) as,

$$\begin{aligned} r_{n,m} &= (1-\nu) r_{n-1,m} + \nu \rho_{n-1,m} \\ &= r_{n-1,m} + \nu(\rho_{n-1,m} - r_{n-1,m}) \\ &= r_{n-1,m} + \nu \left( \frac{(r_{n-1,m})^\alpha \cdot \rho^*_{n-1,m}}{\sum_{m'} (r_{n-1,m'})^\alpha \cdot \rho^*_{n-1,m'}} - r_{n-1,m} \right). \end{aligned}$$

It is clear that the only stochastic term is given by the expressions $\rho^*_{n-1,m}$. Replacing these with their expectations yields the deterministic recursion in $\widehat{r}_{n,m}$,

$$\widehat{r}_{n,m} = \widehat{r}_{n-1,m} + \nu \left( \widehat{\rho}_{n-1,m} - \widehat{r}_{n-1,m} \right),$$

where

$$\begin{aligned} \widehat{\rho}_{n-1,m} = \mathbb{E}[\rho_{n-1,m}] &= p\mu \frac{(\widehat{r}_{n-1,m})^\alpha \cdot \widehat{\rho^*}^{00}_m}{\sum_{m'} (\widehat{r}_{n-1,m'})^\alpha \cdot \widehat{\rho^*}^{00}_{m'}} + p(1-\mu) \frac{(\widehat{r}_{n-1,m})^\alpha \cdot \widehat{\rho^*}^{01}_m}{\sum_{m'} (\widehat{r}_{n-1,m'})^\alpha \cdot \widehat{\rho^*}^{01}_{m'}} \\ &+ (1-p)\mu \frac{(\widehat{r}_{n-1,m})^\alpha \cdot \widehat{\rho^*}^{10}_m}{\sum_{m'} (\widehat{r}_{n-1,m'})^\alpha \cdot \widehat{\rho^*}^{10}_{m'}} + (1-p)(1-\mu) \frac{(\widehat{r}_{n-1,m})^\alpha \cdot \widehat{\rho^*}^{11}_m}{\sum_{m'} (\widehat{r}_{n-1,m'})^\alpha \cdot \widehat{\rho^*}^{11}_{m'}} \end{aligned} \quad \text{(A.1)}$$

is the expected clicking probability of the individual entering in period $n-1$, and where the expected ranking-free values $\widehat{\rho^*}^{00}_m, \widehat{\rho^*}^{01}_m, \widehat{\rho^*}^{10}_m, \widehat{\rho^*}^{11}_m$ are given by the following table, for $L \neq 0, M$:[38]

| | $\widehat{\rho^*}^{00}_m$ | $\widehat{\rho^*}^{01}_m$ | $\widehat{\rho^*}^{10}_m$ | $\widehat{\rho^*}^{11}_m$ |
|---|---|---|---|---|
| $m \in L, y_L \neq y_K$ | $\gamma/L$ | $1/L$ | $0$ | $(1-\gamma)/L$ |
| $m \in L, y_L = y_K$ | $1/L$ | $\gamma/L$ | $(1-\gamma)/L$ | $0$ |
| $m \notin L, y_L \neq y_K$ | $(1-\gamma)/(M-L)$ | $0$ | $1/(M-L)$ | $\gamma/(M-L)$ |
| $m \notin L, y_L = y_K$ | $0$ | $(1-\gamma)/(M-L)$ | $\gamma/(M-L)$ | $1/(M-L)$ |

Here $\widehat{\rho^*}^{00}_m$ represents the expected probability of an individual choosing a website $m$, contingent on having received two correct signals $x_n = \omega$ and $z_n = y_K$, when $\alpha = 0$ (i.e., absent attention bias);

[38] In the extreme cases, $L = 0$ or $L = M$, clearly, $\widehat{\rho^*}^{00}_m = \widehat{\rho^*}^{01}_m = \widehat{\rho^*}^{10}_m = \widehat{\rho^*}^{11}_m = 0$ $(= 1)$ if $m \in L, L = 0$ or $m \notin L, L = M$ (if $m \in L, L = M$ or $m \notin L, L = 0$).

$\widehat{\rho^*}_m^{01}$ represents the expected probability of an individual choosing a website $m$, contingent on having received a correct signal on the state of the world, $x_n = \omega$, and an incorrect signal on the majority, $z_n \neq y_K$, when $\alpha = 0$, and analogously for $\widehat{\rho^*}_m^{10}$ and $\widehat{\rho^*}_m^{11}$. Importantly, $\widehat{\rho^*}_m^{00}, \widehat{\rho^*}_m^{01}, \widehat{\rho^*}_m^{10}$, and $\widehat{\rho^*}_m^{11}$ are fixed coefficients that do not vary with $t$. In order to apply the basic approximation theorem we assume $\nu$ is of the order $O(\frac{1}{n})$ so that, $\nu \to 0$ as $n \to \infty$, and, to guarantee smoothness and avoid boundary problems, we assume there exists $\epsilon > 0$ such that, each $\widehat{r}_{n,m} \geq \epsilon$ for all $n, m$. Moreover, replacing $\widehat{r}_{n-1,m}$ with $x_m$ (and hence the vector $\widehat{r}_{n-1}$ with the vector $x = (x_1, \ldots, x_M)$), we obtain a function $g : \Delta_\epsilon(M) \to \mathbb{R}^M$, defined, for $m = 1, 2, \ldots, M$, by,

$$g_m(x) = \mathbb{E}\left[\rho_{n-1,m} - r_{n-1,m} \,|\, r_{n-1,m} = x_m \text{ for } m = 1, \ldots, M\right] = \theta_m(x) - x_m,$$

where $\theta : \Delta_\epsilon(M) \to \mathbb{R}^M$ is defined by,

$$\begin{aligned} \theta_m(x) \quad = \quad & p\mu \frac{(x_m)^\alpha \cdot \widehat{\rho^*}_m^{00}}{\sum_{m'} (x_{m'})^\alpha \cdot \widehat{\rho^*}_{m'}^{00}} + p(1-\mu) \frac{(x_m)^\alpha \cdot \widehat{\rho^*}_m^{01}}{\sum_{m'} (x_{m'})^\alpha \cdot \widehat{\rho^*}_{m'}^{01}} + \\ & + (1-p)\mu \frac{(x_m)^\alpha \cdot \widehat{\rho^*}_m^{10}}{\sum_{m'} (x_{m'})^\alpha \cdot \widehat{\rho^*}_{m'}^{10}} + (1-p)(1-\mu) \frac{(x_m)^\alpha \cdot \widehat{\rho^*}_m^{11}}{\sum_{m'} (x_{m'})^\alpha \cdot \widehat{\rho^*}_{m'}^{11}}. \end{aligned}$$

Given that the function $g$ is smooth in $x$ on $\Delta_\epsilon(M)$, it can be shown that the expected limit of our stochastic process can be obtained by solving the ordinary differential equation

$$\dot{x} = g(x), \tag{A.2}$$

for some initial condition $x_0 \in \Delta_\epsilon(M)$. In particular, for any given initial condition $r_0$, there is a unique limit, and for small enough values of $\nu$ the stochastic process $r_n$ (and hence also $\rho_n$) tends to follow the unique solution trajectory and linger around the asymptotically stable limit point of the differential equation $\dot{x} = g(x)$. More formally, after verifying that our process $r_n$ satisfies Assumptions 1–4 on p. 261, we apply the following result of Izquierdo and Izquierdo (2013) (summarizing points (i)–(iv) on pp. 261–262):[39]

- For any $x_0 \in \Delta_\epsilon(M)$, the ordinary differential equation $\dot{x} = g(x)$ with initial condition $x(t = 0) = x_0$ has a unique solution with trajectory $x(t, x_0) \in \Delta(M)$.
- For finite $t$, as $\nu \to 0$, the stochastic process $r_n$ in time step $n = [t/\nu]$ converges in probability to the point $x(t, x_0)$ of the trajectory.
- If, for increasing $t$, the trajectory $x(t, x_0)$ approaches an asymptotically stable point, say $x_\infty$, then, for any $\delta > 0$, there exists $N_\delta < \infty$ such that, for any $n > N_\delta$, the probability that the process $r_n$ in time-step $n = [t/\nu]$ is in a small neighborhood $B_\delta$ of $x_\infty$, goes to 1, as $\nu$ goes to 0.
- For fixed $\nu$, the process $r_n$ eventually approaches its asymptotic behavior.

Roughly put, studying the asymptotically stable solution(s) of the differential equation $\dot{x} = g(x)$, allows us to characterize the limit points of the stochastic processes $r_n$ and $\rho_n$ of our search environments defined in Section 2.

[39] The assumptions are straightforward to verify in our relatively simple setting, after making the following change of notation with respect to the notation of Izquierdo and Izquierdo (2013): $\gamma \longleftrightarrow \nu$, $X_n^\gamma \longleftrightarrow r_n$ for given $\nu$, $\Delta X_n^\gamma = \gamma Y_n^\gamma \longleftrightarrow \Delta r_n = \nu(\rho_n - r_n)$, and $I \longleftrightarrow \Delta_\epsilon$.

## B Proofs

**Proof of Proposition 1**. We begin by characterizing the limit ranking probabilities using the differential equation from Appendix A. Since the initial ranking is uniform, we have that the expected ranking probabilities are equal for websites with the same signal, that is, $\widehat{r}_{n,m} = \widehat{r}_{n,m'}$ for any two websites $m, m'$ with $y_m = y_{m'}$. Since we consider partitions of $M$ into sets $J$ and $M \backslash J$, and $\rho_{n,J} + \rho_{n,M\backslash J} = 1$, it suffices to check the case $J = L$. To simplify notation, let $x = \widehat{r}_{n,L} = L \cdot \widehat{r}_{n,m}$ for $m \in L$ be the total expected ranking probability for the websites in $L$, so that $1 - x$ is the total expected ranking probability for the remaining websites in $M \setminus L$. Using the expressions for $\widehat{\rho^*}^{00}_m, \widehat{\rho^*}^{01}_m, \widehat{\rho^*}^{10}_m, \widehat{\rho^*}^{11}_m$ from the table in Appendix A, and assuming that $0 \leq x \leq 1$, we have that the equations defining the limit probabilities can be reduced to two equations of the form:

$$H_L^{minority}(x; \alpha, \mu, \gamma, p) \equiv \theta_L^{minority}(x; \alpha, \mu, \gamma, p) - x = 0, \text{ for } 1 \leq L \leq \frac{M-1}{2} \tag{B.1}$$

$$H_L^{majority}(y; \alpha, \mu, \gamma, p) \equiv \theta_L^{majority}(y; \alpha, \mu, \gamma, p) - y = 0, \text{ for } \frac{M-1}{2} \leq L \leq M-1, \tag{B.2}$$

where $\theta_L^{minority}$ and $\theta_L^{majority}$ are defined as:

$$\theta_L^{minority}(x; \alpha, \mu, \gamma, p) = p(1-\mu) + \frac{p\mu\gamma\left(\frac{x}{L}\right)^\alpha}{\gamma\left(\frac{x}{L}\right)^\alpha + (1-\gamma)\left(\frac{1-x}{M-L}\right)^\alpha} + \frac{(1-p)(1-\mu)(1-\gamma)\left(\frac{x}{L}\right)^\alpha}{(1-\gamma)\left(\frac{x}{L}\right)^\alpha + \gamma\left(\frac{1-x}{M-L}\right)^\alpha} \tag{B.3}$$

and

$$\theta_L^{majority}(y; \alpha, \mu, \gamma, p) = p\mu + \frac{p(1-\mu)\gamma\left(\frac{y}{L}\right)^\alpha}{\gamma\left(\frac{y}{L}\right)^\alpha + (1-\gamma)\left(\frac{1-y}{M-L}\right)^\alpha} + \frac{(1-p)\mu(1-\gamma)\left(\frac{y}{L}\right)^\alpha}{(1-\gamma)\left(\frac{y}{L}\right)^\alpha + \gamma\left(\frac{1-y}{M-L}\right)^\alpha} \tag{B.4}$$

It is easy to check that at $x = 0$ and $x = 1$, we have, respectively,

$$0 \leq p(1-\mu) = \theta_L^{minority}(0; \alpha, \mu, \gamma, p) < \theta_L^{minority}(1; \alpha, \mu, \gamma, p) = p + (1-p)(1-\mu) < 1,$$

and similarly, at $y = 0$ and $y = 1$,

$$0 < p\mu = \theta_L^{majority}(0; \alpha, \mu, \gamma, p) < \theta_L^{majority}(1; \alpha, \mu, \gamma, p) = p + (1-p)\mu \leq 1.$$

In particular, $\theta_L^{minority}$ starts at or above the $x$-function at $x = 0$ and ends below the $x$-function at $x = 1$. Similarly, $\theta_L^{majority}$ starts above the $y$-function at $y = 0$ and ends below the $y$-function at $y = 1$. In Figure B.1, we plot $\theta_L^{majority}$ for different values of $L$ and for $\alpha = 1$ on the panel on the left and for $\alpha = 4$ on the panel on the right. For most parameter values of interest (i.e., while $\alpha$ not too large) there is a unique interior solution to both Equations (B.1) and (B.2); this is the situation depicted on the left panel. However, in general there can be multiple solutions; as depicted in the right panel. Importantly, the solutions of interest are the ones where the functions $\theta_L^{minority}$ and $\theta_L^{majority}$ intersect the $x$- and $y$-functions from above.[40] This is because, the process can only converge to those due to the fact that while clicking probabilities ($\theta_L^{minority}$ or $\theta_L^{majority}$) are above the respective ranking probabilities ($x$ or $y$) then the ranking probabilities will tend to increase and this will continue until the solution (at the intersection) is reached. Similarly, when clicking probabilities are below the respective ranking probabilities, then the ranking probabilities will tend to decrease until the solution

[40] As $\alpha \to \infty$, the stable solutions converge to the bounds $p(1-\mu)$ and $p+(1-p)(1-\mu)$ for $\theta_L^{minority}$ and to $p(1-\mu)$ and $p + (1-p)\mu$ for $\theta_L^{majority}$, where both functions, $\theta_L^{minority}$ and $\theta_L^{majority}$, become arbitrarily flat. Because these solutions do not depend on $L$, this implies that the $AOF$ effect tends to zero in the limit. On the other hand, the unstable solutions converge to $\frac{L}{M}$, where both $\theta_L^{minority}$ and $\theta_L^{majority}$ become arbitrarily steep.

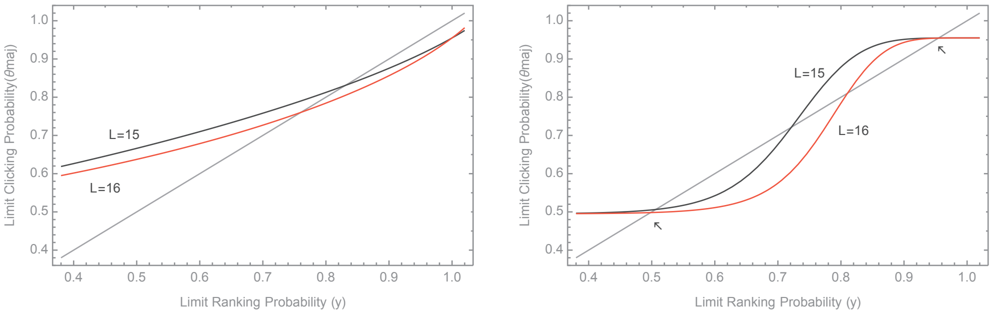


Figure B.1: Limit clicking probability ($\theta_L^{majority}$) as a function of the limit ranking probability ($y$) for different values of $L$ and for $\alpha = 1$ (left) and $\alpha = 4$ (right). In both panels, $M = 20$, $\mu = 0.9$, $\gamma = 0.33$, $p = 0.55$, and $r_1$ is uniform.

is reached. In particular, only the two solutions with arrows are relevant in the panel on the right. The other interior solutions without an arrow (where the $x$- and $y$-functions are intersected from below) are unstable and are never reached by our dynamic process.[41] The key part of the proof is to show that the (stable) solutions $x = x_L^{minority}$ and $y = y_L^{majority}$ to the two equations just above are decreasing in $L$ in the corresponding ranges. Since at the solutions we have $H_L^{minority}(x_L^{minority}; \alpha, \mu, \gamma, p) = 0$ and $H_L^{majority}(y_L^{majority}; \alpha, \mu, \gamma, p) = 0$, we can write the first derivatives with respect to $L$ as:

$$\frac{dx_L^{minority}}{dL} = -\frac{\partial H_L^{minority}(x_L^{minority}; \alpha, \mu, \gamma, p)/\partial L}{\partial H_L^{minority}(x_L^{minority}; \alpha, \gamma, p)/\partial x} = \frac{\partial \theta_L^{minority}/\partial L}{1 - \partial \theta_L^{minority}/\partial x},$$

$$\frac{dy_L^{majority}}{dL} = -\frac{\partial H_L^{majority}(y_L^{majority}; \alpha, \mu, \gamma, p)/\partial L}{\partial H_L^{majority}(y_L^{majority}; \alpha, \gamma, p)/\partial y} = \frac{\partial \theta_L^{majority}/\partial L}{1 - \partial \theta_L^{majority}/\partial y}.$$

To see that both are negaitive, we need to check that $\frac{\partial \theta_L^{minority}}{\partial L} \leq 0$, $\frac{\partial \theta_L^{majority}}{\partial L} \leq 0$ and $\frac{\partial \theta_L^{minority}}{\partial x} < 1$, $\frac{\partial \theta_L^{majority}}{\partial y} < 1$. Straighforward computations yield:

$$\frac{\partial \theta_L^{minority}}{\partial L} = -\frac{\alpha\gamma(1-\gamma)M\left(\frac{x(1-x)}{L(M-L)}\right)^\alpha}{L(M-L)}\left(\frac{p\mu}{\left(\gamma\left(\frac{x}{L}\right)^\alpha + (1-\gamma)\left(\frac{1-x}{M-L}\right)^\alpha\right)^2} + \frac{(1-p)(1-\mu)}{\left((1-\gamma)\left(\frac{x}{L}\right)^\alpha + \gamma\left(\frac{1-x}{M-L}\right)^\alpha\right)^2}\right)$$

$$\frac{\partial \theta_L^{majority}}{\partial L} = -\frac{\alpha\gamma(1-\gamma)M\left(\frac{y(1-y)}{L(M-L)}\right)^\alpha}{L(M-L)}\left(\frac{p(1-\mu)}{\left(\gamma\left(\frac{y}{L}\right)^\alpha + (1-\gamma)\left(\frac{1-y}{M-L}\right)^\alpha\right)^2} + \frac{(1-p)\mu}{\left((1-\gamma)\left(\frac{x}{L}\right)^\alpha + \gamma\left(\frac{1-y}{M-L}\right)^\alpha\right)^2}\right)$$

$$\frac{\partial \theta_L^{minority}}{\partial x} = \frac{\alpha\gamma(1-\gamma)\left(\frac{x(1-x)}{L(M-L)}\right)^\alpha}{x(1-x)}\left(\frac{p\mu}{\left(\gamma\left(\frac{x}{L}\right)^\alpha + (1-\gamma)\left(\frac{1-x}{M-L}\right)^\alpha\right)^2} + \frac{(1-p)(1-\mu)}{\left((1-\gamma)\left(\frac{x}{L}\right)^\alpha + \gamma\left(\frac{1-x}{M-L}\right)^\alpha\right)^2}\right)$$

$$\frac{\partial \theta_L^{majority}}{\partial y} = \frac{\alpha\gamma(1-\gamma)\left(\frac{y(1-y)}{L(M-L)}\right)^\alpha}{y(1-y)}\left(\frac{p(1-\mu)}{\left(\gamma\left(\frac{y}{L}\right)^\alpha + (1-\gamma)\left(\frac{1-y}{M-L}\right)^\alpha\right)^2} + \frac{(1-p)\mu}{\left((1-\gamma)\left(\frac{y}{L}\right)^\alpha + \gamma\left(\frac{1-y}{M-L}\right)^\alpha\right)^2}\right),$$

where the first two are clearly non-positive always, and negative for interior values $\alpha > 0$ and $0 < \gamma < 1$ and $0 < x < 1$. The last two are also clearly non-negative. However, we now show that they are strictly

[41] Even if the process were to start at such a solution, any extra click by an individual will move the ranking probability to the left (or to the right) thereby leading to a situation where the clicking probability is below the ranking probability, leading the ranking probability to decrease and move further to the left, again until a stable solution with an arrow is reached (or similarly, if the extra click increases the ranking probability, this will lead to a situation where the clicking probability is above the ranking probability, leading the ranking probability to increase and move further right until it also reaches a stable solution with an arrow).

less than one for the stable solutions. First, it can be checked that, for any limit clicking probability ($0 \leq x$ (or $y$) $\leq 1$) and for any other parameter values of the model,

$$0 \leq \frac{\gamma\left(\frac{x}{L}\right)^{\alpha}}{\gamma\left(\frac{x}{L}\right)^{\alpha}+(1-\gamma)\left(\frac{1-x}{M-L}\right)^{\alpha}}, \frac{(1-\gamma)\left(\frac{1-x}{M-L}\right)^{\alpha}}{\gamma\left(\frac{x}{L}\right)^{\alpha}+(1-\gamma)\left(\frac{1-x}{M-L}\right)^{\alpha}}, \frac{\gamma(1-\gamma)\left(\frac{x(1-x)}{L(M-L)}\right)^{\alpha}}{\left((1-\gamma)\left(\frac{x}{L}\right)^{\alpha}+\gamma\left(\frac{1-x}{M-L}\right)^{\alpha}\right)^{2}} \leq 1,$$

which, as already seen in Figure B.1 above, implies that for values of $\alpha \geq 1$ there may be more than one solution to Equations (B.1) and (B.2). Moreover, some of the solutions may have a slope greater or equal to one. However, as explained above, only those solutions are stable which intersect the functions $x$ (or $y$) from above. Since both $x$ and $y$ clearly have slope equal to 1 everywhere this implies that both $\theta_L^{minority}$ and $\theta_L^{majority}$ must have a slope less than one at the stable solutions and hence must satisfy $\frac{\partial\theta_L^{minority}}{\partial x} < 1$ and $\frac{\partial\theta_L^{majority}}{\partial y} < 1$. This then implies that the respective solutions $x_L^{minority}$ and $y_L^{majority}$ will satisfy $\frac{\partial\widehat{r}_{\infty,L}}{\partial L} \leq 0$ as well as $\frac{\partial\widehat{\rho}_{\infty,L}}{\partial L} \leq 0$ on the relevant ranges, which will also be negative for interior values of the parameters. This shows $AOF$ for both minority and majority websites with correct signal. To see that $AOF$ also applies to websites $J \subset M$ with incorrect signal, notice that, if increasing $L$, decreases $\widehat{\rho}_{\infty,L}$, then, since the total number of websites $M$ is fixed and $\widehat{\rho}_{\infty,L}+\widehat{\rho}_{\infty,M\setminus L} = 1$, this readily implies that decreasing $M - L$, increases $\widehat{\rho}_{\infty,M\setminus L}$. Therefore, given $AOF$ for websites $L$ with the correct signal and taking $J = M\setminus L$ also shows that $AOF$ applies to minority and majority websites with incorrect signal. This concludes the proof of Proposition 1. □

**Proof of Corollary 1.** The cases of points 1, 2, and 3 are straightforward. To see 4, consider again the slightly generalized equation defining the clicking proabilities:

$$\widetilde{\rho}_{n,m} = \frac{(r_{n,m})^{\alpha}\cdot(\rho^*_{n,m})^{\beta}}{\sum_{m'\in M}(r_{n,m'})^{\alpha}\cdot(\rho^*_{n,m'})^{\beta}}, \text{ for } \alpha \geq 0, \beta \geq 0.$$

For such clicking probabilities define the limit clicking probabilities with equations of the form (B.1) and (B.2), but where now $\theta_L^{minority}$ and $\theta_L^{majority}$ are defined by:

$$\theta_L^{minority}(x;\alpha,\beta,\mu,\gamma,p) = p(1-\mu) + \frac{p\mu L\left(\frac{x}{L}\right)^{\alpha}\left(\frac{\gamma}{L}\right)^{\beta}}{L\left(\frac{x}{L}\right)^{\alpha}\left(\frac{\gamma}{L}\right)^{\beta}+(M-L)\left(\frac{1-x}{M-L}\right)^{\alpha}\left(\frac{1-\gamma}{M-L}\right)^{\beta}} + \frac{(1-p)(1-\mu)L\left(\frac{x}{L}\right)^{\alpha}\left(\frac{1-\gamma}{L}\right)^{\beta}}{L\left(\frac{x}{L}\right)^{\alpha}\left(\frac{1-\gamma}{L}\right)^{\beta}+(M-L)\left(\frac{1-x}{M-L}\right)^{\alpha}\left(\frac{\gamma}{M-L}\right)^{\beta}}$$

$$\theta_L^{majority}(y;\alpha,\beta,\mu,\gamma,p) = p\mu + \frac{p(1-\mu)L\left(\frac{y}{L}\right)^{\alpha}\left(\frac{\gamma}{L}\right)^{\beta}}{L\left(\frac{y}{L}\right)^{\alpha}\left(\frac{\gamma}{L}\right)^{\beta}+(M-L)\left(\frac{1-y}{M-L}\right)^{\alpha}\left(\frac{1-\gamma}{M-L}\right)^{\beta}} + \frac{(1-p)\mu L\left(\frac{y}{L}\right)^{\alpha}\left(\frac{1-\gamma}{L}\right)^{\beta}}{L\left(\frac{y}{L}\right)^{\alpha}\left(\frac{1-\gamma}{L}\right)^{\beta}+(M-L)\left(\frac{1-y}{M-L}\right)^{\alpha}\left(\frac{\gamma}{M-L}\right)^{\beta}},$$

for which we can compute:

$$\frac{\partial\theta_L^{minority}}{\partial L} = (1-\alpha-\beta)\left(\frac{p\mu M\left(\frac{x}{L}\right)^{\alpha}\left(\frac{1-x}{M-L}\right)^{\alpha}\left(\frac{\gamma}{L}\right)^{\beta}\left(\frac{1-\gamma}{M-L}\right)^{\beta}}{\left(L\left(\frac{x}{L}\right)^{\alpha}\left(\frac{\gamma}{L}\right)^{\beta}+(M-L)\left(\frac{1-x}{M-L}\right)^{\alpha}\left(\frac{1-\gamma}{M-L}\right)^{\beta}\right)^{2}} + \frac{(1-p)(1-\mu)M\left(\frac{x}{L}\right)^{\alpha}\left(\frac{1-x}{M-L}\right)^{\alpha}\left(\frac{1-\gamma}{L}\right)^{\beta}\left(\frac{\gamma}{M-L}\right)^{\beta}}{\left(L\left(\frac{x}{L}\right)^{\alpha}\left(\frac{1-\gamma}{L}\right)^{\beta}+(M-L)\left(\frac{1-x}{M-L}\right)^{\alpha}\left(\frac{\gamma}{M-L}\right)^{\beta}\right)^{2}}\right)$$

$$\frac{\partial\theta_L^{majority}}{\partial L} = (1-\alpha-\beta)\left(\frac{(1-p)\mu M\left(\frac{y}{L}\right)^{\alpha}\left(\frac{1-y}{M-L}\right)^{\alpha}\left(\frac{1-\gamma}{L}\right)^{\beta}\left(\frac{\gamma}{M-L}\right)^{\beta}}{\left(L\left(\frac{y}{L}\right)^{\alpha}\left(\frac{\gamma}{L}\right)^{\beta}+(M-L)\left(\frac{1-y}{M-L}\right)^{\alpha}\left(\frac{1-\gamma}{M-L}\right)^{\beta}\right)^{2}} + \frac{p(1-\mu)M\left(\frac{y}{L}\right)^{\alpha}\left(\frac{1-y}{M-L}\right)^{\alpha}\left(\frac{1-\gamma}{L}\right)^{\beta}\left(\frac{\gamma}{M-L}\right)^{\beta}}{\left(L\left(\frac{y}{L}\right)^{\alpha}\left(\frac{1-\gamma}{L}\right)^{\beta}+(M-L)\left(\frac{1-y}{M-L}\right)^{\alpha}\left(\frac{\gamma}{M-L}\right)^{\beta}\right)^{2}}\right).$$

To see that $AOF$ does *not* hold, we check that $\frac{dx_L^{minority}}{dL} = \frac{\partial\theta_L^{minority}/\partial L}{1-\partial\theta_L^{minority}/\partial x} \geq 0$ and $\frac{dy_L^{majority}}{dL} =$

$\frac{\partial\theta_L^{majority}/\partial L}{1-\partial\theta_L^{majority}/\partial y} \geq$. This follows, if $\frac{\partial\theta_L^{minority}}{\partial L} \geq 0$, $\frac{\partial\theta_L^{majority}}{\partial L} \geq 0$ and $\frac{\partial\theta_L^{minority}}{\partial x} < 1$, $\frac{\partial\theta_L^{majority}}{\partial y} < 1$. Clearly $\frac{\partial\theta_L^{minority}}{\partial L} \geq 0$, $\frac{\partial\theta_L^{majority}}{\partial L} \geq 0$ holds for $\alpha+\beta \leq 1$ and both are positive for interior values $\alpha+\beta < 1$, $0 < \gamma < 1$ and $0 < y < 1$. Finally, $\frac{\partial\theta_L^{minority}}{\partial x} < 1$, $\frac{\partial\theta_L^{majority}}{\partial y} < 1$ follow from the same arguments as in the case $\beta = 1$ given in the proof of Proposition 1. □

**Proof of Corollary 2.** Consider Equation (4). Then, when $\alpha = 0$ it is easy to see that:

$$\rho_{n,L} = \begin{cases} 0 & \text{if } L = 0 \\ (1-\mu)p + (1-\mu)(1-p)(1-\gamma) + \mu p\gamma & \text{if } 1 \leq L = \frac{M-1}{2} \\ \mu p + \mu(1-p)(1-\gamma) + (1-\mu)p\gamma & \text{if } \frac{M+1}{2} \leq L \leq M-1 \\ 1 & \text{if } L = M\,, \end{cases}$$

which can be checked is monotonically increasing in $L$ for $\mu \geq \frac{1}{2}$ for any $n$. Now suppose $\alpha > 0$. To see the non-monotonicity, notice again that at $L = 0$, we have $\rho_{n,L} = 0$, and at $L = M-1$, we have $\rho_{n,L} = 1$, for all $n$, so that $\rho_{n,L}$ and hence $\rho_{\infty,L}$ can only increase at $L = 0, M-1$. For the case $L = \frac{M-1}{2}$, ($M - L = \frac{M+1}{2}$) note that the difference $\theta^{majority}_{\frac{M+1}{2}}(y;\alpha,\mu,\gamma,p) - \theta^{minority}_{\frac{M-1}{2}}(x;\alpha,\mu,\gamma,p)$ can be written as:

$$\begin{aligned} \theta_{M-L}^{majority} - \theta_L^{minority} &= (2\mu-1)\left(p\left(1 - \frac{\gamma\left(\frac{x}{L}\right)^\alpha}{\gamma\left(\frac{x}{L}\right)^\alpha + (1-\gamma)\left(\frac{1-x}{M-L}\right)^\alpha}\right) + (1-p)\frac{(1-\gamma)\left(\frac{x}{L}\right)^\alpha}{(1-\gamma)\left(\frac{x}{L}\right)^\alpha + \gamma\left(\frac{1-x}{M-L}\right)^\alpha}\right) \\ &\quad + p(1-\mu)\left(\frac{\gamma\left(\frac{y}{M-L}\right)^\alpha}{\gamma\left(\frac{y}{M-L}\right)^\alpha + (1-\gamma)\left(\frac{1-y}{L}\right)^\alpha} - \frac{\gamma\left(\frac{x}{L}\right)^\alpha}{\gamma\left(\frac{x}{L}\right)^\alpha + (1-\gamma)\left(\frac{1-x}{M-L}\right)^\alpha}\right) \\ &\quad + (1-p)\mu\left(\frac{(1-\gamma)\left(\frac{y}{M-L}\right)^\alpha}{(1-\gamma)\left(\frac{y}{M-L}\right)^\alpha + \gamma\left(\frac{1-y}{L}\right)^\alpha} - \frac{(1-\gamma)\left(\frac{x}{L}\right)^\alpha}{(1-\gamma)\left(\frac{x}{L}\right)^\alpha + \gamma\left(\frac{1-x}{M-L}\right)^\alpha}\right), \end{aligned}$$

where, at the relevant solutions, $x = x_L^{majority}$ and $y = y_{M-L}^{majority}$, we have $y \geq x$, such that the expressions in the second and third lines are both non-negative. And since $\mu > \frac{p}{q} > \frac{1}{2}$, the overall difference $(\theta_{M-L}^{majority} - \theta_L^{minority})$ is non-negative. In all other cases, that is, interior values $L \notin \{0, \frac{M-1}{2}, M-1\}$, $\rho_{\infty,L}$ and hence $\mathcal{P}_L$ are decreasing in $L$ by Proposition 1. □

**Proof of Proposition 2**. From the definition of $\mathcal{P}$ in Equation (8), we have:

$$\frac{\partial\mathcal{P}}{\partial z} = \sum_{L=0}^{M}\binom{M}{L}q^L(1-q)^{M-L}\frac{\partial\mathcal{P}_L}{\partial z},$$

for any given variable $z$. Hence, we can evaluate the comparative statics by looking at the effects on the interim efficiency, $(\frac{\partial\mathcal{P}_L}{\partial z})$. So, for point 1, to see that $\mathcal{P}$ is weakly increasing in $p$, since the initial ranking is uniform, we can use the same reasoning as in the proof of Proposition 1. In particular, it suffices to consider the following derivatives for $\theta_L^{minority}$ and $\theta_L^{majority}$ defined respectively in Equations (B.3) and (B.4) above:

$$\begin{aligned} \frac{\partial\theta_L^{minority}}{\partial p} &= (1-\mu)\left(1 - \frac{(1-\gamma)\left(\frac{x}{L}\right)^\alpha}{\gamma\left(\frac{1-x}{M-L}\right)^\alpha + (1-\gamma)\left(\frac{x}{L}\right)^\alpha}\right) + \mu\frac{\gamma\left(\frac{x}{L}\right)^\alpha}{\gamma\left(\frac{x}{L}\right)^\alpha + (1-\gamma)\left(\frac{1-x}{M-L}\right)^\alpha} \geq 0\,, \\ \frac{\partial\theta_L^{majority}}{\partial p} &= \mu\left(1 - \frac{(1-\gamma)\left(\frac{y}{L}\right)^\alpha}{\gamma\left(\frac{1-y}{M-L}\right)^\alpha + (1-\gamma)\left(\frac{x}{L}\right)^\alpha}\right) + (1-\mu)\frac{\gamma\left(\frac{y}{L}\right)^\alpha}{\gamma\left(\frac{y}{L}\right)^\alpha + (1-\gamma)\left(\frac{1-y}{M-L}\right)^\alpha} \geq 0\,. \end{aligned}$$

This implies that $\rho_{\infty,L} = \mathcal{P}_L$ is increasing in $p$ for all values of $L$ and hence so is $\mathcal{P}$. To see that $\mathcal{P}$ is increasing in $\mu$, notice that:

$$\frac{\partial\theta_L^{minority}}{\partial\mu} = -p\left(1 - \frac{\gamma\left(\frac{x}{L}\right)^\alpha}{\gamma\left(\frac{x}{L}\right)^\alpha + (1-\gamma)\left(\frac{1-x}{M-L}\right)^\alpha}\right) - (1-p)\frac{(1-\gamma)\left(\frac{x}{L}\right)^\alpha}{\gamma\left(\frac{1-x}{M-L}\right)^\alpha + (1-\gamma)\left(\frac{x}{L}\right)^\alpha} \leq 0\,,$$

$$\frac{\partial\theta_L^{majority}}{\partial\mu} = p\left(1 - \frac{\gamma\left(\frac{y}{L}\right)^\alpha}{\gamma\left(\frac{y}{L}\right)^\alpha + (1-\gamma)\left(\frac{1-y}{M-L}\right)^\alpha}\right) + (1-p)\frac{(1-\gamma)\left(\frac{y}{L}\right)^\alpha}{\gamma\left(\frac{1-y}{M-L}\right)^\alpha + (1-\gamma)\left(\frac{y}{L}\right)^\alpha} \geq 0\,.$$

It can be further checked that for $\mu q > p > \frac{1}{2}$, the positive effect of when the websites in $L$ form a majority outweighs the negative effect of when they form a minority. To see this, we can rewrite the above derivatives as:

$$\frac{\partial\theta_L^{minority}}{\partial\mu} = -\theta_{L|_{\mu=1}}^{minority} + \theta_{L|_{\mu=0}}^{minority}, \qquad \frac{\partial\theta_L^{majority}}{\partial\mu} = \theta_{L|_{\mu=1}}^{majority} - \theta_{L|_{\mu=0}}^{majority}\,.$$

Moreover, it can be checked that, at the relevant solutions $x = x_L^{minority}$ and $y = y_{M-L}^{majority}$, taking again as the relevant number of websites with correct signal, $L$ and $M-L$ for the minority and majority case respectively, we have,

$$\theta_{M-L|_{\mu=1}}^{majority}(y) - \theta_{L|_{\mu=1}}^{minority}(x) \geq 0, \qquad \theta_{M-L|_{\mu=0}}^{majority}(y) - \theta_{L|_{\mu=0}}^{minority}(x) \leq 0.$$

Together with the above equations, we can obtain the sign of $\frac{\partial\mathcal{P}}{\partial\mu}$ from:

$$\begin{aligned}
&\sum_{L=1}^{\frac{M-1}{2}} \binom{M}{L} q^L(1-q)^{M-L}\frac{\partial\theta_L^{minority}}{\partial\mu} + \sum_{L=\frac{M+1}{2}}^{M-1} \binom{M}{L} q^L(1-q)^{M-L}\frac{\partial\theta_L^{majority}}{\partial\mu} \\
&= \sum_{L=1}^{\frac{M-1}{2}} \binom{M}{L} q^L(1-q)^{M-L}(-\theta_{L|_{\mu=1}}^{minority} + \theta_{L|_{\mu=0}}^{minority}) + \sum_{L=\frac{M+1}{2}}^{M-1} \binom{M}{L} q^L(1-q)^{M-L}(\theta_{L|_{\mu=1}}^{majority} - \theta_{L|_{\mu=0}}^{majority}) \\
&= \sum_{L=1}^{\frac{M-1}{2}} \binom{M}{L} q^L(1-q)^{M-L}(-\theta_{L|_{\mu=1}}^{minority} + \theta_{L|_{\mu=0}}^{minority}) + \sum_{L=1}^{\frac{M-1}{2}} \binom{M}{M-L} q^{M-L}(1-q)^{L}(\theta_{M-L|_{\mu=1}}^{majority} - \theta_{M-L|_{\mu=0}}^{majority}) \\
&\geq \sum_{L=1}^{\frac{M-1}{2}} \binom{M}{L} q^L(1-q)^{M-L}\left((\theta_{M-L|_{\mu=1}}^{majority} - \theta_{L|_{\mu=1}}^{minority}) - (\theta_{M-L|_{\mu=0}}^{majority} - \theta_{L|_{\mu=0}}^{minority})\right) \geq 0,
\end{aligned}$$

for $\frac{1}{2} < p < q$, which in turn implies that $\frac{\partial\mathcal{P}}{\partial\mu} \geq 0$.

To see points 2 and 3, that $\mathcal{P}$ is both increasing and decreasing in $q$, $\gamma$, and $\alpha$, it suffices to check this by means of examples. As illustrated in Figure 2, $\mathcal{P}$ is increasing in $q$ for low values of $q$ and decreasing for high values of $q$. Figure 3 shows that $\mathcal{P}$ is increasing in $\alpha$ for low values of $\gamma$ and high values of $\mu$ and decreasing in $\alpha$ for high values of $\gamma$ and low values of $\mu$. This is also discussed further in Proposition 3. Finally, to see that $\mathcal{P}$ is also both increasing and decreasing in $\gamma$, notice that in Figure 3 (drawn for $p = 0.55$ and $\mu = 0.9$), $\mathcal{P}$ is clearly decreasing in $\gamma$. Here we have that $p$ is relatively small and $\mu$ relatively large. It is not difficult to see that, in the opposite case, where $p$ is relatively large and $\mu$ relatively small, then $\mathcal{P}$ can be increasing in $\gamma$. The Online Appendix illustrates a case of this (with $p = 0.75$ and $\mu = 0.6$). Basically, whether or not $\mathcal{P}$ is increasing or decreasing in $\gamma$ depends on which of the two signals is ex-ante more informative. □

**Proof of Proposition 3** Assume first that $\mu = 1$. Using the proof of Proposition 1, it can be checked that, for $\alpha = 0$, the solutions to Equations (B.3) and (B.4) take the form, respectively, $x^{minority}_{L|\alpha=0} = \gamma p$, $y^{majority}_{L|\alpha=0} = 1 - \gamma(1-p)$, whereas, for $\alpha = 1$, they take the form:

$$x^{minority}_{L|\alpha=1} = \begin{cases} \frac{(1-\gamma)L-\gamma p(M-L)}{\gamma M-L} & \text{if } L < \frac{\gamma pM}{1-(1-\gamma)p} \\ 0 & \text{if } L \geq \frac{\gamma pM}{1-(1-\gamma)p} \end{cases}, \; y^{majority}_{L|\alpha=1} = \begin{cases} 1 & \text{if } L \leq \frac{(1-\gamma)M}{1-\gamma p} \\ \frac{\gamma pL}{L-(1-\gamma)M} & \text{if } L > \frac{(1-\gamma)M}{1-\gamma p} \end{cases}.$$

It is easy to see that, for $\overline{\gamma} = \frac{1}{p+(1-p)M} > 0$, we have that, for $\gamma \in [0, \overline{\gamma}]$, the solution for $\alpha = 1$ is always $y^{majority}_{L|\alpha=1} = 1$, for $\frac{M+1}{2} \leq L \leq M-1$, which is greater than the corresponding solution for $\alpha = 0$, which is $y^{majority}_{L|\alpha=0} = 1 - \gamma(1-p)$, for any $\frac{M+1}{2} \leq L \leq M-1$. Also, for $\gamma \in [0, \overline{\gamma}]$, the corresponding difference between the solutions $x^{minority}_{L|\alpha=1}$ and $x^{minority}_{L|\alpha=0}$ is bounded below by $-\gamma p$. Recall that the solutions coincide with our interim notion of efficiency for the given number of websites with correct signal, $L$, so that from the definition of $\mathcal{P}$ in Equation (8), we can write:

$$\mathcal{P}' = 0 + \sum_{L=1}^{\frac{M-1}{2}} \binom{M}{L} q^L (1-q)^{M-L} x^{minority}_{L|\alpha=0} + \sum_{L=\frac{M+1}{2}}^{M-1} \binom{M}{L} q^L (1-q)^{M-L} y^{majority}_{L|\alpha=0} + q^M,$$

$$\mathcal{P} = 0 + \sum_{L=1}^{\frac{M-1}{2}} \binom{M}{L} q^L (1-q)^{M-L} x^{minority}_{L|\alpha=1} + \sum_{L=\frac{M+1}{2}}^{M-1} \binom{M}{L} q^L (1-q)^{M-L} y^{majority}_{L|\alpha=1} + q^M,$$

Since $\mu q > p > \frac{1}{2}$, this is enough to imply that, for $q > \frac{p}{\mu}$, while $\gamma \in [0, \overline{\gamma}]$, we have, $\mathcal{P} \geq \mathcal{P}'$ and hence $PoR \geq 0$.

Suppose now $\gamma \geq \overline{\gamma}$. Again, for the given $\gamma$, there are two parts to the solution $y^{majority}_{L|\alpha=1}$ for $\alpha = 1$, namely, a part which is 1 and therefore above the solution for $\alpha = 0$ (for $\frac{M+1}{2} \leq L \leq \frac{(1-\gamma)M}{1-\gamma p}$) and a part which is below the solution for $\alpha = 0$ (for $\frac{(1-\gamma)M}{1-\gamma p} < L \leq M-1$). In particular, while $\gamma \leq \hat{\gamma} =$, there will be $L \geq \frac{M+1}{2}$ for which the solution at $\alpha = 1$ is above the one at $\alpha = 0$. Hence, there exists a level $\overline{q}$ such that, when $q \leq \overline{q}$, then the cases where $L \leq \frac{(1-\gamma)M}{1-\gamma p}$ and hence where the solution $y^{majority}_{L|\alpha=1}$ for $\alpha = 1$ is above the solution $y^{majority}_{L|\alpha=0}$ for $\alpha = 0$ will obtain sufficiently large weight, such that ex ante efficiency for $\alpha = 1$ ($\mathcal{P}$) is greater or equal to ex ante efficiency for $\alpha = 0$ ($\mathcal{P}'$), such that $PoR \geq 0$. At the same time, when $q > \overline{q}$ , then the cases where $L > \frac{(1-\gamma)M}{1-\gamma p}$ and hence where the solution $y^{majority}_{L|\alpha=1}$ for $\alpha = 1$ is below the solution $y^{majority}_{L|\alpha=0}$ for $\alpha = 0$ will obtain sufficiently large weight, such that ex ante efficiency for $\alpha = 1$ ($\mathcal{P}$) is less or equal to ex ante efficiency for $\alpha = 0$ ($\mathcal{P}'$), such that $PoR \leq 0$. This gives the function $\phi$, which is equal to 1 on $[0, \overline{\gamma}]$ and which is decreasing in $\gamma$ on $[\overline{\gamma}, 1]$, since the set of interim realizations, where the solution $y^{majority}_{L|\alpha=1}$ for $\alpha = 1$ is above the solution $y^{majority}_{L|\alpha=0}$, is determined by the cutoff $\frac{(1-\gamma)M}{1-\gamma p}$, which is decreasing in $\gamma$. This shows points 1 and 2 for the case $\mu = 1$. Figure B.2 (left panel) illustrates the case $PoR \geq 0$ for $\mu = 1$.

To see that there exists $\overline{\mu} < 1$, such that points 1 and 2 hold for $\mu \in [\overline{\mu}, 1]$, consider again the functions $\theta^{minority}_L$ and $\theta^{majority}_L$ defining the Equations (B.3) and (B.4). These are linear in $\mu$ and it can be checked that the solutions are arbitrarily close to the ones computed for $\mu = 1$. More specifically, it can be checked that, for general $\mu$, the solutions to Equations (B.3) and (B.4) for $\alpha = 0$ take the form, $x^{minority}_{L|\alpha=0} = (1-\mu)(1-\gamma) + \gamma p$, $y^{majority}_{L|\alpha=0} = \mu(1-\gamma) + \gamma p)$. At the same time, while the solutions for $\alpha = 1$ no longer have a simple closed form, it can be checked that, for any given $L$, they are above the corresponding solutions for $\alpha = 0$, for $L < ((1-\mu)(1-\gamma) + \gamma p)M$, if $L$ is minority, and for

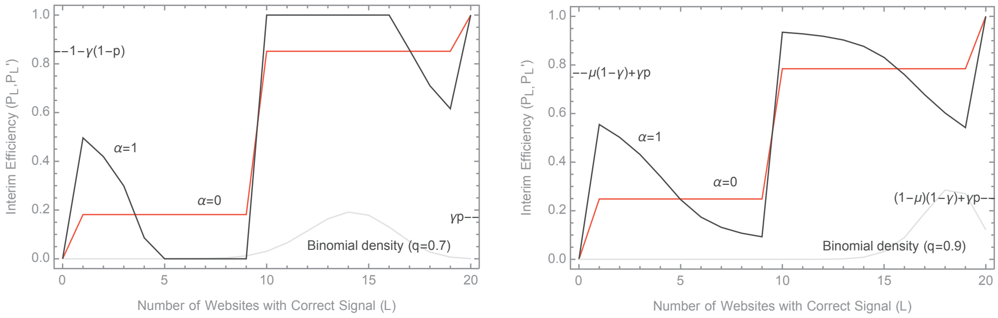


Figure B.2: Interim efficiency $(\mathcal{P}_L, \mathcal{P}'_L)$ for $\alpha = 0$ (black) and $\alpha = 1$ (red dashed) as a function of $L$ for $\mu = 1$ (left) and $\mu = 0.9$ (right). In both panels, $M = 20$, $\gamma = 0.33$, $p = 0.55$, and $r_1$ is uniform. The left panel also shows the density function for the binomial distribution for $q = 0.7 < \phi(\gamma)$ (light gray dashed) at which $PoR > 0$, while the right panel shows the density function for the binomial distribution for $q = 0.9 > \phi(\gamma)$ at which $PoR < 0$.

$L < (\mu(1-\gamma) + \gamma p)M$, if $L$ is majority. In particular, for such $\mu \in [\overline{\mu}, 1]$ there will be corresponding variables $\overline{\gamma}, \hat{\gamma}$ as well as decreasing cutoff levels for $q$ as a function of $\gamma$, such that $PoR \geq 0$ or $PoR \leq 0$ depending on wehther $q$ is above or below the cutoff. Figure B.2 (right panel) illustrates the case $PoR \geq 0$ for $\mu < 1$. □

**Proof of Proposition 4.** For simplicity, we consider the case, where one half of the population has $\gamma_A$ and the other half has $\gamma_B$, and where $\gamma_A \neq \gamma_B$. Recall the equation defining the ranking probabilities in the case of personalization (Equations (10) and (11)), for any $n$ and $m$, and for $\ell = A, B$:

$$r^\ell_{n,m} = (1 - \nu^\ell) r^\ell_{n-1,m} + \nu^\ell \rho^\ell_{n-1,m} ,$$

where:

$$\nu^\ell = \frac{(1-\lambda_n)\nu}{1-\lambda_n \nu} \quad \text{and where } \lambda_n = \begin{cases} 0 & \text{if } n-1 \in \ell \\ \lambda & \text{else .} \end{cases}$$

We can apply the mean dynamics approximation and obtain the deterministic recursions:

$$\begin{aligned} \widehat{r}^A_{n,m} &= \widehat{r}^A_{n-1,m} + \nu\left(\widehat{\rho}^A_{n-1,m} - \widehat{r}^A_{n-1,m}\right) + \frac{(1-\lambda)\nu}{1-\lambda\nu}\left(\widehat{\rho}^B_{n-1,m} - \widehat{r}^A_{n-1,m}\right) \\ \widehat{r}^B_{n,m} &= \widehat{r}^B_{n-1,m} + \nu\left(\widehat{\rho}^B_{n-1,m} - \widehat{r}^B_{n-1,m}\right) + \frac{(1-\lambda)\nu}{1-\lambda\nu}\left(\widehat{\rho}^A_{n-1,m} - \widehat{r}^B_{n-1,m}\right) , \end{aligned} \tag{B.5}$$

where, following Equation (A.1), we can write, for $\ell = A, B$:

$$\begin{aligned} \widehat{\rho}^\ell_{n-1,m} = \mathbb{E}[\rho^\ell_{n-1,m}] = p\mu \frac{(\widehat{r}^\ell_{n-1,m})^\alpha \cdot \widehat{v^*}^{00}_m(\gamma_\ell)}{\sum_{m'}(\widehat{r}^\ell_{n-1,m'})^\alpha \cdot \widehat{v^*}^{00}_{m'}(\gamma_\ell)} + p(1-\mu)\frac{(\widehat{r}^\ell_{n-1,m})^\alpha \cdot \widehat{v^*}^{01}_m(\gamma_\ell)}{\sum_{m'}(\widehat{r}^\ell_{n-1,m'})^\alpha \cdot \widehat{v^*}^{01}_{m'}(\gamma_\ell)} \\ + (1-p)\mu \frac{(\widehat{r}^\ell_{n-1,m})^\alpha \cdot \widehat{v^*}^{10}_m(\gamma_\ell)}{\sum_{m'}(\widehat{r}^\ell_{n-1,m'})^\alpha \cdot \widehat{v^*}^{10}_{m'}(\gamma_\ell)} + (1-p)(1-\mu)\frac{(\widehat{r}^\ell_{n-1,m})^\alpha \cdot \widehat{v^*}^{11}_m(\gamma_\ell)}{\sum_{m'}(\widehat{r}^\ell_{n-1,m'})^\alpha \cdot \widehat{v^*}^{11}_{m'}(\gamma_\ell)}. \end{aligned}$$

Taking the limit $\nu \to 0$ in the equation system (B.5) with $\widehat{r}^\ell_{n,m} = \widehat{r}^\ell_{n-1,m}$, for $\ell = A, B$, yields the equations determining the limit clicking probabilities with personalization, which take the form:

$$\frac{1}{2-\lambda}\widehat{\rho}^A_{n-1,m} + \frac{1-\lambda}{2-\lambda}\widehat{\rho}^B_{n-1,m} - \widehat{r}^A_{n-1,m} = 0 \quad \text{and} \quad \frac{1}{2-\lambda}\widehat{\rho}^B_{n-1,m} + \frac{1-\lambda}{2-\lambda}\widehat{\rho}^A_{n-1,m} - \widehat{r}^B_{n-1,m} = 0.$$

Replacing $\widehat{r}^A_{n-1}$ with $x = (x_1, \ldots, x_M)$ and replacing $\widehat{r}^B_{n-1}$ with $y = (y_1, \ldots, y_M)$, we can study the function $g : \Delta_\epsilon(M) \times \Delta_\epsilon(M) \to \mathbb{R}^{2M}$, defined, for $\ell = A, B$, $m = 1, \ldots, M$, by:

$$g^A_m(x, y) = \theta^A_m(x, y) - x_m \quad \text{and} \quad g^B_m(x, y) = \theta^B_m(x, y) - y_m$$

where $\theta^\ell : \Delta_\epsilon(M) \times \Delta_\epsilon(M) \to \mathbb{R}^M$, $\ell = A, B$, is defined by,

$$\begin{aligned}
\theta^A_m(x,y) = &\frac{1}{2-\lambda}\left(\frac{p\mu \cdot (x_m)^\alpha \cdot \widehat{v^*}^{00}_m(\gamma_A)}{\sum_{m'}(x_{m'})^\alpha \cdot \widehat{v^*}^{00}_{m'}(\gamma_A)} + \frac{p(1-\mu) \cdot (x_m)^\alpha \cdot \widehat{v^*}^{01}_m(\gamma_A)}{\sum_{m'}(x_{m'})^\alpha \cdot \widehat{v^*}^{01}_{m'}(\gamma_A)} + \frac{(1-p)\mu \cdot (x_m)^\alpha \cdot \widehat{v^*}^{10}_m(\gamma_A)}{\sum_{m'}(x_{m'})^\alpha \cdot \widehat{v^*}^{10}_{m'}(\gamma_A)} + \frac{(1-p)(1-\mu) \cdot (x_m)^\alpha \cdot \widehat{v^*}^{11}_m(\gamma_A)}{\sum_{m'}(x_{m'})^\alpha \cdot \widehat{v^*}^{11}_{m'}(\gamma_A)}\right)\\
&+ \frac{1-\lambda}{2-\lambda}\left(\frac{p\mu \cdot (y_m)^\alpha \cdot \widehat{v^*}^{00}_m(\gamma_B)}{\sum_{m'}(y_{m'})^\alpha \cdot \widehat{v^*}^{00}_{m'}(\gamma_B)} + \frac{p(1-\mu) \cdot (y_m)^\alpha \cdot \widehat{v^*}^{01}_m(\gamma_B)}{\sum_{m'}(y_{m'})^\alpha \cdot \widehat{v^*}^{01}_{m'}(\gamma_B)} + \frac{(1-p)\mu \cdot (y_m)^\alpha \cdot \widehat{v^*}^{10}_m(\gamma_B)}{\sum_{m'}(y_{m'})^\alpha \cdot \widehat{v^*}^{10}_{m'}(\gamma_B)} + \frac{(1-p)(1-\mu) \cdot (y_m)^\alpha \cdot \widehat{v^*}^{11}_m(\gamma_B)}{\sum_{m'}(y_{m'})^\alpha \cdot \widehat{v^*}^{11}_{m'}(\gamma_B)}\right),\\
\theta^B_m(x,y) = &\frac{1-\lambda}{2-\lambda}\left(\frac{p\mu \cdot (x_m)^\alpha \cdot \widehat{v^*}^{00}_m(\gamma_A)}{\sum_{m'}(x_{m'})^\alpha \cdot \widehat{v^*}^{00}_{m'}(\gamma_A)} + \frac{p(1-\mu) \cdot (x_m)^\alpha \cdot \widehat{v^*}^{01}_m(\gamma_A)}{\sum_{m'}(x_{m'})^\alpha \cdot \widehat{v^*}^{01}_{m'}(\gamma_A)} + \frac{(1-p)\mu \cdot (x_m)^\alpha \cdot \widehat{v^*}^{10}_m(\gamma_A)}{\sum_{m'}(x_{m'})^\alpha \cdot \widehat{v^*}^{10}_{m'}(\gamma_A)} + \frac{(1-p)(1-\mu) \cdot (x_m)^\alpha \cdot \widehat{v^*}^{11}_m(\gamma_A)}{\sum_{m'}(x_{m'})^\alpha \cdot \widehat{v^*}^{11}_{m'}(\gamma_A)}\right)\\
&+ \frac{1}{2-\lambda}\left(\frac{p\mu \cdot (y_m)^\alpha \cdot \widehat{v^*}^{00}_m(\gamma_B)}{\sum_{m'}(y_{m'})^\alpha \cdot \widehat{v^*}^{00}_{m'}(\gamma_B)} + \frac{p(1-\mu) \cdot (y_m)^\alpha \cdot \widehat{v^*}^{01}_m(\gamma_B)}{\sum_{m'}(y_{m'})^\alpha \cdot \widehat{v^*}^{01}_{m'}(\gamma_B)} + \frac{(1-p)\mu \cdot (y_m)^\alpha \cdot \widehat{v^*}^{10}_m(\gamma_B)}{\sum_{m'}(y_{m'})^\alpha \cdot \widehat{v^*}^{10}_{m'}(\gamma_B)} + \frac{(1-p)(1-\mu) \cdot (y_m)^\alpha \cdot \widehat{v^*}^{11}_m(\gamma_B)}{\sum_{m'}(y_{m'})^\alpha \cdot \widehat{v^*}^{11}_{m'}(\gamma_B)}\right).
\end{aligned}$$

Given that the function $g$ is smooth in $x, y$ on $\Delta_\epsilon(M)^2$, it can be shown that the expected limit of our stochastic process can be obtained by solving the system of ordinary differential equations $(\dot{x}, \dot{y}) = g(x, y)$ (or $(\dot{x}, \dot{y}) = (g^A(x, y), g^B(x, y))$) as done above in the case of $\lambda = 0$. We have that if $\lambda = 0$, then it is as if there were a single group with the same (common) ranking algorithm and where the desirability for reading confirmatory news is $\gamma = \frac{\gamma_A+\gamma_B}{2}$. As $\lambda$ increases, then the rankings of the two groups move apart and it is as if group $A$ had parameter $\frac{1}{2-\lambda}\gamma_A + \frac{1-\lambda}{2-\lambda}\gamma_B$ and group B had parameter $\frac{1-\lambda}{2-\lambda}\gamma_A + \frac{1}{2-\lambda}\gamma_B$ until, when $\lambda = 1$, it is as if there were two separate rankings of two groups with accuracy $\gamma_A$ and $\gamma_B$ respectively. It is then immediate to see that the difference in the levels of desirability for reading confirmatory news of the two groups ($\frac{\lambda}{2-\lambda} \cdot |\gamma_A - \gamma_B|$) is increasing in $\lambda$ when $\gamma_A \neq \gamma_B$, and is increasing in $|\gamma_A - \gamma_B|$ when $\lambda \neq 0$. Finally, since clicking and ranking probabilities coincide in the limit, this translates to increasingly different probabilities of clicking on any given majority website in the two groups and hence, given the definition of $\mathcal{BP}$, also to a measure $\mathcal{BP}(\mathcal{E}_\lambda)$ that is increasing in $\lambda$ when $\gamma_A \neq \gamma_B$, and is increasing in $|\gamma_A - \gamma_B|$ when $\lambda \neq 0$. □

**Proof of Proposition 5**. Let $\mathcal{P}^\lambda$ and $\mathcal{P}^\lambda_L$ denote respectively ex ante and interim efficiency as a function of the personalization parameter $\lambda$. Because the initial ranking is uniform we can look again at the solutions to the Equations (B.3) and (B.4) in the proof of Proposition 1, where we recall again that $x^{minority}_L = \widehat{\rho}_{\infty,L} = \mathcal{P}^\lambda_L$ when websites in $L$ have minority signal and $y^{majority}_L = \widehat{\rho}_{\infty,L} = \mathcal{P}^\lambda_L$ when they have majority signal. Hence to show that $\mathcal{P}^\lambda$ is weakly decreasing in $\lambda$ we can use these solutions to study the interim efficiency levels $\mathcal{P}^\lambda_L$ for $\lambda \in [0, 1]$ and $1 < L < M$. As in the proof of Proposition 3, we have that, for $\alpha = 1$ and $\mu = 1$, the solutions to Equations (B.3) and (B.4) take the form, respectively:

$$x^{minority}_L = \begin{cases} \frac{(1-\gamma)L-\gamma p(M-L)}{\gamma M-L} & \text{if } L < \frac{\gamma pM}{1-(1-\gamma)p} \\ 0 & \text{if } L \geq \frac{\gamma pM}{1-(1-\gamma)p} \end{cases}, \quad y^{majority}_L = \begin{cases} 1 & \text{if } L \leq \frac{(1-\gamma)M}{1-\gamma p} \\ \frac{\gamma pL}{L-(1-\gamma)M} & \text{if } L > \frac{(1-\gamma)M}{1-\gamma p} \end{cases}.$$

Fix a realization of $\mathcal{E}$, say, parametrized by $L$. From the proof of Proposition 4, we have that when $\lambda = 0$, it is as if there were a single group with a common ranking algorithm, with $\gamma = \frac{\gamma_A+\gamma_B}{2}$. As $\lambda$ increases, then the rankings of the two groups move apart and are as if group $A$ had desirability for reading confirmatory news $\frac{1}{2-\lambda}\gamma_A + \frac{1-\lambda}{2-\lambda}\gamma_B$ and group $B$ had $\frac{1-\lambda}{2-\lambda}\gamma_A + \frac{1}{2-\lambda}\gamma_B$, until, when $\lambda = 1$, it is as if there were two separate rankings of two groups with accuracy $\gamma_A$ and $\gamma_B$, respectively. As a result interim efficiency can be written as the average of the interim efficiency of two groups, one with $\gamma^\lambda_A = \frac{1}{2-\lambda}\gamma_A + \frac{1-\lambda}{2-\lambda}\gamma_B$ and another with $\gamma^\lambda_B = \frac{1-\lambda}{2-\lambda}\gamma_A + \frac{1}{2-\lambda}\gamma_B$. Therefore, in the case of intermediate

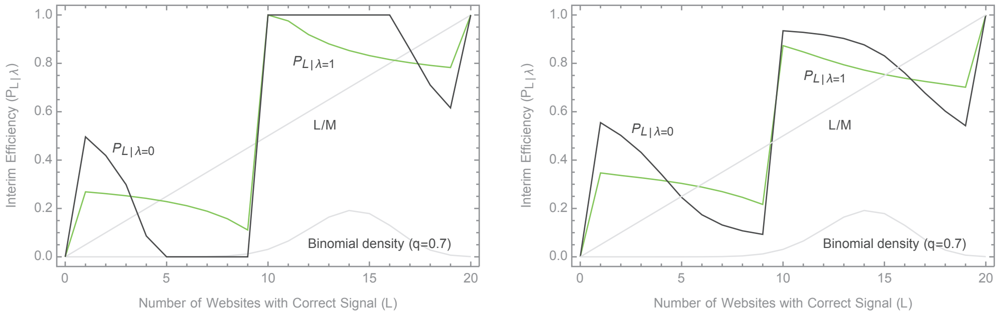


Figure B.3: Interim efficiency with no personalization $\mathcal{P}_L^{\lambda=0}$ (black) and with full personalization $\mathcal{P}_L^{\lambda=1}$ (green dashed) as a function of $L$ for $\mu = 1$ (left) and $\mu = 0.9$ (right). In both panels, $M = 20$, $\gamma_A = 0$, $\gamma_B = 0.66$, $p = 0.55$, and $r_1$ is uniform. Both panels also show (gray dashed) the function $\frac{L}{M}$ as comparison benchmark as well as the density function for the binomial distribution for $q = 0.7 < \phi(\gamma)$ at which PeR$> 0$.

values of $\lambda$ interim efficiency can be seen as the average of two levels of interim efficiency corresponding to two different signal accuracies that are increasingly apart as $\lambda$ increases (that is, go from both signals corresponding to $\frac{\gamma_A+\gamma_B}{2}$ when $\lambda = 0$ to being $\gamma_A$ and $\gamma_B$ respectively when $\lambda = 1$). Since we can always take $\gamma_A, \gamma_B$ to be, respectively, $\gamma_A^\lambda, \gamma_B^\lambda$, it suffices to consider directly the case of $\lambda = 1$. In this case, the interim efficiency levels can be computed from the solutions evaluated at the corresponding levels of $\gamma$:

$$\mathcal{P}_L^{\lambda=0} = \left( x_L^{minority}\left(\frac{\gamma_A+\gamma_B}{2}\right), y_L^{majority}\left(\frac{\gamma_A+\gamma_B}{2}\right)\right),$$

for the non-personalized case ($\lambda = 0$), and from:

$$\mathcal{P}_L^{\lambda=1} = \left( \frac{x_L^{minority}(\gamma_A) + x_L^{minority}(\gamma_B)}{2}, \frac{y_L^{majority}(\gamma_A) + y_L^{majority}(\gamma_B)}{2} \right),$$

for the fully personalized case ($\lambda = 1$). Since the reasoning is very similar to that of the proof of Proposition 3, we focus on the case where the websites with correct signal are a majority. For $\lambda = 0$ we have, as the level of interim efficiency, directly $y_L^{majority}\left(\frac{\gamma_A+\gamma_B}{2}\right)$ from above, while for $\lambda = 1$, recalling $0 \le \gamma_A < \gamma_B \le 1$, and since the cutoff for $L$ ($\frac{\gamma p M}{1-(1-\gamma)p}$) is decreasing in $\gamma$, we can write this as:

$$\frac{y_L^{majority}(\gamma_A) + y_L^{majority}(\gamma_B)}{2} = \begin{cases} 1 & \text{if } \frac{M+1}{2} \le L \le \frac{(1-\gamma_B)M}{1-\gamma_B p} \\ \frac{1}{2} + \frac{\gamma_B p L}{2(L-(1-\gamma_B)M)} & \text{if } \frac{(1-\gamma_B)M}{1-\gamma_B p} < L \le \frac{(1-\gamma_A)M}{1-\gamma_A p} \\ \frac{\gamma_A p L}{2(L-(1-\gamma_A)M)} + \frac{\gamma_B p L}{2(L-(1-\gamma_B)M} & \text{if } \frac{(1-\gamma_A)M}{1-\gamma_A p} < L \le M-1 \end{cases}.$$

Tedious calculations show that, when $y_L^{majority}\left(\frac{\gamma_A+\gamma_B}{2}\right) \ge \frac{L}{M}$, we also have:

$$y_L^{majority}\left(\frac{\gamma_A+\gamma_B}{2}\right) \ge \frac{y_L^{majority}(\gamma_A) + y_L^{majority}(\gamma_B)}{2},$$

and hence $\mathcal{P}_L^{\lambda=0} \ge \mathcal{P}_L^{\lambda=1}$.[42] Using the same reasoning as in the proof of Proposition 3, we can show

[42] To get more intuition for the proof, notice that $\mathcal{P}_L^{\lambda=0}$ for majority values of $L$ is a concave function of $\gamma$, when solutions satisfy $y \ge \frac{L}{M}$, and becomes convex when $y \le \frac{L}{M}$. Interim efficiency being concave implies ex ante efficiency is concave in $\gamma$ such that ex ante efficiency of an average $\gamma = \frac{\gamma_A+\gamma_B}{2}$ will be above the average ex ante efficiency of $\gamma_A$ and $\gamma_B$ (which in turn implies $PeR \le 0$, provided $q < \bar{q}$). On the other hand, the opposite holds ($PeR \ge 0$), when the function becomes

that $y$ (or also $x$) $\geq \frac{L}{M}$ occurs, when $\mu$ is sufficiently large and $\gamma$ sufficiently small, and when $q$ is below a given threshold $\overline{q}(\gamma)$. It can further be shown that the reverse inequality holds for the same values of $\mu$ and $\gamma$ if $q$ is above the threshold $\overline{q}(\gamma)$. Finally, a similar argument as in the proof of Proposition 3 allows to extend to the case where $\mu > \overline{\mu}$ for some $\overline{\mu} < 1$. Figure B.3 illustrates the cases $\mu = 1$ (left panel) and $\mu < 1$ (right panel). □

**Proof of Proposition 6 and Proposition 7.** We prove directly the case with $\gamma \geq 0$. The expected ranking probabilities are then given by:

$$\begin{aligned}
\widehat{r}_{n,m} &= \nu\widehat{r}_{n-1,m} + (1-\nu)\widehat{\rho}_{n-1,m} \\
&= \nu\widehat{r}_{n-1,m} + (1-\nu)\left(\frac{p\mu(\widehat{r}_{t-1,m})^{\alpha}\cdot\widehat{v^*}^{00}_{m}}{\sum_{m'}(\widehat{r}_{t-1,m'})^{\alpha}\cdot\widehat{v^*}^{00}_{m'}} + \frac{p(1-\mu)(\widehat{r}_{t-1,m})^{\alpha}\cdot\widehat{v^*}^{01}_{m}}{\sum_{m'}(\widehat{r}_{t-1,m'})^{\alpha}\cdot\widehat{v^*}^{01}_{m'}} + \right. \\
&\qquad \left. + \frac{(1-p)\mu(\widehat{r}_{t-1,m})^{\alpha}\cdot\widehat{v^*}^{10}_{m}}{\sum_{m'}(\widehat{r}_{t-1,m'})^{\alpha}\cdot\widehat{v^*}^{10}_{m'}} + \frac{(1-p)(1-\mu)(\widehat{r}_{t-1,m})^{\alpha}\cdot\widehat{v^*}^{11}_{m}}{\sum_{m'}(\widehat{r}_{t-1,m'})^{\alpha}\cdot\widehat{v^*}^{11}_{m'}}\right) \\
&= \nu\widehat{r}_{n-1,m} + (1-\nu)\left(\frac{p\mu\cdot\widehat{v^*}^{00}_{m}}{\sum_{m'}(\widehat{r}_{t-1,m'})^{\alpha}\cdot\widehat{v^*}^{00}_{m'}} + \ldots + \frac{(1-p)(1-\mu)\cdot\widehat{v^*}^{11}_{m}}{\sum_{m'}(\widehat{r}_{t-1,m'})^{\alpha}\cdot\widehat{v^*}^{11}_{m'}}\right)\cdot(\widehat{r}_{n-1,m})^{\alpha},
\end{aligned}$$

where $\widehat{v^*}^{00}_{m}$, $\widehat{v^*}^{01}_{m}$, $\widehat{v^*}^{10}_{m}$ and $\widehat{v^*}^{11}_{m}$ are defined in Appendix A. Let $m, m' \in K$ with $m \neq m'$ and $r_{1,m} > r_{1,m'} > 0$. Fix $n > 1$, we first show that the rich-get-richer dynamic applies to the ranking probabilities. To simplify notation, let $x \equiv \widehat{r}_{n-1,m}$ and $y \equiv \widehat{r}_{n-1,m'}$. Because $0 < \nu < 1$ we always have $x > y > 0$ for any $n > 2$, and because the two websites have the same signal they also have the equal coefficients on $(\widehat{r}_{n-1,m})^{\alpha}$ and $\widehat{r}_{n-1,m}$, say, $a$ and $b$ respectively, where $a, b > 0$. Hence we can write:

$$\widehat{r}_{n,m} = ax^{\alpha} + bx \qquad \text{and} \qquad \widehat{r}_{n,m'} = ay^{\alpha} + by.$$

But then it follows that:

$$\frac{\widehat{r}_{n,m}}{\widehat{r}_{n,m'}} > \frac{\widehat{r}_{n-1,m}}{\widehat{r}_{n-1,m'}} \iff \frac{ax^{\alpha}+bx}{ay^{\alpha}+by} > \frac{x}{y} \iff ax^{\alpha}y + bxy > axy^{\alpha} + bxy \iff \left(\frac{x}{y}\right)^{\alpha} > \frac{x}{y} \iff \alpha > 1$$

and similarly

$$\frac{\widehat{r}_{n,m}}{\widehat{r}_{n,m'}} \overset{(=)}{<} \frac{\widehat{r}_{n-1,m}}{\widehat{r}_{n-1,m'}} \iff \frac{ax^{\alpha}+bx}{ay^{\alpha}+by} \overset{(=)}{<} \frac{x}{y} \iff ax^{\alpha}y + bxy \overset{(=)}{<} axy^{\alpha} + bxy \iff \left(\frac{x}{y}\right)^{\alpha} \overset{(=)}{<} \frac{x}{y} \iff \alpha \overset{(=)}{<} 1$$

Finally, to see the claim, notice that $\widehat{\rho}_{n,m} = \frac{a}{1-\nu}(\widehat{r}_{n,m})^{\alpha}$ and $\widehat{\rho}_{n,m'} = \frac{a}{1-\nu}(\widehat{r}_{n,m'})^{\alpha}$, where again $\frac{a}{1-\nu} > 0$, so that:

$$\begin{aligned}
\frac{\widehat{\rho}_{n,m}}{\widehat{\rho}_{n,m'}} > \frac{\widehat{\rho}_{n-1,m}}{\widehat{\rho}_{n-1,m'}} \quad &\iff \quad \frac{\frac{a}{1-\nu}(\widehat{r}_{n,m})^{\alpha}}{\frac{a}{1-\nu}(\widehat{r}_{n,m'})^{\alpha}} > \frac{\frac{a}{1-\nu}(\widehat{r}_{n-1,m})^{\alpha}}{\frac{a}{1-\nu}(\widehat{r}_{n-1,m'})^{\alpha}} \\
&\iff \left(\frac{\widehat{r}_{n,m}}{\widehat{r}_{n,m'}}\right)^{\alpha} > \left(\frac{\widehat{r}_{n-1,m}}{\widehat{r}_{n-1,m'}}\right)^{\alpha} \iff \frac{\widehat{r}_{n,m}}{\widehat{r}_{n,m'}} > \frac{\widehat{r}_{n-1,m}}{\widehat{r}_{n-1,m'}} \iff \alpha > 1,
\end{aligned}$$

and correspondingly for $\alpha < 1$ and $\alpha = 1$. □

convex. This shows why larger levels of $q$ ($> \overline{q}$) can reverse the result.