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Abstract

Many countries, in an effort to address the problem that too many retirees have too little saved up, impose mandatory contributions into retirement accounts, that too, in an age-independent manner. This is puzzling because such funded pension schemes effectively mandate the young, who wish to borrow, to save for retirement. Further, if agents are present-biased, they disagree with the intent of such schemes and attempt to undo them by reducing their own saving or even borrowing against retirement wealth. We establish a welfare case for mandating the middle-aged and the young to contribute to their retirement accounts, even with age-independent contribution rates. We find, somewhat counterintuitively, that even though the young responds by borrowing more that too at a rate higher than offered by pension savings, their life-time utility increases.

JEL-Codes: H550, D910, D030, E600.

Keywords: present-biased preferences, mandatory pensions, pension offsets, crowding out.

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1 Introduction

Most individuals, on their own, do not save adequately for their retirement (Diamond, 1977; Poterba, 2014). Standard models of life-cycle consumption have difficulty rationalizing this undersaving especially in the context of retirement savings. In an effort to overcome this hurdle, researchers have incorporated elements of present-biasedness in the preferences of individuals. Prominent among such attempts have been the inclusion of myopia (Feldstein, 1985; Andersen and Bhattacharya, 2011) and time-inconsistency (Laibson et. al. 1998) in people's preferences. These endeavors – see Chetty (2015) – often utilize a) the notion that individuals are comprised of multiple selves, possibly in conflict with one another, and b) the construct of a chasm between a self's "true preferences" (experienced utility), that which he uses to determine how much he should save, versus his "choice" or "behavioral" preferences (decision utility), that which determines how much he actually saves.[1] The latter can help rationalize the gap between actual and best-intention saving if, for example, the choice preferences of the current self attach a lower weight on future utility than his true preferences do – this is *present-bias* from the standpoint of the true self.

There may be disagreements between the choice preferences of the current self and his future selves. Time-inconsistent preferences (quasi-hyperbolic discounting) help explain the gap between what the current, decision-making self wishes a future self to save and what that self, when his turn to decide arrives, actually does. Here, the choice preferences exhibit *preference reversal*: the future self, for example, may wish to revise downward the previous self's forward-looking, best-intention saving plans – this is undersaving, from the standpoint of the previous self.[2] Cognizant of the impending preference reversal, a sophisticated self may seek commitment devices, such as mandatory pensions, to help his future selves stick to his better judgment about retirement saving – see Summers (1989), Laibson et. al. (1998), and Kaplow (2008). The agent, so the argument goes, uses the commitment device, ends up with more retirement wealth, and is made better off.

Evidently, present-biasedness and time inconsistency preferences can rationalize private undersaving in lifecycle models. What is not apparent, at least not in theory, is whether pension mandates leave the agent with increased retirement wealth. The reason is, with perfect capital markets, present-biased individuals can offset the mandated saving by reducing their own, even one-for-one – if need be, *borrow* against their future pension wealth – leaving total retirement wealth unchanged, possibly lower. (This "ineffectiveness" result is well-known in the theoretical pensions literature – see Gale, 1998.) Which raises the question, why are they so popular? [3] This is

[1] Myopia means the agent places less weight on the future than his true preferences would suggest while time-inconsistent preferences imply *preference reversal*: the relative weight placed by the current self on current versus future utility changes as the lifecycle proceeds.

[2] For expositional ease, the introduction restricts the discussion only to time-inconsistent preferences exhibiting preference-reversal across choice selves. As the body of the paper will make clear, all major assertions will also be true under myopia, present bias from the standpoint of the true self.

[3] Many countries (Australia, Netherlands, Denmark, Sweden, Mexico, Norway, Poland, and many others) have mandatory pension schemes, either mandated by law or via labor market negotiations or contracts, requiring individuals to contribute a certain fraction of their income during their entire work career towards their own retirement. (OECD, 2015). These range from 6% in New Zealand to 33% in Italy, once employer and employee mandates are added up. The contribution rates are typically age-independent, and is, therefore, the same for the young, the

our point of entry into this literature. We seek a welfare rationale for mandated pensions. We find that pension mandates are great commitment devices, effective at raising retirement wealth not by compelling agents to raise their voluntary retirement saving but by ensuring savings at desired levels! This holds even if the young as a response borrow more at a rate exceeding the return on pensions savings (balance expansion).

The customary way to restore policy effectiveness in life cycle models is to assume, implicitly or explicitly, agents face borrowing constraints – see e.g. Feldstein (1985), Laibson et. al (1998), Feldstein and Leibman (2002), Imrohoroglu et. al. (2003), and Cremer et. al. (2008). Under this assumption, the government mandates a sufficiently-high level of pension saving, enough to compel agents to stop saving on their own and, yet, not be allowed to borrow; from this point on, further increases in the mandate generate a proportional increase in retirement saving. We argue the no-borrowing constraint assumption is not entirely innocuous. Our reasoning is simple: in the natural course of a lifecycle, as has been well documented, agents facing a hump-shaped income profile are *net borrowers* when young, net savers when middle aged, and dissavers when old – see, for example, Figure 3.2 in Coeurdacier et. al. 2015. for data on the U.S. In that case, why mandate the young to save for retirement at a time they most want to borrow? We take a quick look at the evidence.

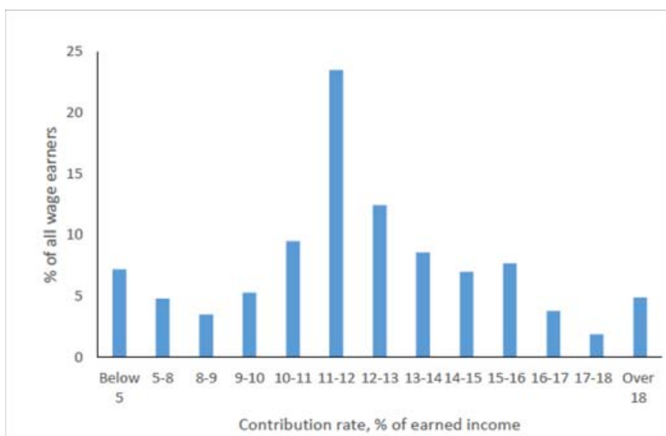

Figure 1a: Fraction contributing to mandated pensions

Note: Contribution rates to mandated labour market pensions for the age group 25-59 in 2014 computed as payments into labour market pension arrangements as a % of wage income. *Source:* Danish Ministry for Economic and Interior Affairs (2014).

In Figure 1a, we document the distribution of the entire Danish population of wage earners in 2014 according to their mandatory contribution rates (into mandated, funded individual retirement accounts). Evidently, more than 70% of wage earners contribute excess of 10% of their earned income into such schemes.

middle-aged and those close to retirement. One exception is Switzerland which has employee pension contribution rates increasing with age (four age groups) rising from 7% for individual in the age group 25-34 to 18% for the age group 55-65 (55-64 for women). Bateman et. al (2001) contains a detailed review of mandatory saving schemes across OECD countries.

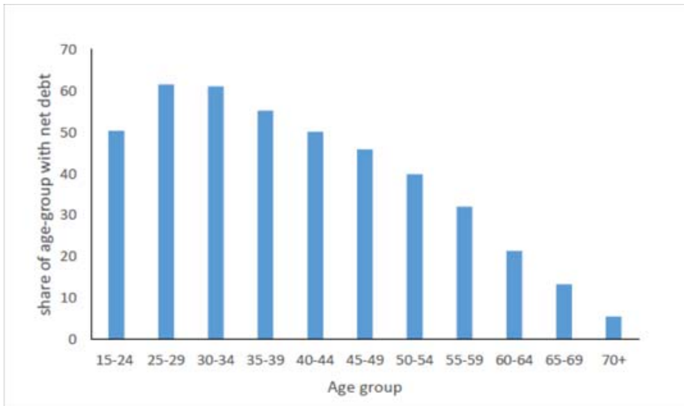

Figure 1b: Share of wage earners with net debt

Note: Share of families with net-debt according to the age of the oldest member of the family in 2012. Pension wealth and housing equity are included. *Source*: Andersen et. al (2012).

Figure 1b illustrates the fraction of people in various age-groups in the entire Danish population with net debt; for example, roughly 60% of 25-29 year olds have net debt. The two figures together suggest many young households are borrowing[4], and at the same time, contributing to mandatory pension schemes.[5] Similar patterns are observed in most OECD countries, including the U.S. – see Figure 1c which suggests a positive correlation between pension wealth and household debt as percentages of income (balance expansion).

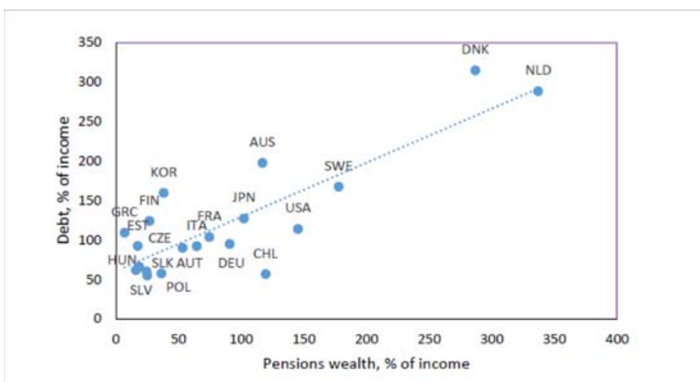

Figure 1c: Cross-country evidence on pension wealth and household debt

Note: Households pension wealth and gross debt in percentage of disposable income, 2012; *Source*: www.oecd-ilibrary.org\database (National accounts at a glance).

A model which imposes a no-borrowing constraint, especially on the young, and uses it to rationalize mandatory pensions is therefore at odds with the well-known borrowing and saving characteristics of the life cycle. The challenge, we take up, is to offer a welfare rationale for mandatory pensions in an otherwise standard life-cycle model with present-biased preferences *without*

[4]Data for 2014 from Statistics Denmark (2017) shows that older households, those above the age of 60, barely have any non-collateralized debt ("credit card debt") in contrast to younger households (those below the age of 50).

[5]In addition to the income their own retirement accounts will generate, Danish wage earners will, upon retirement, receive a means-tested, pay-as-you-pension.

imposing any borrowing constraints.[6] Since most countries that impose mandatory pension contributions do so in an age-independent fashion, our question becomes extra challenging, how to rationalize such saving mandates on the young who also happen to be natural borrowers?[7]

At first glance, it would appear mandates of the defined contribution type (individual accounts) should not matter; after all, they do not change the present value of (disposable) income, only its time profile. If capital markets are imperfect – here, we mean, the direction in which income shifts across time affects its opportunity cost – then the time profile of income matters, and marginal utility of consumption across time is not equalized. In particular, the marginal utility of young-age consumption is higher (relative to the perfect markets case), if the young want to front-load consumption but are restrained from doing so – say, due to a higher interest rate on borrowing relative to saving, and/or an explicit borrowing constraint. Mandating the young to save for retirement, therefore, reduces their current disposable income and utility, misaligning marginal utility of consumption across the life-cycle even further. How, then, can such mandates be welfare enhancing? As we explain below, present-bias in preferences, in conjunction with capital market imperfections, is key.

We conduct our analysis in a standard three-period lifecycle model. Homogeneous agents face an exogenous, hump-shaped income profile and are assumed to have present-biased choice preferences – quasi-hyperbolic (time-inconsistent) preferences and myopia; their true preferences admit standard discounting but no short-sightedness or myopia.[8] We assume the agents are fully sophisticated: they understand their impending preference reversal and respond to it. The consumption-loan market is imperfect – there is an "interest gap", the borrowing rate exceeds the saving rate (which, in turn, exceeds unity), both exogenously specified. The pension scheme is of the mandatory, defined-contribution type with individual accounts and can admit age-specific contribution rates. Agents face no borrowing constraints; indeed, they may borrow against their future income and pension wealth. The government is benevolent and chooses pension contribution rate(s) to maximize lifetime utility of the true young self.

Our results confirm that present-biased preferences imply the young borrow too much and the middle-aged save too little for retirement relative to what the true young self would want. This sets the stage for government intervention. Under mild conditions on the extent of present-biasedness and the size of the interest gap, we show there may be a welfare case for mandating the middle-aged and the young to contribute to their retirement savings, even with age-independent contribution rates.[9] And this is consistent with the young simultaneously borrowing *and* saving (via mandated pensions), albeit at different opportunity costs, i.e. there is a balance-expansion as response to

[6] Goda et.al (2015) present evidence on the ubiquity of present-biasedness (roughly, 55%) seen in Americans. They also find a robust negative relationship between between retirement savings and the extent of present-bias.

[7] By natural borrowers, we mean the young facing a hump-shaped income profile would want to borrow even if their preferences were not present-biased. Attanasio and Weber (2010) provide ample evidence on this lifecycle profile.

[8] Bounded rationality can also be perceived as a so-called self-control problem – see Gul and Pesendorfter (2001, 2004). For an analysis of the design of pension schemes under such preferences in two-period overlapping generations models, see St-Amant and Garon (2015).

[9] If present-biasedness is absent, there is no role for mandated pensions – true and choice preferences agree – and there is no over/under saving problem to correct.

mandated pension savings.

The larger issue is, given the low self-provision for retirement by the middle aged, how should policy be designed to boost old-age consumption? Simply mandating the middle-aged to save more, initially, has no effect: they simply undo the mandate by cutting own saving. As mandated contributions rise, a point is reached where the middle-aged find themselves voluntarily saving nothing for retirement – the zero retirement-saving corner. Further increases in contributions, within limits, raise overall (mandated plus voluntary) retirement saving beyond what the voluntary part could achieve alone. The true young self likes this.

The question is, with the middle-aged pinned at the zero retirement-saving corner, should the young be co-opted in the larger mission of uplifting old-age consumption? The answer is, it depends. A pension mandate on the young raises future pension wealth and helps to raise old-age consumption, something the young true self appreciates. At the same time, though, it reduces current income for the young encouraging them to *borrow more* but not so much – because of the interest gap – as to totally offset the gain in future pension wealth. How is the middle-aged affected? The added cost of debt repayment tends to *reduce* the middle-aged self's consumption, contrary to what his true young self would have wanted. Therein lies the trade-off – boost old-age consumption but hurt middle-age consumption. This is why present-bias is necessary but not sufficient to rationalize mandated retirement saving; the extent of the credit market imperfection is crucial.

What insight do we glean from the discussion above? In our setup, effectiveness of a mandated pension relies on it being aggressive enough to convince the middle-aged to stop saving for retirement on their own! The task of ensuring the retired have sufficient savings is to be taken up solely by the government. This insight must appear counterintuitive; why is it not optimal to have both the government mandate and voluntary saving by the middle-aged? The answer lies in the present-bias. The government wants the young self to contribute to future pension wealth (and hence, old-age consumption) but it is aware the young sophisticated self will borrow against that future wealth and influence the middle-aged self to save more for debt servicing and less for retirement. However, if the middle-aged can be pinned at the zero retirement-saving corner, then, unless the policy is too pushy, the actions of the young self will leave no mark on the retirement saving of the middle-aged. They continue to save nothing for retirement on their own merely contributing to their retirement as mandated. The take-away is that mandatory pensions work by relieving the middle-aged of the task of self-provision for retirement, which potentially may increase lifetime utility. Bear in mind, were heterogeneities in present-bias present, mandatory pensions would work if designed to bind only for those with sufficiently severe present-bias, not those who are already self providing enough for their retirement.

We turn to a quick review of the literature. Imrohoroglu et. al (2003) is a seminal work studying the desirability of pay-as-you-go (PAYG) pension schemes for naive, quasi-hyperbolic discounters, assuming no borrowing is possible. In that case, unfunded schemes yield no benefit and may hurt, as is often the case, they are return-dominated by private saving. Our analysis is an apples-to-apples comparison between private saving and mandatory saving, each bearing the same rate of return; this means a shift from voluntary to mandated saving has no return consequences.[10] In an insightful new paper, Findley and Caliendo (2016a) extend the Imrohoroglu et. al (2003) line of work to allow for borrowing, as well, like us, a spread between borrowing and saving rates. In a lot of ways, their paper is very similar in spirit as ours, except they use a continuous-time OG structure with time-invariant wage income, and as such, cannot make meaningful statements concerning different magnitudes of responses to saving mandates by the young and the middle-aged. Malin (2008) considers the implications of pay-as-you-go social security acting as a floor on savings in a model with heterogenous, time-inconsistent agents. In our setup, a uniform floor will bind for some levels of present-biasedness but not all because the scheme is fully funded. In previous work, Andersen and Bhattacharya (2011) study the desirability of pay-as-you-go pensions in a two-period model under the assumption of agent myopia. There it is shown that for return-dominated unfunded pensions to be welfare enhancing, agent preferences should display sufficiently strong myopia and the pension must be high enough to incentivize agents to not save on their own.

A relatively large empirical literature has analyzed how household savings is affected by incentives and mandated saving requirements. In particular, the Individual Retirement Accounts (IRAs) and 401(k) programs in the US have been much researched. A symposium in the *Journal of Economic Perspectives* summarizes the findings and shows the difference in opinions and results based on analyses of the very same programs – see Hubbard and Skinner (1996), Poterba, Venti and Wise (1996) and Engen, Gale and Scholz (1996). The consensus appears to be, such incentives may increase saving, but the relevant elasticities are small, suggesting many agents are not responding to incentives. More recently, Alessie et. al. (2013) and Blau (2016) find evidence for the displacement effect of pension wealth on household savings. Access to microdata makes it possible to better assess the determinants of savings. In an influential study, Chetty et al. (2014) use Danish data and exploit increases in mandated pension contributions at job shifts to identify the effects of mandated saving on total savings. They find about 85% of individuals are so-called passive savers; for them, an increase in mandated saving leads to a one-to-one increase in total savings, with no adjustment in other forms of saving.[11] About 15% respond to the mandated savings requirement mainly by changing other forms of saving, i.e., their total saving is not much affected.

The paper is organized as follows. Section 2 sets up the three-period model while Section 3 derives the saving decisions of the young and the middle-aged when saving and borrowing rates differ. As a prelude to the full-blown analysis for the entire lifecycle, Section 4 studies a welfare rationale for mandated pensions in a two-period model. The role for mandated pension savings

[10] Cremer and Pestieu (2011) consider a PAYG scheme in a two-period model with homogeneous agents where, in the absence of a no-borrowing constraint, there is no welfare case for such pensions. Their concern is more about redistribution via a PAYG scheme (seen most clearly, in the case where all agents are non-myopic) and not about whether such schemes solve the present-bias problem.

[11] One interpretation of "passive savers" is that they behave like agents at the zero-saving corner alluded to above; there may be other interpretations. It bears emphasis that when we say the zero-saving corner, we have in mind zero savings for lifecycle purposes. These people may have positive levels of savings for other reasons. Chetty et al. (2014) stay away from these nitty-gritties.

in the life-cycle model is considered in Section 5, including an analysis of whether there can be a welfare rationale for mandating the young to save for pensions, and whether there is a welfare rationale even if the contribution rate is age-independent. Section 6 concludes the paper. Proofs and some additional material are relegated to the appendix.

2 The model

2.1 Preliminaries

To capture the essentials of the life-cycle pattern of borrowing and saving, we consider a simple, three-period lifecycle model.[12] Population size is held fixed.[13] A representative agent lives through three phases, young (y), middle-aged (m) and old (o), and is endowed with exogenous incomes $w_y > 0$ and w_m ($> w_y$) during youth and middle-age respectively, and *nothing* when old – in other words, he faces a hump-shaped income profile in accordance with stylized facts. At times, we interchangeably refer to these phases as selves, where Self 1 (young self) is the young phase, and so on. Agents have access to a capital market where the gross return on saving is R (> 1) but the borrowing rate is $R_b \geq R > 1$: borrowing faces a *higher opportunity cost* than saving.[14,15] All borrowing and saving is for consumption purposes only.

We allow agents to act myopically *and* have quasi-hyperbolic preferences (exhibit time-inconsistent behavior). We draw a distinction between the "true" and "choice" utility of agents. Agents' behavior is dictated by their choice utility, but their actual well-being, our measure of welfare, is governed by the true lifetime utility.[16] Let c_y denote consumption as young, c_m denote consumption as middle-aged, and c_o be consumption as old. The "true" preferences, with a "*", defined over consumption in each period of life is the standard, separable

$$\Omega^* \equiv u(c_y) + \delta^* u(c_m) + (\delta^*)^2 u(c_o) \tag{1}$$

where $\delta^* \in [0, 1]$. The felicity function $u(\cdot)$ is assumed to fulfill standard assumptions, including $u'(\cdot) > 0$ and $u''(\cdot) < 0$ and Inada conditions.[17] Our yardstick of welfare is Ω^*, the lifetime true

[12] Three periods are necessary and sufficient to capture the essence of the natural life-cycle pattern (borrowing as young, saving as middle-aged and dissaving as old). The model is deliberately kept barebones so as to reveal the intuition in stark fashion. Ignored are heterogenities in income or present-biasedness (as in Malin, 2008), mortality risk, uncertainty, bequest motives, retirement decisions, transactions costs, among others.

[13] It is unproblematic to allow for young or middle-age mortality when perfect annuities markets are present. In the presence of market imperfections additional issues arise since the mandated pension scheme may (partially) overcome this market failure if it offers life-annuities, see e.g. Eckstein et al (1985). This points to market imperfections as a separate reason for public intervention, which we leave out to focus on the implications of present-biased preferences.

[14] Letting $R_b \to \infty$ generates a no-borrowing constraint as a special case.

[15] To make voluntary and mandatory savings non-perfect substitutes, Gale and Scholz (1994) presents a three-period OLG model with income uncertainty in period 2. This creates a legitimate precautionary demand for saving that is not satisfied by mandated pension savings. Chetty et al (2014) introduces a specific utility gain from the flexibility available with voluntary saving.

[16] Findley and Caliendo (2016) consider other welfare criteria.

[17] In some places below, we illustrate some results for a logarithmic utility function, i.e. $u(\cdot) = \ln(\cdot)$.

utility of Self 1. The choice preferences when young (of Self 1) are given as

$$\Omega \equiv u\left(c_y\right)+\beta\left[\delta u\left(c_m\right)+\delta^2 u\left(c_o\right)\right]=u\left(c_y\right)+\widetilde{\delta}\left[u\left(c_m\right)+\delta u\left(c_o\right)\right] \tag{2}$$

and of Self 2 (when middle-aged) as

$$u\left(c_m\right)+\widetilde{\delta} u\left(c_o\right) \tag{3}$$

where $\beta \in [0,1]$, $\delta \leq \delta^*$ and $\widetilde{\delta} \equiv \beta\delta \leq \delta$.

Note $\beta < 1$ represents quasi-hyperbolic preferences which generate time inconsistency (preference reversal) since the marginal rate of substitution (M.R.S) between consumption as middle-aged and old from the view point of Self 1, $\frac{\partial c_m}{\partial c_o}\mid_{\text{young}}= -\delta\frac{u'(c_o)}{u'(c_m)}$ and the same M.R.S $\frac{\partial c_m}{\partial c_o}\mid_{\text{middle-aged}}= -\beta\delta\frac{u'(c_o)}{u'(c_m)} = -\widetilde{\delta}\frac{u'(c_o)}{u'(c_m)}$ from the point of view of Self 2 are not the same (Laibson,1997). Also, if present bias arises solely due to myopia ($\beta = 1$, $\delta = \widetilde{\delta}$) there is no difference in the MRS's, and therefore, no preference reversal. Similarly, if $\delta = \delta^*$ but $\beta < 1$, there is no myopia but preference reversal persists. In Figure 2, the true preferences of Self 1 for c_m and c_o are shown by the green indifference curve, with discount factor δ^*. The blue indifference curve captures Self 1's preferences for the same with corresponding discount factor $\delta < \delta^*$. For Self 2: the red curve captures his choice preferences over c_m and c_o, that which he as middle-aged uses to make decisions, with an attached discount factor, $\beta\delta = \widetilde{\delta} < \delta$.

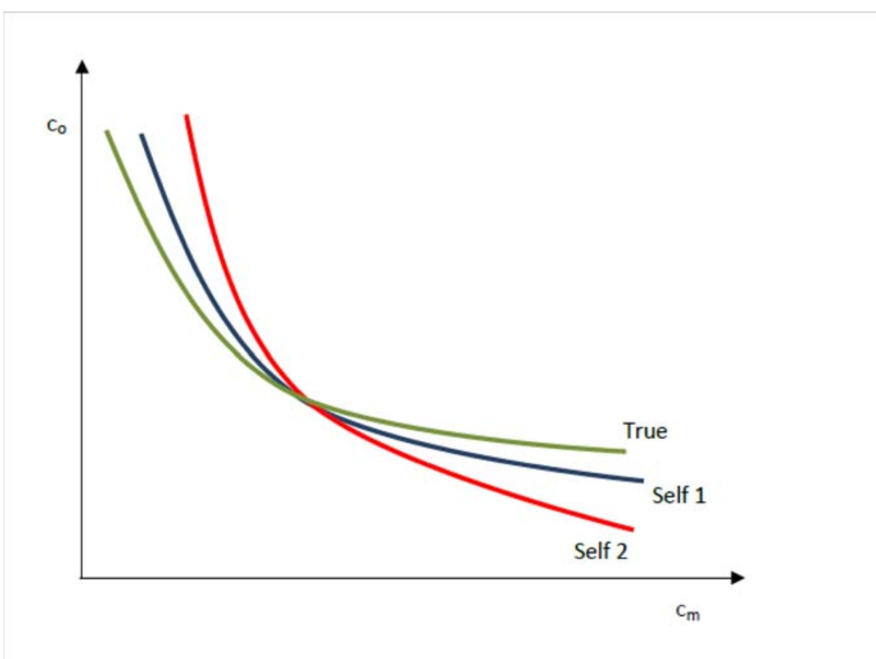

Figure 2: True vs. Choice utility

There is a mandatory, defined-contribution pension scheme with individual accounts to which the young and middle-aged are required to contribute a share $\tau_y \in [0,1]$ and $\tau_m \in [0,1]$ of their respective incomes.[18] This means, pension-mandated (henceforth "mandatory") saving is $\tau_y w_y$ and $\tau_m w_m$ for the young and middle-aged respectively. The gross return on the pension contribution is R ($R > 1$), i.e., the mandatory scheme offers the same return as voluntary saving.[19] The individual is entitled to a pension benefit (P) in the third period where

$$P = R^2 \tau_y w_y + R \tau_m w_m. \tag{4}$$

The agents are assumed to perceive the relationship between their contributions and the benefit they receive. The agent considers his entire future pension wealth, P, as something he can borrow against.[20]

Denoting voluntary saving as young by s_y and saving as middle-aged by s_m, consumption as old is

$$c_o = \begin{cases} P + Rs_m \text{ for } s_m \geq 0 \\ P + R_b s_m \text{ for } s_m < 0 \end{cases},$$

as middle-aged is,

$$c_m = \begin{cases} (1-\tau_m)w_m + Rs_y - s_m \text{ for } s_y \geq 0 \\ (1-\tau_m)w_m \ + R_b s_y - s_m \text{ for } s_y < 0 \end{cases},$$

and as young by

$$c_y = (1-\tau_y)w_y - s_y.$$

If $s_y \geq 0$ and $s_m \geq 0$, (no borrowing as young and middle-aged) the present value of lifetime income is $I^{ss} \equiv (1-\tau_y)w_y + \frac{(1-\tau_m)w_m}{R} + \frac{P}{R^2} = w_y + \frac{w_m}{R}$ which holds because the agent is cognizant of the link between his contributions and eventual benefits. In this case, pension contribution rates do not influence the budget sets of the various selves. However, if $s_y < 0$ and $s_m \geq 0$, the present value of lifetime income is given by $I^{bs} \equiv (1-\tau_y)w_y + \frac{(1-\tau_m)w_m}{R_b} + \frac{P}{RR_b} = \left((1-\tau_y) + \frac{R}{R_b}\tau_y\right) w_y + \frac{w_m}{R_b} < I^{ss}$; evidently, $R \neq R_b$ matters here, which is why the contribution rates influence budget sets. If both $s_y, s_m < 0$, then the present value of lifetime income is $I^{bb} \equiv \left[(1-\tau_y) + \left(\frac{R}{R_b}\right)^2 \tau_y\right] w_y + \left[\frac{R}{R_b}\tau_m + (1-\tau_m)\right] \frac{w_m}{R_b} < I^{bs}$.

[18] The government is assumed to pre-commit to these contribution rates. For an insightful analysis of these issues in the absence of such precommittment, see Findley and Caliendo (2015).

[19] Mandatory savings funds have access to the same capital market products as do private savers, and hence the returns are assumed to be identical. It may be argued that mandated schemes can deliver higher net returns due to lower marketing and transactions (economies of scale) costs. On the other hand, the governance structure may distort the objectives of the investment policies in mandated pension funds.

[20] In reality, that may not be entirely so. If, for example, borrowing requires the agent to put up collateral, it may be that pension wealth is not accepted as collateral by lenders. We touch on this below.

Henceforth, *we restrict attention* to $s_y < 0$ and $s_m > 0$, the most relevant case, as discussed in the introduction. (The appendices admit the case, $s_m < 0$.) To avoid confusion, define $b_y \equiv -s_y > 0$ which means an increase in borrowing is simply an increase in b_y. Then,

$$c_o = P + Rs_m,\ c_m = (1-\tau_m)w_m\ - R_b b_y - s_m,\ c_y = (1-\tau_y)w_y + b_y. \tag{5}$$

3 Voluntary saving

3.1 Middle-aged saving

We focus on so-called *sophisticated myopics*, that is, we posit Self 1 is aware Self 2's decisions are based on (3) and not on $u(c_m) + \delta u(c_o)$.[21] Evidently, it most challenging to rationalize mandatory pensions for sophisticated myopics.

The first order of business is to characterize the individual saving decisions as young and middle-aged for given contribution rates. We will proceed in the usual backward way – the perception-perfect strategy of O'Donoghue and Rabin (1999) – by finding optimal saving s_m for the middle-aged *given* borrowing by the young, b_y, and incorporating that response back into the borrowing decision of the young.

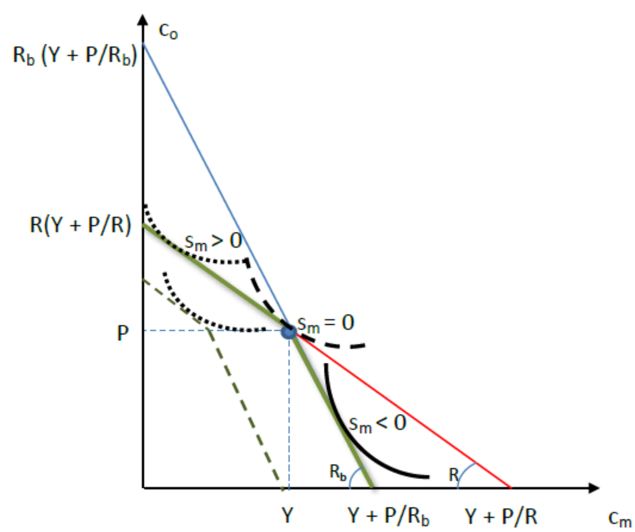

Figure 3: Middle-age voluntary saving

Let $Y \equiv (1-\tau_m)w_m\ - R_b b_y$. Given a $b_y > 0$, the budget set for period 2 and 3 is $c_m + \frac{c_o}{R} = Y + \frac{P}{R}$ if the middle-aged agent is a saver and $c_m + \frac{c_o}{R_b} = Y + \frac{P}{R_b}$ if a borrower, as illustrated in Figure 3. Since $R_b > R$, there is a kink in the budget set (the bold green line segments) at $s_m = 0$. Also note, for a saver $c_m + s_m = Y \Leftrightarrow c_m + D = w_m - R_b b_y$ where $D \equiv (\tau_m w_m + s_m)$. That is a middle-aged

[21] The alternative case has so-called naive myopics, who do not perceive the upcoming change in their preferences. In Appendix G, we briefly consider this case and argue that results remain unchanged qualitatively. When agents choose both saving and their retirement, it is critical whether agents are sophisticated or naive, see Diamond and Köszegi (2003) and Findley and Coliendo (2015).

saver can be thought of as having net income $w_m - R_b b_y$ from which he puts away $D : \tau_m w_m$ is the mandatory savings part and s_m is the voluntary savings part, and importantly, each part earns the same return. Using $P = R^2 \tau_y w_y + R\tau_m w_m$, we can also write $c_o = P + Rs_m = R^2 \tau_y w_y + RD$ which clarifies that a middle aged saver cares about total retirement saving, D, not its composition.

Given a b_y, Self 2's choice utility is given by $u(w_m - R_b b_y - D(b_y)) + \widetilde{\delta} u(P + Rs_m)$ and the budget constraint is $c_m + c_o/R = Y + \frac{P}{R}$. The relevant first order condition is

$$-u'(w_m - R_b b_y - D(b_y)) + \widetilde{\delta} R u' \left(R^2 \tau_y w_y + RD(b_y)\right) = 0; \; s_m > 0 \tag{6}$$

The second-order condition is satisfied given the assumptions made on u. Self 2 is at a corner with zero voluntary retirement savings, i.e., $s_m = 0$, if

$$u'((1-\tau_m)w_m - R_b b_y) > \widetilde{\delta} R_b u' \left(R^2 \tau_y w_y + R\tau_m w_m\right) \tag{7}$$

which, it is noteworthy, holds for a range of b_y.[22] Note, the saving vs. no-saving regime cutoffs depend on the pension contribution rates.[23] This means that increases in contribution rates may change the identity of a middle-aged agent from a retirement saver to a non-saver. We will return to this shortly. Figure 3 illustrates the cases where the middle-aged is a voluntary retirement saver, at a zero saving corner, or is borrowing.[24] Observe, that even if the middle-aged is driven to the zero retirement-saving corner, total savings is not driven to zero. The middle-aged still has consumption less than income due to servicing of the debt incurred when young. Therefore mandated pension savings may lead to balance expansion over the life-cycle; more borrowimg as young, more savings as old (voluntary/mandated retirement savings plus savings for debt servicing) and higher pension savings(wealth).

It can be checked (from (6)),

$$\frac{\partial D}{\partial b_y} = \frac{\partial s_m}{\partial b_y} = \begin{cases} \in (0, -R_b) \text{ if } s_m > 0 \\ 0 \text{ if } s_m = 0 \end{cases} \tag{8}$$

Higher borrowing by Self 1 (higher b_y) reduces starting wealth for Self 2 – reduces Y and shrinks the budget set, see the dotted green lines – who reacts by decreasing his saving for consumption smoothing reasons. This is the wealth effect. To foreshadow, all else same, the government by inducing the young to raise b_y reduces voluntary saving by the middle-aged; this hurts the middle-aged.

[22] To avoid discontinuities, Findley and Caliendo (2016a) "smooth" away the interest rate spread. This allows them to stay away from zero-saving corners that are influential in our discussion. In other words, the agents in their model cannot be "passive" in the sense of Chetty. et. al (2014) as each one of them would actively respond to changes in the policy.

[23] Of course, if $\tau_y = \tau_m = 0$, the middle-aged will save at any R to ensure some consumption when old.

[24] Note the zero retirement saving corner is not due to "prodigality" as in Pestieau and Possen (2008) where some choose not to save knowing they will be bailed out later by the government.

3.2 Borrowing by the young

The sophisticated Self 1 takes the saving behavior of his future self into account and figures out his best response. Lifetime choice utility as perceived by Self 1 is

$$u\left((1-\tau_y)w_y+b_y\right)+\widetilde{\delta}\left[u\left((1-\tau_m)\,w_m-R_b b_y-D\left(b_y\right)\right)+\delta u\left(RD\left(b_y\right)+R^2\tau_y w_y\right)\right]$$

where $D\left(b_y\right)$ is determined by (6). The first-order condition reads

$$\begin{cases} u'\left(c_y\right)+\widetilde{\delta}\left[-u'\left(c_m\right)+\delta R u'\left(c_o\right)\right]\left(\frac{\partial D(b_y)}{\partial b_y}\right)-\widetilde{\delta}R_b u'\left(c_m\right)=0 \text{ if } D\left(b_y\right)>\tau_m w_m \Leftrightarrow s_m>0 \\ u'\left(c_y\right)-\widetilde{\delta}R_b u'\left(c_m\right)=0 \text{ if } D\left(b_y\right)=\tau_m w_m \Leftrightarrow s_m=0 \end{cases} \tag{9}$$

The second order condition is assumed to hold.

At first sight it may appear, given $s_m>0$ was optimally chosen by the middle-aged, the term attached to $\frac{\partial D(b_y)}{\partial b_y}$ – capturing how the saving decision of the middle-aged is influenced by the young – must get washed out by the envelope theorem. Not so here. The reason is the preference reversal: how the young views intertemporal substitution between middle and old age, and sets $-u'\left(c_m\right)+\delta R u'\left(c_o\right)=0$ with attached discount rate δ, is *not* how the middle-aged (see eq.(6)) views the same (and sets $-u'\left(c_m\right)+\widetilde{\delta}R u'\left(c_o\right)=0$ with discount rate $\widetilde{\delta}$).

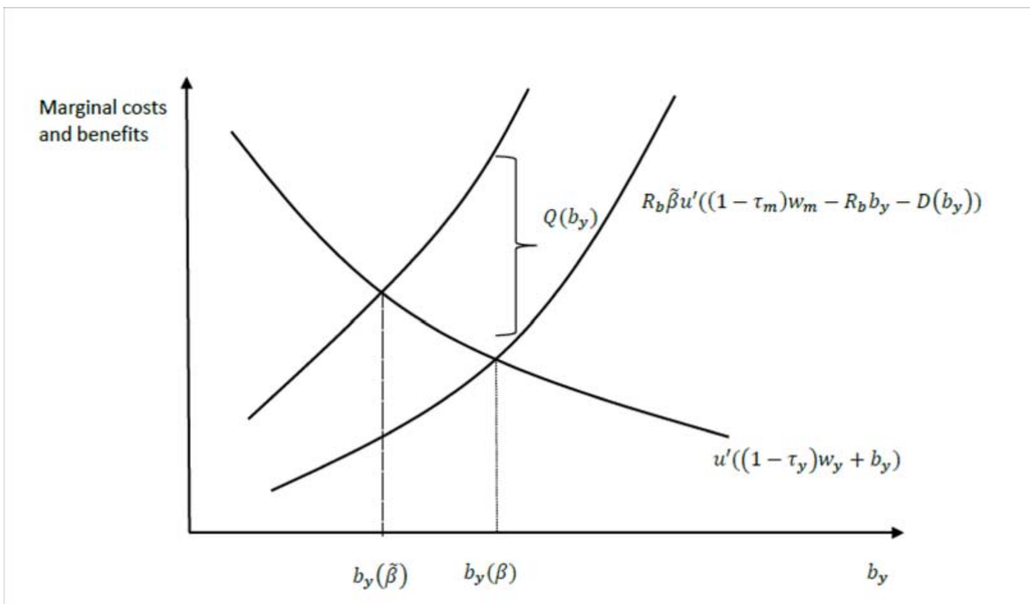

Figure 4: The r.h.s and l.h.s of eq. (10)

Notice from (9) that, in the usual textbook setting, the choice for Self 1 (a borrower) between c_y and c_m would be governed, simply by $\widetilde{\delta}R_b u'\left(c_m\right)=u'\left(c_y\right)$ where the r.h.s is the marginal benefit of borrowing an extra unit for young-age consumption and the l.h.s is the discounted marginal cost of reduced middle-aged consumption. Here, an additional term emerges so that

$$\widetilde{\delta} R_b u'(c_m) \underbrace{-\widetilde{\delta}\left[-u'(c_m)+\delta R u'(c_o)\right]\left(\frac{\partial D(b_y)}{\partial b_y}\right)}_{Q(b_y)} = u'(c_y) \tag{10}$$

Since Self 1 and Self 2 disagree on the correct discount factor, specifically $\delta > \widetilde{\delta}$, we have $-u'(c_m)+\delta R u'(c_o) > 0$ (instead of (9)) and since $\frac{\partial D(b_y)}{\partial b_y} < 0$, the underscored term – call it $Q(b_y)$ – is positive, adding to the previously-discussed marginal cost, thereby raising the total marginal cost of borrowing. This extra cost arises because Self 1's borrowing restricts the feasible set for Self 2 and because Self 1 and Self 2 disagree on the correct discount factor, both of which the sophisticated Self 1 must internalize. This means Self 1 is made better off by choosing a lower level of borrowing, as clear from Figure 4.

Alternatively, use (8) and (9) to get

$$-u'(c_y)+\widetilde{\delta} R\left[\left(\widetilde{\delta}-\delta\right)\left(\frac{\partial D(b_y)}{\partial b_y}\right)+R_b\widetilde{\delta}\right]u'(c_o)=0 \tag{11}$$

For future reference, define

$$\Lambda_a \equiv \left(\widetilde{\delta}-\delta\right)\left(\frac{\partial D(b_y)}{\partial b_y}\right)+R_b\widetilde{\delta} \tag{12}$$

Notice, the term $\left(\widetilde{\delta}-\delta\right)\frac{\partial D(b_y)}{\partial b_y} \geq 0$ appears because the sophisticated Self 1 correctly anticipates the upcoming preference reversal; hence, the envelope theorem does not apply.[25] Curiously, notice if $\frac{\partial D(b_y)}{\partial b_y} = 0$ (which happens when $s_m = 0$, the zero-saving corner), then preference reversal has no bite! If Self 1 cannot influence Self 2's saving by changing his own borrowing, the only instrument in his arsenal, then whether Self 1 anticipates the preference reversal or not makes no difference. To foreshadow, if the government can keep the middle-aged at the zero-saving corner, then, in a sense, it has cured that agent of the preference-reversal problem.

[25] In the special case of "pure" myopia ($\delta = 1, \beta = \widetilde{\beta}$) or for naive agents (see Appendix G) this term drops out.

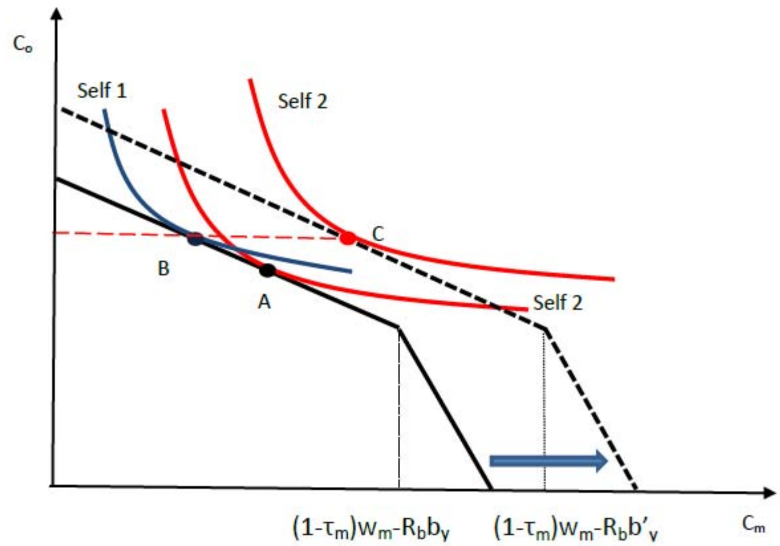

Figure 5: Effect of b_y on s_m

The discussion above makes clear that Self 1 is made better off by cutting borrowing. What impact does this have on Self 2? In Figure 5, Self 1 (using the blue indifference curve) would like Self 2 to consume at point B. However, Self 2 using the red indifference curve (which is steeper, since $\delta > \widetilde{\delta}$) prefers point A. Knowing this, Self 1 can influence the budget set of Self 2 by cutting borrowing (from b_y to b_y'). It is apparent from the figure this shifts out the budget set, allowing Self 2 to, say, choose point C. As drawn, Self 1 has, via this action, ensured Self 2 chooses the same c_0 as Self 1 would have liked (as per point B). Intuitively, what is going on here is that the preference reversal, via the tilting of the indifference curves (from blue to red) causes a "substitution effect" which hurts Self 1; the former responds by transferring some income (reduced b_y) to Self 2, an "income effect", to help Self 2 save more for old-age which, in turn helps Self 1. This comes at a cost: consumption during middle age is higher, and by implication, it is smaller as young – after all, point C may not be optimal.

In passing, recall that the entire analysis above was predicated on Self 1 being a borrower. We show that

Lemma 1 *Self 1 is a borrower if w_y is sufficiently small relative to w_m.*

In Appendix A, we show that a sufficient condition for Self 1 to borrow is that his income as young not be too large.[26] Henceforth, we assume the conditions spelled out in Lemma 1 hold in the rest of the paper.

[26] Obviously, this holds for $w_y = 0$. However, we need a positive wage income for Self 1so as to be able to pose the question, does it make sense for the young to be mandated to save for pensions.

3.3 Changes in the contribution rates

How do the middle-aged react to an increase in their contribution rate? Note

$$\frac{\partial D}{\partial \tau_m} = \begin{cases} 0 \iff \frac{\partial s_m}{\partial \tau_m} = -w_m \text{ if } s_m > 0 \\ -w_m \iff \frac{\partial s_m}{\partial \tau_m} = 0 \text{ if } s_m = 0 \end{cases}$$

meaning, total retirement saving is unaffected – the agent, as alluded to above, does not care about the composition of total retirement saving and therefore offsets any increase in mandatory saving by cutting his voluntary saving. It follows,

$$\frac{\partial c_m}{\partial \tau_m} = 0; \ \frac{\partial c_o}{\partial \tau_m} = 0$$

This is a restatement of a well-known, policy ineffectiveness result — see Gale (1998) – on the neutrality of fully-funded pension schemes. The implication is stark and important: unless the middle-aged is at the zero-saving corner, changes in the pension contribution rate have no effect on his retirement saving (the pension offset is a full 100%). If the agent is a saver for retirement, his present-biasedness and freedom to borrow against the future will goad him to undo any effort by the government aimed at boosting his saving. At the zero corner, however, he is pinned; he cannot continue offsetting the mandate, and hence, his present-biasedness is rendered harmless.[27]

Lemma 2 *When middle-aged saving is positive,*

$$s_m > 0 : \left| \begin{array}{cc} \frac{\partial b_y}{\partial \tau_y} > 0 & \frac{\partial s_m}{\partial \tau_y} < 0 \\ \frac{\partial b_y}{\partial \tau_m} = 0 & \frac{\partial s_m}{\partial \tau_m} = -w_m < 0 \end{array} \right|$$

and when it is at the zero corner,

$$s_m = 0 : \left| \begin{array}{cc} \frac{\partial b_y}{\partial \tau_y} > 0 & \frac{\partial s_m}{\partial \tau_y} = 0 \\ \frac{\partial b_y}{\partial \tau_m} < 0 & \frac{\partial s_m}{\partial \tau_m} = 0 \end{array} \right| .$$

As discussed above, when the middle-aged are savers, a higher τ_m levied on the middle-aged has no effect on b_y of the young or D for middle-aged. On the other hand, a higher contribution rate on the young, τ_y, induced the young to borrow more – lower young income (net of pension contributions) and higher retirement income both induce borrowing, but the crowding out is not complete because borrowing has a higher opportunity return. Self 1 leaves Self 2 (the middle-aged) a lower starting net-wealth inducing the latter to further reduce retirement saving. In passing, note that if the middle aged were borrowing, then an increase in τ_m would cause them to borrow even more, which would bring down their old-age consumption – such an increase in τ_m is, of course, counterproductive, our goal being to raise, not reduce old-age consumption. This explains why a government would not want to raise τ_m so high as to drive the middle-aged to borrow, see below.

[27] See Appendix C for a discussion of the separate role of β and δ.

3.4 Undersaving

We want to establish that in the absence of policy, agents "undersave" both as young and middle-aged, i.e., they borrow "too much" as young and save "too little" for retirement as middle-aged: $b_y > b_y^*$ and $s_m < s_m^*$ where the * denotes solutions derived using *true* utilities, Ω^*.

We continue to restrict focus on young borrowers. Under laissez faire, voluntary retirement saving as middle-aged will be strictly positive (they are "natural savers" – after all, this is the only way to ensure some consumption as old). The saving decision of the middle-aged is determined using $-u'(w_m - R_b b_y - s_m) + \widetilde{\delta} R u'(R s_m) = 0$ while s_m^* is derived from $-u'\left(w_m - R_b b_y^* - s_m^*\right) + \delta^* R u'(R s_m^*) = 0$; the difference between the two problems, given a b_y, is just that the true preferences use δ^* while the choice preferences use $\widetilde{\delta}$. Similarly, borrowing as young, b_y, is derived from $-u'(w_y + b_y) + \widetilde{\delta}\left[-u'(c_m) + \delta R_b u'(c_o)\right] \frac{\partial s_m}{\partial b_y} + \widetilde{\delta} R_b u'(w_m - R_b b_y - s_m) = 0$ and b_y^* is derived using $-u'\left(w_y + b_y^*\right) + \delta^* R_b u'\left(w_m - R_b b_y^* - s_m^*\right) = 0$. In this case, the difference between the two problems is more substantial: the sophisticated young under choice preferences has to contend with the fact that his middle-aged self will attempt to undo his action – a concern that arises only because of time-inconsistency, which the young under true preferences does not have to contend with.

Proposition 1 *$b_y > b_y^*$ and $s_m < s_m^*$ obtain, i.e., agents borrow "too much" as young and save "too little" for retirement as middle-aged compared to what their true selves want.*

Intuitively, a present-biased young agent would want to borrow more than his true young self would. However, this would leave his present-biased Self 2 with a lower starting wealth legitimizing Self 2's choice of lower saving, lower than what true Self 2 would have wanted. If mandatory pensions are to be justified, then they have to help reduce the severity of this "undersaving" problem.

4 A welfare case for mandatory pensions: The two period model

As a prelude to the analysis of the full life-cycle model, it is useful to study the two-period case[28] (middle-aged and old) so as to clarify the key roles played by the crowding-out of private savings and the capital market structure. The two-period model is also the most commonly studied in the pensions literature – see a review of the literature in Cremer and Pestieau (2011). Note, in the two-period setting, quasi-hyperbolic discounting and myopia are indistinguishable. Also, with no old-age income, there is a natural retirement saving (but no borrowing) motive on the part of the middle-aged.

The question is, is there a welfare rationale for mandatory pension saving ($\tau_m > 0$) when welfare assessed in terms of true preferences $\Omega^* = u(c_m) + \delta^* u(c_o)$? Present-biased preferences imply the middle-aged undersave, but can mandated pension saving help solve this problem?[29]

[28] It arises as a special case of the three period model by setting $b_y = \tau_y = 0$ and eliminating consumption when young from the life-time utility function.

[29] Note that $s_m^* > s_m$ follows straightforwardlly in the two-period case, since $\beta^* > \widetilde{\beta}$ and savings are determined by $u'(w_m - s_m^*) = R\beta^* u'(R s_m^*)$ and $u'(w_m - s_m) = R\widetilde{\beta} u'(R s_m)$, respectively.

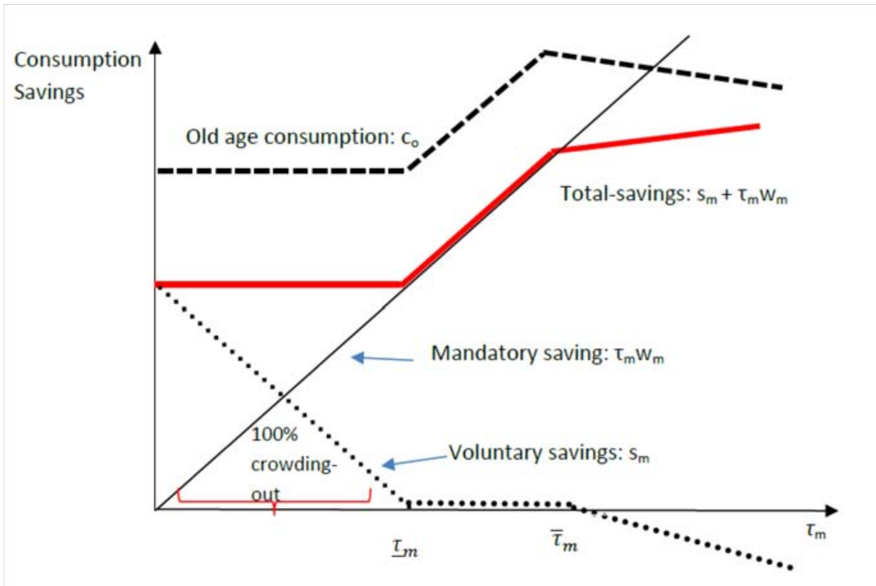

Figure 6: c_o and s_m against τ_m

Voluntary saving depends on mandatory saving (via τ_m), and hence, it is instructive to define critical contributing rates delimiting saving and borrowing regimes. Define $\underline{\tau}_m$ as the contribution rate at which voluntary saving (s_m) is exactly zero, i.e. $u'((1-\underline{\tau}_m)w_m) \equiv R\widetilde{\delta}u'(R\underline{\tau}_m w_m)$. It follows, $s_m > 0$ for $\tau_m < \underline{\tau}_m$ since $u'((1-\tau_m)w_m) < R\widetilde{\delta}u'(R\tau_m w_m)$ for $\tau_m < \underline{\tau}_m$. Next, define $\overline{\tau}_m$ as the contribution rate at which voluntary borrowing is exactly zero, i.e. $u'((w_m(1-\overline{\tau}_m)) \equiv \widetilde{\delta}R_b u'(R\overline{\tau}_m w_m)$, i.e. for $\tau_m > \overline{\tau}_m$ the middle-aged is borrowing. Figure 6 illustrates[30] how savings and old-age consumption depend on the contribution rate τ_m. We show

$$\begin{cases} \tau_m < \underline{\tau}_m & s_m > 0 \quad \frac{\partial s_m}{\partial \tau_m} = -w_m \\ \underline{\tau}_m \le \tau_m \le \overline{\tau}_m & s_m = 0 \quad \frac{\partial s_m}{\partial \tau_m} = 0 \end{cases}$$

Recall, in the absence of mandatory pension saving ($\tau_m = 0$), middle-aged voluntary retirement saving is obviously positive ($s_m > 0$). An increase in τ_m, at first, crowds out voluntary retirement saving one-to-one leaving total-saving ($= s_m + \tau_m w_m$), and thus old-age consumption, unchanged. When τ_m reaches $\underline{\tau}_m$, voluntary retirement saving is driven to zero, and further increases in τ_m up to $\overline{\tau}_m$ increases total-saving and thus old age consumption one-to-one. When τ_m reaches $\overline{\tau}_m$, the middle-aged *switch* to becoming borrowers and further increases in τ_m induce more borrowing, and hence, falling old age consumption ($R\tau_m w_m + R_b s_m$).

For the optimal level of mandatory pensions savings (τ_m) we have:

Proposition 2 *(i) Life-time utility under true preferences (Ω^*) can be increased relative to laissez-faire ($\tau_m = 0$) by setting a mandatory contribution rate for the middle-aged $\tau_m \in (\underline{\tau}_m, \overline{\tau}_m]$, and (ii) The level of saving maximizing true life-time utility, s_m^*, can be implemented by a mandatory pension contribution rate τ_m^* iff $\widetilde{\delta}R_b > \delta^* R$.*

[30] For $\tau_m > \overline{\tau}_m$, voluntary saving and old-age consumption are, of course, not, in general, linear in τ_m.

The logic for the first part of Proposition 2 follows from the discussion above. Voluntary retirement saving by the middle-aged is crowded out one-to-one: present-biased agents do not agree that saving should be increased, and the returns on voluntary and mandatory savings are the same. Mandatory pension savings becomes effective only when voluntary retirement saving is driven to the zero corner, at which point there is no crowding out and total saving (and old age consumption) can be increased.

Can we justify a τ_m at which the middle-aged are driven to borrow? The intuition is that borrowing takes place at a higher rate than what saving earns, which reduces old age consumption and true utility. The need to borrow arises due to the conflict between true and choice utility. The undoing of the mandate is governed by choice utility: the agent is attempting to counter the forced shift of consumption, possibly from middle to old-age, by borrowing to "protect" middle-age consumption even though it results in lower old-age consumption. This is why there is never a welfare case for a τ_m where the middle-aged are driven to borrow.

To see this cleanly, consider Figure 7a which lays out, as a starting point, the budget constraint under laissez faire, $c_m + \frac{c_o}{R} = w_m$ (the segment QK) with endowment point (w_m, Rw_m). The agent, since he has no old-age income, chooses point A, saving an amount $D = s_m$. Now a small τ_m is introduced assuring the agent future pension wealth, $R\tau_m w_m$. His endowment point becomes $((1-\tau_m)\, w_m, R\tau_m w_m)$. Under this arrangement, if he borrows, his budget constraint becomes $c_m + \frac{c_o}{R_b} = (1-\tau_m)\, w_m + \frac{R\tau_m w_m}{R_b}$ but stays unchanged, $c_m + \frac{c_o}{R} = (1-\tau_m)\, w_m + \frac{R\tau_m w_m}{R} = w_m$ if he saves; hence a kink arises at M, meaning the budget set shrinks to QMN. It is clear, for small τ_m, borrowing is not desirable as choosing a point on the segment MN would put the agent on a lower indifference curve (dotted red indifference curve) than at A. As such, the agent continues to choose point A, D is unchanged except $s_m = D - \tau_m w_m$ falls, one for one. As τ_m rises, all the way to $\underline{\tau}_m$, the endowment point moves leftward (his budget set shrinks to QAG). The agent continues to choose point A, but at $\underline{\tau}_m$, he is at a zero voluntary saving corner, $s_m = 0$, with all saving being mandated, $D = \tau_m w_m$.

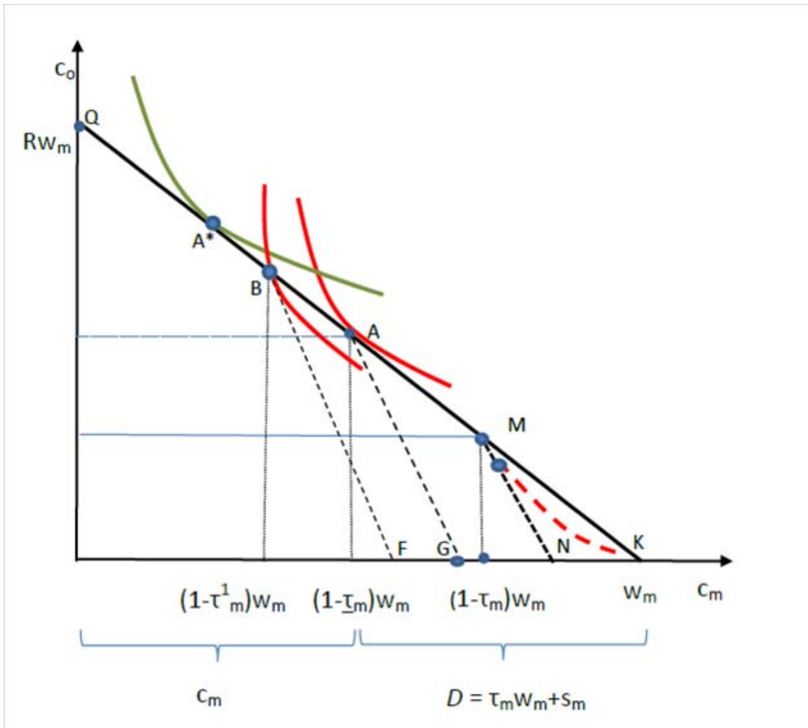

Figure 7a: Changing τ_m

Note, true utility (see the flatter, green indifference curve) is maximized at A* on the segment QAK. It is also apparent true utility can never be maximum except on the segment QAK. In particular, any policy trying to get the agent close to A* cannot hope to succeed if it incentivizes him to locate in the interior of the set QAK. If τ_m is raised slightly beyond $\underline{\tau}_m$ to say, τ_m^1, the budget set shrinks further to QBF. Now the agent chooses point B (since A is no longer attainable), still at the zero-saving corner. Clearly, B has less c_m and more c_o than at point A. B is also closer to A*, the optimal point from the standpoint of true preferences.

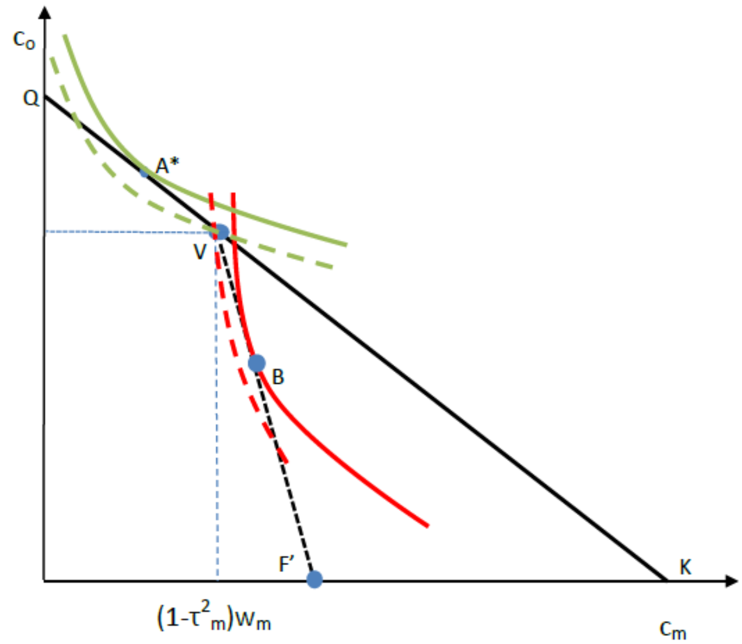

Figure 7b: An over ambitious τ_m

It is also clear why being too aggressive with raising τ_m is a bad idea - see Figure 7b. If τ_m crosses $\overline{\tau}_m$, to say, τ_m^2, the budget set shrinks further to QVF' and now the agent finds point B more desirable than point V (on the lower, dashed red indifference curve; V is at the zero-saving corner). Furthermore, point B necessitates borrowing, leaving the agent with even less c_o than before (hence, defeating the entire purpose of the mandated scheme!) Notice, how the agent is taken further away from A* with such strong a mandate: true utility would have been higher if point V could be chosen.

The second part of Proposition 2 shows, it is possible to implement the optimal level of savings s_m^* (one maximizing true life-time utility Ω^*) iff $\widetilde{\delta} R_b > \delta^* R$ or $\frac{R_b}{R} > \frac{\delta^*}{\widetilde{\delta}}$. Intuitively, any $\tau_m \in (\underline{\tau}_m, \overline{\tau}_m]$ can be implemented; since agents are driven to the zero retirement-saving corner in this range, any such τ_m also delivers higher utility than under laissez-faire. Indeed, the τ_m consistent with optimal saving (s_m^*) must also lie in this interval. The gap between the level of laissez-faire and optimal saving is clearly larger, the larger the ratio of the subjective discount rates, $\frac{\delta^*}{\widetilde{\delta}}$. The zero-saving interval is wider, the bigger the ratio $\frac{R_b}{R}$. Hence, the condition $\left(\widetilde{\delta} R_b > \delta^* R\right)$ essentially renders the zero-saving interval wide enough to place s_m^* in the interior.

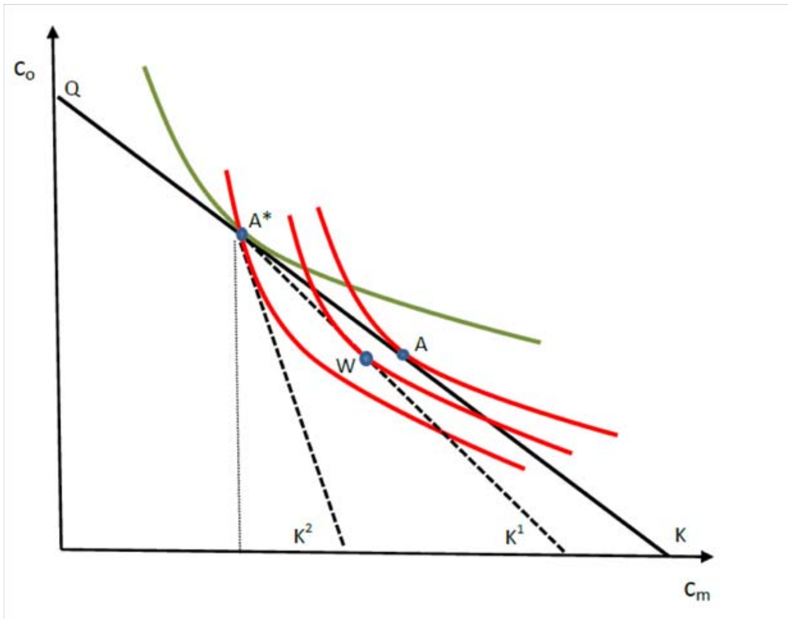

Figure 7c: Importance of gap between R and R_b

Proposition 2 also shows that present-biased preferences are necessary but not sufficient to derive a welfare case for mandatory pensions. The capital market imperfection is also crucial. Consider Figure 7c where the budget set under perfect capital markets is QA*K and the agent chooses point A. If there is a small gap between R and R_b, the budget set becomes QA*K^1 causing the agent to move to W. However, if the market imperfection is sufficiently strong, the relevant budget set would be, say, QA*K^2, in which case, the agent would choose point A* – in this case, the true optimal saving level can be implemented. Increasing R_b for a given R increases the interval where the zero corner arises thereby creating a welfare case for mandatory pensions. In the limit, letting $R_b \to \infty$ (no borrowing allowed, as in most of the literature discussed in the introduction) eliminates borrowing, and as such, old-age consumption is monotonically increasing in the contribution rate (above $\underline{\tau}_m$). In that case, the optimal saving level can always be implemented.

The above discussion makes clear that it matters to what extent agents can undo the mandates on pension saving by borrowing. Our assumption, $R_b > R$, restricts but does not eliminate such undoing. One may think of alternative restrictions on the ability to borrow against future pension wealth. Qualitatively, all such restrictions would effectively yield the same sort of implication, producing a zero-saving corner which, again, would prevent the agent from fully undoing the mandate.[31]

Our finding that there is never a welfare case for policy mandates compelling the middle-aged to borrow suggests the answer to the title of the paper should be, no. The answer to the question, it turns out, is not that simple, for reasons developed in the next section.

[31] If in addition to the return difference, is imposed an upper limit limit on borrowing, one that is dependent on pension wealth, there will be two potential corners, the usual zero-savings corner, and a new one where borrowing is at the maximum allowed level. Even in that case, our central insight that a welfare case for mandated pensions on the middle-aged appears once they are at the zero-saving corner, will continue to hold.

5 Mandatory pension contributions

Could there ever be a welfare case for mandating the young to save when they are simultaneously borrowing, that too, at a higher rate? Since most mandated pension schemes have age-independent contribution rates, there is the additional question of whether such age-constrained schemes can improve welfare.

Any welfare case for mandated pensions must rest on voluntary saving for old age being too low – see Proposition 1. In the two-period model of Section 4, we show it is always possible to increase true life-time utility by mandating contributions for the middle-aged. Also, such mandates work only when voluntary retirement saving is pinned to the zero corner with no possibility of crowding out. By contrast, in a three-period model, saving decisions are made both as young and middle-aged, and as demonstrated above – c.f. (8) – the higher the borrowing as young, the smaller is the borrowing as middle-aged. If the government could restrict borrowing by the young, it could solve their overborrowing problem and be left to contend with the undersaving problem of the middle-aged. This imperative, however, has to be balanced against the consideration that the young are natural borrowers and do not like the borrowing restriction.[32]

To see this more clearly, for the moment set $\tau_y = 0$ and focus on the imposition of τ_m. What governs the choice of such a τ_m? Recall, if the middle-aged have positive voluntary retirement savings, an increase in τ_m is offset one-for-one by a decline in voluntary saving thereby keeping total saving, and hence true utility, unchanged. Once τ_m has been raised sufficiently, voluntary retirement saving is driven to the zero corner, and from there on, crowding out is absent and any further increases in τ_m raises total retirement saving via increases in the mandated part. This increase in total retirement saving helps true utility. What might mandating some contribution from the young do? Since the middle-aged are at the zero voluntary retirement savings corner, ceteris paribus, an increase in future pension wealth induces the young to borrow more. Under our assumptions, the middle-aged remain pinned at the zero voluntary retirement-saving corner while total saving for old age rises. The higher borrowing by the young, that too at the higher rate (R_b), hurts true utility but that is counterbalanced by the gain in true utility resulting from higher total retirement saving. Mandating retirement saving on the middle-aged would provide them much-needed commitment but should the young be left alone or co-opted into this? This *tension* is the subject matter of the rest of the paper.

To set the scene, consider the effect on welfare from changing the pension contribution rates for the young and the middle-aged. Continue to focus on a case with $b_y > 0$ and $s_m > 0$. True utility using (1) and substituting in consumption levels in terms of b_y and s_m is

$$\Omega^* = u\left(w_y(1-\tau_y)+b_y\right)+\delta^* u\left(w_m(1-\tau_m)-R_b b_y - s_m\right)+\left(\delta^*\right)^2 u\left(Rs_m + R^2\tau_y w_y + R\tau_m w_m\right).$$

[32] This sort of simultaneous borrowing and saving, although in very different contexts but for similar underlying reasons, is also centerstage in Morduch (2010) and Basu (2016). In Morduch (2010), for example, it may be desirable for families to borrow even when they have enough savings, just because "it is easier to repay a moneylender than to "repay" oneself."

$$(13)$$

It follows that

$$\frac{\partial \Omega^*}{\partial \tau_y} = u'(c_y)\left(-w_y + \frac{\partial b_y}{\partial \tau_y}\right) + \delta^* u'(c_m)\left(-R_b \frac{\partial b_y}{\partial \tau_y} - \frac{\partial s_m}{\partial \tau_y}\right) + (\delta^*)^2 u'(c_o)\left(R\frac{\partial s_m}{\partial \tau_y} + R^2 w_y\right),$$
$$\frac{\partial \Omega^*}{\partial \tau_m} = -u'(c_y)\frac{\partial b_y}{\partial \tau_y} + \delta^* u'(c_m)\left(-w_m + R_b \frac{\partial b_y}{\partial \tau_y} - \frac{\partial s_m}{\partial \tau_m}\right) + (\delta^*)^2 u'(c_o)\left(R\frac{\partial s_m}{\partial \tau_m} + R w_m\right).$$

Using $-u'(c_y) + \Lambda_a u'(c_o) = 0$ – see (11) and (12) – and $u'(c_m) = \widetilde{\delta} R u'(c_o)$, we can derive

$$\frac{\partial \Omega^*}{\partial \tau_y} = \left[\left(\Lambda_a - R_b \delta^* \widetilde{\delta} R\right)\frac{\partial b_y}{\partial \tau_y} + R\delta^*\left(\delta^* - \widetilde{\delta}\right)\frac{\partial s_m}{\partial \tau_y} + \left((\delta^*)^2 R^2 - \Lambda_a\right) w_y\right] u'(c_o), \tag{14}$$
$$\frac{\partial \Omega^*}{\partial \tau_m} = \left[\left(\Lambda_a - \delta^* \widetilde{\delta} R R_b\right)\frac{\partial b_y}{\partial \tau_m} + \delta^* R\left(\delta^* - \widetilde{\delta}\right)\frac{\partial s_m}{\partial \tau_m} + \delta^* R\left(\delta^* - \widetilde{\delta}\right) w_m\right] u'(c_o). \tag{15}$$

The term $w_y - \frac{\partial b_y}{\partial \tau_y}$ comprises of two parts: the part w_y adds to pension wealth when old; the part $-\frac{\partial b_y}{\partial \tau_y}$ captures the effect of a change in the young's contribution rate on the starting wealth of the middle-aged. The term $w_m + \frac{\partial s_m}{\partial \tau_m}$ captures the effect of a change in the middle-aged's contribution rate on pension wealth of the old. These terms capture the direct budget effects of the contribution rates. In addition, there are indirect effects arising from the interaction between the saving decisions of the young and middle-aged captured in Λ_a. For example, a higher τ_y induces the middle-aged to save less ($\frac{\partial s_m}{\partial \tau_y} < 0$) which hurts true welfare since the middle-aged were undersaving ($\delta^* - \widetilde{\delta} > 0$) to begin with. Similarly, for the other indirect effects.

It is difficult to derive analytical results on the optimal combination of τ_y and τ_m maximizing social welfare satisfying (14)-(15). Part of the reason for this difficulty is the presence of the term, Λ_a, which, at this level of generality, is a function of τ_y and τ_m. Below, we make the assumption that u is homothetic, in which case, Λ_a becomes independent of the contribution rates. In that case, we are able to pursue two manageable questions of great policy significance. First, is there ever a welfare case for imposing mandatory pension contributions on young borrowers ($\tau_y > 0$) if the middle-aged are already mandated (optimally) to save for pensions, $\tau_m > 0$? Put another way, could it be optimal to impose a $\tau_y > 0$, rather than setting a higher τ_m ? If the answer is affirmative, it establishes that the optimal policy has both $\tau_y > 0$ and $\tau_m > 0$ (in general $\tau_y \neq \tau_m$). Second, assuming the contribution rate cannot be made age-dependent (i.e., it must be that $\tau_y = \tau_m = \tau$ must hold), is there a welfare case for introducing a mandatory pension scheme ($\tau > 0$)? If the answer is affirmative, it would imply that the possible net costs from mandating the young to contribute the same as the middle-aged do not dominate the overall welfare gains made by mandatory pensions savings. In either case, the young will, both, save for pensions and borrow, and the mandated pension savings has caused a balanced expansion.

5.1 Should the young be mandated to save for pensions?

We pursue this research question in the following way. Consider a situation where there is only an *optimal* mandated contribution requirement on the middle-aged (age-dependent contribution rates are $\tau_m > 0, \tau_y = 0$). We show in Appendix E that the middle-aged voluntarily save for retirement for $\tau_m < \underline{\tau}_m^a$, and are at the zero retirement-saving corner for $\underline{\tau}_m^a < \tau_m$. As outlined above, we assume henceforth

Assumption u is homothetic.

Under this assumption, it can be checked $D(b_y) = \phi\left(R, \widetilde{\delta}\right)[w_m - R_b b_y + R\tau_y w_y]$ implying $\frac{\partial D(b_y)}{\partial b_y} = -\phi\left(R, \widetilde{\delta}\right) R_b$ and hence, $\Lambda_a = \widetilde{\delta} R\left[\left(\widetilde{\delta} - \delta\right)\left(\frac{\partial D(b_y)}{\partial b_y}\right) + R_b \widetilde{\delta}\right]$ is independent of τ_y and τ_m.

Proposition 3 *(i) If only the middle-aged contribute to the pension scheme ($\tau_y = 0$), there exists a contribution rate $\tau_m \in [\underline{\tau}_m^a, \overline{\tau}_m^a]$ such that middle-aged have zero voluntary savings but total old-age saving is increased and true utility is higher than under laissez-faire.*
(ii) When the middle-aged are at a corner with zero voluntary retirement savings, true utility can be improved by a positive mandated contribution rate for the young, i.e. $\frac{\partial \Omega^*}{\partial \tau_y}|_{\tau_y=0} > 0$ for $\underline{\tau}_m^a < \tau_m < \overline{\tau}_m^a$ under the sufficient condition that $\frac{\delta^*}{\widetilde{\delta}} > \frac{R_b^2}{R^2}$,
(iii) The optimal allocation (b_y^*, s_m^*) cannot be implemented by a choice of (τ_y, τ_m) for $R_b > R$.

The first part of Proposition 3 is a straightforward generalization of Proposition 2 from the two-period case. A contribution rate below $\underline{\tau}_m^a$ is ineffective in raising total retirement saving, since there is complete crowding out. Choosing a $\tau_m > \underline{\tau}_m^a$ drives voluntary retirement saving by the middle-aged to the zero corner, and mandated retirement saving raises total retirement savings, and this improves true utility. Setting $\tau_m > \overline{\tau}_m^a$ is not optimal since it drives the middle-aged to borrow and old-age consumption to fall (see Lemma 2).

The second part of Proposition 3 is more interesting. It finds, in a setting with zero voluntary middle-age retirement saving (meaning any retirement saving is mandated), welfare may be raised by mandating contributions by the young as well. This makes intuitive sense: the middle-aged are already at a corner and requiring the young to save unambiguously increases saving for old age which is desirable due to its sobering effect on the undersaving problem. Of course, mandating the young reduces their consumption, also desirable from a welfare perspective, except it leads them to borrow more (at a rate exceeding that on saving). That curtails middle-age consumption, i.e. they have to save more due to increased debt servicing. Therefore, a trade-off arises between the gain from increasing old-age consumption and the cost in terms of borrowing. A sufficient condition for imposing welfare-enhancing mandatory saving requirements on the young is $\frac{\delta^*}{\widetilde{\delta}} > \frac{R_b^2}{R^2}$, i.e., the present bias in preferences should be sufficiently strong compared to the return difference.

Why is mandating the young to save conceptually different from mandating the middle-aged? Mandating the young to save for pensions increases old-age consumption (with the middle-aged at the zero corner), and that may increase welfare. But requiring the middle-aged to save for pensions may cause them to borrow which, unambiguously, reduces old-age consumption and, hence, defeats the purpose for the mandates.

The final part of Proposition 3 argues that a pre-committed choice of (τ_y, τ_m) cannot simultaneously correct the overborrowing problem of the young and the undersaving problem of the middle-aged. Mandated saving policies can help reduce the severity of the latter, as we have seen, but the overborrowing problem is made worse by the gap between R_b and R. Broadly speaking, which is more crucial, helping with the undersaving problem of the middle-aged or the overborrowing problem of the young? Two issues arise here. First, standard discounting would suggest over/under "errors" matter more when the agent has one period left versus when he has two. But counter to that logic is the notion that borrowing "errors" face a higher opportunity cost since $R_b > R$. [33]

5.2 Age-independent contribution rates

Impose, further, the restriction that $\tau = \tau_y = \tau_m$, that is, the contribution rate has to be age-independent. Is there a welfare case for introducing mandatory pensions savings under this restriction? This restriction makes the task at hand harder. On the other hand, if a welfare case exists even under this restriction, it is a strong validation for imposing mandates on the young.

In line with above, we have that the middle-aged have positive saving if $\tau < \underline{\tau}$, negative savings for $\tau > \overline{\tau}$ and are at the corner with zero savings for $\underline{\tau} \leq \tau \leq \overline{\tau}$, c.f. Appendix E. Note evaluating Ω^* for $\tau = 0$ implies the "starting position" is one where the middle-aged are savers ($\tau < \underline{\tau}$). We have

Proposition 4 *(i) Life-time utility is decreasing in the age-independent contribution rate, when the middle aged are savers*

$$\frac{\partial \Omega^*}{\partial \tau} < 0 \text{ for } \tau < \underline{\tau}$$

(ii) Increasing the age-independent contribution rate increases life-time utility when the middle-aged are at the corner

$$\left.\frac{\partial \Omega^*}{\partial \tau}\right|_{\tau=\underline{\tau}} > 0$$

under the sufficient condition that $\delta^* R > \widetilde{\delta} R_b$.

The result is proved in Appendix E. Following previous insights we find that a potential welfare case for mandatory pension saving arises when the middle-aged are at the zero corner. However, this

[33] Guo and Caliendo (2014) argue that a time-inconsistent mandated saving policy, one which deviates from the stated (τ_y, τ_m) and "misleads" the middle-aged as to their required contribution rate, may deliver the optimal (b_y^*, s_m^*).

is not automatic for two reasons. First, even when the middle-aged are at the zero retirement-saving corner, welfare is not necessarily increasing in the contribution rate. A sufficient condition for this to be the case is that $\frac{\delta^*}{\tilde{\delta}} > \frac{R_b}{R}$, i.e. the present bias should be sufficient strong relative to the return differences. Second, even if this condition holds, a welfare case for a positive contribution rate has not been made; after all, welfare is decreasing in the contribution rate when the middle-aged have positive retirement saving because a higher contribution rate does not raise old-age consumption as the saving of both the young and the middle-aged are crowded out. A welfare case for a positive contribution rate thus requires a $\tau \in (\underline{\tau}, \overline{\tau}]$ such that

$$\Omega^*(\tau = 0) < \Omega^*(\tau) \text{ for } \underline{\tau} < \tau \leq \overline{\tau} \tag{16}$$

Clearly, it is, in general, an open question whether the condition (16) is fulfilled.

The log-utility case (see Eqs. (21)-(22) in Appendix C which hold for $b_y > 0$ and $s_m > 0$) can be used to illustrate the effects of an age-independent contribution rate. Figure 8 shows how true life-time utility depends on the age-independent contribution rate. Starting from zero, increasing the contribution rate decreases welfare. This is the case as long as voluntary savings by the middle-aged is positive. Upon reaching a contribution rate where the middle-aged are at the zero savings corner, welfare starts increasing. Note, for the particular case shown, welfare is higher with a positive contribution rate, i.e. condition (16) holds.

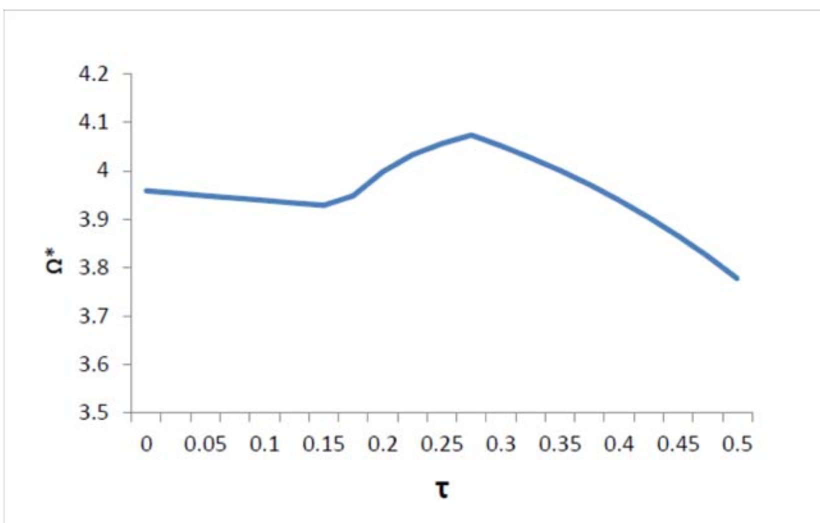

Figure 8: True utility against τ

6 Concluding remarks

The standard life-cycle pattern of consumption and saving is that the young borrow and the middle-aged are net-savers. Present-biased preferences imply the young borrow too much, and the middle-aged save too little compared to the optimal choices under the true preferences of the young. This provides an argument for mandatory pension savings, not only for the middle-aged but also for the young. This may hold even if the contribution rate is constrained to be age-independent. We show why present-bias is necessary but not sufficient to rationalize mandated saving; the extent of the credit market imperfection is crucial. A balance expansion of the portfolio over the life-cycle is a consequence, agents borrow and save more, and despite a higher borrowing than savings rate of return, welfare is higher. Moreover, we show that the welfare case for mandated pension savings does not rely on the specific source (myopia or hyperbolic discounting) of present-biased preferences. This is reassuring from a policy perspective, since it can be difficult to empirically distinguish between the two.

One may wonder if there are important transition issues we are sidestepping, meaning what if the government were to usher in these schemes once agents have made their saving/borrowing decisions? Within the confines of our model, this would not pose a problem. Under laissez faire, even time-inconsistent agents would save a positive amount for old age consumption anyway. Were the government to ask them to make a mandated contribution, they would simply deduct the amount from their voluntary saving (recall, the two offer the same return) leaving them unaffected. As we have shown, once the mandate is high enough, the voluntary retirement saving disappears and further increases in the contribution mandate raises agents' welfare. Problems would emerge if the government mandate was so aggressive as to warrant borrowing by the middle aged, but as we have shown, there is no welfare case to choose such a high mandate. An implication of this idea is the following. Suppose there were some agents who did not suffer from time inconsistency. The welfare of such agents under laissez faire and under the government mandate would be identical.[34]

It may also appear as a concern that the sort of mandatory saving program we discuss, one so large as to force the middle aged to a zero retirement-saving corner, would be too intrusive and arguably infeasible. Bear in mind, though, that the zero-voluntary-retirement-saving corner we speak of is restricted to saving for life-cycle consumption smoothing purposes alone. If there are other legitimate reasons for saving such as a need for flexibility or precautionary – see footnote 15) – then they need to be included as well. Our focus, of course, is on inadequacies in saving-for-retirement alone and the use of mandated schemes to that end.

Are pension mandates Pareto-improving? Consider the notion of Pareto efficiency defined on multiple selves discussed in Luttmer and Mariotti (2007). There, an allocation is Pareto inefficient if there exists another feasible allocation which makes at least one self better off and no self worse off. In our setting, consider as a starting point, the allocation – call it laissez faire – chosen by a sophisticated myopic Self 1. (No feasible allocation, of course, can improve upon the choice utility of the naive Self 1). Our results show, another allocation, one involving mandatory contribution rates on both the young and the middle-aged, improves on laissez faire true utility of Self 1. What about Self 2? Self 2 clearly benefits from the commitment value of mandatory saving but it is also true that, under mandatory pensions, his consumption is lowered (even though his future self, Self 3 has higher consumption and is happier). On net, it is not obvious whether Self 2 is necessarily

[34] In practice, governments could simply exempt those who can show adequate saving-for- retirement from the scheme.

happier than under laissez faire. In short, it is not obvious whether pension mandates constitute a Pareto improvement.[35]

How would our results change if we allowed for endogenous factor prices? Suppose we allowed for neoclassical production using capital (and labor) as inputs so that under competitive markets, the wage rate and the interest rate on saving depended, in a standard way, on the capital-labor ratio as in a Diamond model. In such a setting, a higher contribution rate imposed on the middle-aged would reduce their voluntary retirement saving, one for one, while their pension funds hold proportionately more capital, leaving aggregate capital unchanged.[36] Again, as above, once the middle-aged are driven to a zero voluntary retirement-saving corner, aggregate capital (held entirely by the pension funds) can go up, raising wages but lowering interest rates. These last two general-equilibrium effects will complicate matters by changing the size of the pension itself, and so on. The central insight, that the middle-aged have to be driven to the corner for mandatory pensions to work, is untouched, though.

Our findings suggest optimal contribution rates are age-dependent, somewhat counter to what is observed. Our model is sparse, for one. Besides practical and administrative reasons, there is the argument that the life-cycle pattern for earnings will not be the same for all, and therefore a simple age-dependent system will not be able to capture individual heterogeneities. For this reason an important issue for future research is the implications of heterogeneity not only across the earnings dimension but also with respect to preferences. (On a technical level, the latter may also be used to argue that discontinuities in saving at the individual level – the zero corner – may disappear at the aggregate level.) How should mandatory pension systems be designed if the degree of present-bias is different across the population (including that some may not suffer from present bias). It appears the mandated schemes we study may help those with very strong present biases – the government, by mandating pension contribution can prevent the Samaritan's dilemma of agents leaving too little for old age in the hope they will be "bailed out". It is important to note here that the schemes we study will not hurt the agents with zero or low present-bias: if their savings, to begin with, are high enough, the mandate will simply not be binding.

Finally, balance expansion – the simultaneous expansion of the asset (in our case, pension wealth) and liability side (say, household debt) of a balance sheet – may also have consequences for macroeconomic stability because assets and liabilities have different maturity structures, pension assets being highly illiquid, and available only after retirement and household debt is highly liquid and subject to recall – see Andre (2016). This angle is also worthy of separate inquiry.

[35] Caliendo and Findley (2016b) study the issue of multiple-self Pareto efficiency and find that much depends on the frequency of choice – if large, as in a full-blown lifecycle model, it is more likely that the commitment allocation is preferred by later selves. It would be interesting – an issue we leave to future research – to compute the possible set of commitment allocations that satisy multi-self Pareto efficiency.

[36] This was not the case in Andersen and Bhattacharya (2011) where the alternative to private saving was return-dominated unfunded social security. As such, as private capital got crowded out, its return rose, thereby preventing agents from hitting the zero-capital corner. There too, sufficient myopia was needed to get a welfare role for PAYG pensions.

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Appendix

A Borrowing by the young

The savings decision of the middle-aged (s_m) depends on borrowing as young (b_y), and this is perceived by the (sophisticated) young. Hence, the two decisions are interconnected. The main text works with the case where agents in the absence of mandatory pension savings are borrowers as young ($b_y > 0$) and savers as middle-aged ($s_m > 0$), since this is the case where the welfare rationale for mandatory savings (especially for the young) may be called into question. If the young are savers, there is no essential difference between being young and middle-aged and thus between the two- and three-period model.

In the absence of mandatory savings, it is trivial that the middle-aged are saving ($s_m > 0$) since this is the only way of ensuring consumption when old. From the main text – see eqn. (10), we have that

$$\frac{\partial \Omega}{\partial b_y} = u'(c_y) + \widetilde{\delta}\left[\delta - \widetilde{\delta}\right] R u'(c_o)\frac{\partial s_m}{\partial b_y} - \widetilde{\delta} R_b u'(c_m)$$

If $\frac{\partial \Omega}{\partial b_y}\mid_{b_y=0} > 0$ it follows that $b_y > 0$ which in turn requires

$$u'(w_y) > \widetilde{\delta} R_b u'(w_m - s_m) - \widetilde{\delta}\left[\delta - \widetilde{\delta}\right] R u'(R s_m)\frac{\partial s_m}{\partial b_y}$$

or using that $\widetilde{\delta} R u'(R s_m) = u'(w_m - s_m)$

$$u'(w_y) > u'(w_m - s_m)\left[\widetilde{\delta} R_b - \left[\delta - \widetilde{\delta}\right]\frac{\partial s_m}{\partial b_y}\right]$$

Note that $\widetilde{\delta} R_b - \left[\delta - \widetilde{\delta}\right]\frac{\partial s_m}{\partial b_y} < \delta R_b$ since $\frac{\partial s_m}{\partial b_y} > -R_b$. A sufficient condition ensuring $b_y > 0$ is thus

$$u'(w_y) > u'(w_m - s_m)\delta R_b \tag{17}$$

where s_m is determined from: $u'(w_m - s_m) = R\widetilde{\delta} u'(R s_m)$, hence $0 < s_m < w_m$. Hence, condition (17) holds if the wage as young is not too high, i.e.

$$w_y \leq \overline{w}_y(w_m, \delta, \widetilde{\delta}, R, R_b) \tag{18}$$

B Overborrowing by the young and undersaving by the middle-aged

This appendix proves that agents "overborrow" when young ($b_y > b_y^*$) and "undersave" when middle-aged ($s_m < s_m^*$) in the absence of any pension scheme ($\tau_y = \tau_m = 0$). The savings decision of the middle-aged is determined by: $-u'(w_m - R_b b_y - s_m) + \widetilde{\delta} R u'(R s_m) = 0$ and since $u'(R s_m) \to \infty$ for $s_m \to 0$, it follows that $s_m > 0$. Intuitively, the only way by which the individual can ensure consumption as old is by saving as middle-aged. Under the true preferences we have that savings is determined by $-u'\left(w_m - R_b b_y^* - s_m^*\right) + \delta^* R u'(R s_m^*) = 0$. The savings as middle-aged depends on borrowing as young and we have $s_m = \psi(b_y)$, $s_m^* = \psi^*(b_y^*)$. Notice, that $\psi(b_y) < \psi^*(b_y)$

since $\widetilde{\delta}R < \delta^* R$, and both functions are decreasing in their argument.

The first order condition for the borrowing decision by the young (11)

$$-u'(c_y) + \widetilde{\delta}R\left[\left(\widetilde{\delta} - \delta\right)\left(\frac{\partial D(b_y)}{\partial b_y}\right) + R_b\widetilde{\delta}\right]u'(c_o) = 0$$

can by use of $u'(c_m) = \widetilde{\delta}Ru'(c_o)$ be written

$$-u'(w_y + b_y) + \left[\left(\widetilde{\delta} - \delta\right)\left(\frac{\partial D(b_y)}{\partial b_y}\right) + R_b\widetilde{\delta}\right]u'(w_m - R_b b_y - \psi(b_y)) = 0;$$

denote the solution by b_y^c.

For the true preferences, life-time utility depends on borrowing as young as

$$\frac{\partial\Omega^*}{\partial b_y^*} = u'\left(w_y + b_y^*\right) - \delta^* R_b u'\left(w_m - R_b b_y^* - \psi^*(s_y^*)\right)$$

Evaluate this derivative, for the borrowing decision of the young under the choice preferences b_y^c

$$\begin{aligned}&\frac{\partial\Omega^*}{\partial b_y^*}\mid_{b_y^c} = u'(w_y + b_y) - \delta^* R_b u'(w_m - R_b b_y - \psi^*(b_y))\\ &= \left[-\left(\widetilde{\delta} - \delta\right)\left(-\frac{\partial D(b_y)}{\partial b_y}\right) + R_b\widetilde{\delta}\right]u'(w_m - R_b b_y - \psi(b_y)) - \delta^* R_b u'(w_m - R_b b_y - \psi^*(b_y))\end{aligned} \tag{19}$$

Note that the largest possible value of $-\frac{\partial D(b_y)}{\partial b_y} = R_b$ which implies

$$\left[-\left(\widetilde{\delta} - \delta\right)\left(-\frac{\partial D(b_y)}{\partial b_y}\right) + R_b\widetilde{\delta}\right] \leq -\left(\widetilde{\delta} - \delta\right)(R_b) + R_b\widetilde{\delta} = \delta R_b < \delta^* R_b \tag{20}$$

Since $\psi^*(b_y) > \psi(b_y) \Leftrightarrow u'(w_m - R_b b_y - \psi(b_y)) < u'(w_m - R_b b_y - \psi^*(b_y))$, using (20) in (19) yields

$$\frac{\partial\Omega^*}{\partial b_y^*}\mid_{b_y^c} < 0$$

It follows $b_y^* < b_y$ and, therefore, by implication $s_m^* > s_m$.

C The separate role of δ and β

For future use in determining a welfare case for mandated saving, it is important to understand how saving by the different selves depend on the source of the present bias, i.e., the separate role of myopia ($\delta \leq \delta^*$) and quasi-hyperbolic discounting ($\beta < 1$). It is easiest to see this in the context of a concrete example.

Log utility: If the utility function is $u(c_i) = \ln c_i$ for $i = y, m, o$, then assuming $b_y > 0$ and

$s_m > 0$, it can be checked that

$$b_y = -\frac{1}{1+\frac{1}{\widetilde{\delta}(1+\delta)}}\left[\left\{1-\tau_y\left(1+\frac{1}{\widetilde{\delta}\,(1+\delta)}\frac{R}{R_b}\right)\right\}w_y - \frac{1}{\widetilde{\delta}R_b\,(1+\delta)}w_m\right], \tag{21}$$

$$s_m = \frac{1}{1+\frac{1}{\widetilde{\delta}(1+\delta)}}\frac{1}{1+\widetilde{\delta}}\left[\left\{\widetilde{\delta}-\tau_m\left(\widetilde{\delta}+\frac{1}{\widetilde{\delta}}\frac{1+\widetilde{\delta}}{1+\delta}+1\right)\right\}w_m + \left\{\widetilde{\delta}R_b - \tau_y\left(\widetilde{\delta}R_b + R\frac{1}{\widetilde{\delta}}\frac{1+\widetilde{\delta}}{1+\delta}+R\right)\right\}w_y\right] \tag{22}$$

To see the role of quasi-hyperbolic discounting ($\beta < 1 \Longrightarrow \delta < \widetilde{\delta}$), consider a decrease in β, which, for given $\widetilde{\delta}$, corresponds to an increase in δ. It follows straightforwardly that

$$\left.\frac{\partial b_y}{\partial \delta}\right|_{\widetilde{\delta}} < 0; \quad \left.\frac{\partial s_m}{\partial \delta}\right|_{\widetilde{\delta}} > 0$$

i.e., hyperbolic discounting tends to decrease borrowing as young and raise saving as middle-aged. The intuition is, the sophisticated young perceives her middle-aged self will be present-biased and therefore saves too little. By decreasing borrowing when young, starting wealth as middle-aged increases, which in turn incentivizes the middle-aged to save more. Reduced borrowing (or increased saving) as young thus works as a commitment device which helps with the time inconsistency but not enough to remove the undersaving issue discussed above. Also note, the magnitude of these responses do depend on the contribution rates.

In passing, note that evaulating b_y using (21) at $\tau_y = 0$ yields

$$b_y \geq 0 \Leftrightarrow w_y \leq \frac{1}{\widetilde{\delta}R_b\,(1+\delta)}w_m$$

a precise condition derived implicitly earlier in (18).

D Two period model

Define $\underline{\tau}_m$ as the contribution rate at which voluntary savings for the middle-aged is exactly zero, i.e.

$$u'((1-\underline{\tau}_m)w_m) = R\widetilde{\delta}u'(R\underline{\tau}_m w_m)$$

it follows that $s_m > 0$ for $\tau_m < \underline{\tau}_m$ since $u'((1-\tau_m)w_m) < R\widetilde{\delta}u'(R\tau_m w_m)$ for $\tau_m < \underline{\tau}_m$.

Similarly, define $\overline{\tau}_m$ as the contribution rate at which voluntary borrowing is exactly zero, i.e.

$$u'((w_m(1-\overline{\tau}_m)) = \widetilde{\delta}R_b u'(R\overline{\tau}_m w_m)$$

i.e. for $\tau_m > \overline{\tau}_m$ individuals will borrow since $u'((w_m(1-\tau_m)) > \widetilde{\delta}R_b u'(R\tau_m w_m)$.

For $\tau_m < \underline{\tau}_m$, we have

$$\frac{\partial\Omega^*}{\partial\tau_m}\mid_{s_m>0} = -u'(c_m)\left[w_m + \frac{\partial s_m}{\partial\tau_m}\mid_{s_m>0}\right] + \delta^* u'(c_o)\left[w_m + \frac{\partial s_m}{\partial\tau_m}\mid_{s_m>0}\right]R$$

and hence using that $w_m + \frac{\partial s_m}{\partial \tau_m} \mid_{s_m>0}= 0$ it follows that

$$\frac{\partial \Omega^*}{\partial \tau_m} \mid_{s_m>0}= 0 \text{ for } \tau_m < \underline{\tau}_m$$

For $\underline{\tau}_m \leq \tau_m \leq \overline{\tau}_m$ (implying $s_m = 0$) we have

$$\frac{\partial \Omega^*}{\partial \tau_m} \mid_{s_m=0}= -u'(c_m)w_m + \delta^* u'(c_o)w_m R$$

and hence evaluated for $\tau_m = \underline{\tau}_m$

$$\frac{\partial \Omega^*}{\partial \tau_m} \mid_{\underline{\tau}_m}= \left[\delta^* - \widetilde{\delta}\right] u'(c_o)w_m R > 0$$

This proves that there exists a contribution rate for the middle-aged $\tau_m > \underline{\tau}_m > 0$ which delivers higher welfare than in the absence of mandatory pensions savings.

Finally, consider the case $\tau_m > \overline{\tau}_m$, i.e. the contribution rate is so high that the middle-aged become borrowers ($s_m < 0$) the situation is more complicated since private voluntary savings is no longer zero but negative. Increasing the contribution rate thus makes the middle-aged borrow, and it is not clear that welfare in net terms can be increased. We have

$$\frac{\partial \Omega^*}{\partial \tau_m} \mid_{s_m<0}= -u'(c_m)\left[w_m + \frac{\partial s_m}{\partial \tau_m} \mid_{s_m<0}\right] + \delta^* u'(c_o)\left[w_m R + R_b \frac{\partial s_m}{\partial \tau_m} \mid_{s_m<0}\right] < 0$$

which follows from noting $w_m + \frac{\partial s_m}{\partial \tau_m} \mid_{s_m<0}> 0$ and $w_m R + R_b \frac{\partial s_m}{\partial \tau_m} \mid_{s_m<0}< 0$. In short, welfare can never be improved by choosing a contribution rate implying that the middle-aged borrow to undo part of the mandatory pension saving.

To consider the implementation of the first best, i.e. the savings level under the true preferences s_m^*, define

$$\tau_m^* \equiv \frac{s_m^*}{w_m}$$

i.e. the contribution rate τ_m^* implements the optimal savings level s_m^*, if voluntary savings is zero. Since net saving is unaffected by mandatory pension savings for $\tau_m < \underline{\tau}_m$, it follows from $s_m^* > s_m(\tau_m = 0)$ that $\tau_m^* > \underline{\tau}_m$. For $\underline{\tau}_m \leq \tau_m \leq \overline{\tau}_m$ households will be at the corner, neither voluntary savings nor borrowing – this obtains when τ_m^* obeys

$$\widetilde{\delta} R_b > \frac{u'(w_m(1-\tau_m^*))}{u'(R w_m \tau_m^*)} > \widetilde{\delta} R$$

Recall ($\tau_m^* w_m = s_m^*$) optimal savings is given by

$$\frac{u'(w_m - s_m^*)}{u'(R s_m^*)} = \delta^* R$$

Hence $\underline{\tau}_m \leq \tau_m^* \leq \overline{\tau}_m$ iff

$$\widetilde{\delta} R_b > \delta^* R > \widetilde{\delta} R$$

The last inequality holds always, and the first requires $\frac{\delta^*}{\widetilde{\delta}} < \frac{R_b}{R}$.

E Appendix: Age dependent contribution rates

The following considers whether the young should be mandated to contribute to pension savings, when there is mandatory pension contributions for the middle-aged (τ_m). To this end we first need to establish that it is optimal to make the middle-aged contribute, and next consider whether the young should also be asked to contribute. Similar to above, define $\underline{\tau}_m^a$ and $\overline{\tau}_m^a$ as the critical contribution rates which delineates the positive savings, savings corner and negative savings for the middle-aged, i.e.

$$
\begin{aligned}
\underline{\tau}_m^a \quad &: \quad u'\left(\left(1-\underline{\tau}_m^a\right) w_m - R_b b_y\right) = \widetilde{\delta} R u'\left(R \underline{\tau}_m^a w_m\right) \\
\overline{\tau}_m^a \quad &: \quad u'\left(\left(1-\overline{\tau}_m^a\right) w_m - R_b b_y\right) = \widetilde{\delta} R_b u'\left(R \overline{\tau}_m^a w_m\right)
\end{aligned}
$$

Case I: Middle-aged are saving ($s_m > 0$)

When the middle-aged are savers ($s_m > 0$) we have that savings (b_y, s_m) for $\tau_y = 0$,$\tau_m > 0$ are determined by

$$
\begin{aligned}
u'\left(w_y + b_y\right) &= \widetilde{\delta} R \left[\left(\widetilde{\delta} - \delta\right)\left(\frac{\partial D\left(b_y\right)}{\partial b_y}\right) + R_b \widetilde{\delta}\right] u'\left(R s_m + R \tau_m w_m\right) \\
u'\left(w_m - R_b b_y - \tau_m w_m - s_m\right) &= \widetilde{\delta} R u'\left(R s_m + R \tau_m w_m\right)
\end{aligned}
$$

This system determines borrowing as young b_y and total savings as middled-aged $D \equiv s_m + \tau_m w_m$, hence $\frac{\partial s_m}{\partial \tau_m} = -w_m$ as long as the condition $s_m > 0$ holds, and therefore $\frac{\partial (s_m + \tau_m w_m)}{\partial \tau_m} = 0$ which in turn implies that $\frac{\partial b_y}{\partial \tau_m} = 0$. To obtain the comparative statics with respect to τ_y, differentiate each of the optimality conditions above. Tedious algebra shows

$$
\begin{aligned}
-u''\left(c_y\right) \frac{\partial b_y}{\partial \tau_y} + R \Lambda_a u''\left(c_o\right) \frac{\partial s_m}{\partial \tau_y} &= -w_y u''\left(c_y\right) - u''\left(c_o\right) R^2 w_y \Lambda_a \\
R_b u''\left(c_m\right) \frac{\partial b_y}{\partial \tau_y} + \left[u''\left(c_m\right) + \widetilde{\delta} R^2 u''\left(c_o\right)\right] \frac{\partial s_m}{\partial \tau_y} &= -\widetilde{\delta} R R^2 w_y u''\left(c_o\right)
\end{aligned}
$$

from where the signs of $\frac{\partial b_y}{\partial \tau_y}$ and $\frac{\partial s_m}{\partial \tau_y}$ are established.

Case II: Middle-aged at the corner ($s_m = 0$)

For $\underline{\tau}_m^a \leq \tau_m \leq \overline{\tau}_m^a$, $s_m = 0$ and a change in the contribution rate for the middle-aged affect true life-time utility as

$$
\frac{\partial \Omega^*}{\partial \tau_m} \mid_{s_m=0} = -u'(c_m) w_m + \delta^* u'(c_o) w_m R.
$$

Assessing the marginal welfare effect for $\tau_m = \underline{\tau}_m^a$ using $u'(c_m) = R\widetilde{\delta} u'(c_o)$ we find

$$
\frac{\partial \Omega^*}{\partial \tau_m} \mid_{s_m=0, \tau_m = \underline{\tau}_m} = \left(\delta^* - \widetilde{\delta}\right) u'(c_o) w_m R > 0
$$

i.e., true welfare can be improved by setting a contribution rate $\tau_m > \underline{\tau}_m^a$.

Case III: Middle-aged as borrowers ($s_m < 0$)

Assume that $\tau_m^* > \overline{\tau}_m^a$ in this case the middle-aged are borrowers. The life-time utility as perceived when young reads

$$u\left((1-\tau_y)w_y + b_y\right) + \widetilde{\delta}\left[u\left((1-\tau_m)w_m - R_b b_y - s_m\right) + \delta u\left(b + R_b s_m\right)\right]$$

and the first-order condition reads

$$u'\left(c_y\right) + \widetilde{\delta}\left[-u'\left(c_m\right) + \delta R_b u'\left(c_o\right)\right]\frac{\partial s_m}{\partial b_y}\mid_{s_m<0} - \widetilde{\delta} R_b u'\left(c_m\right) = 0$$

by use of $u'\left(c_m\right) = \widetilde{\delta} R_b u'\left(c_o\right)$ it can be written

$$u'\left(c_y\right) = \Lambda_a u'\left(c_o\right)$$

where

$$\Lambda_a \equiv \left[\left(\widetilde{\delta} - \delta\right)\frac{\partial s_m}{\partial b_y}\mid_{s_m<0} + \widetilde{\delta} R_b\right] R_b \widetilde{\delta}$$

Note that

$$\frac{\partial s_m}{\partial s_y}\mid_{s_m<0} = -\frac{R_b u''\left(c_m\right)}{u''\left(c_m\right) + \widetilde{\delta} R_b^2 u''\left(c_o\right)}$$

implying that $\frac{\partial s_m}{\partial b_y} > -R_b$, and hence it follows that $0 < \Lambda_a < \delta R_b R_b \widetilde{\delta}$. With homothetic preferences Λ_a is independent of c_m and c_o.

Writing the first order condition for the borrowing and savings decisions we have

$$\begin{aligned}
u'\left(\left(1-\tau_y\right)w_y + b_y\right) &= \Lambda_a u'\left(R^2\tau_y w_y + R\tau_m w_m + R_b s_m\right) \\
u'\left(\left(1-\tau_m\right)w_m - R_b b_y - s_m\right) &= \widetilde{\delta} R_b u'\left(R^2\tau_y w_y + R\tau_m w_m + R_b s_m\right)
\end{aligned}$$

Differentiating wrt τ_y yields

$$\begin{aligned}
u''\left(c_y\right)\left[-w_y + \frac{\partial b_y}{\partial \tau_y}\right] &= \Lambda_a u''\left(c_o\right)\left[R^2 w_y + R_b\frac{\partial s_m}{\partial \tau_y}\right] \\
u''\left(c_m\right)\left[-R_b\frac{\partial b_y}{\partial \tau_y} - \frac{\partial s_m}{\partial \tau_y}\right] &= \widetilde{\delta} R_b u''\left(c_o\right)\left[R^2 w_y + R_b\frac{\partial s_m}{\partial \tau_y}\right]
\end{aligned}$$

and wrt τ_m

$$\begin{aligned}
u''\left(c_y\right)\left[\frac{\partial b_y}{\partial \tau_m}\right] &= \Lambda_a u''\left(c_o\right)\left[R w_m + R_b\frac{\partial s_m}{\partial \tau_m}\right] \\
u''\left(c_m\right)\left[-w_m - R_b\frac{\partial b_y}{\partial \tau_m} - \frac{\partial s_m}{\partial \tau_m}\right] &= \widetilde{\delta} R_b u''\left(c_o\right)\left[R w_m + R_b\frac{\partial s_m}{\partial \tau_m}\right]
\end{aligned}$$

Writing this in matrix form we have

$$\begin{bmatrix} u''\left(c_y\right) & -\Lambda_b u''\left(c_o\right) R_b \\ -u''\left(c_m\right) R_b & -\left[u''\left(c_m\right) + \widetilde{\delta} R_b^2 u''\left(c_o\right)\right] \end{bmatrix}\begin{bmatrix} \frac{\partial b_y}{\partial \tau_m} \\ \frac{\partial s_m}{\partial \tau_m} \end{bmatrix} = \begin{bmatrix} \Lambda_b u''\left(c_o\right) R w_m \\ \widetilde{\delta} R_b u''\left(c_o\right) R w_m + u''\left(c_m\right) w_m \end{bmatrix}$$

implying

$$\frac{\partial b_y}{\partial \tau_m} = \frac{\begin{vmatrix} \Lambda_a u''(c_o) R w_m & -\Lambda_a u''(c_o) R_b \\ \widetilde{\delta} R_b u''(c_o) R w_m + u''(c_m) w_m & -\left[u''(c_m) + \widetilde{\delta} R_b^2 u''(c_o)\right] \end{vmatrix}}{\begin{vmatrix} u''(c_y) & -\Lambda_a u''(c_o) R_b \\ -u''(c_m) R_b & -\left[u''(c_m) + \widetilde{\delta} R_b^2 u''(c_o)\right] \end{vmatrix}} < 0$$

It follows from the expressions above

$$\begin{aligned} sign\left[-w_m - R_b \frac{\partial b_y}{\partial \tau_m} - \frac{\partial s_m}{\partial \tau_m}\right] &= sign\left[R w_m + R_b \frac{\partial s_m}{\partial \tau_m}\right] \\ sign\left[\frac{\partial b_y}{\partial \tau_m}\right] &= sign\left[-w_m - R_b \frac{\partial b_y}{\partial \tau_m} - \frac{\partial s_m}{\partial \tau_m}\right] \end{aligned}$$

Since $\frac{\partial b_y}{\partial \tau_m} < 0$ it follows that $-w_m - R_b \frac{\partial b_y}{\partial \tau_m} - \frac{\partial s_m}{\partial \tau_m} < 0$ and $w_m R + R_b \frac{\partial s_m}{\partial \tau_m} < 0$. Next note that

$$\frac{\partial \Omega^*}{\partial \tau_m} = \left[\Lambda_a \frac{\partial b_y}{\partial \tau_m} - \delta^* \widetilde{\delta} R_b \left[w_m + R_b \frac{\partial b_y}{\partial \tau_m} + \frac{\partial s_m}{\partial \tau_m}\right] + (\delta^*)^2 \left[w_m R + R_b \frac{\partial s_m}{\partial \tau_m}\right]\right] u'(c_o)$$

and hence $\frac{\partial \Omega^*}{\partial \tau_m} < 0$ for $\tau_m > \overline{\tau}_m^a$. This proves that the optimal $\tau_m \in [\underline{\tau}_m^a, \overline{\tau}_m^a]$.

Pension contributions by the young

Assume that the optimal $\tau_m^* \in [\underline{\tau}_m^a, \overline{\tau}_m^a]$. Can welfare be improved by setting a $\tau_y > 0$? We have (recall $s_m = 0$)

$$\frac{\partial \Omega^*}{\partial \tau_y} = -u'(c_y)\left(w_y - \frac{\partial b_y}{\partial \tau_y}\right) - \delta^* u'(c_m) R_b \frac{\partial b_y}{\partial \tau_y} + (\delta^*)^2 u'(c_o) R^2 w_y \text{ for } \underline{\tau}_m^a \leq \tau_m \leq \overline{\tau}_m^a$$

or

$$\frac{\partial \Omega^*}{\partial \tau_y} = -u'(c_y) w_y - \left[\delta^* u'(c_m) R_b - u'(c_y)\right] \frac{\partial b_y}{\partial \tau_y} + (\delta^*)^2 u'(c_o) R^2 w_y$$

Since $u'(c_y) = \widetilde{\delta} R_b u'(c_m)$ it follows that

$$\delta^* u'(c_m) R_b - u'(c_y) = \left(\delta^* - \widetilde{\delta}\right) R_b u'(c_m) > 0$$

and therefore

$$\frac{\partial \Omega^*}{\partial \tau_y} > -u'(c_y) w_y - \left[\delta^* u'(c_m) R_b - u'(c_y)\right] w_y + (\delta^*)^2 u'(c_o) R^2 w_y$$

since $\frac{\partial b_y}{\partial \tau_y} < w_y$. Hence a sufficient condition that $\frac{\partial \Omega^*}{\partial \tau_y} > 0$ is

$$-u'(c_y) w_y - \left[\delta^* u'(c_m) R_b - u'(c_y)\right] w_y + (\delta^*)^2 u'(c_o) R^2 w_y > 0$$

or

$$\delta^* w_y \left[\delta^* u'(c_o) R^2 - u'(c_m) R_b\right] > 0$$

This inequality holds if

$$\frac{\delta^* R^2}{R_b} u'(c_o) > u'(c_m) \Leftrightarrow \frac{\delta^* R^2}{\widetilde{\delta} R_b R_b} u'(c_o) \widetilde{\delta} R_b > u'(c_m)$$

For the middle-aged to at the zero savings corner we require

$$u'(c_o)\widetilde{\delta} R_b > u'(c_m) > \widetilde{\delta} R u'(c_o)$$

and hence a sufficient condition that $\frac{\partial \Omega^*}{\partial \tau_y} > 0$ is

$$\frac{\delta^* R^2}{\widetilde{\delta} R_b R_b} > 1 \Leftrightarrow \delta^* R^2 > \widetilde{\delta} R_b^2$$

Implementing (b_y^*, s_m^*)

Is it possible to implement the optimal choice under the true preferences (b_y^*, s_m^*) and the associated consumption levels (c_y^*, c_m^*, c_o^*) by some choice of τ_y and τ_m? To address this quesstion, first write the consumption levels for the optimal choices under the true preferences, i.e.,

$$\begin{aligned} c_y^* &= w_y + b_y^* \\ c_m^* &= w_m - R_b b_y^* - s_m^* \\ c_o^* &= R s_m^* \end{aligned}$$

and under the choice preference for given values of τ_y and τ_m.

$$\begin{aligned} c_y &= (1-\tau_y) w_y + b_y \\ c_m &= (1-\tau_m) w_m - R_b b_y - s_m \\ c_o &= R^2 \tau_y w_y + R \tau_m w_m + R s_m \end{aligned}$$

For $c_y = c_y^*$ we require

$$w_y + b_y^* = (1-\tau_y) w_y + b_y$$

or

$$b_y - b_y^* = \tau_y w_y \tag{23}$$

For $c_m = c_m^*$ we require

$$w_m - R_b b_y^* - s_m^* = (1-\tau_m) w_m - R_b b_y - s_m$$

or

$$s_m - s_m^* = -\tau_m w_m - R_b \left(b_y - b_y^*\right) \tag{24}$$

Finally, for $c_o = c_o^*$ we require

$$Rs_m^* = R^2\tau_y w_y + R\tau_m w_m + Rs_m$$

or

$$s_m - s_m^* = -\left(R\tau_y w_y + \tau_m w_m\right) \tag{25}$$

Combining (24) and (25) gives

$$-\left(R\tau_y w_y + \tau_m w_m\right) = -\tau_m w_m - R_b\left(b_y - b_y^*\right)$$

or $R\tau_y w_y = R_b\left(b_y - b_y^*\right)$ and using (23) this requires $R\tau_y w_y = R_b\tau_y w_y$ which cannot hold for $R < R_b$, showing that it is not possible to implement $\left(b_y^*, s_m^*\right)$ and the associated consumption levels $\left(c_y^*, c_m^*, c_o^*\right)$ by some choice of τ_y and τ_m.

F Age independent contribution rates

When the middle-aged are savers, savings decisions are determined by

$$\begin{aligned} u'\left((1-\tau)w_y + b_y\right) &= \Lambda_i u'\left(Rs_m + R^2\tau w_y + R\tau w_m\right) \\ u'\left((1-\tau)w_m - R_b b_y - s_m\right) &= \widetilde{\delta} R u'\left(Rs_m + R^2\tau w_y + R\tau w_m\right) \end{aligned}$$

where

$$\Lambda_i \equiv \widetilde{\delta} R\left[\left(\widetilde{\delta} - \delta\right)\left(\frac{\partial D\left(b_y\right)}{\partial b_y}\right) + R_b\widetilde{\delta}\right]$$

Note that $0 < \Lambda_i < RR_b\delta\widetilde{\delta}$ and Λ_i is indendent of b_y and s_m under homothetic preferences.

Hence, differentiating the optimality condtions above with respect to τ, we get

$$\begin{aligned} u''\left(c_y\right)\left[-w_y + \frac{\partial b_y}{\partial \tau}\right] &= \Lambda_i u''\left(c_o\right)\left[R\frac{\partial s_m}{\partial \tau} + R^2 w_y + Rw_m\right] \\ u''\left(c_m\right)\left[-w_m - R_b\frac{\partial b_y}{\partial \tau} - \frac{\partial s_m}{\partial \tau}\right] &= \widetilde{\delta} R u''\left(c_o\right)\left[R\frac{\partial s_m}{\partial \tau} + R^2 w_y + Rw_m\right] \end{aligned}$$

implying

$$\begin{aligned} \text{sign}\left[w_y - \frac{\partial b_y}{\partial \tau}\right] &= -\text{sign}\left[R\frac{\partial s_m}{\partial \tau} + R^2 w_y + Rw_m\right] \\ \text{sign}\left[-w_m - R_b\frac{\partial b_y}{\partial \tau} - \frac{\partial s_m}{\partial \tau}\right] &= \text{sign}\left[R\frac{\partial s_m}{\partial \tau} + R^2 w_y + Rw_m\right] \end{aligned}$$

To solve for $\frac{\partial s_y}{\partial \tau}$ and$\frac{\partial s_m}{\partial \tau}$ the above expression can be written in matrix form as

$$\begin{bmatrix} u''\left(c_y\right) & -\Lambda_i u''\left(c_o\right) R \\ -u''\left(c_m\right) R_b & -\left[u''\left(c_m\right) + \widetilde{\delta} R^2 u''\left(c_o\right)\right] \end{bmatrix}\begin{bmatrix} \frac{\partial b_y}{\partial \tau} \\ \frac{\partial s_m}{\partial \tau} \end{bmatrix} = \begin{bmatrix} u''\left(c_y\right) w_y + \Lambda_i u''\left(c_o\right)\left[R^2 w_y + Rw_m\right] \\ u''\left(c_m\right) w_m + \widetilde{\delta} R u''\left(c_o\right)\left[R^2 w_y + Rw_m\right] \end{bmatrix}$$

which implies that

$$\frac{\partial b_y}{\partial \tau} = \frac{\begin{vmatrix} u''(c_y)\, w_y + \Lambda_i u''(c_o)\left[R^2 w_y + R w_m\right] & -\Lambda_i u''(c_o)\, R \\ u''(c_m)\, w_m + \widetilde{\delta} R u''(c_o)\left[R^2 w_y + R w_m\right] & -\left[u''(c_m) + \widetilde{\delta} R^2 u''(c_o)\right] \end{vmatrix}}{\begin{vmatrix} u''(c_y) & -\Lambda_i u''(c_o)\, R \\ -u''(c_m)\, R_b & -\left[u''(c_m) + \widetilde{\delta} R^2 u''(c_o)\right] \end{vmatrix}}$$

implying

$$\frac{\partial b_y}{\partial \tau} = \frac{u''(c_y)\, u''(c_m) + R^2 \widetilde{\delta} u''(c_y)\, u''(c_o) + \Lambda_i u''(c_o u''(c_m))\, R^2}{u''(c_y)\, u''(c_m) + R^2 \widetilde{\delta} u''(c_y)\, u''(c_o) + \Lambda_i u''(c_o)\, u''(c_m)\, R R_b} w_y < w_y$$

From this, it can be inferred by use of the above sign-relations: $w_y - \frac{\partial b_y}{\partial \tau} > 0$; $w_m + R_b \frac{\partial b_y}{\partial \tau} + \frac{\partial s_m}{\partial \tau} > 0$; $R\frac{\partial s_m}{\partial \tau} + R^2 w_y + R w_m < 0$. Considering the welfare effects from a change in τ we have

$$\frac{\partial \Omega^*}{\partial \tau} = -u'(c_y)\left[w_y - \frac{\partial b_y}{\partial \tau}\right] - \delta^* u'(c_m)\left[w_m + R_b \frac{\partial b_y}{\partial \tau} + \frac{\partial s_m}{\partial \tau}\right] + (\delta^*)^2 u'(c_o)\left[R\frac{\partial s_m}{\partial \tau} + R^2 w_y + R w_m\right] < 0$$

and it follows that $\frac{\partial \Omega^*}{\partial \tau} < 0$ for $\tau < \underline{\tau}$.

Consider next $\tau \in [\underline{\tau}, \overline{\tau}]$ where the middle-aged are at a corner. In this case, borrowing as young is determined by

$$u'\left((1-\tau) w_y + b_y\right) = \widetilde{\delta} R_b u'\left((1-\tau) w_m - R_b b_y\right)$$

and the corner condition reads

$$u'(R^2 \tau w_y + R\tau w_m)\widetilde{\delta} R_b > u'((1-\tau) w_m - R_b b_y) > \widetilde{\delta} R u'(R^2 \tau w_y + R\tau w_m)$$

It follows straightforwardly that

$$\frac{\partial b_y}{\partial \tau} = \frac{u''(c_y)\, w_y + \widetilde{\delta} R_b u''(c_m)\, w_m}{u''(c_y) + \widetilde{\delta} R_b^2 u''(c_m)}$$

Note for later

$$w_y - \frac{\partial b_y}{\partial \tau} = \frac{\widetilde{\delta} R_b^2 u''(c_m)\, w_y - \widetilde{\delta} R_b u''(c_m)\, w_m}{u''(c_y) + \widetilde{\delta} R_b^2 u''(c_m)}$$

For $\tau \in [\underline{\tau}, \overline{\tau}]$ and thus $s_m = 0$ we have that welfare is affected by a change in the contribution rate as:

$$\begin{aligned} \frac{\partial \Omega^*}{\partial \tau} &= -u'(c_y)\left[w_y - \frac{\partial b_y}{\partial \tau}\right] - \delta^* u'(c_m)\left[w_m + R_b \frac{\partial b_y}{\partial \tau}\right] + (\delta^*)^2 u'(c_o)\left[R^2 w_y + R w_m\right] \\ &= \left[-\widetilde{\delta} R_b \left[w_y - \frac{\partial b_y}{\partial \tau}\right] - \delta^* \left[w_m + R_b \frac{\partial b_y}{\partial \tau}\right]\right] u'(c_m) + (\delta^*)^2 u'(c_o)\left[R^2 w_y + R w_m\right] \\ &= \left[-\widetilde{\delta} R_b w_y - \delta^* w_m - \left[\delta^* R_b - \widetilde{\delta} R_b\right] \frac{\partial b_y}{\partial \tau}\right] u'(c_m) + (\delta^*)^2 u'(c_o)\left[R^2 w_y + R w_m\right] \end{aligned}$$

evaluated for $\underline{\tau}$ and using $u'(c_m) = \widetilde{\delta} R u'(c_o)$ we get

$$\frac{\partial \Omega^*}{\partial \tau}\mid_{\underline{\tau}} = \left[\left[-\widetilde{\delta} R_b w_y - \delta^* w_m - \left[\delta^* R_b - \widetilde{\delta} R_b\right] \frac{\partial b_y}{\partial \tau}\right] \widetilde{\delta} R + (\delta^*)^2 \left[R^2 w_y + R w_m\right]\right] u'(c_o)$$

or

$$\begin{aligned}
\frac{\frac{\partial \Omega^*}{\partial \tau}\mid_{\underline{\tau}}}{u'(c_o)} &= -\widetilde{\delta} R_b \widetilde{\delta} R w_y - \widetilde{\delta} R \delta^* w_m - \widetilde{\delta} R \left[\delta^* R_b - \widetilde{\delta} R_b\right] \frac{\partial b_y}{\partial \tau} + (\delta^*)^2 \left[R^2 w_y + R w_m\right] \\
&= \left[(\delta^*)^2 R^2 - \widetilde{\delta} R_b \widetilde{\delta} R - \widetilde{\delta} R \left[\delta^* R_b - \widetilde{\delta} R_b\right] \frac{u''(c_y)}{u''(c_y) + \widetilde{\delta} R_b^2 u''(c_m)}\right] w_y \\
&\quad + \left[R\delta^* \left[\delta^* - \widetilde{\delta}\right] + \widetilde{\delta} R \left[\delta^* - \widetilde{\delta}\right] \frac{\widetilde{\delta} R_b^2 u''(c_m)}{u''(c_y) + \widetilde{\delta} R_b^2 u''(c_m)}\right] w_m
\end{aligned}$$

where

$$(\delta^*)^2 R - \widetilde{\delta} R \delta^* + \widetilde{\delta} R \left[\delta^* - \widetilde{\delta}\right] \frac{\widetilde{\delta} R_b^2 u''(c_m)}{u''(c_y) + \widetilde{\delta} R_b^2 u''(c_m)} > 0$$

Consider the term

$$(\delta^*)^2 R^2 - \widetilde{\delta} R_b \widetilde{\delta} R - \widetilde{\delta} R \left[\delta^* - \widetilde{\delta}\right] R_b \frac{u''(c_y)}{u''(c_y) + \widetilde{\delta} R_b^2 u''(c_m)}$$

Since $\frac{u''(c_y)}{u''(c_y)+\widetilde{\delta} R_b^2 u''(c_m)} < 1$ it follows that

$$(\delta^*)^2 R^2 - \widetilde{\delta} R_b \widetilde{\delta} R - \widetilde{\delta} R \left[\delta^* - \widetilde{\delta}\right] R_b \frac{u''(c_y)}{u''(c_y) + \widetilde{\delta} R_b^2 u''(c_m)} > R\delta^* \left[\delta^* R - \widetilde{\delta} R_b\right]$$

Hence, this expression is positive if $\delta^* R - \widetilde{\delta} R_b > 0$, and it follows that a sufficient condition that $\frac{\partial \Omega^*}{\partial \tau}\mid_{\underline{\tau}} > 0$ is $\delta^* R - \widetilde{\delta} R_b > 0$.

G Young-Naive

The sophisticated young realize that they will "change" preferences, the naive do not. Hence, in the case of the naive young we have to distinguish between planned savings as middle-aged (s_m^p) influencing the savings decision as young, and the actual savings as middle-aged (s_m^a). The difference between s_m^p and s_m^a arises from the preference-reversal which the naive does not take into account.

Life-time utility as perceived when young reads

$$u\left((1-\tau_y) w_y + b_y\right) + \widetilde{\delta} \left[u\left((1-\tau_m) w_m - R_b b_y - s_m^p\right) + \delta u\left(R^2 \tau_y w_y + R \tau_m w_m + R s_m^p\right)\right]$$

Hence, the naive expects savings as middle-aged to be determined by (the superscript p refers to planned, which will differ from actual)

$$-u'\left((1-\tau_m)w_m - R_b b_y - s_m^p\right) + \delta R u'\left(R^2\tau_y w_y + R\tau_m w_m + R s_m^p\right) = 0 \text{ if } s_m^p > 0 \quad (26)$$
$$-u'\left((1-\tau_m)w_m - R_b b_y - s_m^p\right) + \delta R_b u'\left(R^2\tau_y w_y + R\tau_m w_m + R_b s_m^p\right) = 0 \text{ if } s_m^p < 0 \quad (27)$$

Hence, the corner condition for middle-aged savings read:

$$\delta R_b u'\left(R^2\tau_y w_y + R\tau_m w_m\right) > u'\left((1-\tau_m)w_m + R_b s_y\right) > \delta R u'\left(R^2\tau_y w_y + R\tau_m w_m\right) \quad (28)$$

or

$$\delta R_b u'\left(c_o^p \mid_{s_m^p=0}\right) > u'\left(c_m^p \mid_{s_m^p=0}\right) > \delta R u'\left(c_o^p \mid_{s_m^p=0}\right)$$

It follows

$$\frac{\partial s_m^p}{\partial b_y} = \begin{cases} -\frac{R_b u''\left(c_m^p\right)}{u''\left(c_m^p\right)+\delta R^2 u''\left(c_o^p\right)} < 0 \text{ for } u'\left(c_m^p \mid_{s_m^p=0}\right) < \delta R u'\left(c_o^p \mid_{s_m^p=0}\right) \\ 0 \text{ for } \delta R_b u'\left(c_o^p \mid_{s_m^p=0}\right) > u'\left(c_m^p \mid_{s_m^p=0}\right) > \delta R u'\left(c_o^p \mid_{s_m^p=0}\right) \\ -\frac{R_b u''\left(c_m^p\right)}{u''\left(c_m^p\right)+\delta R_b^2 u''\left(c_o^p\right)} > 0 \text{ for } \delta R_b u'\left(c_o^p \mid_{s_m^p=0}\right) < u'\left(c_m^p \mid_{s_m^p=0}\right) \end{cases}$$

and

$$\frac{\partial s_m^p}{\partial \tau_m} = \begin{cases} -w_m \text{ for } u'\left(c_m^p \mid_{s_m^p=0}\right) < \delta R u'\left(c_o^p \mid_{s_m^p=0}\right) \\ 0 \text{ for } \delta R_b u'\left(c_o^p \mid_{s_m^p=0}\right) > u'\left(c_m^p \mid_{s_m^p=0}\right) > \delta R u'\left(c_o^p \mid_{s_m^p=0}\right) \\ > -w_m \text{ for } \delta R_b u'\left(c_o^p \mid_{s_m^p=0}\right) < u'\left(c_m^p \mid_{s_m^p=0}\right) \end{cases}$$

The optimal savings decision as young is given as (notice that the envelope theorem applies in this case)

$$u'\left((1-\tau_y)w_y + b_y\right) - \widetilde{\delta} R_b u'\left((1-\tau_m)w_m - R_b b_y - s_m^p\right) = 0$$

It is still the case, that savings is below the level chosen under the true preferences. Finally, actual savings by the middle-aged is determined by

$$-u'\left((1-\tau_m)w_m - R_b b_y - s_m^a\right) + \delta R u'\left(R^2\tau_y w_y + R\tau_m w_m + R s_m^a\right) = 0 \text{ if } s_m^a > 0$$
$$-u'\left((1-\tau_m)w_m - R_b b_y - s_m^a\right) + \delta R_b u'\left(R^2\tau_y w_y + R\tau_m w_m + R_b s_m^a\right) = 0 \text{ if } s_m^a < 0$$

Note that the "corner" condition (28) is unchanged since it depends on b_y which is pre-determined.

It is now straightforward to show the main results of the paper carries over. First, assume $\tau_y = 0$, then

$$\frac{\partial s_y}{\partial \tau_m} = 0; \frac{\partial s_m^a}{\partial \tau_m} = -w_m \text{ for } \tau_m < \underline{\tau}_m^n$$

and hence $\frac{\partial \Omega^*}{\partial \tau_m} = 0$ for $\tau_m < \underline{\tau}_m^n$. If the middle-aged are at the corner, i.e., $\underline{\tau}_m^n < \tau_m < \overline{\tau}_m^n$ ($\overline{\tau}_m^n$

and $\underline{\tau}_m^n$ are defined analogously to those in Appendix D) we have

$$\begin{aligned}\frac{\partial\Omega^*}{\partial\tau_m} &= u'(c_y)\frac{\partial b_y}{\partial\tau_m} - \delta^* u'(c_m)R_b\frac{\partial b_y}{\partial\tau_m} + (\delta^*)^2 u'(c_o)Rw_m \\ &= \left[\widetilde{\delta} - \delta^*\right] R_b u'(c_m)\frac{\partial b_y}{\partial\tau_m} + (\delta^*)^2 u'(c_o)Rw_m > 0\end{aligned}$$

since $\frac{\partial b_y}{\partial\tau_m} < 0$ for $\tau_m < \underline{\tau}_m^n$. Hence, for $\tau_y = 0$ there is a $\tau_m > \underline{\tau}_m^n$ which increases welfare above laissez-faire. Suppose $\underline{\tau}_m^n < \tau_m < \overline{\tau}_m^n$, is there a welfare case for $\tau_y > 0$? We have

$$\frac{\partial\Omega^*}{\partial\tau_y} = u'(c_y)\left(-w_y + \frac{\partial b_y}{\partial\tau_y}\right) - \delta^* u'(c_m)R_b\frac{\partial b_y}{\partial\tau_y} + (\delta^*)^2 u'(c_o)R^2 w_y \lesseqgtr 0 \text{ for } \underline{\tau}_m^n < \tau_m < \overline{\tau}_m^n$$

which is basically the same condition as in the case with the sophisticated young.