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Abstract

We show that repurchase agreements (repos) arise as the instrument of choice to borrow in a competitive model with limited commitment. The repo contract traded in equilibrium provides insurance against fluctuations in the asset price in states where collateral value is high and maximizes borrowing capacity when it is low. Haircuts increase both with counterparty risk and asset risk. In equilibrium, lenders choose to re-use collateral. This increases the circulation of the asset and generates a "collateral multiplier" effect. Finally, we show that intermediation by dealers may endogenously arise in equilibrium, with chains of repos among traders.

JEL-Codes: G190.

Keywords: repos, collateral multiplier, limited commitment, intermediation.

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1 Introduction

[Gorton and Metrick \(2012\)](#) argue that the financial panic of 2007-08 started with a run on the market for repurchase agreements (repos). Lenders stopped lending altogether or drastically increased the haircut requested for some types of collateral. This view was very influential in shaping our understanding of the crisis.¹ Many attempts to understand repos more deeply as well as calls for regulation quickly followed.² The very idea that a run on repos could lead to a financial market meltdown speaks to their importance for money markets. Overall, repo market activity is enormous. Recent surveys estimate outstanding volumes at €5.4 trillion in Europe while \$3.8 trillion to \$5.5 trillion are traded in the US, depending on calculations.³ The main market participants are large dealer banks and other financial institutions who use repos for funding, to finance security purchases, or simply to obtain a safe return on idle cash. For these reasons, repo markets determine the interaction between asset liquidity and funding liquidity, as [Brunnermeier and Pedersen \(2009\)](#) illustrate. Dealer banks also play a major role as repo intermediaries between cash providers and cash borrowers. Finally, most major central banks implement monetary policy using repos, thus contributing to the size and liquidity of these markets.

Repos may be popular because they are simple financial instruments to lend cash against collateral. Precisely, a repo contract is the sale of an asset combined with a forward contract that requires the original seller to repurchase the asset at a future date for a pre-specified (repurchase) price. The seller takes a haircut defined as the difference between the selling price in a repo and the asset's spot market price. Besides the haircut, a repo differs from a sequence of spot trades because the seller commits to buying back the asset at a pre-set repurchase price. A repo contract is not a simple collateralized loan either, because it is a recourse loan and the borrower sells the collateral rather than pledging it. The lender thus acquires the legal title to the asset sold and so the possibility to re-use the collateral before the forward contract with the seller matures. This practice, known as re-use or re-hypothecation, has attracted a lot of attention from

¹Subsequent studies by [Krishnamurty et al. \(2014\)](#) and [Copeland et al. \(2014b\)](#) have qualified this finding by showing that the run was specific to the - large - bilateral segment of the repo market.

²See for example [Acharya and Öncü \(2013\)](#) and [FRBNY \(2010\)](#).

³The number for Europe is from the International Capital Market Association ([ICMA, 2016](#)). The two figures for the US are from [Copeland et al. \(2014a\)](#) and [Copeland et al. \(2012\)](#) where the latter adds reverse repo. These numbers are only estimates because many repo contracts are traded over the counter and thus difficult to account for.

economists and regulators alike.⁴ The following questions then arise: why do traders choose repos as instrument to raise funds? Which economic forces determine haircuts? What are the consequences of collateral re-use? Finally, why would borrowers trade through dealers rather than directly with lenders? To understand repo markets and their potential contribution to systemic risk, a theory of repos should answer these questions while accounting for the basic features of repo contracts.

In this paper we analyze a simple competitive economy where investors can face their funding needs by selling their assets spot or in repo sales, characterized as loan contracts exhibiting the key features of repos described above. In equilibrium, investors trade repos rather than spot. Haircuts increase with counterparty and asset risk but can be negative when collateral is scarce. Furthermore, investors value the option to re-use collateral, that distinguishes repos from standard collateralized loans. In equilibrium they use this option as it allows to expand their borrowing capacity through a multiplier effect. Collateral re-use also shapes the structure of the repo market since intermediation by safer counterparties may endogenously arise.

The model features two types of risk averse investors, a natural borrower and a natural lender. The borrower lacks the ability to commit to future promises but owns some asset whose future payoff is uncertain. A large variety of possible repo contracts, characterized by different values of the repurchase price, are available for trade. Due to borrowers' inability to commit, they may find it optimal to default on these contracts. The punishment for default is the loss of the asset sold in the repo together with a penalty reflecting the borrower's creditworthiness. Hence there is a maximal amount that the borrower can

⁴Aghion and Bolton (1992) argue that securities are characterized by cash-flow rights but also control rights. Collateralized loans grant neither cash-flow rights nor control rights over the collateral to the lender unless the counterparties sign an agreement for this purpose. As a sale of the asset, a repo automatically gives the lender full control rights over the security as well as over its cash-flows. Re-use rights follow directly from ownership rights. As Comotto (2014) explains, there is a subtle difference between US and EU law however. Under EU law, a repo is a transfer of the security's title to the lender. However, a repo in the US falls under New York law which is the predominant jurisdiction in the US. *"Under the law of New York, the transfer of title to collateral is not legally robust. In the event of a repo seller becoming insolvent, there is a material risk that the rights of the buyer to liquidate collateral could be successfully challenged in court. Consequently, the transfer of collateral in the US takes the form of the seller giving the buyer (1) a pledge, in which the collateral is transferred into the control of the buyer or his investor, and (2) the right to re-use the collateral at any time during the term of the repo, in other words, a right of re-hypothecation. The right of re-use of the pledged collateral (...) gives US repo the same legal effect as a transfer of title of collateral."* To conclude, although there are legal differences between re-use and rehypothecation, they are economically equivalent (see e.g. Singh, 2011) and we treat them as such in our analysis.

credibly promise to repay, that depends on the future market value of the asset. This amount determines his borrowing capacity. The recourse nature of repo contracts implies that the borrowing capacity may exceed the future spot market price of the asset.

Risk-averse investors value the ability to borrow but dislike fluctuations in the future value of the asset price. We show that both a hedging and a borrowing motive determine the repurchase price of the repo contract that investors choose to trade in equilibrium. In the states where the market value of the asset is low, the ability to borrow is limited. There, the borrowing motive prevails and the repurchase price equals the borrowing capacity. In the other states, where the asset price is high, the hedging motive implies that the repurchase price is set at a constant level below the borrowing capacity. These motives explain why investors prefer repo contracts over spot trades. The combination of a spot sale and a future repurchase of the asset in the spot market fully exposes investors to the fluctuations in the future asset price. Moreover, as already noticed, a repo allows to pledge more income than the future value of the asset.

We derive comparative statics properties for equilibrium haircuts and liquidity premia. Haircuts increase when counterparty quality decreases, because riskier borrowers can credibly promise to repay lower amounts, or when collateral is more abundant. We also show that riskier assets command higher haircuts and lower liquidity premia, since higher risk entails a worse distribution of collateral value across states relative to collateral needs.

Next, we analyze the benefits of collateral re-use. In equilibrium lenders resell in the spot market the collateral acquired via repos. Borrowers in turn purchase these additional amounts of the asset so as to pledge them again in repo sales to lenders. These trades increase the borrowing capacity of investors. We find that the iteration of these transactions generates a collateral multiplier effect. The benefits of collateral re-use are clear when haircuts on the repos traded in equilibrium are negative, since re-use allows to increase the funds borrowers can get for a given amount of the asset. We show that re-use is also beneficial when haircuts are positive, although the reason is different in this case. Since re-use generates a multiplier effect, the benefits are larger when the asset is scarce. Even though re-use relaxes the borrowing constraint, it may still increase the liquidity premium of the asset used as collateral because the properties of the repo contract traded in equilibrium are also affected.

Finally, we show that collateral re-use has important implications for the pattern of trades observed in markets, as some third parties may emerge as intermediaries between

natural borrowers and natural lenders. In practice, dealer banks indeed make for a significant share of the market by intermediating between hedge funds and money market funds or MMF. This might seem puzzling if direct trading platforms are available for both parties to bypass the dealer bank.⁵ Our model explains the presence of intermediation with differences in counterparty quality and in the ability to re-deploy the collateral. In particular, intermediation may occur via a chain of repo trades. Then, a hedge fund prefers entering a repo with a dealer bank who in turn enters another repo with the MMF. This happens even though there are larger gains from trade with the MMF, when the bank is more creditworthy and more efficient at re-using collateral. Through re-use, one unit pledged to the dealer bank can indeed support more borrowing in the chain of transactions. Our model also helps to explain why dealer banks predominantly fund their operations using repos.

Relation to the literature

Recent theoretical works highlighted some features of repo contracts as sources of funding fragility. As a short-term debt instrument to finance long-term assets, [Zhang \(2014\)](#) and [Martin et al. \(2014\)](#) show that repos are subject to roll-over risk. [Antinolfi et al. \(2015\)](#) show that the benefit of an exemption from automatic stay⁶ granted to repos may be harmful for social welfare in the presence of fire sales, a point also made by [Infante \(2013\)](#) and [Kuong \(2015\)](#).

These papers usually take the trade of repurchase agreements and their specific features as given while we want to understand their emergence as a funding instrument. One natural question is why borrowers do not simply sell the collateral to lenders? A first strand of papers explains the existence of repos using transaction costs (e.g. [Duffie, 1996](#)) or search frictions (e.g. [Narajabad and Monnet, 2012](#), [Tomura, 2016](#), and [Parlatore, 2016](#)). Bundling the sale and the repurchase of the asset in one transaction lowers search costs or mitigates bargaining inefficiencies. [Bigio \(2015\)](#) and [Madison \(2016\)](#) emphasize the role of informational asymmetries regarding the quality of the asset to explain repos: their collateralized debt features reduce adverse selection between the informed seller and the uninformed buyer as in [DeMarzo and Duffie \(1999\)](#) or [Hendel and Lizzeri \(2002\)](#). We show that investors choose to trade repos in an environment with symmetric information,

⁵In the US, Direct RepoTM provides this service

⁶As shown by [Eisfeldt and Rampini \(2009\)](#) for leases, such benefit is in terms of easier repossession of collateral in a default event.

where markets are Walrasian, but where collateral has uncertain payoff. One limitation of the works mentioned above is that the borrower chooses to sell repo if he can obtain more cash than in a spot sale of the asset, that is if the haircut is negative. Our analysis rationalizes the use of repos with positive haircuts when investors are risk-averse. In addition, we account for the possible re-use of collateral in repos by showing its benefits.

To derive the equilibrium repo contract, we follow the competitive approach of [Geanakoplos \(1996\)](#), [Araújo et al. \(2000\)](#), and [Geanakoplos and Zame \(2014\)](#) where the properties of the collateralized promises traded by investors are selected in equilibrium. Unlike these papers where the only cost of default is the loss of the collateral, our model aims to capture the recourse nature of repo transactions. We thus allow for additional penalties for default, some of them non-pecuniary in the spirit of [Dubey et al. \(2005\)](#). While our results on the characterization of repo contracts traded in equilibrium remain valid also in the absence of these additional penalties, the recourse nature of repos is crucial to explain re-use. Indeed, [Maurin \(2017\)](#) showed in a more general environment that the collateral multiplier effect disappears when loans are non-recourse.

Collateral re-use is discussed by [Singh and Aitken \(2010\)](#) and [Singh \(2011\)](#), who claim that it lubricates transactions in the financial system.⁷ At the same time, re-use generates the risk that the lender, who receives the collateral, does not or cannot return it when due, as explained by [Monnet \(2011\)](#). Unlike [Bottazzi et al. \(2012\)](#) or [Andolfatto et al. \(2015\)](#), we account for the double commitment problem induced by re-use. The increase in the circulation of collateral obtained with re-use also arises with pyramiding (see [Gottardi and Kubler, 2015](#)), where collateralized debt claims are themselves used as collateral. However, the mechanism is different: in pyramiding, no two sided commitment problem arises and the recourse nature of loans also plays no role. We stress the role of collateral re-use in explaining the presence of intermediation in the repo market, as in [Infante \(2015\)](#) and [Muley \(2015\)](#). Unlike in these papers, in our analysis intermediation arises endogenously since direct trade between borrowers and lenders is possible.

The structure of the paper is as follows. We present the model and the set of contracts available for trade in [Section 2](#). We characterize the equilibrium and the properties of repo contracts traded in [Section 3](#), where we also derive the properties of haircuts and liquidity premia. In [Section 4](#) we examine the effects of collateral re-use and in [Section 5](#) show that intermediation arises in equilibrium. Finally, [Section 6](#) establishes the robustness of

⁷[Fuhrer et al. \(2015\)](#) estimate an average 10% re-use rate in the Swiss repo market over 2006-2013.

our findings to alternative specifications of the repurchase price and Section 7 concludes. The proofs are collected in the Appendix.

2 The Model

In this section we present a simple environment where risk averse investors have funding needs. To accommodate these needs, they can sell an asset in positive net supply and take short positions in a variety of securities in zero net supply. These trades occur in a competitive financial market. Short positions are subject to limited commitment and require collateral. Trade in these securities capture the main ingredients of repo contracts.

2.1 Setting

The economy lasts three periods, $t = 1, 2, 3$. There is a unit mass of investors of each type $i = 1, 2$ and one consumption good each period. All investors have endowment ω in the first two periods and zero in the last one. Investor 1 is also endowed with a units of an asset while investor 2 has none.⁸ Each unit of the asset pays dividend s in period 3. The dividend is distributed according to a cumulative distribution function $G(\cdot)$ with support $\mathcal{S} = [\underline{s}, \bar{s}]$ and mean $\mathbb{E}[s] = 1$. The realization of s becomes known to all investors in period 2, one period before the dividend is paid. As a consequence, price risk arises in period 2.

Let c_t^i denote investor i 's consumption in period t . Investors have preferences over consumption profiles $\mathbf{c}^i = (c_1^i, c_2^i, c_3^i)$ described by the following utility functions, respectively for $i = 1, 2$:

$$\begin{aligned} U^1(\mathbf{c}^1) &= c_1^1 + v(c_2^1) + c_3^1 \\ U^2(\mathbf{c}^2) &= c_1^2 + u(c_2^2) + \beta c_3^2 \end{aligned}$$

where $\beta < 1$, $u(\cdot)$ and $v(\cdot)$ are respectively strictly and weakly concave functions. We assume $u'(\omega) > v'(\omega)$ and $u'(2\omega) < v'(0)$.⁹ The main role of this specification is to capture

⁸This is for simplicity only and we could easily relax this assumption, as none of the results depend on it.

⁹Observe that a special case of the preferences as specified above is $v(\cdot) = \delta u(\cdot)$, where investors only differ with respect to their discount factor.

the fact that investor 1 wants to borrow short-term in period 1. He wants to borrow because his intertemporal rate of substitution between periods 1 and 2 is lower than that of investor 2. His borrowing should be short-term because investor 2 discounts period 3 cash flow more than investor 1 ($\beta < 1$). In addition, the concavity of the investors' utility over date 2 consumption implies that they dislike variability in repayment terms in period 2. We simplify the analysis by assuming that their utility is linear over consumption at the other dates, but linearity plays no essential role in our results.

2.2 Arrow-Debreu equilibrium

To illustrate the basic features of this economy, it is useful to consider its Arrow-Debreu equilibrium allocation $(\mathbf{c}_*^1, \mathbf{c}_*^2)$. Consumption at date 2 is determined by equating the investors' marginal rates of substitution between period 1 and period 2 while investor 2 does not consume in the last period:¹⁰

$$\begin{cases} u'(c_{2,*}^2) = v'(2\omega - c_{2,*}^2) \\ c_{3,*}^2 = 0 \end{cases} \quad (1)$$

where we used the resource constraint in period 2 to substitute for $c_{2,*}^1 = 2\omega - c_{2,*}^2$. The prices for period 2 and 3 consumption are respectively $u'(c_{2,*}^2)$ and 1, with period 1 consumption as the numeraire. Consumption in period 1 is then obtained from the budget constraints. Thus for investor 2 we have $c_{1,*}^2 = \omega - u'(c_{2,*}^2)(c_{2,*}^2 - \omega)$ and we will assume that

$$\omega \geq u'(c_{2,*}^2)(c_{2,*}^2 - \omega) \quad (2)$$

in the remainder of the text.

In the Arrow-Debreu equilibrium, investor 1 borrows $u'(c_{2,*}^2)(c_{2,*}^2 - \omega)$ from type 2 investors in period 1 and repays with a net interest rate $r^* = 1/u'(c_{2,*}^2) - 1$ in period 2. In the following we refer for simplicity to this equilibrium allocation as the first best allocation. Observe that consumption in period 2 $(c_{2,*}^1, c_{2,*}^2)$ is deterministic even though the asset payoff s is already known. Indeed, risk averse investors prefer a smooth consumption profile.

¹⁰Intuitively, since $\beta < 1$ investor 2 has a lower marginal utility for period 3 consumption utility than investor 1.

2.3 Financial Markets With Limited Commitment

We assume investors can buy or sell the asset each period in the spot market. They can also take long and short positions in financial securities in the initial period 1, before the uncertainty is realized. Unlike in the Arrow-Debreu framework, agents are unable to fully commit to future promised payments. As we will see, this implies that borrowing positions must be collateralized and the first best allocation cannot always be sustained.

Spot Trades

Let p_1 and $p_2(s)$ denote the period 1 and period 2 spot market price of the asset when the realized payoff is s . We let a_1^i (resp. $a_2^i(s)$) be the asset holdings of investor i after trading in period 1 (resp. period 2 and state s). Note that spot trades could be a way for investor 1 to meet his borrowing needs: he could sell the asset in period 1 to carry only $a_1^1 < a$ into period 2 and then buy it back in period 2 to carry $a_2^1(s) > a_1^1$ into period 3. However, a combination of spot trades alone can never sustain the first best allocation. Indeed, since $p_2(s)$ is a function of the state s , such trades generate undesirable consumption variability in period 2 for both investors.¹¹

Repos

In period 1 investors can also trade promises to deliver the consumption good in period 2. We specify these financial securities so as to capture several features of repo contracts, and we will refer to these securities as repos. There are in particular three features of repo contracts we want to match. First, a repo agreement is a sale of an asset combined with a forward repurchase of that asset. The repurchase price is effectively the amount to be repaid by the seller and the asset plays a role as collateral. We thus model securities as collateralized loans. Second, in a repo agreement the lender acquires ownership of the asset and can sell or re-pledge the asset before the repo matures. We then also assume that the collateral backing the security is transferred to the lender who enjoys a right to re-use it. Finally, repos are recourse loans and the non-defaulting counterparty can claim the payment of any remaining shortfall and other expenses. In our environment, default on the securities entails additional costs beyond the loss of the asset pledged as collateral. We now describe in details how we model each of these features.

(i) *Collateralized loans* - We let $f = \{f(s)\}_{s \in \mathcal{S}}$ denote the payoff schedule for a generic security. An investor selling security f promises to repay $f(s)$ in state s of period

¹¹See Online Appendix B.1 for the formal argument.

2 per unit of security sold. We allow for all possible values of f so that the market for financial securities is complete. As we show below, the investors' inability to commit implies that short positions must be backed by the asset as collateral. Without loss of generality, we set the collateral requirement to one unit of asset per unit of security sold. We refer to the payoff schedule $\{f(s)\}_{s \in \mathcal{S}}$ as the repurchase price, which can be state contingent. Repos usually specify a constant repayment but margin calls or repricing of the terms of trade during the lifetime of a repo are ways in which contingencies can arise.¹² In Section 6 we examine the case where repo contracts are restricted to have a constant repurchase price and show that the main qualitative properties of our results still hold.

(ii) *Ownership transfer* - The asset used as collateral is a financial claim. The borrower transfers to the lender both the asset used as collateral and the ownership title to this asset. The lender can then re-use this asset as he pleases.¹³ Specifically, we assume that investor i can re-use a fraction ν_i of the collateral he receives, where $\nu_i \in [0, 1]$. We interpret ν_i as a measure of the operational efficiency of a trader in re-deploying collateral for his own trades.¹⁴

(iii) *Recourse loans* - In a collateralized loan with re-use there is a double commitment problem. The borrower can fail to pay back the lender, but the lender can also fail to return the collateral. In the following, we describe the punishment faced by investors when they default on their obligation. Besides the loss of the collateral pledged or of the right to receive the repayment due, the defaulting party incurs additional costs since the other party can claim compensation. This captures the recourse nature of repo transactions. We start by specifying the penalty for borrowers and then move on to the case of lenders default.

Borrower's Default

When the borrower defaults, the lender can retain or liquidate the collateral. In practice, the lender typically needs to sell the asset at a discount below its fair market value, and we model the cost of liquidation as a linear function of the market value of the

¹²When a trader faces a margin call, he must pledge more collateral to sustain the same level of borrowing. This is equivalent to reducing the amount borrowed per unit of asset pledged.

¹³This distinguishes the situation under consideration from that, for instance, of a mortgage loan where the asset used as collateral is a physical asset and the borrower retains ownership of the collateral

¹⁴Singh (2011) discusses the role played by collateral desks at large dealer banks in channeling these assets across different business lines. These desks might not be available for less sophisticated repo market participants such as money market mutual funds or pension funds.

collateral, that is $\kappa p_2(s)$ per unit of asset, for some $\kappa \geq 0$. Then, the lender can claim the shortfall he faces in a default, equal to the difference (when positive) between the payment due, $f(s)$ and the market value of the collateral, $p_2(s)$, net of the costs associated with the liquidation of the collateral. This is in line with the recourse loan feature of repos and the provisions in the event of default described in standard Repo Master Agreements.¹⁵ We assume that the lender is only able to collect a fraction $\alpha \in [0, 1]$ of this shortfall from the borrower. This partial recovery rate captures various frictions in recouping payments from unsecured claims.¹⁶ We also posit that a defaulting borrower of type i incurs an additional, non-pecuniary cost, equal to a fraction $\pi_i \in [0, 1]$ of the shortfall, measured in consumption units. This non-pecuniary component proxies for legal and reputation costs. It may thus depend on the borrower's type and increases in the size of the default.

To analyze the borrower's incentives to default, consider a trade of one unit¹⁷ of repo contract f between borrower i and lender j . Borrower i prefers to repay rather than default in state s if and only if

$$f(s) \leq p_2(s) + (\alpha + \pi_i) \max \{f(s) - p_2(s)(1 - \kappa), 0\} \quad (3)$$

The borrower will repay whenever the repurchase price $f(s)$ does not exceed the total default cost, given by the expression on the right hand side of (3). The first term in that expression is the market value $p_2(s)$ of the collateral seized by the lender. The second term collects the fraction α of the shortfall recovered by the lender and the non-pecuniary cost $\pi_i \max \{f(s) - p_2(s)(1 - \kappa), 0\}$ for the borrower. We see from equation (3) that a borrower may only choose to default if $f(s) \geq p_2(s)$. Hence (3) can be written as follows:

$$f(s) \leq \frac{1 - (\alpha + \pi_i)(1 - \kappa)}{1 - \alpha - \pi_i} p_2(s) \quad (4)$$

¹⁵See Appendix 7 for further details on the contractual provisions in an event of default according to the two standards Master Agreements. US dealers mostly use the Master Repo Agreement (MRA, 1996) with US domestic counterparties while non-US repos typically use the General Master Repo Agreement (GMRA, 2011). The treatment of default events is similar in the two cases. The full documents can be found at <http://www.sifma.org/services/standard-forms-and-documentation/mra,-gmra,-msla-and-msftas/>.

¹⁶For instance, it might take time for the borrower to make these payments. In addition, such claims have a junior status if the borrower files for bankruptcy.

¹⁷This is without loss of generality since penalties for default are linear in the amount traded, hence incentives to default do not depend on the size of a position.

Lender's Default

We now discuss the punishment faced by a lender (of type j) when he fails to return the collateral. Recall that he can only re-use a fraction ν_j of the asset pledged. We assume that the non re-usable fraction $1 - \nu_j$ is deposited or segregated with a collateral custodian.¹⁸ As a result, the lender may only abscond with the re-usable fraction of the collateral. Absconding with the collateral is a default by the lender.¹⁹ In this event, the borrower gets back the $1 - \nu_j$ units of segregated collateral and may also claim any shortfall remaining after the cancellation of his obligation to repay $f(s)$, equal to $\max\{p_2(s) - f(s) - (1 - \nu_j)p_2(s), 0\}$. Like in a borrower's default, we assume the borrower can only recover a fraction α of the claim and the lender incurs a non-pecuniary cost equal to a fraction $\pi_j \in [0, 1]$ of the shortfall.

Hence, the lender prefers to return the non-segregated collateral rather than default if and only if

$$\nu_j p_2(s) \leq f(s) + (\alpha + \pi_j) \max\{\nu_j p_2(s) - f(s), 0\} \quad (5)$$

The left hand side of (5) is the benefit of defaulting given by the market value of the re-usable collateral held by the lender.²⁰ The expression on the right hand side is the cost of defaulting which includes the foregone payment $f(s)$ from the borrower, the fraction α of the shortfall $\max\{\nu_j p_2(s) - f(s), 0\}$ recovered by the borrower, and the non-pecuniary cost $\pi_j \max\{\nu_j p_2(s) - f(s), 0\}$. Observe that condition (5) can be rewritten as follows:

$$f(s) \geq \nu_j p_2(s) \quad (6)$$

that is, the lender prefers to return the collateral whenever the repurchase price $f(s)$

¹⁸It is easy to understand why this is optimal for the lender. He would not derive ownership benefits from keeping the non re-usable collateral on his balance sheet and segregation reduces his incentives to default. In the tri-party repo market, BNY Mellon and JP Morgan provide these services. If segregation is not available, incentives for the lender are clearly harder to sustain. This can be seen from equation (5) by taking $\nu_j = 1$.

¹⁹Standard Repo Master Agreements allow counterparties to distinguish between outright default by the lender and "failure" to deliver the collateral on time. Late delivery of collateral is not necessarily characterized as an event of default because finding the appropriate security to deliver might take time in practice. We focus here on the first one, in which case the constraints imposed by the limited commitment of the lender are more relevant.

²⁰A lender might re-use the collateral and not have it on his balance sheet when he must return it to the lender. However, observe that he can always purchase the relevant quantity of the asset in the spot market to satisfy his obligation. When he returns the asset, the lender effectively covers a short position $-\nu_j$.

exceeds the value of the re-usable collateral he can abscond with.

No-Default Repos

The environment described extends the framework considered in standard models of collateralized lending, as for instance [Geanakoplos \(1996\)](#), to allow for recourse loans. When a borrower defaults, in addition to the loss of the collateral, he incurs pecuniary and non-pecuniary costs. We assume these costs are sufficiently low and the non-pecuniary cost is not too low compared to the recovery rate. Specifically, for every i :

$$\pi_i + \alpha < 1 \tag{7}$$

$$\alpha(u'(\omega) - v'(\omega)) \leq \pi_i v'(\omega) \tag{8}$$

Assumption (7) implies that a borrower always defaults if the loan is not collateralized. In our environment unsecured borrowing is equivalent to a repo collateralized by an asset with zero value. Under (7) the no-default condition for the borrower, (3), never holds when $p_2(s) = 0$. The second property, condition (8), then ensures that in equilibrium investors prefer to trade default-free repo contracts. When the recovery rate α is positive, the recourse feature of loans in our environment implies that borrowers could make higher payments to lenders with contracts inducing default.²¹ However, by doing so, borrowers incur a non pecuniary penalty which is a deadweight loss. As we will show in the proof of Proposition 1, under (8) such deadweight loss always outweighs the benefits of increasing the income pledged through default. To summarize, investors will use the asset as collateral to sustain borrowers' incentives, and choose to trade securities that do not induce default.

We can now define the set of repo contracts \mathcal{F}_{ij} that can be sold by investor i to investor j so that no default occurs. This set depends on the period 2 spot market price $\mathbf{p}_2 = \{p_2(s)\}_{s \in \mathcal{S}}$. To simplify notation, we let $\theta_i := (\alpha + \pi_i)\kappa / (1 - (\alpha + \pi_i)(1 - \kappa))$. From conditions (4) and (6) we obtain:

$$\mathcal{F}_{ij}(\mathbf{p}_2) = \left\{ f \mid \forall s \in [\underline{s}, \bar{s}], \nu_j p_2(s) \leq f(s) \leq \frac{p_2(s)}{1 - \theta_i} \right\} \tag{9}$$

²¹It is easy to verify that, for f large enough, the actual payment to the lender after a borrower defaults, given by $(1 - \alpha(1 - \kappa))p_2(s) + \alpha f(s)$, exceeds the maximum amount a borrower can repay without defaulting, given by the right hand side of (4).

The upper bound of this set, $\frac{p_2(s)}{1-\theta_i}$, constitutes the *borrowing capacity* of investor i per unit of asset held. It is increasing in θ_i , which we can interpret as a measure of the creditworthiness or counterparty quality of investor i . Observe that the set $\mathcal{F}_{ij}(\mathbf{p}_2)$ is convex and that all contracts have the same collateral requirement given our normalization. Hence, for any combination of multiple contracts sold by i , there exists an equivalent trade of a single repo contract. We can thus focus our attention on equilibria where at most one contract is sold by each agent and we use $f_{ij} \in \mathcal{F}_{ij}(\mathbf{p}_2)$ to denote the (unique) contract sold by investor i to investor j .

Investors' optimization problem.

We can now write the optimization problem of an investor of type i . Let $q_{ij}(f_{ij})$ be the unit price of contract f_{ij} .²² The collection of these repo prices is $\mathbf{q}_{ij} = \{q_{ij}(f_{ij}) \mid f_{ij} \in \mathcal{F}_{ij}(\mathbf{p}_2)\}$. Given the spot prices and the prices of the repo contracts, investor i chooses which contract to sell in $\mathcal{F}_{ij}(\mathbf{p}_2)$, which contract to buy in $\mathcal{F}_{ji}(\mathbf{p}_2)$, the volume of trade for the two contracts as well as the trades of the asset in the spot market. Let b^{ij} (resp. l^{ij}) denote the amount sold (resp. bought) by investor i to investor j using the chosen contract f_{ij} (resp. f_{ji}), that is the amount borrowed and lent. These contracts must be such that investor i does not strictly benefit from trading any other existing contract at the prices he faces. The quantities traded of the two contracts as well as the spot trades must be a solution of the following problem:

$$\max_{a_1^i, a_2^i(s), b^{ij}, l^{ij}} E [U^i(c_1^i, c_2^i(s), c_3^i(s))] \quad (10)$$

$$\text{subject to} \quad c_1^i = \omega + p_1(a_0^i - a_1^i) + q_{ij}(f_{ij})b^{ij} - q_{ji}(f_{ji})l^{ij} \quad (11)$$

$$c_2^i(s) = \omega + p_2(s)(a_1^i - a_2^i(s)) - f_{ij}(s)b^{ij} + f_{ji}(s)l^{ij} \quad (12)$$

$$c_3^i(s) = a_2^i(s)s \quad (13)$$

$$a_1^i + \nu_i l^{ij} \geq b^{ij} \quad (14)$$

$$b^{ij} \geq 0 \quad (15)$$

$$l^{ij} \geq 0 \quad (16)$$

$$a_2^i(s) \geq 0 \quad (17)$$

²²Even in the absence of default the price may depend on the identities of the agents trading the contract, to the extent that investors may have different re-use abilities.

Equation (11) is the budget constraint in period 1 for investor i , where the resources available are $\omega + p_1 a_0^i$. Equation (12) is the budget constraint in period 2 for every realization of s , with the resources available given by the endowment ω , the value of the investor's asset holdings $p_2(s) a_1^i$ and the net value of the repo positions $f_{ji}(s) \ell^{ij} - f_{ij}(s) b^{ij}$. Equation (13) is the budget constraint in period 3. The collateral constraint of investor i is specified in (14). When investor i sells contract f_{ij} (that is $b^{ij} > 0$), he must pledge as collateral one unit of the asset per unit of repo contract sold. He can satisfy this requirement either by acquiring the asset in the spot market (that is $a_1^i > 0$), or in the repo market (that is $l^{ij} > 0$). In the latter case however, only a fraction ν_i of the asset purchased can be re-used.

It is important to observe that, when investor i buys but does not sell a repo contract (that is $l^{ij} > 0$ and $b^{ij} = 0$), the collateral constraint may be satisfied with $a_1^i < 0$ if $\nu_i > 0$. Indeed, with re-use agent i can sell in the spot market an asset that he acquired by purchasing a repo contract. When the repo matures, the investor would then buy the asset to satisfy his obligation to return it to the repo seller, thus covering his short position. Hence (14) shows that a lender can use repo trades to take a short position in the asset. However, investors cannot engage in naked short sales of the asset.

We can now define a competitive equilibrium (in short a repo equilibrium) in the environment described:

Definition.

A repo equilibrium is a system of spot prices p_1 , $\mathbf{p}_2 = \{p_2(s)\}_{s \in \mathcal{S}}$, repo prices \mathbf{q}_{12} , \mathbf{q}_{21} , a pair of repo contracts $(f_{12}, f_{21}) \in \mathcal{F}_{12}(\mathbf{p}_2) \times \mathcal{F}_{21}(\mathbf{p}_2)$ and an allocation $\{c_t^i(s), a_1^i, a_2^i(s), \ell^{ij}, b^{ij}\}$ for $i = 1, 2$, $j \neq i$, $t = 1, 2, 3$ and $s \in \mathcal{S}$ such that

1. $\{c_t^i(s), a_1^i, a_2^i(s), \ell^{ij}, b^{ij}\}_{t=1..3, s \in \mathcal{S}}^{j \neq i}$ solves problem (10) with contracts (f_{ij}, f_{ji}) , $j \neq i$, for investor $i = 1, 2$.
2. Spot markets clear: $a_1^1 + a_1^2 = a$ and $a_2^1(s) + a_2^2(s) = a$ for any s . Repo markets clear: $b^{ij} = l^{ji}$ for $i = 1, 2$ and $j \neq i$.
3. For every other contract $\tilde{f}_{ij} \in \mathcal{F}_{ij}(\mathbf{p}_2) \setminus \{f_{ij}\}$ the price $q_{ij}(\tilde{f}_{ij})$ is such that investors i and j do not wish to trade this contract, for $j \neq i = 1, 2$.

The equilibrium selects the repo contracts that agents trade. Condition 3. ensures that the market for other repo contracts clear with a zero level of trade.

3 Repo markets with no re-use

It is useful to characterize first the equilibrium when investors cannot re-use collateral, that is $\nu_1 = \nu_2 = 0$. We will show that in this case the only repo contract traded in equilibrium is a contract sold by investor 1, who has a funding need. In the remainder of this section, we simply refer to this contract as f and to its price as $q := q_{12}(f)$.

3.1 Equilibrium repo contract

To gain some intuition, recall that, at the first best allocation, investor 1 borrows in period 1 by promising to repay $c_{2,*}^2 - \omega$ in period 2. In a repo equilibrium, the maximum pledgeable income of investor 1 in state s is $ap_2(s)/(1 - \theta_1)$. This expression is the amount investor 1 can promise to repay when he sells all the asset using the repo contract with a repurchase price equal to his borrowing capacity $p_2(s)/(1 - \theta_1)$. For low realizations of $p_2(s)$, this payment may fall short of $c_{2,*}^2 - \omega$. We will see that in equilibrium investor 1 sells all his asset in a repo. At the chosen contract, the repurchase price equals the investor's borrowing capacity in the states where the value of the asset is low, while in the other states it is independent of s and lies below the borrowing capacity. In those states, where $p_2(s)$ is relatively high, the pledgeable income allows to finance the first best allocation: the constant level of consumption $c_{2,*}^2$ in period 2 is then attained with a constant repurchase price.

In this equilibrium agents do not trade in the spot market in period 2. Hence all the asset is held by investor 1 at the end of period 2. Investor 1's consumption in period 2 is then:

$$c_2^1(s) = \omega - af(s)$$

and the equilibrium spot price is determined by the following first order condition

$$p_2(s) = s/v'(c_2^1(s)) \tag{18}$$

As we said above, $f(s)$ is independent of s in some states and equal to $p_2(s)/(1 - \theta_1)$ in other states. This, together with the above expressions, implies that $p_2(s)$ is strictly increasing in s . Hence, there exists a threshold s^* defined by the following equation

$$c_{2,*}^2 = \omega + \frac{ap_2(s^*)}{(1-\theta_1)} = \omega + \frac{as^*}{v'(c_{2,*}^1)(1-\theta_1)}. \quad (19)$$

such that for all $s \geq s^*$, the equilibrium consumption is equal to the first best consumption levels $(c_{2,*}^1, c_{2,*}^2)$. For $s \leq s^*$, the equilibrium consumption of investor 1 is $c_2^1(s) = \omega - ap_2(s)/(1-\theta_1)$. Observe that s^* is decreasing with a and θ_1 . Hence, when the quantity of the asset is large and/or investor 1 is sufficiently creditworthy, s^* lies below \underline{s} and the first best consumption is achieved in all states.

Substituting for $c_2^1(s)$ in equation (18) yields the following expression for the equilibrium spot price in period 2

$$\begin{cases} p_2(s)v' \left(\omega - a \frac{p_2(s)}{1-\theta_1} \right) = s & \text{if } s < s^* \\ p_2(s)v'(c_{2,*}^1) = s & \text{if } s \geq s^* \end{cases} \quad (20)$$

The result is formally stated in the following:

Proposition 1. Repo Equilibrium. *There exists an equilibrium where investors only trade a repo contract f with the following characteristics:*

1. If $s^* \geq \bar{s}$ (a is low), $f(s) = p_2(s)/(1-\theta_1)$ for all $s \in \mathcal{S}$
2. If $s^* \in [\underline{s}, \bar{s}]$ (a is intermediate),

$$f(s) = \begin{cases} \frac{p_2(s)}{1-\theta_1} & \text{for } s \leq s^* \\ \frac{p_2(s^*)}{(1-\theta_1)} & \text{for } s \geq s^* \end{cases} \quad (21)$$

3. If $s^* \leq \underline{s}$ (a is high), $f(s) = f^*$ for all $s \in \mathcal{S}$ where $f^* \in [\frac{p_2(s^*)}{(1-\theta_1)}, \frac{p_2(\bar{s})}{(1-\theta_1)}]$.

where p_2 is defined in (20). The equilibrium allocation is always unique; the pattern of trades is also unique in cases 1. and 2., when $\theta_1 > 0$.

Two forces are shaping the equilibrium repo contract: investor 1's desire to borrow in period 1 and the aversion of both investors to risk in their portfolio return in period 2. When the value of the asset is low, for $s \leq s^*$, the maximum pledgeable income of

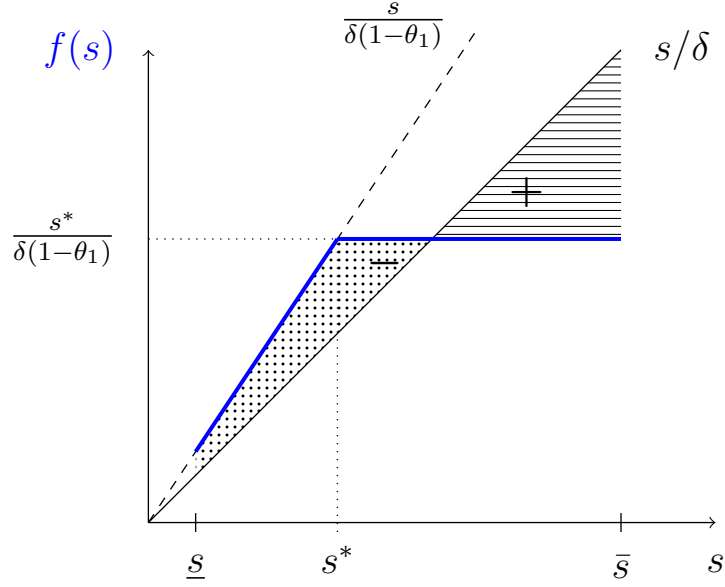


Figure 1: Repo contract ($v(x) = \delta x$).

investor 1 is insufficient to exhaust all gains from trade so that this investor is borrowing constrained. In these states, the repurchase price is equal to this borrowing capacity. Hence, $f(s)$ is increasing in s and is only determined by investor 1's *borrowing motive*. On the other hand, when the collateral value is high, for $s > s^*$, the maximum pledgeable income exceeds investor 1's borrowing needs. Hence, the repurchase price is constant for $s \geq s^*$ and allows investors to perfectly hedge against the price risk in those states. The repurchase price is thus pinned down by this *hedging motive*. Figure 1 plots the equilibrium repo contract in case 2. when $v(x) = \delta x$ for some $\delta \in (0, 1)$.

Note that when $s^* \geq \underline{s}$, there is a unique equilibrium repo contract and investor 1 sells all his asset using this repo. When the collateral is abundant so that $s^* < \underline{s}$, investors attain the first best allocation in equilibrium. In this case, several repo contracts with constant repurchase price or a combination of repo and spot trades allow to support the equilibrium allocation.

As we show in the proof of Proposition 1, when investors trade the repo contract f they do not want to trade other repo contracts nor to engage in spot trades. In addition, there is no other equilibrium where a different contract is traded. To gain some intuition about the first point, suppose instead that investor 1 sells (some of) the asset spot in

period 1 and buys it back at the spot market price $p_2(s)$ in period 2. This is formally equivalent to selling a repo contract \hat{f} with $\hat{f}(s) = p_2(s)$ for each s . This alternative trade is dominated for two reasons. When the collateral value is low, investor 1 can increase the amount he pledges from $p_2(s)$ to $p_2(s)/(1 - \theta_1)$ by selling the equilibrium repo contract. When the collateral value is high, the spot trades expose investors to fluctuations in consumption which they can avoid by trading the equilibrium repo contract. Similar considerations apply to trades involving other repo contracts.

We can associate the equilibrium repurchase price to a repo rate r given by:

$$1 + r = \frac{\mathbb{E}[f(s)]}{q} = \frac{\mathbb{E}[f(s)]}{\mathbb{E}[f(s)u'(c_2^2(s))]} \quad (22)$$

When investors are constrained (cases 1. and 2. of Proposition 1), the borrowing rate is lower than in the first best allocation: $1+r < 1+r^*$ since $u'(c_2^2(s)) > u'(c_{2,*}^2)$ for $s \in [\underline{s}, s^*]$. In the repo equilibrium, investor 1 is borrowing constrained so the equilibrium interest rate must fall to induce type 2 investors to lend an amount compatible with market clearing.

3.2 Haircuts and liquidity premium

In this section we derive the properties of the liquidity premium and the haircut in the repo equilibrium. We define the liquidity premium \mathcal{L} as the difference between the spot price of the asset in period 1 and its fundamental value. Setting the fundamental value of the asset to be its price in the Arrow-Debreu equilibrium, the liquidity premium is:

$$\mathcal{L} \equiv p_1 - \mathbb{E}[s]$$

The liquidity premium is also equal to the shadow price of the collateral constraint. It thus captures the value of the asset as an instrument that facilitates borrowing over and above its holding value. Hence, whenever the asset is scarce and investors are constrained, the asset bears a positive liquidity premium. Using the equilibrium characterization, we can relate the liquidity premium to the repurchase price of the equilibrium contract and the marginal utilities of the borrower and the lender:

$$\mathcal{L} = \mathbb{E}[f(s)(u'(c_2^2(s)) - v'(c_2^1(s)))] \quad (23)$$

When the repo collateral is abundant ($s^* \leq \underline{s}$), investors are not constrained and $c_2^2(s) = c_{2,*}^2$ for all s , so that $\mathcal{L} = 0$. When the repo collateral is scarce ($s^* > \underline{s}$), we have $u'(c_2^2(s)) > v'(c_2^1(s))$ for $s < s^*$, that is some gains from trade are not realized in low states and $\mathcal{L} > 0$.

The repo haircut is the difference between the spot market price of the asset and the repo price in period 1. One unit of the asset can be bought in the spot market at price p_1 and sold at the equilibrium repo price q . So to purchase 1 unit of the asset, an investor needs $p_1 - q$, which is the down payment or haircut:²³

$$\mathcal{H} \equiv p_1 - q = \mathbb{E}[(p_2(s) - f(s))v'(c_2^1(s))] \quad (24)$$

The second equality in (24) follows from the first order condition of investor 1 with respect to spot and repo trades. As Figure 1 shows, the borrowing and hedging motives have opposite effects on the size of the haircut. In the region $s < s^*$, where investor 1 is constrained, the repurchase price is equal to his borrowing capacity $p_2(s)/(1 - \theta_1)$ while the asset trades at price $p_2(s)$. From expression (24) we see that this contributes negatively to the haircut. On the other hand, in states $s \geq s^*$ the repurchase price $f(s)$ is constant while the asset price $p_2(s)$ increases with s . This contributes positively to the haircut (more precisely, this is true when $f(s)$ as specified in (21) is smaller than $p_2(s)$). These two cases correspond respectively to the dotted and dashed regions in Figure 1. The overall sign of the haircut depends on the probability mass attributed to the two regions by the distribution of s . Finally, observe that the haircut is not uniquely pinned down when $s^* \leq \underline{s}$ since several (constant) repurchase prices f are compatible with the unique equilibrium allocation.

3.2.1 Collateral scarcity and counterparty quality

In this section we study the impact of collateral scarcity and counterparty quality on the level of the liquidity premium and the haircut.

Proposition 2. *\mathcal{L} is decreasing and \mathcal{H} is increasing in the amount of collateral a . \mathcal{H} decreases in counterparty quality θ_1 while the effect on \mathcal{L} is ambiguous.*

When a increases, more asset can be sold in a repo. Investor 1 can thus borrow more

²³An alternative but equivalent definition is $(p_1 - q)/q$.

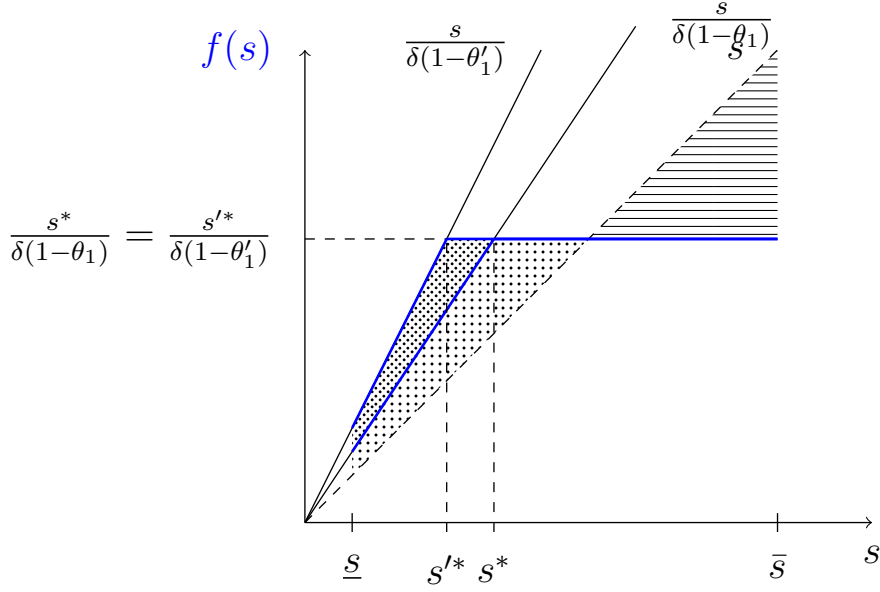


Figure 2: Influence of counterparty quality, with $\theta'_1 > \theta_1$ ($v(x) = \delta x$)

in states $s < s^*$, which reduces the wedge $u'(c_2^2(s)) - v'(c_2^1(s))$ in marginal utilities. As more gains from trade are realized, the shadow price of collateral, that is \mathcal{L} , goes down. Haircuts increase with the quantity a of the asset because s^* declines when a increases. Hence, there are less states where the repurchase price is equal to the borrowing capacity, which contributes negatively to the haircut (see Figure 1).

A higher counterparty quality θ_1 decreases haircuts since the borrowing capacity $p_2(s)/(1 - \theta_1)$ increases. Intuitively, a better counterparty has a higher ability to honor debt, which reduces the required down payment. Figure 2 illustrates the effect of an increase from θ_1 to $\theta'_1 > \theta_1$. The solid line representing the borrowing capacity shifts counterclockwise. This naturally leads to a decrease in the haircut, by increasing the size of the region where $f(s) > p_2(s)$ while leaving the other region unchanged. The increase in the size of the first region corresponds to the area with denser dots on Figure 2.

When it comes to the liquidity premium \mathcal{L} , counterparty quality θ_1 has an ambiguous effect. An increase in θ_1 increases the borrowing capacity in states $s < s^*$ and so allows investor 1 to borrow more,²⁴ reducing the wedge $u'(c_2^2(s)) - v'(c_2^1(s))$ in marginal utilities.

²⁴To assess properly the effect of θ_1 on the borrowing capacity $p_2(s)/(1 - \theta_1)$, one should account for the effect of θ_1 on the equilibrium value of $p_2(s)$. The period 2 spot market price is indeed determined

This effect, similar to the one we found for an increase in the asset available, tends to reduce the liquidity premium. However, the increase in the borrowing capacity due to a higher θ_1 also affects the properties of the equilibrium repo contract in the states $s < s^*$, since $f(s) = p_2(s)/(1 - \theta_1)$. As more income can be pledged when this is most valuable, the asset becomes a better borrowing instrument, which raises its price and so its liquidity premium.

3.2.2 Asset risk

Our model also allows us to compare haircuts and liquidity premia for assets with different risk profiles. To this end, we extend the environment by introducing a second asset. For simplicity, we assume that the second asset has a perfectly correlated payoff with the first asset but carries higher risk. Hence there is no possibility of hedging positions in one asset with an opposite position in the other asset. Therefore the pattern of equilibrium trades as well as the properties of repo contracts are determined by the same principles as before.

The second asset pays a mean preserving spread of the dividend of the first asset dividend,

$$\rho(s) = s + \sigma(s - \mathbb{E}[s]),$$

where $\sigma > 0$. Investor 1 is still endowed with a units of the first asset and also owns b units of the second asset, while investor 2 is not endowed with any of the assets. The set of available contracts consists of all feasible repos using any of the two assets. It is relatively straightforward to extend the equilibrium analysis of the previous section to this new environment. For each asset, the repurchase price of the equilibrium repo contract is equal to the borrowing capacity of that asset in all states where the first best level of consumption cannot be reached and is constant otherwise. We then establish the following result.

Proposition 3. *The safer asset always has a higher liquidity premium and a lower haircut than the riskier asset.*

The key intuition behind the result is that the mean preserving spread of the dividend induces a misallocation of collateral value across states. While the two assets have the same $p_2(s)v'(\omega - ap_2(s)/(1 - \theta_1)) - s = 0$ for $s \leq s^*$, so that $p_2(s)$ decreases with θ_1 . However, one can easily show that the net effect is positive, that is $\partial[p_2(s)/(1 - \theta_1)]/\partial\theta_1 > 0$.

same expected payoff $\mathbb{E}[s]$, the riskier asset pays relatively more in high states (where there is upside risk) and less in low states (downside risk). An asset is particularly valuable as collateral in low states where investor 1 is borrowing constrained. Since the safer asset pays more in these states, it carries a larger liquidity premium. Turning now to the haircut, the riskier asset has a higher dividend in high states, which ensures a higher borrowing capacity in these states compared to the safer asset. However, investor 1 does not benefit by borrowing more in those states where he attains the first best level of consumption. Thus, since a smaller fraction of the asset dividend is pledged in the equilibrium repo for the second asset, the haircut is larger. Observe that, without the hedging motive, the repurchase price would always be equal to the borrowing capacity and so, by the previous argument, asset risk would have no impact on the haircut.

In the analysis above, we compared haircuts and liquidity premia for two assets with different risk when investors can trade them both at the same time, rather than examining how equilibrium prices vary in the one asset economy when the dividend risk is modified.²⁵ An advantage of our approach is that the same stochastic discount factors are used to price both assets. Hence the comparison effectively controls for market conditions and its implications can be brought to the data in a more meaningful way.

So far, the transfer of ownership of the collateral from the borrower to the lender did not play any role in the analysis. Indeed, with $\nu_2 = 0$, the asset is immobile once pledged in a repo to investor 2. The next two sections show that allowing for re-use delivers new predictions. First, re-use increases the borrowing capacity of investor 1. Second, the possibility of re-using collateral may lead to endogenous intermediation in equilibrium.

4 Re-use and the collateral multiplier

In this section, we analyze the impact of collateral re-use on equilibrium contracts and allocations. Various authors (see for instance [Singh and Aitken, 2010](#)) have stressed the importance of this feature of a repo trade where the collateral is sold to the lender. Our model allows to precisely characterize the benefits of re-use and the effects on repo contracts, in the presence of limited commitment. We will show that investors always

²⁵For completeness we also performed this second analysis, finding that a mean preserving spread implies a higher haircut while the effect on the liquidity premium is indeterminate and depends on risk aversion.

want to re-use collateral because it expands their borrowing capacity. The properties of the repo contract traded in equilibrium then need to be suitably adjusted. In particular, we also need to take into account the lender's incentives to return the collateral.

The lender, investor 2, is now able to re-use the collateral, that is $\nu_2 > 0$. We establish first that investor 2 would use this option, if available. Consider the equilibrium without re-use characterized in the previous section. In this equilibrium, investor 1 uses all of his asset as collateral in a repo trade. Hence, investor 2 ends up holding a units of the collateral. Suppose now type 2 investors could sell an infinitesimal amount ϵ of the collateral in the spot market. At the given equilibrium prices, type 1 investors would be happy to buy this amount so as to use it in an additional repo sale under the same terms. These investors neither benefit nor lose from these two transactions as they were already available to them in the absence of re-use. The marginal gain for investor 2 from the opposite transactions (spot sale and repo purchase) is instead:

$$\begin{aligned} \frac{\partial U^2}{\partial \epsilon} &= p_1 - E[p_2(s)u'(c_2^2(s))] - q + E[f(s)u'(c_2^2(s))] \\ &= \frac{\theta_1}{1 - \theta_1} \int_{\underline{s}}^{s^*} p_2(s) (u'(c_2^2(s)) - v'(c_2^1(s))) dG(s) \end{aligned}$$

where, to derive the second equality,²⁶ we used the expressions for the haircut in (24) and the equilibrium repurchase price in (21). Hence when $s^* > \underline{s}$ (collateral is too scarce to satisfy fully the borrowing needs of investor 1) and $\theta_1 > 0$ we have $\partial U^2 / \partial \epsilon > 0$, thus investor 2 strictly benefits from the possibility of carrying out these trades. In other words, whenever investor 1 is borrowing constrained and since repos are recourse loans ($\theta_1 > 0$), investor 2 always benefits by selling an infinitesimal amount ϵ of the collateral in the spot market and buying it back in a repo. These trades are not feasible without

²⁶To understand the expression above, note that investor 2 gets $f(s)\epsilon$ in state s of period 2 from the additional repo purchase of ϵ units. However, since he sold some of the collateral he received, to cover this short position he must also purchase ϵ units of the asset spot in period 2. The net additional payoff to investor 2 in period 2 in the states $s < s^*$ is then

$$-p_2(s)\epsilon + \frac{p_2(s)}{1 - \theta_1}\epsilon = \frac{\theta_1}{1 - \theta_1}p_2(s)\epsilon.$$

This expression is strictly positive if $\theta_1 > 0$. Hence in this case the additional trades allowed by re-use increase the income pledged by investor 1, which is beneficial since the investor is borrowing constrained in those states. In states $s > s^*$ the marginal impact of the net payoff of these trades is null since gains from trade are exhausted.

re-use because all the asset is segregated as collateral in the repo and hence investor 2 has no asset to sell.

Having shown that a marginal re-use of collateral relaxes the borrowing constraint of investor 1, we now determine the total effect of re-use on his borrowing capacity. To this end, we should take into account that collateral re-use can occur repeatedly over several rounds of trade within period 1. At the end of the first round, for every unit purchased from investor 1 in a repo, a type 2 investor has ν_2 units of the asset which he can re-use to sell spot. Investor 1 can then buy spot and resell these units in a new repo, which generates a net additional payoff $\frac{\theta_1}{1-\theta_1}\nu_2 p_2(s)$ for investor 2 in state s of period 2. At the end of this second round, type 2 investors 2 have $(\nu_2)^2$ units of asset they can re-use. Iterating this process over infinitely many rounds, we obtain the new expression of investor 1's borrowing capacity in state s with re-use:

$$\frac{p_2(s)}{1-\theta_1} + \sum_{r=1}^{\infty} (\nu_2)^r \frac{\theta_1}{1-\theta_1} p_2(s) = \frac{1-\theta_1}{1-\nu_2} \left[\frac{1}{1-\theta_1} - \nu_2 \right] \frac{p_2(s)}{1-\theta_1} \quad (25)$$

The term

$$M_{12} \equiv \frac{1-\theta_1}{1-\nu_2} \left[\frac{1}{1-\theta_1} - \nu_2 \right] \quad (26)$$

constitutes the *collateral multiplier*, that is the increase in borrowing capacity generated by the infinite sequence of collateral re-use. This multiplier is greater than 1 and strictly increasing in ν_2 as long as $\theta_1 > 0$. This clearly shows that the effectiveness of re-use in expanding the borrowing capacity crucially depends on the recourse nature of repo loans. Indeed, re-use would have no effect if the only punishment for default were the loss of collateral.²⁷

We now characterize the new properties of the equilibrium allocation and the repo contract. Re-use induces two changes to the properties of the equilibrium contract. First, it lowers the threshold s^* : the collateral multiplier expands the borrowing capacity, thus increasing the set of states where investors can attain the first best allocation. Let $s^*(\nu_2)$ denote the minimal state above which investor 1 can pledge enough income to finance the first best allocation when investor 2 can re-use a fraction ν_2 of the collateral. The

²⁷In line with our result, [Maurin \(2017\)](#) proved in a more general setting that when loans are non recourse, re-use is redundant unless the market for financial securities is incomplete.

new threshold $s^*(\nu_2)$ is determined by the following equation:

$$c_{2,*}^2 = \omega + aM_{12} \frac{p_2(s^*(\nu_2))}{1 - \theta_1} = \omega + aM_{12} \frac{s^*(\nu_2)}{(1 - \theta_1)v'(c_{2,*}^1)} \quad (27)$$

which is similar to (19) except for the presence of the multiplier.

The structure of the repo contract also changes because investor 2 effectively shorts the asset when he re-sells the collateral in the spot market. To unwind his short position and be able to return the collateral he received, investor 2 has to purchase the asset in the spot market in period 2, which exposes him to price risk. Hence, to hedge this risk when $s > s^*(\nu_2)$, the repurchase price should vary with s so as to perfectly offset the cost $\nu_2 p_2(s)$ of unwinding the short position. On the other hand, when $s < s^*(\nu_2)$ the borrowing motive dominates the hedging motive as before, so that the structure of the contract does not change.

Proposition 4. *Equilibrium with re-use.* *Let $\nu_1, \nu_2 \in (0, 1)$, $\theta_1 > 0$ and $s^*(0) = s^* > \underline{s}$ (the first-best allocation cannot be achieved without re-use). There is a unique equilibrium allocation where investor 1 borrows using repo contract $f(\nu_2)$ satisfying:*

$$f(s, \nu_2) = \begin{cases} \frac{p_2(s)}{1 - \theta_1} & \text{if } s < s^*(\nu_2) \\ \frac{p_2(s^*(\nu_2))}{(1 - \theta_1)} + \nu_2(p_2(s) - p_2(s^*(\nu_2))) & \text{if } s \geq s^*(\nu_2) \end{cases} \quad (28)$$

where $p_2(s)$ is determined by an expression analogous to (20). Investor 2 re-sells collateral spot in equilibrium. There exists $\nu^* < 1$ such that for $\nu_2 \geq \nu^*$ the first-best allocation is attained in equilibrium.

The repo contract specified in (28) is again such that the borrower never wants to default. In addition, we also need to check the incentives of the lender to comply with his promise to return the asset. This is immediate. The payment from the repo contract $f(s, \nu_2)$ is in fact always higher than the value of the re-usable collateral $\nu_2 p_2(s)$ that investor 2 can abscond with. Hence, the lender never wants to default with this contract since the no-default constraint (6) is satisfied for any value of ν_2 .

From the expression of the collateral multiplier M_{12} in (26) it is clear that, the higher ν_2 , the higher the multiplier and ultimately the borrowing capacity of investor 1. The final claim in the proposition states that, when the re-usable fraction of the collateral is

sufficiently high ($\nu_2 \geq \nu^*$), the first-best allocation can be financed even in the lowest state \underline{s} . One can obtain the expression for ν^* simply by setting $s^*(\nu_2) = \underline{s}$ in equation (27) (see the Appendix for details):

$$\nu^* = \frac{s^* - \underline{s}}{s^* - (1 - \theta_1)\underline{s}}.$$

We showed that investors always want to re-use collateral when they can do so, and this is true whether the haircut is positive or negative. Buying 1 unit of asset spot and selling it back in a repo increases investor 1's income in period 1 by $-p_1 + p_F = -\mathcal{H}$, that is minus the haircut. When $\mathcal{H} < 0$, these transactions relax investor 1 borrowing constraint to capture some of the unexploited gains from trade. It may thus seem that buying spot to sell repo is not desirable when $\mathcal{H} > 0$ since in that case the period 1 income of investor 1 decreases. But this line of argument ignores other gains from transferring income across states in period 2. Indeed, in period 2 in state s investor 1 will re-purchase one unit of the asset at price $f(s)$, as agreed in the repo contract, and will sell it spot at price $p_2(s)$, thus netting a gain $p_2(s) - f(s)$. This gain is negative for $s < s^*$, but from expression (24) we see that, when $\mathcal{H} > 0$, it must be positive for s sufficiently large. In words, these trades allow investor 1 to reduce his income in the low states where his marginal utility for consumption is low (and the one of investor 2 is high) while increasing his income in the high states. Therefore, re-use with $\mathcal{H} > 0$ will allow investor 1 to smooth (albeit imperfectly) his consumption across states in period 2. Our analysis shows that this additional smoothing effect in period 2 compensates for the reduction in investor 1's income in period 1. Note these possible benefits do not depend on the fact that the repurchase price $f(s)$ is contingent on s (as further discussed in the last section).

Consider now to the effect of re-use on the liquidity premium \mathcal{L} . Since re-use expands the borrowing capacity of investor 1, its effect is similar to that of an increase in counterparty quality in which case, as we saw in Section 3.2, the overall impact on \mathcal{L} is ambiguous. Finally, our model predicts that the benefits of re-use are larger when collateral is most scarce (that is $s^* > \bar{s}$) and there is evidence that this is indeed the case (see [Fuhrer et al., 2015](#)).

5 Collateral Re-use and Intermediation

In practice, cash is intermediated among market participants through chains of repos.²⁸ For example, as Figure 3 illustrates, a hedge fund borrows cash through a repo from a dealer bank who finances this transaction by tapping a cash pool, say a money market fund (MMF), via another repo. This is surprising because platforms such as Direct Repo™ in the US grant hedge funds direct access to cash pools. So why do traders resort to repo intermediation? In this section we show that these chains of repos may arise in equilibrium. A remarkable feature of our analysis is that intermediation arises endogenously: although the hedge fund is free to trade directly with the MMF, he still prefers to trade instead with a dealer bank. We explain this feature with differences in counterparty quality for the hedge fund and the dealer bank.²⁹

In this section we extend the economy introducing a third type of investors labeled B , for dealer Banks. Investor B is endowed with no asset and ω units of the consumption good in periods 1 and 2 and has the following preferences:

$$U^B(c_1, c_2, c_3) = c_1 + \delta_B c_2 + c_3$$

For simplicity, as a special case of our general specification, we assume here that investor 1 has linear preferences too, that is $v(x) = \delta x$ or:

$$U^1(c_1, c_2, c_3) = c_1 + \delta c_2 + c_3$$

We posit $\delta \leq \delta_B < u'(\omega)$. This implies that investor B would also like to borrow from investor 2 in the first period but has no asset to use as collateral, and has weakly lower gains from trade than investor 1. We assume $\theta_B > \theta_1$ so investor B is more creditworthy than investor 1. His greater borrowing capacity will explain why investor B can play a role as an intermediary. All investors are free to participate in the spot market and engage in repo trades with any type of counterparty. We will say that there is intermediation when

²⁸In their guide to the repo market, Baklanova et al. (2015) state that “dealers operate as intermediaries between those who lend cash collateralized by securities, and those who seek funding”.

²⁹In practice, the transaction between the dealer bank and the MMF could take place using a Tri-Party agent as a custodian. We abstract from modeling the services provided by the Tri-Party agent. See Federal Reserve Bank of New York (2010) for a discussion of this segment of the repo market. We thus focus on the intermediation provided by the dealer bank to the hedge fund and the MMF.

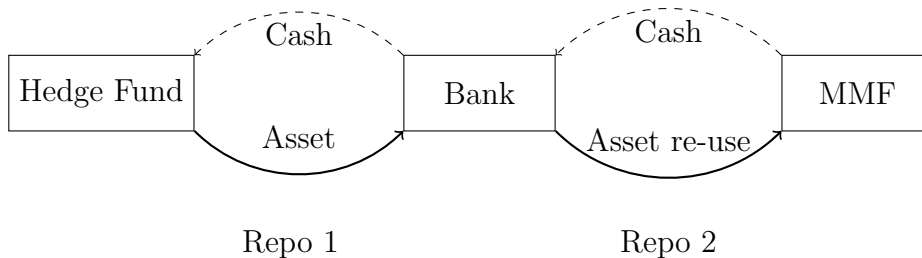


Figure 3: Intermediation with Repo

investor 1 sells his asset to B and B re-sells it to investor 2. We show that intermediation indeed arises in equilibrium. It may take place via a spot or a repo sale from investor 1 to B depending on the relative values of δ and δ_B . Thus our notion of intermediation encompasses more than just chains of repos and we derive below the conditions for each pattern of intermediation to arise. In what follows, it is useful to refer sometimes to agent 1 as the natural borrower, to agent 2 as the natural lender, and to agent B as the intermediary.

5.1 Intermediation via spot trades

We assume first that the natural borrower and the intermediary have the same preferences, that is $\delta = \delta_B$ and only differ in their creditworthiness. We show that in equilibrium intermediation takes place via a spot sale from 1 to B . Note that in this case, there are no direct gains from trade between 1 and B . Hence, the trades between these investors are only driven by the intermediation role played by B .

Proposition 5. *Intermediation Equilibrium.* *Let $\delta = \delta_B$ and $\theta_1 < \theta_B$. When $s^*(\nu_2) > \underline{s}$ (the first best allocation cannot be achieved in the equilibrium with re-use of Proposition 4), in equilibrium, investor 1 sells his asset spot to investor B , who then re-sells it in a repo to investor 2.*

The striking feature in Proposition 5 is that investor 1, who is endowed with the asset, no longer sells it in a repo contract to investor 2, the natural lender. Instead, in equilibrium, investor 1 sells the asset spot to B . Once investor B gains possession of the asset, he finds himself in the same position as investor 1 in the last section via a vis

investor 2. He then engages in an infinite sequence of repo sales and spot purchases of the re-usable collateral with 2. The equilibrium repo contract f_{B2} is specified as in (28), replacing θ_1 with θ_B .

If investor B were not present, we saw in the previous sections that investor 1 would borrow in a repo from investor 2. However, since $\theta_B > \theta_1$, investor B can borrow more than 1 from investor 2 for each unit of the asset. Thus investor B values the asset more and bids up the spot market price. As a result, investor 1 prefers to sell his asset in the spot market, as if he were delegating borrowing to a more creditworthy investor.

Intermediation takes place via a spot sale from investors 1 to B and not via a repo sale. To understand this, observe that, since 1 and B have the same preferences, they cannot benefit from a redistribution of income among them between periods 1 and 2. With a repo, investor 1 would in fact be able to obtain from B more income to be spent in period 1 as compared to a spot sale. However, investor 1's benefit equals what he must pay to B for the transfer. In addition, trading a repo entails a cost because a fraction $1 - \nu_B$ of every unit of the asset transferred to B could not be used to borrow from 2. Hence, investor B would pay a lower price to acquire the asset through a repo purchase, which implies the preference for a spot transaction.

Finally, investor B could be inactive in equilibrium if investor 1 is endowed with a sufficiently high quantity of the asset (that is, $s^*(\nu_2) < \underline{s}$). In the case, investor 1 can attain the first best allocation by trading directly with investor 2 in spite of his lower creditworthiness. An interesting implication of our result is thus that intermediation should be observed precisely when collateral is scarce.

5.2 Chain of repos

We show next that when $\delta < \delta_B$ intermediation may occur via a chain of repos. We call *intermediation equilibrium with a chain of repos* an equilibrium where the following pattern of trades is observed: investor 1 sells the asset in a repo to investor B , who re-uses the asset to sell it in a repo to a type 2 investor. Since $\delta < \delta_B$, there are now direct gains from trade between 1 and B . However, since $\delta_B < u'(\omega)$, these gains are still smaller than those between 1 and 2. Hence, trades between 1 and B must still be at least partially driven by the intermediation role of B .

It is useful to compare first the chain of repos with alternative patterns of trades. This

discussion will shed some light on the conditions stated in the repo chain equilibrium of Proposition 6. When $\delta < \delta_B$ a redistribution of income from period 2 to period 1 in favor of investor 1 is beneficial. It follows from the discussion in the previous section that investor 1 could capture these benefits by using a repo, instead of a spot sale, at the cost of immobilizing collateral. Thus a trade-off emerges now. For investors 1 and B to prefer a repo sale over a spot sale, the direct gains from trade between 1 and B, given by $\delta_B - \delta$, must be sufficiently large relative to the fraction of collateral segregated $1 - \nu_B$. At the same time, the direct gains from trade between B and 2 must be sufficiently large for B to be willing to re-use the asset he acquires from 1 in a repo trade with 2. Otherwise, he will use all the asset in trades with investor 1. This imposes an upper bound on $\delta_B - \delta$. Finally observe that, unlike with a spot sale from investor 1 to B, intermediation with a repo chain involves collateral segregation. Hence, intermediation is preferred to direct trade between investors 1 and 2 if the difference in counterparty quality $\theta_B - \theta_1$ between B and 1 offsets the cost of segregation $1 - \nu_B$.

Investor 2's ability to re-use collateral does not affect qualitatively any of the trade-offs described above so for clarity we set $\nu_2 = 0$ in what follows.³⁰ We can now state the exact conditions under which a chain of repo arises in equilibrium.

Proposition 6. Chain of Repos. *Let $\nu_2 = 0$. There exists $\bar{\delta}_B > \underline{\delta}_B > \delta$ such that the equilibrium features intermediation with a chain of repos if and only if $\delta_B \in [\underline{\delta}_B, \bar{\delta}_B]$ and*

$$\frac{\nu_B}{1 - \theta_B} \geq \frac{1}{1 - \theta_1} \quad (29)$$

Investor 1 sells all the asset in a repo f_{1B} to B with

$$f_{1B}(s) = \frac{s}{1 - \theta_1} \quad \forall s \in [\underline{s}, \bar{s}] \quad (30)$$

Investor B sells part of the asset in a repo f_{B2} to 2 with

$$f_{B2}(s) = \begin{cases} \frac{p_2(s)}{1 - \theta_B} & \text{if } s < s_{B2}^* \\ \frac{p_2(s_{B2}^*)}{1 - \theta_B} & \text{if } s \geq s_{B2}^* \end{cases}$$

³⁰In Online Appendix B.3, we show that an analogous result holds when ν_2 is positive but sufficiently smaller than ν_B .

for some $s_{B2}^* \in [\underline{s}, \bar{s}]$ and the remaining part in a spot sale to investor 1.

The lower bound $\underline{\delta}_B$ on δ_B ensures investor 1 prefers to sell the asset in a repo rather than spot to investor B . The upper bound $\bar{\delta}_B$ ensures that the direct gains from trade between investors B and 2 are sufficiently large that B prefers to re-use part of the asset to trade with 2. We actually show that, in equilibrium, investor B must be indifferent at the margin between re-selling collateral spot to investor 1 and selling it in a repo to investor 2. For instance, suppose to the contrary that investor B strictly prefers to re-pledge the collateral to investor 2. Then we show that, at the margin, investors 1 and B would rather engage in a spot trade than in a repo. Intuitively, a marginal switch from a repo sale to a spot sale from 1 to B is beneficial since it frees up some of the segregated collateral, allowing B to borrow more from 2.

Condition (29) ensures that intermediation dominates direct trade between investors 1 and 2. It states that $\frac{1}{1-\theta_1}$, the borrowing capacity of investor 1 per unit of asset, is lower than $\frac{\nu_B}{1-\theta_B}$, the borrowing capacity of investor B with one unit of asset acquired in a repo. Since only a fraction ν_B can be re-used by investor B , his higher creditworthiness must compensate for the cost of segregation.

Finally, observe that the repo contract f_{1B} between investors 1 and B does not reflect any hedging motive since both investors are risk neutral. For investors B and 2, the repo contract is instead essentially the same as in Proposition 1 (since $\nu_2 = 0$).

To sum up, intermediation via a chain of repos will arise in equilibrium if a third party is more creditworthy than the natural borrower and more efficient at re-deploying collateral than the natural lender. Our analysis thus shows that repo intermediation arises endogenously out of fundamental heterogeneity between traders. Existing models of repo intermediation typically take the chain of possible trades as exogenous. Our approach is helpful to rationalize several features of the repo market. First, we can explain why intermediating repos is still popular despite the emergence of direct trading platforms. Second, in exogenous intermediation models, dealers typically gain and collect fees by charging higher haircuts to borrowers. In our model, the haircut paid by the borrower to the bank may very well be smaller than the one paid by the bank to the lender. Using data from the Australian repo market, Issa and Jarnecic (2016) show that this is indeed the case in most transactions.

6 Constant Repurchase Price

So far, we allowed the repurchase price in a repo contract to be state-contingent. As we argued in Section 2.3, this feature can be justified by margin calls or loan repricing. However, these events may not occur for short-term maturity repos. We thus extend the analysis in this section to the case where investors can only trade contracts with a constant repurchase price. We show that default may occur in equilibrium but - under some additional condition - investors still prefer trading repos rather than spot and value the ability to re-use the collateral. To keep the analysis simple we consider, as in Section 5, the situation where type 1 investors have linear preferences :

$$U^1(c_1, c_2, c_3) = c_1 + \delta c_2 + c_3.$$

and focus on the case where investor 2 cannot re-use collateral, that is $\nu_2 = 0$.

Let now \bar{f} denote the constant payoff of a repo contract. This contract does not induce default in any state if investor 1 finds it optimal to repay \bar{f} in state \underline{s} , where the value of the collateral is lowest: in particular, condition (3) implies that we must have $\bar{f} \leq \bar{f}_{min} = \frac{p_2(\underline{s})}{1-\theta_1}$. Thus, the maximum income investor 1 can pledge per unit of asset without defaulting is now given by \bar{f}_{min} .

We show in what follows that investors may now prefer to trade contracts with a higher payoff $\bar{f} > \bar{f}_{min}$, which induces default in low states, even when the assumed condition (8) holds. The key difference with the case of repos with state contingent repurchase prices is that, by trading contract \bar{f} , investors can pledge a higher income in the high states, where default does not occur. More specifically, when investor 1 sells a contract $\bar{f} > \bar{f}_{min}$, he defaults in all states $s \leq s_d(\bar{f})$, with the threshold $s_d(\bar{f})$ defined as the state such that the promised repayment equals the borrowing capacity in that state:

$$\bar{f} = \frac{p_2(s_d(\bar{f}))}{1-\theta_1}$$

The payoffs from this contract are depicted in Figure 4. For $s \geq s_d(\bar{f})$, investor 1 repays \bar{f} to the lender. Whenever default occurs, the payoffs for the lender and the borrower differ. Indeed, below $s_d(\bar{f})$, investor 1 incurs a total cost of $p_2(s) + (\alpha + \pi_1)(\bar{f} - (1 - \kappa)p_2(s))$, represented by the upward sloping dash and dotted line. The lender (investor 2) then

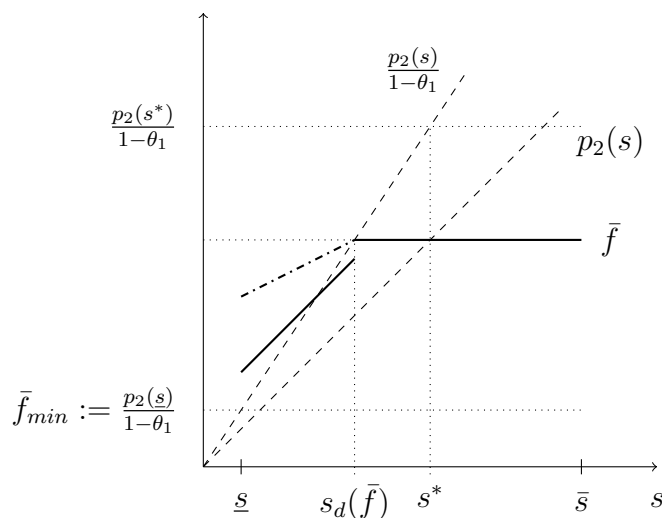


Figure 4: Repo contract with constant repayment \bar{f} . The default threshold is $s_d(\bar{f})$.

obtains the collateral and a fraction α of the shortfall and the liquidation costs $\kappa p_2(s)$ for a total payment of $p_2(s) + \alpha(\bar{f} - (1 - \kappa)p_2(s))$ represented by the upward sloping solid line. The difference between these two lines is the value of the non-pecuniary cost of default $\pi_1(\bar{f} - (1 - \kappa)p_2(s))$ for investor 1.

The trade-off between the benefits from the higher income pledged by the borrower in high states and the default costs incurred in low states appears clearly when comparing the payoffs for contract \bar{f} in Figure 4 to those of contract \bar{f}_{min} . In the states $s \geq s_d(\bar{f})$ no default occurs so that the increase in income pledged $\bar{f} - \bar{f}_{min}$ clearly goes in favor of \bar{f} when investors' borrowing constraints bind. In contrast, in the low states $s < s_d(\bar{f})$ investors lose from trading \bar{f} since under condition (8) the deadweight costs from default outweigh the benefits from the increase in income pledged. As a consequence the relative profitability of the two contracts with constant repurchase price, and hence the equilibrium analysis now depends on the probability distribution G of the asset dividend.

This trade-off between default costs and borrowing capacity also affects the choice of investors to trade repo rather than spot. As we discussed, with a spot sale of the asset in period 1 followed by a spot purchase in period 2, investor 1 effectively pledges $p_2(s_0)$ per unit of the asset in state s_0 . With a repo, investor 1 can instead promise to repay up to the borrowing capacity $\frac{p_2(s_0)}{1 - \theta_1}$ and thus pledge more income in that state. This explains why repos with state contingent repurchase prices dominate spot trades. If the

constant repurchase price of a repo is set equal to $\frac{p_2(s_0)}{1-\theta_1}$, however, investor 1 will default and incur the deadweight costs in states below s_0 . As a result, agents will now strictly prefer trading repo over spot only if the higher pledgeable income in good states more than compensates the default costs in bad states.

It follows from the previous discussion that the set of unit payoffs attainable by trading repos with constant repurchase prices is no longer convex and does not include the payoff of spot trades. Hence we cannot guarantee anymore that investors will trade a single contract in equilibrium. As a consequence, the complete characterization of the equilibrium with constant repurchase prices is difficult. Still, we show in the following proposition that, under suitable conditions, repo trades have similar advantages over spot trades as those established for state contingent repos in Section 3. In addition, there is default in equilibrium, so that actual payoffs still vary with the state:

Proposition 7. *Suppose investors can only trade repos with constant repurchase prices. Under the following conditions: $g(s) = G'(s)$ exists for all s ,*

$$\frac{\underline{s}}{1-\theta_1} > E[s], \quad (31)$$

$$\frac{u' \left(\omega + a \frac{\underline{s}}{\delta(1-\theta_1)} \right) - \delta}{\delta} > \frac{\pi_1 \kappa \underline{s} g(\underline{s})}{1 - (\alpha + \pi_1)(1 - \kappa) - \pi_1 \kappa \underline{s} g(\underline{s})} \quad (32)$$

in equilibrium investors trade repo contracts that induce default in some states.

The first condition ensures that investor 1 strictly prefers selling the asset in a repo than in the spot market. Consider again contract \bar{f}_{min} , with the highest possible repayment such that investors never default. Since the payoff with a spot transaction is $p_2(s)$ and the equilibrium price is $p_2(s) = s/\delta$, condition (31) implies that the income pledged by investor 1 is higher on average when he sells contract \bar{f}_{min} than with a spot sale. Since the contract \bar{f}_{min} is such that default never occurs, the first claim follows. The second condition, inequality (32), then ensures that investors prefer to trade, at least partly, some contract $f > \bar{f}_{min}$, so that in equilibrium investor 1 defaults on some of the repos traded in the states where collateral value is low. Observe that this condition holds if the density $g(\underline{s})$ of the dividend distribution at \underline{s} is sufficiently small, so that the marginal increase in default costs when the repurchase price \bar{f} is raised above \bar{f}_{min} is smaller than the benefit of the increase in pledgeable income. Finally, it can be verified that condition

(32) is consistent with (8), provided that α and π_1 are sufficiently close to 0.

We have shown in Section 4 that when investor 1 strictly prefers to trade repos over spot in equilibrium, investor 2 will re-use collateral if he can. It then follows from the result in Proposition 7 that investors would also benefit from the ability to re-use collateral with constant repurchase price repos. Note that the claim in Proposition 7 requires that $\theta_1 > 0$, needed for condition (31) to hold. Hence the analysis in this section again points to the recourse nature of repo contracts to explain the benefits from collateral re-use. In contrast, we have seen that our main findings survive if the repurchase price cannot be set contingent on the realization of the state.

7 Conclusion

We analyzed a simple model of repurchase agreements with limited commitment and price risk. Unlike a combination of a sale and future repurchase in the spot market, a repo contract provides insurance against price fluctuations. We introduced counterparty risk as the risk of defaulting on the promised repurchase price, in turn determined by - possibly heterogeneous - default costs. We showed that the repo haircut is an increasing function of counterparty risk and of the asset inherent risk. Safe assets also command a higher liquidity premium than risky ones. We model repos as recourse loans and allow investors to re-use collateral, thus capturing the distinctive aspects of repos from standard collateralized loans. We showed that re-use increases borrowing through a collateral multiplier effect. In addition, it can explain intermediation whereby creditworthy investors borrow on behalf of riskier counterparties.

Our simple model delivers rich implications about the repo market but leaves many venues for future research. We argued that counterparty risk is a fundamental determinant of the terms of trade in repo contracts. In Europe, over the past few years, bilateral repo transactions between banks moved increasingly to the centrally cleared segment of the market (see [Mancini et al., 2015](#)). In this case, clearing implies novation of trades by the central counterparties. Novation bears some similarities with intermediation although terms of trades cannot be adjusted and risk may end up being concentrated on a single agent. We believe our model could be extended to account for this evolution. When it comes to re-use, besides the limit on the amount of collateral that can be re-deployed, we assumed a frictionless process. Traders establish and settle positions smoothly although

many rounds of re-use may be involved. Although we did not investigate this aspect in the present work, we believe that in the presence of frictions such as bilateral trading, collateral re-use may contribute to market fragility. This extension would complement the recent literature, such as [Biais et al. \(2015\)](#), who have shown the negative impact of spot market fire sales on secured lending markets.

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Appendix A: Default procedure in Master Agreements

This Appendix describes the default procedure as stated in paragraph 10 of the General Master Repo Agreement (GMRA) of the Securities Industry and Financial Markets Association (SIFMA) and the International Capital Market Association (ICMA). The GMRA, governed by English law is the most widely used Agreement. According to [Baklanova et al. \(2015\)](#), most U.S. repo dealers would rather use SIFMA's Master Repurchase Agreement (MRA), governed by New York state law, for domestic U.S. counterparties but the provisions for default stated in paragraph 11 of the MRA are essentially the same as those in the GRMA.

The default procedure is initiated by one of the parties sending a default notice to the other, after an event of default occurred. Events of default include, for example, the borrower failing to repay the lender at the agreed date, the lender failing to deliver equivalent securities on the repurchase date, either party failing to transfer margins when margins are due, or either party being in a general state of insolvency with respect to other obligations triggering bankruptcy.

Within 20 days following an event of default, if the event is continuing, the non-defaulting party may specify an Early Termination Date (ETD) for all outstanding obligations pertaining to the contract. These obligations and their values are determined by the non-defaulting party. The latter has to present a statement to the defaulting party detailing the amount due under each obligation, as soon as reasonably practicable. The amounts due in relation to the repo contract are netted against each other. The party with a positive balance is liable to pay the other party the day after the statement is provided and interests shall apply between the ETD and the effective payment date following usual market conventions. A delay between the ETD and the statement date may occur, for instance, as the non-defaulting counterparty needs to value the collateral and possibly other securities used as margins in the transaction, as explained below.

To determine the market value of the securities used in the repo, the non-defaulting party can³¹ (1) choose the realized price when (s)he has traded these securities on or

³¹According to the ICMA's summary of the Master Agreement, this value should be a measure of the

about the early termination date, (2) use the arithmetic means of price quotes by two or more market dealers, (3) determine a value for the securities whenever s(he) deems it not reasonable to trade the securities at the quotes provided by the market dealers, e.g. if the securities are illiquid. In any of these scenarios all reasonable transaction costs, realized or expected, related to the purchase or sale of these securities (including commissions, fees, any mark-up, mark-down or premium paid for guaranteed delivery) can be deducted from the securities' value to obtain the default market value of the securities.

The defaulting party shall also be liable for the amount of all reasonable legal and other professional expenses incurred by the non-defaulting party in connection with, or as a consequence of, an event of default. In addition, the non-defaulting party can claim any loss or expense s(he) incurred in replacing transactions or in hedging its exposure in connection with the termination of the repo contract. However, no party may claim any amount related to consequential losses or damages due to an event of default. The default market value of the securities and the other expenses specified above play a key role in the determination of the balance due.

Note that the GMRA distinguishes between an outright default by the lender and a settlement fail whereby the borrower expects the lender to return the collateral but with some delay. The latter event is not often treated as an event of default (see paragraph 10(a)(ii)), because delay may result from internal miscommunication on the type of securities to be transferred, or genuine difficulties for the lender to find these securities (see [Fleming and Garbade, 2005](#)). Still, settlement fails disrupt the functioning of the repo market and the Treasury Market Practices Group (TPMG) introduced a fails charge for U.S. Treasuries in 2009, while the European Union is planning on introducing similar penalties.

Finally, while it is not explicitly stated in the GMRA it is worth pointing out that, in case a party is insolvent, any unsecured claim resulting from the procedure described above will be treated as a junior claim in the bankruptcy proceedings.

securities' fair market value, calculated using whatever pricing sources and methods the non-defaulting party deems appropriate in his reasonable opinion. Sources can include, without limitation, securities with similar maturities, terms and credit characteristics. In effect, the calculation of the Net Value is marking-to-model (calculating a theoretical fundamental price).

Appendix B: Proofs

B.1 Proof of Proposition 1

In the absence of re-use, the set of no-default repo contracts for investor $i \in \{1, 2\}$, for a given spot market price schedule $\mathbf{p}_2 = \{p_2(s)\}_{s \in \mathcal{S}}$, is:

$$\mathcal{F}_{ij}(\mathbf{p}_2) = \left\{ f_{ij} \in \mathcal{F}[\underline{s}, \bar{s}] \mid 0 \leq f_{ij}(s) \leq \frac{p_2(s)}{1 - \theta_i} \right\}$$

where $\mathcal{F}[\underline{s}, \bar{s}]$ is the set of integrable functions over $[\underline{s}, \bar{s}]$. We show here that a competitive equilibrium exists where investors do not trade spot in period 1, investor 1 only sells one contract and investor 2 does not sell a repo contract to investor 1. The proof has several steps. In step 1, we derive the first order conditions for the individual problem (10). In step 2, we determine the conditions under which investors do not wish to trade other repos. This allows us to characterize the equilibrium repo contract and the spot market price in period 2 (Step 3). Finally, we prove in Step 4 our claim that, when condition (8) holds, investors do not trade repos inducing default. In the online Appendix B.2, we then show that the equilibrium allocation is unique, thus completing the proof of the claim in Proposition 1.

Step 1: First order conditions for the individual choice problem (10).

Let γ_1^i denote the Lagrange multiplier of the collateral constraint (14) in problem (10), for investor $i = 1, 2$. The variable $\gamma_2^i(s)$ denotes the Lagrange multiplier on the no short sale constraint (17) in period 2 and state s for investor $i = 1, 2$. As we wrote in the main text, it is convenient here to simply write f for the contract f_{12} sold in equilibrium by investor 1 to 2 and $q = q_{12}(f)$ for its price. The first order conditions of problem (10)

with respect to $a_1^i, a_2^i(s)$ for $i = 1, 2$ and b^{12}, l^{21} are:

$$-p_1 + \mathbb{E}[p_2(s)v'(c_2^1(s))] + \gamma_1^1 = 0, \quad (33)$$

$$q - \mathbb{E}[f(s)v'(c_2^1(s))] - \gamma_1^1 = 0, \quad (34)$$

$$-p_1 + \mathbb{E}[p_2(s)u'(c_2^2(s))] + \gamma_1^2 = 0, \quad (35)$$

$$-q + \mathbb{E}[f(s)u'(c_2^2(s))] = 0, \quad (36)$$

$$-p_2(s)v'(c_2^1(s)) + \gamma_2^1(s) + s = 0, \quad (37)$$

$$-p_2(s)u'(c_2^2(s)) + \gamma_2^2(s) + \beta s = 0. \quad (38)$$

We will verify that in equilibrium investor 2 does not sell a repo contract to investor 1. Hence the first order conditions with respect to l^{12}, b^{21} are not reported above.

Step 2: Conditions on the equilibrium contract f

We determine then the conditions f must satisfy to ensure investors do not trade other repo contracts. Consider an arbitrary repo contract $\tilde{f}_{12} \in \mathcal{F}_{12}(\mathbf{p}_2)$ different from f . For f to be the only traded contract, the price $q_{12}(\tilde{f}_{12})$ must be such that investor 1 does not wish to sell \tilde{f}_{12} and investor 2 does not wish to buy it. Observe that investor 1 prefers not to sell \tilde{f}_{12} as long as its price is lower than $\mathbb{E}\left[\tilde{f}_{12}(s)v'(c_2^1(s))\right] + \gamma_1^1$, where we used the marginal rate of substitution of investor 1, evaluated at the equilibrium allocation, to determine his marginal willingness to sell \tilde{f}_{12} . Similarly, investor 2 prefers not to buy this contract if the price is higher than $\mathbb{E}\left[\tilde{f}_{12}(s)u'(c_2^2(s))\right]$. Hence, it is possible to find a price $q_{12}(\tilde{f}_{12})$ such that there is no trade in equilibrium of repo contract \tilde{f}_{12} iff the following condition holds:

$$\mathbb{E}\left[\tilde{f}_{12}(s)v'(c_2^1(s))\right] + \gamma_1^1 \geq \mathbb{E}\left[\tilde{f}_{12}(s)u'(c_2^2(s))\right] \quad (39)$$

The above inequality can be rewritten, using (34) and (36) above to substitute for γ_1^1 , as:

$$\mathbb{E}\left[\left(f(s) - \tilde{f}_{12}(s)\right)\left(u'(c_2^2(s)) - v'(c_2^1(s))\right)\right] \geq 0 \quad (40)$$

which must hold for all $\tilde{f}_{12} \in \mathcal{F}_{12}(\mathbf{p}_2)$. Similarly there is no trade for repo contract $\tilde{f}_{21} \in \mathcal{F}_{21}(\mathbf{p}_2)$ sold by investor 2 to investor 1 if:

$$\mathbb{E}\left[\tilde{f}_{21}(s)u'(c_2^2(s))\right] + \gamma_1^2 \geq \mathbb{E}\left[\tilde{f}_{21}(s)v'(c_2^1(s))\right] \quad (41)$$

Substituting for γ_1^2 using equations (33)-(36), the condition becomes:

$$\mathbb{E} \left[\left(\tilde{f}_{21}(s) + f(s) - p_2(s) \right) \left(u'(c_2^2(s)) - v'(c_2^1(s)) \right) \right] \geq 0 \quad (42)$$

for all $\tilde{f}_{21} \in \mathcal{F}_{21}(\mathbf{p}_2)$.

Step 3: Equilibrium contract f and spot market price \mathbf{p}_2

We first prove that if (40) and (42) hold, then we must have $u'(c_2^2(s)) \geq v'(c_2^1(s))$ for all s of any subset of $[\underline{s}, \bar{s}]$ of positive measure.³² This means that investor 1 never over-borrows in period 1. Suppose this were not the case on a subset \mathcal{S}_0 of $[\underline{s}, \bar{s}]$. To establish a contradiction we need to consider two cases.

Suppose first there exists a subset $\mathcal{S}_1 \subseteq \mathcal{S}_0$ of positive measure such that $\epsilon_1 := \min_{s \in \mathcal{S}_1} f(s) > 0$. Consider a repo contract with payoff $\tilde{f}_{12}(s) = f(s) - \epsilon_1$ for $s \in \mathcal{S}_1$ and $\tilde{f}_{12}(s) = f(s)$ otherwise. Condition (40) for such contract is clearly incompatible with $u'(c_2^2(\cdot)) < v'(c_2^1(\cdot))$ on \mathcal{S}_0 .

If no such subset exists, this implies that $f = 0$ almost surely on \mathcal{S}_0 . Using investor 1 budget constraint (12) in period 2 and states $s \in \mathcal{S}_0$ with $f = 0$, we obtain:

$$c_2^1(s) = \omega + p_2(s)(a_1^1 - a_2^1(s)) = \omega + p_2(s)(a - a_2^1(s)) = \omega + p_2(s)a_2^2(s)$$

The second equality follows from the claimed property that there is no spot trade in period 1 and hence $a_1^1 = a_0^1 = a$. To derive the third equality, we used the spot market clearing condition in period 2. From the investors' short sale constraint (17) in period 2 it follows that $a_2^2(s) \geq 0$. Hence, $c_2^1(s) \geq \omega$ for all $s \in \mathcal{S}_0$, which together with the assumption that $u'(\omega) > v'(\omega)$ implies that $u'(c_2^2(s)) > v'(c_2^1(s))$. This again contradicts the claim $u'(c_2^1(s)) < v'(c_2^2(s))$ for all $s \in \mathcal{S}_0$.

We next show that agents do not trade spot in period 2, that is $a_2^2(s) = 0$. Using equations (37)-(38), we have

$$\gamma_2^2(s) = \gamma_2^1(s) + p_2(s) [u'(c_2^2(s)) - v'(c_2^1(s))] + (1 - \beta)s$$

Since the Lagrange multiplier $\gamma_2^1(s)$ is non negative, $u'(c_2^2(s)) \geq v'(c_2^1(s))$ and $\beta < 1$, it follows that $\gamma_2^2(s) > 0$ in any state s . Since $\gamma_2^2(s)$ is the Lagrange multiplier on the no

³²All similar assertions in this proof require the qualification "on any subset of $[\underline{s}, \bar{s}]$ of positive measure", though we sometimes omit the statement in what follows.

short sale constraint (17) of agent 2 in period 2 and state s , we have that $a_2^2(s) = 0$. By market clearing, $a_2^1(s) = a - a_2^2(s) = a$ and hence $\gamma_2^1(s) = 0$. Using the budget constraint (12) of investor 1 in period 2, state s and the property that investors do not trade spot in period 2, we get:

$$c_2^1(s) = \omega - b^{12}f(s) + (a_1^1 - a_2^1(s))p_2(s) = \omega - b^{12}f(s) \quad (43)$$

Plugging this expression in equation (37), we obtain:

$$\forall s, \quad p_2(s)v'(\omega - b^{12}f(s)) = s \quad (44)$$

To derive the equilibrium contract f and the spot market prices $p_2(s)$, p_1 , we distinguish two cases which correspond to case 3 and cases 1 and 2 of Proposition 1 respectively.

i) $\gamma_1^1 = 0$: investor 1 collateral constraint does not bind.

In this case, we show that investors reach the first best allocation and $s^* \leq \underline{s}$. Since $\gamma_1^1 = 0$, condition (39) may hold for any positive function \tilde{f}_{12} only if $u'(c_2^2(s)) = v'(c_2^1(s))$ for all s . This implies that $(c_2^1(s), c_2^2(s)) = (c_{2,*}^1, c_{2,*}^2)$ for all s , by definition of $(c_{2,*}^1, c_{2,*}^2)$. Using equation (44), we obtain $p_2(s) = s/v'(c_{2,*}^1)$ for all $s \in [\underline{s}, \bar{s}]$, which is strictly increasing in s .

From equation (43) it then follows that the repurchase price f of the equilibrium contract must be constant. For f to lie in $\mathcal{F}_{12}(\mathbf{p}_2)$ the constant value of f must be less or equal than $p_2(\underline{s})/(1 - \theta_1)$. Since $b^{12} \leq a$ we have, for all s ,

$$c_2^2(s) \leq \omega + \frac{as}{(1 - \theta_1)v'(c_{2,*}^1)}$$

Hence, $c_2^2(s) = c_{2,*}^2$ can hold if and only if the right hand side of this inequality is larger than $c_{2,*}^2$. By definition of s^* in (19), this is equivalent to $s^* \leq \underline{s}$. Since $c_2^2(s) = \omega + b^{12}f$, any repo contract $f^* \in [\frac{p_2(s^*)}{1-\theta_1}, \frac{p_2(\underline{s})}{1-\theta_1}]$ traded in quantity $b^{12} = \frac{c_{2,*}^2 - \omega}{f^*}$ implements the first-best allocation. Observe that, since $u'(c_2^2(s)) = v'(c_2^1(s))$ for all s , conditions (40) and (42) hold and the first order condition with respect to spot trades in period 1 hold with

$$p_1 = \mathbb{E}[p_2(s)v'(c_{2,*}^1)] = \mathbb{E}[p_2(s)u'(c_{2,*}^2)] = \mathbb{E}[s]$$

This proves case 3 of Proposition 1.

ii) $\gamma_1^1 > 0$: investor 1 collateral constraint binds.

This means that $b^{12} = a$. Suppose there exists a subset \mathcal{S}_0 of $[\underline{s}, \bar{s}]$ where $u'(c_2^2(s)) > v'(c_2^1(s))$ for $s \in \mathcal{S}_0$. Then for (40) to hold, for $s \in \mathcal{S}_0$, $f(s)$ must take the maximum possible value in $\mathcal{F}_{12}(\mathbf{p}_2)$, that is $f(s) = p_2(s)/(1 - \theta_1)$, and equation (44) becomes

$$p_2(s)v' \left(\omega - a \frac{p_2(s)}{1 - \theta_1} \right) = s \quad (45)$$

so that $p_2(s)$ is strictly increasing in s . In addition, since $u'(c_2^2(s)) > v'(c_2^1(s))$ by definition of $c_{2,*}^1$ we have $c_2^1(s) = \omega - a \frac{p_2(s)}{1 - \theta_1} > c_{2,*}^1$. On the other hand, if for some states $s \in [\underline{s}, \bar{s}]$ the inequality $u'(c_2^2(s)) \geq v'(c_2^1(s))$ holds as an equality, we have $c_{2,*}^1 = c_2^1(s) = \omega - af(s)$, f must be constant and $p_2(s)$ is given by $p_2(s)v'(c_{2,*}^1) = s$.

We next show that there exists a threshold \hat{s} such that $c_2^1(s) > c_{2,*}^1$ for $s < \hat{s}$ and $c_2^1(s) = c_{2,*}^1$ for $s \geq \hat{s}$. Suppose this were not true, that is we can find two states $(s_1, s_2) \in [\underline{s}, \bar{s}]^2$ such that $s_1 < s_2$ and $c_2^1(s_1) = c_{2,*}^1$ while $c_2^1(s_2) > c_{2,*}^1$. Then, from the argument in the previous paragraph, we get $f(s_2) = p_2(s_2)/(1 - \theta_1)$, with $p_2(s_2)$ as specified in (45). Since by assumption $c_2^1(s_2) > c_{2,*}^1$, from (45) we obtain

$$p_2(s_2) > \frac{s_2}{v'(c_{2,*}^1)} > \frac{s_1}{v'(c_{2,*}^1)} = p_2(s_1)$$

where the last equality follows from the fact that $c_2^1(s_1) = c_{2,*}^1$. But then

$$c_2^1(s_2) = \omega - af(s_2) = \omega - a \frac{p_2(s_2)}{1 - \theta_1} < \omega - a \frac{p_2(s_1)}{1 - \theta_1} \leq c_2^1(s_1) = c_{2,*}^1$$

a contradiction, which establishes that the claimed threshold \hat{s} exists.

Since $p_2(s) = s/v'(c_{2,*}^1)$ is increasing in s for $s \geq \hat{s}$, the threshold \hat{s} is the minimum state where the first best allocation can be financed given that the spot market price verifies $p_2(s)v'(c_{2,*}^1) = s$. Hence, by the definition of s^* , the threshold \hat{s} coincides with s^* . We thus have:

$$f(s) = \begin{cases} \frac{p_2(s)}{1 - \theta_1} & \text{if } s \leq s^* \\ \frac{p_2(s^*)}{1 - \theta_1} & \text{otherwise} \end{cases}, \quad \text{with} \quad \begin{cases} p_2(s)v' \left(\omega - a \frac{p_2(s)}{1 - \theta_1} \right) - s = 0 & \text{if } s \leq s^* \\ p_2(s) = \frac{s}{v'(c_{2,*}^1)} & \text{otherwise} \end{cases}$$

Finally, it is immediate to verify that investors do not engage in other repo trades: for any $\tilde{f}_{12} \in \mathcal{F}_{12}(\mathbf{p}_2)$, we have in fact $f(s) > \tilde{f}_{12}(s)$ for any s such that $u'(c_2^2(s)) > v'(c_2^1(s))$, while in the other states $u'(c_2^2(s)) = v'(c_2^1(s))$. This proves that condition (40) holds. Moreover, we have $f(s) \geq p_2(s)$ when $u'(c_2^2(s)) > v'(c_2^1(s))$, which proves that (42) also holds. Finally, the first order conditions with respect to spot trades in period 1 hold with

$$\begin{aligned} p_1 &= \mathbb{E}[p_2(s)v'(c_2^1(s))] + \gamma_1^1 = \mathbb{E}[s] + \mathbb{E}[f(s)(u'(c_2^2(s)) - v'(c_2^1(s)))] \\ \gamma_1^2 &= \mathbb{E}[(f(s) - p_2(s))(u'(c_2^2(s)) - v'(c_2^1(s)))] \end{aligned}$$

This proves case 1 and 2 of Proposition 1.

Step 4: No default-prone contracts

We show that the equilibrium contract f we derived is also preferred to any other contract inducing default. Let us denote such a repo contract by f_{12}^d . Recall that in the claimed equilibrium the first best allocation is attained for $s \geq s^*$. Hence, there is no possible gain in trading a contract with a different payoff on this region. We can thus set $\tilde{f}_{12}^d(s) = f(s)$ for $s \geq s^*$ without loss of generality. Let $\mathcal{S}_d \subseteq [\underline{s}, s^*]$ denote the set of states in the region $s < s^*$ where \tilde{f}_{12}^d violates (3). Then, $\mathcal{S}_{nd} = [\underline{s}, s^*] \setminus \mathcal{S}_d$ is the set of states where investor 1 does not default. Building on our argument earlier, investors do not trade contract f_{12}^d in equilibrium if and only if

$$\begin{aligned} & \int_{\mathcal{S}_{nd}} (f(s) - \tilde{f}_{12}^d(s)) (u'(c_2^2(s)) - v'(c_2^1(s))) dG(s) \\ & + \int_{\mathcal{S}_d} (f(s) - \alpha \tilde{f}_{12}^d(s) - (1 - \alpha(1 - \kappa))p_2(s)) (u'(c_2^2(s)) - v'(c_2^1(s))) dG(s) \\ & + \pi \int_{\mathcal{S}_d} (\tilde{f}_{12}^d(s) - (1 - \kappa)p_2(s)) v'(c_2^1(s)) dG(s) \geq 0 \end{aligned}$$

Since the equilibrium contract is such that $f(s)$ equals the borrowing capacity without default for $s \leq s^*$, we must have $\tilde{f}_{12}^d(s) \leq f(s)$ for $s \in \mathcal{S}_{nd} \subseteq [\underline{s}, s^*]$. Since $u'(c_2^2(s)) - v'(c_2^1(s)) > 0$ for $s \leq s^*$, the term on the first line in the above expression must be positive. It suffices then to show that the expression in the second line is also positive. Rearranging terms, this property is equivalent to:

$$\begin{aligned} \int_{\mathcal{S}_d} (f(s) - p_2(s)) (u'(c_2^2(s)) - v'(c_2^1(s))) dG(s) & \geq \int_{\mathcal{S}_d} \alpha (\tilde{f}_{12}^d(s) - (1 - \kappa)p_2(s)) (u'(c_2^2(s)) - v'(c_2^1(s))) dG(s) \\ & - \pi \int_{\mathcal{S}_d} (\tilde{f}_{12}^d(s) - (1 - \kappa)p_2(s)) v'(c_2^1(s)) dG(s) \quad (46) \end{aligned}$$

Using the fact that $u'(c_2^2(s)) \leq u'(\omega)$ and $v'(c_2^1(s)) \geq v'(\omega)$ for all s and $\tilde{f}_{12}^d(s) \geq p_2(s)$ for

$s \in \mathcal{S}_d$, the term on the right hand side of the above inequality is bounded above by:

$$[\alpha(u'(\omega) - v'(\omega)) - \pi v'(\omega)] \int_{\mathcal{S}_d} \left(\tilde{f}_{12}^d(s) - (1 - \kappa)p_2(s) \right) dG(s)$$

which is negative, under assumption (8). Since $f(s) = p_2(s)/(1 - \theta_1) \geq p_2(s)$ for $s \in [\underline{s}, s^*]$, and thus for all $s \in \mathcal{S}_d$, the term on the left-hand side of inequality (46) is positive. This proves that investors do not trade any repo contract inducing default.

B.2 Proof of Proposition 2

Using conditions (33)-(38) derived in the proof of Proposition 1, we obtain:

$$\begin{aligned} \mathcal{L} &= p_1 - \mathbb{E}[s] = \mathbb{E}[p_2(s)v'(c_2^1(s))] + \gamma_1^1 - \mathbb{E}[p_2(s)v'(c_2^1(s))] \\ &= \mathbb{E} [f(s) (u'(c_2^2(s)) - v'(c_2^1(s)))] \\ &= \int_{\underline{s}}^{s^*} \frac{p_2(s)}{1 - \theta_1} (u'(c_2^2(s)) - v'(c_2^1(s))) dG(s) \\ &= \int_{\underline{s}}^{s^*} \frac{1}{1 - \theta_1} (p_2(s)u'(c_2^2(s)) - s) dG(s) \end{aligned}$$

The second line justifies expression (23) in the main text. In the third line, $f(s)$ is replaced by its equilibrium value, while the expression in the last line follows from (37). Differentiating \mathcal{L} with respect to a and using (45), we obtain:

$$\frac{\partial \mathcal{L}}{\partial a} = \frac{1}{1 - \theta_1} \int_{\underline{s}}^{s^*} \left[\frac{\partial p_2(s)}{\partial a} u'(c_2^2(s)) + \frac{\partial [ap_2(s)]}{\partial a} \frac{p_2(s)}{1 - \theta_1} u''(c_2^2(s)) \right] dG(s)$$

From (45), we have that:

$$\begin{aligned} \frac{\partial p_2(s)}{\partial a} &= \frac{v'' \left(\omega - a \frac{p_2(s)}{1 - \theta_1} \right)}{v' \left(\omega - a \frac{p_2(s)}{1 - \theta_1} \right) - \frac{a}{1 - \theta_1} v'' \left(\omega - a \frac{p_2(s)}{1 - \theta_1} \right)} \leq 0 \\ \frac{\partial [ap_2(s)]}{\partial a} &= \frac{s}{v' \left(\omega - a \frac{p_2(s)}{1 - \theta_1} \right) - \frac{1}{1 - \theta_1} v'' \left(\omega - a \frac{p_2(s)}{1 - \theta_1} \right)} > 0 \end{aligned}$$

Since u is strictly concave, this proves that $\frac{\partial \mathcal{L}}{\partial a} < 0$.

Turning then to the haircut, using the equilibrium expression for the repo contract f and the consumption of investor 1 in period 2, we obtain:

$$\begin{aligned}\mathcal{H} &= - \int_{\underline{s}}^{s^*} \frac{\theta_1}{1-\theta_1} p_2(s) v'(c_2^1(s)) dG(s) + \int_{s^*}^{\bar{s}} \left(p_2(s) - \frac{p_2(s^*)}{1-\theta_1} \right) v'(c_{2,*}^1) dG(s) \\ &= - \int_{\underline{s}}^{s^*} \frac{\theta_1}{1-\theta_1} s dG(s) + \int_{s^*}^{\bar{s}} \left(s - \frac{s^*}{1-\theta_1} \right) dG(s)\end{aligned}\quad (47)$$

where, to derive the expression in the second line, we used again (45). Observe that \mathcal{H} only depends on a through s^* . Hence:

$$\frac{\partial \mathcal{H}}{\partial a} = - \frac{1}{1-\theta_1} \frac{\partial s^*}{\partial a} [1 - G(s^*)]$$

This expression is positive because, from equation (19), s^* is decreasing in a . The effect of counterparty quality θ_1 is clearly negative since:

$$\frac{\partial \mathcal{H}}{\partial \theta_1} = - \frac{1}{(1-\theta_1)^2} \left[\int_{\underline{s}}^{s^*} s dG(s) + s^* [1 - G(s^*)] \right] < 0$$

B.3 Proof of Proposition 3

In the proof, we refer to the first asset as asset A and to the second asset as asset B , with dividend, respectively $\rho^A(s) = s$ and $\rho^B(s) = s + \sigma(s - E[s])$. The repo equilibrium with two assets is similar to the one asset case. Investor 1 sells his holdings of asset $i = A, B$ in a repo f^i . Let s^{**} be the minimal state where the first best allocation can be reached, defined by:

$$\omega + \frac{a\rho^A(s^{**}) + b\rho^B(s^{**})}{(1-\theta_1)v'(c_{2,*}^1)} = c_{2,*}^2.$$

The repurchase price for the equilibrium repo on asset $i \in \{A, B\}$ is:

$$f^i(s) = \begin{cases} \frac{p_2^i(s)}{1-\theta_1} & \text{if } s \leq s^{**} \\ \frac{\rho^i(s^{**})}{(1-\theta_1)v'(c_{2,*}^1)} & \text{if } s > s^{**} \end{cases},$$

where p_2^i is the spot market price of asset $i = A, B$ in period 2, given by:

$$\begin{cases} p_2^i(s)v' \left(\omega - \frac{ap_2^B(s)+bp_2^B(s)}{(1-\theta_1)} \right) - \rho^i(s) = 0 & s \leq s^{**} \\ p_2^i(s)v'(c_{2,*}^1) = \rho^i(s) & s > s^{**} \end{cases}$$

Using the derivations in the proof of Proposition 2, the liquidity premium for asset $i = A, B$ is then

$$\mathcal{L}^i = \int_{\underline{s}}^{s^{**}} \frac{\rho^i(s)}{1-\theta_1} \left[\frac{u'(c_2^2(s))}{v'(c_2^1(s))} - 1 \right] dG(s)$$

Let us define $l(s) := u'(c_2^2(s))/v'(c_2^1(s)) - 1$. We obtain:

$$\mathcal{L}^A - \mathcal{L}^B = \int_{\underline{s}}^{s^{**}} \frac{s - \rho^B(s)}{1-\theta_1} l(s) dG(s) = -\frac{\sigma}{1-\theta_1} \int_{\underline{s}}^{s^{**}} (s - \mathbb{E}[s]) l(s) dG(s)$$

We need to show that the integral in the above expression has a negative sign. Note that $l(s)$ is strictly decreasing in s on $[\underline{s}, s^*]$. This follows from the fact that $c_2^2(s) = \omega + a\frac{p_2(s)}{1-\theta_1}$ and $p_2(s)$ is increasing in s , $c_2^2(s)$ is increasing in s while $c_2^1(s)$ is decreasing in s , while u' and v' are decreasing since u and v are concave functions. This implies that, for all s , $[l(s) - l(\mathbb{E}[s])][s - \mathbb{E}[s]] \leq 0$. We thus obtain

$$\int_{\underline{s}}^{s^{**}} (s - \mathbb{E}[s]) l(s) dG(s) \leq l(\mathbb{E}[s]) \int_{\underline{s}}^{s^{**}} (s - \mathbb{E}[s]) dG(s)$$

Since $\int_{\underline{s}}^{s^{**}} (s - \mathbb{E}[s]) dG(s)$ is negative for any value of $s^{**} \in [\underline{s}, \bar{s}]$, the expression on the right hand side of the inequality above is negative and so $\mathcal{L}^A - \mathcal{L}^B > 0$, which proves our claim.

The haircut for the equilibrium repo on asset B can be obtained proceeding along

similar lines to the argument in the proof of Proposition 2:

$$\begin{aligned}
\mathcal{H}_B &= \mathbb{E}[(p_2^B(s) - f^B(s))v'(c_2^1(s))] \\
&= \mathbb{E}[p_2^B(s)v'(c_2^1(s))] - \int_{\underline{s}}^{s^*} \frac{p_2^B(s)}{1-\theta_1} v'(c_2^1(s)) dF(s) - \int_{\underline{s}}^{s^*} \frac{p_2^B(s^*)}{1-\theta_1} v'(c_{2,*}^1) dF(s) \\
&= \mathbb{E}[s] - \int_{\underline{s}}^{s^{**}} \frac{(1+\sigma)s - \sigma\mathbb{E}[s]}{1-\theta_1} dF(s) - \int_{s^{**}}^{\bar{s}} \frac{(1+\sigma)s^{**} - \sigma\mathbb{E}[s]}{1-\theta_1} dF(s) \\
&= \mathbb{E}[s] + \frac{\sigma\mathbb{E}[s]}{1-\theta_1} - \frac{(1+\sigma)}{1-\theta_1} \left[\int_{\underline{s}}^{s^{**}} s dF(s) + \int_{s^{**}}^{\bar{s}} s^{**} dF(s) \right]
\end{aligned}$$

The haircut for the repo on asset A is given by expression (47), which we can rewrite as follows:

$$\mathcal{H}_A = \mathbb{E}[s] - \frac{1}{1-\theta_1} \left[\int_{\underline{s}}^{s^{**}} s dF(s) + \int_{s^{**}}^{\bar{s}} s^{**} dF(s) \right]$$

Hence, we obtain:

$$\mathcal{H}^B - \mathcal{H}^A = \frac{\sigma}{1-\theta_1} \left(\mathbb{E}[s] - \left[\int_{\underline{s}}^{s^{**}} s dF(s) + \int_{s^{**}}^{\bar{s}} s^{**} dF(s) \right] \right) \leq 0$$

that is the safer asset A always commands a lower haircut than the risky asset. The inequality is strict if $s^{**} > \underline{s}$, that is investor 1 is borrowing constrained.

B.4 Proof of Proposition 4

As in Proposition 1, the claim states that in equilibrium, investors trade only one repo contract $f(\nu_2) \in \mathcal{F}_{12}(\mathbf{p}_2)$ sold by investor 1 to investor 2. We need then to verify that investor 2 does not sell a repo contract to investor 1, nor buys other repo contracts. We can use the results established in Proposition 1 to characterize the equilibrium repo contract $f(\nu_2)$. The equilibrium repurchase price must be equal to $p_2(s)/(1-\theta_1)$ when the first best allocation cannot be attained. Otherwise, $f(s, \nu_2)$ must be such that $c_2^2(s) = c_{2,*}^2$. We defined this threshold $s^*(\nu_2)$ in equation (27). The maximum pledgeable income $aM_{12}p_2(s)/(1-\theta_1)$, where M_{12} is defined in (26), obtains when investor 1 sells in a repo all the asset acquired in the spot market and investor 2 re-sells in the spot market all the

re-usable collateral. Hence, both investors collateral constraint bind, that is:

$$a_1^1 = b^{12}, \quad a_1^2 = -\nu_2 l^{21} = -\nu_2 b^{12}$$

where $l^{21} = b^{12}$ follows from repo market clearing. Spot market clearing imposes $a_1^2 + a_1^1 = a$ so that $b^{12} = a/(1 - \nu_2)$. Investor 2 consumption in period 2 is:

$$c_2^2(s) = \omega + b^{12} f(s, \nu_2) + (a_1^2 - a_2^2(s)) p_2(s) = \omega + \frac{a}{1 - \nu_2} (f(s, \nu_2) - \nu_2 p_2(s)) \quad (48)$$

Then, for $c_2^2(s)$ to be equal to $c_{2,*}^2$ for $s \geq s^*(\nu_2)$, using (27), it must be that $f(\nu_2)$ is equal to the expression defined in (28).

We are left to prove that $f(\nu_2) \in \mathcal{F}_{12}(\mathbf{p}_2)$, that is no investor wants to default on the repo contract. Observe first that, by construction, $f(s, \nu_2) \leq p_2(s)/(1 - \theta_1)$ for all s , with equality for $s \leq s^*(\nu_2)$. Hence, investor 1 does not default as a repo seller. For $s \geq s^*(\nu_2)$, observe that

$$f(s, \nu_2) = \nu_2 p_2(s) + \left[\frac{1}{1 - \theta_1} - \nu_2 \right] p_2(s^*(\nu_2)) \geq \nu_2 p_2(s)$$

where the right hand side is the lowest possible payoff in $\mathcal{F}_{12}(\mathbf{p}_2)$. Hence, investor 2 does not default as the repo buyer.

Like in Proposition 1, we can determine the spot market price in period 2 using the relationship $p_2(s) v'(c_2^1(s)) = s$. Hence, we obtain

$$\begin{cases} p_2(s) v' \left(\omega - \frac{a}{1 - \nu_2} \left[\frac{1}{1 - \theta_1} - \nu_2 \right] p_2(s) \right) = s & \text{if } s < s^*(\nu_2) \\ p_2(s) v'(c_{2,*}^1) = s & \text{if } s \geq s^*(\nu_2) \end{cases}$$

Finally, we need to check that investor 2 should not want to sell the asset in a repo. Adapting equation (41) in the proof of Proposition 1 to the case $\nu_1 > 0$, we must have that for all $\tilde{f}_{21} \in \mathcal{F}_{21}(\mathbf{p}_2)$:

$$\mathbb{E}[\tilde{f}_{21}(s) u'(c_2^2(s))] + \gamma_1^2 \geq \mathbb{E}[\tilde{f}_{21}(s) v'(c_2^1(s))] + \nu_1 \gamma_1^1$$

where (γ_1^1, γ_1^2) are the Lagrange multipliers on investor 1 and 2's collateral constraint

respectively. Hence, substituting for γ_1^1 and γ_1^2 , the inequality above becomes:

$$\begin{aligned} \mathbb{E} \left[\tilde{f}_{21}(s) (u'(c_2^2(s)) - v'(c_2^1(s))) \right] &\geq \frac{\nu_1}{1 - \nu_2} \mathbb{E} [(f(s, \nu_2) - \nu_2 p_2(s)) (u'(c_2^2(s)) - v'(c_2^1(s)))] \\ &\quad - \frac{1}{1 - \nu_2} \mathbb{E} [(f(s, \nu_2) - p_2(s)) (u'(c_2^2(s)) - v'(c_2^1(s)))] \end{aligned}$$

Since $u'(c_2^2(s)) - v'(c_2^1(s)) \geq 0$, this inequality holds for any $\tilde{f}_{21} \in \mathcal{F}_{21}(\mathbf{p}_2)$ if it holds for $f_{21} = \nu_1 p_2(s)$, the contract with the minimal repayment in this set. Using the expression for the equilibrium contract $f(s, \nu_2)$ sold by investor 1, we obtain:

$$\begin{aligned} \nu_1 &\geq \frac{(1 - \nu_1 \nu_2)(1 - \theta_1) - (1 - \nu_1)}{(1 - \theta_1)(1 - \nu_2)} \\ \Leftrightarrow \quad \nu_1(1 - \nu_2)(1 - \theta_1) &\geq \nu_1(1 - \nu_2) - \theta_1(1 - \nu_1 \nu_2) \\ \Leftrightarrow \quad \theta_1(1 - \nu_1 \nu_2) &\geq \theta_1 \nu_1(1 - \nu_2) \end{aligned}$$

The last inequality holds since $\nu_1 < 1$.

We characterized the equilibrium in the case where $s^*(\nu_2) > \underline{s}$, that is when the first-best allocation cannot be attained in every state. We are left to prove that it can be attained if ν_2 is high enough. Observe from (27) that $s^*(\nu_2)$ is decreasing in ν_2 and that $\lim_{\nu_2 \rightarrow 1} s^*(\nu_2) < 0$. Hence there exists $\nu^* \in (0, 1)$ such that $s^*(\nu^*) = \underline{s}$. To find the expression for ν^* , from equations (19) and (27), we have

$$\frac{s^*(\nu_2)}{1 - \nu_2} [1 - \nu_2(1 - \theta_1)] = s^*(0)$$

Using $s^*(0) = s^*$ as well as $s^*(\nu^*) = \underline{s}$ we get $\nu^* = \frac{s^* - \underline{s}}{s^* - (1 - \theta_1)\underline{s}}$.

B.5 Proof of Proposition 5.

The proposition states that in equilibrium, investor B acquires at least part of the asset in the spot market and sells it in a repo with 2. Using the results derived in Proposition 4, the repo contracted traded by investors B and 2 is given by:

$$f_{B2}(s) = \begin{cases} \frac{p_2(s)}{1 - \theta_B} & s \leq s_{B2}^*(\nu_2) \\ \frac{p_2(s_{B2}^*(\nu_2))}{1 - \theta_B} + \nu_2(p_2(s) - p_2(s_{B2}^*(\nu_2))) & s > s_{B2}^*(\nu_2) \end{cases}$$

with $s_B^*(\nu_2)$ determined by the following equation, analogous to (27):

$$c_{2,*}^2 = \omega + aM_{B2} \frac{p_2(s_B^*(\nu_2))}{1 - \theta_B} = \omega + aM_{B2} \frac{s_B^*(\nu_2)}{(1 - \theta_B)\delta}$$

where M_{B2} is the collateral multiplier between B and 2 specified as in (26), replacing θ_1 with θ_B . Observe in particular that $s_B^*(\nu_2) < s^*(\nu_2)$ where $s^*(\nu_2)$ is defined by (27) since $\theta_B > \theta_1$. The spot market price is determined by $p_2(s)v'(c_2^1(s)) = s$ for all s . By a similar argument as in the proof of Proposition 1 we can show that only investor 1, who has the greatest marginal utility for consumption in period 3, carries the asset into period 3. Since $v(x) = \delta x$ here, we obtain $p_2(s) = \frac{s}{\delta}$.

We need to prove that investor 1 sells at least part of his asset spot to investor B . Observe that, under the assumption $s^*(\nu_2) > \underline{s}$, it is still possible to attain the first-best allocation in every state with intermediation whenever $s_B^*(\nu_2) \leq \underline{s}$. We consider first the case where this is not possible or $s_B^*(\nu_2) > \underline{s}$. In this case, we actually show that investor 1 sells all his asset spot to investor B and does not sell any repo to investor 2. The first order conditions for the period 1 spot trades of the three types of investors and for the repo sales by investor B and repo purchases by investor 2 are:

$$-p_1 + \delta \mathbb{E}[p_2(s)] + \gamma_1^1 = 0 \quad (49)$$

$$-p_1 + \delta \mathbb{E}[p_2(s)] + \gamma_1^B = 0 \quad (50)$$

$$q_{B2} - \delta \mathbb{E}[f_{B2}(s)] - \gamma_1^B = 0 \quad (51)$$

$$-p_1 + \mathbb{E}[p_2(s)u'(c_2^2(s))] + \gamma_1^2 = 0 \quad (52)$$

$$-q_{B2} + \mathbb{E}[f_{B2}(s)u'(c_2^2(s))] + \nu_2 \gamma_1^2 = 0 \quad (53)$$

From equations (49) to (59), we obtain:

$$\gamma_1^1 = \gamma_1^B = \frac{1}{1 - \nu_2} \mathbb{E}[(f_{B2}(s) - p_2(s))(u'(c_2^2(s)) - \delta)] > 0$$

where the sign follows from the fact that $u'(c_2^2(s)) > \delta$ for $s \in [\underline{s}, s_B^*(\nu_2)]$. Using (39), we obtain that investor 1 does not sell the asset in a repo to investor B if, for all $\tilde{f}_{1B} \in \mathcal{F}_{1B}(\mathbf{p}_2)$,

$$\delta \mathbb{E}[\tilde{f}_{1B}(s)] + \gamma_1^1 \geq \delta \mathbb{E}[\tilde{f}_{1B}(s)] + \nu_B \gamma_B^1$$

This inequality is actually strict since $\gamma_1^1 = \gamma_B^1 > 0$. Also, investor 1 does not wish to sell a repo $\tilde{f}_{12} \in \mathcal{F}(\mathbf{p}_2)$ to investor 2 if

$$\delta \mathbb{E}[\tilde{f}_{12}(s)] + \gamma_1^1 \geq \mathbb{E} \left[\tilde{f}_{12}(s) u'(c_2^2(s)) \right] + \nu_2 \gamma_1^2$$

Using equations (51) and (53), we can replace the Lagrange multipliers to obtain:

$$\mathbb{E} [f_{B2}(s) (u'(c_2^2(s)) - \delta)] \geq \mathbb{E} \left[\tilde{f}_{12}(s) (u'(c_2^2(s)) - \delta) \right] \quad (54)$$

Observe that for all $\tilde{f}_{12} \in \mathcal{F}(\mathbf{p}_2)$ and $s \leq s_B^*(\nu_2)$,

$$\tilde{f}_{12}(s) \leq \frac{p_2(s)}{1 - \theta_1} < \frac{p_2(s)}{1 - \theta_B} = f_{B2}(s)$$

since $\theta_B > \theta_1$. When $s > s_B^*(\nu_2)$, $u'(c_2^2(s)) = \delta$ so that inequality (54) holds for any $\tilde{f}_{12} \in \mathcal{F}(\mathbf{p}_2)$.

In the alternative case where $s_B^*(\nu_2) \leq \underline{s}$, that is the first-best allocation can be attained in every state, the pattern of trades described above is still an equilibrium. However, as discussed in the proof of Case 3 of Proposition 1, other patterns of trade can also implement this allocation.

B.6 Proof of Proposition 6

We show in what follows that there exists an equilibrium where type 1 investors sell all their asset in a repo f_{1B} to investor B , who in turn re-uses the asset acquired as collateral to sell it partly spot (to investor 1) and partly in a repo f_{B2} to type 2 investors. The first order condition for investor 1 spot trade in period 1 is again given by (49) while that with respect to the repo trade of contract f_{B1} is given by (55) below. The first order conditions of investor B with respect to spot trades and repo trades of contract f_{B1} and f_{B2} are then given by equations (56)-(58) below, and those of investor 2 with respect to

spot trades and repo trades of contract f_{B2} are given, respectively, by (52) and (59):

$$q_{1B} - \delta \mathbb{E}[f_{1B}(s)] - \gamma_1^1 = 0 \quad (55)$$

$$-p_1 + \delta_B \mathbb{E}[p_2(s)] + \gamma_1^B = 0 \quad (56)$$

$$-q_{1B} + \delta_B \mathbb{E}[f_{1B}(s)] + \nu_B \gamma_1^B = 0 \quad (57)$$

$$q_{B2} - \delta_B \mathbb{E}[f_{B2}(s)] - \gamma_1^B = 0 \quad (58)$$

$$-q_{B2} + \mathbb{E}[f_{B2}(s)u'(c_2^2(s))] = 0 \quad (59)$$

The argument used in the proof of Proposition 5 applies to show that $p_2(s) = s/\delta$.

Step 1: Equilibrium Repo Contracts

It follows from the analysis in Proposition 1 and the fact that investors 1 and B are both risk neutral, that the repo contract f_{1B} sold by investor 1 to B must be given by:

$$f_{1B}(s) = \frac{p_2(s)}{1 - \theta_1}, \quad \forall s. \quad (60)$$

We now characterize the repo contract f_{B2} sold by investor B to investor 2. From equations (55) to (57), we obtain

$$\gamma_1^B = \frac{\delta_B - \delta}{1 - \nu_B} \mathbb{E}[f_{1B}(s) - p_2(s)], \quad \gamma_1^1 = \gamma_1^B + (\delta_B - \delta) \mathbb{E}[p_2(s)] \quad (61)$$

Hence, since $f_{1B}(s) > p_2(s)$ for all s , we get $\gamma_1^B > 0$ and thus $\gamma_1^1 > 0$. This implies that the collateral constraints of both investors 1 and B bind or:

$$a_1^1 = b^{1B}, \quad a_1^B + \nu_B b^{1B} = b^{B2}$$

The first equation states that investor 1 sells in a repo the amount of asset he is endowed plus what he buys in the spot market. The second equation states that investor B re-uses all the collateral acquired in the repo with investor 1, that is $\nu_B b^{1B}$, both to sell it in the spot market (since $a_1^B < 0$ in the claimed equilibrium) and to sell it in a repo to investor 2. By spot market clearing in period 1 we have $a = a_1^1 + a_1^B + a_1^2$ and, since in the claimed

equilibrium investor 2 does not trade spot, that is $a_1^2 = 0$, we obtain:

$$a = (1 - \nu_B)b^{1B} + b^{B2} \quad (62)$$

From equation (62) it follows that the possible values of b^{B2} compatible with equilibrium are $[0, \nu_B a]$. The highest possible value is obtained by setting $b^{1B} = a$ and corresponds to the situation where investor B does not re-sell spot any of the re-usable collateral bought in repo f_{1B} so that investor 1 may only sell repo his endowment $a_0^1 = a$. For any given value of b^{B2} , the pattern of trades between investors B and 2 is given as in Proposition 1. Hence, the equilibrium repo contract f_{B2} sold by B to 2 is:

$$f_{B2}(b^{B2}, s) = \begin{cases} \frac{p_2(s)}{1-\theta_B} & \text{if } s < s^*(b^{B2}) \\ \frac{p_2(s^*(b^{B2}))}{1-\theta_B} & \text{if } s \geq s^*(b^{B2}) \end{cases}$$

where $s^*(b^{B2})$ is defined by an expression analogous to (19):

$$c_{2,*}^2 = \omega + b^{B2} \frac{p_2(s^*(b^{B2}))}{1-\theta_B} = \omega + b^{B2} \frac{s^*(b^{B2})}{\delta(1-\theta_B)}$$

For $s \geq s^*(b^{B2})$ investor 2 consumption in period 2 equals the first best level $c_{2,*}^2$.

Step 2: Re-use of collateral

We now determine the quantity b^{B2} sold in the repo by investor B to investor 2. This will also pin down the amount b^{1B} sold in the repo by investor 1 to investor B via equation (62). From equations (60) and (61), we obtain

$$\gamma_1^B = \frac{(\delta_B - \delta)\theta_1}{(1 - \nu_B)(1 - \theta_1)} \mathbb{E}[p_2(s)] \quad (63)$$

while from (58) and (59) we get:

$$\gamma_1^B = \int_{\underline{s}}^{s_{B2}^*(b^{B2})} [u'(c_2^2(b^{B2}, s)) - \delta_B] \frac{p_2(s)}{1-\theta_B} dG(s) \quad (64)$$

where $c_2^2(b^{B2}, s) = \omega + b^{B2} f_{B2}(b^{B2}, s)$. Substituting (63) above for γ_1^B in (64) yields:

$$\int_{\underline{s}}^{s_{B2}^*(b^{B2})} [u'(c_2^2(b^{B2}, s)) - \delta_B] \frac{p_2(s)}{1 - \theta_B} dG(s) = \frac{(\delta_B - \delta)\theta_1}{(1 - \nu_B)(1 - \theta_1)} \mathbb{E}[p_2(s)] \quad (65)$$

This relationship allows to prove the property stated below Proposition 6 that investor B is indifferent between re-using the collateral to sell it in a repo to investor 2 (the left hand side) or to sell it spot to investor 1 (the right hand side). Since

$$\int_{\underline{s}}^{s_{B2}^*(b)} [u'(c_2^2(b, s)) - \delta_B] p_2(s) dG(s)$$

is strictly decreasing in b , there is at most one value of b^{B2} satisfying equation (65). To establish the claimed property of the equilibrium, we have to prove that the solution lies in the feasible range for b^{B2} , which we showed is $[0, \nu_B a]$. The condition that $b^{B2} \geq 0$ yields

$$\frac{u'(\omega) - \delta_B}{1 - \theta_B} \geq \frac{(\delta_B - \delta)\theta_1}{(1 - \nu_B)(1 - \theta_1)} \quad (66)$$

or equivalently

$$\delta_B \leq \bar{\delta}_B := \frac{\frac{\theta_1}{(1 - \nu_B)(1 - \theta_1)} \delta + \frac{u'(\omega)}{1 - \theta_B}}{\frac{\theta_1}{(1 - \nu_B)(1 - \theta_1)} + \frac{1}{1 - \theta_B}}$$

Observe in particular that $\bar{\delta}_B \leq u'(\omega)$. The condition $b^{B2} \leq \nu_B a$ is equivalent to:

$$\int_{\underline{s}}^{s_{B2}^*(\nu_B a)} \frac{u'\left(\omega + \nu_B a \frac{s}{\delta(1 - \theta_B)}\right) - \delta_B}{1 - \theta_B} s dF(s) \leq \frac{(\delta_B - \delta)\theta_1}{(1 - \nu_B)(1 - \theta_1)} \quad (67)$$

or

$$\delta \geq \underline{\delta}_B := \frac{\frac{\theta_1}{(1 - \nu_B)(1 - \theta_1)} \delta + \int_{\underline{s}}^{s_{B2}^*(\nu_B a)} \frac{u'\left(\omega + \nu_B a \frac{s}{\delta(1 - \theta_B)}\right)}{1 - \theta_B} s dF(s)}{\frac{\theta_1}{(1 - \nu_B)(1 - \theta_1)} + \int_{\underline{s}}^{s_{B2}^*(\nu_B a)} \frac{1}{1 - \theta_B} s dF(s)}$$

with $\underline{\delta}_B \geq \delta$. Since $\delta_B \leq \bar{\delta}_B$ and $\delta_B \geq \underline{\delta}_B$ are respectively equivalent to (66) and (67), it is easy to see from these expressions that $\underline{\delta}_B \leq \bar{\delta}_B$

Step 3: No other profitable trades

We are left to show that investors do not wish to engage in other trades. The argument in the proof of Proposition 4 still applies to show that investor 2 does not wish to sell a

repo to any other investor and that investor B does not sell a repo to 1. Hence, we are left to verify that investor 1 does not wish to bypass investor B . In other words, there should be no repo contract that investor 1 desires to sell to investor 2. Hence, for any $\tilde{f}_{12} \in \mathcal{F}_{12}(\mathbf{p}_2)$ the following inequality must hold:

$$\delta \mathbb{E}[\tilde{f}_{12}(s)] + \gamma_1^1 \geq \mathbb{E} \left[\tilde{f}_{12}(s) u'(c_2^2(s)) \right]$$

Using equations (49) to (59) to substitute for γ_1^1 , we obtain:

$$\mathbb{E} \left[\left(\tilde{f}_{12}(s) - p_2(s) \right) \left(u'(c_2^2(s)) - \delta \right) \right] \leq \mathbb{E} \left[\left(f_{B2}(s) - p_2(s) \right) \left(u'(c_2^2(s)) - \delta_B \right) \right]$$

This inequality holds for all $\tilde{f}_{12} \in \mathcal{F}_{12}(\mathbf{p}_2)$ if it holds for the highest value of the payoff in $\mathcal{F}_{12}(\mathbf{p}_2)$, $\frac{p_2(s)}{1-\theta_1}$. Substituting this value into the inequality above and rearranging terms we obtain:

$$\begin{aligned} 0 &\leq \mathbb{E} \left[\left(f_{B2}(s) - \frac{p_2(s)}{1-\theta_1} \right) \left(u'(c_2^2(s)) - \delta_B \right) \right] - \mathbb{E} \left[\left(f_{12}(s) - p_2(s) \right) \left(\delta_B - \delta \right) \right] \\ \Leftrightarrow 0 &\leq \left[\frac{1}{1-\theta_B} - \frac{1}{1-\theta_1} \right] \int_{\underline{s}}^{s_{B2}^*(b^{B2})} \left[u'(c_2^2(b^{B2}, s)) - \delta_B \right] p_2(s) dG(s) - \frac{\theta_1(\delta_B - \delta)}{1-\theta_1} \mathbb{E}[p_2(s)] \\ \Leftrightarrow 0 &\leq \left[1 - \frac{1-\theta_B}{1-\theta_1} \right] \gamma_1^B - \gamma_1^B (1-\nu_B) \\ \Leftrightarrow 0 &\leq \frac{\nu_B}{1-\theta_B} - \frac{1}{1-\theta_1} \end{aligned}$$

where the last inequality is condition (29).

B.7 Proof of Proposition 7

We argue by contradiction. Suppose in equilibrium investors did not trade any repo contract inducing default, that is they only trade spot and repo contracts with payoff $\bar{f} \leq \bar{f}_{min}$. If so, proceeding along similar lines to the proof of Proposition 1, we can show that in equilibrium investors only trade contract \bar{f}_{min} . In particular, this pattern of trades constitutes an equilibrium if and only if the Lagrange multiplier on the short sale constraint of investor 2, γ_1^2 , is strictly positive. From the first order conditions with respect to spot and repo trades in \bar{f}_{min} , and using the fact that $c_2^2(s) = c_2^2 = \omega + a\bar{f}_{min}$

for all s , we obtain:

$$\gamma_1^2 = \mathbb{E}[(\bar{f}_{min} - p_2(s))(u'(c_2^2) - \delta)] = \left(\frac{\underline{s}}{\delta(1 - \theta_1)} - \frac{\mathbb{E}[s]}{\delta} \right) (u'(c_2^2) - \delta)$$

where we used the fact that $p_2(s) = s/\delta$. Hence, γ_1^2 is strictly positive if condition (31) holds since it states that $\frac{\underline{s}}{1 - \theta_1} > \mathbb{E}[s]$.

In addition, for any $\bar{f} > f_{min}$ there should be a price $q_{12}(\bar{f})$ such that investors do not want to trade this contract. Let $\bar{f}^1(s)$ and $\bar{f}^2(s)$ denote the effective payoff, respectively for investor 1 and investor 2, in period 2 in each state s , per unit of contract traded. In the states $s < s_d(\bar{f})$, where investor 2 defaults on contract \bar{f} , we have

$$\bar{f}^2(s) = p_2(s) + \alpha(\bar{f} - (1 - \kappa)p_2(s)), \quad \bar{f}^1(s) = \bar{f}^2(s) + \pi(\bar{f} - (1 - \kappa)p_2(s))$$

while in the other states $s \geq s_d(\bar{f})$ we have $\bar{f}^1(s) = \bar{f}^2(s) = \bar{f}$. Building on our previous analysis, a price $q_{12}(\bar{f})$ compatible with no trade in \bar{f} exists if the minimum price at which investor 1 is willing to sell, at the margin, contract \bar{f} exceeds the maximum price at which investor 2 is willing to purchase it:

$$\delta \mathbb{E}[\bar{f}^1(s)] + \gamma_1^1 \geq \mathbb{E}[\bar{f}^2(s)u'(c_2^2)]$$

In the conjectured equilibrium, the Lagrange multiplier on the collateral constraint of investor 1 is $\gamma_1^1 = E[\bar{f}_{min}(u'(c_2^2) - \delta)]$. The above inequality can thus be rewritten as follows:

$$M(s_d(\bar{f})) := \mathbb{E}[(\bar{f}^2(s) - \bar{f}_{min})u'(c_2^2) - (\bar{f}^1(s) - \bar{f}_{min})\delta] \leq 0$$

Using the expressions for $\bar{f}^1(s)$ and $\bar{f}^2(s)$ specified above yields:

$$\begin{aligned} M(s_d(\bar{f})) &= (u'(c_2^2) - \delta) \int_{\underline{s}}^{s_d(\bar{f})} (\alpha\bar{f} + (1 - \alpha(1 - \kappa))p_2(s) - \bar{f}_{min})dG(s) \\ &\quad - \pi\delta \int_{\underline{s}}^{s_d(\bar{f})} (\bar{f} - (1 - \kappa)p_2(s))dG(s) + (u'(c_2^2) - \delta)(\bar{f} - \bar{f}_{min})(1 - G(s_d(\bar{f}))) \leq 0 \end{aligned}$$

This inequality should hold for all \bar{f} , that is for all possible values of $s_d(\bar{f})$. Since $M(\underline{s}) = 0$, to find a contradiction it suffices to show that the derivative of M at \underline{s} is strictly positive. Differentiating M with respect to the value of $s_d(\bar{f})$ and evaluating it at $s_d(\bar{f}) = \underline{s}$ we

get:

$$\begin{aligned}
M'(\underline{s}) &= -\frac{u'(c_2^2) - \delta}{\delta} \left(\frac{\theta_1(1 - \alpha)}{(1 - \theta_1)} - \kappa\alpha \right) \underline{s}g(\underline{s}) - \pi_1 \frac{(1 - (1 - \theta_1)(1 - \kappa))\underline{s}g(\underline{s})}{1 - \theta_1} + \frac{(u'(c_2^2) - \delta)}{\delta(1 - \theta_1)} \\
&= -\frac{u'(c_2^2) - \delta}{\delta} \frac{\pi_1(\kappa + \theta_1(1 - \kappa))}{1 - \theta_1} \underline{s}g(\underline{s}) - \pi_1 \frac{(\kappa + \theta_1(1 - \kappa))\underline{s}g(\underline{s})}{1 - \theta_1} + \frac{(u'(c_2^2) - \delta)}{\delta(1 - \theta_1)}
\end{aligned} \tag{68}$$

where we used the definition of θ_1 to obtain the second line. We are left to show that under the assumed condition (32), we have $M'(\underline{s}) > 0$. Using equation 63, we obtain that $M'(\underline{s}) > 0$ if and only if

$$\frac{(u'(c_2^2) - \delta)}{\delta} [1 - \pi_1(\kappa + \theta_1(1 - \kappa))\underline{s}g(\underline{s})] > \pi_1(\kappa + \theta_1(1 - \kappa))\underline{s}g(\underline{s})$$

Using the definition of θ_1 , the inequality above is equivalent to condition (32). This completes the proof

It is then immediate to verify that condition (32) is consistent with the maintained assumption (8), as claimed in the text. Take any pair (π, α) that satisfy (8): letting α and π go to 0 at the same rate, condition (8) remains valid, while (32) clearly holds, since the term on the right hand side of that inequality goes to 0 and the one on the left hand side is strictly positive.