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Salomon Faure, Hans Gersbach

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Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
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Money Creation and Destruction

Abstract

We study money creation and destruction in today's monetary architecture within a general equilibrium setting. Two types of money are created and destructed: bank deposits, when banks grant loans to firms or to other banks, and central bank money, when the central bank grants loans to private banks. We show that symmetric equilibria yield the first-best allocation when prices are exible, regardless of the monetary policy or capital regulation. When prices are rigid, we identify the circumstances in which money creation is excessive or breaks down and how an adequate combination of monetary policy and capital regulation may restore efficiency.

JEL-Codes: D500, E400, E500, G210.

Keywords: money creation, bank deposits, capital regulation, zero lower bound, monetary policy, price rigidities.

Salomon Faure
CER-ETH – Center of Economic Research at ETH Zurich
Zürichbergstrasse 18
Switzerland – 8092 Zurich
sfaure@ethz.ch

Hans Gersbach
CER-ETH – Center of Economic Research at ETH Zurich
Zürichbergstrasse 18
Switzerland – 8092 Zurich
hgersbach@ethz.ch

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1 Introduction

Motivation and approach

Money is predominantly held by the public in the form of bank deposit contracts.[1] These deposits—which are claims on banknotes—are typically created by the banks' lending decisions. How is such inside money creation controlled, and how can it be steered towards socially desirable levels? These long-standing questions are the focus of this paper.[2]

For several reasons the constraints on asset and inside money creation—thereafter simply called "money creation"—in the commercial banking system in today's architectures have received renewed attention recently (see McLeay et al. (2014)). First, the price of reserves, i.e. the short-term interest rate, has widely replaced traditional quantity instruments in the form of reserve requirements, which do not restrict lending directly.[3] Moreover, at exceptional times some central banks purchase securities or lend to banks at low and even negative interest rates. Whether such policies trigger corresponding money creation and foster economic activities is unclear.

In our paper we develop a sequential general equilibrium model to study these issues. In particular, we build the simplest general equilibrium model for which the feature that competitive commercial banks create money by granting loans is crucial. In this setting, we investigate the functioning of money creation in various circumstances and we examine which combinations of central bank policy rates and capital requirements lead to a socially efficient money creation and intermediation of households' endowments to the production sectors.

In our model, bank deposits are essential to buy physical goods, and these deposits

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[1]The use of banknotes and coins in daily transactions today is low. For instance Bennett et al. (2014) estimate the share of the volume of payments made in cash in the US at 14%.

[2]Gurley and Shaw (1960) and Tobin (1963) are well-known contributions. Tobin (1963), for instance, established the so-called "new view" by stressing that there are natural economic limits to the amount of assets and liabilities the commercial banking industry can create.

[3]Based on a 2010 IMF survey of 121 central banks, Gray (2011) describes the main purposes of reserve requirements and points out that nine countries do not have any reserve requirements, including the United Kingdom, Australia, Mexico, and Canada. Similarly, Carpenter and Demiralp (2012) show that the standard money multiplier model cannot explain the relationship between reserves and money. For instance, they point out that reserve balances held at the Fed increased dramatically—by a factor of at least 50—from July 2007 to December 2008 and that no similar increase in any measure of money could be observed during this time frame.

are created in the lending process by banks for firms that can only obtain funds through monitored lending. The central bank sets an interest rate (or policy rate) at which banks are able to refinance themselves and which they can earn by holding reserves at the central bank, and regulatory authorities impose bank capital requirements. Households sell their endowment of investment goods to firms and choose a portfolio of bank equity, bank deposits, and bonds. Consumption goods are produced by firms and sold to and consumed by households. With the proceeds, banks and firms pay dividends and reimburse bonds and loans. Money in the form of bank deposits is destroyed, when firms repay their loans, and money in the form of central bank reserves is destroyed, when banks repay their central bank liabilities.[4]

Relation to the literature

Our paper is inspired by the long-standing issue of the limits on money creation by commercial banks in a world where money is fiat. Independently of this paper, Jakab and Kumhof (2015) construct a DSGE model in which a bank can create money. They show quantitatively that shocks have larger effects on bank lending and on the real economy than in the corresponding loanable funds model in which banks are constrained by resources provided by depositors. We focus on the welfare properties of general equilibrium models when private banks compete with regard to money creation—both in the absence and presence of price rigidities.

Conceptually, our research is connected to four further strands of the literature.

First, one important line of reasoning and the corresponding models show that fiat money can have positive value in a finite-horizon model when, first, there are sufficiently large penalties when debts to governments—such as tax liabilities—are not paid and, second, there are sufficiently large gains from using and trading money.[5] To this literature we add the two-tier structure with privately and publicly

[4]We note that banks are able to repay their central bank liabilities, because we assume that the central bank deposit rate is equal to the central bank loan rate. A difference between these rates would result either in a net liability or a net asset against the central bank.

[5]See for example Shubik and Wilson (1977), Dubey and Geanakoplos (1992), Dubey and Geanakoplos (2003a,b), Shapley and Shubik (1977), and Kiyotaki and Moore (2003). There are various important approaches to constructing general equilibrium models with money to which we cannot do justice in this paper. We refer to Huber et al. (2014) for a summary of the reasons why the value of fiat money can be positive in finite and infinite horizon models. Shubik and Tsomocos (1992) extend this type of models by introducing a mutual bank with fractional

created monies. Commercial banks create bank deposits (privately created money) when they grant loans to firms enabling them to buy investment goods. Bank deposits will be used later by households to buy consumption goods.[6] The central bank creates reserves (publicly created money) when it grants loans to commercial banks enabling them to settle claims on privately created money among banks. The publicly created money is often called "central bank money".

Second, beside their role in money creation, the existence of banks in our model is justified by their role as delegated monitors.[7] In this respect, our paper builds on the seminal work by Diamond (1984), whose rationale for the existence of financial intermediaries relies on economies of scale in monitoring borrowers under moral hazard. Furthermore, Boot and Thakor (1997) provide a rationale explaining why financial markets and banks can coexist. They show that high-quality firms can borrow directly from the financial markets and that the moral hazard problem can be alleviated by banks' monitoring activities. Similarly, Bolton and Freixas (2000) develop a model based on asymmetric information with equity and bond issues as well as bank loans. They show that safe firms borrow from the bond market, whereas riskier firms are financed by banks. Based on these insights we construct our model on the assumption that there are two different types of firms. The first type encompasses small and opaque firms, which are risky and need to be monitored by banks to get financing. The second type assembles large firms, which are safe and can obtain financing directly from households through bond issues.

Third, a large body of literature on banks in partial or general equilibrium has provided important insights on how appropriate capital regulation may reduce excessive risk-taking, stabilize credit cycles, and undermine liquidity provision.[8]

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reserves.

[6] For simplicity, we will neglect payments via banknotes and thus all consumption goods will be bought via bank deposits. Therefore, this setting is equivalent to a model with a deposit-in-advance constraint. Such constraints—usually in the form of cash-in-advance constraints—have been introduced by Clower (1967) and Lucas (1982). For a discussion of their foundations, see Shi (2002).

[7] For a complete account of the role of banks as delegated monitors, see Freixas and Rochet (2008).

[8] Diamond and Rajan (2000) show that the optimal bank capital ratio balances its negative effects on liquidity creation with its positive effects on the costs of bank distress. Recent general equilibrium models are developed by Gersbach and Rochet (2017) to provide a foundation for counter-cyclical capital regulation and Gersbach et al. (2015b) on the role of capital regulation as an equilibrium selection device. Cao and Illing (2015) model banks' incentives to overinvest

We examine the role of capital regulation with regard to money creation.

Fourth, our modeling of heterogeneous banks and of an interbank market relates to the approach of Tsomocos (2003) and Goodhart et al. (2006), who develop a tractable general equilibrium model to study financial fragility and derive conclusions regarding monetary, regulatory, and fiscal policies. While in their model, banks lend to firms the money they have first borrowed from the central bank, we develop a general equilibrium model in which banks create inside money by granting loans to firms before any borrowing from the central bank. Banks then have to borrow from the central bank or from the interbank market to finance any outflow of deposits that is greater than the inflow.

Main insights

The analysis of our model produces three main insights. First, with perfectly flexible prices, i.e. prices adjusting perfectly to macroeconomic conditions, equilibria with money creation are associated with the first-best allocation, regardless of the central bank's monetary policy. If prices are rigid, there exist central bank policies for which money creation collapses or explodes. In the only equilibrium possible, in these cases, there is no financial intermediation, and an inefficient allocation occurs. Appropriate central bank policy can restore socially efficient money creation and lending. Second, with price rigidities and the zero lower bound, there may not exist a feasible central bank monetary policy inducing socially efficient money creation and lending. Capital regulation in the form of a minimum equity ratio and monetary policy can jointly limit money creation and under normal economic conditions restore the existence of equilibria with socially efficient money creation and lending. Third, when prices are rigid, the central bank's choice of zero interest rates[9] and appropriate capital regulation can only avoid a slump in money creation and lending if economic conditions are sufficiently favorable.[10] The working of the economy is illustrated in a simple example in Appendix H.

We also investigate how these insights translate (*i*) in the presence of financial frictions at the bankers' level, (*ii*) when bonds are denominated in nominal terms,

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in illiquid assets and provide a rationale for ex ante liquidity coverage requirements.

[9]Since the central bank chooses its interest rate before the shock is realized, such monetary policy commitment can be called Forward Guidance.

[10]Formally, this means that there is a positive probability that the real interest rate is above zero.

(*iii*) when there are more than two states of the world, (*iv*) when we also consider asymmetric equilibria with banks, (*v*) when there are real costs for monitoring activities, (*vi*) when the lending rates or the real deposit rates cannot be written contingently on the state of the economy, and (*vii*) when a reserve requirement and a haircut rule for borrowing against the central bank are imposed by government authorities. While our results continue to hold for extensions (*ii*), (*iii*), (*v*), and (*vi*), we obtain three further main insights: First, in the presence of financial frictions, we are able to show that there are equilibria with banks only when capital regulation is adequately combined with monetary policy. Second, we demonstrate that there are inefficient asymmetric equilibria with banks when prices are flexible and that capital requirements that are sufficiently high eliminate these inefficient equilibria with banks, so that only efficient equilibria with banks remain. Finally, we prove that the impact of a reserve requirement coupled with a haircut rule on money creation is identical to the one of a minimum equity ratio requirement.

One important remark is in order. The features of our model entail results of the knife-edge type. For instance, money creation is either at optimal level, or explodes, or collapses to zero. This has the advantage of illustrating in the simplest and most transparent way both the forces at work and appropriate monetary policy and capital regulation. Moreover, it should motivate to construct smoother versions of the model.[11]

Structure of the paper

The set-up of the model is outlined in Section 2. Section 3 derives the resulting equilibria and their welfare properties. Section 4 analyzes the role of capital regulation when prices are perfectly rigid and the central bank policy rate is constrained by the zero lower bound. Section 5 presents extensions and generalizations of the model, and Section 6 concludes. The Appendices A to I contain detailed analyses of the stages, proofs, an example, as well as a description of the notations.

[11] Smoother versions might involve, for instance, risk-averse households, transaction costs, and costs of monitoring and deposit creation, in which cases money creation may react more smoothly to interest rate changes, for example.

2 Model

2.1 Overview

We consider a two-period general equilibrium model with two production sectors and one investment good. In Period $t = 0$, investment takes place in both sectors. In one sector, firms can obtain direct financing from the bond market and thus from households. In the second sector, firms can only be financed by bank loans. At the beginning of Period $t = 1$, the production technologies transform the investment good into a *consumption good*. The gross rates of return are impacted by a macroeconomic shock. At the end of Period $t = 1$, households consume the consumption good.

Banks grant loans to firms in one sector, thereby creating money in the form of deposits, which serve as a means of payment and as a store of value. Whether or not such deposits will have value has to be determined in equilibrium. Households, who are initially endowed with the investment good, sell some amount of it to the latter firms in exchange for deposits enabling households to invest in bank equity and bank deposits. Households then directly provide the remaining amount of the investment good to the firms in the other sector in exchange for bonds promising the delivery of some amount of consumption good after production in the next period.

The payment processes are supported by a central bank that sets the policy rate and bank capital requirements are imposed by regulatory authorities. Banks facing an outflow of deposits to other banks that is higher than the inflow—and hence net debt against other banks—can refinance themselves at the policy rate. These banks can fulfill the claims of other banks by paying with central bank money. Banks that have net claims against other banks will thus receive reserves at the central bank and interest payments according to the policy rate.

Figure 1 summarizes the agents' interactions during Period $t = 0$.

At the beginning of Period $t = 1$, a macroeconomic shock occurs and affects the output from production. The firms' production technologies transform the amount of investment good acquired in the previous period into some amount of consumption good. The firms directly financed by bonds repay them by delivering the amount of consumption good due, and the other firms sell the amount of con-

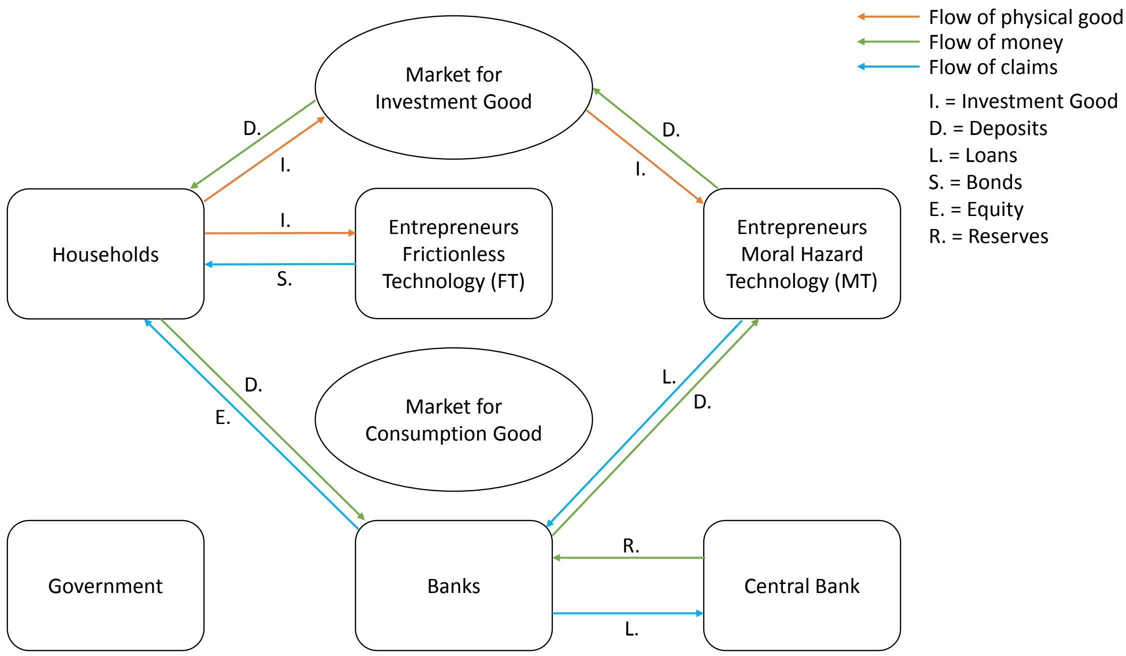

Figure 1: Flows and interactions between agents during Period $t = 0$.

sumption good produced to households in exchange for deposits. These firms use the deposits to repay bank loans. When borrowers pay back loans, the deposits originally created during Period $t = 0$ are destroyed. Since repayments by borrowers depend on the macroeconomic shock, the banks' balance sheets are risky. As banks' shareholders are protected by limited liability, some banks may default on depositors. The households' deposits are fully insured by government authorities. To guarantee the value of deposits, the government resorts to lump-sum taxation if some banks default. The dividends of non-defaulting banks are paid to households in the form of deposits. At the end of Period $t = 1$, households consume the consumption good.

Figure 2 summarizes the agents' interactions during Period $t = 1$.

We focus on a complete market setting in the sense that all contracts can be conditioned on macroeconomic events.[12] All nominal contracts are denominated in terms of a currency unit. To differentiate nominal from real variables—investment or consumption goods—we express the latter in bold characters. Furthermore, to

[12]The market setting is incomplete in two other respects. Payments must be made with bank deposits, and households cannot invest directly in all the firms. Firms in one sector rely on financial intermediation.

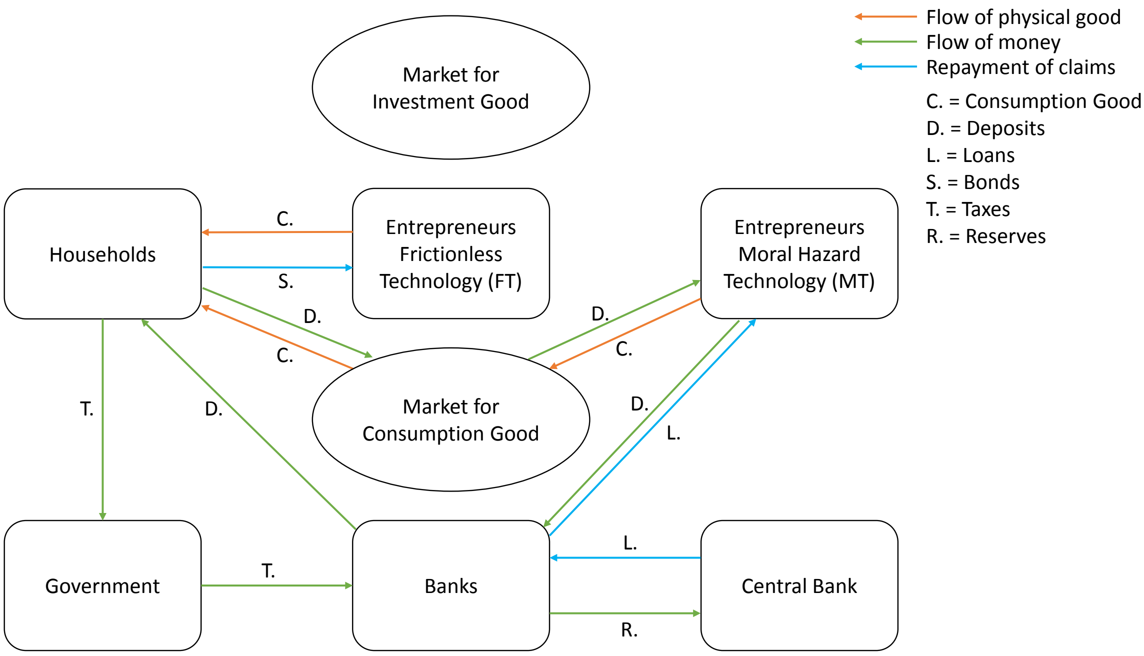

Figure 2: Flows and interactions between agents during Period $t = 1$.

distinguish individual quantities from aggregate quantities, the former are denoted by small letters, the latter by capitals.

The economic activities of the four types of agent—entrepreneurs, bankers, households, and the government—are described in Subsections 2.2 and 2.4.2. Subsection 2.3 describes the macroeconomic shock. The institutional set-up is given in Subsection 2.4. The sequence of decisions by the agents and the markets across the two periods ($t = 0, 1$), including all payment processes, are detailed in Subsection 2.5. Subsection 2.6 defines the notion of equilibrium.

2.2 Agents

In Subsection 2.2 we describe the agents in the economy in more detail.

2.2.1 Entrepreneurs

Two different technologies are employed by firms to transform the investment good into a consumption good. These firms are run by entrepreneurs, who only play a passive role and simply maximize the value of shareholders.

There is a moral hazard technology called hereafter Sector MT or simply MT. Entrepreneurs running the firms employing this technology are subject to moral hazard and need to be monitored.[13] We use $\mathbf{K_M} \in [\mathbf{0}, \mathbf{W}]$ to denote the aggregate amount of investment good invested in MT, where $\mathbf{W} > 0$ denotes the total amount of the investment good in the economy. An investment of $\mathbf{K_M}$ produces $\mathbf{K_M R_M}$ units of the consumption good, where $\mathbf{R_M} > 0$ denotes the real gross rate of return.[14]

There is a frictionless technology referred to hereafter as Sector FT or simply FT. Entrepreneurs running the firms employing this technology are not subject to any moral hazard problem.[15] We use $\mathbf{K_F} \in [\mathbf{0}, \mathbf{W}]$ to denote the aggregate amount of investment good invested in FT and $\mathbf{f}(\mathbf{K_F})$ to denote the amount of consumption good produced by FT. We assume $\mathbf{f}' > 0$ and $\mathbf{f}'' < 0$ as well as the following conditions:

Assumption 1

$$\mathbf{f}'(\mathbf{W}) < \mathbf{R_M} < \mathbf{f}'(\mathbf{0}).$$

In words, the above assumption ensures that the expected total production can never be maximized by allocating the entire amount of the investment good to one sector of production.

Firms in MT and FT are owned by households, and as long as they are positive, the resulting profits from both technologies, denoted by Π_M and Π_F, are paid to owners as dividends. The shareholders' values are given by $\max(\Pi_M, 0)$ and $\max(\Pi_F, 0)$, respectively.

[13] Typically, Sector MT comprises small or opaque firms that cannot obtain direct financing.

[14] We define a real gross rate of return—also called hereafter real gross rate or simply gross rate—as being the amount of the consumption good produced by investing one unit of the investment good. Similarly, we define a nominal gross rate of return—also called hereafter nominal gross rate or simply gross rate—as being the amount of money which has to be repaid to the creditor by the debtor per unit of nominal investment.

[15] Typically, these entrepreneurs run well-established firms that do not need to be monitored for repayment after having borrowed money.

2.2.2 Bankers

There is a continuum of banks labeled $b \in [0, 1]$ and operated by shareholders' value-maximizing bankers. At the very beginning, banks are only labels or indices and offer equity contracts. We assume that each bank receives the same amount of equity financing, denoted by e_B.[16] The aggregate amount is denoted by E_B. As the measure of banks is 1, the aggregate amount is numerically identical to the individual amount e_B. For the time being, we will concentrate on constellations with $E_B > 0$ and thus on circumstances in which banks are founded[17] and can engage in money creation and lending activities.[18] For simplicity, we assume that banks can perfectly alleviate the moral hazard problem when investing in MT by monitoring borrowers and enforcing contractual obligations. Moreover, we assume that monitoring costs are zero. Banks provide (nominal) loans to firms in Sector MT at a nominal lending gross rate R_L. The individual and aggregate amounts of loans are denoted by l_M^b and L_M, respectively. We can express the ratio of individual lending by Bank b to average lending by banks as $\alpha_M^b := \frac{l_M^b}{L_M}$.[19]

By granting loans to firms in MT, Bank b simultaneously creates deposits $d_M^b = l_M^b$.[20] We use $D_M = L_M$ to denote aggregate private deposits. d_M^b (or α_M^b) is the distribution of MT firms' deposits across banks. In the course of economic activities, these deposits will be transferred to households that will keep them to buy some amount of the consumption good. We assume that households keep deposits evenly distributed across all banks at all times. For example, they never transfer money from their account at one bank to another bank. Bank owners are protected by limited liability, and as long as they are positive, the resulting profits of Bank b, denoted by Π_B^b, are paid as dividends to owners. The bank shareholders' value and the nominal gross rate of return on equity are given by $\max(\Pi_B^b, 0)$ and $\frac{\max(\Pi_B^b, 0)}{E_B}$, respectively.

[16]As households are indifferent regarding their equity investment across banks in equilibrium, we can directly assume that they allocate their equity investment symmetrically across banks.

[17]Typically, banks need to have some minimal equity to obtain a banking license.

[18]The case $E_B = 0$ will be discussed in Subsection 2.5.2.

[19]As the continuum of banks is of a measure equal to one, the aggregate lending L_M can also be interpreted as the average lending per bank and α_M^b as the ratio of individual lending to average lending.

[20]There are three reasons in practice why banks do not issue equity when granting loans. The value of equity is much more volatile than the value of deposits. Moreover, the division of shares is not as fine as with deposits. Finally, only deposits are supported by central banks in the payment process.

2.2.3 Households

There is a continuum of identical and risk-neutral[21] households represented by $[0, 1]$. They are the only consuming agents in the economy. We can focus on a representative household initially endowed with $\mathbf{W}$ units of the investment good and ownership of all firms in the economy. It sells a part of its endowment of the investment good to firms in MT against bank deposits. Then it chooses a portfolio of bank equity and bank deposits and lends the remaining endowment of the investment good directly to firms in FT against bonds.[22] The dividends from firm ownership and bank equity investment as well as the repayments from bonds and bank deposits are used to buy the consumption good. The details of this process are set out in Subsection 2.5.

2.3 Macroeconomic Shock

A macroeconomic shock $s = l, h$ occurs at the beginning of Period $t = 1$ after the investment good has been allocated to the two technologies during Period $t = 0$. It affects the real gross rate of return from production in Sector MT.[23] Specifically, an investment of $\mathbf{K_M}$ in MT produces $\mathbf{K_M R_M^h}$ and $\mathbf{K_M R_M^l}$ with probability σ in the good state and $1 - \sigma$ in the bad state of the world, respectively ($0 < \sigma < 1$), where $\mathbf{R_M^s}$ is the real gross rate of return in State s ($s = l, h$). We assume that $0 < \mathbf{R_M^l} < \mathbf{R_M^h}$.

Banks monitor entrepreneurs running firms in MT and plagued by moral hazard (see Subsection 2.2.1) and offer state-contingent loans with nominal lending gross rates $(R_L^s)_{s=l,h}$. The lending interest rates are given by $(R_L^s - 1)_{s=l,h}$.

In general, we assume that in all contracts during Period $t = 0$ all nominal gross rates to be repaid during Period $t = 1$ can be written contingently on the outcome of the macroeconomic shock. This reflects our assumption of complete markets. As the output in FT is not stochastic, the real gross rate of return on bonds $\mathbf{R_F}$

[21]Household risk aversion would require more elaborate portfolio decisions. This is left to future research.

[22]Alternatively, we could assume that firms in FT are only financed by equity. Since households are the only agents financing firms in FT and financing is frictionless, they are indifferent between different capital structures, and this would not affect our results.

[23]Letting the macroeconomic shock impact Sector FT would not change our results qualitatively but would complicate the analysis.

is risk-free.

We will use interchangeably the notations $\mathbb{E}[X]$ and $\overline{X}$ to denote the expected value of some real or nominal variable X. Finally, taking into account the occurrence of a macroeconomic shock, we restate Assumption 1 as follows:

$$\mathbf{f}'(\mathbf{W}) < \overline{\mathbf{R}}_{\mathbf{M}} < \mathbf{f}'(\mathbf{0}).$$

2.4 Institutional Set-up

We purposely impose favorable conditions on the working of the monetary architecture and the public authorities involved.

2.4.1 Monies and Interbank Market

Two types of money (privately created and publicly created monies) and three forms of money creation are representative of the modern money architecture.[24] A first type of money is privately created by commercial banks through loans to firms, held at banks in the form of deposits by households or firms and destroyed when households buy bank equity and when firms repay loans. This type of money can also be privately created by commercial banks when they grant loans to other banks. It is held at the former banks by the latter in the form of deposits. We call the first type of money “private deposits”. A second type of money is publicly created by the central bank—called hereafter CB—via loans to banks. It is held at the central bank in the form of deposits by banks. We call this second type of money “CB deposits”.

The essential rules linking publicly created and privately created monies are illustrated as follows. When households use private deposits to make payments, these deposits typically move from one bank (account of buyer, say b_j) to another bank (account of seller, say b_i). To settle the transfer of private deposits, Bank b_j becomes liable to b_i. These banks now have two options. Either b_j obtains a loan from Bank b_i, or it refinances itself at the CB and transfers the central bank money received, CB deposits, to Bank b_i. The institutional rule is that one unit of

[24] We abstract from banknotes and coins, as they are not used in our economy and all payments are done by transferring deposits.

central bank money settles one unit of liabilities of privately created money, and both types of money have the same unit. This fixes the "exchange rate" between central bank money and privately created money at 1.[25] Finally, we assume that there are no transaction costs for paying with private or CB deposits.

The prices of the investment and the consumption goods in units of both privately created and publicly created monies are denoted by p_I and $(p_C^s)_{s=l,h}$, respectively.

We integrate an interbank market. In this market banks can lend to and borrow from each other at the same nominal gross rate. This lending / borrowing gross rate and the amount of lending can be conditioned on the macroeconomic shock. Banks cannot discriminate between deposits owned by households and deposits owned by other banks. As a consequence, the gross rate at which banks can lend to, and borrow from, each other is equal to the households' deposit gross rate, which we denote by $(R_D^s)_{s=l,h}$. The interbank market works as follows: At any time, banks can repay their CB liabilities by using their deposits at other banks, repay their interbank liabilities by using CB deposits, and require their debtor banks to repay their interbank liabilities with CB deposits.[26] Accordingly, as long as banks can refinance themselves at the central bank, interbank borrowing is not associated with default risk. Moreover, we assume that no bank participating in the interbank market makes any loss by doing so. Finally, the following tie-breaking rule simplifies the analysis: If banks are indifferent between lending to other banks and depositing money at the central bank, they will choose the latter.

2.4.2 Role of Public Authorities

Two public authorities—a central bank and a government—ensure the functioning of the monetary architecture. These authorities fulfill three roles.

First, banks can obtain loans from the central bank and can thus acquire CB deposits at the same policy gross rates $(R_{CB}^s)_{s=l,h}$ at any stage of economic activities, where $(R_{CB}^s - 1)_{s=l,h}$ are the central bank interest rates. This assumption implies that banks do not have to worry about the exact flow of funds at any particular stage. Only their net position at the final stage matters.[27] Banks can also borrow

[25] In principle, this exchange rate could be fixed at any other level.

[26] The mechanisms by which banks become liable to other banks or hold assets against them are explained in detail in Appendix D.

[27] Note that this assumption also rules out the possibility of bank runs.

from, or deposit at, the central bank contingently on the state of the world s.

Second, the government impose heavy penalties on those bankers whose bank defaults on obligations to any public authority.[28] As a consequence, no bank will default on its liabilities against the central bank in any state of the economy. Moreover, we assume that the central bank ensures the repayment of interbank loans by taking them on its balance sheet if the counterparty bank were to default. By this assumption, heavy penalties on bankers whose bank defaults against the central bank translate directly into heavy penalties on bankers whose bank defaults on other banks. A bank, however, may default on households' deposits.

In such cases, the government has a third role. It makes deposits safe by levying lump-sum taxes on households to bail out banks that default on households' deposits. In practice, making deposits safe is a necessary condition for their use as money and it protects unsophisticated households. Thus, we integrate implicit insurance of bank deposits into our framework. Later we will introduce a third public authority, i.e. bank regulators, and bank regulation in the form of a capital requirement.

We explore equilibrium outcomes for different policies—the central bank policy gross rate and the capital requirement—and for each combination of these outcomes we determine the associated level of welfare expressed in terms of household consumption. We assume that the central bank and the bank regulators aim at maximizing the welfare of households.

2.5 Timeline of Events

An overview of the timeline of events is given in Figure 3.

We next describe the timeline of events in detail. For this purpose we divide each period into several stages.

2.5.1 Period t = 0

It is convenient to describe the sequence of economic activities via the balance sheets of households and banks. The economy starts with the following balance

[28] As banks are able to borrow from the CB at any time, it is sufficient to assume that heavy penalties are imposed on those bankers whose bank defaults on obligations to the CB.

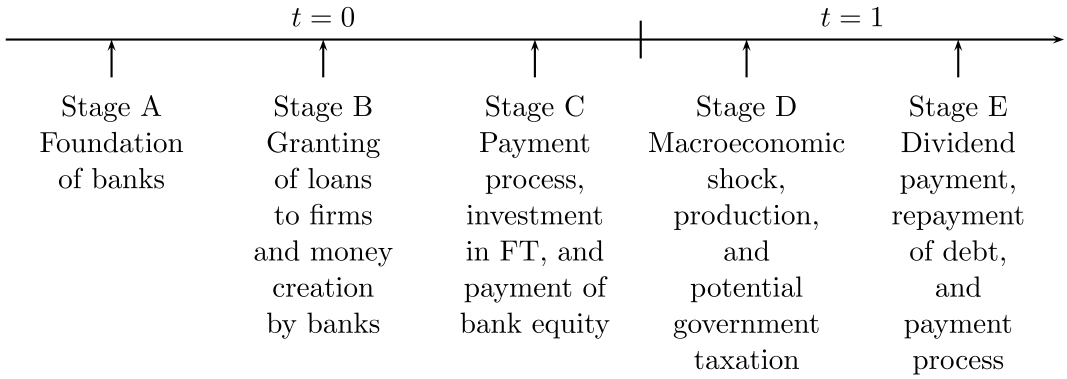

Figure 3: Timeline of events.

sheets:

Households		Bank b	
W	E_H	0	0

Table 1: Balance sheets at the beginning of Period $t = 0$.

E_H denotes the households' equity, which represents the ownership of the investment good and both production technologies at the beginning of Period $t = 0$.[29]

Stage A: Foundation of Banks. Either banks are not founded because no household invests in bank equity and the only possible allocation is given in Subsection 2.5.2, or households found banks by pledging to convert a predefined share $\varphi \in (0, 1]$ of their initial deposits D_M into an amount $E_B = \varphi D_M$ of bank equity before production in Stage C. When banks are founded, the gross rate of return on equity is equal to shareholders' value per unit of equity, and it is denoted by $R_E^{b,s} = \frac{\max(\Pi_B^{b,s},0)}{e_B}$. In the remainder of Subsection 2.5, we focus on the case where banks are founded (unless specified otherwise).

Stage B: Granting of Loans by Banks. Bank b grants loans $l_M^b = \alpha_M^b L_M$ to firms in MT at the contingent nominal lending gross rates $(R_L^s)_s$, which simultane-

[29]Note that households also own firms in Sectors MT and FT and may receive dividends from firms' profits.

ously creates d_M^b private deposits at Bank b and aggregate private deposits D_M.[30] The resulting balance sheets are given in Table 2.

Households		Bank b	
$\mathbf{W}$	E_H	l_M^b	d_M^b

Table 2: Balance sheets at the end of Stage B.

Stage C: Payment Process, Investment in FT, and Payment of Bank Equity. Households sell an amount of the investment good to firms in MT. Then they invest in FT by buying S_F bonds denominated in real terms at the real gross rate of return $\mathbf{R_F}$, meaning that such a bond costs one unit of investment good and promises the delivery of $\mathbf{R_F}$ units of the consumption good once production has occurred.[31] Finally, at the end of Period $t = 0$, households pay for the equity E_B pledged in Stage A with deposits, which reduces the amount of deposits in the economy. The resulting amount of deposits is denoted by d_H for an individual bank and $D_H = L_M - E_B$ for the aggregate banking system. At the end of Stage C and depending on their lending decisions, some banks labeled b_i have claims $d_{CB}^{b_i}$, and the other banks have liabilities $l_{CB}^{b_j}$ against the central bank. These processes are detailed in Appendix A. The balance sheets are displayed in Table 3.[32]

A summary of the agents' interactions during Period $t = 0$ is given in Figure 1, page 7.

2.5.2 Period t = 1

In Period $t = 1$ we distinguish between two cases, when either no bank is founded by households, or banks are founded by households. The latter case can again be

[30]These deposits will be used in Stage C to buy some amount of investment good. We do not consider constellations, for which an infinite amount of loans and money are created, which would only be compatible with a price of the investment good equal to zero, as such constellations cannot represent equilibria with banks.

[31]In practice, such bonds are called "inflation-indexed bonds". Using bonds denominated in nominal terms does not change the results qualitatively but significantly complicates the analysis, as one has to verify that firms do not default. Details are given in Subsection 5.2.

[32]The banks creating more money than the average automatically force the other banks to hold claims against the creators of high levels of money. We call such an externality "a money creation externality".

Households		Bank b_i		Bank b_j	
S_F		$d_{CB}^{b_i}$			$l_{CB}^{b_j}$
D_H	E_H	$l_M^{b_i}$	d_H	$l_M^{b_j}$	d_H
E_B			e_B		e_B

Table 3: Balance sheets at the end of Stage C.

divided into two subcases: Either no bank defaults, or some banks default.

Case I: No Bank Is Founded. When no bank is founded, we have $E_B = 0$. This could constitute an equilibrium, as no household can found a bank individually. We call this an *equilibrium without banks.* In such circumstances, no money creation takes place, the central bank is inactive, no investment in MT is possible, and the investment good is allocated entirely to Sector FT, which leads to the following allocation:[33]

$$\mathbf{K}^*_{\mathbf{M}} = \mathbf{0},$$
$$\mathbf{K}^*_{\mathbf{F}} = \mathbf{W},$$

where * denotes equilibrium variables. This is an inefficient allocation, as households are risk-neutral and Assumption 1 stipulates that $\mathbf{f}'(\mathbf{W}) < \overline{\mathbf{R}}_{\mathbf{M}}$.[34]

Case II: Banks are Founded. When banks are founded, they grant loans to firms in MT, and we can considerably simplify the description of Period $t = 1$ by making the observation given by Lemma 1:[35]

[33]Note that no bank deposits are needed to buy the output from Sector FT, as bonds are in real terms and are repaid in terms of the output.

[34]Remember that we use the notation $\overline{X}$ to denote the expected value of some real or nominal variable X.

[35]This observation enables us to rule out considerations in which firms in MT would make positive profits or go bankrupt.

Lemma 1

An equilibrium with banks and hence with positive lending to Sector MT requires

$$\mathbf{R_M^s} p_C^s = R_L^s p_I$$

and implies $\Pi_M^s = 0$ *for* $s = l, h$.

Lemma 1 is a direct consequence of the MT technology. If for some state s $\mathbf{R_M^s} p_C^s > R_L^s p_I$, firms in MT would demand an infinite amount of loan, as their shareholders' value per loan unit would be positive in one state, be at least zero in the other state,[36] and scale with the level of borrowing. If $\mathbf{R_M^s} p_C^s < R_L^s p_I$ for both states of the world, firms would forgo borrowing from banks.[37]

Subcase II.a: No Bank Defaults. Suppose next that no bank defaults. Then the following stages occur:

Stage D: Production. The macroeconomic state s is realized. Firms produce and repayments contingent on s fall due. Using bank balance sheets in Table 3 as well as the expression of the net position of Bank b against the CB given by Equation (12) in Appendix A, we derive the expression of Bank b's profits as follows:[38]

$$\begin{aligned}
\Pi_B^{b,s} &= (1-\alpha_M^b)L_M R_{CB}^s + \alpha_M^b L_M R_L^s - d_H R_D^s \\
&= (1-\alpha_M^b)L_M R_{CB}^s + \alpha_M^b L_M R_L^s - (L_M - E_B) R_D^s \\
&= \alpha_M^b L_M (R_L^s - R_{CB}^s) + L_M (R_{CB}^s - R_D^s) + E_B R_D^s. \qquad (1)
\end{aligned}$$

[36]Since entrepreneurs running firms in Sector MT do not have any wealth, they have zero profit if they cannot repay and thus default against banks.

[37]Other arguments could be used to derive the zero profit condition in Sector MT. As banks monitor entrepreneurs running firms in Sector MT, they can offer them state-contingent repayment gross rates of return, and are thus able to extract the entrepreneurs' entire surplus.

[38]Note that profits are non-negative here, as we have assumed that banks do not default. In the case of default by Bank b, $\Pi_B^{b,s}$ will be negative, but shareholders' value will be equal to zero, and bank shareholders will not be affected by the magnitude of $\Pi_B^{b,s}$, as they are protected by limited liability.

Profits from firms in the real sector are given by

$$\Pi^s_M = \mathbf{K_M}(\mathbf{R^s_M} p^s_C - R^s_L p_I),$$
$$\Pi^s_F = (\mathbf{f}(\mathbf{K_F}) - \mathbf{K_F}\mathbf{R_F})p^s_C.$$

The balance sheets are given in Table 4, where R^s_H denotes the resulting nominal gross rate of return on household ownership of the investment good and of both production technologies.

Households	
$S_F\mathbf{R_F}$	$E_H R^s_H$
$D_H R^s_D$	
$E_B R^s_E$	
Π^s_F	

Bank b_i	
$d^{b_i}_{CB} R^s_{CB}$	
$l^{b_i}_M R^s_L$	$d_H R^s_D$
	$e_B R^{b_i,s}_E$

Bank b_j	
	$l^{b_j}_{CB} R^s_{CB}$
$l^{b_j}_M R^s_L$	$d_H R^s_D$
	$e_B R^{b_j,s}_E$

Table 4: Balance sheets at the end of Stage D if no bank defaults.

Stage E: Dividend Payment, Repayment of Debt, and Payment Process. Households obtain dividends from their equity investment[39] and buy the amount of consumption good produced. All debts are paid back. These processes are detailed in Appendix B. The resulting balance sheets are given in Table 5.

Households	
$\mathbf{K_M}\mathbf{R^s_M}$	$E_H R^s_H$
$\mathbf{f}(\mathbf{K_F})$	

Bank b	
0	0

Table 5: Balance sheets at the end of Stage E if no bank defaults.

Subcase II.b: Some Banks Default. Finally, we consider the scenario where some banks default. In this case, Stages D and E have to be modified as follows:

[39]Banks pay dividends to households in anticipation of the repayment of loans by firms in Sector MT.

Stage D: Production and Government Taxation. The macroeconomic state s is realized. Firms produce, and repayments fall due. Two cases can occur. First, if $-d_H R_D^s \leq \Pi_B^{b,s} < 0$, Bank b defaults on households but not on the central bank. Second, if $\Pi_B^{b,s} > 0$, Bank b does not default. We note that the case $\Pi_B^{b,s} < -d_H R_D^s < 0$ cannot occur, as banks would default on households and the central bank. Due to the heavy penalties incurred for default against governmental authorities banks will avoid the latter case under all circumstances.

Consider now a non-defaulting bank b. If $R_{CB}^s > R_L^s$ for some state s, there then exists an upper bound on α_M^b given by

$$\alpha_M^b \leq \alpha_{DH}^s := \frac{R_{CB}^s - (1-\varphi)R_D^s}{R_{CB}^s - R_L^s},$$

such that this bank does not default on households in State s. α_{DH}^s is the critical amount of money creation at which a bank is just able to pay back depositors in State s. α_{DH}^s is obtained from Equation (1) by setting $\Pi_B^{b,s} = 0$ and using $\varphi = \frac{E_B}{L_M}$. From now on, consider a defaulting bank b. If $R_{CB}^s > R_L^s$ for some state s, there exist a lower bound α_{DH}^s and an upper bound α_{DCB}^s for α_M^b given by

$$\alpha_{DH}^s < \alpha_M^b \leq \alpha_{DCB}^s := \frac{R_{CB}^s}{R_{CB}^s - R_L^s},$$

which mark two default points. For $\alpha_M^b \in (\alpha_{DH}^s, \alpha_{DCB}^s]$, Bank b defaults against households but not against the central bank in State s. For $\alpha_M^b > \alpha_{DCB}^s$, the bank would default against households *and* the central bank in State s. α_{DCB}^s is the critical amount of money creation at which a bank is just able to pay back the central bank in State s. α_{DCB}^s is obtained from Equation (1) by setting $\Pi_B^{b,s} = -D_H R_D^s$. The lump-sum tax levied to bail out Bank b in State s is denoted by $t^{b,s}$. Aggregate tax payments in State s by households are then given by

$$T^s = \int_{b\in[0,1]} t^{b,s} \mathrm{d}b.$$

Furthermore, we use $\Pi_B^{+,s}$ to denote the aggregate profits of non-defaulting banks in State s. The balance sheets possible are given in Table 6.

In Table 6, the labels $b_{i'}$ and $b_{j'}$ denote banks with a non-negative and negative net position against the CB, respectively. The exact expressions of $d_{CBT}^{b_{i'}}$ and $l_{CBT}^{b_{j'}}$

Households		Bank $b_{i'}$		Bank $b_{j'}$	
$S_F \mathbf{R_F}$	$E_H R_H^s$	$d_{CBT}^{b_{i'}}$			$l_{CBT}^{b_{j'}}$
$D_H R_D^s - T^s$		$l_M^{b_{i'}} R_L^s$	$d_H R_D^s - T^s$	$l_M^{b_{j'}} R_L^s$	$d_H R_D^s - T^s$
$\Pi_B^{+,s}$			$\Pi_B^{b_{i'},s} + t^{b_{i'},s}$		$\Pi_B^{b_{j'},s} + t^{b_{j'},s}$
Π_F^s					

Table 6: Balance sheets at the end of Stage D if some banks default.

are not needed for the subsequent analysis, but for completeness they are given in Appendix C. We note that the balance sheets in Table 6 are structurally identical to the ones in Subcase II.a of Subsection 2.5.2. Therefore, the description of Stage E is similar to the one laid out in Appendix B.

A summary of the agents' interactions during Period $t = 1$ is given in Figure 2, page 8.

2.6 Definition of an Equilibrium with Banks

We look for symmetric equilibria with banks in the sequential market process described in Subsection 2.5. In a symmetric equilibrium with banks, all banks take the same decision regarding money creation and lending and thus have identical balance sheets in equilibrium. Moreover, the policy gross rates $(R_{CB}^s)_s$ are set by the central bank, so equilibria with banks are dependent on this choice.

Definition 1
Given the central bank policy gross rates $(R_{CB}^s)_s$, a symmetric equilibrium with banks in the sequential market process described in Subsection 2.5 is defined as a

tuple

$$\mathcal{E} := \Big((R_E^s)_s, (R_D^s)_s, (R_L^s)_s, \mathbf{R_F},$$
$$p_I, (p_C^s)_s,$$
$$E_B, D_H, (\widetilde{D}_H^s)_s, L_M, S_F,$$
$$\mathbf{K_M}, \mathbf{K_F}\Big)$$

consisting of positive and finite gross rates of return, prices, savings, bank deposits D_H *at the end of Stage C of Period* $t = 0$*, bank deposits* $(\widetilde{D}_H^s)_s$ *in Stage E of Period* $t = 1$*, and the corresponding physical investment allocation, such that*

- *households hold some private deposits* $D_H > 0$ *at the end of Stage C,*[40]
- *households maximize their expected utility*

$$\max_{\{D_H, E_B, S_F\}} \left\{ E_B \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] + D_H \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] + \mathbf{f}(S_F) \right\}$$
$$s.t. \quad E_B + D_H + p_I S_F = p_I \mathbf{W},$$

  taking gross rates of return $(R_E^s)_s$ *and* $(R_D^s)_s$ *as well as prices* p_I *and* $(p_C^s)_s$ *as given,*

- *firms in MT and FT as well as each bank* $b \in [0, 1]$ *maximize their expected shareholders' value,*[41] *given respectively by*

$$\max_{\mathbf{K_M} \in [\mathbf{0}, \mathbf{W}]} \{\mathbb{E}[\max(\mathbf{K_M}(\mathbf{R_M^s} p_C^s - R_L^s p_I), 0)]\},$$
$$s.t.\ \mathbf{R_M^s} p_C^s = R_L^s p_I \ for\ s = l, h,$$
$$\max_{\mathbf{K_F} \in [\mathbf{0}, \mathbf{W}]} \{\mathbb{E}[\max((\mathbf{f}(\mathbf{K_F}) - \mathbf{K_F} \mathbf{R_F}) p_C^s, 0)]\},$$
$$and \quad \max_{\alpha_M^b \geq 0} \{\mathbb{E}[\max(\alpha_M^b L_M (R_L^s - R_{CB}^s) + L_M (R_{CB}^s - R_D^s) + E_B R_D^s, 0)]\},$$

  taking gross rates of return $(R_D^s)_s$*,* $(R_L^s)_s$*, and* $\mathbf{R_F}$ *as well as prices* p_I *and* $(p_C^s)_s$ *as given,*

- *all banks choose the same level of money creation, and*

[40] As deposits are the only means of payment, we rule out knife-edge equilibria with banks in which private money creation at the end of Period $t = 0$ is zero.

[41] In our setting the maximization of profits in nominal terms by firms and by banks is qualitatively equivalent to the maximization of profits in real terms. Details are available on request.

– *markets for investment and consumption goods clear in each state.*

In the remainder of the paper we will use Superscript * to denote equilibrium variables. Henceforth, for ease of presentation, an equilibrium with banks given $(R^s_{CB})_s$ is a symmetric equilibrium with banks given $(R^s_{CB})_s$ in the sense of Definition 1.

3 Equilibria with Banks

3.1 Individually Optimal Choices

In this subsection we prepare the characterization of equilibria with banks by determining the individually optimal choices of banks, households, and firms. We first establish the way in which deposit gross rates are related to policy gross rates. Since banks can grant loans to, or borrow from, other banks, we obtain

Lemma 2

In any equilibrium with banks, the nominal lending gross rates on the interbank market satisfy

$$R^{s*}_D = R^s_{CB} \quad \textit{for all states } s = l, h.$$

The proof of Lemma 2 can be found in Appendix G. It is based on a simple arbitrage argument: any differential in the gross rates could be used in the interbank market by borrowing or lending to infinitely increase expected shareholders' value.

We next investigate the optimal choice of money creation by an individual bank. For convenience, we denote circumstances in which no finite amount of money creation is optimal by "∞". Then we obtain

Proposition 1

If $R^s_D = R^s_{CB}$ in all states $s = l, h$, the privately optimal amounts of money creation and lending by an individual bank are represented by a correspondence denoted by[42]

[42] If X denotes a set, we use $\mathcal{P}(X)$ to denote the power set of X.

$\hat{\alpha}_M : \mathbb{R}^4_{++} \times (0,1) \to \mathcal{P}(\mathbb{R} \cup \{+\infty\})$ *and given by*

$$\hat{\alpha}_M\big((R^s_L)_s, (R^s_{CB})_s, \varphi\big) =$$
$$\begin{cases} \{+\infty\} & \text{if } R^s_L \geq R^s_{CB} \text{ for all states } s = l,h \\ & \text{with at least one strict inequality,} \\ \{\alpha^l_{DCB}\} & \text{if } (\overline{R}_L \geq \overline{R}_{CB},\ R^l_L < R^l_{CB}, \text{ and } R^h_{CB} < R^h_L) \text{ or} \\ & \text{if } (\overline{R}_L < \overline{R}_{CB},\ R^h_{CB} < R^h_L, \text{ and } \varphi < \left(\frac{\sigma}{1-\sigma}\right) \frac{R^h_L - R^h_{CB}}{R^l_{CB} - R^l_L}), \\ \{\alpha^h_{DCB}\} & \text{if } (\overline{R}_L \geq \overline{R}_{CB},\ R^h_L < R^h_{CB}, \text{ and } R^l_{CB} < R^l_L) \text{ or} \\ & \text{if } (\overline{R}_L < \overline{R}_{CB},\ R^l_{CB} < R^l_L, \text{ and } \varphi < \left(\frac{1-\sigma}{\sigma}\right) \frac{R^l_L - R^l_{CB}}{R^h_{CB} - R^h_L}), \\ [0, +\infty) & \text{if } R^s_L = R^s_{CB} \text{ for all states } s = l,h, \\ \{0, \alpha^l_{DCB}\} & \text{if } \overline{R}_L < \overline{R}_{CB},\ R^h_{CB} < R^h_L, \text{ and } \varphi = \left(\frac{\sigma}{1-\sigma}\right) \frac{R^h_L - R^h_{CB}}{R^l_{CB} - R^l_L}, \\ \{0, \alpha^h_{DCB}\} & \text{if } \overline{R}_L < \overline{R}_{CB},\ R^l_{CB} < R^l_L, \text{ and } \varphi = \left(\frac{1-\sigma}{\sigma}\right) \frac{R^l_L - R^l_{CB}}{R^h_{CB} - R^h_L}, \\ \{0\} & \text{if } (R^s_L \leq R^s_{CB} \text{ for all states } s = l,h \\ & \text{with at least one strict inequality) or} \\ & \text{if } (\overline{R}_L < \overline{R}_{CB},\ R^h_{CB} < R^h_L, \text{ and } \left(\frac{\sigma}{1-\sigma}\right) \frac{R^h_L - R^h_{CB}}{R^l_{CB} - R^l_L} < \varphi) \text{ or} \\ & \text{if } (\overline{R}_L < \overline{R}_{CB},\ R^l_{CB} < R^l_L, \text{ and } \left(\frac{1-\sigma}{\sigma}\right) \frac{R^l_L - R^l_{CB}}{R^h_{CB} - R^h_L} < \varphi). \end{cases}$$

The proof of Proposition 1 can be found in Appendix G. There are several observations to make. First, the banks' behavior depends only on $(R^s_L - R^s_{CB})_{s=l,h}$, which is the intermediation margin, on average lending by banks, and on their capital structure φ. If the intermediation margin is zero in all states, it is obvious that banks are indifferent between all lending levels. For positive intermediation margins in all states, banks would like to grant as many loans as possible. For negative intermediation margins, banks are not willing to grant any loans. Finally, if the intermediation margin is positive in one state and negative in the other state, banks can use shareholders' limited liability and depositors' bail-out by the government to maximize their expected gross rate of return on equity by defaulting against households in one state and by making large profits in the other. This strategy is only profitable in the following two cases: (i) when the expected intermediation margin is non-negative, meaning that banks can weakly increase their expected shareholders' value even if they do not use limited liability and depositors' bail-out by the government, (ii) when the expected intermediation margin is negative and banks can sufficiently leverage on limited liability, which occurs when the banks' equity ratio is sufficiently low. Next we turn to the households' investment behavior. We obtain

Lemma 3
The representative household's optimal portfolio choice depends solely on the comparison of expected real gross rates of return $\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]$, $\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]$, $\frac{\mathbf{f}'(0)}{p_I}$, *and* $\frac{\mathbf{f}'(\mathbf{W})}{p_I}$ *when choosing* E_B, D_H, *and* S_F.

The correspondences representing households' optimal choices for different constellations of these expected real gross rates of return are given in Lemma 6 in Appendix E.

We next turn to the firms' behavior.

Lemma 4
Demands for the investment good by firms in MT and FT are represented by two correspondences denoted by $\hat{\mathbf{K}}_{\mathbf{M}} \in \mathcal{P}(\mathbb{R} \cup \{+\infty\})$ *and* $\hat{\mathbf{K}}_{\mathbf{F}} : \mathbb{R}_{++} \to \mathcal{P}([\mathbf{0}, \mathbf{W}])$, *respectively and given by*

$$\hat{\mathbf{K}}_{\mathbf{M}} = [\mathbf{0}, +\infty]$$

$$\text{and} \quad \hat{\mathbf{K}}_{\mathbf{F}}(\mathbf{R}_{\mathbf{F}}) = \begin{cases} \{\mathbf{0}\} & \text{if } \mathbf{f}'(\mathbf{0}) \leq \mathbf{R}_{\mathbf{F}}, \\ \{\mathbf{f}'^{-1}(\mathbf{R}_{\mathbf{F}})\} & \text{otherwise.} \end{cases}$$

The proof of Lemma 4 can be found in Appendix G.[43] We note that in Sector MT, firms are indifferent between any investment level $\mathbf{K}_{\mathbf{M}}$, as the condition in Lemma 1, $\mathbf{R}_{\mathbf{M}}^{\mathbf{s}} p_C^s = R_L^s p_I$ for $s = l, h$, implies that these firms make zero profits at any level of $\mathbf{K}_{\mathbf{M}}$.

3.2 Characterization of Equilibria with Banks

The preceding lemmata enable us to characterize all equilibria with banks.

Theorem 1
Given the policy gross rates $(R_{CB}^s)_{s=l,h}$, *all equilibria with banks take the following*

[43]We note that firms do not take the scarcity of the investment good into account when applying for loans in Sector MT and when issuing bonds in Sector FT, since we assume that markets are competitive.

form:

$$R_E^{s*} = R_D^{s*} = R_L^{s*} = R_{CB}^s, \quad \mathbf{R_F^*} = \overline{\mathbf{R}}_\mathbf{M}, \tag{2}$$

$$p_I^* = p, \quad p_C^{s*} = p\frac{R_{CB}^s}{\mathbf{R_M^s}}, \tag{3}$$

$$E_B^* = \varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big), \quad D_H^* = (1-\varphi^*)p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big), \tag{4}$$

$$\tilde{D}_H^{s*} = p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big)R_{CB}^s, \tag{5}$$

$$L_M^* = p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big), \quad S_F^* = \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big), \tag{6}$$

$$\mathbf{K_M^*} = \mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big), \quad \mathbf{K_F^*} = \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big), \tag{7}$$

where the price of the investment good denoted by $p \in (0, +\infty)$ *and the aggregate equity ratio* $\varphi^* \in (0,1)$ *are arbitrary. Equilibrium profits of firms and banks are given by*

$$\Pi_M^{s*} = 0, \quad \Pi_F^{s*} = p\frac{R_{CB}^s}{\mathbf{R_M^s}}\left(\mathbf{f}\Big(\mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big) - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\overline{\mathbf{R}}_\mathbf{M}\right), \tag{8}$$

$$\Pi_B^{s*} = \varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big)R_{CB}^s. \tag{9}$$

The proof of Theorem 1 can be found in Appendix G.

We now look at the equilibrium conditions in detail. First, all nominal gross rates are equal to the policy gross rates set by the central bank, as expressed in (2). The equilibrium with banks is unique in real terms, i.e. the physical investments in both sectors expressed in (7), and thus with respect to the real values of lending and saving expressed in (6), where we divide L_M^* by p.

As expressed in (4) the initial split of investments in banks into deposits and equity is indeterminate. In fact, in an equilibrium with banks any capital structure of banks can occur. Equation (5) reflects macroeconomic uncertainty, as the dividends and the deposit gross rates depend on the state of the world. Equations (8) and (9) represent the profits of firms and banks. The representative firm's profits in Sector FT are paid in terms of the consumption good, while banks' dividends are paid in the form of bank deposits.

Finally, the second equation in (3) relates the prices of the consumption good in different states to the price of the investment good. The latter is not determinate.

The economic system is nominally anchored by the price of the investment good and by the central bank interest rate. While these parameters determine prices and interest rates, the asset structure and the payment processes are additionally determined by the capital structure of banks.

Here, more remarks are in order. First, no bank defaults in equilibrium. Indeed, the profits of any bank in State s are given by $\varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big)R^s_{CB}$ and are thus positive. The reason is twofold. On the one hand, loan interest rates equal deposit interest rates in each state of the world. On the other hand, low gross rates of return $\mathbf{R}^\mathbf{l}_\mathbf{M}$ trigger a high price p^{l*}_C for the consumption good, which enables firms in Sector MT to pay back their loans, which, in turn, enables banks to pay back depositors.

Second, the theorem shows that in any equilibrium with banks, private money creation is naturally limited. Since $R^{s*}_L = R^s_{CB}$ in both states $s = l, h$, banks have no incentive to increase money creation, as they would be forced to refinance themselves at the gross rates $(R^s_{CB})_s$ to cover additional money creation.

Third, the capital structure of banks has no impact on the physical investment allocation, so there is no need to regulate bank equity capital. Fourth, the physical investment allocation is independent of the central bank's policy gross rates. Monetary policy is neutral.

There are important implications and a variety of further consequences of Theorem 1, which we summarize in the next subsection.

3.3 Welfare Properties and Implications

We start with the characterization of the optimal investment allocation. The social planner's problem is given by

$$\begin{aligned} &\max_{(\mathbf{K_M},\mathbf{K_F})} \mathbb{E}[\mathbf{K_M}\mathbf{R^s_M} + \mathbf{f}(\mathbf{K_F})] \\ \text{s.t.} \quad &\mathbf{K_M} + \mathbf{K_F} = \mathbf{W}. \end{aligned}$$

It is clear that household utility is maximized at $\mathbf{K^{FB}_F} := \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})$. From Theorem 1 we immediately obtain

Corollary 1
Given any policy gross rates $(R_{CB}^s)_s$, the equilibria with banks yield the first-best allocation.

As a direct consequence, the central bank is indifferent between policy gross rates $(R_{CB}^s)_{s=l,h}$, as they all implement the first-best allocation. Essentially, Theorem 1 is a first welfare theorem for an economy with private money creation. It is a benchmark for the results we derive in the next section.

We stress that the welfare theorem does not depend on whether the policy gross rates—and as a consequence, all nominal interest rates—depend on the state of the world. Indeed, another immediate consequence is given by

Corollary 2
Suppose that R_{CB}^s is the same in both states s of the world. Then the nominal lending and deposit gross rates are not contingent on the states of the world, and the resulting allocation is first-best.

The corollary implies that the nominal gross rate of return on deposits does not need to depend on the macroeconomic shock to guarantee the first-best allocation. The reason is that in the event of a negative macroeconomic shock, firms in Sector MT compensate for lower real production gross rates of return by higher prices for the consumption good, thereby avoiding default against banks and rendering non-contingent deposits safe even without government intervention.[44] The reason why the prices of the consumption good increase when a negative macroeconomic shock occurs is detailed below.

The equilibria with banks described in Theorem 1 are indeterminate in two respects, with regard to (a) the price of the investment good and (b) the capital structure of banks. Regarding the former, it simply represents a price normalization problem, and we can set $p_I = 1$ without loss of generality. The indeterminacy of the capital structure in equilibrium is a macroeconomic manifestation of the Modigliani-Miller Theorem. As banks do not default in equilibrium and the gross rates of return on equity and deposits are the same, households are indifferent between equity and deposits. Moreover, different capital structures of banks have no

[44] The conclusion would not hold if the real deposit gross rates of return were independent of the state of the world and thus if deposit interest rates were inflation-linked. This is addressed in Subsection 5.7 below.

impact on money creation and lending by banks. Finally, we note in the following corollary that with price normalization $p_I = 1$ and some capital structure choice φ^*, all equilibrium values are uniquely determined.

Corollary 3
Given $p_I = 1$ and some $\varphi^* \in (0,1)$, all equilibrium values are uniquely determined when the central bank sets the policy gross rates $(R^s_{CB})_s$.

The relationship between the policy gross rates and the prices of the consumption good in different states of the world is contained in the following corollary:

Corollary 4

(i) If R^s_{CB} does not depend on the state s of the economy, i.e. if $R^l_{CB} = R^h_{CB}$,

$$\textit{then} \quad p^h_C < p^l_C \quad \textit{and} \quad \frac{p^l_C}{p^h_C} = \frac{\mathbf{R^h_M}}{\mathbf{R^l_M}}.$$

(ii) For central bank policy gross rates $(R^s_{CB})_s$ characterized by

$$\frac{R^h_{CB}}{R^l_{CB}} = \frac{\mathbf{R^h_M}}{\mathbf{R^l_M}},$$

the price of the consumption good is independent of the state of the world ($p^h_C = p^l_C$).

We note that central bank policy gross rates described in (ii) imply $R^l_{CB} < R^h_{CB}$. Corollary 4 stems from the equilibrium condition in (3) and is based on the following intuition: If R^s_{CB} is independent of the state of the world and State l occurs, the households possess a comparatively large amount of deposits in Period $t = 1$ when production has occurred, which causes the price of the consumption good to rise, as its supply is low. When the central bank chooses lower interest rates in bad states, the amount of privately created money declines in line with the supply of the consumption good. As a consequence, the price of the consumption good remains constant across states.

In the next section we explore potential cases of friction that may move allocations away from the first-best allocation and may even cause a collapse of the monetary system. We also explore whether monetary policy or capital regulation might help to restore efficiency. We note that the explosion of money creation and lending could not happen in a banking model that only comprises a real sector, as in such models lending is constrained by the funding of banks with the investment good.

4 Price Rigidities and Capital Requirements

4.1 Absence of Capital Requirements

In Section 4 we explore what happens when money creation is affected by price rigidities and the zero lower bound. In such a setting we also examine how a capital requirement can improve the possible equilibrium allocations. For this purpose, it is useful to introduce three types of situation:

(*i*) Money creation is positive and limited, but aggregate investment is distorted between sectors,

(*ii*) money creation is zero, and physical investment occurs only in Sector FT, and

(*iii*) money creation explodes without limit, the monetary system collapses, and physical investment remains viable in Sector FT only.[45]

In Section 4, without loss of generality, we normalize the price of the investment good to $p_I = 1$. We assume in this section that nominal prices are perfectly rigid in the sense that they do not depend on the state of the world, and we assume that they are equal to some value p_C, which for convenience we set to 1.[46]

From Corollary 4 we obtain that when

$$R^s_{CB} = \mathbf{R^s_M}, \tag{10}$$

which means that the central bank chooses the real gross rates of return as its policy gross rates, we recover the first-best equilibria with banks in Theorem 1. We next

[45] Essentially, no equilibrium with banks with finite money creation exists. However, there exists an equilibrium in which no household offers equity to banks, all investment goods are channeled to Sector FT, and no lending to Sector MT occurs.

[46] Of course, this is a strong assumption. The results could be extended to models with multiple consumption goods, where a subset of firms would face such rigidities in the sense of Calvo (1983). Throughout Section 4 the concept of price rigidities refers to $p^s_C = p_I = 1$ for both states $s = l, h$.

investigate circumstances where the central bank does not or cannot choose the policy gross rates according to (10). This occurs, for example, if $\mathbf{R^l_M} < 1$, i.e. the real gross rate of return in the bad macroeconomic state is sufficiently low, since due to the zero lower bound the policy gross rate R^l_{CB} cannot be set smaller than one.[47] It could also occur if the central bank—for example because of uncertainty about the underlying real gross rates of return—does not or cannot choose the policy gross rates according to (10). From Proposition 1 we immediately obtain

Proposition 2
Suppose prices are rigid and $R^s_{CB} \neq \mathbf{R^s_M}$ for some state s of the world. Then either there is no money creation, or it explodes.[48] In both cases all investments are channeled to Sector FT.

In a symmetric equilibrium with banks, an individual bank cannot grant more loans and generate more money creation than the average. Otherwise money creation would be explosive, as all banks would try to create more money than the average and as a consequence, the monetary system would break down. In the case where no loan is granted, no money is created and only investment in Sector FT is possible, which constitutes an inefficient allocation. Moreover, we note that the equilibrium allocation of Proposition 2 is inefficient, as expected output is maximized only when investment is channeled to both sectors. Expected loss in output is given by

$$\big(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})\big)\overline{\mathbf{R}}_\mathbf{M} + \mathbf{f}\big(\mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})\big) - \mathbf{f}(\mathbf{W}).$$

The possible constellations with price rigidities are depicted in Table 7.

[47] In practice, banks can exchange central bank deposits for banknotes and coins. By storing cash, banks could in principle bypass negative central bank policy interest rates. The same possibility protects depositors from negative interest rates. Accordingly, the presence of banknotes and coins is essential in rationalizing the zero lower bound. In our model we assume that the central bank is constrained by the zero lower bound by the threat of private agents to withdraw deposits and store banknotes, but we do not explicitly model banknotes and coins.

[48] We say that there is no money creation when all elements of $\hat{\alpha}_M\big((\mathbf{R^s_M})_s, (R^s_{CB})_s, \varphi\big)$ are smaller than 1 and we say that money creation explodes when all elements of $\hat{\alpha}_M\big((\mathbf{R^s_M})_s, (R^s_{CB})_s, \varphi\big)$ are larger than 1. Finally, we say that either there is no money creation or it explodes if there exist $\alpha_1, \alpha_2 \in \hat{\alpha}_M\big((\mathbf{R^s_M})_s, (R^s_{CB})_s, \varphi\big)$ such that $(\alpha_1 - 1)(\alpha_2 - 1) < 0$.

	$R^l_{CB} < \mathbf{R^l_M}$	$R^l_{CB} = \mathbf{R^l_M}$	$R^l_{CB} > \mathbf{R^l_M}$
$R^h_{CB} < \mathbf{R^h_M}$	Money Explosion	Money Explosion	Money Crunch or Money Explosion
$R^h_{CB} = \mathbf{R^h_M}$	Money Explosion	Efficient Equilibrium	Money Crunch, No Banking
$R^h_{CB} > \mathbf{R^h_M}$	Money Crunch or Money Explosion	Money Crunch, No Banking	Money Crunch, No Banking

Table 7: Possible constellations with price rigidities.

4.2 Capital Requirements

We next investigate the extent to which whenever there is a difference between R^s_{CB} and $\mathbf{R^s_M}$ for some state s a capital requirement can restore both the existence of an equilibrium with banks in the sense of Theorem 1 as well as efficiency. A capital requirement is defined as follows:

Definition 2
A minimum bank equity ratio φ^{reg} ($\varphi^{reg} \in (0,1)$) requires each bank to hold more equity at the end of Period $t = 0$ than the fraction φ^{reg} of its total assets. In other words, the realized equity ratio of each bank b, which we denote by φ^b, has to be larger than φ^{reg}.

We first establish a lemma describing how a capital requirement impacts money creation by an individual bank.

Lemma 5
Suppose the average capital structure in the economy is φ and $\varphi^{reg} \leq \varphi$. Then the capital requirement φ^{reg} imposes an upper bound on individual money creation:

$$\alpha^b_M \leq \frac{\varphi}{\varphi^{reg}} \quad \textit{for all banks } b.$$

The proof of Lemma 5 can be found in Appendix G. We next determine the optimal money creation choice by banks when the government sets a capital requirement. When $R^s_{CB} \neq \mathbf{R^s_M}$ for some state s of the economy, money creation is either limited by the threat of default against the central bank, by the capital requirement, or it is not profitable. The detailed characterization of the correspondence describing these three situations is given in Lemma 7 in Appendix F. We use Lemma 7 to derive general conditions under which equilibria with banks exist when $R^s_{CB} \neq \mathbf{R^s_M}$ for some state s of the world.

Proposition 3
Suppose that prices are rigid and $R^s_{CB} \neq \mathbf{R^s_M}$ for some state s. Then there exists an equilibrium with banks if the central bank policy gross rates $(R^s_{CB})_s$ and the capital requirement level φ^{reg} are set as either (i) or (ii):

(i) $\overline{R}_{CB} = \overline{\mathbf{R}}_{\mathbf{M}}$ *and* $\max\left(\frac{R^h_{CB}-\mathbf{R^h_M}}{R^h_{CB}}, \frac{R^l_{CB}-\mathbf{R^l_M}}{R^l_{CB}}\right) \leq \varphi^{reg}$.

(ii) $\overline{R}_{CB} > \overline{\mathbf{R}}_{\mathbf{M}}$ *and* $0 < \varphi^{reg} = \max\left(\frac{1-\sigma}{\sigma}\frac{\mathbf{R^l_M}-R^l_{CB}}{R^h_{CB}}, \frac{\sigma}{1-\sigma}\frac{\mathbf{R^h_M}-R^h_{CB}}{R^l_{CB}}\right) < 1$.

The proof of Proposition 3 is given in Appendix G. From Proposition 3 and its proof we can derive the welfare properties of equilibria with banks when a capital requirement is imposed. These welfare properties are summarized in the following corollary:

Corollary 5
Suppose that prices are rigid and $R^s_{CB} \neq \mathbf{R^s_M}$ for some state s. Then the central bank policy gross rates $(R^s_{CB})_s$ and the capital requirement level φ^{reg} implement a socially efficient equilibrium with banks if and only if

$$\overline{R}_{CB} = \overline{\mathbf{R}}_{\mathbf{M}} \quad \text{and} \quad \max\left(\frac{R^h_{CB} - \mathbf{R^h_M}}{R^h_{CB}}, \frac{R^l_{CB} - \mathbf{R^l_M}}{R^l_{CB}}\right) \leq \varphi^{reg}.$$

The intuition for Proposition 3 and Corollary 5 runs as follows: If in some state s, $R^s_{CB} < \mathbf{R^s_M}$, banks would like to expand money creation to high, if not infinite, levels because potential losses in the other state $s' \neq s$ would be bounded due to limited shareholder liability. In such cases, the capital requirement constrains money creation. Two cases may occur.

When $\overline{R}_{CB} = \overline{\mathbf{R}}_{\mathbf{M}}$, no bank has any incentive to push money creation above average, since first, losses in some state s' exactly offset gains from money creation in the other state $s \neq s'$, and second, the minimum capital requirement is set at a level that prevents banks from defaulting against depositors and thus from leveraging on limited shareholder liability. By preventing default against depositors, such a minimum capital requirement induces socially efficient money creation and lending.

When $\overline{R}_{CB} > \overline{\mathbf{R}}_{\mathbf{M}}$, banks would expand money creation above average in the absence of a capital requirement, since for an increasing money creation level, the shareholders' value increases in some state s, while it stays at zero in the other state s'. Thus, the capital requirement directly limits money creation by preventing banks from granting any above-average amount of loans.

In this case, even though such a minimum capital requirement restores a potential equilibrium with banks, it does not implement a socially efficient allocation. The inefficiency results from banks' default against depositors. When they make their investment decision, households do not take into account the impact of banks' default on the lump-sum taxes levied to bail them out. From the proof of Proposition 3 it is straightforward that the equilibria with banks' default can be ranked in terms of welfare according to the capital requirement level φ^{reg}. The intuition runs as follows: A larger equity ratio reduces the amount of taxes levied to bail out banks, which in turn improves households' investment decision making. Therefore, the intensity of the inefficiency associated with banks' default declines in the capital requirement level φ^{reg}.

4.3 The Zero Lower Bound and Capital Requirements

We next explore the case where the central bank is constrained by the zero lower bound and prices are assumed to be rigid, i.e. when $p_C^{s*} = p_I^* = 1$ for all states $s = l, h$. From Corollary 5 we obtain

Corollary 6
Suppose that prices are rigid, $\mathbf{R}_{\mathbf{M}}^{\mathbf{l}} < 1 \leq \overline{\mathbf{R}}_{\mathbf{M}}$, and the central bank is constrained by the zero lower bound ($R_{CB}^s \geq 1$ for all states $s = l, h$). Then there exist central bank policy gross rates $(R_{CB}^s)_s$ and capital requirement levels φ^{reg} such that the allocation of the resulting equilibrium with banks is socially efficient.

(i) The central bank policy gross rates have to satisfy $\overline{R}_{CB} = \overline{\mathbf{R}}_{\mathbf{M}}$. One example is

$$R_{CB}^l = 1, \quad R_{CB}^h = \frac{\overline{\mathbf{R}}_{\mathbf{M}} - (1-\sigma)}{\sigma}.$$

(ii) The regulatory capital requirement levels φ^{reg} have to satisfy

$$\varphi^{reg} \geq \frac{R^l_{CB} - \mathbf{R^l_M}}{R^l_{CB}}.$$

The proof of Corollary 6 can be found in Appendix G. Corollary 6 shows that price rigidities and the zero lower bound can be countered by a suitable combination of monetary policy and capital regulation. The capital requirement ensures that money creation is sufficiently constrained for no individual bank to default. The central bank policy gross rates $R^l_{CB} = 1$, $R^h_{CB} = \frac{\overline{\mathbf{R}}_\mathbf{M} - (1-\sigma)}{\sigma}$ ensure that in the good state gains from money creation are sufficiently high to offset losses in the bad state. In other words, setting $R^h_{CB} < \mathbf{R^h_M}$ generates sufficient incentives for banks to lend and to create money. The capital requirement, in turn, ensures that money creation does not become excessive. We note that any monetary policy that satisfies $\overline{R}_{CB} = \overline{\mathbf{R}}_\mathbf{M}$ achieves the same purpose and induces a socially efficient allocation. In Appendix H we illustrate our results with a simple numerical example.

From Corollary 5 and the proof of Proposition 3 we also immediately obtain

Proposition 4
Suppose that prices are rigid, $\overline{\mathbf{R}}_\mathbf{M} < 1$, and the central bank is constrained by the zero lower bound ($R^s_{CB} \geq 1$ for all states $s = l, h$). Then there exist no central bank policy gross rates $(R^s_{CB})_s$ and capital requirement level φ^{reg} making the allocation of the resulting equilibrium with banks socially efficient. We derive two cases:

- *If $1 < \mathbf{R^h_M}$, there exist central bank policy gross rates $(R^s_{CB})_s$ and a capital requirement level φ^{reg} implementing equilibria with banks.*
  (a) The central bank policy gross rates have to satisfy $R^h_{CB} < \mathbf{R^h_M}$. An example is

  $$R^l_{CB} = R^h_{CB} = 1.$$

  (b) The regulatory capital requirement level φ^{reg} has to satisfy

  $$\varphi^{reg} = \frac{\sigma}{1-\sigma} \frac{\mathbf{R^h_M} - R^h_{CB}}{R^l_{CB}}.$$

– *If* $\mathbf{R}^{\mathbf{h}}_{\mathbf{M}} \leq 1$*, there are no central bank policy gross rates* $(R^s_{CB})_s$ *and capital requirement level* φ^{reg} *implementing an equilibrium with banks.*

Proposition 4 states that in a depressed economy characterized by $\overline{\mathbf{R}}_{\mathbf{M}} < 1$, where prices are rigid and the central bank is constrained by the zero lower bound, money creation can only be induced by a suitable combination of monetary policy and capital regulation if $\mathbf{R}^{\mathbf{h}}_{\mathbf{M}} > 1$.

If $\mathbf{R}^{\mathbf{h}}_{\mathbf{M}} \leq 1$, the only possible equilibrium is the equilibrium without banks, which is inefficient, as all investments are channeled to FT.[49] The reason is that under any feasible monetary policy and even with no capital requirement, money creation and lending are not profitable in such cases.

If $\overline{\mathbf{R}}_{\mathbf{M}} < 1$ but $1 < \mathbf{R}^{\mathbf{h}}_{\mathbf{M}}$, the central bank and the bank regulators can only make banking profitable and thus trigger money creation and lending by inducing profits in the good state and letting them default against depositors in the bad state. From the proof of Proposition 3 we deduce that the policy gross rates inducing the equilibrium with banks with highest welfare are given by $R^s_{CB} = 1$ for $s = l, h$. Moreover, a capital requirement has to be imposed on banks to prevent money creation from exploding.

The equilibria associated with the policy gross rates $R^s_{CB} = 1$ for $s = l, h$ in the case $1 < \mathbf{R}^{\mathbf{h}}_{\mathbf{M}}$ are inefficient. Hence, the central bank and the bank regulators will implement such a policy only if the welfare induced by the policy described in Proposition 4, (a) and (b) is higher than the welfare associated to the equilibrium without banks. A sufficient condition for this is $\mathbf{f}(\mathbf{W}) < \overline{\mathbf{R}}_{\mathbf{M}}\mathbf{W}$.

The above result in the cases $\overline{\mathbf{R}}_{\mathbf{M}} < 1$ and $\mathbf{R}^{\mathbf{h}}_{\mathbf{M}} > 1$ can be interpreted in terms of Forward Guidance.[50] The central bank announces that it will set the policy gross rates at 1 in both states of the world, even if the real gross rate $\mathbf{R}^{\mathbf{h}}_{\mathbf{M}}$ is larger than one. This announcement means that banks can expect positive profits in the good state of the world, thereby making money creation and lending profitable. This stimulates money creation and lending at the zero lower bound. However, in the bad state of the world money creation is associated with bank failures, so expected

[49] In such a case, other kinds of policies such as Quantitative Easing might be useful to stimulate money creation and lending. We leave this to future research.

[50] In our two-period model the central bank does not face a time-inconsistent problem regarding such announcements. For the implementation of Forward Guidance at the zero lower bound, see e.g. Woodford (2013) and Gersbach et al. (2015a).

social welfare is lower than in the first-best allocation.

In summary, price rigidity does not cause a welfare loss unless the central bank does not or cannot set the policy rates appropriately. The latter situation occurs when the zero lower bound prevents the equalization of the policy rate and the real rate of return in the bad state. In such an environment, money creation either explodes or it is not attractive. Both cases result in a collapse of the banking system and in an equilibrium without banks. When money creation is profitable, capital requirements are a suitable tool to control the incentive to create money and to restore the existence of equilibria with banks.

5 Extensions and Generalizations

5.1 Financial Frictions

We can introduce a well-known case of financial friction into our model as follows: Bankers cannot pledge the entire return from their investment to depositors or shareholders, so for carrying out their monitoring activities they receive a non-pledgeable income proportional to the repayments $\theta \alpha_M^b L_M R_L^s$ at $t = 1$, where $\theta \in (0, 1)$. This financial friction arises from several theories on the micro-foundation of such frictions, such as moral hazard in the sense of Holmström and Tirole (1997), asset diversion (Gertler and Karadi, 2011; Gertler and Kiyotaki, 2011), and inalienability of human capital (Diamond and Rajan, 2001; Hart and Moore, 1994).

In line with these approaches, we integrate the financial friction into our model in the following form: Bankers need to be paid the amount $\theta \alpha_M^b L_M R_L^s$ in the form of deposits in Period $t = 1$ to ensure that they behave as they should (monitoring entrepreneurs and not diverting assets, for instance). Like households bankers will use these deposits to buy an amount of the consumption good. We assume that bankers are risk-neutral, i.e. they aim at maximizing their own expected consumption instead of their expected shareholders' value:

$$\mathbb{E}\left[\frac{\theta \alpha_M^b L_M R_L^s}{p_C^s}\right]. \tag{11}$$

Since the price of physical goods and aggregate lending cannot be influenced by an individual banker, bankers will aim to maximize their expected consumption by choosing α_M^b under the constraint that their bank does not default against the central bank.[51]

In such a setting, no equilibrium with banks exists, since a banker has an incentive to increase money creation to earn more rents for any given level of average money creation. We are able to show that money creation can be limited by a suitable combination of capital regulation and monetary policy. Such a policy mix can restore the existence of equilibria with banks and can implement the second-best allocation, i.e. the allocation for which households' welfare is maximized under the bankers' incentive-compatibility constraint.[52]

5.2 Nominal Bonds

In our model, bonds are indexed to inflation, meaning that the gross rate of return they promise is denominated in real terms. We could also allow for bonds denominated in nominal terms. The results would stay the same, but there would be an additional consideration and condition. In a setting with nominal bonds, firms in FT would be subject to uncertainty of their profits in nominal terms, meaning that, if the price of the consumption good is sufficiently small in the bad state for example, firms may not be able to repay the bonds in this state. Therefore, with nominal bonds, we have to add a non-default condition that itself is a condition on the gross rates of return on bonds and thus ultimately on central bank policy rates.[53]

5.3 Multiple States of the World

Assume now that there are N ($N \geq 2$) states of the world denoted by $s = 1, ..., N$ and occurring with probability $\sigma^s \in (0, 1)$. The analysis can be easily generalized to this setting. Typically, banks will choose the highest possible lending level for

[51]We continue to assume that bankers will face severe penalties if they default against the central bank. These penalties are assumed to be higher than expected consumption. One such penalty could be the seizure of the bankers' income.

[52]Details available on request.

[53]Details available on request.

which they do not default against the CB in any state. This is the lending level for which they default entirely on depositors in some state s_D, which we choose as being the lowest state with the smallest positive value $\alpha_{DCB}^{s_D}$. When prices are rigid and the central bank cannot choose the policy gross rates equal to some multiple of the nominal lending gross rates, a sufficiently high minimum equity ratio imposed on banks can restore allocation efficiency.[54]

5.4 Asymmetric Equilibria

In equilibria with banks, for which we allow them to have different strategies in equilibrium, the privately optimal relative amount of lending no longer needs to be $\alpha_M^b = 1$ for all banks $b \in [0,1]$. Even with perfectly flexible prices, equilibria with banks exist in which some positive measure of banks defaults in one state and the corresponding allocation is inefficient.[55] However, these types of equilibria with banks disappear when sufficiently high minimum equity ratios are imposed on banks, so that only the efficient equilibria with banks persist.[56]

5.5 Costs of Monitoring

Costs of monitoring typically constitute a major part of a private bank's expenses, a statement supported by empirical evidence from Philippon (2012) and Gropp et al. (2013). Moreover, economizing on these costs constitutes one rationale for the existence of banks. We can integrate monitoring costs into our model as follows: Suppose there are costs $\mathbf{c} > \mathbf{0}$ in terms of the consumption good per unit of investment good. By setting $\mathbf{R}_{\mathbf{M}}^{\mathbf{s}'} = \mathbf{R}_{\mathbf{M}}^{\mathbf{s}} - \mathbf{c}$, we obtain the original model with revised gross rates of return in Sector MT. We can apply all our results to the original model with the revised gross rates of return in the MT sector.

[54] Details available on request.

[55] Note that there even exist asymmetric equilibria with banks for which some banks do not grant any loan to firms in MT and hold only central bank deposits.

[56] Details are available on request.

5.6 Non-contingent Lending Rates

From Theorem 1 we deduce that, when the lending gross rate of return cannot be written contingently on the state of the world, an equilibrium with banks exists if and only if the policy gross rates $(R^s_{CB})_{s=l,h}$ are independent of the macroeconomic shock s. However, when prices are rigid, i.e. when $p_I = p^s_C = 1$ for all states $s = l, h$, the lending gross rate of return R_L must be equal to $\mathbf{R^h_M}$, as otherwise firms in MT would demand an infinite amount of loan, as their profits per loan unit would be positive in one state and non-negative in the other. With lending gross rates equal to $\mathbf{R^h_M}$, firms in MT default in State l, as they are only able to repay $\mathbf{R^l_M}$ per unit of loan. In anticipation of these defaults, banks will expect repayment $\mathbf{R^l_M}$ per unit of loan in State l. As a consequence, the realized repayments per unit of loan are identical to the ones with contingent lending gross rates, so all our results continue to hold.

5.7 Non-contingent Real Deposit Rates

We next examine an economy in which the real deposit gross rate offered by banks cannot be written contingently on the state of the world. We obtain

Corollary 7
If the deposit gross rate of return in real terms is independent of the state of the world, no equilibrium with banks exists.

The proof of Corollary 7 is given in Appendix G. Corollary 7 follows from the following considerations: With a non-contingent real gross rate of return on deposits, either banks have an incentive to increase money creation beyond the average level, or they have no incentive to lend at all. This can be seen from the following equations, which hold in any potential equilibrium with banks:

$$\frac{R^s_L}{p^s_C} = \frac{\mathbf{R^s_M}}{p_I},$$
$$\frac{R^s_{CB}}{p^s_C} = \frac{R^s_D}{p^s_C} = \mathbf{R_D},$$

where $\mathbf{R_D}$ denotes the deposit gross rate of return in real terms, which is independent of the state of the world. Following Proposition 1, three cases can occur depending on the value of $\mathbf{R_D}$ compared to $\frac{\mathbf{R^s_M}}{p_I}$. Either there is no privately optimal finite amount of money creation, or the privately optimal individual amount of money creation is the level at which a bank is just able to pay back the central bank in the state where the bank makes losses and defaults against households, or no bank grants any loan.

In the first two cases, an individual bank would grant a larger amount of loans than the average lending level in the economy and would borrow from the central bank the amount it does not receive in the form of households' deposits. However, in a symmetric equilibrium with banks, an individual bank cannot grant more loans and generate more money creation than the average, which means that money creation in these two cases is explosive and the monetary system breaks down. In the case where no loan is granted, no money is created and only investment in Sector FT is possible, which constitutes an inefficient allocation.

5.8 Reserve Requirements and Haircuts

We study the impact of reserve requirements coupled with haircuts on money creation. We introduce reserve requirements as follows:

Definition 3
A minimum reserve requirement r^{reg} ($r^{reg} \in (0,1)$) requires each bank to hold more central bank reserves at the end of Period $t = 0$ than the fraction r^{reg} of its deposits. If we denote the reserve ratio of Bank b by $r^b = \frac{d^b_{CB}}{d_H}$, a reserve requirement imposes the following relationship on central bank reserves d^b_{CB}:

$$r^{reg} \leq r^b = \frac{d^b_{CB}}{d_H}.$$

We define a haircut rule h as follows:

Definition 4
A haircut rule h ($h \in (0,1)$) requires each bank to hold more loans to Sector MT at the end of Period $t = 0$ than a multiple $\frac{1}{1-h}$ of its CB liabilities.

The balance sheets of banks b_i and b_j that are complying with some reserve requirement r^{reg} and some haircut rule h are given in Table 8. In these balance sheets, we use the following notations:

$$\begin{aligned} d_{CB}^{\Delta_i} &= l_{CB}^{\Delta_i} = \max(0, r^{reg} d_H - d_{CB}^{b_i}), \\ \text{and} \quad d_{CB}^{\Delta_j} &= l_{CB}^{\Delta_j} = r^{reg} d_H. \end{aligned}$$

In the following proposition, we investigate the impact of a minimum reserve

Bank b_i		Bank b_j	
$d_{CB}^{\Delta_i}$	$l_{CB}^{\Delta_i}$	$d_{CB}^{\Delta_j}$	$l_{CB}^{\Delta_j}$
$d_{CB}^{b_i}$			$l_{CB}^{b_j}$
$l_M^{b_i}$	d_H	$l_M^{b_j}$	d_H
	e_B		e_B

Table 8: Balance sheets at the end of $t = 0$, with a combination of a minimum reserve requirement r^{reg} and a haircut rule h.

requirement r^{reg} coupled with a haircut rule h on money creation α_M^b by a Bank b. We obtain

Proposition 5

A combination of a minimum reserve requirement r^{reg} and a haircut rule h imposes the following constraint on money creation by Bank b:

$$\alpha_M^b \leq \frac{1 - r^{reg}(1-\varphi)}{h}.$$

Equilibria with banks exist if and only if the equity ratio φ fulfills

$$1 - \frac{1-h}{r^{reg}} \leq \varphi.$$

The proof of Proposition 5 is given in Appendix G. From Lemma 5 and Proposition 5, we directly deduce that the impact of the reserve requirement coupled with the haircut rule on money creation by commercial banks given in Proposition 5 is identical to the one of the minimum equity ratio requirement given in Lemma 5.

We summarize this observation in the following Proposition:

Proposition 6
A combination of a reserve requirement r^{reg} and a haircut rule h imposes the same constraint on the banks' behavior as a minimum equity ratio requirement φ^{reg} if and only if

$$\varphi^{reg} = \frac{\varphi h}{1 - r^{reg}(1-\varphi)}.$$

The condition on the bank capital structure for which an equilibrium with banks exists then writes

$$\varphi^{reg} \leq \varphi,$$

or alternatively

$$1 - \frac{1-h}{r^{reg}} \leq \varphi.$$

As a consequence, it is sufficient to focus on the impact of a minimum equity ratio requirement on the banks' incentives to create money, since imposing a reserve requirement coupled with a haircut rule yields the same properties.

6 Conclusion

The integration of money creation by commercial banks into a general equilibrium setting allows to investigate the interaction between monetary policy and capital regulation. Our main findings are as follows: In a general equilibrium economy without price rigidities, any policy rate set by the central bank implements equilibria with banks that are first-best. In these equilibria, money creation is naturally limited, so there is no need for capital regulation. However, if prices are rigid, equilibria with banks only exist for certain values of the central bank policy rates. Moreover, the central bank policy rates may cause unfavorable situations: Either money creation may explode, or lending may not be profitable and money may not be created at all. Both types of failure are associated with inefficient equilibria. In addition, when the central bank policy is constrained by the zero lower bound, there may not even exist a central bank monetary policy with positive and finite money creation. Capital regulation in the form of a minimum equity ratio is an effective tool for limiting money creation, so it can restore the existence of equilibria with banks and also social efficiency. Finally, when prices are rigid, Forward Guidance together with capital regulation can only stimulate money creation and lending if economic conditions are sufficiently favorable.

Numerous further extensions deserve scrutiny. We outline the main ones here. Risk aversion of households and more sophisticated portfolio decisions between bank equity and deposits is an obvious candidate. Integrating an active government that provides public goods financed by taxation and issuance of bonds would provide an opportunity to examine the potential and limits of Quantitative Easing. A more elaborate model of this kind could also provide insights into the role of collaterals and haircuts, as well as their impact on physical investment allocation. Further down the line, variants of the model could be used in a dynamic setting with more than one period. This would be useful for investigating the impact of monetary policy and capital regulation on inflation and price stability.

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A Appendix – Stage C

We examine the detailed payment process, investment in FT, and payment of bank equity in Stage C through a series of substages. For this purpose, we index all variables changing in some substage by an integer starting from 1.

Stage C, Substage 1: Borrowing of Banks from the CB

In order to have enough CB deposits to guarantee payments using bank deposits, Bank b borrows from the central bank the amount of[57]

$$d^b_{CB_1} := l^b_M = \alpha^b_M D_M.$$

As a result, an aggregate amount of CB deposits amounting to $D_{CB_1} := D_M > 0$ is created. The balance sheets of banks and households are given in Table 9.

Households	
$\mathbf{W}$	E_H

Bank b	
$d^b_{CB_1}$	$l^b_{CB_1}$
l^b_M	d^b_M

Table 9: Balance sheets at the end of Stage C, Substage 1.

Stage C, Substage 2: Sale of an Amount of Investment Good to MT

We assume that firms in MT buy the largest possible amount of investment good they can afford and do not hold deposits in the production stage D:[58]

$$\mathbf{K_M} = \frac{L_M}{p_I}.$$

In order to settle these payments, each bank b transfers $d^b_M = \alpha^b_M D_M$ to other banks and receives the same amount $d_{H_1} := D_M$ from other banks in the form of CB deposits. We note that d_{H_1} does not depend on the individual bank b

[57]As the description of the interbank lending process is formally identical to that of depositing at and borrowing from the central bank, we limit the description to the case where all banks deposit at, and borrow exclusively from, the central bank.

[58]Note that relaxing this assumption would not change the equilibrium allocation of the investment good, as firms would not be able to improve shareholders' value in equilibrium by holding deposits.

due to our assumption that households keep deposits evenly distributed across all banks at all times. The corresponding aggregate amount is denoted by D_{H_1}. This transaction impacts CB deposits of Bank b as follows:

$$d^b_{CB_2} := d^b_{CB_1} - \alpha^b_M D_M + D_M = D_M.$$

The balance sheets of banks and households are given in Table 10.

Households		Bank b	
$\mathbf{K_F}$	E_H	$d^b_{CB_2}$	$l^b_{CB_1}$
D_{H_1}		l^b_M	d_{H_1}

Table 10: Balance sheets at the end of Stage C, Substage 2.

Stage C, Substage 3: Investment in FT

When buying S_F bonds from firms in FT, households deliver $\mathbf{K_F} = S_F$ units of the investment good against the promise to obtain $\mathbf{K_F R_F}$ units of the consumption good from FT after production has taken place. The balance sheets of banks and households are given in Table 11.

Households		Bank b	
S_F	E_H	$d^b_{CB_2}$	$l^b_{CB_1}$
D_{H_1}		l^b_M	d_{H_1}

Table 11: Balance sheets at the end of Stage C, Substage 3.

Stage C, Substage 4: Netting of CB Deposits and CB Loans

Now banks can net their CB deposits and CB loans, as no further payment has to be made before production. We use

$$\delta^b := d^b_{CB_2} - l^b_{CB_1} = (1 - \alpha^b_M) L_M \tag{12}$$

to denote the net position of Bank b against the CB. We distinguish banks with claims against the central bank from banks that are debtors of the central bank:

$$B_I := \{b_i \in [0,1] \quad \text{s.t.} \quad \delta^{b_i} \geq 0\}$$
$$\text{and} \quad B_J := \{b_j \in [0,1] \quad \text{s.t.} \quad \delta^{b_j} < 0\}.$$

Net claims against the central bank are denoted by $d^{b_i}_{CB} := \delta^{b_i}$ for all $b_i \in B_I$ and net liabilities by $l^{b_j}_{CB} := -\delta^{b_j}$ for all $b_j \in B_J$. The balance sheets of banks and households are given in Table 12.

Households		Bank b_i		Bank b_j	
S_F		$d^{b_i}_{CB}$			$l^{b_j}_{CB}$
D_{H_1}	E_H	$l^{b_i}_M$	d_{H_1}	$l^{b_j}_M$	d_{H_1}

Table 12: Balance sheets at the end of Stage C, Substage 4.

Stage C, Substage 5: Payment of Bank Equity

Now households pay the equity $E_B = \varphi D_M > 0$ pledged in $t = 1$, thereby destroying the corresponding amount of bank deposits. We use $D_H = (1 - \varphi)D_M$ to denote the remaining amount of deposits. Accordingly, $D_{H_1} = E_B + D_H$. The balance sheets of two typical banks representing a net depositor and a net borrower from the central bank are displayed in Table 3.

B Appendix – Stage E – No Bank Defaults

We examine the detailed dividend payment, payback of debt, and payment process of Stage E through a series of substages. Similarly to Appendix A, when a variable changes in some substage, we increase the index by 1 starting with the last index from Appendix A.

Stage E, Substage 1: Borrowing of Banks from the CB

In order to have enough CB deposits to guarantee payments using bank deposits, Bank b borrows from the central bank the amount of $l^{b,s}_{CB_3} = d^{b,s}_{CB_3} := D_H R^s_D + \Pi^{b,s}_B$.

We use the notations

$$\begin{aligned} d^{b_i,s}_{CB_4} &:= d^{b_i,s}_{CB_3} + d^{b_i}_{CB} R^s_{CB} \\ \text{and} \quad l^{b_j,s}_{CB_4} &:= l^{b_j,s}_{CB_3} + l^{b_j}_{CB} R^s_{CB}. \end{aligned}$$

The balance sheets of banks and households are given in Table 13.

Households		Bank b_i		Bank b_j	
$S_F \mathbf{R_F}$	$E_H R^s_H$	$d^{b_i,s}_{CB_4}$	$l^{b_i,s}_{CB_3}$	$d^{b_j,s}_{CB_3}$	$l^{b_j,s}_{CB_4}$
$D_H R^s_D$		$l^{b_i}_M R^s_L$	$d_H R^s_D$	$l^{b_j}_M R^s_L$	$d_H R^s_D$
$E_B R^s_E$			$\Pi^{b_i,s}_B$		$\Pi^{b_j,s}_B$
Π^s_F					

Table 13: Balance sheets at the end of Stage E, Substage 1.

Stage E, Substage 2: Dividend Payment

Bank profits are paid as dividends to households. This creates bank deposits, and households' deposits at Bank b become $\tilde{d}^s_H := D_H R^s_D + \Pi^s_B$. The aggregate amount of households' deposits is then denoted by $\tilde{D}^s_H$. In order to settle these payments, each bank b transfers $\Pi^{b,s}_B$ to other banks and receives Π^s_B from other banks in the form of CB deposits. These processes impact CB deposits of Banks b_i and b_j as follows:

$$\begin{aligned} d^{b_i,s}_{CB_6} &:= d^{b_i,s}_{CB_4} - \Pi^{b_i,s}_B + \Pi^s_B = d^{b_i}_{CB} R^s_{CB} + D_H R^s_D + \Pi^s_B \\ \text{and} \quad d^{b_j,s}_{CB_5} &:= d^{b_j,s}_{CB_3} - \Pi^{b_j,s}_B + \Pi^s_B = D_H R^s_D + \Pi^s_B. \end{aligned}$$

The balance sheets of banks and households are given in Table 14.

Stage E, Substage 3: Repayment of Debt and Distribution of Profits

From the repayment of debt $S_F \mathbf{R_F}$ and the distribution of profits Π^s_F, both in terms of the consumption good, households obtain $\mathbf{f}(\mathbf{K_F})$ units of the consumption good. The balance sheets of banks and households are given in Table 15.

Stage E, Substage 4: Sale of the Consumption Good Produced by MT

Households		Bank b_i		Bank b_j	
$S_F\mathbf{R_F}$	$E_H R_H^s$	$d_{CB_6}^{b_i,s}$	$l_{CB_3}^{b_i,s}$	$d_{CB_5}^{b_j,s}$	$l_{CB_4}^{b_j,s}$
$\tilde{D}_H^s$		$l_M^{b_i} R_L^s$	$\tilde{d}_H^s$	$l_M^{b_j} R_L^s$	$\tilde{d}_H^s$
Π_F^s					

Table 14: Balance sheets at the end of Stage E, Substage 2.

Households		Bank b_i		Bank b_j	
$\tilde{D}_H^s$	$E_H R_H^s$	$d_{CB_6}^{b_i,s}$	$l_{CB_3}^{b_i,s}$	$d_{CB_5}^{b_j,s}$	$l_{CB_4}^{b_j,s}$
$\mathbf{f}(\mathbf{K_F})$		$l_M^{b_i} R_L^s$	$\tilde{d}_H^s$	$l_M^{b_j} R_L^s$	$\tilde{d}_H^s$

Table 15: Balance sheets at the end of Stage E, Substage 3.

Firms in MT sell the entire amount of the consumption good they have produced. Households buy it with their private deposits consisting of their wealth in terms of equity and deposits.[59] The supply of $\mathbf{K_M R_M^s}$ units of the consumption good meets the real demand $\frac{\tilde{D}_H^s}{p_C^s}$. Hence, the equilibrium price is given by

$$p_C^s = \frac{\tilde{D}_H^s}{\mathbf{K_M R_M^s}}.$$

In order to settle these payments, each bank b transfers $\tilde{d}_H^s$ to other banks and receives an amount $d_{M_1}^{b,s} := \alpha_M^b \tilde{d}_H^s$ from other banks in the form of CB deposits. By summing over all banks $b \in [0,1]$ in the expression of banks' profits in Equation (1), we obtain $L_M R_L^s = D_H R_D^s + \Pi_B^s$, which means that $d_{M_1}^{b,s} = \alpha_M^b L_M R_L^s$. This transaction impacts CB deposits of Banks b_i and b_j as follows:

$$\begin{aligned} & d_{CB_8}^{b_i,s} := d_{CB_6}^{b_i,s} - \tilde{d}_H^s + d_{M_1}^{b_i,s} = \alpha_M^{b_i} L_M R_L^s + d_{CB}^{b_i} R_{CB}^s \\ \text{and} \quad & d_{CB_7}^{b_j,s} := d_{CB_5}^{b_j,s} - \tilde{d}_H^s + d_{M_1}^{b_j,s} = \alpha_M^{b_j} L_M R_L^s. \end{aligned}$$

The balance sheets of banks and households are given in Table 16.

[59]The household receives additional deposits from the banks' dividend payments.

Households		Bank b_i		Bank b_j	
$\mathbf{f(K_F)}$	$E_H R_H^s$	$d_{CB_8}^{b_i,s}$	$l_{CB_3}^{b_i,s}$	$d_{CB_7}^{b_j,s}$	$l_{CB_4}^{b_j,s}$
$\mathbf{K_M R_M^s}$		$l_M^{b_i} R_L^s$	$d_{M_1}^{b_i,s}$	$l_M^{b_j} R_L^s$	$d_{M_1}^{b_j,s}$

Table 16: Balance sheets at the end of Stage E, Substage 4.

Stage E, Substage 5: Repayment of Loans by Firms in MT

Firms in MT pay back their loans, and bank deposits are destroyed. The balance sheets of banks and households are given in Table 17.

Households		Bank b_i		Bank b_j	
$\mathbf{f(K_F)}$	$E_H R_H^s$	$d_{CB_8}^{b_i,s}$	$l_{CB_3}^{b_i,s}$	$d_{CB_7}^{b_j,s}$	$l_{CB_4}^{b_j,s}$
$\mathbf{K_M R_M^s}$					

Table 17: Balance sheets at the end of Stage E, Substage 5.

Stage E, Substage 6: Netting of CB Deposits and CB Loans

Banks net their CB deposits and CB loans. Using the expression of bank profits given by Equation (1), we obtain

$$\begin{aligned}
d_{CB_8}^{b_i,s} - l_{CB_3}^{b_i,s} &= \alpha_M^{b_i} L_M R_L^s + (1 - \alpha_M^{b_i}) L_M R_{CB}^s - \big((L_M - E_B) R_D^s + \Pi_B^{b_i,s}\big) = 0, \\
d_{CB_7}^{b_j,s} - l_{CB_4}^{b_j,s} &= \alpha_M^{b_j} L_M R_L^s - (\alpha_M^{b_j} - 1) L_M R_{CB}^s - \big((L_M - E_B) R_D^s + \Pi_B^{b_j,s}\big) = 0.
\end{aligned}$$

C Appendix – Net Positions of Banks against the CB after a Bail-out

In Table 6 the label $b_{i'}$ denotes banks with a non-negative net position against the CB. For completeness, the net position is given by

$$d^{b_{i'}}_{CBT} := \begin{cases} d^{b_{i'}}_{CB} R^s_{CB} - T^s & \text{if } d^{b_{i'}}_{CB} R^s_{CB} - T^s \geq 0 \text{ and } \Pi^{b_{i'},s}_B \geq 0, \\ d^{b_{i'}}_{CB} R^s_{CB} - T^s + t^{b_{i'},s} & \text{if } d^{b_{i'}}_{CB} R^s_{CB} - T^s + t^{b_{i'},s} \geq 0 \text{ and } \Pi^{b_{i'},s}_B < 0, \text{ and} \\ t^{b_{i'},s} - T^s - l^{b_{i'}}_{CB} R^s_{CB} & \text{if } l^{b_{i'}}_{CB} R^s_{CB} + T^s - t^{b_{i'},s} \leq 0 \text{ and } \Pi^{b_{i'},s}_B < 0, \end{cases}$$

where T^s are the tax payments introduced in Subsection 2.5.2 representing the households' deposit withdrawals to pay taxes in State $s = l, h$ and $t^{b_{i'},s}$, the possible bail-out in State $s = l, h$ if Bank $b_{i'}$ defaults against households. Similarly, the label $b_{j'}$ denotes banks with a negative net position against the CB:

$$l^{b_{j'}}_{CBT} := \begin{cases} l^{b_{j'}}_{CB} R^s_{CB} + T^s & \text{if } \Pi^{b_{j'},s}_B \geq 0, \\ T^s - d^{b_{j'}}_{CB} R^s_{CB} & \text{if } d^{b_{j'}}_{CB} R^s_{CB} - T^s < 0 \text{ and } \Pi^{b_{j'},s}_B \geq 0, \\ T^s - t^{b_{j'},s} - d^{b_{j'}}_{CB} R^s_{CB} & \text{if } d^{b_{j'}}_{CB} R^s_{CB} - T^s + t^{b_{j'},s} < 0 \text{ and } \Pi^{b_{j'},s}_B < 0, \text{ and} \\ l^{b_{j'}}_{CB} R^s_{CB} + T^s - t^{b_{j'},s} & \text{if } l^{b_{j'}}_{CB} R^s_{CB} + T^s - t^{b_{j'},s} > 0 \text{ and } \Pi^{b_{j'},s}_B < 0. \end{cases}$$

D Appendix – Interbank Borrowing and Lending

In Appendix D we describe how banks settle payments between agents and how banks can borrow and lend to each other, thereby creating bank assets and liabilities. Ultimately, we will be able to investigate the implications of this process for the gross rates of return on private and CB deposits in equilibrium. For ease of presentation, we omit the superscript s as the same considerations hold for both states of the world.

We use an example with two banks, b_j and b_i. Assume that Bank b_i grants a loan to Bank b_j. Then four entries in the balance sheets are created, as shown in Table 18.

<table>
<tr><th colspan="2">Bank b_j</th><th colspan="2">Bank b_i</th></tr>
<tr><td>D_j</td><td>L_i</td><td>L_i</td><td>D_j</td></tr>
</table>

Table 18: Balance sheets representing interbank lending and borrowing (1/4).

L_i represents the amount of loans granted by Bank b_i to Bank b_j, and D_j the amount of deposits held by Bank b_j at Bank b_i. We have assumed a competitive interbank market with a single gross rate of return for lending and borrowing. Since banks cannot discriminate between deposits owned by households and deposits owned by other banks, the corresponding gross rates are both equal to R_D.

We next investigate the relationship between R_{CB} and R_D. Assume first that some buyers pay with their deposits at Bank b_j and that the sellers deposit the money at Bank b_i. In order to settle the transfer, Bank b_j has two options. If $R_{CB} < R_D$, it will borrow from the CB and transfer CB deposits to Bank b_i. Suppose now that $R_{CB} > R_D$. Then Bank b_j directly becomes liable to Bank b_i. The buyers' deposits at Bank b_j are replaced by a loan Bank b_i grants to Bank b_j. This loan is an asset for Bank b_i that is matched by the liability corresponding to the new sellers' deposits. As assumed in Subsection 2.4.1, Bank b_i has the right to require Bank b_j to repay its liabilities with CB deposits, which Bank b_i will do as $R_{CB} > R_D$. The balance sheets at the end of the process look exactly the same, no matter whether or not Bank b_j became liable to Bank b_i in the first place. Therefore, independently of R_D, the refinancing gross rate is equal to R_{CB}. However, assuming that no bank participating in the interbank market makes any loss by doing so requires $R_D = R_{CB}$, which we show next.

Here we prove that $R_D = R_{CB}$. By contradiction, assume first that $R_D < R_{CB}$. Bank b_j, for example, would borrow from Bank b_i at the gross rate of return R_D and from the central bank at the gross rate of return R_{CB}, as shown in the balance sheets in Table 19.

Bank b_j	
D_j	L_i
D_{CB}	L_{CB}

Bank b_i	
L_i	D_j

Table 19: Balance sheets representing interbank lending and borrowing (2/4).

Using deposits at Bank b_i, Bank b_j can now repay CB liabilities. To carry out this payment, Bank b_i has to borrow from the central bank at the gross rate of return R_{CB}. The balance sheets are given in Table 20.

Bank b_j	
D_{CB}	L_i

Bank b_i	
L_i	L_{CB}

Table 20: Balance sheets representing interbank lending and borrowing (3/4).

Bank b_j would make positive profits from this operation, while Bank b_i would make losses. As we assumed that no bank participating in the interbank market makes any loss by doing so, $R_D < R_{CB}$ cannot occur in any equilibrium with banks.

Now assume that $R_{CB} < R_D$. Then Bank b_j would like to repay its liabilities against Bank b_i using CB deposits. This would result in the balance sheets given in Table 21.

Bank b_j	
D_j	L_{CB}

Bank b_i	
D_{CB}	D_j

Table 21: Balance sheets representing interbank lending and borrowing (4/4).

Bank b_j would make positive profits from this operation, while Bank b_i would make losses. As we assumed that no bank participating in the interbank market makes any loss by doing so, $R_D > R_{CB}$ cannot occur in any equilibrium with banks.[60]

[60] Otherwise, we could have constellations with $R_{CB} > R_D$ and an inactive interbank market, as no bank would lend in such a market. An active interbank market requires $R_D = R_{CB}$.

E Appendix – Households' Optimal Investment Choices

Lemma 6

The representative household's optimal portfolio choices are represented by three correspondences denoted by

$$\begin{aligned}
\hat{E}_B &: \mathbb{R}^7_{++} \times [0, \mathbf{W}] \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\}), \\
\hat{D}_H &: \mathbb{R}^7_{++} \times \mathbb{R}_+ \times [0, \mathbf{W}] \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\}), \\
\hat{S}_F &: \mathbb{R}^7_{++} \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\}),
\end{aligned}$$

and given by

$$\begin{aligned}
&\Big(\hat{E}_B\big((R^s_E)_s, (R^s_D)_s, p_I, (p^s_C)_s, S_F\big), \\
&\hat{D}_H\big((R^s_E)_s, (R^s_D)_s, p_I, (p^s_C)_s, E_B, S_F\big), \\
&\hat{S}_F\big((R^s_E)_s, (R^s_D)_s, p_I, (p^s_C)_s\big)\Big) = \\
&\begin{cases}
\big(\{0\}, \{0\}, \{\mathbf{W}\}\big) \\
\qquad \text{if } \max\left(\mathbb{E}\left[\frac{R^s_D}{p^s_C}\right], \mathbb{E}\left[\frac{R^s_E}{p^s_C}\right]\right) \leq \frac{\mathbf{f}'(\mathbf{W})}{p_I}, \\
\big(\{0\}, \{p_I \mathbf{W}\}, \{0\}\big) \\
\qquad \text{if } \max\left(\frac{\mathbf{f}'(\mathbf{0})}{p_I}, \mathbb{E}\left[\frac{R^s_E}{p^s_C}\right]\right) < \mathbb{E}\left[\frac{R^s_D}{p^s_C}\right], \\
\big(\{p_I \mathbf{W}\}, \{0\}, \{0\}\big) \\
\qquad \text{if } \max\left(\frac{\mathbf{f}'(\mathbf{0})}{p_I}, \mathbb{E}\left[\frac{R^s_D}{p^s_C}\right]\right) < \mathbb{E}\left[\frac{R^s_E}{p^s_C}\right], \\
\big([0, p_I \mathbf{W}], \{p_I \mathbf{W} - E_B\}, \{0\}\big) \\
\qquad \text{if } \frac{\mathbf{f}'(\mathbf{0})}{p_I} < \mathbb{E}\left[\frac{R^s_D}{p^s_C}\right] = \mathbb{E}\left[\frac{R^s_E}{p^s_C}\right], \\
\big(\{0\}, \{p_I (\mathbf{W} - S_F)\}, \{\mathbf{f}'^{-1}\left(p_I \mathbb{E}\left[\frac{R^s_D}{p^s_C}\right]\right)\}\big) \\
\qquad \text{if } \max\left(\frac{\mathbf{f}'(\mathbf{W})}{p_I}, \mathbb{E}\left[\frac{R^s_E}{p^s_C}\right]\right) < \mathbb{E}\left[\frac{R^s_D}{p^s_C}\right] \leq \frac{\mathbf{f}'(\mathbf{0})}{p_I}, \\
\big(\{p_I (\mathbf{W} - S_F)\}, \{0\}, \{\mathbf{f}'^{-1}\left(p_I \mathbb{E}\left[\frac{R^s_E}{p^s_C}\right]\right)\}\big) \\
\qquad \text{if } \max\left(\frac{\mathbf{f}'(\mathbf{W})}{p_I}, \mathbb{E}\left[\frac{R^s_D}{p^s_C}\right]\right) < \mathbb{E}\left[\frac{R^s_E}{p^s_C}\right] \leq \frac{\mathbf{f}'(\mathbf{0})}{p_I}, \\
\big([0, p_I (\mathbf{W} - S_F)], \{p_I (\mathbf{W} - S_F) - E_B\}, \{\mathbf{f}'^{-1}\left(p_I \mathbb{E}\left[\frac{R^s_D}{p^s_C}\right]\right)\}\big) \\
\qquad \text{if } \frac{\mathbf{f}'(\mathbf{W})}{p_I} < \mathbb{E}\left[\frac{R^s_E}{p^s_C}\right] = \mathbb{E}\left[\frac{R^s_D}{p^s_C}\right] \leq \frac{\mathbf{f}'(\mathbf{0})}{p_I}.
\end{cases}
\end{aligned} \tag{13}$$

The proof of Lemma 6 is given in Appendix G.

F Appendix – Optimal Choice of Money Creation by Banks with Capital Regulation

Lemma 7

Suppose that banks have to comply with a minimum equity ratio φ^{reg} at the end of Period $t = 0$. If $R_D^s = R_{CB}^s$ in all states $s = l, h$, the privately optimal amounts of money creation and lending by an individual bank are represented by a correspondence denoted by $\hat{\alpha}_M^{reg} : \mathbb{R}_{++}^4 \times [\varphi^{reg}, 1) \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\})$ and given by

$$
\hat{\alpha}_M^{reg}\big((R_L^s)_s, (R_{CB}^s)_s, \varphi\big) =
$$

$$
\begin{cases}
\{\frac{\varphi}{\varphi^{reg}}\} & \text{if } (R_L^s \geq R_{CB}^s \text{ for all states } s = l, h \\
& \text{with at least one strict inequality) or} \\
& \text{if } (\overline{R}_L > \overline{R}_{CB},\ R_L^l < R_{CB}^l,\ R_{CB}^h < R_L^h, \text{ and } \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DCB}^l) \text{ or} \\
& \text{if } (\overline{R}_L > \overline{R}_{CB},\ R_L^h < R_{CB}^h,\ R_{CB}^l < R_L^l, \text{ and } \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DCB}^h) \text{ or} \\
& \text{if } (\overline{R}_L = \overline{R}_{CB},\ R_L^l < R_{CB}^l,\ R_{CB}^h < R_L^h, \text{ and } \alpha_{DH}^l < \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DCB}^l) \text{ or} \\
& \text{if } (\overline{R}_L = \overline{R}_{CB},\ R_L^h < R_{CB}^h,\ R_{CB}^l < R_L^l, \text{ and } \alpha_{DH}^h < \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DCB}^h) \text{ or} \\
& \text{if } (\overline{R}_L < \overline{R}_{CB},\ R_L^l < R_{CB}^l,\ R_{CB}^h < R_L^h,\ \alpha_{DH}^l < \frac{\varphi}{\varphi^{reg}} < \alpha_{DCB}^l, \\
& \text{and } \varphi^{reg} < \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l}) \text{ or} \\
& \text{if } (\overline{R}_L < \overline{R}_{CB},\ R_L^h < R_{CB}^h,\ R_{CB}^l < R_L^l,\ \alpha_{DH}^h < \frac{\varphi}{\varphi^{reg}} < \alpha_{DCB}^h, \\
& \text{and } \varphi^{reg} < \frac{1-\sigma}{\sigma}\frac{R_L^l - R_{CB}^l}{R_{CB}^h}), \\
\{\alpha_{DCB}^l\} & \text{if } (\overline{R}_L < \overline{R}_{CB},\ R_L^l < R_{CB}^l,\ R_{CB}^h < R_L^h,\ \alpha_{DCB}^l \leq \frac{\varphi}{\varphi^{reg}}, \\
& \text{and } \varphi < \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}), \text{ or} \\
& \text{if } (\overline{R}_L \geq \overline{R}_{CB},\ R_L^l < R_{CB}^l,\ R_{CB}^h < R_L^h, \text{ and } \alpha_{DCB}^l \leq \frac{\varphi}{\varphi^{reg}}), \\
\{\alpha_{DCB}^h\} & \text{if } (\overline{R}_L < \overline{R}_{CB},\ R_L^h < R_{CB}^h,\ R_{CB}^l < R_L^l,\ \alpha_{DCB}^h \leq \frac{\varphi}{\varphi^{reg}}, \\
& \text{and } \varphi < \frac{1-\sigma}{\sigma}\frac{R_L^l - R_{CB}^l}{R_{CB}^h - R_L^h}), \text{ or} \\
& \text{if } (\overline{R}_L \geq \overline{R}_{CB},\ R_L^h < R_{CB}^h,\ R_{CB}^l < R_L^l, \text{ and } \alpha_{DCB}^h \leq \frac{\varphi}{\varphi^{reg}}), \\
\left[0, \frac{\varphi}{\varphi^{reg}}\right] & \text{if } (R_L^s = R_{CB}^s \text{ for all states } s = l, h) \text{ or} \\
& \text{if } (\overline{R}_L = \overline{R}_{CB},\ R_L^l < R_{CB}^l,\ R_{CB}^h < R_L^h, \text{ and } \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DH}^l) \text{ or} \\
& \text{if } (\overline{R}_L = \overline{R}_{CB},\ R_L^h < R_{CB}^h,\ R_{CB}^l < R_L^l, \text{ and } \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DH}^h),
\end{cases}
$$

$$
\hat{\alpha}_M^{reg}\big((R_L^s)_s,(R_{CB}^s)_s,\varphi\big) =
\begin{cases}
\{0,\frac{\varphi}{\varphi^{reg}}\} & \textit{if } (\overline{R}_L < \overline{R}_{CB},\, R_L^l < R_{CB}^l,\, R_{CB}^h < R_L^h,\, \alpha_{DH}^l < \frac{\varphi}{\varphi^{reg}} < \alpha_{DCB}^l, \\
& \textit{and } \varphi^{reg} = \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l}) \textit{ or} \\
& \textit{if } (\overline{R}_L < \overline{R}_{CB},\, R_L^h < R_{CB}^h,\, R_{CB}^l < R_L^l,\, \alpha_{DH}^h < \frac{\varphi}{\varphi^{reg}} < \alpha_{DCB}^h, \\
& \textit{and } \varphi^{reg} = \frac{1-\sigma}{\sigma}\frac{R_L^l - R_{CB}^l}{R_{CB}^h}), \\
\{0,\alpha_{DCB}^l\} & \textit{if } (\overline{R}_L < \overline{R}_{CB},\, R_L^l < R_{CB}^l,\, R_{CB}^h < R_L^h,\, \alpha_{DCB}^l \leq \frac{\varphi}{\varphi^{reg}}, \\
& \textit{and } \varphi = \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}), \\
\{0,\alpha_{DCB}^h\} & \textit{if } (\overline{R}_L < \overline{R}_{CB},\, R_L^h < R_{CB}^h,\, R_{CB}^l < R_L^l,\, \alpha_{DCB}^h \leq \frac{\varphi}{\varphi^{reg}}, \\
& \textit{and } \varphi = \frac{1-\sigma}{\sigma}\frac{R_L^l - R_{CB}^l}{R_{CB}^h - R_L^h}), \\
\{0\} & \textit{if } (R_L^s \leq R_{CB}^s \textit{ for all states } s = l,h \\
& \textit{with at least one strict inequality) or} \\
& \textit{if } (\overline{R}_L < \overline{R}_{CB},\, R_L^l < R_{CB}^l,\, R_{CB}^h < R_L^h,\, \alpha_{DCB}^l \leq \frac{\varphi}{\varphi^{reg}}, \\
& \textit{and } \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l} < \varphi) \textit{ or} \\
& \textit{if } (\overline{R}_L < \overline{R}_{CB},\, R_L^h < R_{CB}^h,\, R_{CB}^l < R_L^l,\, \alpha_{DCB}^h \leq \frac{\varphi}{\varphi^{reg}}, \\
& \textit{and } \frac{1-\sigma}{\sigma}\frac{R_L^l - R_{CB}^l}{R_{CB}^h - R_L^h} < \varphi) \textit{ or} \\
& \textit{if } (\overline{R}_L < \overline{R}_{CB},\, R_L^l < R_{CB}^l,\, R_{CB}^h < R_L^h,\, \alpha_{DH}^l < \frac{\varphi}{\varphi^{reg}} < \alpha_{DCB}^l, \\
& \textit{and } \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l} < \varphi^{reg}) \textit{ or} \\
& \textit{if } (\overline{R}_L < \overline{R}_{CB},\, R_L^h < R_{CB}^h,\, R_{CB}^l < R_L^l,\, \alpha_{DH}^h < \frac{\varphi}{\varphi^{reg}} < \alpha_{DCB}^h, \\
& \textit{and } \frac{1-\sigma}{\sigma}\frac{R_L^l - R_{CB}^l}{R_{CB}^h} < \varphi^{reg}) \textit{ or} \\
& \textit{if } (\overline{R}_L < \overline{R}_{CB},\, R_L^l < R_{CB}^l,\, R_{CB}^h < R_L^h,\, \textit{and } \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DH}^l) \textit{ or} \\
& \textit{if } (\overline{R}_L < \overline{R}_{CB},\, R_L^h < R_{CB}^h,\, R_{CB}^l < R_L^l,\, \textit{and } \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DH}^h).
\end{cases}
$$

The proof of Lemma 7 is given in Appendix G.

G Appendix – Proofs

Proof of Lemma 2

As set out in Subsection 2.4.1, banks can lend to, and borrow from, each other at the gross rates $(R_D^{s*})_s$ contingently on State s. Similarly, as explained in Subsection 2.4.2, they can also borrow from, or deposit at, the central bank at the policy gross rates $(R_{CB}^s)_s$ contingently on State s. Suppose now, by contradiction, that $R_D^{s*} \neq R_{CB}^s$ for some state s. If $R_D^{s*} < R_{CB}^s$, all banks would like to become liable to other banks and use the money obtained to hold assets against the central bank, contingently on State s. Similarly, if $R_D^{s*} > R_{CB}^s$, all banks would like to become liable to the central bank and use the money obtained to hold assets against other banks, contingently on State s. As we assumed that no bank participating in the interbank market makes any loss by doing so, both cases cannot hold in an equilibrium with banks.[61] $\square$

Proof of Proposition 1

Let $b \in [0,1]$ denote a bank. As $R_D^s = R_{CB}^s$ in all states $s = l, h$ by Lemma 2, the expected shareholders' value of Bank b is given by

$$\mathbb{E}[\max(\alpha_M^b L_M (R_L^s - R_{CB}^s) + E_B R_D^s, 0)].$$

Suppose that $\overline{R}_L < \overline{R}_{CB}$.

- Suppose first that $R_L^s \leq R_{CB}^s$ for all states $s = l, h$ with at least one strict inequality. In this case, Bank b's expected shareholders' value is decreasing in the volume of loans. Therefore, its choice is $\alpha_M^b = 0$.
- Suppose now that $R_L^l < R_{CB}^l$ and $R_{CB}^h < R_L^h$. For these constellations Figure 4 depicts three typical cases representing the expected gross rate of return on equity as a function of α_M^b. The three different cases are given by the comparison between the capital ratio φ and $\frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}$.

  For $\alpha_M^b \leq \alpha_{DH}^l$, Bank b does not default on depositors, and its expected shareholders' value is decreasing with α_M^b, as illustrated in Figure 4. However, for $\alpha_{DH}^l < \alpha_M^b$, Bank b defaults on depositors in the bad state. Then Bank b can further increase expected shareholders' value by granting more loans, as illustrated in Figure 4. The reason is that shareholders are protected by limited liability and due to depositors' bail-out by the government, the deposit gross rate of return of Bank b received by households in the bad state is R_D^l.

[61] The mechanisms by which banks become liable to other banks or hold assets against them are explained in detail in Appendix D.

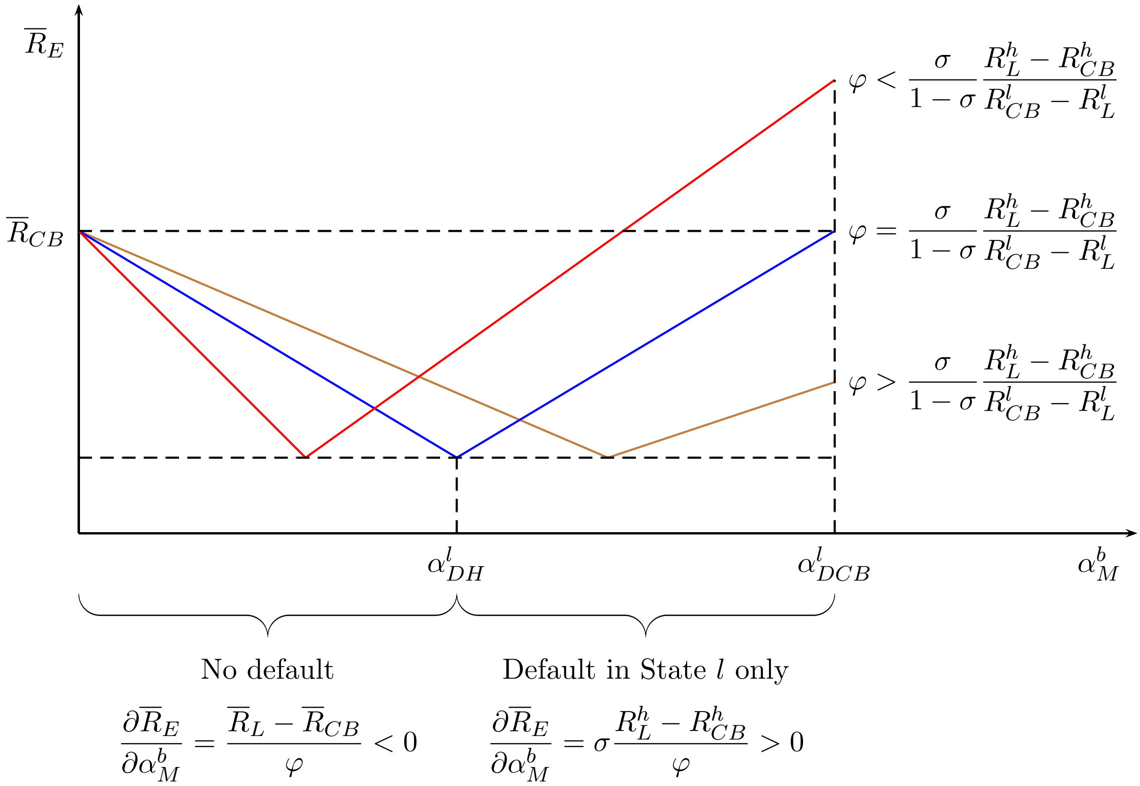

Figure 4: Expected gross rate of return on equity of Bank b as a function of α_M^b when $\overline{R}_L < \overline{R}_{CB}$ and $R_{CB}^h < R_L^h$ for three typical relationships between the capital ratio φ and $\frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}$. The corresponding areas of default and no default are depicted for $\varphi = \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}$, including the critical value α_{DH}^l.

However, money creation levels $\alpha_M^b > \alpha_{DCB}^l$ cannot be optimal for Bank b, as it would default on the CB and its banker would be subject to heavy penalties. Therefore, Bank b compares expected shareholders' value with $\alpha_M^b = 0$ given by

$$E_B \overline{R}_{CB}$$

and expected shareholders' value with $\alpha_M^b = \alpha_{DCB}^l$ given by

$$\sigma\left(\alpha_{DCB}^l L_M (R_L^h - R_{CB}^h) + E_B R_{CB}^h\right).$$

This comparison leads to the threshold of the equity ratio φ

$$\frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l},$$

below which Bank b chooses $\alpha_M^b = \alpha_{DCB}^l$ and above which it chooses $\alpha_M^b = 0$.

– Suppose now that $R_L^h < R_{CB}^h$ and $R_{CB}^l < R_L^l$. Analogously to the previous case,

$$\frac{1-\sigma}{\sigma}\frac{R_L^l - R_{CB}^l}{R_{CB}^h - R_L^h}$$

is the equity ratio below which Bank b chooses $\alpha_M^b = \alpha_{DCB}^h$ and above which it chooses $\alpha_M^b = 0$.

Suppose now that $\overline{R}_L = \overline{R}_{CB}$.

– Suppose first that $R_L^s = R_{CB}^s$ for all states $s = l, h$. In this case, Bank b cannot influence its expected shareholders' value by varying its amount of loans. Therefore, $[0, +\infty)$ constitutes the set of Bank b's optimal choices.

– Suppose now that $R_L^l < R_{CB}^l$ and $R_{CB}^h < R_L^h$. In this case, for $\alpha_M^b \leq \alpha_{DH}^l$, Bank b does not default on depositors, and its expected shareholders' value is constant and equal to $E_B\overline{R}_D$. However, for $\alpha_{DH}^l < \alpha_M^b$, Bank b defaults on depositors in the bad state. Then Bank b can further increase expected shareholders' value by granting more loans. The reason is that shareholders are protected by limited liability and due to depositors' bail-out by the government, the deposit gross rate of return of Bank b received by households in the bad state is R_D^l. However, levels of money creation $\alpha_M^b > \alpha_{DCB}^l$ cannot be optimal for Bank b, as it would default on the CB and its banker would be subject to heavy penalties. Therefore, Bank b chooses the highest level of lending for which it does not default on the CB. This means that Bank b chooses $\alpha_M^b = \alpha_{DCB}^l$.

– Suppose finally that $R_L^h < R_{CB}^h$ and $R_{CB}^l < R_L^l$. Analogously to the previous case, Bank b chooses $\alpha_M^b = \alpha_{DCB}^h$.

Suppose finally that $\overline{R}_L > \overline{R}_{CB}$.

– Suppose first that $R_{CB}^s \leq R_L^s$ for all states $s = l, h$ with at least one strict inequality. In this case, Bank b can increase expected shareholders' value by granting more loans. Accordingly, its choice is $\alpha_M^b = +\infty$.

– Suppose now that $R_L^l < R_{CB}^l$ and $R_{CB}^h < R_L^h$. In this case, for $\alpha_M^b \leq \alpha_{DH}^l$, Bank b does not default on depositors, and it can increase expected shareholders' value by increasing its lending level. However, for $\alpha_{DH}^l < \alpha_M^b$, Bank b defaults on depositors in the bad state. Then Bank b can further increase expected shareholders' value by granting more loans. The reason is that shareholders are protected by limited liability and due to depositors' bail-out by the government, the deposit gross rate of return of Bank b received by households in the bad state is R_D^l. However, levels of money creation $\alpha_M^b > \alpha_{DCB}^l$ cannot be optimal for Bank b, as it would default on the CB and its banker would be subject to heavy penalties. Therefore, Bank b chooses the highest level of lending for which it does not default on the CB. This means that Bank b chooses $\alpha_M^b = \alpha_{DCB}^l$.

– Suppose finally that $R_L^h < R_{CB}^h$ and $R_{CB}^l < R_L^l$. Analogously to the previous case, Bank b chooses $\alpha_M^b = \alpha_{DCB}^h$.

We can summarize the choices of lending levels by banks, given gross rates $(R_L^s)_s$, policy choices $(R_{CB}^s)_s$, and their equity ratio φ, with the correspondence $\hat{\alpha}_M\big((R_L^s)_s, (R_{CB}^s)_s, \varphi\big)$ given in the proposition. $\square$

Proof of Lemma 6

Suppose first that $\max\left(\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right], \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right) \leq \frac{\mathbf{f}'(\mathbf{W})}{p_I}$. Now we define the auxiliary function

$$g_1(S_F) := \mathbf{f}(\mathbf{W}) - \left(\mathbf{f}(S_F) + p_I(\mathbf{W} - S_F)\max\left(\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right], \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right)\right).$$

It is easy to verify that, for all $S_F \in [0, \mathbf{W})$, $g_1'(S_F) < 0$. Moreover, $g_1(\mathbf{W}) = 0$. Therefore, $g_1(S_F) > 0$ for all $S_F \in [0, \mathbf{W})$, which establishes the first case in Equation (13).

Suppose now that $\max\left(\frac{\mathbf{f}'(\mathbf{0})}{p_I}, \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right) < \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]$. Next we consider the function

$$g_2(S_F) := p_I\mathbf{W}\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] - \left(\mathbf{f}(S_F) + p_I(\mathbf{W} - S_F)\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right),$$

which shares similar properties to g_1: for all $S_F \in [0, \mathbf{W}]$, $g_2'(S_F) > 0$, $g_2(0) = 0$, and thus $g_2(S_F) > 0$ for all $S_F \in (0, \mathbf{W}]$. Accordingly, we can apply the same argument to g_2 as previously for g_1 and obtain the second case in Equation (13).

With similar arguments we also obtain the third and fourth cases.

Suppose finally that $\max\left(\frac{\mathbf{f}'(\mathbf{W})}{p_I}, \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right) < \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \leq \frac{\mathbf{f}'(\mathbf{0})}{p_I}$. Now we consider

$$
\begin{aligned}
g_3(S_F) :=& \mathbf{f}\left(\mathbf{f}'^{-1}\left(p_I\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)\right) + p_I\left(\mathbf{W} - \mathbf{f}'^{-1}\left(p_I\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)\right)\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \\
&- \left(\mathbf{f}(S_F) + p_I(\mathbf{W} - S_F)\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right).
\end{aligned}
$$

We observe that g_3 is strictly convex in S_F, $g_3'(0) = -\mathbf{f}'(\mathbf{0}) + p_I\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \leq 0$, and $g_3'(\mathbf{W}) = -\mathbf{f}'(\mathbf{W}) + p_I\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] > 0$. Hence, on $[\mathbf{0}, \mathbf{W}]$, g_3 takes the minimum at $S_F = \mathbf{f}'^{-1}\left(p_I\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)$, and it holds that $g_3\left(\mathbf{f}'^{-1}\left(p_I\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)\right) = 0$. Therefore, $g_3(S_F) > 0$ for all $S_F \neq \mathbf{f}'^{-1}\left(p_I\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)$, which proves the fifth case in Equation (13). With similar arguments we also obtain the last two cases. $\square$

Proof of Lemma 4

Demands for the investment good by firms in MT and FT are directly derived from the following shareholders' value-maximization problems:

$$
\begin{aligned}
&\max_{\mathbf{K_M}\in[\mathbf{0},\mathbf{W}]}\{\mathbb{E}[\max(\mathbf{K_M}(\mathbf{R^s_M}p_C^s - R_L^s p_I), 0)]\} \\
&\text{s.t. } \mathbf{R^s_M}p_C^s = R_L^s p_I \text{ for all states } s = l, h \\
\text{and} \quad &\max_{\mathbf{K_F}\in[\mathbf{0},\mathbf{W}]}\{\mathbb{E}[\max((\mathbf{f}(\mathbf{K_F}) - \mathbf{K_F}\mathbf{R_F})p_C^s, 0)]\}.
\end{aligned}
$$

$\square$

Proof of Theorem 1

Let $\mathcal{E}^*$ be an equilibrium with banks.

Then all banks choose the same level of money creation and lending denoted by α_M^*. At the aggregate level, however, the amount borrowed by banks from the CB has to equal the amount deposited by banks at the CB, meaning that $\int_0^1 \alpha_M^b \mathrm{d}b = 1$, which translates into $\alpha_M^* = 1$. The result of Lemma 2 implies that we can apply Proposition 1. Thus, given gross rates of return $(R_L^{s*})_s$, policy choices $(R_{CB}^s)_s$, and the equity ratio φ^*, all banks $b \in [0, 1]$ choose a lending level $\alpha_M^b \in \hat{\alpha}_M\big((R_L^{s*})_s, (R_{CB}^s)_s, \varphi^*\big)$, as given in Proposition 1. The only gross rates of return in Proposition 1 rationalizing $\alpha_M^* = 1$ are

$$R_L^{s*} = R_{CB}^s$$

for all states $s = l, h$. A direct consequence of this relation, Lemma 2, and the expression of profits in Equation (1) is that

$$R_E^{s*} = R_D^{s*} = R_L^{s*} = R_{CB}^s \tag{14}$$

for all states $s = l, h$. Moreover, due to Lemma 2 and the tie-breaking rule introduced in Subsection 2.4.1, the interbank lending market is not used in an equilibrium with banks. Finally, $\Pi_M^{s*} = 0$ for all states $s = l, h$ (see Subsection 2.5.2), which translates into

$$\mathbf{R_M^s} p_C^{s*} = R_L^{s*} p_I^*$$

for all states $s = l, h$. Given gross rates of return $(R_E^{s*})_s$ and $(R_D^{s*})_s$ as well as prices p_I^* and $(p_C^{s*})_s$, households choose $E_B^* \in \hat{E}_B\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, S_F^*\big)$ given S_F^*, $D_H^* \in \hat{D}_H\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, E_B^*, S_F^*\big)$ given E_B^* and S_F^*, and $S_F^* \in \hat{S}_F\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s\big)$. These correspondences are given in Lemma 6 in Appendix E. Only the first, the fourth, and the seventh cases of the definition of the correspondences $\hat{E}_B$, $\hat{D}_H$, and $\hat{S}_F$ correspond to equal nominal gross rates of return R_E^{s*} and R_D^{s*} and are hence consistent with the equality of nominal gross rates of return in Equation (14). However, the assumption $\mathbf{f}'(\mathbf{W}) < \overline{\mathbf{R}}_\mathbf{M} < \mathbf{f}'(\mathbf{0})$ plus $\mathbf{R_M^s} p_C^{s*} = R_L^{s*} p_I^*$ rule out the first and fourth cases. As in an equilibrium with banks $E_B^*, D_H^* > 0$, we thus obtain

$$\begin{aligned}
&E_B^* \in (0, p_I^* \left(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})\right)),\\
&D_H^* = p_I^* \left(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})\right) - E_B^*, \text{ and}\\
&S_F^* = \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M}).
\end{aligned}$$

Finally, $\mathbf{R}_\mathbf{F}^*$ can be determined by using Lemma 4 and equating the demand for the investment good $\mathbf{K}_\mathbf{F}^*$ to its supply S_F^*. With the help of the equity ratio φ^*, we can then rewrite all equilibrium variables as given in Theorem 1.

In turn, it is straightforward to verify that the tuples given in Theorem 1 constitute equilibria with banks as defined in Subsection 2.6. □

Proof of Lemma 5

Let $\mathcal{E}^*$ be an equilibrium with banks for which a minimum equity ratio φ^{reg} is imposed on banks at the end of Period $t = 0$. If $\alpha^b_M \geq 1$ for some bank $b \in [0, 1]$, the minimum equity ratio imposes the following constraint on money creation α^b_M:

$$\begin{aligned}&\frac{E^*_B}{\alpha^b_M L^*_M} \geq \varphi^{reg}, \quad \text{or equivalently}\\ &\alpha^b_M \leq \frac{\varphi^*}{\varphi^{reg}}.\end{aligned}$$

If $\alpha^b_M \leq 1$, the previous constraint becomes

$$\begin{aligned}&\frac{E^*_B}{L^*_M} \geq \varphi^{reg}, \quad \text{or equivalently}\\ &\varphi^* \geq \varphi^{reg}.\end{aligned}$$

□

Proof of Lemma 7

Let $b \in [0, 1]$ denote a bank and assume that a minimum equity ratio $\varphi^{reg} \leq \varphi$ is imposed on banks at the end of Period $t = 0$. Using Lemma 5 and the property $R^s_D = R^s_{CB}$ for all states $s = l, h$, Bank b's maximization problem simplifies to

$$\max_{\alpha^b_M \in \left[0, \frac{\varphi}{\varphi^{reg}}\right]} \left\{\mathbb{E}[\max(\alpha^b_M L_M(R^s_L - R^s_{CB}) + E_B R^s_{CB}, 0)]\right\}.$$

As the arguments used in this proof to investigate the impact of lending on shareholders' value are similar to the ones given in the proof of Proposition 1, we refer readers to the proof of Proposition 1 for further details.

Suppose that $\overline{R}_L < \overline{R}_{CB}$.

- Suppose first that $R^s_L \leq R^s_{CB}$ for all states $s = l, h$ with at least one strict inequality. In this case, expected shareholders' value of Bank b is decreasing in the volume of loans. Therefore, its choice is $\alpha^b_M = 0$.
- Suppose now that $R^l_L < R^l_{CB}$ and $R^h_{CB} < R^h_L$.
  - Suppose first that $\alpha^l_{DCB} \leq \frac{\varphi}{\varphi^{reg}}$. Then the equity ratio requirement does not impose an additional constraint on Bank b, and its optimal choice

of money creation is

$$
\begin{aligned}
&\alpha_M^b = 0 && \text{if } \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l} < \varphi,\\
&\alpha_M^b \in \{0, \alpha_{DCB}^l\} && \text{if } \varphi = \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l},\\
\text{and} \quad &\alpha_M^b = \alpha_{DCB}^l && \text{if } \varphi < \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}.
\end{aligned}
$$

- Suppose now that $\alpha_{DCB}^l > \frac{\varphi}{\varphi^{reg}}$. Then either $\alpha_{DH}^l < \frac{\varphi}{\varphi^{reg}}$ and expected shareholders' value of Bank b is decreasing for $\alpha_M^b \in [0, \alpha_{DH}^l]$ and increasing for $\alpha_M^b \in [\alpha_{DH}^l, \frac{\varphi}{\varphi^{reg}}]$, or $\alpha_{DH}^l \geq \frac{\varphi}{\varphi^{reg}}$ and expected shareholders' value is decreasing for $\alpha_M^b \in [0, \frac{\varphi}{\varphi^{reg}}]$. Therefore, if $\alpha_{DH}^l \geq \frac{\varphi}{\varphi^{reg}}$, the choice of Bank b is $\alpha_M^b = 0$. Suppose that $\alpha_{DH}^l < \frac{\varphi}{\varphi^{reg}}$. Then the choice of Bank b can be derived by comparison between expected shareholders' value for $\alpha_M^b = 0$ and for $\alpha_M^b = \frac{\varphi}{\varphi^{reg}}$. Using the expression for profits in Equation (1) and rearranging terms establishes that the choice for Bank b is

$$
\begin{aligned}
&\alpha_M^b = 0 && \text{if } \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l} < \varphi^{reg},\\
&\alpha_M^b \in \{0, \tfrac{\varphi}{\varphi^{reg}}\} && \text{if } \varphi^{reg} = \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l},\\
\text{and} \quad &\alpha_M^b = \tfrac{\varphi}{\varphi^{reg}} && \text{if } \varphi^{reg} < \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l}.
\end{aligned}
$$

- The analysis for $R_L^h < R_{CB}^h$ and $R_{CB}^l < R_L^l$ is similar to the previous one.

Suppose now that $\overline{R}_L = \overline{R}_{CB}$.

- Suppose first that $R_L^s = R_{CB}^s$ for all states $s = l, h$. Then $[0, \frac{\varphi}{\varphi^{reg}}]$ constitutes the set of Bank b's optimal choices.
- Suppose now that $R_L^l < R_{CB}^l$ and $R_{CB}^h < R_L^h$.
  - Suppose now that $\alpha_{DH}^l < \frac{\varphi}{\varphi^{reg}}$. Then the expected shareholders' value of Bank b is constant for all $\alpha_M^b \in [0, \alpha_{DH}^l]$ and increases with α_M^b in the interval $[\alpha_{DH}^l, \frac{\varphi}{\varphi^{reg}}]$. Therefore, Bank b chooses $\alpha_M^b = \min(\alpha_{DCB}^l, \frac{\varphi}{\varphi^{reg}})$.
  - Suppose now that $\alpha_{DH}^l \geq \frac{\varphi}{\varphi^{reg}}$. Then Bank b's expected shareholders' value is constant for all $\alpha_M^b \in [0, \frac{\varphi}{\varphi^{reg}}]$. Therefore, $[0, \frac{\varphi}{\varphi^{reg}}]$ constitutes the set of Bank b's optimal choices.

– The analysis for $R_L^h < R_{CB}^h$ and $R_{CB}^l < R_L^l$ is similar to the previous case.

Suppose finally that $\overline{R}_L > \overline{R}_{CB}$.

– Suppose first that $R_L^s \geq R_{CB}^s$ for all states $s = l, h$ with at least one strict inequality. In this case, Bank b can increase expected shareholders' value by granting more loans. Therefore, its choice is $\alpha_M^b = \frac{\varphi}{\varphi^{reg}}$.

– Suppose now that $R_L^l < R_{CB}^l$ and $R_{CB}^h < R_L^h$. In this case, Bank b can increase expected shareholders' value by granting more loans. Therefore, its choice is $\alpha_M^b = \min(\alpha_{DCB}^l, \frac{\varphi}{\varphi^{reg}})$.

– The analysis for $R_L^h < R_{CB}^h$ and $R_{CB}^l < R_L^l$ is similar to the previous case.

We can summarize our findings with the correspondence $\hat{\alpha}_M^{reg}$ given in the lemma.

□

Proof of Proposition 3

Let $\mathcal{E}^*$ be an equilibrium with banks for which a minimum equity ratio φ^{reg} is required to be held by banks at the end of Period $t = 0$. We first note that a direct consequence is that $\varphi^* \in [\varphi^{reg}, 1)$.

Then all banks choose the same level of money creation and lending denoted by α_M^*. At the aggregate level, however, the amount borrowed by banks from the CB has to equal the amount deposited by banks at the CB, meaning that $\int_0^1 \alpha_M^b \mathrm{d}b = 1$, which translates into $\alpha_M^* = 1$. The result of Lemma 2 implies that we can apply Lemma 7. Thus, given gross rates of return $(R_L^{s*})_s$, policy choices $(R_{CB}^s)_s$, and the equity ratio φ^*, all banks $b \in [0, 1]$ choose a lending level $\alpha_M^b \in \hat{\alpha}_M^{reg}\big((R_L^{s*})_s, (R_{CB}^s)_s, \varphi^*\big)$ as given in Lemma 7. Therefore, the only gross rates of return and capital structure

φ^* in Lemma 7 in Appendix F rationalizing $\alpha_M^* = 1$ are such that

either Case a) $(R_L^{s*} = R_{CB}^s$ for all states $s = l, h)$,
or Case b) $(\overline{R}_L^* = \overline{R}_{CB}$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, and $\alpha_{DH}^l \geq \frac{\varphi^*}{\varphi^{reg}})$,
or Case c) $(\overline{R}_L^* = \overline{R}_{CB}$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, and $\alpha_{DH}^h \geq \frac{\varphi^*}{\varphi^{reg}})$,
or Case d) $(\overline{R}_L^* < \overline{R}_{CB}$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, $\alpha_{DH}^l < 1$,
and $\varphi^* = \varphi^{reg} = \frac{\sigma}{1-\sigma}\frac{R_L^{h*} - R_{CB}^h}{R_{CB}^l})$,
or Case e) $(\overline{R}_L^* < \overline{R}_{CB}$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, $\alpha_{DH}^h < 1$,
and $\varphi^* = \varphi^{reg} = \frac{1-\sigma}{\sigma}\frac{R_L^{l*} - R_{CB}^l}{R_{CB}^h})$,
or Case f) $(R_L^{s*} \geq R_{CB}^s$ for all states $s = l, h$ with at least one strict
inequality, and $\varphi^* = \varphi^{reg})$,
or Case g) $(\overline{R}_L^* = \overline{R}_{CB}$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, $\alpha_{DH}^l < 1$, and $\varphi^* = \varphi^{reg})$,
or Case h) $(\overline{R}_L^* = \overline{R}_{CB}$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, $\alpha_{DH}^h < 1$, and $\varphi^* = \varphi^{reg})$,
or Case i) $(\overline{R}_L^* < \overline{R}_{CB}$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, $\alpha_{DH}^l < 1$,
and $\varphi^* = \varphi^{reg} < \frac{\sigma}{1-\sigma}\frac{R_L^{h*} - R_{CB}^h}{R_{CB}^l})$,
or Case j) $(\overline{R}_L^* < \overline{R}_{CB}$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, $\alpha_{DH}^h < 1$,
and $\varphi^* = \varphi^{reg} < \frac{1-\sigma}{\sigma}\frac{R_L^{l*} - R_{CB}^l}{R_{CB}^h})$,
or Case k) $(\overline{R}_L^* > \overline{R}_{CB}$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, and $\varphi^* = \varphi^{reg})$,
or Case l) $(\overline{R}_L^* > \overline{R}_{CB}$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, and $\varphi^* = \varphi^{reg})$.

Note first that in Cases f) to l), the expected gross rate of return on equity achieved by any bank b when choosing $\alpha_M^b = 1$ is higher than the expected gross rate of return on equity when choosing $\alpha_M^b = 0$. Since the latter is equal to the expected deposit gross rate, we can conclude that in all cases f) to l) the expected gross rate of return on equity is larger than the expected deposit gross rate. Moreover, for Cases a) to e), the expected gross rate of return on equity is equal to the expected deposit gross rate.

Given gross rates of return $(R_E^{s*})_s$ and $(R_D^{s*})_s$ as well as prices $p_I^* = p_C^* = 1$, households choose $E_B^* \in \hat{E}_B\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^* = 1, p_C^* = 1, S_F^*\big)$ given S_F^*, $D_H^* \in \hat{D}_H\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^* = 1, p_C^* = 1, E_B^*, S_F^*\big)$ given E_B^* and S_F^*, and $S_F^* \in \hat{S}_F\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^* = 1, p_C^* = 1\big)$. These correspondences are given in Lemma 6 in Appendix E.

In Cases f) to l), Lemma 6 implies that $D_H^* = 0$, which is excluded from the definition of an equilibrium with banks. Therefore, Cases f) to l) do not correspond to possible equilibria with banks.

In Cases a) to e), expected gross rates of return $\overline{R}^*_E$ and $\overline{R}^*_D$ are equal, and only the first, the fourth, and the seventh cases of the definition of the correspondences $\hat{E}_B$, $\hat{D}_H$, and $\hat{S}_F$ in Appendix E are consistent with $\overline{R}^*_E = \overline{R}^*_D$.

In Cases a) to c), the assumption $\mathbf{f}'(\mathbf{W}) < \overline{\mathbf{R}}_\mathbf{M} < \mathbf{f}'(\mathbf{0})$ together with $\overline{\mathbf{R}}_\mathbf{M} = \overline{R}^*_E = \overline{R}^*_D$ rule out the first and fourth cases. As in an equilibrium with banks $E^*_B, D^*_H > 0$, we obtain

$$\begin{aligned}
&E^*_B \in (0, \big(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})\big)),\\
&D^*_H = \big(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})\big) - E^*_B, \text{ and}\\
&S^*_F = \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M}).
\end{aligned}$$

In Cases d) and e), the assumption $\mathbf{f}'(\mathbf{W}) < \overline{\mathbf{R}}_\mathbf{M}$ together with $\overline{\mathbf{R}}_\mathbf{M} < \overline{R}^*_E = \overline{R}^*_D$ rule out the first case. As in an equilibrium with banks $E^*_B, D^*_H > 0$, we obtain

$$\begin{aligned}
&E^*_B \in (0, (\mathbf{W} - S^*_F)),\\
&D^*_H = (\mathbf{W} - S^*_F) - E^*_B,\\
&S^*_F = \begin{cases} \mathbf{f}'^{-1}\big(\overline{R}^*_{CB}\big) & \text{if } \mathbf{f}'(\mathbf{0}) \geq \overline{R}^*_{CB},\\ 0 & \text{otherwise.}\end{cases}
\end{aligned}$$

In turn, it is straightforward to verify that the tuples found in this proof constitute equilibria with banks as defined in Subsection 2.6. □

Proof of Corollary 6

Corollary 6 immediately results from Corollary 5 and from the observation that $\overline{R}_{CB} = \overline{\mathbf{R}}_\mathbf{M}$, $\mathbf{R}^\mathbf{l}_\mathbf{M} < 1 \leq \overline{\mathbf{R}}_\mathbf{M}$, and $R^s_{CB} \geq 1$ for all $s = l, h$ together imply that $R^h_{CB} > \mathbf{R}^\mathbf{h}_\mathbf{M}$.

□

Proof of Corollary 7

Suppose that there is an equilibrium with banks denoted by $\mathcal{E}^*$ for which the deposit gross rate of return in real terms is independent of the state of the world. We use $\mathbf{R}^*_\mathbf{D}$ to denote the deposit gross rate of return in terms of the consumption good:

$$\mathbf{R_D^*} = \frac{R_D^{l*}}{p_C^{l*}} = \frac{R_D^{h*}}{p_C^{h*}}.$$

Theorem 1 implies that

$$\mathbf{R_D^*} = \frac{\mathbf{R_M^s}}{p_I^*}$$

in all states $s = l, h$, which contradicts $\mathbf{R_M^l} < \mathbf{R_M^h}$. □

Proof of Proposition 5

Suppose that a minimum reserve requirement $r^{reg} \in (0,1)$ and a haircut rule $h \in (0,1)$ are imposed on each Bank b at the end of Period $t = 0$.

Then a Bank b_i has to borrow the amount $\max(0, r^{reg} d_H - d_{CB}^{b_i})$ of central bank money at the end of Period $t = 0$ to fulfill the reserve requirement r^{reg}. The maximum amount of reserves which Bank b_i can borrow from the central bank is given by $(1-h) l_M^{b_i}$.[62] Therefore, the following constraint holds in equilibrium at the end of Period $t = 0$:

$$\max(0, r^{reg} d_H - d_{CB}^{b_i}) \leq (1-h) l_M^{b_i},$$

which is equivalent to

$$0 \leq \alpha_M^{b_i} \leq \frac{1 - r^{reg}(1-\varphi)}{h},$$

where $\alpha_M^{b_i} \leq 1$.

Similarly, a Bank b_j has to borrow the amount $r^{reg} d_H$ of central bank money at the end of Period $t = 0$ to fulfill the reserve requirement r^{reg}. The maximum amount of reserves which Bank b_j can borrow from the central bank is given by $(1-h) l_M^{b_j}$. Therefore, the following constraint holds in equilibrium at the end of Period $t = 0$:

$$r^{reg} d_H + l_{CB}^{b_j} \leq (1-h) l_M^{b_j},$$

[62] Note that banks are indifferent between borrowing any lower reserve level as soon as it fulfills the reserve requirement, as the gross rate of return charged for CB liabilities is equal to the gross rate of return for holding CB deposits.

which is equivalent to

$$\alpha_M^{b_j} \leq \frac{1 - r^{reg}(1-\varphi)}{h},$$

where $\alpha_M^{b_j} \geq 1$. We note that for any Bank b, the constraint is given by

$$\alpha_M^{b} \leq \frac{1 - r^{reg}(1-\varphi)}{h}.$$

□

H Appendix – Example

We illustrate our results with an example. In this example we use the normalization $p_I^* = 1$, and we set households' portfolio choice to $\varphi^* = 0.4$. We use the parameter values given in Table 22.

$\mathbf{W}$	1
$(\mathbf{R_M^l}, \mathbf{R_M^h})$	$(0.98, 1.06)$
σ	0.5
$\mathbf{f(K_F)}$	$2(\mathbf{K_F} - \frac{\mathbf{K_F}^2}{2})$

Table 22: Parameter values.

We note that all assumptions on parameters and the function $\mathbf{f}$ are fulfilled, including Assumption 1. We now distinguish two cases:

- Either the central bank sets $(R_{CB}^l, R_{CB}^h) = (1.02, 1.02)$. Then we obtain the variable values given on the left side of Table 23.
- Or the central bank sets $(R_{CB}^l, R_{CB}^h) = (\mathbf{R_M^l}, \mathbf{R_M^h})$. Then we obtain the variable values given on the right side of Table 23.

In the case of price rigidities characterized by $p_C^{s*} = 1$ for $s = l, h$, the policy presented in Corollary 6 yields the following values:

$$R_{CB}^l = 1, \quad R_{CB}^h = 1.04, \quad \text{and} \quad \varphi^{reg} = 0.02.$$

(R_D^l, R_D^h) $= (R_L^l, R_L^h)$ $= (R_E^l, R_E^h)$	$(1.02, 1.02)$
$\mathbf{R_F}$	1.02
(p_C^l, p_C^h)	$(1.04, 0.96)$
$L_M = \mathbf{K_M}$	0.51
$S_F = \mathbf{K_F}$	0.49
D_H	0.31
E_B	0.20
$(\widetilde{D}_H^l, \widetilde{D}_H^h)$	$(0.52, 0.52)$
Π_M^s	0
(Π_F^l, Π_F^h)	$(0.25, 0.23)$
(Π_B^l, Π_B^h)	$(0.21, 0.21)$

(R_D^l, R_D^h) $= (R_L^l, R_L^h)$ $= (R_E^l, R_E^h)$	$(0.98, 1.06)$
$\mathbf{R_F}$	1.02
(p_C^l, p_C^h)	$(1.00, 1.00)$
$L_M = \mathbf{K_M}$	0.51
$S_F = \mathbf{K_F}$	0.49
D_H	0.31
E_B	0.20
$(\widetilde{D}_H^l, \widetilde{D}_H^h)$	$(0.50, 0.54)$
Π_M^s	0
(Π_F^l, Π_F^h)	$(0.24, 0.24)$
(Π_B^l, Π_B^h)	$(0.20, 0.22)$

Table 23: Variable values with policy gross rates $(R_{CB}^l, R_{CB}^h) = (1.02, 1.02)$ on the left side and $(R_{CB}^l, R_{CB}^h) = (0.98, 1.06)$ on the right side.

I Appendix – List of Notations

Symbol	***Meaning***
FT	Frictionless technology exhibiting decreasing marginal returns
MT	Moral hazard technology exhibiting constant returns to scale
CB	Central Bank
H	Representative household
t	Period $t = 0, 1$ of the economy
$\mathbf{W}$	Initial endowment of investment good
$\mathbf{K_F}$	Amount of investment good invested in FT
$\mathbf{K_F^{FB}}$	Socially efficient amount of investment good invested in FT
$\mathbf{\hat{K}_F}$	Correspondence matching the real gross rate of return $\mathbf{R_F}$ on bonds to FT firms' optimal demand for investment good
$\mathbf{K_M}$	Amount of investment good invested in MT
$\mathbf{\hat{K}_M}$	Correspondence giving the optimal demand for investment good by MT firms
p_I	Price of one unit of investment good
p_C^s	Price of one unit of consumption good in State s
$\mathbf{f}(\mathbf{K_F})$	Amount of consumption good produced by investing $\mathbf{K_F}$ in FT
$\mathbf{R_F}$	Gross rate of return on investment in FT in terms of the consumption good per unit of investment good
$\mathbf{R_D}$	Deposit gross rate of return on investment in FT in terms of the consumption good per unit of investment good in Subsection 5.7
$\mathbf{R_M}$	Real gross rate of return on investment in MT in terms of consumption good per unit of investment good
$\mathbf{R_M^s}$	Real gross rate of return on investment in MT in terms of consumption good per unit of investment good in State s
R_H^s	Nominal gross rate of return on households' assets in State s
R_{CB}^s	Nominal policy gross rate of return on CB deposits and CB loans in State s
R_D^s	Nominal gross rate of return on investment in deposits in State s

R_L	Nominal gross rate of return on bank loans granted to MT
R_L^s	Nominal gross rate of return on bank loans granted to MT in State s
R_E^s	Aggregate gross rate of return on bank equity in State s
$R_E^{b,s}$	Gross rate of return on equity of Bank b in State s
Π_B^b	Profits of Bank b
$\Pi_B^{b,s}$	Profits of Bank b in State s
$\Pi_B^{+,s}$	Aggregate profits of non-defaulting banks in State s
Π_F	Profits of firms in FT
Π_M	Profits of firms in MT
Π_F^s	Profits of firms in FT in State s
Π_M^s	Profits of firms in MT in State s
s, s'	State l or h of the world
s_D	In Subsection 5.3 smallest state with the smallest value $\alpha_{DCB}^{s_D}$
l	Bad state of the world
h	Good state of the world
σ	Probability that State $s = h$ occurs
σ^s	In Subsection 5.3 probability that State s occurs
$\overline{}$	Overline to denote the expected value of variables depending on the state of the world (for example, $\overline{\mathbf{R}}_{\mathbf{M}} = \mathbb{E}[\mathbf{R}_{\mathbf{M}}^{\mathbf{s}}]$)
*	Superscript denoting equilibrium variables
$\mathcal{P}(X)$	Power set of Set X
N	In Subsection 5.3 number of states of the world
$\mathcal{E}$	Tuple of variables used to define an equilibrium in Subsection 2.6
$\mathbf{c}$	Costs of monitoring in Subsection 5.5 in terms of the consumption good per unit of investment good
$\mathbf{R}_{\mathbf{M}}^{\mathbf{s}'}$	Real gross rate of return on investment in MT in terms of consumption good per unit of investment good in State s in Subsection 5.5
S_F	Amount of bonds purchased by households
$\hat{S}_F$	Correspondence matching gross rates of return $(R_E^s)_s$ and $(R_D^s)_s$ and prices p_I and $(p_C^s)_s$ to the optimal choices of households regarding investment in FT
E_H	Amount denominated in terms of the currency unit denoting households' equity

b	Label in $[0, 1]$ denoting a bank
δ^b	Net assets of Bank b against the CB if positive and liability against the CB if negative
$b_i \in B_I$	Variable denoting banks with assets against the CB at the end of Stage C, i.e. for which $\delta^i \geq 0$
$b_j \in B_J$	Variable denoting banks with liabilities against the CB at the end of Stage C, i.e. for which $\delta^j < 0$
e_B	Amount in terms of the currency unit invested by households in bank equity of an individual bank
E_B	Aggregate amount in terms of the currency unit invested in bank equity
$\hat{E}_B$	Correspondence matching gross rates of return $(R_E^s)_s$ and $(R_D^s)_s$, prices p_I and $(p_C^s)_s$, and investment S_F to the optimal choices of households regarding investment in bank equity
d_{H_1}	Interim amount of deposits held by households at an individual bank (also in the form of $\tilde{d}_H$)
D_{H_1}	Aggregate interim amount of deposits held by households (also in the form of $\tilde{D}_H$)
d_H	Amount in terms of the currency unit invested in deposits at an individual bank by households
D_H	Aggregate amount in terms of the currency unit invested in deposits by households
$\hat{D}_H$	Correspondence matching gross rates of return $(R_E^s)_s$ and $(R_D^s)_s$, prices p_I and $(p_C^s)_s$, and investments E_B and S_F to the optimal choices of households regarding investment in bank deposits
d_M^b	Amount in terms of the currency unit of deposits held by MT at Bank b
$d_{M_1}^{b,s}$	Amount in terms of the currency unit of deposits held by MT at Bank b in State s in Stage E, Substage 4
D_M	Aggregate amount in terms of the currency unit of deposits held by MT
l_M^b	Amount in terms of the currency unit invested by Bank b in MT
L_M	Aggregate amount in terms of the currency unit invested in MT
d_{CB}^b	Amount in terms of the currency unit borrowed by Bank b

	from the CB
l^b_{CB}	Amount in terms of the currency unit lent by the CB to Bank b
$d^b_{CB_1}, d^b_{CB_2}$	CB deposits of Bank b at different stages of the economy
$l^b_{CB_1}, l^b_{CB_2}$	Amount in terms of the currency unit borrowed by Bank b at different stages of the economy
$d^{b,s}_{CB_3}, ..., d^{b,s}_{CB_8}$	CB deposits of Bank b at different stages of the economy in State s
$l^{b,s}_{CB_3}, l^{b,s}_{CB_4}$	Amount in terms of the currency unit borrowed by Bank b at different stages of the economy in State s
D_{CB_1}	Aggregate amount of CB deposits in Stage C, Substage 1
d^b_{CBT}	CB deposits of Bank b after lump-sum taxation and bail-out
l^b_{CBT}	Bank b's liability against the CB after lump-sum taxation and bail-out
D_{CB}	In Appendix D deposits held by banks b_i or b_j at the CB
L_{CB}	In Appendix D debt due by bank b_i or b_j to the CB
D_i	In Appendix D amount of deposits held by bank b_i at bank b_j
L_j	In Appendix D amount of loans granted by bank b_j to bank b_i
α^b_M	Ratio of individual lending by Bank b to aggregate lending by banks, or equivalently, ratio of individual to average lending
$\hat{\alpha}_M$	Correspondence matching gross rates of return $(R^s_L)_s$ and $(R^s_{CB})_s$ and capital structure φ to the set of privately optimal levels of money creation by banks
$\hat{\alpha}^{reg}_M$	Correspondence matching gross rates of return $(R^s_L)_s$ and $(R^s_{CB})_s$ and capital structure φ to the set of privately optimal levels of money creation by banks when prices are perfectly rigid and a minimum equity ratio φ^{reg} has to be held at the end of Period $t = 0$
α^s_{DCB}	Threshold for α^b_M above which Bank b defaults on the CB in State s
α^s_{DH}	Threshold for α^b_M above which Bank b defaults on households in State s
φ	Share of households' deposits converted into equity in Stage C of Period 0

φ^b	Equity ratio of Bank b
φ^{reg}	Minimum equity ratio imposed by government authorities that has to be held by banks at the end of Period $t = 0$
θ	In Subsection 5.1 fraction of the balance sheet that a banker cannot pledge to investors
r^b	Reserve ratio of Bank b in Subsection 5.8
r^{reg}	Reserve requirement in Subsection 5.8
h	Haircut regulation in Subsection 5.8
Δd^b_{CB}	CB deposits used to fulfill the reserve requirement in Subsection 5.8
Δl^b_{CB}	Amount in terms of the currency unit lent by the CB to Bank b to fulfill the reserve requirement in Subsection 5.8
T^s	Aggregate tax burden borne by households if some bank defaults in State s
$t^{b,s}$	Tax households have to pay in order to bail out depositors at Bank b in State s
g_1, g_2, g_3	Auxiliary functions used in the proof of Lemma 6
A,B,C,D,E	Stages of economic activity