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6704
2017

October 2017

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Wolfgang Buchholz, Dirk Rübbelke

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editors: Clemens Fuest, Oliver Falck, Jasmin Gröschl
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CESifo *Working Paper No. 6704*
Category 1: Public Finance

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Abstract

In this paper, we show that progressivity (regressivity) of burden sharing in a Lindahl equilibrium is a direct consequence of gross complementarity (substitutability) between the private and the public good when the public good is taken as the numéraire. We, moreover, link the respective conditions for gross complementarity to the more familiar ones in which the private good serves as the numéraire.

JEL-Codes: D630, H230, H410.

Keywords: Lindahl equilibrium, progressive (regressive) burden sharing, complements and substitutes.

Wolfgang Buchholz
University of Regensburg
Universitäts Str. 31
Germany - 93040 Regensburg
wolfgang.buchholz@ur.de

Dirk Rübbelke
TU Bergakademie Freiberg
Lessingstraße 45
Germany - 09596 Freiberg
Dirk.Ruebbelke@vwl.tu-freiberg.de

October 10, 2017
We thank Theresa Stahlke for helpful comments.

1. Motivation

Fair burden-sharing among agents is a precondition for a successful cooperation on public good provision. One fairness principle that has already been prominent in the classical theory of public finance (e.g. Musgrave 1959) is the benefit principle, which means that an agent's cost/"tax" share in financing a public good should depend on her (marginal) willingness to pay for it. In this note, we reconsider (see Aaron and McGuire 1970, Kovenock and Sadka 1981, and Snow and Warren 1983) specific distributional effects that are caused by the application of the benefit principle. Specifically, we consider the Lindahl equilibrium that is entailed by the benefit principle in a standard public good economy and explore the conditions under which a progressive pattern of public good contributions results in this equilibrium. Then burden-sharing complies with an apparent normative postulate flowing from the ability-to-pay principle.

The structure of this note will be as follows: After presenting the framework of the analysis in Section 2, we show in Section 3 that progressivity (regressivity) of burden-sharing is a direct consequence of gross complementarity (substitutability) between the public and the private good, if the public good is the numéraire and the price of the private good varies. In Section 4 we further discuss how this basic criterion can be related to the more familiar notions of gross complementarity (substitutability) where the cross-price effect of changes of the public good price is considered while income in units of the private good is fixed.

2. The Framework

There are n agents with the same preferences $u(x_i, G)$, where x_i is country i's private consumption and G is the public good supply. The utility function $u(x_i, G)$ has the standard properties, i.e. twice continuous differentiability and strict monotonicity in both variables and strict quasi-concavity. Both goods are moreover assumed to be strictly non-inferior. At any point (x_i, G) the marginal rate of substitution between the public and the private good (i.e. the marginal willingness to pay for the public good = its "shadow price") is

(1) $$m(x_i, G) = \frac{\partial u / \partial G}{\partial u / \partial x_i}(x_i, G).$$

The partial derivatives of $m(x_i, G)$ w.r.t. the first and the second variable are denoted by $m_1(x_i, G)$ and $m_2(x_i, G)$, respectively. Normality implies $m_1(x_i, G) > 0$ and $m_2(x_i, G) < 0$. The initial endowment of country i measured in units of the private good is w_i. Countries are ranked according to their incomes, i.e. $w_1 \leq ... \leq w_n$.

The public good is produced by a summation technology for which the marginal rate of transformation mrt between the public and the private good is identical for all countries and normalized to one. Thus, an allocation $(x_1,...,x_n,G)$ is feasible if for $g_i = w_i - x_i$ we have

(2) $$G = \sum_{i=1}^{n} g_i \qquad \text{or} \qquad G + \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} w_i \ .$$

Given some allocation $(x_1,...,x_n,G)$ country i's cost share for public good provision is denoted by $p_i = \frac{g_i}{G}$ so that $x_i + p_i G = w_i$.

A feasible allocation $(\tilde{x}_1,...,\tilde{x}_n,\tilde{G})$ is said to satisfy the benefit principle if $\tilde{p}_i = m(\tilde{x}_i,\tilde{G})$ and thus $\tilde{g}_i = m(\tilde{x}_i,\tilde{G})\tilde{G} = \tilde{p}_i\tilde{G}$ holds for each $i = 1,...,n$, i.e. if evaluated by the mrs at $(\tilde{x}_i,\tilde{G})$, public good supply $\tilde{G}$ represents the equivalent to i's public good contribution $\tilde{g}_i$. Given $\tilde{p}_i = m(\tilde{x}_i,\tilde{G})$ as hypothetical personal public good price, agent i as a price-taker would choose the public good level $\tilde{G}$, i.e. $\tilde{G}$ maximizes $u(w_i - \tilde{p}_i G, G)$. A feasible allocation $(\tilde{x}_1,...,\tilde{x}_n,\tilde{G})$, which satisfies the benefit principle, hence is the Lindahl equilibrium in which all agents confronted with their individual Lindahl prices $\tilde{p}_i$ demand the same level of the public good. Given normality, $w_k > w_j$ implies $\tilde{p}_k > \tilde{p}_j$ for the Lindahl prices (since otherwise agent k would demand more of the public good than agent j) and also $\tilde{x}_k < \tilde{x}_j$ for private consumption (since, again by normality, $\tilde{x}_k \geq \tilde{x}_j$ would imply $\tilde{p}_k = m(\tilde{x}_k,\tilde{G}) \leq m(\tilde{x}_j,\tilde{G}) = \tilde{p}_j$).

3. The Basic Progressivity Criterion

Let $G^{MG}(\omega, q)$ and $x^{MG}(\omega, q)$ be Marshallian demand functions for the public and the private good, respectively, when, other than usual, the public good is taken as the numéraire so that income ω and the private good price q are measured in units of the public good. These Marshallian demand functions result from maximizing utility $u(x,G)$ under the budget constraint $qx+G=\omega$. Normality implies that $G^{MG}(\omega,q)$ and $x^{MG}(\omega,q)$ are increasing in ω and that $x^{MG}(\omega,q)$ is falling in q. If public good demand $G^{MG}(\omega,q)$ is falling (rising) in q, so that an increase of the private good price has a negative (positive) cross-price effect on public good demand, we label the public and the private good as gross x-*price complements* (x-*price substitutes*).

Now fix any level of public good supply $\bar{G}>0$ and consider the auxiliary function $\Phi(x,\bar{G})=\dfrac{x}{m(x,\bar{G})}$ depending on the level of private consumption x. As a starting point for our analysis of progressivity (regressivity) conditions, we then get the following result:

Proposition 1: If the private and the public good are gross x-price complements (substitutes) then $\Phi(x,\bar{G})$ is decreasing (increasing) in x.

Proof: Assume that x increases from x' to x''. Let $q'=\dfrac{1}{m(x',\bar{G})}$, $q''=\dfrac{1}{m(x'',\bar{G})}$, $\omega'=q'x'+\bar{G}$ and $\omega''=q''x''+\bar{G}$.

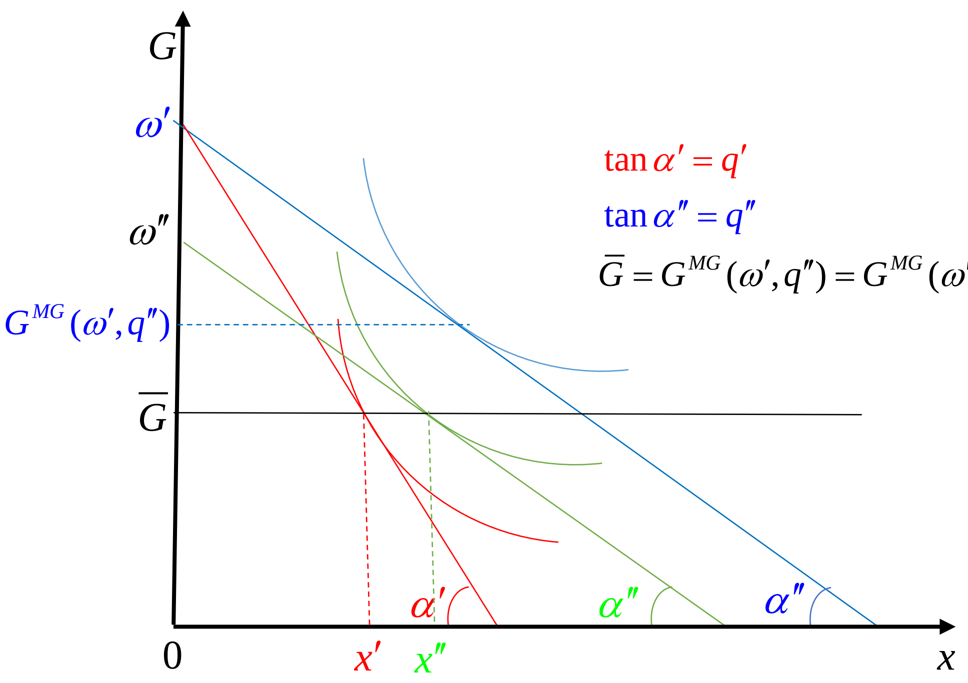

Figure 1

As $q'' < q'$, gross x-price complementarity gives (see Figure 1 above)

$$(3) \qquad G^{MG}(\omega', q'') > G^{MG}(\omega', q') = \overline{G}$$

Since $G^{MG}(\omega'', q'') = \overline{G}$ has to hold, $\omega'' < \omega'$ is required by normality, which implies $\Phi(x'', \overline{G}) = q''x'' = \omega'' - \overline{G} < \omega' - \overline{G} = q'x' = \Phi(x', \overline{G})$. The case of x-price substitutability is treated analogously. QED

We now apply Proposition 1 to formulate a basic criterion for progressivity (regressivity) of burden-sharing in a Lindahl equilibrium.

Proposition 2: If the public and the private good are gross x-price complements (substitutes), burden-sharing in a Lindahl equilibrium $(\tilde{x}_1, ..., \tilde{x}_n, \tilde{G})$ is progressive (regressive), i.e. the contribution-income ratio $\frac{\tilde{g}_i}{w_i}$ is increasing (decreasing) in w_i.

Proof: For each country i it is implied by $\tilde{p}_i = m(\tilde{x}_i, \tilde{G})$ that

$$(4) \qquad \frac{\tilde{g}_i}{w_i} = \frac{\tilde{p}_i \tilde{G}}{\tilde{x}_i + \tilde{p}_i \tilde{G}} = \frac{\tilde{G}}{\Phi(\tilde{x}_i, \tilde{G}) + \tilde{G}}.$$

Therefore, the assertion is an immediate consequence of Proposition 1 as private good consumption in a Lindahl equilibrium $\tilde{x}_i$ is increasing in income w_i QED

Proposition 2 shows that further conditions on progressive (regressive) burden sharing[1] completely boil down to far more general conditions on gross complementarity (substitutability), one of which is the following:

Proposition 3: The public and the private good are gross x-price complements (substitutes) if

$$(5) \qquad \frac{m_1(x,G)x}{m(x,G)} > 1 \qquad (<1)$$

holds for the elasticity of the marginal willingness to pay for the public good $m(x,G)$ w.r.t. private consumption x.

[1] Explicit conditions on preferences for a proportional burden sharing in the Lindahl equilibrium are provided by Cornes and Sandler (1996, pp. 204-205).

Proof: Since $\frac{d\Phi(x,\bar{G})}{dx} = \frac{m(x,G) - m_1(x,G)x}{m(x,G)^2}$ the result follows from Proposition 1. QED

Condition (5)[2] refers to the cross-price elasticity of the private good price while it seems more natural to use the private good as the numéraire and to make gross complementarity (substitutability) dependent on the cross-price elasticity of the public good price. Therefore, the question arises how this alternative form of complementarity (substitutability) is related to gross x-price complementarity (x-price substitutability).

4. Complementarity/Substitutability with the Private Good as Numeraire

Let $G^{Mx}(w, p)$ and $x^{Mx}(w, p)$ denote the Marshallian demand functions for the public and the private good given the private good endowment w and the public good price p measured in units of the private good. Similar as before we now call the private and the public good gross G-*price complements* (*substitutes*) if $x^{Mx}(w, p)$ is decreasing (increasing) in p, i.e. if $\frac{dx^{Mx}(w,p)}{dp} = \frac{d(w - pG^{Mx}(w,p))}{dp} = -(pG_2^{Mx}(w,p) + G^{Mx}(w,p)) < 0$ (>0) or, equivalently,

(6) $$-\frac{G_2^{Mx}(w,p)p}{G^{Mx}(w,p)} < 1 \qquad (>1)$$

holds as a condition on the price elasticity of Marshallian public-good demand.

As in the proof of Proposition 3, it is shown that the public and the private good are gross G-price complements (substitutes), if $\frac{m_2(x,G)G}{m(x,G)} > 1$ (<1) holds at all (x,G). But this elasticity of the marginal willingness to pay is completely independent from that appearing in Proposition 3 that provides a condition for gross x-price complementarity (substitutability). For an additively separable utility function $u(x,G) = f(x) + h(G)$, for example, we have $m_1(x,G) = -\frac{f''(x)x}{f'(x)}$ and $m_2(x,G) = -\frac{h''(x)x}{h'(x)}$, which makes it clear that gross x-price and

[2] This condition has already been implicit in Kovenock and Sadka (1981, p. 97) and also plays a central role in Ebert and Tillmann (2007) who investigate the progressivity issue in a more general setting in which public good supply is exogenously given and a budget surplus may arise.

gross G -price complementarity do not amount to the same thing (see, e.g. Samuelson 1974, p. 1268). At first sight, we thus cannot expect to get progressivity (regressivity) conditions from assumptions on gross G -price complementarity (substitutability). Nevertheless, some relationship between the two versions of gross complementarity (substitutability) emerges when the income elasticity of public good demand is brought into play.

For an explanation note that $G^{MG}(\omega,q)=G^{Mx}(\frac{\omega}{q},\frac{1}{q})$ for $q=\frac{1}{p}$ so that the public and the private good are x -price complements (substitutes) if

(7) $$\frac{dG^{Mx}(\frac{\omega}{q},\frac{1}{q})}{dq}=-\frac{1}{q^2}(G_1^{Mx}(\frac{\omega}{q},\frac{1}{q})\omega+G_2^{Mx}(\frac{\omega}{q},\frac{1}{q}))<0 \quad (>0).$$

where G_1^{Mx} and G_2^{Mx} denote the first derivatives of G^{Mx} w.r.t. income w and public good price p. Letting $p=\frac{1}{q}$ and $w=p\omega$ we then obtain from inequality (7) and our central Proposition 2 the following result[3]:

Proposition 5: The private and the public good are gross x -price complements (substitutes) if for the income and price elasticity of standard public good demand we have

(8) $$-\frac{G_2^{Mx}(w,p)p}{G^{Mx}(w,p)}<\frac{G_1^{Mx}(w,p)x}{G^{Mx}(w,p)} \quad (>).$$

Proposition 5 shows that a small price elasticity and a large income elasticity of Marshallian public good demand are favorable for having gross x -price complements and thus a progressive Lindahl contribution scheme. The income effect then is strong and the price effect is weak in this case.

Combining (6) and (8) and applying our basic criterion Proposition 2 then yields the following progressivity (regressivity) criterion, which is based on the familiar gross G -price complementarity (substitutability) assumptions:

[3] A slightly different version of this condition is provided and discussed by Snow and Warren (1983, p. 321) and Lambert (2001, p. 177; 2012, p. 487).

Proposition 6: If the public and the private good are gross G -price complements (substitutes) and the elasticity of public good demand w.r.t. income in private good units lies above (below) one then burden-sharing in a Lindahl equilibrium is progressive (regressive).

From the line of argument as presented in this paper, it is obvious that the message of Proposition 6 goes far beyond the question whether burden sharing in a Lindahl equilibrium is progressive or regressive. Rather, Proposition 6 provides a condition which, in a general household model, makes it possible to conclude from the sign of one cross-price elasticity to the sign of the other cross-price elasticity and thus to interlink the two corresponding versions of gross complementarity (substitutability) between two goods.

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