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Climate Policies under Climate Model Uncertainty: Max-Min and Min-Max Regret

Abstract

Temperature responses and optimal climate policies depend crucially on the choice of a particular climate model. To illustrate, the temperature responses to given emission reduction paths implied by the climate modules of the well-known integrated assessments models DICE, FUND and PAGE are described and compared. A dummy temperature module based on the climate denialists' view is added. Using a simple welfare-maximising growth model of the global economy, the sensitivity of the optimal carbon price, renewable energy subsidy and energy transition to each of these climate models is discussed. The paper then derives max-min, max-max and min-max regret policies to deal with this particular form of climate (model) uncertainty and with climate scepticism. The max-min or min-max regret climate policies rely on a non-sceptic view of global warming and lead to a substantial and moderate amount of caution, respectively. The max-max leads to no climate policies in line with the view of climate sceptics.

JEL-Codes: H210, Q510, Q540.

Keywords: carbon price, renewable energy subsidy, temperature modules, climate model uncertainty, climate sceptics, max-min, max-max, min-max regret.

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1. Introduction

The complex interactions between greenhouse gas emissions (GHGs) and the earth's climate are still highly uncertain, as has become clear from the excellent work by the Intergovernmental Panel on Climate Change (IPCC) on comparing the temperature responses resulting from various emission reduction paths to different Integrated Assessment Models (IAMs). Much effort has gone into elucidating the effects of the many uncertainties (about key parameters such as the climate sensitivity or the transient climate response, positive feedback loops such as release of CH_4 from ocean floors, catastrophic shocks, etc.) in the climate models of these IAMs by generating fan charts for the temperature responses to emission reduction paths for each of the main climate models. It is important, however, to distinguish between all the *statistical* uncertainties concerning the parameters, equation errors, shocks and initial conditions of a particular model, on the one hand, and *scientific* uncertainty about which particular climate model with all the scientific uncertainties that are associated with it is the right one, on the other hand (cf. Arrow, 1951).[4] Our objective in this paper is to analyse the effects of the second type of uncertainty, scientific or more precisely climate model uncertainty, on the optimal price of carbon and optimal energy transition and to suggest suitable ways of dealing with these fundamental uncertainties.

To create a testbed to illustrate the effects of climate model uncertainty on optimal policy formulation, we use the carbon cycles and temperature modules of three prominent IAMs that are used most by economists and in policy debates: the "Dynamic Integrated model of Climate and the Economy" or DICE (Nordhaus, 2014); the climate "Framework for Uncertainty, Negotiation and Distribution" or FUND (Anthoff and Tol, 2013); and the "Policy Analysis model of the Greenhouse Effect" or PAGE (Hope, 2006, 2011). For purposes of our analysis we abstract from the many statistical uncertainties captured by these models by using their deterministic versions of the carbon cycle and temperature responses. We justify this, since we wish to illustrate how to deal with climate model uncertainty and therefore focus on this type of scientific uncertainty only. To make our testbed more relevant for the current policy debate, we also add a fourth climate model, namely the one adhered to by climate sceptics. President Trump has elected one of the most prominent climate sceptics, Scott Pruitt, to be in charge of the Environmental Protection Agency and has nominated the CEO of ExxonMobil, Rex Tillerson, to be Secretary of the State Department. He has also claimed that climate change is a (Chinese) hoax and the expectation is that the United States will withdraw from the Paris climate change agreement or treat it as a dead letter. Many share Trump's opinion and believe that global warming is not caused by humans, so we add a model, contrary to the main body of scientific evidence, which states that burning fossil fuel does not contribute to global warming at all. It is not relevant whether we believe climate sceptics are scientifically correct or not. Neither does it matter whether climate sceptics are driven or captured by fossil fuel business interests or not. These views are clearly present, so one needs take of such views when formulating policy.

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[4] See Heal and Millner (2014, 2015) for an overview of the different types of uncertainty facing climate policy.

To illustrate our interpretation of the climate modules of DICE, PAGE and FUND, we discuss their special features and compare the temperature responses to pre-specified business-as-usual and de-carbonisation emission paths taken from the IPCC for each of these three models. We also compare these temperature responses to those generated by the MAGICC emulator based on a large ensemble of detailed large-scale carbon and temperature models (cf. Meinshausen, et al., 2011). To study how such different temperature responses affect the optimal global price of carbon and transition from fossil fuel to renewable energy, we specify a very simple welfare-maximising Ramsey economic growth model with these two types of energy and a specification of temperature-dependent climate damages. Our economic module also allows for two market failures resulting from not internalising global warming damages caused by burning fossil fuel and not internalising learning by doing externalities in the production of renewable energy. Hence, the globally first-best optimal policies require a carbon price *and* a renewable energy subsidy (cf. Rezai and van der Ploeg, 2017a).

We thus use a simple common economic module rather than the different economic modules of the DICE, FUND and PAGE models, again to focus all our attention at climate model uncertainty. The common economic block of our IAM is hooked up with our best possible interpretation of the deterministic versions of the carbon cycle and temperature modules of each of the three climate models. Our choice of economic module and of temperature modules is somewhat arbitrary. We justify this on the grounds that we are more interested in the illustration of our proposed methods for deriving optimal carbon prices under climate model uncertainty than in the precise value of the numbers that are generated by our optimal policy simulations. Previous studies have focused on comparing outputs across standardised inputs and IAMs (cf. IAWG, 2016; Gillingham et al., 2016) but have not addressed climate model uncertainty within a uniform welfare-maximising framework.[5]

First, we show the sensitivity of the optimal climate policies and timing of the optimal energy transition to the particular climate model that is used. We then substitute the optimal climate policies derived from each of the four climate models into the other three models and see how well or badly they perform. This is not a trivial task, since the fundamental theorem of welfare economics no longer holds and therefore one is required to maximise welfare subject to the constraints of the decentralised market economy instead of the easier approach of solving for the command optimum (cf. Kalkuhl et al., 2013; Rezai and van der Ploeg, 2016, 2017a). In this sense, our approach is an illustration of second-best welfare economics.

Second, we use our framework of an economic module with four climate modules to derive robust climate policies. So we do not maximise expected welfare, but consider and derive the max-min optimal climate policy. This is the policy that yields the highest welfare if the welfare of each policy is evaluated under the worst possible outcome as has been originally suggested by Wald (1945) and has been given an axiomatic foundation by Gilboa and Schmeidler (1989). For each climate module we

[5] Cai and Sanstad (2016) use a similar decision framework to study the policy implications of uncertainty over technological change in an energy-climate system.

first calculate the optimal climate policy, then evaluate the welfare under each of the other climate modules, and then note the lowest of these welfare outcomes (i.e., the worst possible outcome for that policy). The max-min policy then corresponds to the one that gives the highest welfare under the worst possible outcome. An early application of max-min to climate policy can be found in Woodford and Bishop (1997), who consider a catastrophic and a non-catastrophic scenario in DICE and find that the max-min policy is to assume that the catastrophic scenario holds (until it is proven not to hold any longer). Strictly speaking, this is an exercise not in climate model uncertainty but in statistical uncertainty and techniques have since been developed to deal with catastrophic Poisson shocks in versions of the DICE integrated assessment model (e.g., Lemoine and Traeger, 2014; Lontzek et al., 2015). Climate model uncertainty or scientific uncertainty is, however, conceptually very different from statistical uncertainty and we analyse for the first time the use of max-min to derive optimal climate policy in this context.

We find that the max-min policy is the optimal policy derived from the model with the climate model of DICE. This is not surprising, since the DICE model has in our calibration of the various temperature modules the most adverse temperature responses and the max-min policy is a prudent policy that avoids bad outcomes when the world turns out to be different. In contrast, the max-max policy assumes that every policy rule is evaluated in the best possible view of the world, i.e., President Trump's climate sceptic view, in which case it is best not to price carbon at all. Arrow and Hurwicz (1977) suggest some average of max-min and max-max policies, which introduces some robustness into the optimal climate policy but less than for the max-min policy.

Less conservative policies are obtained under the min-max regret policy. Regret is defined as the difference between the welfare that would have been obtained if the right optimal climate policy was used for the climate model under consideration minus the welfare that prevails under this climate model with the climate policy under consideration. Clearly, regrets are zero if the right optimal climate policy is implemented for the climate model that happens to be correct. The min-max regret policy is then the policy that gives least regret across all the different climate models and was originally proposed by Savage (1951, 1954). So the objective is not to do as well as possible under the worst outcome as with max-min, but to minimise how much better one could have been off. Min-max regret thus leads to less ambitious climate policies than max-min, since what could have been had rather than the worst possible outcome is highlighted. The min-max regret policy in our simulations corresponds to the optimal price of carbon under the PAGE or FUND climate model depending on whether the sceptic view is included or not. Optimal policy based on President Trump's climate sceptic view, i.e., not pricing carbon at all, never prevails under min-max regret or max-min.

Section 2 discusses the climate and temperature models of DICE, FUND, PAGE and the denialists' and compares temperature responses for each of these to IPPC emission-reduction and business-as-usual paths and compares them with the emulated responses in MAGICC. Section 3 describes the common economic block of our integrated assessment model. Section 4 compares the optimal climate policy and energy transitions across these three different climate models. Section 5 derives and discusses the optimal max-min, max-max and min-max regret climate policies under climate model uncertainty. Section 6 concludes with a summary of results and a discussion of alternatives for calculating robust climate policies.

2. Temperature modelling in three prominent IAMs

Integrated assessment models (IAMs) combine sets of economic and geophysical assumptions in order to understand the complex interactions between the processes driving anthropogenic GHG emissions and their implications for the climate and the global economy, and sometimes regional economies, through temperature deviations. In this section we take a closer look under the hood of the prominent IAMs that are used in the study of the Interagency Working Group on the social cost of carbon (IAWG, 2016), and present and discuss the climate modules adopted in each of the study's IAMs. We use this to obtain three simplified temperature modules for each of these three IAMs. We also add a climate-sceptic module. Given the different time scales across the IAMs considered here, we rescale all models to an annual time grid. In this section we will give a qualitative discussion of the various climate models and refer the interested reader to appendix A where the mathematical details of these models are presented. Table 1 present a summary of the models' key characteristics.

	DICE	FUND	PAGE
Number of carbon boxes	3	5	2
Number of heat bodies	2	1	7
Non-CO_2 GHGs	No	Yes	Yes
% of emissions permanent	4.9%	13%	19%
Equilibrium climate sensitivity	2.9°C	3°C	3°C
Regional climate dynamics	No	Yes	Yes

Table 1: Features of DICE, FUND, and PAGE

2.1. Temperature module of the DICE-2013R IAM

We employ the DICE-2013R version of DICE (Nordhaus, 2014). DICE-2013R approximates the geophysical dynamics with a three-box carbon cycle, radiative forcing, and a two-box temperature cycle. In the carbon cycle, carbon diffuses between the atmosphere and the upper and lower parts of the oceans following a Markov process. Carbon emissions from industrial and exogenous land use change enter the atmosphere from where it can transition to the lower oceans via the upper oceans. However, some fraction of the emitted carbon will remain in the atmosphere indefinitely. In equilibrium, the distribution of carbon is 4.9% of emissions in the atmosphere, 11.3% in the upper, and 83.8% in the lower oceans.

A higher stock of carbon in the atmosphere increases the amount of energy retained on Earth, which translates into increases in global mean temperature. Nordhaus (2014) takes a first-order approximation of radiative forcing – the difference between energy received from the sun and the energy radiated back from Earth – resulting from deviations from the pre-industrial stock of atmospheric carbon and also captures the evolution of other greenhouse gases (e.g., CH_4, N_2O, SO_2 and SF_6) and forcing components such as ozone and albedo by the exogenous forcing components. Nordhaus (2014) acknowledges the difficulty of accounting for the forcing components not directly related to CO_2 in DICE.

Temperature dynamics are modelled with two heat bodies: the atmosphere and the oceans. Heat moves between the two in a linear fashion and radiative forcing increases atmospheric temperature. The equilibrium climate sensitivity (ECS) is set to 2.9°C so that a doubling of the CO2 stock leads to a rise in temperature by 2.9°C in DICE-2013R. Nordhaus (2014) calibrates the temperature dynamics to give a transient climate response (TCR) at 2.9°C of 1.7°C.[6]

2.2. Temperature module of the FUND IAM version 3.7

We discuss the FUND model (version 3.7; Anthoff and Tol, 2013) next. Although FUND disaggregates the world economy into 16 regions and allows for rich regional dynamics, we take from the FUND model only the global geophysical component governing changes in temperature which is uniform across regions.

The climate cycle put forward in FUND is elaborate. It includes the effects of all major greenhouse gases, i.e., CH_4, N_2O, SF_6, SO_2 and CO_2, on radiative forcing. The transition dynamics of each gas are approximated using boxes with geometric depletion rates. There are five boxes to describe the dynamics of CO_2, only one box for CH_4, N_2O and SF_6, and no box for SO_2 (as the residence time of SO_2 is very short and can be taken to be zero). 50% of CO_2 emissions is removed within 45 years and 13% of emissions remains in the atmosphere forever.

Non-carbon GHGs emissions and their abatement are endogenous in FUND. For purposes of our numerical simulations, however, we set emissions of non-CO_2 greenhouse gases to zero to increase comparability across climate cycles and to abstract from exogenous scenarios and additional abatement options. While this ensures comparability across the various climate cycles, it biases the social cost of carbon of FUND downward, since non-CO_2 emissions will remain positive in all but the most ambitious mitigation scenarios. The modelling of dynamics of multiple GHGs improves the modelling of radiative forcing which also allows for the interaction between CH_4 and N_2O. This also eliminates the need for exogenous forcing components as in DICE.

[6] The Transient Climate Response (TCR) measures the sensitivity of temperature to increases in atmospheric carbon and is defined as the temperature increase due at the point of CO2 doubling if concentration was to increase by 1% per year until it reaches twice the pre-industrial level.

Temperature is approximated linearly in FUND by introducing a lagged temperature response relative to pre-industrial levels to radiative forcing. The equilibrium climate sensitivity is set to 3°C and the lag in temperature to 66 years, implying that half of a temperature impulse is left dissipated within slightly less than 50 years. Regional temperature levels are scaled to global mean temperature in fixed proportions.

2.3. Temperature module of the PAGE09 IAM

The third temperature module we consider is based on the deterministic version of the PAGE09 model (Hope, 2006, 2011). This is the only of the three climate models which traces global mean temperature through regional temperatures.

Four greenhouse gases are considered: CO_2, CH_4, N_2O and linear gases including SF_6. Emissions of these gases emanate in regions and are combined with emissions from national sources. For CO_2 emissions natural emissions increase in temperature due reductions in the system's absorptive capacity, introducing positive feedback. For CO_2 38% of emissions is absorbed immediately and of the rest entering the atmosphere 30% stay there forever. All other gases fully enter the atmosphere and dissipate over time.

The abatement of all greenhouse gases is endogenous in PAGE. For purposes of our numerical simulations, however, we set emissions of non-CO_2 greenhouse gases to zero to increase comparability across climate cycles and to abstract from exogenous scenarios and additional abatement options. While this ensures comparability across climate cycle, it again biases the social cost of carbon downward, since non-CO_2 emissions will remain positive in all but the most ambitious mitigation scenarios.

Radiative forcing results from all greenhouse gases, including interaction between CH_4 and N_2O. PAGE also allows for an additional exogenous forcing component and regional cooling effects from sulphate aerosols. We assume that aerosol emissions is zero and therefore abstract from this cooling effect and regional variations in radiative forcing.

Temperature dynamics in PAGE are regional across which global mean temperature is aggregated. Regional equilibrium temperature in each of the seven regions of the global economy is a linear function of total radiative forcing with a transient climate response of 1.7 °C and an equilibrium climate sensitivity of 3 °C per doubling of atmospheric CO_2. Regional temperature slowly adjusts to its equilibrium level given current levels of radiative forcing. Global mean temperature is an average of all regional temperatures adjusted by their land mass and latitude.

2.4. Dummy temperature module based on denial of global warming

Finally, we add a fourth temperature module based on the view that climate denialists are correct in their belief that global warming change is not caused by fossil fuel emissions due to human activities. We capture this notion by assuming that anthropogenic carbon emissions from burning fossil fuel do not contribute to global warming damages. To capture this as simple as possible, we assume that temperature remains constant at its current level of 0.88°C.[7] This implies that emission dynamics are irrelevant to the temperature response and the economy. We refer to this module as the DENIAL model. An alternative way of modelling temperature under the denialist view is to take one of the more scientifically grounded temperature modules, or an average thereof, and calculate the projected temperature paths when carbon emissions from now on are zero. This does not affect the qualitative nature of our core results regarding max-min and min-max regret optimal climate policies (section 5).

2.5. Temperature responses implied by IPCC emission reduction paths

It is standard practice in inter-model comparison exercises to take existing climate models and compare their temperature responses to given or closely matched exogenous trajectories for economic growth, energy dynamics, and regional and global emissions (e.g., Weyant and Kriegler, 2014). Here we perform a similar exercise and compare the temperature responses for each of our temperature modules taken from DICE, FUND and PAGE, respectively, to paths of carbon emissions resulting from fossil fuel emissions given by two representative concentration pathways (RCPs): rapid stabilisation and de-carbonisation scenario where fossil fuel emissions vanish by 2070 and then turn negative to almost –1 GtC/year (RCP3PD) and the business-as-usual emissions scenario (RCP8.5).[8] Radioactive forcing reaches 3 W/m^2 before 2100 and then falls in the RCP3D scenario but reaches 8.5 W/m^2 in 2100 and then continues to rise in the RCP8.5 scenario.

Temperature responses for each of these two scenarios are given in panels (a) and (b) of figure 1 to give a first feel for the climate models used in our study. In the DICE model temperature increases to 2 °C and 6.4 °C, respectively. Peak warming levels are lower for the FUND and PAGE model as non-carbon GHG emissions are modelled explicitly in these IAMs but omitted in our comparison. Peak warming levels fall to 1.7°C for both models in the case of rapid stabilisation and de-carbonisation and 4.7 °C and 4.3 °C, respectively, in BAU. Our rendition of the temperature modules of these IAMs thus shows the biggest temperature responses for DICE and weaker responses for FUND and PAGE except for a slightly slower temperature response for the latter under rapid de-carbonisation. The temperature responses for the denialist climate view are constant under both scenarios.

Temperature responses are for the RCP3PD scenario very comparable with those under the default parameter values of the live MAGICC emulator (Meinshausen, et al. 2011; www.live.magicc.org). MAGICC indicates that temperature starts at a higher level, rises to a lower maximum of 1.7 °C during the late 2050s, and then drops off ever so slowly. From that time onwards DICE, PAGE and FUND give somewhat higher temperature responses for the RCP3PD scenario than MAGICC. However, for the RCP8.5 scenario the global mean temperature responses of our renditions of especially the PAGE and FUND temperature modules are smaller than indicated by MAGICC which

[7] Global mean temperature in 2010 as given by MAGIC C in its standard parametrisation.
[8] We use the extended representative pathway scenarios for 2100 onward. These are not available for MAGICC live where projections end in 2100. For details see http://www.pik-potsdam.de/~mmalte/rcps/.

has global mean temperature rising to 3.4 °C in 2070, to 4.8 °C in 2100 and continues to rise thereafter. For RCP8.5 DICE and MAGICC seem to give consistent projections.

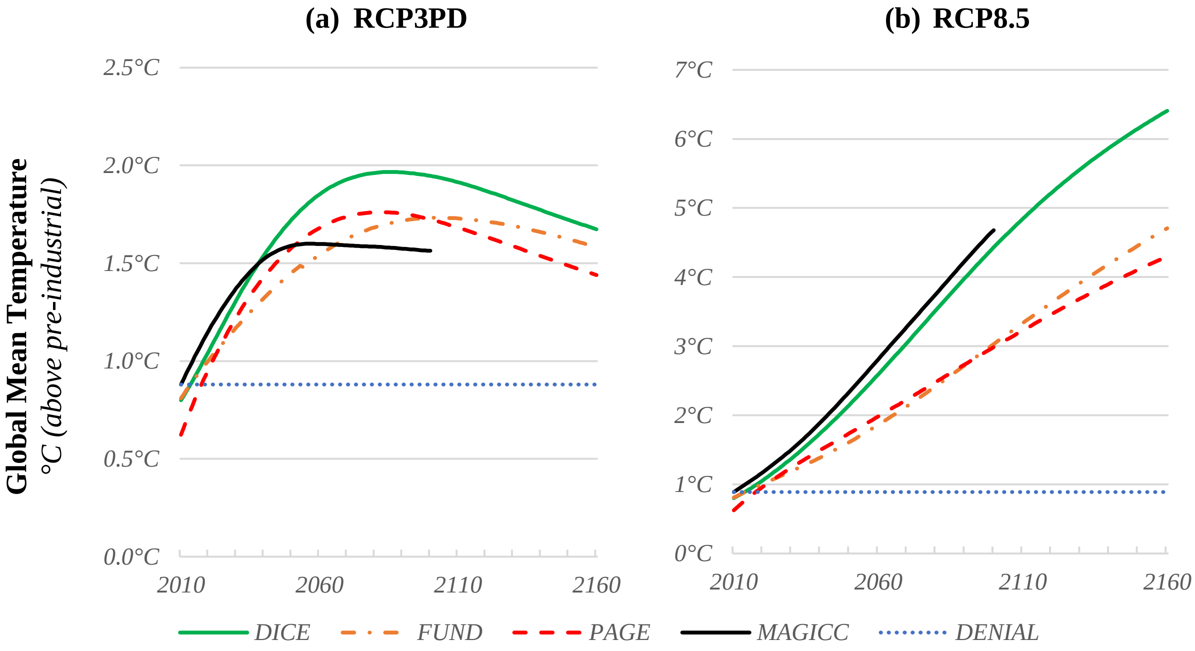


Figure 1: Temperature responses to RCP fossil carbon emissions across climate models

3. A global model of economic growth, energy transitions and climate damages

In previous model-comparison exercises temperature responses for given emission paths and economic growth paths were studied for each of the temperature modules. Here we take a complementary approach and use the same emissions generating processes, i.e., economic module, to study the effects on climate change. By harmonising economic assumptions, we can isolate and pinpoint the differences in estimates for the optimal price of carbon due to different ways of modelling the climate, which arguably is outside the economists' expertise. Given that we are using a global model of economic growth and energy transitions, we are also the first to study optimal mitigation policy across different climate models and temperature modules.

Our model of the global economy is neoclassical and follows Rezai and van der Ploeg (2017a). All firms maximise profits under perfect competition. Producers of final goods employ labour, L_t, capital, K_t, and energy, E_t, to produce output, $Y(K_t, L_t, E_t)$ with $Y(.)$ constant returns to scale, concave and satisfying the Inada conditions. They have the choice of sourcing energy from fossil fuel firms, F_t, and clean energy firms, R_t, so $E_t = F_t + R_t$. Fossil fuel entails carbon emissions but renewables do not. Coal, gas, and oil reserves are limited, so the stock of fossil fuel, S_t, is limited and gradually depletes as fossil fuel is extracted, $S_{t+1} = S_t - F_t$ with S_0 given. As the stock of fossil fuel falls, less accessible reserves have to be accessed and the unit cost of extraction in terms of units of final goods, $G(S_t)$, rises, hence we assume that $G'(S_t) < 0$. Although renewable energy is initially more expensive due to their relative infancy and such problems as intermittency, its unit cost, $W(B_t)$, however, has the potential to fall as learning and efficiency increase in cumulative use, B_t, where $B_{t+1} = B_t + R_t$ with B_0 given and $W'(B_t) < 0$. This together with the rising cost of fossil fuel as reserves are depleted implies there comes a point where renewable energy becomes cheaper than fossil fuel at which point the economy transitions from fossil fuel to renewable energy.

Past fossil fuel emissions lead to global warming as outlined for each of the temperature modules discussed in section 2. A short-hand description of this relationship is captured by the function M_m where m = DICE, FUND, PAGE or DENIAL stands for the climate model that is used:

$$T_{t,m} = M_m\left(\{F_i\}_{i=0}^{t-1}\right) \tag{1}$$

Global warming negatively impacts economic output through direct and indirect material and immaterial damages. Following DICE we assume that higher temperature increases the share of output lost due to climate change, so usable output is $O(L_t, K_t, F_t + R_t, T_t) = Z(T_t)Y(L_t, K_t, F_t + R_t)$ with $Z' < 0$. What remains after costs of energy generation is used for aggregate consumption, C_t, and gross investment which consists of net investment $K_{t+1} - K_t$, and depreciation δK_t where δ is the depreciation rate of manmade physical capital:

$$K_{t+1} = (1-\delta)K_t + O(K_t, L_t, F_t + R_t, T_t) - G(S_t)F_t - W(B_t)R_t - C_t. \tag{2}$$

Households have rational expectations and make investment-consumption decisions to maximise their discounted stream of utility from consumption, $\sum_{t=0}^{\infty}(1+\rho)^{-t} L_t \left[\frac{(C_t / L_t)^{1-\eta} - 1}{1-\eta}\right]$, where $\rho > 0$ is the rate of pure time preference and $\eta > 0$ is the constant coefficient of relative intergenerational inequality aversion under CES preferences. We suppose that the government has a sufficient set of instruments available to implement the social optimum in a decentralised market economy, so if it sets the global price of carbon equal to the social cost of carbon (SCC) and the clean energy subsidy to the social benefit of learning by doing in renewable energy production (SBL), the market outcome will correspond to the first best. The government can set the price of carbon to the SCC in various ways; for example, by setting the specific carbon tax or setting up a competitive market for carbon emission permits. Any excess of carbon tax or carbon emissions permits revenue over the cost of financing renewable energy subsidies is rebated as lump sums to the private sector.

The first-best global optimum satisfies the efficiency conditions for energy use

$$O_{E_t}(K_t, L_t, E_t, T_t) \le G(S_t) + \theta_t^S + \theta_t^E, \qquad F_t \ge 0, \quad \text{c.s.}, \tag{3a}$$

$$O_{E_t}(K_t, L_t, E_t, T_t) \le W(B_t) - \theta_t^B, \qquad R_t \ge 0, \quad \text{c.s.}, \tag{3b}$$

where the scarcity rent, θ_t^S, the $\text{SCC} = \theta_t^E$ and the $\text{SBL} = \theta_t^B$ are, respectively,

$$\theta_t^S = -\sum_{s=0}^{\infty}\left[G'(S_{t+1+s})F_{t+1+s}\Delta_{t+s}\right], \tag{4a}$$

$$\theta_t^E = -\sum_{s=0}^{\infty}\left[O_{T_{t+1+s}}(K_{t+1+s}, L_{t+1+s}, E_{t+1+s}, T_{t+1+s})M_{F_{t+s}}\Delta_{t+s}\right], \tag{4b}$$

$$\theta_t^B = -\sum_{s=0}^{\infty}\left[W'(B_{t+1+s})R_{t+1+s}\Delta_{t+s}\right], \tag{4c}$$

with the compound discount factors given by $\Delta_{t+s} \equiv \prod_{s'=0}^{s}(1+r_{t+1+s'})^{-1},\ s \geq 0$. In equilibrium firms hire energy until the marginal revenue product equals marginal cost (see equations (3)). The unit social cost of fossil fuel is extraction cost, $G(S_t)$, plus scarcity rent, θ_t^S, plus the SCC (see (3a)). Here the scarcity rent is the present discounted value of all future increases in extraction costs resulting from depleting one unit of fossil fuel today (see equation (4a)) and the SCC is present discounted value of all global warming damages from burning one ton of carbon today (see equation (4b)). The SCC is conditional on the climate model *M*, most importantly global mean temperature's response to emissions. Fossil fuel is thus used as long as the marginal product of fossil fuel equals the extraction cost of fossil fuel plus the scarcity rent on fossil fuel plus the social cost of carbon (or the carbon tax in decentralised market equilibrium as we will see later). If the marginal product of fossil fuel is too low, fossil fuel is not used.

Similarly, the unit cost of renewable energy consists of the unit cost of generation which falls over time due to learning by doing minus the SBL (see (3b)), which is the present discounted value of the all future learning-by-doing cuts in the cost from using one unit of renewable energy today (see equation (4c)). If the marginal product of renewable energy falls short of the production cost of renewable energy minus the social benefit of learning (or the renewable energy subsidy in decentralised market equilibrium), then renewable energy is not used in final goods production. As soon the marginal product of renewable energy equals the cost of renewable energy minus the SBL, renewable energy is used in the production process. Our cost parameters are set in such a way that it is optimal to have a phase of fossil fuel use followed by a phase of renewable energy use (the carbon-free era). The transition time between the two phases is determined optimally.

The Keynes-Ramsey rule for the per-capita consumption growth rate is given by

$$\frac{C_{t+1}/L_{t+1}}{C_t/L_t} = \left(\frac{1+r_{t+1}}{1+\rho}\right)^{1/\eta}, \quad r_{t+1} \equiv O_{K_{t+1}} - \delta \tag{5}$$

The Keynes-Ramsey rule (5) indicates that per-capita consumption growth is high if the interest rate, *r*, is high, the pure rate of time preference is low, and intergenerational inequality aversion is small, because then it is worthwhile more to save and postpone consumption. This saving motive is especially strong if intergenerational inequality aversion is small or equivalently intertemporal substitution is high. On a steady growth path (5) implies that the interest rate to be used for calculating the SCC and the SBL equals (approximately) the rate of time preference plus intergenerational inequality aversion times the growth rate in GDP per capita. Hence, more rapid growth implies that future generations are richer than current generations and current generations have less appetite to price carbon or subsidise renewable energy, especially if the coefficient of relative intergenerational inequality aversion is large. This follows directly from the higher discount rate that must be used to calculate the SCC and the SBL.

For a mathematical derivation of the social optimum for a particular climate model *M*, see Rezai and van der Ploeg (2017a). The emission of fossil fuel results in global warming damages to world economic production. These damages are not internalised by the agents making the emission decision which is why the government needs to force the price of emissions on those agents either through levying a tax on emissions, setting a cap on the overall permissible amount, or direct regulation and control. Similarly, the benefits of learning by doing in the production of renewable energy are not internalised by the market. Hence, the government needs to step in and offer a renewable energy subsidy that is set equal to the SBL. Revenue from the carbon tax minus net expenditures on renewable energy subsidies are rebated to the private sector in non-distorting lump-sum transfers. With these policies in place the government is able to replicate the social optimum in a decentralised market economy.

For purposes of our illustrative numerical simulations, table 2 summarises the functional forms and calibration assumptions we have made for our economic module. Preferences and final goods production are described by CES functions. Intergenerational inequality aversion is set to its conventional value of 2 and the elasticity of factor substitution is 0.5 which is the midrange of values reported in Kander and Stern (2014) and Frieling and Madlener (2016). Population starts at its current value of 7 billion and flattens out at 11 billion people consistent with recent UN projections (UN, 2015). Harrod-neutral productivity grows at a rate of 1% per year initially and stabilises at 3 times its current value.

The calibration of the energy sector follows Rezai and van der Ploeg (2017b). Fossil fuel extraction costs are calibrated to give an initial share of energy in GDP around 6% depending on the policy scenario, which translates to fossil production costs of $350/tC ($35/barrel of oil), and assumes that production costs double if a total 2000 GtC is extracted from the initial stock of reserves of 4000 GtC. We suppose that the cost of complete de-carbonisation is 8% of world GDP and that 14% of world GDP would go to the energy sector if all carbon emissions where mitigated today, hence with $\sigma = 0.15$ we have $W(0) = b_0 + b_\infty = 0.94$. Through learning by doing this cost can be reduced by 60%, so that $W(\infty) = b_\infty = 0.564$ and thus $b_0 = 0.376.$ We suppose a mere 1.25% reduction in cost would occur if all of world energy use would be supplied by renewable sources in the initial period, so that $b_1 = 0.00375$. Together with the assumption about fossil energy, this biases against a process of rapid de-carbonisation of the global economy.

CES preferences: Pure rate of time preference ρ = 1 % per year, coefficient of relative inequality aversion η = 2
Global warming damages: $Z(T_t) = \left(1 + 0.00245 T_t^2 + 5.02110^{-6} T_t^{6.76}\right)^{-1}$
CES final goods production function: $Y(L_t, K_t, E_t) = A_t \left[(1-\beta)\left(AK_t^{\alpha}(L_t)^{1-\alpha} \right)^{1-1/\vartheta} + \beta(\frac{F_t + R_t}{\sigma})^{1-1/\vartheta} \right]^{\frac{1}{1-1/\vartheta}}$ Initial capital stock \$150 trillion, depreciation rate of the capital stock δ = 10% per year Capital share α = 30%, energy share β = 6%, elasticity of factor substitution ϑ = 0.5 Initial world GDP and energy use in 2010 are \$63 trillion and 9 GtC Calibrate A = 3.78 and σ = 0.15 GtC/\$T
Drivers of economic growth: Population growth $L_t = 11 - 4e^{-7\,0.01/2.1t}$ billion, so L_0 in 2010 is 7 billion Rate of technical progress is 2%/year, $A_t = 1.02^t$
Energy costs: Initial stock of fossil fuel reserves S_0 = 4000 GtC, fossil fuel extraction cost $G(S_t) = 0.35 S_0 / S_t$ Renewable production cost $W(B_t) = 0.564 + 0.376\mathrm{e}^{-0.00375 B_t}$

Table 2: Functional forms and calibration of our module of the global economy

Finally, our specification of global warming damages are taken from Weitzman (2010) and Dietz and Stern (2015) who argue that damages rise more rapidly at higher levels of temperature than suggested by Nordhaus (2008), who has combined detailed micro cost estimates to get aggregate macro costs of 1.7% of world GDP when global warming is 2.5 ^{o}C. Ackerman and Stanton (2012) recalibrate Nordhaus (2008) assuming that damages are 50% of world GDP at 6^{o} C and 99% at 12.5^{o} C.

In contrast, to DICE2013R our IAM described above has two market failures, namely the lack of internalising the external benefits from learning by doing as well those from carbon emissions, and consequently the optimal policy requires to supplement the carbon tax with a renewable energy subsidy. Our IAM has compared to DICED2013R also more substitution possibilities between energy and the other factors of production and highlights the scarcity rents on fossil fuel reserves. The latter are crucial, since these will be strongly eroded by climate policies. The insights of our analysis carry over to other economic modules, so our choice of module should be seen as illustrative.

4. Optimal climate policy and carbon price across the four temperature modules

We now combine the economic module presented in section 4 with each of the four temperature modules discussed in sections 2 and 3. We thus derive the optimal climate policies for each of these four temperature modules and compare the resulting equilibrium trajectories. We derive the optimal policies by maximising welfare given the decentralised equilibrium conditions in equations (3)-(5) and subject to the transition equations of the stocks of conventional capital and renewable knowledge and one of the climate and temperature modules of section 2.

Table 3 and figure 2 present the welfare-maximising carbon taxes and renewable energy subsidies for each of the DICE, PAGE, FUND and DENIAL temperature modules. Carbon prices rise sharply in the next 100 years and plateau well into the post-carbon era once global mean temperature increases have stabilised and receded. Carbon taxes under the DENIAL view are zero. Optimal renewable subsidies are aggressive and short-lived with peak subsidies occurring mostly within the next 30 years.

Temperature Module	Carbon price, τ_t		Renewable subsidy, v_t		End of Fossil Era	Carbon Budget	Peak Warming
	Initial	max.	initial	max.			
DICE	76 $/tC	432 $/tC	154 $/tCe	403 $/tCe	2045	401 GtC	2.2 °C
PAGE	47 $/tC	125 $/tC	122 $/tCe	426 $/tCe	2056	581 GtC	2.1 °C
FUND	51 $/tC	146 $/tC	127 $/tCe	423 $/tCe	2054	546 GtC	2.0 °C
DENIAL	0 $/tC	0 $/tC	72 $/tCe	408 $/tCe	2082	1094 GtC	0.9 °C

Table 3: Optimal policy, transition times, carbon budget and peak warming

The initial carbon price and renewable energy subsidy are highest under the DICE temperature module (76$/tC and $154 $/tCe) and significantly lower under the PAGE (47$/tC and $122 $/tCe) and the FUND temperature module (51$/tC and 127/tCe). This is most likely due to the omission of non-carbon greenhouse gases in our rendition of the various temperature modules. The inclusion of other greenhouse gases in our calculation would bring policy in the PAGE and FUND temperature modules in line with those of the DICE module. The finding that the FUND and PAGE temperature modules yield similar policy levels is surprising, because differences between these two models were largest as used in the Inter-Agency Working Group summary report (IAWG, 2013) in which also the economic motors were allowed to differ across models.

Under the DENIAL climate module, carbon prices are zero because there is no anthropogenic climate change. Renewable energy subsidies are still relevant to facilitate learning in the renewable energy sector. But the optimal renewable subsidy starts off at a lower level (72 $/tCe) and leads to a later phasing out of fossil fuel energy than when there is human-made climate change.

The decentralised equilibrium trajectories implied by the cases of optimal climate policies and business as usual with no climate policies are presented in figure 3. Optimal policies manage to limit climate change to slightly above 2 °C while having no significant impact on the trajectory aggregate consumption and welfare. Under business as usual temperature rises to almost 5 °C under the DICE temperature module but less than 4 °C under the PAGE module, and somewhere in between for the FUND module. These temperature increases are significantly less than under the RCP8.5 scenario.

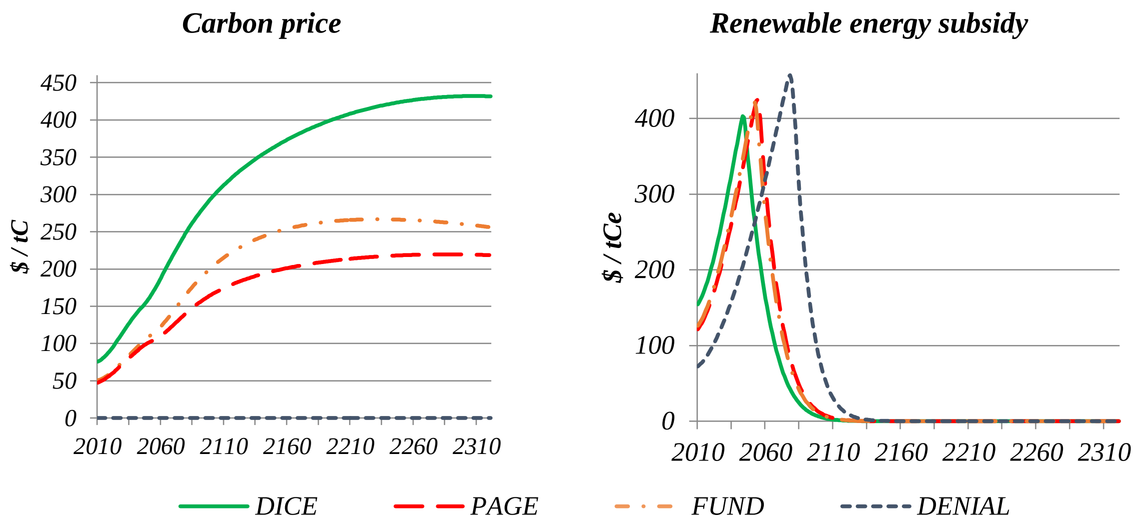


Figure 2: Optimal carbon price and renewables subsidies for various temperature modules

Cumulative carbon emissions under the optimal climate policies are significantly lower; lowest under the DICE temperature module with only 401 GtC. Since the FUND and PAGE temperature modules are less responsive to carbon emissions in the absence of other GHGs, the carbon budgets under the optimal policies for these temperature modules increase by 546 GtC and 581 GtC, respectively. Still, due to the smaller temperature responses of the FUND and PAGE temperature modules, peak warming levels are a little lower than with the DICE temperature module.

For the DENIAL climate module, the learning externality is the only market failure and carbon emissions are not priced. Consequently, cumulative carbon emissions under the climate sceptic view rise to 1,094 GtC. Note that this denialist policy introduces a green paradox in the sense that fossil fuel use under the renewable energy subsidy leads to slightly higher fossil fuel use and consequently more short-run global warming than under business as usual (Rezai and van der Ploeg, 2017a). The optimal climate policies with the other temperature policies consist of both a carbon price and a renewable energy subsidy, so do not suffer from green paradox effects.

Given our assumption of substitutability between energy and the labour-capital composite, energy use is price-sensitive and responds to differences in carbon prices and renewable energy subsidies. Fossil fuel use is lowest and phased out earliest in 2048 under the DICE temperature module with optimal climate policies. The FUND and PAGE temperature modules have similar patterns in energy use and final transition times in 2054 and 2056, respectively. The absence of carbon pricing in the DENIAL module leads to fossil fuel being used at the highest rate and the longest, with the post-carbon era starting in 2082. The patterns of fossil fuel use translate into similar scarcity rents of the owners of fossil fuel. Climate deniers may have genuine opinions given the ambiguity of scientific evidence, but they can also be viewed as using this as an excuse to protect the huge fossil fuel rents when climate deniers represent vested interests in coal, oil and gas. Climate policies work by virtue of killing off these rents as is evident from comparing the outcomes with those under business as usual.

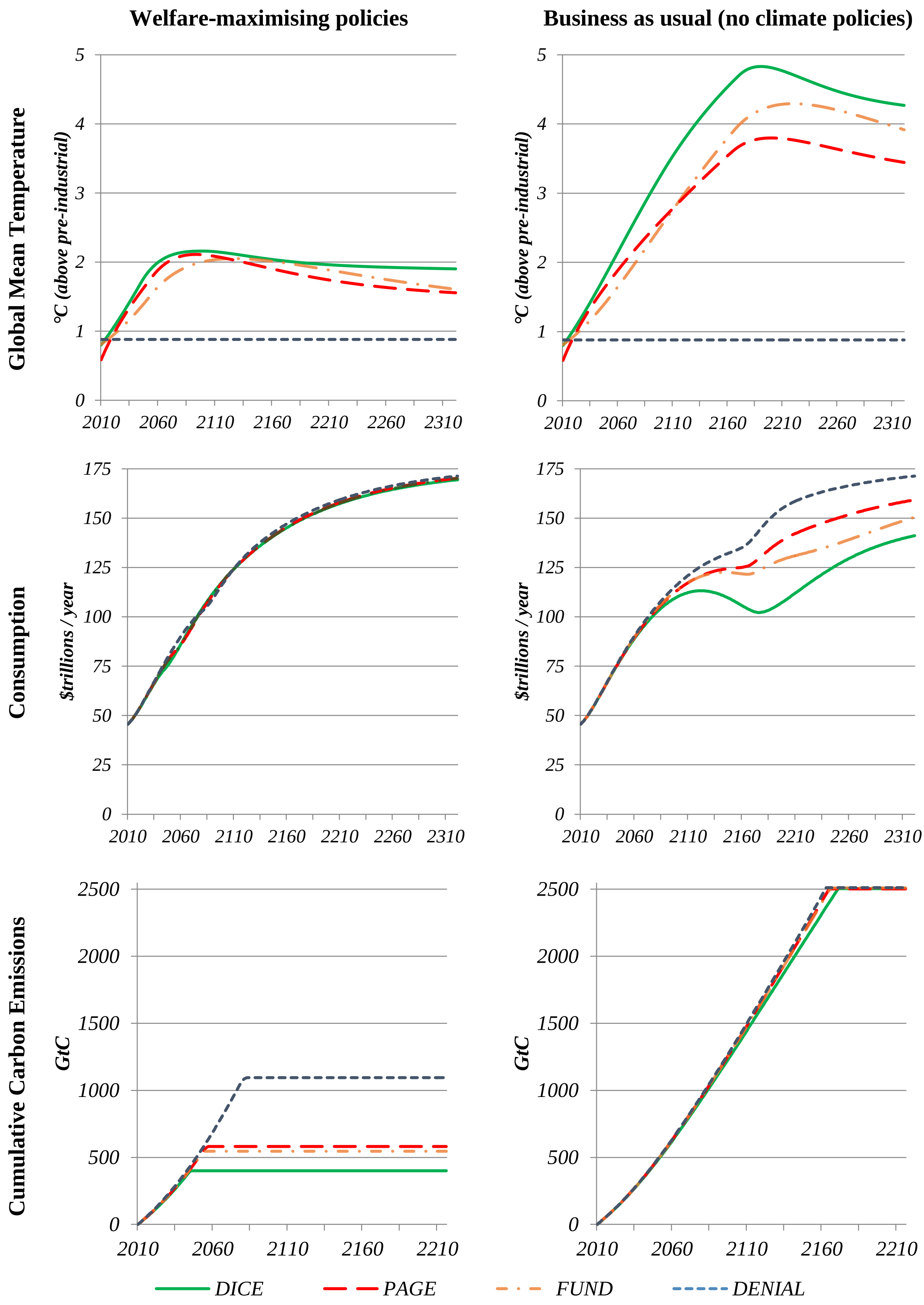
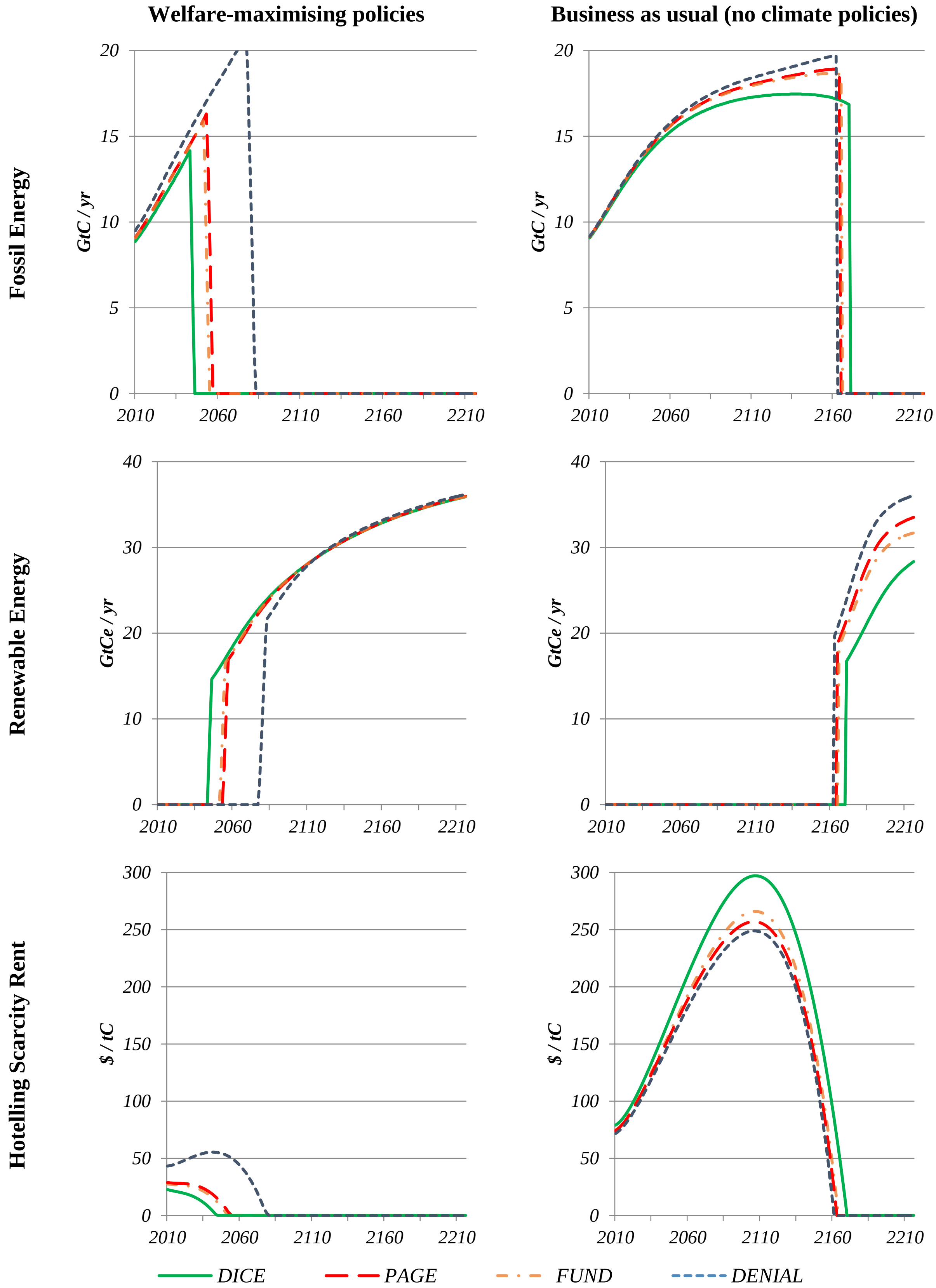


Figure 3: Policy simulations for optimal (left panel) and business as usual (right panel)



Key Baseline (RTI = 0.1%, IIA = 1, g = 2%) yields rapid de-carbonisation mid-century, limiting global warming to slightly above 2°C. Conventional economic parameters (RTI = 1%, IIA = 2, g = 2%) delay the transition by one decade and lead to temperature increases of 3°C.

Figure 3 (cont'd): Policy simulations for optimal (left panel) and business as usual (right panel)

Equilibrium trajectories of the decentralised market economy under business as usual also differ markedly across the different climate and temperature modules. In the absence of any carbon price or renewable energy subsidy, the transition to carbon-free energy is driven solely by technical cost considerations: once extraction costs reach the initial cost of renewable energy, the transition takes place. As a result, the carbon budgets are uniform across climate models. The implications of these cumulative emissions are, however, significant. Peak warming levels differ by 1 °C. Our IAM with the DICE temperature module experiences an increase in global mean temperature of 4.8 °C and severe climate damages, leading to an economic climate crisis with temporary falls in consumption and capital stock (not pictured). Peak levels are lower in our IAM with the FUND or PAGE temperature module and the implications for consumption and capital accumulation moderate. The inefficient energy transition has significant repercussions even under the DENIAL climate sceptic view where fossil fuel use does not boost global warming. The absence of industrial policy to spur technological innovation in the renewable energy sector depresses economic growth and welfare and boosts cumulative carbon emissions. Fossil fuel owners benefit from longer fossil fuel use across all climate models as can be seen from their scarcity rents on fossil fuel which increase considerably.

5. Robust climate policies under climate model uncertainty

We now evaluate the four optimal climate policies derived from our common economic module and each of the four temperature policies in each of the other three IAMs corresponding to the economic module and each of the other three temperature modules. To save space, we omit the plots and simply focus on welfare and peak global warming for each of these 16 combinations of policies and models. For completeness we also add welfare and peak global warming for the business as usual case.

5.1. The max-min and max-max optimal climate policies

Table 4 gives both the welfare levels and peak global warming in the decentralised market economy of each of the optimal policies in its own model and in case another model of the climate turns out to be correct model. The min-max policy turns out to be the optimal policy with the DICE temperature module, which is not surprising as this module gives the biggest temperature responses.

Interestingly, this is also the min-max policy in terms of peak global cooling. So taking the worst outcome for each of the four optimal policies across each of the temperature modules, the DICE optimal policy is the best one. It ensures that peak global warming stays below 2.2 ^{o}C.

The max-min policy gives a prudent and robust response to climate model uncertainty. The max-max policy assumes that whatever policy is devised it is evaluated under the most optimistic possible view of the world. Both in terms of welfare and in terms of global cooling it is then clear that the policy under the DENIAL climate sceptic view, i.e., no pricing of carbon and only correction for the market failures in the renewable energy market, is the max-max policy. Peak global warming would then be

2.6 ^{o}C under the best scenario. However, if business as usual is an option too (i.e., no renewable subsidy either), this is the max-max policy in terms of global cooling (but not in terms of welfare).

(a) Welfare (utils)

	DICE policies	PAGE policies	FUND policies	DENIAL policies	Business as usual
DICE module	729.134	729.080	729.095	728.518	724.207
PAGE module	729.345	729.387	729.385	729.202	727.693
FUND module	729.394	729.425	729.426	729.186	726.980
DENIAL module	729.712	729.853	729.837	729.985	729.451
Maximum	729.712	729.853	729.837	**729.985**	729.451
Minimum	**729.134**	729.080	729.095	728.519	724.207
0.5 Max Min	729.423	**729.467**	729.466	729.252	726.829

(b) Peak global warming (°C)

	DICE policies	PAGE policies	FUND policies	DENIAL policies	Business as usual
DICE module	2.2	2.5	2.4	3.2	4.8
PAGE module	1.9	2.1	2.1	2.6	3.8
FUND module	1.8	2.1	2.0	2.7	4.3
DENIAL module	0.8	0.8	0.8	0.8	0.8
Maximum	**2.2**	2.5	2.4	3.3	4.9
Minimum	0.8	0.8	0.8	0.8	0.8
excluding DENIAL	1.8	2.1	2.0	**2.6**	3.8

Table 4: Max-min, max-max and 0.5-max-min climate policies

Finally, Arrow and Hurwicz (1977) suggest the α-min-max policy with $0 \leq \alpha \leq 1$, which is a mix of min-max and max-max policies. For example, taking a value of $\alpha = 0.5$, the best policy is to take the optimal climate policies derived from using the PAGE (very closely followed by the FUND) temperature module. This is not surprising, since these modules have temperature responses in between that of the DICE and DENIAL temperature modules.

5.2. The min-max regret optimal climate policies

Table 5 calculates from table 4 the regret from implementing a particular set of policies when the climate turns out to be described by a different model from what was assumed when deriving this set of policies. The optimal climate policies derived with the PAGE temperature module then correspond to the max-min regret policies. If the DENIAL view is excluded the optimal climate policies derived with the FUND temperature module turn out to be the min-max regret policies.

Max-min regret policies induce less ambitious climate policies than min-max and to less ambitious policies than max-max policies. Min-max thus introduces more caution than max-min regret policies.

The qualitative conclusions regarding the nature of the max-min and the min-max regret policies when considering welfare (or min-max and max-min regret when considering peak warming) are unaffected when parameters such as the pure rate of time preference, the coefficient of relative intergenerational inequality aversion, the elasticity of substitution between factors of production, the energy costs and their rates of declines and the rate of future population growth are varied.

	DICE policies	PAGE policies	FUND policies	DENIAL policies	Business as usual
DICE module	0	0.054	0.039	0.616	4.927
PAGE module	0.042	0	0.002	0.185	1.694
FUND module	0.032	0.001	0	0.240	2.446
DENIAL module	0.273	0.132	0.148	0	0.534
Maximum	0.273	**0.132**	0.148	0.616	4.927
excluding DENIAL	0.042	0.054	**0.039**	0.616	4.927

Table 5: Welfare and min-max regret climate policies

6. Conclusions

The main insight arising from our analysis is that robust climate policies, whether min-max or max-min regret, price carbon even if President Trump's climate sceptic view is taken account of. The danger in terms of lost production and welfare of not taking action to fight global warming is simply too high in the event that the climate sceptic view turns out to be wrong whilst the efficiency losses of pricing carbon when the climate sceptic view turns out to be correct are only modest. The max-min approach leads in our illustrative calibration of the various temperature modules and economic modules to the optimal policy predicted by the DICE temperature module, since this is the one with the largest temperature responses. The min-max regret approach leads to less ambitious climate policies corresponding to the optimal climate policies predicted by the PAGE module (or by the FUND temperature module if the climate sceptic view is excluded). The max-min or min-max regret climate policies thus rely on a non-sceptic view of global warming and lead to a substantial and moderate amount of caution in climate policies, respectively.

Our economic module has been chosen for its ability to starkly distinguish the switch from the fossil fuel era to the carbon-free era, to allow for technical change in the production of renewable energies, and to predict the determinants of scarcity rents on fossil fuel. The latter is crucial, since effective climate policies will destroy these rents and thus threaten the vested interests of the fossil fuel lobby and possibly those of climate sceptics active in industry and politics too. A potential weakness of our analysis is that all types of fossil fuel are lumped into one. However, President Trump and the climate sceptics that follow him are big advocates of reviving the coal industry, which is per unit of energy much more harmful to global warming. If coal is separated out from oil and gas, rents on it will be much lower and therefore under the DENIAL policy there will be much more exhaustion of coal up to the limit and a corresponding later arrival of the carbon-free era.

Our non-probabilistic analysis is only illustrative, but we hope that in future inter-comparison analyses of large-scale IAMs more attention will be paid to max-min and min-max regret policies. It is straightforward to then extend our analysis to include not only economic model uncertainty as well as climate model uncertainty and also to include more generally scientific uncertainties. It is also feasible to extend our analysis to allow for statistical uncertainties about the future growth of the economy, equilibrium climate sensitivity, the transient climate response, the onset of catastrophic shocks, damages to health and aggregate production, etc. Some of these uncertainties can be modelled by normal distributions or Brownian motion but many of them will need to take account of fat tails in distributions and need to be modelled by Poisson shocks or regime shifts.

In future research one could deal with scientific uncertainties by assigning prior subjective probabilities to each of the IAMs being the correct one (and make the bold assumption that there are no other models describing climate-economy interactions) and employing recursive smooth ambiguity preferences (Klibanoff et al., 2009).[9] Rather than maximising expected utility, policy makers then maximise a concave transformation of utility. The optimal robust climate policies then depend on the coefficient of relative ambiguity aversion (or climate model uncertainty aversion) and these prior probabilities.[10] This approach is a way of hedging against model uncertainty.

With infinite climate model uncertainty aversion one gets the max-min climate policy over all possible climate policies (instead of just the optimal policies from the DICE, FUND and PAGE temperature modules). No climate model uncertainty aversion corresponds to expected utility maximisation. In general, climate model uncertainty aversion lies in between these extremes. Smooth ambiguity preferences assumes that one can assign prior probabilities to each of the climate models being the correct one and thus that one can summarise all there is to know about the climate with a single probability density function. This is unlikely to be feasible. Hence, the max-min approach which does not need these prior probabilities is an attractive and pragmatic approach for getting climate policies that are robust to climate model uncertainty.

Finally, our analysis needs to be extended to allow for learning about the subjective prior probabilities of each model being the correct one. This is especially relevant for climate sceptics. They have an incentive to price carbon and cut emissions, possibly even more than believers, as this generates information on the causes of global warming and is less costly than boosting emissions due to the irreversibility of climate change (van Wijnbergen and Willems, 2015). However, the slow response in the climate system, both in terms of warming and cooling due to decarbonisation, make learning difficult. For example, with emissions being high, the oceans have been storing heat which in future

[9] An alternative is to use a more sophisticated max-min approach with non-unique priors developed by Gilboa and Schmeidler (1989). The axiomatic foundations and their relevance to climate policy of these various ways of dealing with ambiguity in climate policy are discussed by Heal and Millner (2014, 2015) and Millner (2013).

[10] In macroeconomics model uncertainty and the design of robust monetary policies has been studied intensively following the seminal work of Hansen and Sargent (2007). In climate economics this approach has been applied by Xepapadeus (2012) and Li et al. (2016) to obtain robust paths for the optimal price of carbon.

will lead to heating of the planet. Depending on the existing model priors, climate sceptics could see a failure of changes in temperature dynamics under climate policy as validation of their views. We leave the study of such potentially perverse effects of learning for future research.

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Appendix - Details on Temperature modelling in three prominent IAMs

A.1. Temperature module of the DICE-2013R IAM

In the carbon cycle, carbon diffuses between the atmosphere (M_{AT}), and the upper (M_{UP}) and lower (M_{LO}) parts of the oceans following a Markov process. Total carbon emissions E entering the atmosphere are the sum of emissions from fossil fuel use (E_{ind}) and exogenous land use (E_{land}). According to IPCC (2014, Science, Chapter 6), land use changes currently contribute 3 GtC/yr. The IAM DICE-2013R assumes that these emissions continuously fall by 4.3% per year.

From the atmosphere carbon emissions transition according to the carbon cycle described by

$$\begin{pmatrix} M_{\mathrm{AT}}(t) \\ M_{\mathrm{UP}}(t) \\ M_{\mathrm{LO}}(t) \end{pmatrix} = E(t)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \mathbf{\Phi}\begin{pmatrix} M_{\mathrm{AT}}(t-1) \\ M_{\mathrm{UP}}(t-1) \\ M_{\mathrm{LO}}(t-1) \end{pmatrix}, \tag{A.1}$$

where the state transition matrix is $\mathbf{\Phi} \cong \begin{pmatrix} 0.981436 & 0.0080939 & -1.123\ 10^{-6} \\ 0.018583 & 0.991397 & 0.0000687 \\ -1.91\ 10^{-5} & 0.0005089 & 0.999932 \end{pmatrix}$.[11] The carbon cycle is structured such that interaction between the atmosphere and the lower oceans only occurs through the upper oceans. Since the columns of $\mathbf{\Phi}$ add up to 1, one has $M(t) = E(t) + M(t-1)$ with $M(t)$ the total stock of carbon in the system at time t. Any carbon resulting from taking and burning fossil fuel out of the ground thus ends up in the atmosphere or the oceans. In equilibrium only 4.9% of emissions remains in the atmosphere, 11.3% transitions to the upper, and 83.8% transitions to the lower oceans (from the top row of $(\mathrm{I} - \mathbf{\Phi})^{-1}$). Initial conditions for the carbon stocks in 2010 are $M_{\mathrm{AT}}(0) = 830.4$ GtC, $M_{\mathrm{UP}}(0) = 1527$ GtC, and $M_{\mathrm{LO}}(0) = 10010$ GtC.

Nordhaus (2014) takes a first-order approximation of radiative forcing, $F(t)$, resulting from deviations from the pre-industrial stock of atmospheric carbon and captures the evolution of other greenhouse

[11] The semi-decadal matrix in DICE is $\mathbf{\Phi}_5 = \{\{0.912, 0.0383289, 0\}, \{0.088, 0.959171, 0.0003375\}, \{0, 0.0025, 0.999663\}\}$. Since this is a closed carbon cycle, we can diagonalise $\mathbf{\Phi}_5 = \mathrm{P\ D\ P}^{-1}$, where D and P are the diagonal matrix of eigenvectors and the matrix of eigenvectors. The annual transition matrix $\mathbf{\Phi} = \mathrm{P\ D}^{1/5}\ \mathrm{P}^{-1}$.

gases (e.g., CH_4, N_20, SO_2 and SF_6) and forcing components such as ozone and albedo by the exogenous forcing component, $F_{\mathrm{EX}}(t)$:

$$F(t) = \eta_2 \mathrm{Ln}\left[\frac{M_{\mathrm{AT}}(t)}{M_{\mathrm{AT1750}}}\right] / \mathrm{Ln}[2] + F_{\mathrm{EX}}(t) \tag{A.2}$$

with $\eta_2 = 3.8$ W/m² and $M_{\mathrm{AT1750}} = 588$ GtC.[12] Exogenous forcing amounts to at most 15% of total forcing and increases linearly from 0.25 W/m² in 2010 to 0.7 W/m² in 2200 after which it is assumed constant (based on the RCP 6.0 W/m² representative scenario). Nordhaus (2014) acknowledges the difficulty of accounting for the forcing components not directly related to CO_2 in modelling DICE.

Radiative forcing directly increases global atmospheric temperature, $T_{\mathrm{AT}}(t)$, while some of this additional energy is taken up by the ocean over time, increasing oceanic temperature, $T_{\mathrm{LO}}(t)$. This second temperature box for the deep oceans approximates oceanic diffusion process in DICE, while atmospheric temperature in FUND and PAGE is modelled more simply as a lag relationship to capture the delayed temperature response to increases in carbon levels. As heat moves between the two media, temperatures converge. Radiative forcing increases atmospheric temperature linearly, so that

$$\begin{pmatrix} T_{\mathrm{AT}}(t) \\ T_{\mathrm{LO}}(t) \end{pmatrix} = \begin{pmatrix} T_{\mathrm{AT}}(t-1) \\ T_{\mathrm{LO}}(t-1) \end{pmatrix} - \xi_1 \left[\Delta(t-1) \begin{pmatrix} \xi_3 \\ -\xi_4/\xi_1 \end{pmatrix} + \begin{pmatrix} F(t) - \xi_2 T_{\mathrm{AT}}(t-1) \\ 0 \end{pmatrix} \right] \tag{A.3}$$

where $\Delta(t) \equiv T_{\mathrm{AT}}(t) - T_{\mathrm{LO}}(t)$ defines the difference in atmospheric and oceanic temperature. Equilibrium climate sensitivity (ECS) is set to 2.9°C for doubling of the CO2 stock in DICE-2013R and Nordhaus (2014) calibrates the temperature dynamics to give a transient climate response (TCR) at 2.9°C of 1.7°C.[13] In equilibrium one has $T_{\mathrm{AT}} = T_{\mathrm{LO}} = F/\xi_2$. We restate (3) in order to annualise the temperature dynamics,

$$\begin{pmatrix} T_{\mathrm{AT}}(t) \\ T_{\mathrm{LO}}(t) \end{pmatrix} = \Upsilon \begin{pmatrix} T_{\mathrm{AT}}(t-1) \\ T_{\mathrm{LO}}(t-1) \end{pmatrix} + \Theta\, F(t) \tag{A.3'}$$

where the state transition matrix is $\Upsilon \cong \begin{pmatrix} 0.970932 & 0.00535475 \\ 0.00184718 & 0.99493 \end{pmatrix}$ and $\Theta \cong \begin{pmatrix} 0.0207734 \\ -0.0002172 \end{pmatrix}$.[14]

A.2. Temperature module of the FUND IAM version 3.7

The FUND model (version 3.7; Anthoff and Tol, 2013) disaggregates the world economy into 16 regions and allows for region-specific growth rates of the economy, emissions, and damage trajectories. Global warming damages are disaggregated to account for changes in agriculture,

[12] Radiative forcing equations are gas-specific. IPCC (2001, Chapter 6) provides a general overview and lists the most important relationships in Table 6.2. FUND and PAGE model the warming contributions of gases other than CO_2 explicitly.
http://www.grida.no/publications/other/ipcc_tar/?src=/climate/ipcc_tar/wg1/222.htm.
[13] The Transient Climate Response (TCR) measures the sensitivity of temperature to increases in atmospheric carbon and is defined as the temperature increase due at the point of CO2 doubling if concentration was to increase by 1% per year until it reaches twice the pre-industrial level.
[14] The semi-decadal matrix in DICE is $\Upsilon_5 = \{\{1 - \xi_1\xi_2 - \xi_{1\xi_3}, \xi_{1\xi_3}\}, \{\xi_4, 1 - \xi_4\}\}$ with $\xi_2 = 1.31$, $\xi_1 = 0.98$, $\xi_3 = 0.088$, and $\xi_4 = 0.025$.. We again diagonalise Υ_5 =P D P^{-1}. The annual transition matrix Υ =P D$^{1/5}$ P^{-1}.

availability of water resource, and loss of wetland due to sea level rises. FUND also captures the impacts of climate change on ecosystems and biodiversity and on human health and mortality rates, the latter through estimating increases in vector-borne diseases such as malaria and diarrhoea, and cardio-vascular and respiratory mortality. Given our interest in the global climate model, we borrow from the FUND model only the global geophysical component governing changes in temperature which is uniform across regions.

The climate cycle put forward in FUND is elaborate. It includes the effects of all major greenhouse gases, i.e., CH_4, N_2O, SF_6, SO_2 and CO_2, on radiative forcing. The transition dynamics of each gas are approximated using boxes with geometric depletion. There are five boxes to describe the dynamics of CO_2, only one box for CH_4, N_2O and SF_6, and no box for SO_2 (as the residence time of SO_2 is very short with 95% dissipated within 1 to 4 days). With E(t) still denoting carbon emissions, B_i denote the stock of carbon in box i, $(1 - b_i)$ the yearly depreciation rate and e_i the share of emissions entering box i, then the 5-box CO_2 cycle is described by

$$B_i(t+1) = b_i\, B_i(t) + e_i E(t) \tag{A.4}$$

with $b = (1,\ 0.9972,\ 0.9866,\ 0.9429,\ 0.6065)$ and $e = (0.13,\ 0.2,\ 0.32,\ 0.25,\ 0.1)$ where dissipation rates are based on mean lifetimes of $(\infty,\ 363,\ 74,\ 17,\ 2)$ years. This parametrisation implies that 50% of CO_2 emissions is removed within 45 years and 13% of emissions remains in the atmosphere forever. The stock of CO_2 in the atmosphere is the sum of CO_2 in all of the 5 boxes,

$$CO_2(t) = \textstyle\sum_i B_i(t). \tag{A.5}$$

Concentrations of CH_4, N_2O and SF_6, measured relative to pre-industrial levels, follow from

$$C_i(t+1) = c_i\, C_i(t) + f_i\, F_i \tag{A.6}$$

with $c = (0.883721,\ 0.991667,\ 0.999688)$ and $f = (0.3597,\ 0.2079,\ 0.3984064)$ for $i =$ CH_4, N_2O, and SF_6, where depreciation rates are based on mean lifetimes of $(8.6,\ 120,\ 3200)$ years. Pre-industrial levels are $C_i^{pre} = (790,\ 285,\ 0.04)$. CH_4 is a more damaging greenhouse gas than CO_2, but has a much shorter mean lifetime. SF_6 is much longer lived than CO_2. Emissions for CH_4 and N_2O are taken from the IS92a scenario in Leggett et al. (1992) and can be abated in FUND using quadratic cost functions. Emissions of SF_6 are modelled to increase with absolute and per capita GDP. Emissions of SO_2 increase with population but decrease with per capita income. FUND has no direct options for curbing SF_6 and SO_2 emissions. Abatement of SO_2, however, is coupled to overall energy efficiency improvements and de-carbonisation efforts. For purposes of our numerical simulations, we set emissions of non-CO_2 greenhouse gases to zero to increase comparability across climate cycles and to abstract from exogenous scenarios and additional abatement options. This ensures comparability across climate cycle and gives an overly optimistic assessment of the social cost of carbon, since non-CO_2 emissions will remain positive in all but the most ambitious mitigation scenarios.

Given atmospheric concentrations of greenhouse gases, radiative forcing, $RF(t)$, is the sum of gas-specific forcing terms

$$RF(t) = \eta_2 \mathrm{Ln}\left[\frac{CO_2(t)}{CO_2^{pre}}\right] / \mathrm{Ln}[2]$$

$$+\mathfrak{r}_{\mathrm{CH}_4}\left(1+\mathrm{CH}_4^{ind}\right)\left(\sqrt{\mathrm{CH}_4(\mathrm{t})}-\sqrt{\mathrm{CH}_4^{\mathrm{pre}}}\right)-\left(\mathfrak{R}[\mathrm{CH}_4(\mathrm{t}),\mathrm{N}_2\mathrm{O}^{\mathrm{pre}}]-\mathfrak{R}[\mathrm{CH}_4^{\mathrm{pre}},\mathrm{N}_2\mathrm{O}^{\mathrm{pre}}]\right)$$

$$+\mathfrak{r}_{\mathrm{N}_2\mathrm{O}}\left(\sqrt{\mathrm{N}_2\mathrm{O}(\mathrm{t})}-\sqrt{\mathrm{N}_2\mathrm{O}^{\mathrm{pre}}}\right)-\left(\mathfrak{R}[\mathrm{N}_2\mathrm{O}(\mathrm{t}),\mathrm{CH}_4^{\mathrm{pre}}]-\mathfrak{R}[\mathrm{CH}_4^{\mathrm{pre}},\mathrm{N}_2\mathrm{O}^{\mathrm{pre}}]\right)$$

$$+\mathfrak{r}_{\mathrm{SF}_6}\left(\sqrt{\mathrm{SF}_6(\mathrm{t})}-\mathrm{SF}_6^{\mathrm{pre}}\right)-\mathfrak{S}[\mathrm{SO}_2(\mathrm{t})]+c_{RF} \tag{A.7}$$

with $\eta_2 = 3.7,\ \mathfrak{r} = (0.036, 0.12),\ \mathfrak{R}[\mathrm{x},\mathrm{y}] = 0.47\ \mathrm{Ln}[1 + 2.01\ 10^{-5}\ (\mathrm{x\,y})^{0.75} + 5.31\ \mathrm{x}(\mathrm{x\,y})^{1.52}]$, and $\mathfrak{S}[\mathrm{SO}_2(\mathrm{t})] = 0.03\frac{\mathrm{SO}_2(\mathrm{t})}{14.6} + 0.08\ \mathrm{Ln}\left[1+\frac{\mathrm{SO}_2(\mathrm{t})}{34.4}\right]/\mathrm{Ln}\left[1+\frac{14.6}{34.4}\right]$.[15] As we are only considering carbon emissions in our simulations and assume that sulphur dioxide emissions are fully abated, we can ignore its forcing component and therefore set $\mathfrak{S}[\mathrm{SO}_2(\mathrm{t})] = 0$. The values for c_{RF} and CH_4^{ind} differ between the model source code and its documentation. We follow the source code with $c_{RF} = 0$ and $\mathrm{CH}_4^{ind} = 0.4$.

The FUND model linearly approximates the complex interactions of different heat reservoirs by introducing a lagged temperature response relative to pre-industrial levels to radiative forcing:

$$T(t+1) = \left(1-\frac{1}{\varphi}\right)T(t) + \frac{1}{\varphi}\frac{CS}{\eta_2}RF(t). \tag{A.8}$$

The equilibrium climate sensitivity, *CS*, is set to 3 and φ to 66 years, implying that half of a temperature impulse is left dissipated within slightly less than 50 years.[16] Equilibrium temperature is $T^* = \frac{CS}{\eta_2}RF^*$ which depends on all radiative forcing components. Ceteris paribus, doubling of atmospheric carbon leads to a CS °C increase in temperature. In general, equilibrium temperature is determined by the share of past emissions accumulated in the non-depreciating carbon box, hence $T^* = CS\ \mathrm{Ln}\left[\frac{0.13\ \sum_{\theta=0}^{\infty} E(\theta)}{CO_2^{pre}}\right]/\mathrm{Ln}[2] > 0$. Regional temperature levels are scaled to global mean temperature in fixed proportions and play no role in our study of global climate-economy interactions.

Returning to emissions, the FUND model also incorporates a positive feedback from temperature to CO_2 emissions (e.g., the release of methane due to melting of the permafrost). Drawing on an initial stock of potential emissions, D^{max}, biosphere emissions and the stock of potential emissions are

[15] These equations capture the physical gas-specific impacts of higher concentrations of greenhouse gases on the earth's ability to deflect energy. Higher concentrations increase the amount of energy absorbed, which gradually translates into higher atmospheric temperature. For more details see IPCC (2001), TAR, WG1, Chapter 6 and Table 6.2 in particular: https://www.ipcc.ch/ipccreports/tar/wg1/ .

[16] The FUND model assumes random distributions of various variables such as the CS. Here FUND assumes $\varphi = \max(-31.9 + 32.7\ CS - 0.00993\ CS^2, 1)$.

$$E^D(t) = e^D(T(t) - T^{2010})\frac{D(t)}{D^{max}} \tag{A.9}$$

$$D(t+1) = \max(\, D(t) - E^D(t),\ 0) \tag{A.10}$$

with $e^D = 2.6\, GtC/°C$ and $D^{max} = 1900\, GtC$. Due to the lag in global mean temperature, past emissions are projected to increase temperature by another 0.5°C even if no more fossil fuels were emitted.[17] With the positive feedback effect captured by (9) and (10), temperature would increase by an additional whopping 1.5°C. In our simulations we ignore this positive feedback and set $e^D = 0$.

A.3. Temperature module of the PAGE09 IAM

The third temperature module we consider is based on the deterministic version of the PAGE09 model (Hope, 2006, 2011). This model traces the interaction of the global economy and the climate via the interaction of four greenhouses, *g*: CO_2, CH_4, N_2O and low-concentration gases such as SF_6. Climate dynamics are regional and compound to global mean temperature. Time steps in PAGE09 increase along the time horizon with evaluations in periods *Y*: 2008, 2009, 2010, 2020, 2030, 2040, 2050, 2075, 2100 and 2150. For purposes of our illustrative simulations we choose a uniform annual time across the whole horizon. To maintain transparency, we follow the exposition of Hope (2006) for PAGE02 and its revision for PAGE09 in Hope (2011). Model time in PAGE09 starts in 2008. Initial concentrations, *C*(g,0), of all GHGs are expressed as excess concentration, *EXC*(g,0), to their pre-industrial concentration, *PIC*(g,0): $EXC(g,0) = C(g,0) - PIC(g,0)$ and translated into the amount of GHGs in the atmosphere: *RE*(g,0) = $DEN_g\ EXC(g,0)$ with DEN_g the density of each gas. The pre-industrial concentrations of CO_2, CH_4, N_2O and linear gases including SF_6 are *PIC*(g) = (278000, 700, 270, 0) ppbv and initial concentrations in 2008 are *C*(g,0) = (395000, 1860, 322, 0.11) ppbv. The specific densities for converting into megatons (Mt) are: *DEN*(g) = (7.8, 2.78, 7.8, 100000).

Emissions of greenhouse gases emanate in regions *r*.[18] They are aggregated and combined with natural emissions, *NtE* to allow for the earth's declining capacity to absorb gases and potential positive feedback effects. Natural emissions follow linearly from last periods' global temperature, *GRT*(t-1), and a gas-specific coefficient, $STIM_g$: $NtE(g,t) = STIM_g\ GRT(t-1)$ with initial natural emissions following initial temperature, $NtE(g,0) = STIM_g\ GRT(0)$. In the revision from PAGE02 to PAGE09 the positive feedback from natural CO_2 emissions has been changed to a linear feedback from excess temperature to percentage increase in atmospheric carbon. As a result in PAGE09, $NtE(CO_2) = 0$, and atmospheric carbon is multiplied by the *GAIN* factor (15) given below. For CO_2 38% of emissions is absorbed immediately and the rest enters the atmosphere, *TEA*(g,t):

$$TEA(g,t) = \frac{AIR_g}{100}\,(\mathrm{E(g,t) + NtE(g,t)}) \tag{A.11}$$

[17] Our simulations of FUND give a committed increase of 0.4 °C.
[18] PAGE expresses emissions in growth rates over the base year. We take emissions from our global economic model, so we disregard equations (4) and (5) in appendix A.1 of Hope (2006).

with *AIR* = 62 % for CO_2 but 100 % for all other gases.[19]

All gases except CO_2 dissipate fully over time. With mean residence times *RES*(g) = (10.5, 114, 1000) years, the amount of CH_4, N_2O, and linear gases remaining in the atmosphere at end of period are[20]

$$RE(g,t) = RE(g,t-1)e^{-1/RES(g)} + TEA(g,t)\left(1 - e^{-\frac{1}{RES(g)}}\right)RES(g). \tag{A.12}$$

A fraction of all CO_2 emissions remain in the atmosphere forever. The permanent component is tracked by cumulative emissions, *CEA*(t):

$$CEA(t) = CEA(t-1) + TEA(\mathrm{CO_2}, t) \tag{A.13}$$

with the inherited stock of past emissions $CEA(0) = CE(0)\frac{AIR(\mathrm{CO_2})}{100} = 2{,}050{,}000\,\frac{62}{100} = 1{,}271{,}000\ \mathrm{MtCO_2}$. Total carbon in the atmosphere is the sum of permanent emissions (cumulative emissions times fraction STAY = 30%) plus the transient component which is decaying with mean lifetime, *RES*(CO_2), of 73.33 years.[21]

$$RE(\mathrm{CO_2},t) = \left(1 + \frac{GAIN(\mathrm{t})}{100}\right)\left\{\begin{matrix}\left[RE(\mathrm{CO_2},t-1)e^{-\frac{1}{RES(\mathrm{CO_2})}} + TEA(\mathrm{CO_2},t)\, e^{-\frac{1}{2\,RES(\mathrm{CO_2})}}\right] \\ +STAY\left(1 - e^{-\frac{1}{RES(\mathrm{CO_2})}}\right)CEA(t)\end{matrix}\right\}. \tag{A.14}$$

The *GAIN* factor captures the reduction in natural carbon sinks as temperature rises and replaces the natural emissions from CO_2 in previous versions of PAGE, so *NtE*(CO_2) = 0 in the PAGE09 model:

$$GAIN(t) = \min[CCF\ GRT(t-1), CCCFFmax]. \tag{A.15}$$

The climate carbon feedback is a linear function of past global mean temperature with coefficient *CCF* = 9.67 %/°C and capped at the upper limit *CCFFmax* = 53.33 %/°C. $GAIN(0) = 9.67 \times 0.93 = 9$, so that past reductions in absorptive capacity have increased the current atmospheric stock of carbon by 9%. Greenhouse gas levels are converted to concentrations in order to compute changes radiative forcing and temperature. Radiative forcing, $F(g,t)$, follows from

$$F(CO_2,t) = F(CO_2,0) + FSLOPE_{CO_2}\ \mathrm{Ln}\left[\frac{C(CO_2,\mathrm{t})}{C(CO_2,0)}\right] \tag{A.16a}$$

$$F(CH_4,t) = F(CH_4,0) + FSLOPE_{CH_4}\left(\sqrt{\mathrm{CH_4(t)}} - \sqrt{C(CH_4,0)}\right) + \mathrm{OVER}(CH_4,\mathrm{t}) - \mathrm{OVER}(CH_4,0) \tag{A.16b}$$

$$F(N_2O,t) = F(N_2O,0) + FSLOPE_{N_2O}\left(\sqrt{N_2O(\mathrm{t})} - \sqrt{C(N_2O,0)}\right) + \mathrm{OVER}(N_2O,\mathrm{t}) - \mathrm{OVER}(N_2O,0) \tag{A.16c}$$

$$F(Lin,t) = F(Lin,0) + FSLOPE_{Lin}\,(C(Lin,\mathrm{t}) - C(Lin,0)) \tag{A.16d}$$

[19] PAGE smooths the emission flow between time periods by linear interpolation (Hope, 2006, Appendix A.1, equation (7)). Since time steps are uniformly annual in our analysis, we disregard this interpolation.
[20] PAGE models emissions throughout the period while *RE* captures the remaining stock at the end of the period. The term multiplying *TEA* in (12) captures within-period dissipation. At our annual time scale where *RES* is at least an order of magnitude greater than the time step, this effect is not relevant and could be ignored.
[21] Given the long half-life of transient CO_2, PAGE models emissions occurring in the middle of each time period, introducing the ½ fraction in the emissions exponential multiplying *TEA* in (5).

with $F(g,0) = (1.735,\ 0.55,\ 0.18,\ 0.022)$, $FSLOPE_g = (5.5,\ 0.036,\ 0.12,\ 0.2)$ and $\mathrm{OVER}[x,y] = -0.47\,\mathrm{Ln}[1 + 2.01\ 10^{-5}\ (x\,y)^{0.75} + 5.31\ x(x\,y)^{1.52}]$.

Total forcing, *FT*(t), is the sum of gas-specific forcing and an exogenous component, *EXF*(t), which increases linearly from 0.84 W/m² in 2008 to 1.89 W/m² in 2100 after which is it is constant (as in the PAGE02 model). The PAGE09 model allows exogenous forcing to vary with policy (without additional abatement costs), $FT(t) = \sum_g F(g,t) + EXF(t)$.

Temperature dynamics in PAGE are regional across which global mean temperature is averaged. Seven world regions, *r*, are considered: Europe (EU), USA (US), Other OECD (OT), Former Soviet Union and Eastern Europe (EE), China and Centrally Planned Asia (CA), India and South East Asia (IA), Africa and the Middle East (AF), and Latin America (LA). Regional equilibrium temperature, $ET(r,t)$, in each of the seven regions of the global economy *r*[22] is a linear function of total forcing, *FT*(t), and forcing from sulphate aerosols, *FS*(t):

$$ET(r,t) = \frac{SENS}{Ln[2]}\,\frac{FT(t)+FS(r,t)}{FSLOPE_{CO_2}}. \tag{A.17}$$

Equilibrium climate sensitivity, *SENS*, is calibrated to the transient climate response, *TCR* = 1.7 °C, and the median lifetime of global warming, *FRT* = 35 years, hence the equilibrium climate sensitivity is $SENS = TCR\left(1-\frac{FRT}{70}\left(1-e^{-\frac{70}{FRT}}\right)\right)^{-1}$ = 2.995 °C per doubling of atmospheric CO_2. We abstract from the cooling effect of sulphate aerosols, $FS(r,t) = 0$, and suppose that equilibrium temperature is uniform across regions $ET(r,t) = ET(t)$.

Regional temperature, $RT(r,t) = RT(r,t-1) + \left(1-e^{-\frac{1}{FRT}}\right)[ET(t) - RT(r,t-1)]$, slowly adjusts to equilibrium temperature. In PAGE02 global realised temperature, *GRT*(t), is the area-weighted sum of all regional temperatures: $GRT(t) = \frac{\sum_r RT(r,t)\,AREA(r)}{\sum_r AREA(r)}$. This was revised in PAGE09 to adjust realised regional and global temperatures, *RT*(r,t) and *GRT*(r,t), by latitude and other land-based effects of each region. To achieve this adjustment, several geographical parameters are introduced and presented in table A.1: each region's area, *AREA*(r), regional temperature, *RTL*0(r), and average latitude of each region, *LAT*(r). Further parameters include, area-weighted mean global latitude, $LAT_g = \frac{\sum_r LAT(r)\,AREA(r)}{\sum_r AREA(r)}$, the proportion of Earth's 510 10^6 km² of surface covered by oceans, $OCEAN_{prop} = 1 - \frac{(\sum_r AREA(r))}{510\ 10^6}$, and the difference in temperature increase between equator and pole, $POLE$ = 1.5 °C.

	AREA (10^6 km²)	*RTL*0	*LAT*
EU	4.50	1.00°C	45°
US	9.36	1.00°C	40°
OT	14.20	1.20°C	40°
EE	22.90	1.40°C	55°

[22] The regional modelling of temperature plays an important role in the PAGE IAM (see appendix).

CA	11.70	0.60°C	30°
IA	8.90	0.80°C	15°
AF	36.30	0.70°C	20°
LA	34.70	0.85°C	20°

Table A.1: Region-specific geographical data

Region-specific adjustment coefficients for latitude and land-mass are $RT_{adj}(r) = \frac{POLE}{90}(LAT(r) - LAT_g)$. Together with the ratio of mean temperature increases between land and ocean, *RLO* = 1.4, latitude-adjusted regional temperature, *RTL*(r), is $RTL(r,t) = RT(r,t)\left(1 + \frac{OCEAN_{prop}}{RLO} - OCEAN_{prop}\right) + RT_{adj}(r)$ with table A.1 giving the initial regional temperatures *RTL*(r,0) = *RTL0*(r). Global realised temperature, *GRT*(t), of equation (A.14) is now used to compute global mean land temperature, RTL_g(t) = *GRT*(t), from which global mean ocean temperature is $RTO_g(t) = \frac{RTL_g(t)}{RLO}$. Global mean temperature is the area-weighted mean between ocean and land temperatures:

$$RT_g(t) = OCEAN_{prop}\, RTO_g(t) + \left(1 - OCEAN_{prop}\right) RTL_g(t). \tag{A.18}$$