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Trade and Investment in the Global Economy

Abstract

We develop a dynamic multi-country trade model with foreign direct investment (FDI) in the form of non-rival technology capital. The model nests structural gravity subsystems for FDI and trade, with accumulation/decumulation of phyisical and technology capital in transition to the steady state. The empirical importance of the FDI channel is demonstrated comparing actual aggregate cross-section data for 89 countries in 2011 to a hypothetical world without FDI. The gains from FDI amount to 9% of world's welfare and to 11% of world's trade, unevenly distributed among winners and losers. Net exports of FDI substitute for export trade in the results.

JEL-Codes: F100, F430, O400.

Keywords: foreign direct investment, trade, trade liberalization, capital accumulation.

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All errors are our own.

"Foreign direct investment (FDI) is an integral part of an open and effective international economic system and a major catalyst to development. [...] With most FDI flows originating from OECD countries, developed countries can contribute to advancing this agenda. They can facilitate developing countries' access to international markets and technology."

OECD (2002)

"Today, FDI is not only about capital, but also –and more important– about technology and know-how, [...] International patterns of production are leading to new forms of cross-border investment, in which foreign investors share their intangible assets such as know-how or brands in conjunction with local capital or tangible assets of domestic investors."

The World Bank (2015)

1 Introduction and Motivation

Foreign Direct Investment (FDI) is viewed as a key driver of prosperity in policy circles. For example, FDI had a prominent role in many recent integration agreements such as Transatlantic Trade and Investment Partnership (TTIP).[1] Moreover, both opening quotes demonstrate concern for FDI as transfers of intangible assets such as know-how, brands, patents, etc. in addition to traditional forms of FDI as physical capital. Policy makers' enthusiasm is based on the obvious partial equilibrium evidence of FDI impacts on sectoral output, employment and capital returns, but there is relatively little structural evidence for the *economy-wide* importance of FDI and its role in knowledge transfer. This paper generates evidence on the importance of FDI from counter-factual simulation of a dynamic multi-country general equilibrium model with costly international trade and investment.

The model traces the relationships between trade, domestic investment in physical capital accumulation, and FDI in the form of technology capital. On the trade side, our model is

[1]Policy makers and academics alike see high potential and promise that such agreements will not only liberalize trade but also facilitate FDI. For example, on the policy side, EU analysts hope that TTIP will "liberalise *trade and investment* between the EU and the US and will result in more jobs and growth." (Press release, Brussels, 28 January 2014). Academics share the hopes for positive impact of integration agreements on welfare through FDI. "If successfully negotiated, [TTIP and TPP] would deepen and strengthen ties with many of the most significant U.S. economic partners. A large majority of inward FDI in the United States already originates from TTIP and TPP countries, making these deals particularly important in the broader effort to recruit global business investment." (p. 3, Slaughter, 2013).

a member of the wide class of new quantitative general equilibrium trade models described in detail by Arkolakis et al. (2012). The novelty of our theory comes on the supply side. Production uses FDI in the form of non-rival technology capital along with labor and physical capital stocks. Thus countries can use their technology capital in potentially all countries while using at home the technology capital of potentially all countries.[2]

A byproduct of our model is an intuitive structural FDI gravity system that very much resembles the traditional gravity system from the trade literature. First, the value of FDI stock is proportional to the size of the country of origin, as measured by expenditure. The intuition for this result is that expenditure in the country of origin is proportional to the value of marginal product of technology capital. Second, bilateral FDI is proportional to the size of the host country, as measured by nominal output. The intuition for this result is that nominal output is proportional to the returns to FDI in the host country. Third, the stock value of FDI is inversely related to FDI barriers. Fourth, bilateral FDI stock values are linked to trade via the inward multilateral resistance (IMR) in an intuitive way. Specifically, higher IMRs in the country of origin lead to less FDI. The intuition for this result is that higher IMRs mean higher direct and opportunity cost of investing in technology capital. We view the structural interdependence between trade and FDI in our model as an important contribution to the existing FDI literature, where, despite significant interest, the relationships between trade and FDI are not clearly established. Finally, our theory suggests that the value of FDI stock is inversely related to the amount of technology capital in the country of its origin. This relationship is also intuitive because it is a reflection of the law of diminishing returns to investment into technology capital.

FDI liberalization increases FDI, with interesting implications for trade, income, and

[2] In the spirit of McGrattan and Prescott (2009, 2010, 2014) and McGrattan and Waddle (2017) we characterize technology capital as *'non-rival'*. However, our interpretation of technology capital is also consistent with the notion of *'jointness'* of capital from Markusen (2002), who defines *'jointness'* as "the ability to use the engineer or other headquarters asset in multiple production locations without reducing the services provided in any single location. A blueprint is the classical example of a joint input. Jointness inherently refers to the costs of running two plants rather than one." Markusen (2002, p. 130). Thus, depending on preference, throughout the analysis the reader may use the terms *'non-rival technology capital'* and *'joint technology capital'* interchangeably.

expenditure. An increase in bilateral FDI directly leads to higher income and to higher expenditure in the liberalizing countries. Since higher expenditure leads to more accumulation of technology capital, FDI liberalization between two countries will also trigger positive spillover effects on output and expenditure in third countries. Through its impact on output and expenditure, increases in FDI will also translate into increases in trade flows. Moreover, the changes in FDI will affect trade indirectly, by the effect of outputs and expenditures on the multilateral resistances. Finally, since multilateral resistances are general equilibrium indexes, i.e. they capture the effects of trade liberalization between any two countries on consumer and on producer prices in any other country in the model, the FDI changes will transmit throughout the world.

The results of the counterfactual comparative (steady state) statics show that net export of FDI substitutes for export trade. The modular structure of the model allows an intuitive explanation, despite the unavailability of analytic results. In partial equilibrium with incomes held constant, net FDI importers pay for their use of foreign technology on balance, hence elimination of FDI raises expenditure to the level of income. The share of income spent on home goods rises so exports must fall. Net FDI exporters lose net payments from foreign users, hence elimination of FDI lowers expenditure to the constant level of income. The share of income spent on home goods falls, so exports must rise. A general equilibrium force modifies partial equilibrium insight for countries with large (in absolute value) net FDI positions. Net exporters of FDI lose income from FDI elimination (less technology capital is accumulated in response to loss of technology capital exports) while net FDI importers gain income (more domestic technology capital is accumulated in response to absence of foreign technology capital). This force dampens the partial effects but does not eliminate it.

Trade and trade liberalization affect FDI via two channels in our model. First, changes in trade costs lead to changes in expenditure, which in turn shift the value of marginal product of FDI and thus directly affect investment in technology capital. Second, trade liberalization shifts prices, which affects both FDI and trade. Specifically, FDI is inversely related to the prices of consumer and investment goods in the country of origin, because these prices are part of the direct and the opportunity costs, respectively, of investment in technology capital. To the extent that trade liberalization leads to increased expenditure and to lower prices of consumption and investment goods, our model predicts that lower trade costs will stimulate the accumulation of technology capital in the liberalizing countries. Moreover, the increased stock of technology capital in the liberalizing countries will lead to positive spillover effects in the rest of the world, due to the non-rival nature of technology.

The empirical importance of the novel FDI channel is demonstrated in a counterfactual experiment that describes a hypothetical world without FDI. Three main findings stand out. First, we establish that FDI is indeed an important component of the modern world economic system. Our estimates reveal that, on average, the gains from FDI amount to 9% of world's welfare and to 11% of world's trade in 2011, which is the year of our counterfactual analysis. Second, we find that the impact of FDI has been very heterogeneous across the 89 countries in our data. While most countries in the world have benefitted from FDI, some have incurred losses due to FDI. We offer a series of explanations for this result. Third, the large magnitude of the FDI effects combines with its wide hetorogeneity to suggest implications of FDI for changes in cross-country income inequality. For example, our results suggest that FDI has led to significant increases in the real GDP and the welfare of some of the poorest economies in the European Union. Given the renewed interest in the determinants of inequality, cf. European Commission (2010), our results have potentially important implications for regional policy.

Our work is related to several prominent strands in the literature. Our model nests the standard structural gravity model of trade, so its static module is a member of the wide class of quantitative general equilibrium (GE) trade models described by Arkolakis et al. (2012) and Costinot and Rodríguez-Clare (2014). The dynamic model of domestic investment in the form of physical capital accumulation is in the spirit of Olivero and Yotov

(2012), Anderson et al. (2015) and Eaton et al. (2016).[3] The novelty in our analysis adds to the previous static and dynamic GE trade models by allowing for FDI on the transition to a steady (stationary) state. Thus, our work is related to the literature that studies the determinants and implications of foreign direct investment.[4] Following McGrattan and Prescott (2009, 2010, 2014) and McGrattan and Waddle (2017), and motivated by empirical evidence, FDI takes the form of non-rival technology capital.[5] Relative to McGrattan et al., our methodological contribution is the modeling of the joint interactions between FDI and bilateral trade in an asymmetric many country world. Finally, our work contributes to the empirical literature that studies the determinants of FDI, e.g. Eicher et al. (2012) and Blonigen and Piger (2014). Even though FDI estimations are beyond the scope of this study, we believe that, owing to its similarities with the very successful gravity system of trade, our structural FDI gravity system will be useful to researchers and policy makers.

The remainder of the paper is organized as follows. Section 2 develops the theoretical framework. This section also introduces the structural FDI gravity system as a byproduct of our model. Section 3 presents the empirical analysis. Section 3.1 describes the data and the calibration methods and parameters, while Section 3.2 presents and discusses our main findings of the counterfactual analysis. Section 4 concludes and offers directions for improvements and extensions.

[3]The empirical relevance of physical capital accumulation as a key determinant of various economic outcomes is demonstrated in Wacziarg (2001), Baldwin and Seghezza (2008), Wacziarg and Welch (2008), and Cuñat and Maffezzoli (2007).

[4]Related FDI papers include Head and Ries (2008) and Bergstrand and Egger (2007, 2010). In addition, we refer the reader to Antras and Yeaple (2014) for an excellent survey of the literature on the decisions of multinational firms.

[5]As noted earlier, our interpretation of technology capital is akin to the 'jointness' of knowledge capital (i.e. patents, blue-prints, management skills/practices, etc.) from Markusen (1997, 2002) and Markusen and Maskus (2002). Sampson (2016) models non-rival technology capital accumulation with dynamic selection of heterogeneous firms in a highly symmetric setting with uniform country size and uniform trade costs.

2 Theoretical Foundation

We build a dynamic model of trade, domestic capital accumulation and foreign direct investment. The world consists of N countries, and each of them produces a single tradeable good, differentiated by place of origin. Each country purchases goods from every source (as in Armington, 1969), which are used for final consumption and for domestic investment in physical and non-rival technology capital that may be 'leased' to all other countries. The quotes enclosing 'leased' connote that technology transfer includes both within firm Foreign Direct Investment (FDI) and arms length licensed technology transfer, equivalently leading to payments across borders for the use of technology.[6] Following McGrattan and Prescott (2009), we abstract from FDI in the form of physical means of production. The basic building blocks are set out below.

Production. Total nominal output in country j at time t ($Y_{j,t}$) is produced subject to the following constant returns to scale (CRS) Cobb-Douglas production function:

$$Y_{j,t} = p_{j,t} A_{j,t} \left(L_{j,t}^{1-\alpha} K_{j,t}^{\alpha} \right)^{1-\phi} \mathcal{M}_{j,t}^{\phi} \quad \alpha, \phi \in (0,1), \tag{1}$$

where $p_{j,t}$ denotes the factory-gate price of good (country) j at time t. Production in country j at time t relies on local technology ($A_{j,t}$), global technology stock applied locally ($\mathcal{M}_{j,t}$) and country-specific (internationally immobile) resources including labor endowment ($L_{j,t}$) and physical capital stock ($K_{j,t}$).

Technology capital is 'non-rival' or 'joint', i.e. country i can use its technology capital ($M_{i,t}$) at home and in all other countries. Possible examples include patents, blue-prints, management skills/practices, etc. Formally global technology capital $\mathcal{M}_{j,t}$ applied in j at

[6] Payments by affiliates for use of parent firm technology often differ from the true internal value, for tax and strategic reasons beyond the scope of this study, so the neutral term 'licensing' is used to more accurately describe the economically relevant value of the technology transfer.

time t in production function (1) is defined as

$$\mathcal{M}_{j,t} \equiv \left(\prod_{i=1}^{N} \left(\max\{1, \omega_{ij,t} M_{i,t}\} \right)^{\eta_i} \right), \tag{2}$$

comprising domestic technology capital $M_{j,t}$ and foreign direct investments:

$$FDI_{ij,t} \equiv \omega_{ij,t} M_{i,t}. \tag{3}$$

Here, $M_{i,t}$ is the technology capital stock in country i at time t, and $\omega_{ij,t}$ measures the openness for foreign technology of country i in country j at time t, thus encompassing all possible bilateral FDI frictions between i and j. If $\omega_{ij,t} = 0$, no foreign technology from country i can be used in country j at time t. If $\omega_{ij,t} > 0$, then every unit of foreign technology from country i at time t has $\omega_{ij,t}$-times the use in country j.[7] Finally, constant returns to scale is imposed by $\sum_{i=1}^{N} \eta_i = 1$. The *max*-function, which we use to introduce FDI as a component in national production implements the notion that there is some world knowledge of technology capital freely available to all countries and normalized to one. The stock of world knowledge is normalized in the second argument of the *max*-function. A technical advantage of the *max*-function is that it ensures that we can capture zero bilateral FDI flows between countries, which we observe regularly in the data.[8]

Physical capital in each country accumulates according to a standard linear transition function:

$$K_{j,t+1} = (1 - \delta_K) K_{j,t} + \Omega_{j,t}, \tag{4}$$

where δ_K are the physical capital adjustment costs and $\Omega_{j,t}$ denotes the aggregate flow of investment in physical capital in country j at time t, which we model as a CES aggregate of

[7] The FDI frictions are "dark" like iceberg trade costs. Sampson (2016) lays out a dynamic selection device for heterogeneous firms that potentially lights some of the darkness.

[8] Our approach is similar to McGrattan and Prescott (2009, 2010). In Appendix B we provide an alternative specification where technology capital across all countries is as in McGrattan and Prescott (2009) summed rather than combined via a Cobb-Douglas function.

investment goods ($I^K_{ij,t}$) from all possible countries in the world, including j itself:

$$\Omega_{j,t} = \left(\sum_{i=1}^{N} \gamma_i^{\frac{1-\sigma}{\sigma}} \left(I^K_{ij,t} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \tag{5}$$

Here, γ_i is a positive distribution parameter, and $\sigma > 1$ is the elasticity of substitution across goods varieties from different countries.

Technology capital in each country accumulates with a linear transition function similar to physical capital :

$$M_{j,t+1} = (1 - \delta_M)\, M_{j,t} + \chi_{j,t}, \tag{6}$$

where δ_M are the adjustment costs for technology capital and $\chi_{j,t}$ denotes the CES-aggregated flow of investments in technology capital ($I^M_{ij,t}$) in country j at time t from all possible countries in the world, including j itself:

$$\chi_{j,t} = \left(\sum_{i=1}^{N} \gamma_i^{\frac{1-\sigma}{\sigma}} \left(I^M_{ij,t} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \tag{7}$$

To facilitate analysis, we use the definition of nominal output from Equation (1) to obtain the value marginal product of technology capital at home:[9]

$$\frac{\partial Y_{j,t}}{\partial M_{j,t}} = \begin{cases} \frac{\phi \eta_j Y_{j,t}}{M_{j,t}} & \text{if} \quad \omega_{jj,t} M_{j,t} > 1, \\ 0 & \text{if} \quad \omega_{jj,t} M_{j,t} \leq 1, \end{cases} \tag{8}$$

and the value marginal product of $M_{j,t}$ abroad:

$$\frac{\partial Y_{i,t}}{\partial M_{j,t}} = \begin{cases} \frac{\phi \eta_j Y_{i,t}}{M_{j,t}} & \text{if} \quad \omega_{ji,t} M_{j,t} > 1, \\ 0 & \text{if} \quad \omega_{ji,t} M_{j,t} \leq 1. \end{cases} \tag{9}$$

Consumption. Consumer preferences are identical across countries and represented by a

[9]See for a derivation Appendix A.

logarithmic utility function with a subjective discount factor $\beta < 1$:

$$U_{j,t} = \sum_{t=0}^{\infty} \beta^t \ln(C_{j,t}), \tag{10}$$

where aggregate consumption ($C_{j,t}$) includes domestic and foreign goods ($c_{ij,t}$) from all possible countries in the world, including country j, subject to:

$$C_{j,t} = \left(\sum_{i=1}^{N} \gamma_i^{\frac{1-\sigma}{\sigma}} c_{ij,t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \tag{11}$$

The assumption that consumption and investment goods are both a combination of all world varieties subject to the same CES aggregation is very convenient analytically. Allowing for heterogeneity in preferences and prices between and within consumption and investment goods will open additional channels for the interaction between trade, FDI and domestic investment. However, such treatment requires the introduction of an additional sectoral dimension which is beyond the scope of this project.

Agent's Problem. Representative agents in each country work, invest and consume. At every point in time consumers in country j choose aggregate consumption ($C_{j,t}$) and aggregate investment into physical ($\Omega_{j,t}$) and technology ($\chi_{j,t}$) capital to maximize the present discounted value of lifetime utility subject to a sequence of constraints:

$$
\begin{aligned}
\max_{\{C_{j,t},\Omega_{j,t},\chi_{j,t}\}} & \quad \sum_{t=0}^{\infty} \beta^t \ln(C_{j,t}) && (12)\\
K_{j,t+1} &= (1-\delta_K)\, K_{j,t} + \Omega_{j,t} \quad \text{for all } t, && (13)\\
M_{j,t+1} &= (1-\delta_M)\, M_{j,t} + \chi_{j,t} \quad \text{for all } t, && (14)\\
Y_{j,t} &= p_{j,t} A_{j,t} \left(L_{j,t}^{1-\alpha} K_{j,t}^{\alpha}\right)^{1-\phi} \left(\prod_{i=1}^{N} (\max\{1, \omega_{ij,t} M_{i,t}\})^{\eta_i}\right)^{\phi} \quad \text{for all } t, && (15)\\
E_{j,t} &= P_{j,t} C_{j,t} + P_{j,t}\Omega_{j,t} + P_{j,t}\chi_{j,t} \quad \text{for all } t, && (16)\\
E_{j,t} &= Y_{j,t} + \phi\eta_j \sum_{i\in\mathbb{N}_{ji,t}} Y_{i,t} - \phi Y_{j,t} \sum_{i\in\mathbb{N}_{ij,t}} \eta_i \quad \text{for all } t, && (17)\\
& K_{j,0},\ M_{j,0} \text{ given.} && (18)
\end{aligned}
$$

Here, for expositional convenience, we have defined a set of constraints $\mathbb{N}_{ij,t} \equiv \{i \neq j, \omega_{ij,t} M_{i,t} > 1\}$. Equation (12) is the representative agent's intertemporal utility function. Equations (13), (14) and (15) define the law of motion for physical capital stock, the law of motion for technology capital stock, and the value of production, respectively. Equation (16) gives total spending in country j at time t, $E_{j,t}$, as the sum of spending on consumption $(P_{j,t}C_{j,t})$, spending on investments in physical capital $(P_{j,t}\Omega_{j,t})$, and spending on investments in technology capital $(P_{j,t}\chi_{j,t})$. Finally, Equation (17) defines disposable income as the sum of total nominal output $(Y_{j,t})$ plus rents from foreign investments $\left(\phi\eta_j \sum_{i\in\mathbb{N}_{ji,t}} Y_{i,t} = \sum_{i\in\mathbb{N}_{ji,t}} M_{j,t} \times \frac{\partial Y_{i,t}}{\partial M_{j,t}}\right)$, minus rents accruing to foreign investments $\left(\phi Y_{j,t} \sum_{i\in\mathbb{N}_{ij,t}} \eta_i = \sum_{i\in\mathbb{N}_{ij,t}} M_{i,t} \times \frac{\partial Y_{j,t}}{\partial M_{i,t}}\right)$, as defined in Equations (8) and (9), respectively.

2.1 A Model of Trade and Investment

Solving the representative agent's problem delivers a structural system that describes the relationships between trade, domestic investment and FDI. We solve the agent's optimization problem in two steps. First, we solve the optimal demand of $c_{ij,t}$, $I^K_{ij,t}$ and $I^M_{ij,t}$ for given aggregate variables. Following Anderson et al. (2015), we label this stage the '*lower level*'.

Then, we solve the dynamic optimization problem for $C_{j,t}$, $\Omega_{j,t}$ and $\chi_{j,t}$. This is what we call the '*upper level*'.

'Lower Level' Equilibrium. Let $p_{ij,t} = p_{i,t}t_{ij,t}$ denote the delivered price of country i's goods for country j consumers, where $t_{ij,t}$ is the variable bilateral trade cost factor on shipments from i to j at time t.[10] Let $X_{ij,t} = p_{ij,t}(c_{ij,t} + I^K_{ij,t} + I^M_{ij,t})$ denote country j's total nominal spending on goods from country i at time t. Solving the representative agent's optimization of (5), (7), and (11), subject to (16) and taking $C_{j,t}$, $\Omega_{j,t}$, and $\chi_{j,t}$ for all j as given, delivers the familiar structural system of Anderson and van Wincoop (2003):

$$X_{ij,t} = \frac{Y_{i,t}E_{j,t}}{Y_t}\left(\frac{t_{ij,t}}{\Pi_{i,t}P_{j,t}}\right)^{1-\sigma}, \tag{19}$$

$$P_{j,t}^{1-\sigma} = \sum_{i=1}^{N}\left(\frac{t_{ij,t}}{\Pi_{i,t}}\right)^{1-\sigma}\frac{Y_{i,t}}{Y_t}, \tag{20}$$

$$\Pi_{i,t}^{1-\sigma} = \sum_{j=1}^{N}\left(\frac{t_{ij,t}}{P_{j,t}}\right)^{1-\sigma}\frac{E_{j,t}}{Y_t}. \tag{21}$$

In terms of notation, system (19)-(21) is virtually identical to the original structural gravity system of Anderson and van Wincoop (2003) and, therefore, it is representative of the whole family of micro-founded gravity models described by Arkolakis et al. (2012). The introduction of FDI implies that $Y_{i,t} \neq E_{i,t}$. In turn, this means that in our framework trade and prices respond endogenously to changes in FDI. This will become clear next, when we solve for the '*upper level*' equilibrium in our model.

'Upper Level' Equilibrium. Our model of trade, domestic capital accumulation, and FDI does not have analytical solutions for the transition functions for physical and technology capital. Thus we analyze the steady-state in order to be able to offer a clear discussion of the key structural relationships and mechanisms in our framework. To solve for the upper level

[10]Trade costs thus can be interpreted by the standard iceberg melting metaphor: It is as if goods melt away in distribution so that 1 unit shipped becomes $1/t_{ij,t} < 1$ units on arrival. Technologically, a unit of distribution services required to ship goods uses resources in the same proportions as does production. The units of distribution services required on each link vary bilaterally.

steady state equilibrium, we set up the Lagrangian and we obtain the first order conditions for the key variables in our model, including the first order condition for physical capital and the first-order condition for technology capital.[11] Then, we combine the first-order conditions with the production function, the budget constraint, the expressions for expenditure and factory-gate prices, and the equations from the lower-level equilibrium:

$$X_{ij} = \frac{Y_i E_j}{Y}\left(\frac{t_{ij}}{\Pi_i P_j}\right)^{1-\sigma} \quad \text{for all } i \text{ and } j \tag{22}$$

$$P_j^{1-\sigma} = \sum_{i=1}^{N}\left(\frac{t_{ij}}{\Pi_i}\right)^{1-\sigma}\frac{Y_i}{Y} \quad \text{for all } j, \tag{23}$$

$$\Pi_i^{1-\sigma} = \sum_{j=1}^{N}\left(\frac{t_{ij}}{P_j}\right)^{1-\sigma}\frac{E_j}{Y} \quad \text{for all } i, \tag{24}$$

$$p_j = \frac{\left(Y_j / \sum_{j=1}^{N} Y_j\right)^{\frac{1}{1-\sigma}}}{\gamma_j \Pi_j} \quad \text{for all } j, \tag{25}$$

$$Y_j = p_j A_j \left(L_j^{1-\alpha} K_j^{\alpha}\right)^{1-\phi}\left(\prod_{i=1}^{N}\left(\max\{1, FDI_{ij}\}\right)^{\eta_i}\right)^{\phi} \quad \text{for all } j, \tag{26}$$

$$E_j = Y_j + \phi\eta_j \sum_{i\in\mathbb{N}_{ji,t}} Y_i - \phi Y_j \sum_{i\in\mathbb{N}_{ij,t}} \eta_i \quad \text{for all } j, \tag{27}$$

$$K_j = \frac{\alpha\beta(1-\phi)\left(1-\phi\sum_{i\in\mathbb{N}_{ij,t}}\eta_i\right)}{1-\beta+\beta\delta_K}\frac{Y_j}{P_j} \quad \text{for all } j, \tag{28}$$

$$FDI_{ji}^{value} = \frac{\beta\phi\eta_j}{1-\beta+\beta\delta_M}\omega_{ji}\frac{E_j}{P_j}\phi\eta_j\frac{Y_i}{M_j} \quad \text{for all } j. \tag{29}$$

Most of the equations in system (22)-(29) are familiar because they have already been derived and discussed in the existing literature. Therefore, here we discuss them only briefly and intuitively with references to relevant papers. For example, as noted above, Equations (22)-(25) represent the structural gravity system of Anderson and van Wincoop (2003). (22) is the well-known gravity equation of trade, cf. Arkolakis et al. (2012) and Head and Mayer (2014). Equations (23)-(24) define the multilateral resistance, which consistently aggregate bilateral trade costs to the country level and decompose their incidence on the consumers and

[11]See for details of the derivations Appendix A.

producers, cf. Anderson and van Wincoop (2003) and Anderson and Yotov (2010). Equation (25) is a restatement of the market-clearing condition according to which, at delivered prices, the value of output in country j should equal the value of total sales across all destinations, including j itself. In its current version, Equation (25) captures the inverse relationship between the outward multilateral resistance faced by the producers in country j and their factory-gate prices.

As introduced earlier, Equation (26) defines the value of the production in country j. Importantly for our purposes, (26) includes foreign direct investment as a key factor of production. The implications for the general equilibrium analysis and for the relationships between trade and FDI, which are key in our setting, are that by stimulating production, through (26), FDI will also influence trade directly, as captured by Equation (22), and indirectly, through the multilateral resistances, (23)-(24). FDI will also influence trade directly and indirectly by affecting expenditure. This is captured by Equation (27), where the inflow of income in response to outward FDI increases expenditure in country j, while the payments to FDI coming from abroad will decrease disposable income.

Equation (28) defines domestic capital as a function of model parameters and two endogenous variables, namely the value of national output (Y_j) and the inward multilateral resistance (P_j). The links between domestic investment and trade and their implications for welfare have been the object of interest in several recent studies, e.g. Anderson et al. (2015) and Eaton et al. (2016), which generate equations similar to (28). The positive relationship between K_j and Y_j in (28) reflects the fact that a higher value of marginal product of physical capital would lead to more domestic investment. To see the link between domestic investment and trade liberalization, note that trade policies will affect the factory-gate price (through (25)) and, therefore, the value of production in j. The inverse relationship between K_j and P_j in (28) is a reflection of the law of demand, when P_j is thought of as the price of investment goods. Alternatively, if P_j is the price of consumption goods, the inverse relationship K_j and P_j is explained with the higher opportunity cost of investment. Note that since P_j is a general equilibrium trade cost index, changes in trade policy anywhere in the system may impact domestic investment in j.

Finally, Equation (29) defines the value of outward FDI from country j to destination i as a function of other variables and parameters in our framework. Since modeling FDI is the key innovation of our work in relation to the existing literature and because our theory leads to a convenient and intuitive gravity presentation of FDI that is remarkably similar to the familiar trade gravity system, we devote a separate sub-section to it.

2.2 A Structrural Gravity System of FDI

Equation (29) defines positive FDI. However, in many cases, bilateral FDI is indeed zero in the data. Therefore, we explicitly account for this possibility using our theory to define zero FDI flows. In addition, we combine our FDI equation with the definitions of the multilateral resistance terms P_j and Π_j given by Equations (23) and (24), respectively, to obtain the following standalone FDI gravity system:

$$FDI_{ij}^{value} = \begin{cases} \frac{\beta\phi^2\eta_i^2}{1-\beta+\beta\delta_M}\omega_{ij}\frac{E_i}{P_i}\frac{Y_j}{M_i} & \text{if} \quad FDI_{ij} = \omega_{ij}M_i > 1, \\ 0 & \text{if} \quad FDI_{ij} = \omega_{ij}M_i \leq 1, \end{cases} \tag{30}$$

$$P_i = \left[\sum_{j=1}^{N}\left(\frac{t_{ji}}{\Pi_j}\right)^{1-\sigma}\frac{Y_j}{Y}\right]^{\frac{1}{1-\sigma}}, \tag{31}$$

$$\Pi_j = \left[\sum_{i=1}^{N}\left(\frac{t_{ji}}{P_i}\right)^{1-\sigma}\frac{E_i}{Y}\right]^{\frac{1}{1-\sigma}}. \tag{32}$$

There is a clear resemblance between system (30)-(32) to the structural trade gravity system. First, the gravity equation for FDI, Equation (30), reveals that FDI is directly related to the size of the country of origin, as measured by expenditure E_i. The intuition for this relationship is that the expression for expenditure in our model reflects the value of marginal product of technology capital M_i. Second, Equation (30) captures the positive relationship between FDI and the size of the host country, as captured by nominal output Y_j. The intuition for this relationship is that Y_j is a proxy for the value of marginal product of technology capital in the host country. Third, Equation (30) accounts for the fact that the stock value of FDI is inversely related to FDI barriers ω_{ij}. In sum, the three relationships that we established thus far are analogous to the familiar dependencies in physics and in international trade. Specifically, the closer and the larger the host and the source economies are, the larger the stock value of FDI between them will be.

Fourth, an important feature of our FDI gravity system is that it links bilateral FDI stock values to trade via the multilateral resistance in an intuitive way. Specifically, higher inward multilateral resistance in the country of origin i, P_i, lead to less FDI abroad and at destination j in particular. The intuition for this result is that higher P_i means higher direct and opportunity cost of investing in knowledge capital in i. A key difference of (30) from the trade gravity model is the absence of outward multilateral resistance. The reason is the non-rival nature of technology capital, in contrast to goods sales: goods sold to j from i cannot be used elsewhere whereas i's technology used in j has no effect on its utilization elsewhere. Our model assumes that the origin sells use of its technology capital to the destination at its value to the buyer at zero cost to itself. In arms length transactions this assumption is consistent with bargaining where all the power lies with the seller.[12]

Fifth, Equation (30) reveals that the value of the FDI stock of country i in country j depends negatively on the amount of technology capital in country i. This relationship is also intuitive and it is a reflection of the diminishing returns to investments into technology capital. Finally, we note that the similarities between our structural FDI gravity system and the corresponding trade equations suggest that system (30)-(32) can be estimated using the well-established techniques from the trade literature. While we see significant potential in estimating system (30)-(32), this is beyond the scope of the current paper and we leave

[12]We abstract from intermediate bargaining power that splits the surplus between seller and buyer parametrically because it adds nothing useful to the model. We also abstract from various tax avoidance FDI motives. Our gravity model of FDI also contrasts to the gravity FDI model of Head and Ries (2008) in the same respect: the 'non-rival' nature of technology in our model means there is no role for outward multilateral resistance.

it for future work. Instead, in order to demonstrate the effectiveness of our methods, we follow leading studies from the new quantitative trade literature (e.g. Caliendo and Parro, 2015; Eaton et al., 2016) and we use our theoretical framework to perform a calibration experiment, which we describe in the next section.

3 Empirical Analysis: A World Without FDI

The goal of this section is to study the impact of FDI on welfare and inequality in the world. To do so, we perform a counterfactual experiment that is similar to the standard exercise of moving to autarky in the trade literature, cf. Costinot and Rodríguez-Clare (2014). However, instead we simulate a move to a world without FDI, while allowing for all other channels and relationships (e.g. trade and domestic investment) in our model to be active. Our focus will be exclusively on the FDI channel since this is our main contribution in relation to the existing literature. The benefits of this experiment are threefold. First, from a methodological perspective, it will enable us to demonstrate the effectiveness of our methods and the empirical relevance of the novel FDI channel that we model. Second, as motivated by the opening quote of our paper, policy makers and analysis see FDI as a key driver of prosperity for many countries, especially in the developed world. Our analysis will offer quantitative evidence in support or against such claims and expectations. Finally, we will be able to compare the contributions of international trade vs. FDI as two of the main drivers of globalization in the past quarter century. In subsection 3.1 we describe our data and calibration methods. Then, section 3.2 presents our findings.

3.1 Data and Calibration

In order to perform the empirical analysis, we compile a novel balanced panel data set for 89 countries in 2011,[13] which includes data on foreign direct investment, trade flows, gross

[13]The list of countries and their respective labels in parentheses includes Angola (AGO), Argentina (ARG), Australia (AUS), Austria (AUT), Bangladesh (BGD), Belarus (BLR), Belgium (BEL), Brazil (BRA), Bul-

domestic product (GDP), employment, and physical capital. The 89 countries accounted for more than 96 percent of world GDP and for more than 94 percent of FDI during the sample period. The set of parameters that are needed to perform the counterfactual experiment are either taken from the literature or calibrated. To facilitate presentation, all variables and parameters as well as the data sources and the methods used to construct the calibrated data are summarized in Table 1.

Data on employment and capital stocks are from the Penn World Tables (PWT) 8.0.[14] The Penn World Tables 8.0 include data that enables us to measure employment in effective units for all countries in our sample. To do this we multiply the *Number of persons engaged in the labor force* with the *Human capital index*, which is based on average years of schooling. Capital stocks in the Penn World Tables 8.0 are constructed based on accumulating and depreciating past investments using the perpetual inventory method. For more detailed information on the construction and the original sources for the PWT series see Feenstra et al. (2013).

Aggregate trade data come from the United Nations Statistical Division (UNSD) Commodity Trade Statistics Database (COMTRADE). In order to construct internal aggregate trade, which is needed for the counterfactual analysis, we used the ratio between aggregate manufacturing in gross values and total exports of manufacturing goods in order to construct

garia (BGR), Canada (CAN), Chile (CHL), China (CHN), Colombia (COL), Croatia (HRV), Czech Republic (CZE), Cyprus (CYP), Denmark (DNK), Ecuador (ECU), Egypt (EGY), Estonia (EST), Finland (FIN), France (FRA), Germany (DEU), Ghana (GHA), Greece (GRC), Hong Kong (HKG), Hungary (HUN), India (IND), Indonesia (IDN), Iran (IRN), Ireland (IRL), Israel (ISR), Italy (ITA), Japan (JPN), Kazakhstan (KAZ), Kenya (KEN), Korea, Republic of (KOR), Kuwait (KWT), Lebanon (LBN), Lithuania (LTU), Latvia (LVA), Luxembourg (LUX), Macedonia (MKD), Malaysia (MYS), Malta (MLT), Mexico (MEX), Morocco (MAR), Netherlands (NLD), New Zealand (NZL), Nigeria (NGA), Norway (NOR), Pakistan (PAK), Peru (PER), Philippines (PHL), Poland (POL), Portugal (PRT), Qatar (QAT), Romania (ROU), Russia (RUS), Saudi Arabia (SAU), Singapore (SGP), Slovak Republic (SVK), Slovenia (SVN), South Africa (ZAF), Spain (ESP), Sweden (SWE), Switzerland (CHE), Syria (SYR), Thailand (THA), Turkey (TUR), Ukraine (UKR), United Kingdom (GBR), United States (USA), Uzbekistan (UZB), Venezuela (VEN), and Vietnam (VNM). 2011 was determined by the availability of capital stock data.

[14]These series are now maintained by the Groningen Growth and Development Centre and reside at http://www.rug.nl/research/ggdc/data/pwt/. We refer the reader to Feenstra et al. (2013) for details on the PWT dataset and we are very grateful to Robert Inklaar and Marcel Timmer for useful insights and clarifications regarding the PWT data.

a multiplier at the country-time level.[15] We then used this multiplier along with data on aggregate exports to project the values for intra-national trade. Data on gross manufacturing production, which came from the United Nations' IndStat database, enabled us to construct multiplier indexes for half of the counties in our sample. We used a rest-of-the-world (ROW) multiplier index to construct the rest of the internal trade data.

FDI data come from two sources. The main source is the newly constructed Bilateral FDI Statistics database of the United Nations Conference on Trade and Development (UNCTAD).[16] UNCTAD's FDI data covers inflows, outflows, inward stock, and outward stock for 206 countries over the period 1990-2011. Data are collected from national sources and international organizations. In order to ensure maximum coverage, mirror data from partner countries is used as well. The second source of FDI data is the International Direct Investment Statistics database, which is constructed and maintained by the Organization for Economic Co-operation and Development (OECD).[17] OECD's data offers detailed statistics for inward and outward foreign direct investment flows and positions (stocks) of the OECD countries, including transactions between the OECD members and non-member countries. We used the OECD data to ensure consistency and maximum coverage. Finally, we note that the focus throughout the analysis is on FDI stocks (positions), which is the FDI category for which most data are available and which is also the theoretically consistent measure.

The parameters needed to run counterfactuals with our model come from three sources. Parameters borrowed from the literature include the elasticity of substitution ($\sigma = 6$), which

[15] An alternative approach is to construct intra-national trade flows as the difference between GDP data, which are widely available, and data on total exports. However this procedure is inconsistent with our theory because GDP is a measure of value added while total exports are a gross measure. Another alternative is to use the World Input-Output Database (WIOD) data, which includes consistently constructed intra-national trade flows. The main disadvantage of the WIOD data is its limited country coverage (43 countries), which will prevent us from focusing on many of the developed countries, where we expect the impact of FDI to be significant.

[16] UNCTAD's Bilateral FDI Statistics database can be found and accessed from UNCTAD's web site at http://unctad.org/en/Pages/DIAE/FDI%20Statistics/FDI-Statistics-Bilateral.aspx. We are extremely grateful to Marco Fugazza who answered many questions and offered useful insights about the FDI data. We also thank colleagues at Global Affairs Canada, and Felix Stips from the University of Bayreuth who helped with the downloading and the formatting of earlier versions of the UNCTAD FDI data.

[17] We thank George Pinel from Drexel University for his help with the downloading and formatting of the OECD FDI data.

is about the average and very close to the preferred reported value by Head and Mayer (2014);[18] the consumer discount factor ($\beta = 0.98$), which is from Yao et al. (2012); the country-specific capital shares of production (α_j); and the country-specific adjustment costs of capital (δ_j), which come from the Penn World Tables.[19] The exact values for the α's and δ's are reported in columns (2) and (3) of Table 2, respectively.

A second group of parameters are constructed directly from the data. To calculate η_i, i.e., the share of technology capital of a country to all destinations as a share from total world technology capital, we use the data on FDI_{ij}^{value} and calculate η_i as follows:

$$\eta_i = \frac{\sum_j FDI_{ij}^{value}}{\sum_i \sum_j FDI_{ij}^{value}}. \tag{33}$$

In addition, we calculate the production share of FDI using the relationship between inward FDI ($FDI_j^{in} = \sum_i FDI_{ij}^{value}$) and physical capital in the production function along with FDI and physical capital data and data on the capital shares:[20]

$$\phi_j = \frac{\alpha_j \times (FDI_j^{in}/K_j)}{1 + \alpha_j (FDI_j^{in}/K_j)}. \tag{34}$$

The values for η and ϕ are reported in columns (3) and (4) of Table 2.

The third group of parameters that we use are calibrated to fit the model. These parameters include the bilateral trade frictions (t_{ij}), which are calibrated from Equations (22)-(24); the FDI frictions (ω_{ij}), which are constructed from Equation (29); the preference adjusted technology parameters (A_j/γ_j), which is calibrated using Equations (25) and (26); and the endogenous values for technology capital, which are calibrated using the following theory-

[18]Head and Mayer (2014) report elasticities of trade with respect to trade costs, which are given by $1 - \sigma$ in our framework. Hence, the reported average value of -4.51 translates to a σ of 5.51, while the preferred estimate based on tariff data of -5.03 implies $\sigma = 6.03$.

[19]Due to the lack of data, we use the same country-specific depreciation rates for physical and technology capital.

[20]Specifically, we use that $\phi_j Y_j$ is the share of spending on inward FDI and $(1 - \phi_j)\alpha_j Y_j$ the share of spending on capital, which also enables us to construct country-specific ϕ's.

consistent equation for technology capital:

$$M_j = \frac{\beta\phi\eta_j}{1-\beta+\beta\delta_M}\frac{E_j}{P_j} \quad \text{for all } j. \tag{35}$$

Finally, we construct the inward and the outward multilateral resistance indexes using Equations (23) and (24), respectively. In order to perfectly match the differences between the value of domestic income and expenditure in the data, we define $\psi_j \equiv E_j / \Big(Y_j + \eta_j \sum_{i \in \mathbb{N}_{ji,t}} \phi_i Y_i - \phi_j Y_j \sum_{i \in \mathbb{N}_{ij,t}} \eta_i \Big)$ as an exogenous country-specific parameter that accounts for any residual trade imbalances that are not due the payments to inward and outward FDI, which are already captured in our theory. We then keep this exogenous part of the trade imbalances ($[(\psi_j - 1)/\psi_j]E_j$) constant in our counterfactual analysis, while the part due to payments to inward and outward FDI endogenously adjusts.[21]

3.2 Empirical Findings

Armed with our structural system of trade and investment from Section 2 and with the data and parameters from the previous subsection, we proceed in this section to perform a counterfactual experiment that simulates the steady state equilibrium of a hypothetical world economy without FDI. Mechanically, we start in a baseline scenario, where trade, domestic investment, and FDI are all active channels and then we increase FDI frictions to eliminate all FDI in the world, while still allowing for trade and domestic investment.

In order to keep the presentation and the discussion of our results manageable, we focus only on two outcomes of the counterfactual analysis. Specifically, first, we investigate the implications of FDI for international trade. Then, we quantify the impact of FDI on welfare, i.e. 'the gains from FDI', which is a standard experiment with respect to trade policy in the related empirical trade literature. Given the dynamic nature of our model, in order to measure welfare we follow Lucas (1987) and we obtain the welfare indexes by calculating

[21] Note that it is ensured that world trade imbalances are zero in the baseline and the counterfactual.

the constant fraction ζ of aggregate consumption that consumers would need to be paid in the baseline case to give them the same utility they obtain from consumption stream in the counterfactual (C_j^c). Specifically, we calculate:

$$\sum_{t=0}^{\infty} \beta^t \ln\left(C_j^c\right) = \sum_{t=0}^{\infty} \beta^t \ln\left[\left(1+\frac{\zeta}{100}\right) C_j^b\right] \Rightarrow$$
$$\zeta = \left(\exp\left[\ln\left(C_j^c\right) - \ln\left(C_j^b\right)\right] - 1\right) \times 100. \qquad (36)$$

Our findings with respect to the trade and welfare effects are reported in Table 2. Focus first on the trade effects of FDI. The changes in trade due to no FDI are in column (6) of Table 2, revealing a very substantial impact on trade. On average, trade would have been 11% lower without FDI in the world. Second, the impact of FDI on international trade is quite heterogeneous across the countries in our sample. The countries whose trade has benefitted the most from FDI, i.e. the countries with the most negative trade indexes in column (6), are Luxembourg with a decrease of total exports of 90% after the hypothetical elimination of FDI, followed by Belgium with a decrease in exports of 61%, and Hong Kong with a fall in exports of 57%. In contrast, there are countries such as Pakistan, China and India, that actually see an increase in their total exports of more than 10% due to the elimination of FDI in the world.

The variation in the net FDI position of the countries in our sample offers a clue about the reason for the heterogeneous results regarding the relationship between FDI and trade. *Net export of FDI and export trade are substitutes in the cross-section.* Figure 1 plots the counterfactual change in total exports from the elimination of FDI against the net FDI position of countries. The Figure shows a best fit line with negative slope in the plot of trade change against net FDI. This regularity has a simple partial equilibrium intuition. The general equilibrium forces of the model modify the intuition, leading further insight into the scatter of data points, especially the outliers.

The partial equilibrium reasoning is based on the effects of FDI elimination on expenditures. FDI net position directly signs the capital services account: net inward FDI implies that $E_j < Y_j$. Elimination of FDI, all else equal, implies that E_j moves to equal Y_j, rising for net inward FDI and falling for net outward FDI.

Now put this relationship to work on the shift in the exports of country i:

$$\sum_{j \neq i} X_{ij} = Y_i \sum_{j \neq i} \frac{E_j}{Y} \left(\frac{t_{ij}}{\Pi_i P_j} \right)^{1-\sigma} = Y_i \left[1 - \frac{E_i}{Y} \left(\frac{t_{ii}}{\Pi_i P_i} \right)^{1-\sigma} \right]. \tag{37}$$

The elimination of FDI implies, all else equal, that E_i rises for net inward FDI and thus i's export trade falls according to the rightmost equality in (37). For net outward FDI E_i falls, hence exports rise. This is the pattern of Figure 1.

The negative slope is most pronounced in the steeply sloped data cluster around net FDI =0 in Figure 1. In contrast, the outliers (China, USA, Japan and Belgium) dampen the relationship and suggest general equilibrium forces at work. The modularity of the model makes tracing the effect of FDI elimination straightforward. Essentially, countries with negative net FDI lose global income share from FDI elimination (technology capital is less profitable so less is accumulated) while countries with positive net FDI gain income share (domestic technology capital accumulation becomes more profitable following the loss of imported technology capital). Our simulation results strongly show this effect. By (37) this effect on the right and left outliers flattens the best fit line in Figure 1.

A secondary general equilibrium effect is the effect of FDI elimination on multilateral resistances. Our simulations show that net FDI is negatively associated with outward multilateral resistance Π_i, with a scatter plot closely resembling Figure 1. Net FDI is positively associated with inward multilateral resistance, but less strongly than the negative association with outward multilateral resistance. All else equal, these multilateral resistance effects in (37) would *raise* exports, so the full general equilibrium effect in Figure 1 arises because the direct effects of FDI elimination on E_i and Y_i discussed in the preceding two paragraphs dominates.

Thus while general equilibrium forces of the model act to blur the negative association of net FDI and trade changes, the direct link between expenditures and the net FDI position predominates enough to explain the negative slope in Figure 1.

Next, we turn to the welfare effects of FDI. The estimates of the Lucas-discounted indexes are reported in column (7) of Table 2. Our results suggest that, on average, the gains from FDI amount to an impressive 9% of world's welfare in 2011. This result confirms that FDI is indeed an important component and a key driver of prosperity in today's global economy. In addition, and similar to our results for trade, we find that the welfare impact of FDI has also been quite heterogeneous across the 89 countries in our sample. Most countries experience losses from shutting off FDI. The largest negative impact of the removal of FDI on welfare are for some developed countries (e.g. Ireland and Luxembourg) as well as for some developing economies (e.g. Philippines and India). Interesting and important, we also obtain some positive welfare effects from the elimination of FDI. For example, we find that welfare in Ethiopia, Lithuania, Estonia and, especially in Azerbaijan would have been larger without FDI in the world.

In order to shed more light on the driving forces behind the welfare effects of abolishing FDI and to confirm some intuitive explanations for our results, we plot the welfare changes that we obtain against a series of parameters and variables. Figure 2 plots the welfare changes against the country-specific production shares of FDI (ϕ_j's). As expected, Figure 2 confirms that higher ϕ's imply larger welfare losses. Intuitively, the more important FDI is in the production function for a country, the more heavily the country would be affected by changes in FDI. Figures 3-5 plot the welfare changes against inward, outward and net (inward-outward) FDI, respectively. As can be seen from these figures, there is a slight negative relationship between the welfare effects and inward and outward FDI, and a more pronounced negative relationship between the welfare effects and net FDI. Intuitively, this suggests that if a country has more inward than outward FDI, restricting FDI will lead to larger welfare losses, and vice versa. Finally, we plot in Figure 6 the relationship between welfare and the change in total exports induced by restricting FDI. Smaller negative or larger positive changes of trade lead to smaller welfare losses. Thus, the figure suggests that for some countries, as Ethiopia, Pakistan and China, for example, restricting FDI leads to an increase of total exports, mitigating the negative welfare effects from the removal of FDI.

In sum, the analysis in this section and the estimates from Table 2 reveal that FDI has had significant but uneven impact on trade and welfare across the countries in the world. Importantly, our estimates suggest that, while international trade and welfare in the world on average and in most of the countries in our sample have been affected positively by FDI, there are countries that have incurred losses due to FDI. In addition to pointing to the specific need for those countries to re-evaluate their national policies toward FDI, our results have implications for cross country inequality and for regional policies, which is of significant policy interest, c.f. European Commission (2010). For example, according to our results, some of the smaller and poorer nations in the European Union (including Cyprus, Bulgaria, Ireland, and Malta) are among the countries that have enjoyed the largest positive impact of FDI on both trade and welfare. This implies that FDI has lead to a decrease in income inequality in Europe. In addition, we find that the majority of the countries that have lost the most from FDI are former Soviet Union republics.

4 Conclusion

In this paper we developed a structural dynamic model that accounts for the economic impact of foreign direct investment (FDI) in the global economy and decomposes the relationships between trade, domestic investment via physical capital accumulation and FDI in the form of non-rival technology capital. Our counterfactual experiment of characterizing a hypothetical world without FDI demonstrated the effectiveness of our methods and the empirical importance of FDI as a key component of the global economy with significant contributions to welfare and inequality.

We see several possibilities to extend our analysis in new directions and to offer meaningful additional contributions. First, since we were limited by FDI data availability, we did not incorporate sectors in our model. In light of recent developments in the related trade literature, e.g. Caliendo and Parro (2015), who demonstrate the importance of allowing for input-output linkages, we expect that a sectoral model of trade and investment with intermediates will prove quantitatively important and will generate new insights about the impact of FDI. Another possible direction for further research that is motivated by our analysis is in the area of FDI estimations. Specifically, we expect that in combination with availability of new reliable bilateral FDI data, the similarity between our structural FDI gravity system and its popular trade counterpart will generate interest among researchers and policy makers who are interested in studying the impact of various determinants of FDI.

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Table 1: Summary of the Model's Endogenous Variables, Exogenous Variables, and Parameters

Variables/Parameters	Value	Description	Data/Calibration
Endogenous Variables			
X_{ij}	Matrix	Trade flows	UNIDO, COMTRADE
K_j	Vector	Physical capital	Penn World Tables 8.0
M_j	Vector	Technology capital	Calibrated using Equation (35)
Y_j	Vector	Income	UNIDO, COMTRADE
P_j	Vector	Inward MRT	Calibrated using Equation (23)
Π_j	Vector	Outward MRT	Calibrated using Equation (24)
E_j	Vector	Expenditure	Calibrated using Equation (27)
Exogenous Variables			
L_j	Vector	Labor Endowment	Penn World Tables 8.0
A_j/γ_j	Vector	Technology Parameter	Calibrated using Equations (25)-(26)
Parameters			
N	89	Number of countries	Given by data set
t_{ij}	Matrix	Trade cost matrix	Calibrated using system (22)-(24)
ω_{ij}	Matrix	FDI friction matrix	Calibrated using Equation (29)
σ	6	Elasticity of substitution	Head and Mayer (2014)
β	0.98	Discount factor	Yao et al. (2012)
α_j	Vector	Capital share in production	Penn World Tables 8.0
δ_{jK}	Vector	Adjustment costs for physical capital	Penn World Tables 8.0
δ_{jM}	Vector	Adjustment costs for technology capital	Penn World Tables 8.0
η_j	Vector	Share of technology capital of a country to all destinations as a share from total world technology capital	Constructed from bilateral FDI data
ϕ_j	Vector	FDI share in production	Calibrated to match contribution of FDI vs. capital in production

Table 2: Calibration Results and Results of Counterfactual Analysis

(1) Country	(2) α	(3) δ	(4) η	(5) ϕ	(6) Trade	(7) Welfare
AGO	0.47	0.05	0.00	0.06	-20.25	-2.78
ARG	0.57	0.04	0.01	0.02	-5.35	-13.94
AUS	0.44	0.04	0.01	0.05	-21.38	-9.12
AUT	0.43	0.04	0.01	0.06	-12.91	-14.10
AZE	0.79	0.07	0.00	0.04	-16.05	5.61
BEL	0.38	0.05	0.01	0.24	-60.54	-12.50
BGD	0.47	0.04	0.00	0.00	4.12	-1.47
BGR	0.51	0.06	0.00	0.09	-17.22	-13.38
BLR	0.48	0.05	0.00	0.03	-4.76	-0.10
BRA	0.44	0.05	0.03	0.04	-8.76	-11.73
CAN	0.39	0.04	0.02	0.06	-12.99	-18.40
CHE	0.35	0.06	0.01	0.19	-42.87	-22.55
CHL	0.55	0.04	0.00	0.06	-13.40	-13.86
CHN	0.46	0.05	0.18	0.01	11.49	-11.77
COL	0.39	0.04	0.01	0.01	2.26	-3.68
CYP	0.48	0.04	0.00	0.19	-39.58	-32.51
CZE	0.49	0.04	0.00	0.06	-18.06	-1.98
DEU	0.39	0.04	0.04	0.03	-5.73	-7.69
DNK	0.37	0.04	0.00	0.05	-12.04	-12.35
DOM	0.34	0.03	0.00	0.01	-2.23	-0.95
ECU	0.55	0.05	0.00	0.01	-0.49	-1.38
EGY	0.62	0.06	0.00	0.03	-6.95	-9.44
ESP	0.39	0.04	0.02	0.05	-7.45	-13.31
EST	0.42	0.05	0.00	0.09	-24.93	0.07
ETH	0.47	0.05	0.00	0.00	8.37	0.30
FIN	0.39	0.04	0.00	0.05	-9.78	-7.72
FRA	0.37	0.04	0.03	0.04	-4.86	-10.61
GBR	0.39	0.04	0.03	0.08	-19.63	-17.16
GHA	0.47	0.06	0.00	0.02	-4.61	-3.08
GRC	0.47	0.03	0.00	0.01	2.54	-8.01
GTM	0.58	0.05	0.00	0.03	-9.98	-3.86
HKG	0.48	0.04	0.01	0.23	-57.24	-20.34
HRV	0.34	0.04	0.00	0.04	-8.26	-4.41
HUN	0.41	0.04	0.00	0.07	-12.73	-10.68
IDN	0.54	0.04	0.01	0.02	-3.30	-7.51
IND	0.50	0.06	0.04	0.01	10.76	-10.68

Continued on next page

Table 2 – *Continued from previous page*

(1) Country	(2) α	(3) δ	(4) η	(5) ϕ	(6) Trade	(7) Welfare
IRL	0.52	0.05	0.00	0.29	-48.47	-54.89
IRN	0.74	0.06	0.01	0.00	3.38	-4.44
IRQ	0.70	0.06	0.00	0.00	-1.99	-1.18
ISR	0.45	0.04	0.00	0.03	-7.33	-8.87
ITA	0.46	0.04	0.03	0.02	-1.07	-8.89
JPN	0.39	0.05	0.07	0.00	9.53	-8.52
KAZ	0.58	0.04	0.00	0.06	-23.65	-7.65
KEN	0.57	0.05	0.00	0.02	-2.53	-2.46
KOR	0.50	0.05	0.02	0.01	0.40	-6.10
KWT	0.75	0.06	0.00	0.01	1.02	-7.07
LBN	0.56	0.04	0.00	0.00	7.28	-7.51
LKA	0.31	0.04	0.00	0.00	3.65	-2.47
LTU	0.53	0.04	0.00	0.06	-14.59	1.85
LUX	0.46	0.05	0.01	0.63	-89.80	-43.53
LVA	0.45	0.03	0.00	0.05	-14.03	-2.66
MAR	0.51	0.05	0.00	0.05	-8.96	-8.18
MEX	0.61	0.04	0.02	0.05	-11.05	-17.42
MKD	0.47	0.04	0.00	0.03	-6.14	-1.77
MLT	0.46	0.05	0.00	0.22	-32.55	-16.43
MYS	0.47	0.06	0.01	0.03	-7.47	-10.07
NGA	0.50	0.06	0.00	0.05	-19.69	-1.70
NLD	0.41	0.04	0.02	0.11	-16.98	-17.43
NOR	0.48	0.04	0.00	0.09	-23.38	-14.17
NZL	0.43	0.04	0.00	0.08	-26.59	-6.05
OMN	0.70	0.06	0.00	0.03	-10.65	-1.34
PAK	0.47	0.06	0.00	0.01	15.73	-9.54
PER	0.69	0.04	0.00	0.02	-5.06	-3.06
PHL	0.64	0.05	0.00	0.01	-2.03	-17.84
POL	0.44	0.05	0.01	0.05	-7.27	-8.81
PRT	0.39	0.04	0.00	0.04	-8.03	-11.80
QAT	0.81	0.10	0.00	0.03	-8.51	-12.50
ROM	0.53	0.05	0.00	0.05	-11.00	-9.85
RUS	0.26	0.04	0.03	0.01	5.80	-2.87
SAU	0.72	0.05	0.01	0.01	-0.94	-2.85
SDN	0.41	0.07	0.00	0.01	-4.07	-0.86
SER	0.42	0.04	0.00	0.03	-9.74	-1.42
SGP	0.56	0.05	0.00	0.18	-40.51	-23.60

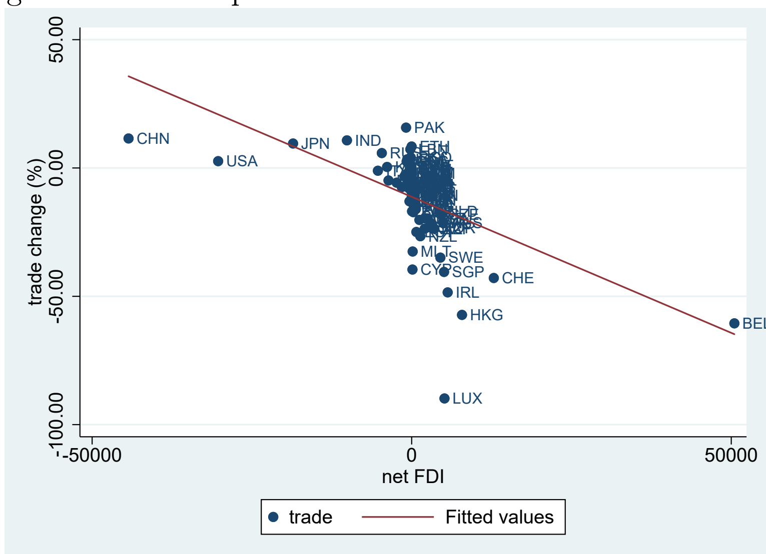
Continued on next page

Table 2 – *Continued from previous page*

(1) Country	(2) α	(3) δ	(4) η	(5) ϕ	(6) Trade	(7) Welfare
SVK	0.46	0.05	0.00	0.07	-21.45	1.65
SVN	0.33	0.04	0.00	0.02	-5.39	-0.11
SWE	0.45	0.05	0.00	0.18	-34.93	-22.49
SYR	0.47	0.06	0.00	0.00	-0.20	-0.99
THA	0.61	0.07	0.01	0.04	-5.61	-13.54
TKM	0.47	0.04	0.00	0.00	-4.38	-1.30
TUN	0.50	0.05	0.00	0.01	0.71	-4.35
TUR	0.56	0.06	0.01	0.04	-5.81	-6.11
TZA	0.57	0.04	0.00	0.03	-12.48	-2.18
UKR	0.44	0.03	0.01	0.01	-0.84	-2.52
USA	0.40	0.05	0.18	0.02	2.66	-7.11
UZB	0.47	0.03	0.00	0.00	-1.17	-0.19
VEN	0.63	0.04	0.00	0.02	-6.45	-3.25
VNM	0.47	0.05	0.01	0.01	-1.67	-0.77
ZAF	0.46	0.05	0.00	0.07	-23.66	-6.38
ZWE	0.44	0.04	0.00	0.08	-16.88	-0.47

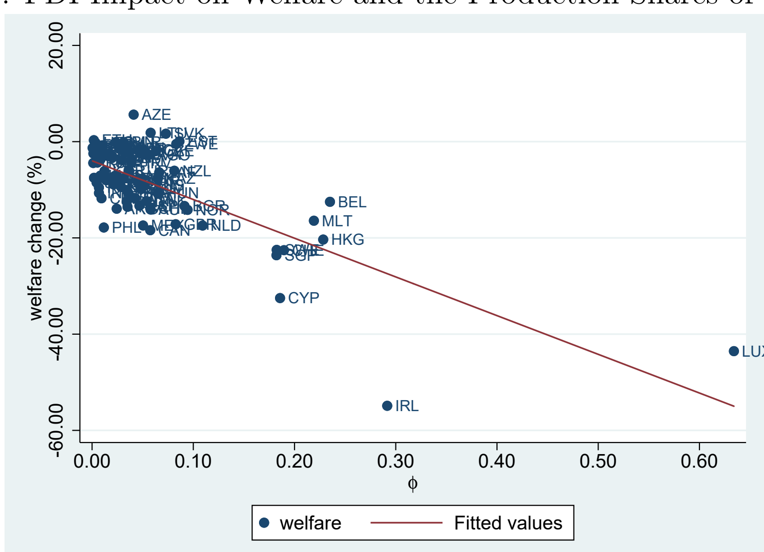
Notes: This table reports results from our FDI counterfactual. Column (1) lists the country abbreviations. Columns (2) reports the calculated capital shares α. In column (3) we report the calculated depriciation rates δ. Column (4) gives the shares of technology capital of a country to all destinations as a share from total world technology capital η, while column (5) reports the production shares of FDI ϕ. Column (6) and (7) report percentage changes in total exports and welfare for the FDI counterfactual. See text for further details.

Figure 1: FDI Impact on Trade and the Net FDI Position.



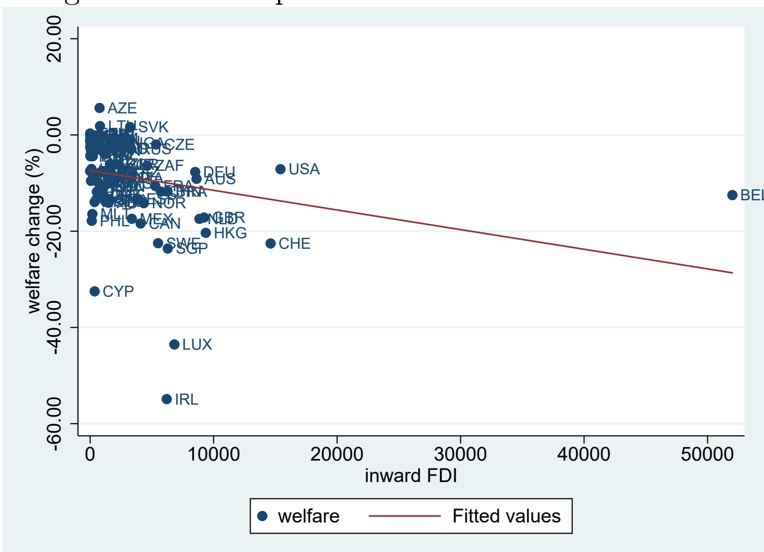
Notes: This figure plots the change in total exports in response to the elimination of FDI in the world against net FDI position, i.e. inward FDI − outward FDI, in the baseline scenario. See text for further details.

Figure 2: FDI Impact on Welfare and the Production Shares of FDI (ϕ).



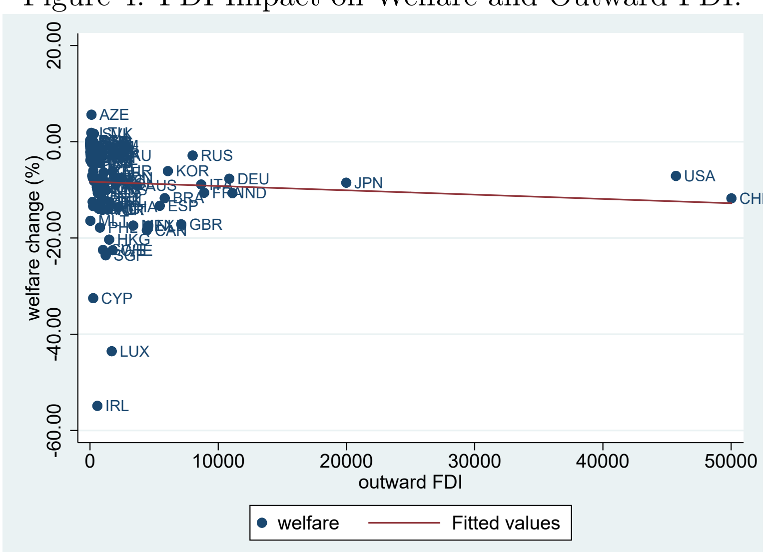
Notes: This figure plots the change in welfare in response to the elimination of FDI in the world against the FDI share in production (ϕ). See text for further details.

Figure 3: FDI Impact on Welfare and Inward FDI.



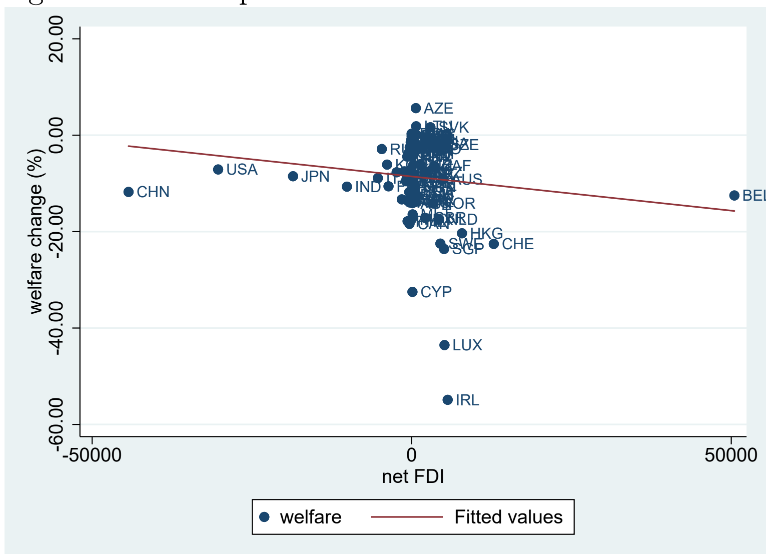
Notes: This figure plots the change in welfare in response to the elimination of FDI in the world against inward FDI in the baseline. See text for further details.

Figure 4: FDI Impact on Welfare and Outward FDI.



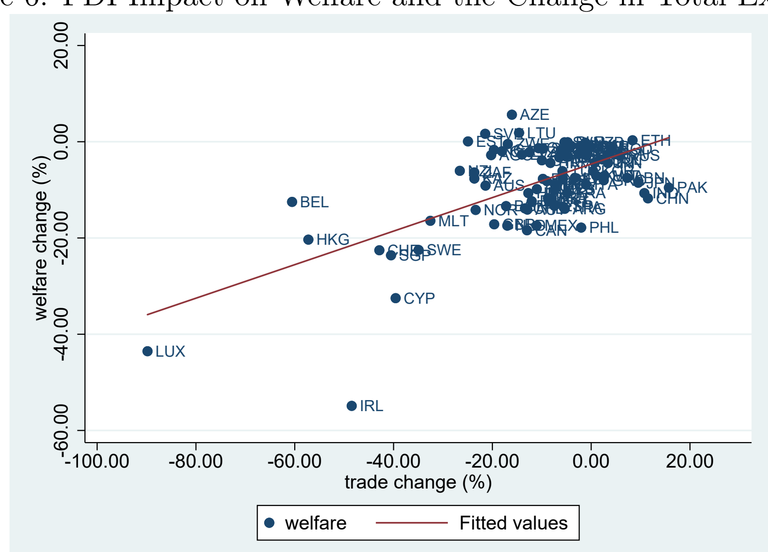
Notes: This figure plots the change in welfare in response to the elimination of FDI in the world against outward FDI in the baseline. See text for further details.

Figure 5: FDI Impact on Welfare and Net FDI Position.



Notes: This figure plots the change in welfare in response to the elimination of FDI in the world against the net FDI position. See text for further details.

Figure 6: FDI Impact on Welfare and the Change in Total Exports.



Notes: This figure plots the change in welfare in response to the elimination of FDI in the world against the change in exports. See text for further details.

Appendix

A Derivation of System of Equations (22)-(29)

This appendix gives derivation details for our system of equations (22)-(29).

First, let us re-state our production function as given in Equation (1):

$$Y_{j,t} = p_{j,t} A_{j,t} \left(L_{j,t}^{1-\alpha} K_{j,t}^{\alpha} \right)^{1-\phi} \left(\prod_{i=1}^{N} \left(\max\{1, \omega_{ij,t} M_{i,t}\} \right)^{\eta_i} \right)^{\phi} \quad \alpha, \phi \in (0,1). \tag{A1}$$

Note that we can write $\max\{1, \omega_{ij,t} M_{i,t}\} = (1 + \omega_{ij,t} M_{i,t} + |1 - \omega_{ij,t} M_{i,t}|)/2 = (1 + \omega_{ij,t} M_{i,t} + ((1 - \omega_{ij,t} M_{i,t})^2)^{1/2})/2$.[22] The derivative of $\max\{1, \omega_{ij,t} M_{i,t}\}$ with respect to $M_{i,t}$ is given by:[23]

$$\begin{aligned} &\left(\omega_{ij,t} - \frac{(1 - \omega_{ij,t} M_{i,t})}{((1 - \omega_{ij,t} M_{i,t})^2)^{1/2}} \omega_{ij,t} \right) /2 \\ &= \left(1 - \frac{(1 - \omega_{ij,t} M_{i,t})}{|1 - \omega_{ij,t} M_{i,t}|} \right) \omega_{ij,t}/2. \end{aligned} \tag{A2}$$

Using this definition of nominal output, the value marginal product of technology capital at home is given by:

$$\frac{\partial Y_{j,t}}{\partial M_{j,t}} = \frac{\phi \eta_j Y_{j,t}}{\max\{1, \omega_{jj,t} M_{j,t}\}} \left(1 - \frac{(1 - \omega_{jj,t} M_{j,t})}{|1 - \omega_{jj,t} M_{j,t}|} \right) \frac{\omega_{jj,t}}{2}, \tag{A3}$$

and the value marginal product of $M_{j,t}$ abroad by:

$$\frac{\partial Y_{i,t}}{\partial M_{j,t}} = \frac{\phi \eta_j Y_{i,t}}{\max\{1, \omega_{ji,t} M_{j,t}\}} \left(1 - \frac{(1 - \omega_{ji,t} M_{j,t})}{|1 - \omega_{ji,t} M_{j,t}|} \right) \frac{\omega_{ji,t}}{2}. \tag{A4}$$

Note that an alternative way of writing these two conditions is the following:

[22] See for example https://math.stackexchange.com/questions/1633071/absolute-value-and-max-min-function-why-a-b-a-b-2-maxa-b.

[23] See for example http://organicchemist.us/organichem/my_pagesabsvalder.html.

$$\frac{\partial Y_{j,t}}{\partial M_{j,t}} = \begin{cases} \frac{\phi\eta_j Y_{j,t}}{M_{j,t}} & \text{if} \quad \omega_{jj,t}M_{j,t} > 1, \\ 0 & \text{if} \quad \omega_{jj,t}M_{j,t} \leq 1. \end{cases}$$

$$\frac{\partial Y_{i,t}}{\partial M_{j,t}} = \begin{cases} \frac{\phi\eta_j Y_{i,t}}{M_{j,t}} & \text{if} \quad \omega_{ji,t}M_{j,t} > 1, \\ 0 & \text{if} \quad \omega_{ji,t}M_{j,t} \leq 1. \end{cases}$$

With this new expressions for the value marginal products, disposable income can be written as:

$$\begin{aligned} E_{j,t} = Y_{j,t} + \phi\eta_j M_{j,t} \sum_{i\neq j} \frac{Y_{i,t}}{\max\{1, \omega_{ji,t}M_{j,t}\}} \left(1 - \frac{(1-\omega_{ji,t}M_{j,t})}{|1-\omega_{ji,t}M_{j,t}|}\right) \frac{\omega_{ji,t}}{2} \\ - \phi Y_{j,t} \sum_{i\neq j} \frac{\eta_i M_{i,t}}{\max\{1, \omega_{ij,t}M_{i,t}\}} \left(1 - \frac{(1-\omega_{ij,t}M_{i,t})}{|1-\omega_{ij,t}M_{i,t}|}\right) \frac{\omega_{ij,t}}{2}, \end{aligned} \tag{A5}$$

which describes expenditure as the sum of total nominal output ($Y_{j,t}$) plus rents from foreign investments ($\sum_{i\neq j} M_{j,t} \times \frac{\partial Y_{i,t}}{\partial M_{j,t}}$), minus rents accruing to foreign investments ($\sum_{i\neq j} M_{i,t} \times \frac{\partial Y_{j,t}}{\partial M_{i,t}}$), which are part of nominal output. Rewriting a bit further, we end up with:

$$E_{j,t} = Y_{j,t} + \phi\eta_j \sum_{\substack{i\neq j, \\ \omega_{ji,t}M_{j,t}>1}} Y_{i,t} - \phi Y_{j,t} \sum_{\substack{i\neq j, \\ \omega_{ij,t}M_{i,t}>1}} \eta_i. \tag{A6}$$

In the next subsection, we first derive the solution of the dynamic problem. Afterward, we state the steady-state of the system.

A.1 Solving the 'Upper Level'

This section details the Lagrangian problem and the corresponding first-order conditions for the 'upper level' optimization problem leading to the structural dynamic system of trade, growth, and FDI.

We assume a log-intertemporal utility function:

$$U_{j,t} = \sum_{t=0}^{\infty} \beta^t \ln(C_{j,t}), \tag{A7}$$

and combine the budget constraint given by Equation (16) with the expenditure function given by Equation (A6):

$$\begin{aligned} P_{j,t}C_{j,t} + P_{j,t}\Omega_{j,t} + P_{j,t}\chi_{j,t} &= Y_{j,t} + \phi\eta_j \sum_{\substack{i \neq j, \\ \omega_{ji,t}M_{j,t} > 1}} Y_{i,t} - \phi Y_{j,t} \sum_{\substack{i \neq j, \\ \omega_{ij,t}M_{i,t} > 1}} \eta_i \\ &= \left(1 - \phi \sum_{\substack{i \neq j, \\ \omega_{ij,t}M_{i,t} > 1}} \eta_i \right) Y_{j,t} + \phi\eta_j \sum_{\substack{i \neq j, \\ \omega_{ji,t}M_{j,t} > 1}} Y_{i,t}. \end{aligned}$$

Further, we replace $Y_{j,t}$ with the production function as formulated in Equation (A1), leading to:

$$P_{j,t}C_{j,t}+P_{j,t}\Omega_{j,t}+P_{j,t}\chi_{j,t}=$$

$$\left(1-\phi\sum_{\substack{i\neq j,\\ \omega_{ij,t}M_{i,t}>1}}\eta_i\right)p_{j,t}A_{j,t}\left(L_{j,t}^{1-\alpha}K_{j,t}^{\alpha}\right)^{1-\phi}\left(\prod_{i=1}^{N}\left(\max\{1,\omega_{ij,t}M_{i,t}\}\right)^{\eta_i}\right)^{\phi}$$

$$+\phi\eta_j\sum_{\substack{i\neq j,\\ \omega_{ji,t}M_{j,t}>1}}p_{i,t}A_{i,t}\left(L_{i,t}^{1-\alpha}K_{i,t}^{\alpha}\right)^{1-\phi}\left(\prod_{k=1}^{N}\left(\max\{1,\omega_{ki,t}M_{k,t}\}\right)^{\eta_k}\right)^{\phi}.$$

In order to end up with only one constraint, we also replace $\Omega_{j,t}$ and $\chi_{j,t}$ by using:

$$\Omega_{j,t}=K_{j,t+1}-\left(1-\delta_K\right)K_{j,t},$$

$$\chi_{j,t}=M_{j,t+1}-\left(1-\delta_M\right)M_{j,t},$$

leading to the following budget constraint:

$$P_{j,t}C_{j,t}+P_{j,t}\left(K_{j,t+1}-\left(1-\delta_K\right)K_{j,t}\right)+P_{j,t}\left(M_{j,t+1}-\left(1-\delta_M\right)M_{j,t}\right)=$$

$$(1-\phi\sum_{\substack{i\neq j,\\ \omega_{ij,t}M_{i,t}>1}}\eta_i)p_{j,t}A_{j,t}\left(L_{j,t}^{1-\alpha}K_{j,t}^{\alpha}\right)^{1-\phi}\left(\prod_{i=1}^{N}\left(\max\{1,\omega_{ij,t}M_{i,t}\}\right)^{\eta_i}\right)^{\phi}$$

$$+\phi\eta_j\sum_{\substack{i\neq j,\\ \omega_{ji,t}M_{j,t}>1}}p_{i,t}A_{i,t}\left(L_{i,t}^{1-\alpha}K_{i,t}^{\alpha}\right)^{1-\phi}\left(\prod_{k=1}^{N}\left(\max\{1,\omega_{ki,t}M_{k,t}\}\right)^{\eta_k}\right)^{\phi}.$$

The corresponding expression for the Lagrangian is:

$$
\begin{aligned}
\mathcal{L}_j \;=\; & \sum_{t=0}^{\infty} \beta^t \Big[\ln(C_{j,t}) + \lambda_{j,t} \Big((1 - \phi \sum_{\substack{i \neq j, \\ \omega_{ij,t} M_{i,t} > 1}} \eta_i) p_{j,t} A_{j,t} \left(L_{j,t}^{1-\alpha} K_{j,t}^{\alpha} \right)^{1-\phi} \\
& \times \left(\prod_{i=1}^{N} (\max\{1, \omega_{ij,t} M_{i,t}\})^{\eta_i} \right)^{\phi} \\
& + \phi \eta_j \sum_{\substack{i \neq j, \\ \omega_{ji,t} M_{j,t} > 1}} p_{i,t} A_{i,t} \left(L_{i,t}^{1-\alpha} K_{i,t}^{\alpha} \right)^{1-\phi} \left(\prod_{k=1}^{N} (\max\{1, \omega_{ki,t} M_{k,t}\})^{\eta_k} \right)^{\phi} \\
& - P_{j,t} C_{j,t} - P_{j,t} \left(K_{j,t+1} - (1 - \delta_K) K_{j,t} \right) - P_{j,t} \left(M_{j,t+1} - (1 - \delta_M) M_{j,t} \right) \Big) \Big].
\end{aligned}
$$

Take derivatives with respect to $C_{j,t}$, $K_{j,t+1}$, $M_{j,t+1}$ and $\lambda_{j,t}$ to obtain the following set of first-order conditions:

$$
\frac{\partial \mathcal{L}_j}{\partial C_{j,t}} = \frac{\beta^t}{C_{j,t}} - \beta^t \lambda_{j,t} P_{j,t} \overset{!}{=} 0 \quad \text{for all } j \text{ and } t. \tag{A8}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}_j}{\partial K_{j,t+1}} = \; & \beta^{t+1} \lambda_{j,t+1} (1 - \phi \sum_{\substack{i \neq j, \\ \omega_{ij,t+1} M_{i,t+1} > 1}} \eta_i)(1 - \phi) \alpha \frac{Y_{j,t+1}}{K_{j,t+1}} \\
& - \beta^t \lambda_{j,t} P_{j,t} \\
& + \beta^{t+1} \lambda_{j,t+1} P_{j,t+1} (1 - \delta_K) \overset{!}{=} 0 \quad \text{for all } j \text{ and } t.
\end{aligned} \tag{A9}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}_j}{\partial M_{j,t+1}} = \beta^{t+1}\lambda_{j,t+1}(1-\phi & \sum_{\substack{i \neq j, \\ \omega_{ij,t+1}M_{i,t+1}>1}} \eta_i)\frac{\phi\eta_j Y_{j,t+1}}{\max\{1,\omega_{jj,t+1}M_{j,t+1}\}} \\
& \times \left(1-\frac{(1-\omega_{jj,t+1}M_{j,t+1})}{|1-\omega_{jj,t+1}M_{j,t+1}|}\right)\frac{\omega_{jj,t+1}}{2} \qquad \text{(A10)} \\
& + \beta^{t+1}\lambda_{j,t+1}\phi\eta_j \\
& \times \sum_{\substack{i \neq j, \\ \omega_{ji,t+1}M_{j,t+1}>1}} \frac{\phi\eta_j Y_{i,t+1}}{\max\{1,\omega_{ji,t+1}M_{j,t+1}\}}\left(1-\frac{(1-\omega_{ji,t+1}M_{j,t+1})}{|1-\omega_{ji,t+1}M_{j,t+1}|}\right)\frac{\omega_{ji,t+1}}{2} \\
& \qquad \text{(A11)} \\
& - \beta^{t}\lambda_{j,t}P_{j,t} \\
& + \beta^{t+1}\lambda_{j,t+1}P_{j,t+1}\left(1-\delta_M\right) \overset{!}{=} 0 \quad \text{for all } j \text{ and } t. \qquad \text{(A12)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}_j}{\partial \lambda_{j,t}} = & (1-\phi \sum_{\substack{i \neq j, \\ \omega_{ij,t}M_{i,t}>1}} \eta_i)p_{j,t}A_{j,t}\left(L_{j,t}^{1-\alpha}K_{j,t}^{\alpha}\right)^{1-\phi}\left(\prod_{i=1}^{N}\left(\max\{1,\omega_{ij,t}M_{i,t}\}\right)^{\eta_i}\right)^{\phi} \\
& +\phi\eta_j \sum_{\substack{i \neq j, \\ \omega_{ji,t}M_{j,t}>1}} p_{i,t}A_{i,t}\left(L_{i,t}^{1-\alpha}K_{i,t}^{\alpha}\right)^{1-\phi}\left(\prod_{k=1}^{N}\left(\max\{1,\omega_{ki,t}M_{k,t}\}\right)^{\eta_k}\right)^{\phi} \\
& -P_{j,t}C_{j,t}-P_{j,t}\left(K_{j,t+1}-\left(1-\delta_K\right)K_{j,t}\right)-P_{j,t}\left(M_{j,t+1}-\left(1-\delta_M\right)M_{j,t}\right) \\
& \overset{!}{=} 0 \quad \text{for all } j \text{ and } t. \qquad \text{(A13)}
\end{aligned}
$$

Use the first-order condition for consumption to express $\lambda_{j,t}$ as:

$$
\lambda_{j,t} = \frac{1}{C_{j,t}P_{j,t}}. \qquad \text{(A14)}
$$

Replace this in the first-order condition for physical capital:

$$\begin{aligned}\frac{\partial \mathcal{L}_j}{\partial K_{j,t+1}} &= \beta^{t+1}\frac{1}{C_{j,t+1}P_{j,t+1}}(1-\phi\sum_{\substack{i \neq j,\\ \omega_{ij,t+1}M_{i,t+1}>1}}\eta_i)(1-\phi)\alpha\frac{Y_{j,t+1}}{K_{j,t+1}} \\ &\quad -\beta^t\frac{P_{j,t}}{C_{j,t}P_{j,t}} \\ &\quad +\beta^{t+1}\frac{P_{j,t+1}}{C_{j,t+1}P_{j,t+1}}\left(1-\delta_K\right) \overset{!}{=} 0 \quad \text{for all } j \text{ and } t. \end{aligned} \tag{A15}$$

Simplify and re-arrange to obtain:

$$\begin{aligned}\beta(1-\phi\sum_{\substack{i \neq j,\\ \omega_{ij,t+1}M_{i,t+1}>1}}\eta_i)\alpha(1-\phi)\frac{Y_{j,t+1}}{K_{j,t+1}} - \frac{C_{j,t+1}P_{j,t+1}}{C_{j,t}} &= \\ -\beta\left(1-\delta_K\right)P_{j,t+1} \quad \text{for all } j \text{ and } t.\end{aligned} \tag{A16}$$

Now replace λ_j with the expression from the first-order condition for consumption given in

Equation (A14) in the first-order condition for technology capital given in Equation (A12):

$$\begin{aligned}
\frac{\partial \mathcal{L}_j}{\partial M_{j,t+1}} = \; & \beta^{t+1} \frac{1}{C_{j,t+1} P_{j,t+1}} \Big(1 - \phi \sum_{\substack{i \neq j, \\ \omega_{ij,t+1} M_{i,t+1} > 1}} \eta_i\Big) \\
& \times \frac{\phi \eta_j Y_{j,t+1}}{\max\{1, \omega_{jj,t+1} M_{j,t+1}\}} \left(1 - \frac{(1-\omega_{jj,t+1} M_{j,t+1})}{|1-\omega_{jj,t+1} M_{j,t+1}|}\right) \frac{\omega_{jj,t+1}}{2} \\
& + \beta^{t+1} \frac{1}{C_{j,t+1} P_{j,t+1}} \phi \eta_j \\
& \times \sum_{\substack{i \neq j, \\ \omega_{ji,t+1} M_{j,t+1} > 1}} \frac{\phi \eta_j Y_{i,t+1}}{\max\{1, \omega_{ji,t+1} M_{j,t+1}\}} \left(1 - \frac{(1-\omega_{ji,t+1} M_{j,t+1})}{|1-\omega_{ji,t+1} M_{j,t+1}|}\right) \frac{\omega_{ji,t+1}}{2} \\
& - \beta^t \frac{1}{C_{j,t}} \\
& + \frac{\beta^{t+1}}{C_{j,t+1}} (1-\delta_M) \stackrel{!}{=} 0 \quad \text{for all } j \text{ and } t. \qquad \text{(A17)}
\end{aligned}$$

Simplify and re-arrange to obtain:

$$\begin{aligned}
& \beta \Big(1 - \phi \sum_{\substack{i \neq j, \\ \omega_{ij,t+1} M_{i,t+1} > 1}} \eta_i\Big) \frac{\phi \eta_j Y_{j,t+1}}{\max\{1, \omega_{jj,t+1} M_{j,t+1}\}} \left(1 - \frac{(1-\omega_{jj,t+1} M_{j,t+1})}{|1-\omega_{jj,t+1} M_{j,t+1}|}\right) \frac{\omega_{jj,t+1}}{2} \\
& + \beta \phi \eta_j \sum_{\substack{i \neq j, \\ \omega_{ji,t+1} M_{j,t+1} > 1}} \frac{\phi \eta_j Y_{i,t+1}}{\max\{1, \omega_{ji,t+1} M_{j,t+1}\}} \left(1 - \frac{(1-\omega_{ji,t+1} M_{j,t+1})}{|1-\omega_{ji,t+1} M_{j,t+1}|}\right) \frac{\omega_{ji,t+1}}{2} \\
& - \frac{C_{j,t+1} P_{j,t+1}}{C_{j,t}} = \\
& \beta (1-\delta_M) P_{j,t+1} \quad \text{for all } j \text{ and } t. \qquad \text{(A18)}
\end{aligned}$$

Assuming that for sure $\omega_{jj,t+1} M_{j,t+1} > 1$, i.e., technology stock at home is positive and frictions small, we may simplify as follows:

$$\beta(1-\phi \sum_{\substack{i \neq j, \\ \omega_{ij,t+1} M_{i,t+1} > 1}} \eta_i)\frac{\phi \eta_j Y_{j,t+1}}{M_{j,t+1}} + \beta \phi \eta_j \sum_{\substack{i \neq j, \\ \omega_{ji,t+1} M_{j,t+1} > 1}} \frac{\phi \eta_j Y_{i,t+1}}{M_{j,t+1}} - \frac{C_{j,t+1} P_{j,t+1}}{C_{j,t}} = -\beta \left(1-\delta_M\right) P_{j,t+1} \quad \text{for all } j \text{ and } t. \qquad \text{(A19)}$$

Combining the production function given by Equation (A1), the budget constraint given by Equation (16), the expression for $E_{j,t}$ given in Equation (A6), the expressions for $p_{j,t}$ for each t from Equation (25), and the equations for the trade MRTs $P_{j,t}$ and $\Pi_{j,t}$ given by Equations (20) and (21), respectively, with the two first order conditions for $K_{j,t+1}$ and $M_{j,t+1}$ as given by Equations (A16) and (A18), respectively, we end up with the following system:

$$Y_{j,t} = p_{j,t} A_{j,t} \left(L_{j,t}^{1-\alpha} K_{j,t}^{\alpha}\right)^{1-\phi} \left(\prod_{i=1}^{N} \left(\max\{1, \omega_{ij,t} M_{i,t}\}\right)^{\eta_i}\right)^{\phi} \quad \text{for all } j \text{ and } t, \tag{A20}$$

$$\begin{aligned} E_{j,t} &= P_{j,t} C_{j,t} + P_{j,t} \left(K_{j,t+1} - (1-\delta_K) K_{j,t}\right) \\ &+ P_{j,t} \left(M_{j,t+1} - (1-\delta_M) M_{j,t}\right) \quad \text{for all } j \text{ and } t, \end{aligned} \tag{A21}$$

$$E_{j,t} = Y_{j,t} + \phi \eta_j \sum_{\substack{i \neq j, \\ \omega_{ji,t} M_{j,t} > 1}} Y_{i,t} - \phi Y_{j,t} \sum_{\substack{i \neq j, \\ \omega_{ij,t} M_{i,t} > 1}} \eta_i \quad \text{for all } j \text{ and } t, \tag{A22}$$

$$p_{j,t} = \frac{\left(Y_{j,t}/Y_t\right)^{\frac{1}{1-\sigma}}}{\gamma_j \Pi_{j,t}} \quad \text{for all } j \text{ and } t, \tag{A23}$$

$$Y_t = \sum_{j=1}^{N} Y_{j,t} \quad \text{for all } t, \tag{A24}$$

$$P_{j,t}^{1-\sigma} = \sum_{i=1}^{N} \left(\frac{t_{ij,t}}{\Pi_{i,t}}\right)^{1-\sigma} \frac{Y_{i,t}}{Y_t} \quad \text{for all } j \text{ and } t, \tag{A25}$$

$$\Pi_{i,t}^{1-\sigma} = \sum_{j=1}^{N} \left(\frac{t_{ij,t}}{P_{j,t}}\right)^{1-\sigma} \frac{E_{j,t}}{Y_t} \quad \text{for all } i \text{ and } t, \tag{A26}$$

$$\begin{aligned} &\beta(1 - \phi \sum_{\substack{i \neq j, \\ \omega_{ij,t+1} M_{i,t+1} > 1}} \eta_i) \alpha (1-\phi) \frac{Y_{j,t+1}}{K_{j,t+1}} - \frac{C_{j,t+1} P_{j,t+1}}{C_{j,t}} = \\ &\beta \left(\delta_K - 1\right) P_{j,t+1} \quad \text{for all } j \text{ and } t. \end{aligned} \tag{A27}$$

$$\begin{aligned} &\beta(1 - \phi \sum_{\substack{i \neq j, \\ \omega_{ij,t+1} M_{i,t+1} > 1}} \eta_i) \frac{\phi \eta_j Y_{j,t+1}}{M_{j,t+1}} + \beta \phi \eta_j \sum_{\substack{i \neq j, \\ \omega_{ji,t+1} M_{j,t+1} > 1}} \frac{\phi \eta_j Y_{i,t+1}}{M_{j,t+1}} \\ &- \frac{C_{j,t+1} P_{j,t+1}}{C_{j,t}} = \beta \left(\delta_M - 1\right) P_{j,t+1} \quad \text{for all } j \text{ and } t. \end{aligned} \tag{A28}$$

This is a system of $(8 \times N + 1) \times T$ equations in the $(8 \times N + 1) \times T$ unknowns $C_{j,t}$, $K_{j,t}$,

$M_{j,t}$, $Y_{j,t}$, Y_t, $p_{j,t}$, $P_{j,t}$, $\Pi_{j,t}$, $E_{j,t}$ and given parameters and exogenous variables $A_{j,t}$, $\omega_{ij,t}$, $L_{j,t}$, α, β, ϕ, η_j, γ_j, σ, $t_{ij,t}$, δ_K, and δ_M.

A.2 Derivation of the Steady-State

In steady-state, values for $t+1$ and t have to be equal. Hence, we can express physical and technology capital as:

$$K_j = \frac{\Omega_j}{\delta_K}, \tag{A29}$$

$$M_j = \frac{\chi_j}{\delta_M}. \tag{A30}$$

Further, we can drop the time index for all variables. Let us first drop time indices and use $K_j = \Omega_j/\delta_K$ and $M_j = \chi_j/\delta_M$ in the first-order condition for physical capital as given in Equation (A27):

$$\beta(1-\phi \sum_{\substack{i \neq j, \\ \omega_{ij,t+1}M_{i,t+1} > 1}} \eta_i)\alpha(1-\phi)\frac{Y_j}{K_j} - \frac{C_j P_j}{C_j} = \beta\left(\delta_K - 1\right) P_j \quad \text{for all } j. \Rightarrow$$

$$\beta(1-\phi \sum_{\substack{i \neq j, \\ \omega_{ij,t+1}M_{i,t+1} > 1}} \eta_i)\alpha(1-\phi)\frac{Y_j}{P_j K_j} - 1 = \beta\left(\delta_K - 1\right) \quad \text{for all } j. \Rightarrow$$

$$\beta(1-\phi \sum_{\substack{i \neq j, \\ \omega_{ij,t+1}M_{i,t+1} > 1}} \eta_i)\alpha(1-\phi)/(1-\beta+\beta\delta_K)\frac{Y_j}{P_j} = K_j \quad \text{for all } j.$$

Let us next drop time indices and use $K_j = \Omega_j$ and $M_j = \chi_j$ in the first-order condition for technology capital as given in Equation (A28):

$$\beta(1-\phi \sum_{\substack{i \neq j, \\ \omega_{ij} M_i > 1}} \eta_i)\frac{\phi\eta_j Y_j}{M_j} + \beta\phi\eta_j \sum_{\substack{i \neq j, \\ \omega_{ji} M_j > 1}} \frac{\phi\eta_j Y_i}{M_j} - \frac{C_j P_j}{C_j} = \beta\left(\delta_M - 1\right) P_j \quad \text{for all } j \Rightarrow$$

$$\beta(1-\phi \sum_{\substack{i \neq j, \\ \omega_{ij} M_i > 1}} \eta_i)\frac{\phi\eta_j Y_j}{P_j M_j} + \beta\phi\eta_j \sum_{\substack{i \neq j, \\ \omega_{ji} M_j > 1}} \frac{\phi\eta_j Y_i}{P_j M_j} - 1 = \beta\left(\delta_M - 1\right) \quad \text{for all } j \Rightarrow$$

$$\frac{\beta\phi\eta_j}{1-\beta+\beta\delta_M}((1-\phi \sum_{\substack{i \neq j, \\ \omega_{ij} M_i > 1}} \eta_i)\frac{Y_j}{P_j} + \sum_{\substack{i \neq j, \\ \omega_{ji} M_j > 1}} \frac{\phi\eta_j Y_i}{P_j}) = M_j \quad \text{for all } j \Rightarrow$$

$$\frac{\beta\phi\eta_j}{1-\beta+\beta\delta_M}\frac{E_j}{P_j} = M_j \quad \text{for all } j \Rightarrow$$

Hence, the equation system given by Equations (A20)-(A28) simplifies to:

$$Y_j = p_j A_j \left(L_j^{1-\alpha} K_j^{\alpha}\right)^{1-\phi} \left(\prod_{i=1}^{N} (\max\{1, \omega_{ij} M_i\})^{\eta_i}\right)^{\phi} \quad \text{for all } j, \tag{A31}$$

$$E_j = P_j C_j + P_j K_j + P_j M_j \quad \text{for all } j, \tag{A32}$$

$$E_j = Y_j + \phi \eta_j \sum_{\substack{i \neq j, \\ \omega_{ji} M_j > 1}} Y_i - \phi Y_j \sum_{\substack{i \neq j, \\ \omega_{ij} M_i > 1}} \eta_i \quad \text{for all } j, \tag{A33}$$

$$p_j = \frac{(Y_j/Y)^{\frac{1}{1-\sigma}}}{\gamma_j \Pi_j} \quad \text{for all } j, \tag{A34}$$

$$Y = \sum_{j=1}^{N} Y_j, \tag{A35}$$

$$P_j^{1-\sigma} = \sum_{i=1}^{N} \left(\frac{t_{ij}}{\Pi_i}\right)^{1-\sigma} \frac{Y_i}{Y} \quad \text{for all } j, \tag{A36}$$

$$\Pi_i^{1-\sigma} = \sum_{j=1}^{N} \left(\frac{t_{ij}}{P_j}\right)^{1-\sigma} \frac{E_j}{Y} \quad \text{for all } i, \tag{A37}$$

$$K_j = \beta(1 - \phi \sum_{\substack{i \neq j, \\ \omega_{ij,t+1} M_{i,t+1} > 1}} \eta_i) \alpha (1-\phi)/(1 - \beta + \beta \delta_K) \frac{Y_j}{P_j} \quad \text{for all } j, \tag{A38}$$

$$M_j = \frac{\beta \phi \eta_j}{1 - \beta + \beta \delta_M} \frac{E_j}{P_j} \quad \text{for all } j. \tag{A39}$$

Note that trade flows in steady-state are then given by $X_{ij} = \frac{Y_i E_j}{Y} \left(\frac{t_{ij}}{\Pi_i P_j}\right)^{1-\sigma}$.

The steady state system above yields a convenient gravity representation of FDI that is remarkably similar to the familiar trade gravity system. To obtain it, recall the (steady-state) definition of bilateral FDI stock:

$$FDI_{ij} \equiv \omega_{ij} M_i. \tag{A40}$$

Use the steady-state solution for technology capital M_j from Equation (A39):

$$M_j = \frac{\beta\phi\eta_j}{1-\beta+\beta\delta_M}\frac{E_j}{P_j}. \tag{A41}$$

Substitute for M_j in Equation (A40) to obtain:

$$FDI_{ij} = \omega_{ij}\frac{\beta\phi\eta_i}{1-\beta+\beta\delta_M}\frac{E_i}{P_i}. \tag{A42}$$

Equation (A42) resembles a gravity equation, except that it describes physical FDI stocks. To translate (A42) into a stock value FDI equation needed for estimation with data on FDI stock *values*, define the value of FDI from country i to country j as the product of the FDI stock times its value marginal product:

$$\begin{aligned} FDI_{ij}^{value} &\equiv FDI_{ij} \times \frac{\partial Y_j}{\partial M_i} && \text{(A43)}\\ &= \omega_{ij}\frac{\beta\phi\eta_i}{1-\beta+\beta\delta_M}\frac{E_i}{P_i}\phi\eta_i\frac{Y_j}{M_i} && \text{(A44)}\\ &= \frac{\beta\phi^2\eta_i^2}{1-\beta+\beta\delta_M}\omega_{ij}\frac{E_i}{P_i}\frac{Y_j}{M_i}. && \text{(A45)} \end{aligned}$$

Combine Equation (A42) with the definitions of the multilateral resistance terms P_j and Π_j given by Equations (20) and (21), respectively, to obtain the following FDI gravity system:

$$FDI_{ij}^{value} = \frac{\beta\phi^2\eta_i^2}{1-\beta+\beta\delta_M}\omega_{ij}\frac{E_i}{P_i}\frac{Y_j}{M_i}, \tag{A46}$$

$$P_i = \left[\sum_{j=1}^{N}\left(\frac{t_{ji}}{\Pi_j}\right)^{1-\sigma}\frac{Y_j}{Y}\right]^{\frac{1}{1-\sigma}}, \tag{A47}$$

$$\Pi_j = \left[\sum_{i=1}^{N}\left(\frac{t_{ji}}{P_i}\right)^{1-\sigma}\frac{E_i}{Y}\right]^{\frac{1}{1-\sigma}}. \tag{A48}$$

B Sum as Functional Form for FDI Aggregation

Our production function as given in Equation (1) combined technology capital across all countries of the world via a max-Cobb-Douglas function. The original formulation by McGrattan and Prescott (2009, 2010) summed over all technology capital stock. In this section, we will explore the implications of this alternative specification.

Specifically, the production function is now assumed to be given by:

$$Y_{j,t} = p_{j,t}A_{j,t}\left(L_{j,t}^{1-\alpha}K_{j,t}^{\alpha}\right)^{1-\phi}\left(\sum_{i=1}^{N}\omega_{ij,t}M_{i,t}\right)^{\phi} \quad \alpha,\phi\in(0,1). \tag{A49}$$

Using this definition of nominal output, the value marginal product of technology capital at home is given by:

$$\frac{\partial Y_{j,t}}{\partial M_{j,t}} = \phi\omega_{jj,t}\frac{Y_{j,t}}{\sum_{i=1}^{N}\omega_{ij,t}M_{i,t}}, \tag{A50}$$

and the value marginal product of $M_{j,t}$ abroad by:

$$\frac{\partial Y_{i,t}}{\partial M_{j,t}} = \phi\omega_{ji,t}\frac{Y_{i,t}}{\sum_{k=1}^{N}\omega_{ki,t}M_{k,t}}. \tag{A51}$$

With this new expressions for the value marginal products, disposable income reads as follows:

$$E_{j,t} = Y_{j,t} + \phi M_{j,t}\sum_{i\neq j}\left(\frac{\omega_{ji,t}Y_{i,t}}{\sum_{k=1}^{N}\omega_{ki,t}M_{k,t}}\right) - \phi\frac{Y_{j,t}}{\sum_{k=1}^{N}\omega_{kj,t}M_{k,t}}\sum_{i\neq j}\omega_{ij,t}M_{i,t}, \tag{A52}$$

which describes expenditure as the sum of total nominal output ($Y_{j,t}$) plus rents from foreign investments ($\sum_{i\neq j}M_{j,t}\times\frac{\partial Y_{i,t}}{\partial M_{j,t}} = \sum_{i\neq j}M_{j,t}\phi\omega_{ji,t}\frac{Y_{i,t}}{\sum_{k=1}^{N}\omega_{ki,t}M_{k,t}} = \phi M_{j,t}\sum_{i\neq j}(\frac{\omega_{ji,t}Y_{i,t}}{\sum_{k=1}^{N}\omega_{ki,t}M_{k,t}})$), minus rents accruing to foreign investments ($\sum_{i\neq j}M_{i,t}\times\frac{\partial Y_{j,t}}{\partial M_{i,t}} = \sum_{i\neq j}M_{i,t}\phi\omega_{ij,t}\frac{Y_{j,t}}{\sum_{k=1}^{N}\omega_{kj,t}M_{k,t}} = \phi\frac{Y_{j,t}}{\sum_{k=1}^{N}\omega_{kj,t}M_{k,t}}\sum_{i\neq j}\omega_{ij,t}M_{i,t}$), which are part of nominal output.

All other assumptions are unchanged.

B.1 First-Order Conditions

The next step is to look at the Lagrangian and the corresponding first-order conditions. Specifically, we again assume a log-linear intertemporal utility function:

$$U_{j,t} = \sum_{t=0}^{\infty} \beta^t \ln(C_{j,t}),$$

and combine the budget constraint given by Equation (16) (not replacing investment) with the expenditure function given by Equation (A52):

$$\begin{aligned} P_{j,t}C_{j,t} + P_{j,t}\Omega_{j,t} + P_{j,t}\chi_{j,t} = Y_{j,t} & \left(1 - \frac{\phi}{\sum_{k=1}^{N} \omega_{kj,t} M_{k,t}} \sum_{i \neq j} \omega_{ij,t} M_{i,t}\right) \\ & + \phi M_{j,t} \sum_{i \neq j} \left(\frac{\omega_{ji,t} Y_{i,t}}{\sum_{k=1}^{N} \omega_{ki,t} M_{k,t}}\right). \end{aligned}$$

Further, we replace $Y_{j,t}$ with the production function as formulated in Equation (A49), leading to:

$$\begin{aligned} P_{j,t}C_{j,t} + P_{j,t}\Omega_{j,t} + P_{j,t}\chi_{j,t} = p_{j,t} A_{j,t} \left(L_{j,t}^{1-\alpha} K_{j,t}^{\alpha}\right)^{1-\phi} & \left(\sum_{i=1}^{N} \omega_{ij,t} M_{i,t}\right)^{\phi} \times \\ & \left(1 - \frac{\phi}{\sum_{k=1}^{N} \omega_{kj,t} M_{k,t}} \sum_{i \neq j} \omega_{ij,t} M_{i,t}\right) \\ & + \phi M_{j,t} \sum_{i \neq j} \left(\frac{\omega_{ji,t} p_{i,t} A_{i,t} \left(L_{i,t}^{1-\alpha} K_{i,t}^{\alpha}\right)^{1-\phi} \left(\sum_{k=1}^{N} \omega_{ki,t} M_{k,t}\right)^{\phi}}{\sum_{k=1}^{N} \omega_{ki,t} M_{k,t}}\right). \end{aligned}$$

In order to end up with only one constraint, we also replace $\Omega_{j,t}$ and $\chi_{j,t}$ by:

$$\Omega_{j,t} = K_{j,t+1} - (1 - \delta_K) K_{j,t},$$

$$\chi_{j,t} = M_{j,t+1} - (1 - \delta_M) M_{j,t},$$

leading to the following budget constraint:

$$
\begin{aligned}
P_{j,t}C_{j,t} + P_{j,t}\left(K_{j,t+1} - (1-\delta_K)K_{j,t}\right) + P_{j,t}\left(M_{j,t+1} - (1-\delta_M)M_{j,t}\right) = \\
p_{j,t}A_{j,t}\left(L_{j,t}^{1-\alpha}K_{j,t}^{\alpha}\right)^{1-\phi}\left(\sum_{i=1}^{N}\omega_{ij,t}M_{i,t}\right)^{\phi} \times \\
\left(1 - \frac{\phi}{\sum_{k=1}^{N}\omega_{kj,t}M_{k,t}}\sum_{i\neq j}\omega_{ij,t}M_{i,t}\right) \\
+\phi M_{j,t}\sum_{i\neq j}\left(\frac{\omega_{ji,t}p_{i,t}A_{i,t}\left(L_{i,t}^{1-\alpha}K_{i,t}^{\alpha}\right)^{1-\phi}\left(\sum_{k=1}^{N}\omega_{ki,t}M_{k,t}\right)^{\phi}}{\sum_{k=1}^{N}\omega_{ki,t}M_{k,t}}\right).
\end{aligned}
$$

The corresponding expression for the Lagrangian is:

$$
\begin{aligned}
\mathcal{L}_j &= \sum_{t=0}^{\infty}\beta^t\Big[\ln(C_{j,t}) + \lambda_{j,t}\Big(p_{j,t}A_{j,t}\left(L_{j,t}^{1-\alpha}K_{j,t}^{\alpha}\right)^{1-\phi}\left(\sum_{i=1}^{N}\omega_{ij,t}M_{i,t}\right)^{\phi} \\
&\quad \times\left(1 - \frac{\phi}{\sum_{k=1}^{N}\omega_{kj,t}M_{k,t}}\sum_{i\neq j}\omega_{ij,t}M_{i,t}\right) \\
&\quad +\phi M_{j,t}\sum_{i\neq j}\left(\frac{\omega_{ji,t}p_{i,t}A_{i,t}\left(L_{i,t}^{1-\alpha}K_{i,t}^{\alpha}\right)^{1-\phi}\left(\sum_{k=1}^{N}\omega_{ki,t}M_{k,t}\right)^{\phi}}{\sum_{k=1}^{N}\omega_{ki,t}M_{k,t}}\right) \\
&\quad -P_{j,t}C_{j,t} - P_{j,t}\left(K_{j,t+1} - (1-\delta_K)K_{j,t}\right) - P_{j,t}\left(M_{j,t+1} - (1-\delta_M)M_{j,t}\right)\Big)\Big].
\end{aligned}
$$

Take derivatives with respect to $C_{j,t}$, $K_{j,t+1}$, $M_{j,t+1}$ and $\lambda_{j,t}$ to obtain the following set of first-order conditions:

$$\frac{\partial \mathcal{L}_j}{\partial C_{j,t}} = \frac{\beta^t}{C_{j,t}} - \beta^t \lambda_{j,t} P_{j,t} \overset{!}{=} 0 \quad \text{for all } j \text{ and } t. \tag{A53}$$

$$\begin{aligned} \frac{\partial \mathcal{L}_j}{\partial K_{j,t+1}} = \; & \beta^{t+1} \lambda_{j,t+1} p_{j,t+1} A_{j,t+1} \left(\sum_{i=1}^{N} \omega_{ij,t+1} M_{i,t+1} \right)^{\phi} \\ & \left(1 - \frac{\phi}{\sum_{k=1}^{N} \omega_{kj,t+1} M_{k,t+1}} \sum_{i \neq j} \omega_{ij,t+1} M_{i,t+1} \right) L_{j,t+1}^{(1-\alpha)(1-\phi)} \alpha (1-\phi) K_{j,t+1}^{\alpha(1-\phi)-1} \\ & - \beta^t \lambda_{j,t} P_{j,t} \\ & + \beta^{t+1} \lambda_{j,t+1} \left(1 - \delta_K\right) P_{j,t+1} \overset{!}{=} 0 \quad \text{for all } j \text{ and } t. \end{aligned} \tag{A54}$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}_j}{\partial M_{j,t+1}} &= \beta^{t+1}\lambda_{j,t+1}\omega_{jj,t+1}p_{j,t+1}A_{j,t+1}\left(L_{j,t+1}^{1-\alpha}K_{j,t+1}^{\alpha}\right)^{1-\phi}\phi \\
&\left(1-\frac{\phi}{\sum_{k=1}^{N}\omega_{kj,t+1}M_{k,t+1}}\sum_{i\neq j}\omega_{ij,t+1}M_{i,t+1}\right)\left(\sum_{i=1}^{N}\omega_{ij,t+1}M_{i,t+1}\right)^{\phi-1} \\
&+\beta^{t+1}\lambda_{j,t+1}\omega_{jj,t+1}p_{j,t+1}A_{j,t+1}\left(L_{j,t+1}^{1-\alpha}K_{j,t+1}^{\alpha}\right)^{1-\phi}\left(\sum_{i=1}^{N}\omega_{ij,t+1}M_{i,t+1}\right)^{\phi} \\
&\times\left(\frac{\phi}{\left(\sum_{k=1}^{N}\omega_{kj,t+1}M_{k,t+1}\right)^2}\sum_{i\neq j}\omega_{ij,t+1}M_{i,t+1}\right) \\
&+\beta^{t+1}\lambda_{j,t+1}\phi\sum_{i\neq j}\left(\frac{\omega_{ji,t+1}p_{i,t+1}A_{i,t+1}\left(L_{i,t+1}^{1-\alpha}K_{i,t+1}^{\alpha}\right)^{1-\phi}\left(\sum_{k=1}^{N}\omega_{ki,t+1}M_{k,t+1}\right)^{\phi}}{\sum_{k=1}^{N}\omega_{ki,t+1}M_{k,t+1}}\right) \\
&+\beta^{t+1}\lambda_{j,t+1}\phi M_{j,t+1} \\
&\times\sum_{i\neq j}\left(\frac{\omega_{ji,t+1}^2p_{i,t+1}A_{i,t+1}\left(L_{i,t+1}^{1-\alpha}K_{i,t+1}^{\alpha}\right)^{1-\phi}\phi\left(\sum_{k=1}^{N}\omega_{ki,t+1}M_{k,t+1}\right)^{\phi-1}}{\sum_{k=1}^{N}\omega_{ki,t+1}M_{k,t+1}}\right. \\
&\left.-\frac{\omega_{ji,t+1}^2p_{i,t+1}A_{i,t+1}\left(L_{i,t+1}^{1-\alpha}K_{i,t+1}^{\alpha}\right)^{1-\phi}\left(\sum_{k=1}^{N}\omega_{ki,t+1}M_{k,t+1}\right)^{\phi}}{\left(\sum_{k=1}^{N}\omega_{ki,t+1}M_{k,t+1}\right)^2}\right) \qquad \text{(A55)} \\
&-\beta^{t}\lambda_{j,t}P_{j,t} \\
&+\beta^{t+1}\lambda_{j,t+1}\left(1-\delta_M\right)P_{j,t+1}\overset{!}{=}0 \quad \text{for all } j \text{ and } t. \qquad \text{(A56)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}_j}{\partial \lambda_{j,t}} \;=\;& p_{j,t}A_{j,t}\left(\sum_{i=1}^{N}\omega_{ij,t}M_{i,t}\right)^{\phi}\left(L_{j,t}^{1-\alpha}K_{j,t}^{\alpha}\right)^{1-\phi} \\
&\times\left(1-\frac{\phi}{\sum_{k=1}^{N}\omega_{kj,t}M_{k,t}}\sum_{i=1}^{N}\omega_{ij,t}M_{i,t}\right)+\phi M_{j,t}\sum_{i\neq j}\left(\frac{\omega_{ji,t}Y_{i,t}}{\sum_{k=1}^{N}\omega_{ki,t}M_{k,t}}\right) \\
&-P_{j,t}C_{j,t}-P_{j,t}\left(K_{j,t+1}-\left(1-\delta_K\right)K_{j,t}\right) \\
&-P_{j,t}\left(M_{j,t+1}-\left(1-\delta_M\right)M_{j,t}\right)\overset{!}{=}0 \quad \text{for all } j \text{ and } t. \qquad \text{(A57)}
\end{aligned}
$$

Use the first-order condition for consumption to express $\lambda_{j,t}$ as:

$$\lambda_{j,t} = \frac{1}{C_{j,t}P_{j,t}}. \tag{A58}$$

Replace this in the first-order condition for physical capital:

$$\begin{aligned}
\frac{\partial \mathcal{L}_j}{\partial K_{j,t+1}} \quad = \quad & \beta^{t+1}\frac{1}{C_{j,t+1}P_{j,t+1}} p_{j,t+1}A_{j,t+1}\left(\sum_{i=1}^{N}\omega_{ij,t+1}M_{i,t+1}\right)^{\phi} \\
& \left(1-\frac{\phi}{\sum_{k=1}^{N}\omega_{kj,t+1}M_{k,t+1}}\sum_{i\neq j}\omega_{ij,t+1}M_{i,t+1}\right)L_{j,t+1}^{(1-\alpha)(1-\phi)}\alpha(1-\phi)K_{j,t+1}^{\alpha(1-\phi)-1} \\
& -\beta^{t}\frac{1}{C_{j,t}P_{j,t}}P_{j,t} \\
& +\beta^{t+1}\left(1-\delta_K\right)\frac{1}{C_{j,t+1}P_{j,t+1}}P_{j,t+1} \overset{!}{=} 0 \quad \text{for all } j \text{ and } t.
\end{aligned} \tag{A59}$$

Simplify and re-arrange to obtain:

$$\begin{aligned}
& \beta\frac{1}{C_{j,t+1}P_{j,t+1}} p_{j,t+1}A_{j,t+1}\left(\sum_{i=1}^{N}\omega_{ij,t+1}M_{i,t+1}\right)^{\phi} \\
& \left(1-\frac{\phi}{\sum_{k=1}^{N}\omega_{kj,t+1}M_{k,t+1}}\sum_{i\neq j}\omega_{ij,t+1}M_{i,t+1}\right)L_{j,t+1}^{(1-\alpha)(1-\phi)}\alpha(1-\phi)K_{j,t+1}^{\alpha(1-\phi)-1} = \\
& \frac{1}{C_{j,t}} + \frac{\left(\delta_K-1\right)\beta}{C_{j,t+1}} \quad \text{for all } j \text{ and } t.
\end{aligned} \tag{A60}$$

Use the definition of Y_t to re-write the left-hand side of the above expression as:

$$\begin{aligned}
& \frac{\alpha(1-\phi)\beta Y_{j,t+1}}{K_{j,t+1}C_{j,t+1}P_{j,t+1}}\left(1-\frac{\phi}{\sum_{k=1}^{N}\omega_{kj,t+1}M_{k,t+1}}\sum_{i\neq j}\omega_{ij,t+1}M_{i,t+1}\right) = \\
& \frac{1}{C_{j,t}} + \frac{\beta\left(\delta_K-1\right)}{C_{j,t+1}} \quad \text{for all } j \text{ and } t.
\end{aligned} \tag{A61}$$

Now replace λ_j with the expression from the first-order condition for consumption given in Equation (A58) in the first-order condition for technology capital given in Equation (A56):

$$
\begin{aligned}
\frac{\partial \mathcal{L}_j}{\partial M_{j,t+1}} = {} & \beta^{t+1}\frac{1}{C_{j,t+1}P_{j,t+1}}\omega_{jj,t+1}p_{j,t+1}A_{j,t+1}\left(L_{j,t+1}^{1-\alpha}K_{j,t+1}^{\alpha}\right)^{1-\phi}\phi \\
& \left(1-\frac{\phi}{\sum_{k=1}^{N}\omega_{kj,t+1}M_{k,t+1}}\sum_{i\neq j}\omega_{ij,t+1}M_{i,t+1}\right)\left(\sum_{i=1}^{N}\omega_{ij,t+1}M_{i,t+1}\right)^{\phi-1} \\
& +\beta^{t+1}\frac{1}{C_{j,t+1}P_{j,t+1}}\omega_{jj,t+1}p_{j,t+1}A_{j,t+1}\left(L_{j,t+1}^{1-\alpha}K_{j,t+1}^{\alpha}\right)^{1-\phi}\left(\sum_{i=1}^{N}\omega_{ij,t+1}M_{i,t+1}\right)^{\phi} \\
& \times\left(\frac{\phi}{\left(\sum_{k=1}^{N}\omega_{kj,t+1}M_{k,t+1}\right)^2}\sum_{i\neq j}\omega_{ij,t+1}M_{i,t+1}\right) \\
& +\beta^{t+1}\frac{1}{C_{j,t+1}P_{j,t+1}}\phi\sum_{i\neq j}\left(\frac{\omega_{ji,t+1}Y_{i,t+1}}{\sum_{k=1}^{N}\omega_{ki,t+1}M_{k,t+1}}\right) \\
& +\beta^{t+1}\frac{1}{C_{j,t+1}P_{j,t+1}}\phi M_{j,t+1} \\
& \times\sum_{i\neq j}\left(\frac{\omega_{ji,t+1}^2p_{i,t+1}A_{i,t+1}\left(L_{i,t+1}^{1-\alpha}K_{i,t+1}^{\alpha}\right)^{1-\phi}\phi\left(\sum_{k=1}^{N}\omega_{ki,t+1}M_{k,t+1}\right)^{\phi-1}}{\sum_{k=1}^{N}\omega_{ki,t+1}M_{k,t+1}}\right. \\
& \left.-\frac{\omega_{ji,t+1}^2p_{i,t+1}A_{i,t+1}\left(L_{i,t+1}^{1-\alpha}K_{i,t+1}^{\alpha}\right)^{1-\phi}\left(\sum_{k=1}^{N}\omega_{ki,t+1}M_{k,t+1}\right)^{\phi}}{\left(\sum_{k=1}^{N}\omega_{ki,t+1}M_{k,t+1}\right)^2}\right) \qquad \text{(A62)} \\
& -\frac{\beta^t}{C_{j,t}} \\
& +\frac{\beta^{t+1}}{C_{j,t+1}}(1-\delta_M)\stackrel{!}{=}0 \quad \text{for all } j \text{ and } t. \qquad \text{(A63)}
\end{aligned}
$$

Simplify and re-arrange to obtain:

$$
\begin{aligned}
& \frac{\beta\omega_{jj,t+1}}{C_{j,t+1}P_{j,t+1}} p_{j,t+1} A_{j,t+1} \left(L_{j,t+1}^{1-\alpha} K_{j,t+1}^{\alpha}\right)^{1-\phi} \phi \\
& \left(1 - \frac{\phi}{\sum_{k=1}^{N} \omega_{kj,t+1} M_{k,t+1}} \sum_{i \neq j} \omega_{ij,t+1} M_{i,t+1}\right) \left(\sum_{i=1}^{N} \omega_{ij,t+1} M_{i,t+1}\right)^{\phi-1} \\
& + \frac{\beta\omega_{jj,t+1}}{C_{j,t+1}P_{j,t+1}} p_{j,t+1} A_{j,t+1} \left(L_{j,t+1}^{1-\alpha} K_{j,t+1}^{\alpha}\right)^{1-\phi} \left(\sum_{i=1}^{N} \omega_{ij,t+1} M_{i,t+1}\right)^{\phi} \\
& \times \left(\frac{\phi}{\left(\sum_{k=1}^{N} \omega_{kj,t+1} M_{k,t+1}\right)^2} \sum_{i \neq j} \omega_{ij,t+1} M_{i,t+1}\right) \\
& + \frac{\beta\phi}{C_{j,t+1}P_{j,t+1}} \sum_{i \neq j} \left(\frac{\omega_{ji,t+1} Y_{i,t+1}}{\sum_{k=1}^{N} \omega_{ki,t+1} M_{k,t+1}}\right) \\
& + \frac{\beta\phi}{C_{j,t+1}P_{j,t+1}} M_{j,t+1} \\
& \times \sum_{i \neq j} \left(\frac{\omega_{ji,t+1}^2 p_{i,t+1} A_{i,t+1} \left(L_{i,t+1}^{1-\alpha} K_{i,t+1}^{\alpha}\right)^{1-\phi} \phi \left(\sum_{k=1}^{N} \omega_{ki,t+1} M_{k,t+1}\right)^{\phi-1}}{\sum_{k=1}^{N} \omega_{ki,t+1} M_{k,t+1}}\right. \\
& \left. - \frac{\omega_{ji,t+1}^2 p_{i,t+1} A_{i,t+1} \left(L_{i,t+1}^{1-\alpha} K_{i,t+1}^{\alpha}\right)^{1-\phi} \left(\sum_{k=1}^{N} \omega_{ki,t+1} M_{k,t+1}\right)^{\phi}}{\left(\sum_{k=1}^{N} \omega_{ki,t+1} M_{k,t+1}\right)^2}\right) \\
= \; & \frac{1}{C_{j,t}} + \frac{\beta\left(\delta_M - 1\right)}{C_{j,t+1}}.
\end{aligned}
$$

Use the definition of $Y_{j,t}$ to re-write the left-hand side of the above expression as:

$$
\begin{aligned}
& \frac{\beta \omega_{jj,t+1} \phi Y_{j,t+1}}{C_{j,t+1} P_{j,t+1} \left(\sum_{i=1}^{N} \omega_{ij,t+1} M_{i,t+1} \right)} \\
& \times \left(1 - \frac{\phi}{\sum_{k=1}^{N} \omega_{kj,t+1} M_{k,t+1}} \sum_{i \neq j} \omega_{ij,t+1} M_{i,t+1} \right) \\
& + \frac{\beta \omega_{jj,t+1} Y_{j,t+1}}{C_{j,t+1} P_{j,t+1}} \left(\frac{\phi}{\left(\sum_{k=1}^{N} \omega_{kj,t+1} M_{k,t+1} \right)^2} \sum_{i \neq j} \omega_{ij,t+1} M_{i,t+1} \right) \\
& + \frac{\beta \phi}{C_{j,t+1} P_{j,t+1}} \sum_{i \neq j} \left(\frac{\omega_{ji,t+1} Y_{i,t+1}}{\sum_{k=1}^{N} \omega_{ki,t+1} M_{k,t+1}} \right) \\
& + \frac{\beta \phi}{C_{j,t+1} P_{j,t+1}} M_{j,t+1} \sum_{i \neq j} \left(\frac{\phi \omega_{ji,t+1}^2 Y_{i,t+1}}{\left(\sum_{k=1}^{N} \omega_{ki,t+1} M_{k,t+1} \right)^2} \right. \\
& \left. - \frac{\omega_{ji,t+1}^2 Y_{i,t+1}}{\left(\sum_{k=1}^{N} \omega_{ki,t+1} M_{k,t+1} \right)^2} \right) \\
= \; & \frac{1}{C_{j,t}} \left(\frac{1}{M_{j,t}^{1-\delta_M}} \right)^{\frac{1}{\delta_M}} \frac{1}{\delta_M} M_{j,t+1}^{\frac{1}{\delta_M}-1} + \beta \frac{1}{C_{j,t+1}} M_{j,t+2}^{\frac{1}{\delta_M}} \frac{\delta_M - 1}{\delta_M} M_{j,t+1}^{-\frac{1}{\delta_M}}.
\end{aligned}
$$

Now multiply with $C_{j,t+1}P_{j,t+1}$ to end up with:

$$\begin{aligned}
& \frac{\beta\phi\omega_{jj,t+1}Y_{j,t+1}}{\left(\sum_{i=1}^{N}\omega_{ij,t+1}M_{i,t+1}\right)} \\
& \times\left(1-\frac{\phi}{\sum_{k=1}^{N}\omega_{kj,t+1}M_{k,t+1}}\sum_{i\neq j}\omega_{ij,t+1}M_{i,t+1}\right) \\
& +\beta\omega_{jj,t+1}Y_{j,t+1}\left(\frac{\phi}{\left(\sum_{k=1}^{N}\omega_{kj,t+1}M_{k,t+1}\right)^{2}}\sum_{i\neq j}\omega_{ij,t+1}M_{i,t+1}\right) \\
& +\beta\phi\sum_{i\neq j}\left(\frac{\omega_{ji,t+1}Y_{i,t+1}}{\sum_{k=1}^{N}\omega_{ki,t+1}M_{k,t+1}}\right) \\
& -\beta\phi(\phi-1)M_{j,t+1}\sum_{i\neq j}\left(\frac{\omega_{ji,t+1}^{2}Y_{i,t+1}}{\left(\sum_{k=1}^{N}\omega_{ki,t+1}M_{k,t+1}\right)^{2}}\right) \\
= \quad & \frac{C_{j,t+1}P_{j,t+1}}{C_{j,t}}+\beta\left(\delta_{M}-1\right)P_{j,t+1}. \qquad \text{(A64)}
\end{aligned}$$

Combining the production function given by Equation (A49), the budget constraint given by Equation (16), the expression for $E_{j,t}$ given in Equation (A52), the expressions for $p_{j,t}$ for each t from Equation (25), and the equations for the trade MRTs $P_{j,t}$ and $\Pi_{j,t}$ given by Equations (20) and (21), respectively, with the two first order conditions for $K_{j,t+1}$ and $M_{j,t+1}$ as given by Equations (A61) and (A64), respectively, we end up with the following system:

$$Y_{j,t} = p_{j,t} A_{j,t} \left(L_{j,t}^{1-\alpha} K_{j,t}^{\alpha}\right)^{1-\phi} \left(\sum_{i=1}^{N} \omega_{ij,t} M_{i,t}\right)^{\phi} \quad \text{for all } j \text{ and } t, \tag{A65}$$

$$E_{j,t} = P_{j,t} C_{j,t} + P_{j,t} \left(\frac{K_{j,t+1}}{K_{j,t}^{1-\delta_K}}\right)^{\frac{1}{\delta_K}} + P_{j,t} \left(\frac{M_{j,t+1}}{M_{j,t}^{1-\delta_M}}\right)^{\frac{1}{\delta_M}} \quad \text{for all } j \text{ and } t, \tag{A66}$$

$$E_{j,t} = Y_{j,t} + \phi M_{j,t} \sum_{i \neq j} \left(\frac{\omega_{ji,t} Y_{i,t}}{\sum_{k=1}^{N} \omega_{ki,t} M_{k,t}}\right) \tag{A67}$$

$$- \frac{\phi Y_{j,t}}{\sum_{k=1}^{N} \omega_{kj,t} M_{k,t}} \sum_{i \neq j} \omega_{ij,t} M_{i,t} \quad \text{for all } j \text{ and } t, \tag{A68}$$

$$p_{j,t} = \frac{(Y_{j,t}/Y_t)^{\frac{1}{1-\sigma}}}{\gamma_j \Pi_{j,t}} \quad \text{for all } j \text{ and } t, \tag{A69}$$

$$Y_t = \sum_{j=1}^{N} Y_{j,t} \quad \text{for all } t, \tag{A70}$$

$$P_{j,t}^{1-\sigma} = \sum_{i=1}^{N} \left(\frac{t_{ij,t}}{\Pi_{i,t}}\right)^{1-\sigma} \frac{Y_{i,t}}{Y_t} \quad \text{for all } j \text{ and } t, \tag{A71}$$

$$\Pi_{i,t}^{1-\sigma} = \sum_{j=1}^{N} \left(\frac{t_{ij,t}}{P_{j,t}}\right)^{1-\sigma} \frac{E_{j,t}}{Y_t} \quad \text{for all } i \text{ and } t, \tag{A72}$$

$$\frac{\alpha(1-\phi)\beta Y_{j,t+1}}{K_{j,t+1} C_{j,t+1} P_{j,t+1}}$$

$$\times \left(1 - \frac{\phi}{\sum_{k=1}^{N} \omega_{kj,t+1} M_{k,t+1}} \sum_{i \neq j} \omega_{ij,t+1} M_{i,t+1}\right) = \frac{1}{C_{j,t}}$$

$$+ \frac{\beta\left(\delta_K - 1\right)}{C_{j,t+1}} \quad \text{for all } j \text{ and } t, \tag{A73}$$

$$
\begin{aligned}
&\frac{\beta\phi\omega_{jj,t+1}Y_{j,t+1}}{\left(\sum_{i=1}^{N}\omega_{ij,t+1}M_{i,t+1}\right)}\left(1-\frac{\phi}{\sum_{k=1}^{N}\omega_{kj,t+1}M_{k,t+1}}\sum_{i\neq j}\omega_{ij,t+1}M_{i,t+1}\right)\\
&+\beta\omega_{jj,t+1}Y_{j,t+1}\left(\frac{\phi}{\left(\sum_{k=1}^{N}\omega_{kj,t+1}M_{k,t+1}\right)^2}\sum_{i\neq j}\omega_{ij,t+1}M_{i,t+1}\right)\\
&+\beta\phi\sum_{i\neq j}\left(\frac{\omega_{ji,t+1}Y_{i,t+1}}{\sum_{k=1}^{N}\omega_{ki,t+1}M_{k,t+1}}\right)\\
&-\beta\phi(\phi-1)M_{j,t+1}\sum_{i\neq j}\left(\frac{\omega_{ji,t+1}^2Y_{i,t+1}}{\left(\sum_{k=1}^{N}\omega_{ki,t+1}M_{k,t+1}\right)^2}\right)\\
&=\frac{C_{j,t+1}P_{j,t+1}}{C_{j,t}}+\beta\left(\delta_M-1\right)P_{j,t+1}\quad\text{for all } j \text{ and } t. \qquad \text{(A74)}
\end{aligned}
$$

This is a system of $(8\times N+1)\times T$ equations in the $(8\times N+1)\times T$ unknowns $C_{j,t}$, $K_{j,t}$, $M_{j,t}$, $Y_{j,t}$, Y_t, $p_{j,t}$, $P_{j,t}$, $\Pi_{j,t}$, $E_{j,t}$ and given parameters and exogenous variables $A_{j,t}$, $\omega_{ij,t}$, $L_{j,t}$, α, β, ϕ, γ_j, σ, $t_{ij,t}$, δ_K, and δ_M. We can again replace $p_{j,t}$ and add trade flows to end up with a very similar system as in our main case.

B.2 Derivation of the Steady-State

In steady-state, values for $t+1$ and t have to be equal. Hence, we can express physical and technology capital as:

$$
\begin{aligned}
K_j &= \frac{\Omega_j}{\delta_K}, &\text{(A75)}\\
M_j &= \frac{\chi_j}{\delta_M}. &\text{(A76)}
\end{aligned}
$$

Further, we can drop the time index for all variables. Let us first drop time indices and use $K_j=\Omega_j/\delta_K$ and $M_j=\chi_j/\delta_M$ in the first-order condition for physical capital as given in

Equation (A73):

$$
\begin{aligned}
&\frac{\alpha\beta(1-\phi)Y_j}{K_jC_jP_j} \times \left(1 - \frac{\phi}{\sum_{k=1}^{N}\omega_{kj}M_k}\sum_{i\neq j}\omega_{ij}M_i\right) \\
&\qquad = \frac{1}{C_j} + \frac{\beta\left(\delta_K - 1\right)}{C_j} \quad \text{for all } j \Rightarrow \\
&\frac{\alpha\beta(1-\phi)Y_j}{K_jP_j} \times \left(1 - \frac{\phi}{\sum_{k=1}^{N}\omega_{kj}M_k}\sum_{i\neq j}\omega_{ij}M_i\right) \\
&\qquad = 1 + \beta\left(\delta_K - 1\right) \quad \text{for all } j \Rightarrow \\
&\frac{\alpha\beta(1-\phi)Y_j}{P_j(1-\beta+\beta\delta_K)} \times \left(1 - \frac{\phi}{\sum_{k=1}^{N}\omega_{kj}M_k}\sum_{i\neq j}\omega_{ij}M_i\right) \\
&\qquad = K_j \quad \text{for all } j.
\end{aligned}
$$

Let us next drop time indices and use $K_j = \Omega_j/\delta_K$ and $M_j = \chi_j/\delta_M$ in the first-order condition for technology capital as given in Equation (A74):

$$
\begin{aligned}
&\frac{\beta\phi\omega_{jj}Y_j}{\left(\sum_{i=1}^{N}\omega_{ij}M_i\right)}\left(1 - \frac{\phi}{\sum_{k=1}^{N}\omega_{kj}M_k}\sum_{i\neq j}\omega_{ij}M_i\right) \\
&\quad + \beta\omega_{jj}Y_j\left(\frac{\phi}{\left(\sum_{k=1}^{N}\omega_{kj}M_k\right)^2}\sum_{i\neq j}\omega_{ij}M_i\right) \\
&\quad + \beta\phi\sum_{i\neq j}\left(\frac{\omega_{ji}Y_i}{\sum_{k=1}^{N}\omega_{ki}M_k}\right) \\
&\quad - \beta\phi(\phi-1)M_j\sum_{i\neq j}\left(\frac{\omega_{ji}^2Y_i}{\left(\sum_{k=1}^{N}\omega_{ki}M_k\right)^2}\right) \\
&\quad = \frac{C_jP_j}{C_j} + \beta\left(\delta_M - 1\right)P_j \quad \text{for all } j \Rightarrow
\end{aligned}
$$

$$\frac{\beta\phi\omega_{jj}Y_j}{P_j(1-\beta+\beta\delta_M)\left(\sum_{i=1}^N \omega_{ij}M_i\right)}\left(1-\frac{\phi}{\sum_{k=1}^N \omega_{kj}M_k}\sum_{i\neq j}\omega_{ij}M_i\right)$$
$$+\frac{\beta\omega_{jj}Y_j}{P_j(1-\beta+\beta\delta_M)}\left(\frac{\phi}{\left(\sum_{k=1}^N \omega_{kj}M_k\right)^2}\sum_{i\neq j}\omega_{ij}M_i\right)$$
$$+\frac{\beta\phi}{P_j(1-\beta+\beta\delta_M)}\sum_{i\neq j}\left(\frac{\omega_{ji}Y_i}{\sum_{k=1}^N \omega_{ki}M_k}\right)=$$
$$1+\frac{\beta\phi(\phi-1)M_j}{P_j(1-\beta+\beta\delta_M)}\sum_{i\neq j}\left(\frac{\omega_{ji}^2 Y_i}{\left(\sum_{k=1}^N \omega_{ki}M_k\right)^2}\right) \quad \text{for all } j.$$

Hence, the equation system given by Equations (A65)-(A74) simplifies to:

$$Y_j = p_j A_j \left(L_j^{1-\alpha} K_j^{\alpha}\right)^{1-\phi} \left(\sum_{i=1}^{N} \omega_{ij} M_i\right)^{\phi} \quad \text{for all } j, \tag{A77}$$

$$E_j = P_j C_j + P_j K_j + P_j M_j \quad \text{for all } j, \tag{A78}$$

$$\begin{aligned} E_j = Y_j + \phi M_j \sum_{i \neq j} \left(\frac{\omega_{ji} Y_i}{\sum_{k=1}^{N} \omega_{ki} M_k}\right) \\ - \frac{\phi Y_j}{\sum_{k=1}^{N} \omega_{kj} M_k} \sum_{i \neq j} \omega_{ij} M_i \quad \text{for all } j, \end{aligned} \tag{A79}$$

$$p_j = \frac{(Y_j/Y)^{\frac{1}{1-\sigma}}}{\gamma_j \Pi_j} \quad \text{for all } j, \tag{A80}$$

$$Y = \sum_{j=1}^{N} Y_j, \tag{A81}$$

$$P_j^{1-\sigma} = \sum_{i=1}^{N} \left(\frac{t_{ij}}{\Pi_i}\right)^{1-\sigma} \frac{Y_i}{Y} \quad \text{for all } j, \tag{A82}$$

$$\Pi_i^{1-\sigma} = \sum_{j=1}^{N} \left(\frac{t_{ij}}{P_j}\right)^{1-\sigma} \frac{E_j}{Y} \quad \text{for all } i, \tag{A83}$$

$$K_j = \frac{\alpha\beta(1-\phi)Y_j}{P_j(1-\beta+\beta\delta_K)} \times \left(1 - \frac{\phi}{\sum_{k=1}^{N} \omega_{kj} M_k} \sum_{i \neq j} \omega_{ij} M_i\right) \quad \text{for all } j, \tag{A84}$$

$$\begin{aligned} \frac{(1+\beta\delta_M-\beta)\,P_j}{\phi\beta} = \frac{\omega_{jj} Y_j}{\left(\sum_{i=1}^{N} \omega_{ij} M_i\right)} \left(1 - \frac{1-\phi}{\sum_{k=1}^{N} \omega_{kj} M_k} \sum_{i \neq j} \omega_{ij} M_i\right) \\ + \sum_{i \neq j} \left(\frac{\omega_{ji} Y_i}{\sum_{k=1}^{N} \omega_{ki} M_k}\right) - (\phi-1) M_j \sum_{i \neq j} \left(\frac{\omega_{ji}^2 Y_i}{\left(\sum_{k=1}^{N} \omega_{ki} M_k\right)^2}\right) \quad \text{for all } j. \end{aligned} \tag{A85}$$