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Abstract

I investigate the interaction between a country that imports a commodity whose production contributes to a stock pollution, such as electricity, from a country that produces that commodity. If the transboundary externality is priced improperly, the application of a feed-in tariff or border tax adjustment can provide an indirect policy instrument. But the imposition of such a tariff or tax creates an incentive for the producing country to deploy some sort of pollution controlling instrument. This, in turn, creates a strategic interaction between the two countries. Because the externality is inked to a stock pollutant, this strategic interaction will play out over time, which induces a dynamic game. In this modeling context, I describe the nature of the strategic interaction, and characterize the Markov-perfect equilibrium.

JEL-Codes: C730, Q540, Q580.

Keywords: transboundary pollution, differential game, tariff, tax.

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1. Introduction

International environmental problems featuring pollution spillovers such climate change pose a special challenge since they have special features that distinguish them from national environmental problems. Electricity production provides a natural example of this sort of interaction: imports of electricity into countries like England and Germany can come from Eastern Europe countries such as Poland; likewise, electricity produced in India can be exported to Bangladesh. In each of these examples, production of the exported commodity contributes to a flow of emissions, such as CO_2, that accumulate in a stock that generates damages; moreover, there are likely to be differences between the valuation of these damages by the exporting and importing countries. In this context, feed-in tariffs or border tax adjustments can serve as a "second-best" instrument to limit the transnational pollution, playing a role for cross-border externalities that is somewhat similar to the role played by a Pigouvian tax.[1]

A handful of authors have explored to the potential use of trade instruments to control environmental externalities (Baumol and Oates, 1988; Markusen, 1975a,b; Parry and Oates, 2000); as a general rule, this extant literature has employed a static framework. But by its very nature the climate change problem is dynamic: environmental damages depend primarily on an accumulated stock, and they play out over a long time frame. There is also an existing literature that investigates the dynamic strategic incentives in transboundary pollution problems by using dynamic games. In these games, the players, usually the governments of countries, care about the stock of pollution, *i.e.*, as pollution accumulates it affects the payoff to each country. Generally, the focus of these studies is a comparison between the cooperative scenario, which assumes a high degree of commitment to follow the agreed-upon pollution regulations, and non-cooperative scenario, in which each country's environmental policy is selected to promote its own interest, given the other country's emission standards (Bayramoglu, 2006; Dockner et al., 2000; Dockner and van Long, 1993; List and Mason, 2001; Maler and de Zeeuw, 1998; Mason, 1997). This literature typically neglects the potential for a trade relationship among the countries involved in the transboundary pollution stock control. Notable exceptions include Fernandez (2002), who explores empirically dynamic solutions to transboundary pollution through trade liberalization and environmental institutions for multilateral pollution control, Cabo et al. (2001), who analyzed strategies that lead to a self-enforcing agreement on transboundary pollution problem within a North-South framework, and Cabo et al. (2006), who study a model similar to ours, but with fixed output levels, Mason et al. (2015), who focus on the use of a feed-in tariff by the importing country, and Mason et al. (2017), who study the potential for the upstream country to lobby the downstream government to lower the import tariff. None of these papers explore the potential use of a climate-based strategy by the exporting country, such as a carbon tax, to blunt the effect of the importing country's strategy. My goal in this paper is to evaluate such a strategic interaction.

I develop a dynamic model to investigate this problem. There are two countries, one of whom – country 1 – produces a commodity that contributes to a transboundary externality; this commodity is consumed in both countries. Because these damages become important over time, the relative weighting of future effects between the two countries is a potentially important concern. This distinction manifests itself in terms of divergent evaluations of the future damages associated with the stock pollutant; I adopt the limiting view of this asymmetry wherein the importing country – country 2 – bears damages but the exporting country does not.[2] In this framework, country 2 applies a tariff (or a border tax adjustment) against imports from country 1 as an indirect measure to control the carbon externality. In response, country 1 applies a tax on consumption within its borders. The interaction thus described induces a differential game between the two countries. I assume the strategic choices are indexed by the pollution stock, *i.e.*, that the players use Markov strategies, and derive the Markov-perfect equilibrium to this differential game.

The rest of the paper is organized as follows. Section 2 develops the dynamic model. In section 3 I describe the solution to the cooperative problem. The non-cooperative game is evaluated in section 4. I illustrate the application of this model with a linear-quadratic example in section 5. Concluding remarks are offered in section 6.

2. Modeling preliminaries

There are two countries, 1 and 2. I will often refer to country 1 as the "upstream" country and country 2 as the "downstream" country. A single consumption good is produced only in 1 with a given fixed endowment of factors of production and a given technology. The associated aggregate cost is described by the increasing and convex function $C(Q)$, where Q is the amount produced in country 1. Some of this output is exported to country 2; I denote the exported amount by X. Consumers are homogeneous within each country, but may be heterogeneous across countries. At every instant, production in 1, $Q(t)$, results in a flow of emissions, $E(t)$. I assume these emissions are proportional to output, and without loss of generality set units so that they equal output. Emissions contribute to the stock of pollution, Z, which evolves according to the following equation of motion:[3]

$$\dot{Z} = Q - kZ, \tag{1}$$

where k represents the rate of pollution decay. The initial stock of the pollutant is Z_0.

Although pollution is generated by emissions in country 1, for expositional clarity I assume that the environmental damages from the stock of pollution are only suffered in country 2.[4] I also assume that there are no damages from the flow of emissions. Damages suffered in 2, $d(Z)$, are an increasing and convex function of the pollution stock.

When these pollution externalities are exported from one country to another, no authority has the ability to intervene and enforce cooperation. Thus countries will act only if their efforts ultimately serve their own interest (Sigman, 2002). While the downstream country can not address the externality directly by imposing a Pigouvian tax on upstream producers, it can indirectly tackle the externality by imposing a tariff, τ, on imports X from 1. The tariff lowers the upstream price, which induces firms to reduce their production – and with it the flow of emissions. Because this implies lower upstream net surplus, country 1 may be motivated to use some sort of pollution control instrument, so as to blunt the effect of the tariff. I assume they do so by taxing upstream consumption, σ.[5]

Following the standard assumption of the "second-best" trade and environment literature, I assume that country 2 is able to influence the terms of trade through its tariff, which implies it cares about tariff revenues. To avoid any complications associated with the impact price effects might have upon consumption in country 2, I assume that any tariff revenues collected by 2 are redistributed in a lump-sum fashion to downstream consumers. Likewise, any tax revenues collected by 1 are redistributed in a lump-sum fashion to upstream consumers. I note also that there is a tariff that maximizes country 2's static net surplus, which I denote by $\hat{\tau}$.

The equilibrium price received by sellers is determined by the combination of tariff and upstream tax, and so this price can be expressed as $p(\tau,\sigma)$. With a tariff in place, producers would only be willing to sell in both countries if the price in 2 equals $p(\tau,\sigma)-\tau$. Accordingly, the price received by producers in country 1 is $p(\tau,\sigma)-\tau$, while consumers pay $p(\tau,\sigma)-\tau+\sigma$. Production sets marginal cost equal to price:

$$C'(Q) = p(\tau,\sigma)-\tau, \tag{2}$$

which implies that production is a function of both the downstream tariff and the upstream tax. Further, production falls with either an increase in the tariff or the upstream tax – which then implies lower emissions.

For each country $i = 1,2$, I denote gross consumer benefits (*i.e.*, the area under the inverse demand curve) by U_i. Then net benefits for 1 are given by net surplus (the sum of consumer surplus and profit), which can be expressed as

$$W_1 \equiv U_1(Q-X) - C(Q) + (p-\tau)X. \tag{3}$$

Net benefits for 2 are given by the sum of its consumer surplus and the tariff revenue, less the damages from the pollution stock:

$$W_2 \equiv U_2(X) - (p-\tau)X - d(Z). \tag{4}$$

Given any value of the tariff and the standard in 1, the market-clearing conditions are eq. (2) and:

$$U_1'(Q - X) = p - \tau + \sigma, \tag{5}$$

$$U_2'(X) = p. \tag{6}$$

Because equilibrium exports and the upstream price are all function of the tariff and the standard, net payoffs can be regarded as functions of τ and σ.

3. The socially optimal solution

Before analyzing the equilibrium of the non-cooperative game between the two countries, I first briefly describe the cooperative solution. To that end, suppose there is a global social planner whose goal is to choose the time paths of output and exports so as to maximize the discounted flow of the two countries' combined payoffs. These combined payoffs equal the sum of the two countries' utilities, less combined production costs, less pollution damages.

It is clear that this solution requires equating marginal utilities in the two countries must be equated; this ensures that maximal gross benefits are obtained. One may therefore define the aggregate benefit function

$$U(Q) = U_1(Q - X) + U_2(X),$$

where X is chosen to set $U_1' = U_2'$. Accordingly, the cooperative problem can be re-cast as the choice of a time path of output that maximizes the discounted flow of aggregate benefits less production costs less pollution damages.

Following this reinterpretation, the current-value Hamiltonian for the cooperative problem can be written as

$$\mathcal{H}_c = U(Q) - C(Q) - d(Z) + m(Q - kZ),$$

where m is the (cooperative) shadow value of the pollution stock; as pollution is a bad, one presumes that m is negative. Let $\Omega(Q) = U(Q) - C(Q)$, combined utility net of production costs. The corresponding optimality rule for the cooperative solution is

$$\Omega'(Q) + m = 0. \tag{7}$$

Interpreting marginal utility as price, the optimality conditions boil down to setting rents (price less MC) equal to $-m$, the imputed marginal damage from a one-unit increase in emissions. The solution also requires the evolution of the shadow value satisfy

$$\dot{m} = (r + k)m + d'(Z), \tag{8}$$

where r is the (common) discount rate.[6]

The evolution of the socially optimal quantity may be derived by time-differentiating eq. (7), which yields

$$\begin{aligned} & \dot{Q}\Omega''(Q) + \dot{m} = 0 \\ \iff \quad & \dot{Q} = (r+k)Q\eta - \frac{d'(Z)}{\Omega''(Q)}, \end{aligned} \tag{9}$$

where $\eta = \frac{\Omega\,'(Q)}{Q\Omega''(Q)}$, the elasticity of net marginal flow payoffs with respect to the quantity produced. The steady-state associated with this problem entails the combination (Q^*, Z^*) that solve

$$\Omega'(Q^*) = \frac{d'(Z^*)}{r+k}; \tag{10}$$

$$Q^* = kZ^*. \tag{11}$$

In addition, I note that the long-run shadow value of the pollution stock for this problem is $m^* = -\frac{d'(Z^*)}{r+k}$. Interpreting $-m^*$ as the level of a pollution tax, the long run equilibrium condition for the socially optimal level of production requires setting marginal utility equal to the sum of marginal production cost and this tax.[7]

4. The non-cooperative equilibrium

I now turn to an analysis of the non-cooperative equilibrium of the strategic interaction between the two countries. Because each country's policy instrument is likely to depend on the level of the pollution stock, the natural focus is on Markov strategies. The solution concept I apply is Markov-perfect equilibrium, which requires each player's action to be optimal given the other player's strategy, for every stock level; as such, the strategy combination is subgame-perfect. I begin by describing the two countries' optimization problems, starting with country 2.

4.1. *Country 2's optimization problem*

The optimization problem for country 2 is to choose the time path of the tariff so as to maximize the discounted flow of its net benefit function over time, given the strategy that 1 employs. Letting $\sigma(Z)$ denote 1's Markov strategy, and r the discount rate, 2's optimization problem is:

$$\max_{\tau} \int_0^{\infty} W_2\Big(X(\tau,\sigma(Z)),Z\Big)e^{-rt}\mathrm{dt}$$

$$\text{subject to } \dot{Z}(t) = Q(\tau,\sigma(Z)) - kZ; Z(0) = Z_0.$$

The current-value Hamiltonian for this optimization problem is

$$\mathcal{H}_2 = W_2\Big(X(\tau,\sigma(Z)),Z\Big) + \theta\left[Q(\tau,\sigma(Z)) - kZ\right],$$

where θ is 2's shadow value of pollution and $\sigma(Z)$ is country 1's Markov strategy. The necessary conditions for the solution to this dynamic optimization problem are give by Pontryagin's maximum principle; in light of eqs. (4) and (6), these conditions can be written as:

$$X + \tau\frac{\partial X}{\partial \tau} + \theta\frac{\partial Q}{\partial \tau} = 0, \tag{12}$$

$$\dot{\theta} = (r+k)\theta + d'(Z) - \left(X + \tau\frac{\partial X}{\partial \sigma} + \theta\frac{\partial Q}{\partial \sigma}\right)\sigma'(Z), \tag{13}$$

as well as the transversality condition $\lim_{t\to\infty} \theta(t)Z(t)e^{-rt} = 0$.[8]

Eq. (12) describes country 2's best-reply to country 1's tax strategy, as an implicit funciton. Eq. (13) illustrates the rate of change in the shadow value, θ. Were the shadow value positive, the transversality condition would be violated;[9] it follows that the shadow value is negative, and that it tends to a long-run equilibrium level.

Upon totally differentiating eq. (12) with respect to time, the evolution of the optimal tariff can be characterized by

$$\left(2\frac{\partial X}{\partial \tau} + \tau\frac{\partial^2 X}{\partial \tau^2}\right)\dot{\tau} + \left(\frac{\partial X}{\partial \sigma} + \frac{\partial^2 X}{\partial \tau \sigma}\right)\sigma\,'(Z)\dot{Z} + \theta\left[\frac{\partial^2 Q}{\partial \tau^2}\dot{\tau} + \frac{\partial^2 Q}{\partial \tau \partial \sigma}\sigma'(Z)\dot{Z}\right] + \frac{\partial Q}{\partial \sigma}\dot{\theta} = 0. \qquad (14)$$

Next, eq. (13) can be used to replace $\dot{\theta}$ with an expression involving θ, Z, $\frac{\partial X}{\partial \sigma}$ and $\frac{\partial Q}{\partial \sigma}$ (the latter two of which are functions of τ and σ). Then, by applying eq. (12), θ can be replaced by an expression involving τ and σ. Finally, by employing eq. (1) one can replace $\dot{Z}$ with an expression involving Z and $Q(\tau,\sigma)$. The end result is a (non-linear) first-order differential equation for τ, in terms of Z and σ. As σ is described by country 1's Markov strategy, this end result can be boiled down to an ordinary differential equation describing τ in terms of Z.

4.2. *Country 1's optimization problem*

The optimization problem for country 1 is to choose the time path of the consumption tax so as to maximize the discounted flow of its net benefit function over time, given the strategy that 2 employs:

$$\max_{\sigma} \int_0^{\infty} W_1\Big(X(\tau(Z),\sigma), Q(\tau(Z),\sigma)\Big) e^{-rt} \mathrm{dt}$$

$$\text{subject to } \dot{Z}(t) = Q(\tau(Z),\sigma) - kZ; Z(0) = Z_0.$$

The current-value Hamiltonian for this optimization problem is

$$\mathcal{H}_1 = W_1\Big(X(\tau(Z),\sigma), Q(\tau(Z),\sigma)\Big) + \xi\,[Q(\tau(Z),\sigma) - kZ],$$

where ξ is 1's shadow value of pollution and $\tau(Z)$ is country 2's Markov strategy. The necessary conditions for the solution to this dynamic optimization problem are give by Pontryagin's maximum principle; in light of eqs. (2), (3), and (6), these conditions can be written as:

$$\sigma \frac{\partial(Q-X)}{\partial\sigma} + X\frac{\partial(p-\tau)}{\partial\sigma} + \xi\frac{\partial Q}{\partial\sigma} = 0, \tag{15}$$

$$\dot{\xi} = (r+k)\xi - \left[\sigma\frac{\partial(Q-X)}{\partial\tau} + X\frac{\partial(p-\tau)}{\partial\tau} + \xi\frac{\partial Q}{\partial\tau}\right]\tau'(Z), \tag{16}$$

as well as the transversality condition $\lim_{t\to\infty} \xi(t)Z(t)e^{-rt} = 0$.

As with country 2's problem, one can proceed by totally differentiating the optimization condition governing 1's optimal policy choice, which here is eq. (15), with respect to time. Then, using eq. (1) to eliminate $\dot{Z}$, eq. (16) to eliminate $\dot{\xi}$, and eq. (15) to eliminate ξ, one arrives at a (non-linear) first-order differential equation for σ in terms of Z and τ. Since τ is described by a Markov strategy, this result can be boiled down to an ordinary differential equation describing σ in terms of Z.

The Markov-perfect equilibrium is then determined by the solution to a pair of differential equations, one for τ and one for σ. This pair of equations defines a system of two (non-linear) first order differential equations in Z; existence of a solution over some compact set of stocks follows from standard theorems (see, *e.g.*, Boyce and DiPrima (2005, pp.68-70)). In general, it is not possible to determine the resultant strategies in closed form without imposing considerable structure on the problem. A particular structure that allows one can to say more is a linear-quadratic framework, which I investigate in the next section.

5. Linear-Quadratic Example

To further characterize the time paths of the border tax and carbon stock, I next present a simplified variant of the model, in which demand and marginal cost are linear functions, and damages are a quadratic function. With these assumptions, the two countries' payoff functions are quadratic functions. Linear-quadratic models are considered to be a good approximation for more general games and are characterized by equations of motion being linear in state and control variables and objective functionals being quadratic in state and control variables.

To minimize notational clutter, I assume demand in the two countries differs only via the intercept, *i.e.*, the slope of inverse demands in the two countries are equal. Thus, quantity demanded in country $k = 1,2$ is then $Q_k = (a_k - p_k)/b$, where p_k is the price paid by consumers in country k. Utility in country k, the area under inverse demand, is the quadratic in Q_k:

$$U_k = a_k Q_k - \frac{b}{2} Q_k^2, \quad k = 1, 2. \tag{17}$$

I also assume that supply is linear. Letting p the price received by sellers (all of whom are located in country 1), quantity supplied is $Q = p/c$; this corresponds to assuming that marginal costs are linear: $C'(Q) = cQ$. Finally, I assume the damage function is $d(Z) = \frac{\delta}{2} Z^2$, which implies linear marginal damage $d'(Z) = \delta Z$.

With a consumption tax in country 1 and a tariff in country 2, firms receive the price $p - \tau$, where p is the price paid by consumers in 2. Accordingly, consumer in country 1 pay $p - \tau + \sigma$. The supply relation is

$$Q = \frac{p - \tau}{c}, \tag{18}$$

while the demand relations are

$$Q_1 = \frac{a_1 - (p - \tau + \sigma)}{b};$$
$$Q_2 = \frac{a_2 - p}{b}.$$

At each point in time, market clearing requires that combined quantity demanded equals combined quantity supplied: $Q_1 + Q_2 = Q$. It is straightforward to derive the equilibrium price paid by consumers in country 2, which is a linear function of the consumption tax in 1 and tariff in 2:

$$p = \hat{p} + \alpha\tau - (1 - \alpha)\sigma, \tag{19}$$

where $\alpha = \frac{b+c}{b+2c}$ and $\hat{p} = (1-\alpha)(a_1 + a_2)$ is the equilibrium price in the absence of any taxes or tariffs. Based on the equilibrium price paid in country 2, the quantity produced in country 1 and the volume exported to country 2 are easily derived as

$$Q = \hat{Q} - \frac{(1-\alpha)(\tau+\sigma)}{c}; \tag{20}$$

$$X = \hat{X} - \frac{\alpha\tau - (1-\alpha)\sigma}{b}, \tag{21}$$

where $\hat{Q} = \hat{p}/c$ is the amount produced in country 1 and $\hat{X} = (a_2 - \hat{p})/b$ is the amount exported to country 2 in the absence of any taxes or tariffs.[10]

5.1. *Socially optimal solution*

In the globally optimal solution, quantities consumed in the two countries' outputs are linked by the equi-marginal principle – namely, marginal utilities are equal. (Equivalently, consumers pay the same price irrespective of which country they consume in). Using eq. (17), this observation implies

$$X = \frac{a_2 - a_1}{2b} + \frac{Q}{2}.$$

The global flow of utility is $U_1(Q-X) + U_2(X)$. Using eq. (17) then leads to $U'(Q) = \frac{a_1+a_2-bQ}{2}$, so that the difference between marginal utility and marginal production cost is $\Omega'(Q) = \frac{a_1+a_2}{2} - \frac{b+2c}{2}Q$. Then using eq. (7), the socially optimal production rate is

$$Q = \frac{a_1 + a_2}{b+2c} + \frac{2}{b+2c}m, \tag{22}$$

where m is the global shadow value of the pollution stock.

It is easy to see that this problem is saddle-point stable, and so there is a unique asymptotically stable steady state. The steady state shadow value is $m^* = -\frac{\delta Z^*}{r+k} = -\frac{\delta Q^*}{k(r+k)}$; inserting this relation into eq. (22) yields the long-run equilibrium output:

$$Q^* = \frac{k(r+k)(a_1+a_2)}{k(r+k)(b+2c)+2\delta}. \tag{23}$$

This value may then be used to determine the long-run stock as

$$Z^* = \frac{k(r+k)(a_1+a_2)}{\delta k(r+k)(b+2c)+2\delta^2}; \tag{24}$$

As I noted above, the global planner can obtain this outcome by charging all firms a pollution tax equal to the capitalized value of long-run imputed pollution stock marginal damages in this linear-quadratic framework: $\tau^* = \frac{\delta\phi Z^*}{r+k}$. Then using eq. (24), we obtain

$$\tau^* = \frac{k(a_1+a_2)}{k(r+k)(b+2c)+2\delta}. \tag{25}$$

5.2. *Non-cooperative equilibrium*

Now suppose that the two countries make their policy choices non-cooperatively, with each player adopting a Markov strategy. A key result here is that there exists a Markov-perfect equilibrium in linear strategies. I proceed by incorporating the information from eqs. (19)–(21) into the material developed in section 4.[11]

If there is a Markov-perfect equilibrium in linear strategies, the two countries' policies can be described by

$$\tau(Z) = \tau_0 + \tau_1 Z; \tag{26}$$

$$\sigma(Z) = \sigma_0 + \sigma_1 Z. \tag{27}$$

With linear marginal costs, and taking note of eq. (21), the equation characterizing country 2's optimal tariff (eq. (12)) can be re-written as

$$\tau = \frac{\hat{X}}{b\alpha} + \left(\frac{1-\alpha}{\alpha}\right)\sigma(Z) - \frac{\theta}{bc\alpha}. \tag{28}$$

Similarly, because $U_1'' = U_2'' = b$, the equation characterizing country 1's optimal tax (eq. (15)) can be re-written as

$$\sigma = \frac{\hat{p}}{1-\alpha} + \frac{c-b}{1-\alpha} - \tau(Z) + \frac{1}{1-\alpha}\xi. \tag{29}$$

It is straightforward to show that the determinant of the Jacobian matrix of the system eqs. (52)-(53) is negative and thus the steady state is a saddle point. Given the configuration of the $\dot{\tau} = 0$ and $\dot{Z} = 0$ isoclines, the steady state is unique. Figure 1 presents a phase diagram for our analysis.

A comparison of eqs. (10) and (52) is instructive. In the socially optimal solution, the difference between the marginal impact of a change in Q upon U and marginal cost equals the capitalized value of marginal damages from the socially optimal steady state pollution stock; because marginal utilities are equated across countries, as are marginal costs, the difference between marginal utility and marginal cost for D equals the capitalized value of marginal damages. In D's con-cooperative solution, the ratio $\frac{W_d'(Q_d(\tau^e))}{Q_d'(\tau^e)}$ equals the capitalized value of marginal damages from D's privately optimal steady state pollution stock. As the global planner's flow payoffs correspond to the sum to the two countries' payoffs, the left-hand side of eq. (52) can be written as

$$\begin{aligned}
&\frac{W_d'(\tau^e)}{Q'(\tau^e)} = U'(Q(\tau^e)) - C'(Q(\tau^e)) - \frac{W_u'(\tau^e)}{Q\,'(\tau^e)} \Longleftrightarrow \\
&\frac{W_d'(\tau^e)}{Q'(\tau^e)} > U'(Q(\tau^e)) - C'(Q(\tau^e)),
\end{aligned} \tag{30}$$

where the inequality follows from the observation that both W_u and Q are decreasing in τ. Suppose, for the sake of argument, that $Z^e = Z^*$, *i.e.*, that the steady state pollution stocks under the global optimal plan and D's privately optimal plan are equal. In that case, the equilibrium market level of output in steady state is identical under the two optimization plans: $Q(\tau^e) = Q^*$. But the relation (30) implies $d'(Z^e) > (r-k)[U'(Q(\tau^e)) - C'(Q(\tau^e))]$, which under the maintained hypothesis must equal $d'(Z^*)$; accordingly, $Z^e > Z^*$, a contradiction. Thus, the two plans deliver different long-run outcomes, *i.e.*, D's privately optimal plan is second-best; indeed, the long-run pollution stock is larger under D's privately optimal scheme.

The observation that D may be strictly better off in the non-cooperative regime is reminiscent of a central theme in (List and Mason, 2001), who show that the equilibrium payoffs under an over-arching regulatory authority may not dominate those obtained via unilateral (non-cooperative) actions of a single policymaker (here, D). This arises in List and Mason's paper whenever there are sharp asymmetries between the two countries. In our setting, the incentives facing D and U are similarly asymmetric, inasmuch as only D suffers damages from the pollution stock. As a result, it is unclear that the two countries would be able to successfully negotiate a mutually beneficial treaty.

6. Concluding Remarks

Concerns about limiting carbon emissions have recently led some OECD countries to invoke feed-in tariffs and border tax adjustments. These instruments serve two purposes: they increase the cost of associated products in the importing country, reducing the level of consumption; they also increase the cost of doing business in the importing country, inducing the exporting country to adjust its behavior. Under an optimistic view of these incentives, the exporting country will limit emissions, and perhaps invoke some sort of climate policy; the potential for such adaptation is most intriguing for transitioning countries such as those found in Eastern Europe.

In this paper I showed existence of Markov-Perfect equilibrium in a game between two countries, a country (which I call "country 1") whose production generates a flow pollutant, such as carbon dioxide, and a country (which I call "country 2") that suffers harm from the stock of that pollutant. Country 1?s production trades internationally, and so country 2 has an indirect method to influence country 1?s emissions: by imposing a tariff, country 2 effectively taxes the source of pollution. In light of this structure, the country 1 is motivated to impose a tax its product – even if country 1 suffers little or no harm – as this will induce country 2 to lower its tariff. In this setting, the combination of importing country tariff and producing country tax serves to reduce the flow of emissions below the level that would otherwise be observed. Even so, the resultant flow of emissions exceeds the socially optimal level.

The essential feature of this set of results is that the importing country is able to induce lower emissions through two channels: there is a direct effect, as the imposition of a tariff is akin to an emissions tax. But there is also an indirect effect: by creating a financial environment wherein the country 1 has an incentive to curtail production (so as to lower emissions), country 2 motivates country 2 to tax its own product. This effect arises because the two instruments are strategic substitutes. Accordingly, a policy environment that allows the country suffering harm from the stock pollutant to tax the associated (imported) production can be doubly beneficial.

Appendix A: Details of Linear-quadratic Model

With linear marginal costs, and taking note of eq. (21), the equation characterizing country 2's optimal tariff (eq. (12)) can be re-written as

$$\tau = \frac{X^0}{b\alpha} + \frac{1-\alpha}{\alpha}\sigma(Z) - \frac{1}{bc\alpha}\theta. \tag{31}$$

Similarly, because $U_1'' = U_2'' = b$, the equation characterizing country 1's optimal tax (eq. (15)) can be re-written as

$$\sigma = \frac{p^0}{1-\alpha} + \frac{c-b}{1-\alpha} - \tau(Z) + \frac{1}{1-\alpha}\xi. \tag{32}$$

To work out the instantaneous equilibrium in this setting, we note that $Q_d = a_d - bp$ and $Q = c(p-\tau) \leftrightarrow p = Q/c + \tau$. Hence $Q_u + a_d - bp = c(p-\tau) \leftrightarrow p = \frac{a_d + Q_u + c\tau}{b+c}$. Substituting into the expressions for Q_d and Q, we obtain

$$Q_d = \omega a_d - (1-\omega)Q_u - \omega b\tau, \tag{33}$$

$$Q = \omega[a_d + Q_u - b\tau], \tag{34}$$

where $\omega = \frac{c}{b+c}$. Note that $\partial Q/\partial Q_u = \omega$ and $\partial Q_d/\partial Q_u = \omega - 1$. It follows that

$$\partial W_u/\partial Q_u = \frac{a_u - Q_u}{b} - \left(\frac{Q}{c}\right)\frac{\partial Q}{\partial Q_u} + (p-\tau)\frac{\partial Q_d}{\partial Q_u} + Q_d\frac{\partial(p-\tau)}{\partial Q_u} \tag{35}$$

$$= a_u - \left(\frac{b+c}{bc}\right)Q_u. \tag{36}$$

I now turn to the derivation of the Markov Perfect equilibrium in the non-cooperative differential game. The current-value Hamiltonian for U's problem is $\mathcal{H}_u = W_u + \theta(Q - kZ)$, so that the first-order condition is $\partial W_u/\partial Q_u + \theta \partial Q/\partial Q_u = 0$; this implies

$$Q_u = \omega a_u + b\theta\omega^2. \tag{37}$$

The maximum principle also gives:

$$\dot{\theta} = r\theta - \frac{\partial \mathcal{H}_u}{\partial Z} = \left(r + k - \frac{\partial Q}{\partial \tau}\tau'(Z)\right)\theta - \frac{\partial W_u}{\partial \tau}\tau'(Z). \tag{38}$$

With linear Markov strategies, $\tau(Z) = \tau_0 + \tau_1 Z$, so that $\tau'(Z) = \tau_1$. Moreover, $\partial Q/\partial \tau = \partial Q_d/\partial \tau = -b\omega$. It follows that $\partial W_u/\partial \tau = (\omega - 1)Q_d$, and thus

$$\dot{\theta} = (r + k + \omega b \tau_1)\theta + (1-\omega)\tau_1 Q_d. \tag{39}$$

Time-differentiating eq. (37) yields:

$$\begin{aligned} \dot{Q}_u &= b\omega^2\dot{\theta} \\ &= b\omega^2 \left(r + k + b\omega\tau_1\right)\theta + b\omega^2(1-\omega)\tau_1 Q_d. \end{aligned} \tag{40}$$

Upon substituting in for Q_d, one has

$$\dot{Q}_u = (r+k)[Q_u - \omega a_u] + \omega b \tau_1 \Big(Q_u - \omega a_u + (1-\omega)\omega\left[\omega a_d - (1-\omega)Q_u - \omega b\tau\right]\Big). \tag{41}$$

Now, $\dot{Q}_u = Q_u'(Z)\dot{Z}$, where $Q_u(Z)$ is the linear Markov strategy $\rho_0 + \rho_1 Z$. Thus,

$$\dot{Q}_u = \rho_1\omega(a_d + Q_u - b\tau) - k\rho_1 Z.$$

Combined with eq. (41), this yields a linear relation that must hold for all values of Z; in turn, this imposes constraints on the intercept and slope:

$$\rho_0 = \frac{\omega\left(b\omega\tau_1\left[\omega(1-\omega)(a_d-b\tau_0)-a_u\right]-\rho_1(a_d-b\tau_0)-(r+k)a_u\right)}{\omega\rho_1-(r+k)-\omega b\tau_1+\omega^2(1-\omega)^2 b\tau_1}; \tag{42}$$

$$\omega\rho_1^2-\left(2\omega b\tau_1+(r+2k)-\omega^2(1-\omega)^2\tau_1)\right)\phi_b+(1-\omega)(c\tau_1)^2=0. \tag{43}$$

Payoffs to D in this context are consumer surplus plus tariff revenues, less damages:

$$W_d = \frac{Q_d^2}{2b}+\tau Q_d-\frac{sZ^2}{2}.$$

Using the expression for Q_d above, we have

$$\frac{\partial W_d}{\partial \tau} = (1-\omega)\left[Q_d - c\tau\right].$$

The current-value Hamiltonian for D's problem is $\mathcal{H}_u = W_d + \eta(Q-kZ)$, so that the first-order condition implies

$$\tau = Q_d/c-\eta. \tag{44}$$

Then using the expression for Q_d above, we obtain:

$$\tau = \frac{a_d-(b+c)\eta}{c+2b}-\frac{b}{c+2b}Q_u. \tag{45}$$

The maximum principle also gives:

$$\dot{\eta} = r\eta-\frac{\partial \mathcal{H}_d}{\partial Z} = \left(r+k-\frac{\partial Q}{\partial Q_u}Q_u'(Z)\right)\theta-\frac{\partial W_u}{\partial Q_u}Q_u'(Z). \tag{46}$$

With U playing the linear Markov strategy $Q_u(Z) = \rho_0 + \rho_1 Z$ we have $Q_u'(Z) = \rho_1$, so that

$$\dot{\eta} = (r + k - \omega\rho_1)\eta + \left(\frac{1-\omega}{b+c}\right)\left[\frac{ca_d}{b} - Q_u + b\tau\right]\rho_1 + sZ. \tag{47}$$

As with the characterization of U's strategy, I proceed by time-differentiating the first order condition (here, eq. (44)); this yields:

$$\dot{\tau} = \frac{-(b+c)}{c+2b}\dot{\eta} - \frac{-b}{c(c+2b)}Q_u'\dot{Z}.$$

Then using eq. (47) to substitute for $\dot{\eta}$, and recalling that $Q_u'(Z) = \rho_1$, one has

$$\dot{\tau} = -\left(\frac{b+c}{c+2b}\right)\left[r + k - \frac{c\rho_1}{b+c}\right]\eta - \left(\frac{b+c}{c+2b}\right)\left(\frac{b}{(b+c)^2}\left[\frac{ca_d}{b} - Q_u + b\tau\right]\rho_1 + sZ\right) - \frac{b\rho_1}{c(c+2b)}\dot{Z}. \tag{48}$$

Because D is playing the linear Markov strategy $\tau(Z) = \tau_0 + \tau_1 Z$ and U is playing the linear Markov strategy $Q_u(Z) = \rho_0 + \rho_1 Z$, one can write

$$\dot{Z} = \omega(a_d - b\tau_0 + \rho_0) + [\omega(\rho_1 - b\tau_1) - k]Z. \tag{49}$$

Then combining eqs. (48) and (49), and using eq. (44) to substitute for $\frac{-(b+c)\eta}{c+2b}$, we obtain a linear relation that must hold for all values of Z. This relation imposes constraints on the intercept and slope:

$$(r + k + \omega(b\tau_1 - \rho_1))\tau_0 = \omega(a_d + \rho_0)\tau_1 - (r + k - \omega\rho_1)\left(\frac{b\rho_o - ca_d}{c(c+2b)}\right) + \frac{a_d\rho_1}{(c+2b)(b+c)^2}; \tag{50}$$

$$b\omega\tau_1^2 - (2\omega\rho_1 - (r+2k))\tau_1 - \frac{b}{c(c+2b)}\left(\omega\rho_1^2 - (r+2k)\rho_1 + \frac{cs}{1-\omega}\right) = 0. \tag{51}$$

The Markov Perfect equilibrium is given by the four parameters $\tau_0, \tau_1, \rho_0, \rho_1$ that solve the system of four equations (42), (43), (50) and (51).

Using the time paths of pollution and tariff, the steady state can be found as the solution of the system of two equations, $\dot{\tau} = 0$ and $\dot{Z} = 0$:

$$\frac{W_d'(\tau^e)}{Q'(\tau^e)} = \frac{d'(Z^e)}{r+k}, \tag{52}$$

$$Z^e = Q(\tau^e)/k. \tag{53}$$

The long-run equilibrium shadow price of carbon is negative and equal to

$$\theta^e = -\frac{d'(Z^e)}{r+k}. \tag{54}$$

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Notes

[1] See Copeland (1994, 1996); Snape (1992). It is not clear whether border tax adjustments in energy markets are compatible with World Trade Organization laws governing international trade; feed-in tariffs might be justified by the World Trade Organization regulations (Article XX which allows for exceptions to general GATT principles) since production of the traded good is a direct cause of environmental damages occurring in the importing country. Biermann and Brohm (2005) provides a detailed discussion of the potential legal ramifications. For a model of dynamic interactions among countries that value environmental damages identically, see Yanase (2010).

[2] Alternatively, one could envision a situation where the exporting country places little weight on future effects, while the importing country cares very much about the future. Adopting a similar simplification, List and Mason (2001) investigate the potential for differing national policies to produce preferable outcomes to a common pollution control measure.

[3] From now on, unless otherwise stated, we will suppress the time argument t.

[4] List and Mason (2001) take a similar approach. As they note, one can think of this scenario as characterizing a situation where one country bears the brunt of the damages; allowing for damages in both countries greatly complicates the analysis without changing the qualitative results. An alternative approach would be to assume the government in country 1 does not care about the damages borne by its citizenry.

[5] One might think it would be more natural for 1 to impose a tax on production. But as noted by Mason et al. (2015), such a policy can be preempted by 2, in the sense that the tariff that is optimal for country 2 will generally drive the upstream tax to zero. In light of this result, country 1 needs to find an alternative instrument; since taxing local consumption can influence something that country 2 can not impact, using such a policy can generate net gains for country 1.

[6] With a convex damage function, eqs. (7)-(8) are also sufficient.

[7] The globally optimal path is comparable to those found in earlier papers on transboundary pollution problems (*e.g.*, Dockner and van Long (1993)).

[8] The optimal tariff is also subject to the constraint that $\tau \geq \hat{\tau}$; in practice, this constraint never binds.

[9] If θ were positive, it would grow at least as fast as $e^{(r+kt)}$. As a result, $\theta(t)Z(t)e^{-rt}$ would grow at least as fast as $Z(t)e^{kt}$, which grows without bound as t goes to ∞.

[10] Based on these values, the quantity consumer in country 1 is easily seem to equal $Q_1 = \hat{Q} - \hat{X} - (\alpha\sigma - (1-\alpha)\tau)/b$.

[11] For expositional clarity, details of the derivations are relegated to the Appendix.