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Fiscal Federalism in a Monetary Union: The Cooperation Pitfall

Abstract

Fiscal federalism may not be a panacea in a monetary union if it does not address the non-cooperative behaviour between fiscal policymakers. To prove this, we assess the relative merits of a fiscal federalism scheme in a monetary union and intergovernmental fiscal cooperation without such a federal authority. Using a standard macroeconomic model commonly used for policy analysis we show that it is impossible to conclude that one solution is always preferable to the other. The benefits from an extra instrument and a policymaker with union-wide objectives may not compensate the adding of a non-cooperative player to the policy game. This result is sustained when an active monetary policy is introduced in the model or when shocks affect the functioning of the economy. The welfare ranking of these two options depends on the cross-border spillover effects, the objectives of policymakers and the variances of shocks.

JEL-Codes: E620, E630, F450.

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1 Introduction.

The current difficulties of the European monetary union following the debt crisis of the early 2010s and the painful discussion around the Greek debt have put to the fore the shortcomings of its initial institutional framework. The necessity to complement the monetary side of the EMU by a stronger fiscal apparatus is not contested. However its precise contours are painfully discussed and an agreement between the European member countries is not yet foreseen.

In general economists are more likely to support the proposal of a fiscal federation flanking the EMU and the creation of a proper supranational or "federal" Treasury.[1] Political European leaders, likely expressing the feeling of their electorates, see such a step ahead with caution and prefer (or behave according to) an intergovernmental cooperation scheme.[2] This approach to the current woes of the EMU is seen with skepticism by academics and pundits alike.

The current paper aims at analyzing the debate between the advocates of these two options by means of an analytical comparison of their respective merits rooted in a simple macroeconomic model of a monetary union commonly used in policymaking studies. Two variants of the model are developed, one featuring a fiscal federalism scheme and the other one a cooperative approach to fiscal policy leading to an agreement between the "country" or "national" policymakers.

The outcome supports an "agnostic" view on the problem: it is impossible to state without further explanations that a fiscal federalism scheme is to be preferred to an intergovernmental approach in a monetary union. The reason is that fiscal federalism introduces a dilemma. On the one hand, the introduction of an extra fiscal authority has two benefits: first it enlarges the set of policy instruments to tackle the policy issues, second this authority (likely) adopts a union-wide perspective and thus takes into account cross-border spillovers. On the other hand, it does not address the non-cooperative behaviour of fiscal authorities which generates non-desirable effects; to the contrary it adds fiscal externalities to the game between non-cooperating authorities which may be negative. As a consequence, it cannot be claimed that the positive aspects of fiscal federalism always outweigh its negative one of not addressing the cooperation issue. This is the cooperation pitfall of fiscal federalism.

In other words, the strenghtening of a monetary union by means of an adequate design of fiscal institutions does not rule out a priori either one of the two options we focus upon. In particular intergovernmental cooperation, setting aside the complexity of the bargaining process, cannot be dismissed as an unsound design of a monetary union. More troublesome, it may happen that a fiscal federalism scheme is dominated by a setting without any cooperation between fiscal players. Which option is preferable (from the point of view of national authorities) actually depends on the macroeconomic configuration of the union. More precisely, what matters is the extent of cross-border spillovers, the differing objectives of the various policymakers

[1] A recent proposal to be applied to the European Union (Wolff, 2012) is representative of this view. See also Bureau and Champsaur (1992) and Persson, Roland, Tabellini (1996) for standard expositions of the stakes of fiscal federalism applied to the European Union. The recent paper by Farhi and Werning (2012) has renewed the arguments in favor of a fiscal federal scheme complementing a currency union. See also the constitutional approach of Mueller (1997).

[2] The recent book by Van Middelaar (2013) is an illuminating analysis of the workings of the European Council which embodies the intergovernmental cooperation approach.

active in the union and the variances of shocks.

The issue at stake comes from the fact that a policy mix covering the entire union is required in a monetary union. From a theoretical point of view, two arguments have been put forward to stress the heightened role of fiscal instruments in a monetary union. First the traditional argument is that the reduction in the number of monetary instrument compared to a multi-currency world with flexible exchange rates gives a larger role to the remaining instruments, that is, the fiscal ones for stabilization purposes; a second more recent argument is that cross-border fiscal spillovers cannot be nullified by any monetary policy rule: insulation is impossible.[3] Empirically the evidence clearly shows that the role of interjurisdictional public transfers cannot be ignored even though the magnitude of these spillovers is subject to discussion.[4]

Thus there is no disagreement on the need of overcoming the defects of fiscal policies when they are taken at the country level by national authorities which neglect or downplay the impact of their decisions on the rest of the union. The real issue is about the proper design of the fiscal side of a monetary union.[5] Specifically, given the historical precedent of the United States, the discussion concentrates over the benefits of a fiscal federalism option.[6]

To explore the cooperation pitfall, our model of a two-country monetary union incorporates cross-border fiscal spillovers, both federal and inter-country ones. Policymakers make their decision according to standard macroeconomic loss functions. We study three games which correspond to three different institutional variants: the case without a federal Treasury and with cooperating fiscal authorities, which serves as a benchmark; the case without a federal Treasury but where fiscal authorities cooperate; and finally, the case with a federal Treasury materializing a fiscal federation but no cooperation among policymakers. For each variant, we are able to compute the corresponding equilibrium and the implied (expected) loss for each policymaker. This allows us to compare from a normative point of view the three variants under study. We concentrate on the comparison between these variants on the (expected) losses of the national fiscal authorities.

Formally our analysis relies on the undetermined coefficients method, commonly used to solve equations with rational expectations. We use it to solve a system of interdependent equations with rational expectations. It could be used to solve more complex models than ours. The model studied here is simple and could easily be extended either in complexity with more policymakers or a richer economic structure.[7]

In the next section we set up a deterministic model of a two-country monetary union, which is a variant of the Dixit and Lambertini (2003) model and discuss fiscal policies under the three different fiscal institutional designs: no-cooperation (used as a benchmark), cooperation and fiscal federalism. In the following section, we assess the robustness of our "agnostic" result by introducing first an active central bank and then shocks,

[3]See Cooper, Peled and Kempf (2014).

[4]The seminal study is Asdrubali, Sorensen and Yosha (1996). Melitz (2004) provides a very detailed discussion of the empirical difficulties encountered in the attempts to quantify these spillovers. See also Evers (2006).

[5]Dixit and Lambertini (2003) have shown in a simple model, similar to the one set-up here, that the choice of a fiscal scheme can be irrelevant. Kempf and von Thadden (2014) have generalized this "symbiosis" result and proven that the required conditions are extremely restrictive.

[6]Several studies have developed this precedent with the aim of drawing lessons from this experience for Europe. See Bordo, Jonung and Martkiewicz (2011) and Henning and Kessler (2012).

[7]A proper dynamic analysis including state variables would imply different game-theoretical tools.

so as to study the stabilization issue. Section 4 concludes. Proofs are contained in an Appendix.

2 Fiscal institutions and policies in a model of a monetary union.

We set up a model of a two-country monetary union where the equal-sized countries are endowed with autonomous fiscal authorities, possibly coexisting with a federal fiscal authority. The central bank is entirely passive.[8] Each fiscal national authority $i, i = 1, 2$, controls a fiscal instrument g_i. The federal Treasury when active controls its own instrument denoted by g_F. In the variant of the economy where this policymaker is absent (inactive), that is, in the absence of fiscal federalism, this instrument is set to 0. In this section we focus on the deterministic version of the model for ease of presentation.[9]

The aggregate output level in country i, denoted by y_i, is given by the following equation:

$$y_i = \hat{y} + \alpha g_i + \beta g_j + \gamma g_F + b(\pi - \pi^e) \tag{1}$$

where $\hat{y}$ denotes the natural aggregate output level, equal in both countries, π the inflation rate in the union, π^e the expected inflation rate rationally anticipated by private agents. α, β and γ measure the impacts on the output of each country of fiscal impulses decided by its national fiscal authority, the other national fiscal authority and the federal authority, respectively. β measures the magnitude of cross-border spillovers between countries and γ the magnitude of the federal ones. It is reasonable to assume that $\alpha > \beta$: the national fiscal policy is a stronger impact than the other country's policy. This may be justified by stronger and more direct transmission channels.[10] However we do not rank β and γ. It may be that the federal spillover is larger than the neighbouring country's one, either because it has a more direct impact or because it is more efficient. Finally we assume $\alpha \neq \gamma$.

The (average) union's aggregate output level is equal to:

$$\overline{y} = \hat{y} + \frac{1}{2}(\alpha + \beta)(g_1 + g_2) + \gamma g_F + b(\pi - \pi^e) \tag{2}$$

where $\hat{y}$ is the (average) natural aggregate output level in the union, assumed to the same as the natural level in each country.

Inflation at the union's level is given by the following equation:

$$\pi = c(g_1 + g_2 + g_F) \tag{3}$$

We assume that the various fiscal instruments have the same impact on inflation. It does not depend on any policy instrument controlled by the union's central bank as we assume it to be totally inactive.

This basic structure is a variant of the model set by Dixit and Lambertini (2003): the two differences with their model are the absence of a central bank actively controlling a monetary instrument and the possible introduction of a federal Treasury. There are no lagged terms in this economy. The endogenous variables of the model are thus functions of the objectives of the various policymakers.

As far as policymaking is concerned, three options are possible, within this simple macromodel:

[8] We shall relax this assumption in section 3.

[9] In Section 3, we analyze a stochastic version of it.

[10] Relaxing the assumption of equal size may lead to relaxing this assumption too.

1. the union is not a federation and there is no cooperation among policymakers. National fiscal authorities make their decisions non-cooperatively.

2. the union is not a federation but national fiscal authorities cooperate and aim at minimizing a (weighted) sum of their losses. As we assume symmetrical countries, we consider a simple average of losses.

3. the union is a federation but there is no cooperation among policymakers. A federal Treasury actively controls its instrument and attempts to minimize a loss function the arguments of which are the union's output gap and inflation. National fiscal authorities keep their autonomy and their control over their own fiscal instrument.

The assumption that the central bank is inactive (and sets its instrument π_m to 0) allows us to simplify the solutions of these variants and concentrate on fiscal policymakers. We abstract from informational and fiscal policy implementation issues.

The welfare criteria used by policymakers are simple and in line with the standard approach to macroeconomic policy. The various policymakers present in the monetary union aim at stabilizing output and inflation, however with different views on the ideal macroeconomic configuration. The two national fiscal authorities are concerned with national aggregate output, whereas the central bank and the federal Treasury worry about the union's aggregate output. All of them focus on the union inflation rate. Moreover their output and inflation objectives may differ. Each authority wants to minimize the squared gap of output (country or union-wide) to a desired "natural" level as well as the squared value of inflation (meaning that their desired inflation rate is 0).

Formally, the welfare criterion of a policymaker is given by a quadratic loss function the arguments of which are the output gap and inflation. These loss functions are the following:

Fiscal authority of country i: The loss function of the national Treasury in country i writes

$$L_i = \frac{1}{2}\theta\left(y_i - \tilde{\chi}\right)^2 + \frac{1}{2}\pi^2 \tag{4}$$

where $\tilde{\chi}$ is the level of union's aggregate output level desired by any national fiscal authority. As is traditionally done in this literature, we assume $\tilde{\chi} > \hat{y}$. θ expresses the relative weight given to the output objective compared to the inflation one by the national fiscal authorities. The inflation objective is set to 0 for all policymakers.

Federal fiscal authority: The loss function of the federal Treasury writes

$$L_F = \frac{1}{2}\theta_F\left(\overline{y} - \tilde{\chi}_F\right)^2 + \frac{1}{2}\pi^2 \tag{5}$$

where $\tilde{\chi}_F$ is the union's aggregate output level desired by the federal authority. We assume $\tilde{\chi}_F \neq \tilde{\chi}$. For simplicity reasons, we further assume: $\tilde{\chi}_F = \hat{y}$ (The federal authority has no desire to push the union's output beyond its natural level). θ_F expresses the relative weight given to the output objective compared to the inflation one by the federal Treasury. This loss function can be rewritten as

$$L_F = \frac{1}{2}\theta_F\left(\frac{1}{2}\left(\alpha+\beta\right)\left(g_1+g_2\right) + \gamma g_F + b\left(\pi - \pi^e\right)\right)^2 + \frac{1}{2}\left(c\left(g_1+g_2+g_F\right)\right)^2$$

In the sequel, we assume $\theta_F = \theta$.

2.1 Options.

Two stages constitute the economy:

1. Private agents form their expectations of inflation;
2. Policymakers make their decisions.

2.1.1 No fiscal cooperation.

We start with this benchmark case characterized by the absence of a federal Treasury and no cooperation between fiscal authorities. It is equivalent to assume that $g_F = 0$.

Country aggregate outputs are equal to:

$$y_i = \hat{y} + \alpha g_i + \beta g_{-i} + b(\pi - \pi^e) = \hat{y} + \alpha g_i + \beta g_{-i}$$

and the union's aggregate output:

$$\overline{y}_n^* = \hat{y} + \frac{1}{2}(\alpha + \beta)(g_1 + g_2) + b(\pi - \pi^e).$$

Inflation is

$$\pi = c(g_1 + g_2)$$

We define $\chi = \tilde{\chi} - \tilde{\chi}_F = \tilde{\chi} - \hat{y}$ and we refer to it as the "objective gap" between the federal and the country fiscal authorities. We denote by g_{in}^* the optimal decision of the fiscal authority in country i. Solving this game generates the following reduced forms:

$$g_{in}^* = g_n^* = \frac{\theta(\alpha + bc)}{\theta(\alpha + bc)(\alpha + \beta + 2bc) + 2c^2}\chi, \quad \forall i = 1, 2. \tag{6}$$

The resulting inflation, which we denote by π_n^*, is equal to:

$$\pi_n^* = \frac{2c\theta(\alpha + bc)}{\theta(\alpha + bc)(\alpha + \beta) + 2c^2}\chi. \tag{7}$$

The reduced forms for national outputs are:

$$y_n^* = \hat{y} + (\alpha + \beta) g_n^* = \hat{y} + \frac{\theta(\alpha + bc)(\alpha + \beta)}{\theta(\alpha + bc)(\alpha + \beta) + 2c^2}\chi \tag{8}$$

and thus equal to the union's aggregate (average) output level $\overline{y}_n^*$.

2.1.2 Fiscal cooperation.

This variant corresponds to intergovernmental cooperation in the absence of a federation, that is, without a federal Treasury enpowered with a fiscal instrument and macroeconomic objectives. The structure of the economy is the same as before.

In these circumstances, intercountry cooperation seeks to minimize the unweighted sum of the losses incurred by the two cooperating players. The assumption of an unweighted sum is consistent with the symmetry of the two countries and the absence of size effects in this model. Formally the loss function in this case is:

$$L_C = \frac{1}{2}\left\{\frac{\theta}{2}\left[(y_1-\tilde{\chi})^2+(y_2-\tilde{\chi})^2\right]\right\}+\frac{1}{2}\pi^2 \tag{9}$$

The two fiscal authorities cooperatively set the levels of the two fiscal instruments, g_1 et g_2, in order to minimize this joint loss. We denote g^*_{iC} the optimal decision for country i in a cooperative setting.

The solutions of this minimization program are:

$$g^*_{iC} = g^*_C = \frac{\frac{\theta}{2}(\alpha+\beta+2bc)}{\frac{\theta}{2}(\alpha+\beta+2bc)(\alpha+\beta)+2c^2}\chi, \quad \forall i=1,2. \tag{10}$$

The corresponding inflation is given by the following equation:

$$\pi^*_C = \frac{\theta c(\alpha+\beta+2bc)}{\frac{\theta}{2}(\alpha+\beta+2bc)(\alpha+\beta)+2c^2}\chi \tag{11}$$

Country aggregate outputs are equal to:

$$y^*_C = \hat{y} + \frac{\theta c(\alpha+\beta+2bc)(\alpha+\beta)}{\frac{\theta}{2}(\alpha+\beta+2bc)(\alpha+\beta)+2c^2}\chi \tag{12}$$

As they are equal, they are equal to the union's output $\overline{y}^*_C$.

2.1.3 Fiscal federalism.

Fiscal federalism is based on an active federal Treasury. The objectives of this Treasury, as formalized by the loss function (5), are union-wide. Yet the three fiscal authorities act non-cooperatively and simultaneously (the central bank is still assumed to be inactive). We look for the solution to this three-player Nash game. We denote by g^*_{iF} the optimal decision of the fiscal authority in country i in the presence of fiscal federalism and g^*_{FF} the optimal decision of the federal Treasury.

The outcomes of the Nash equilibrium of this game are:

$$g^*_{iF} = g^*_F = \frac{(\alpha+bc)\left(\theta\gamma(\gamma+bc)+c^2\right)}{c^2(\gamma-\alpha)(2\gamma-\alpha-\beta)}\chi, \quad \forall i=1,2. \tag{13}$$

$$g^*_{FF} = -\frac{\left(2c^2+\theta(\gamma+bc)(\alpha+\beta)\right)(\alpha+bc)}{c^2(\gamma-\alpha)(2\gamma-\alpha-\beta)}\chi \tag{14}$$

Inflation, denoted by π^*_F, is given by:

$$\pi^*_F = \frac{(\alpha+bc)\left[\theta(\gamma+bc)(2\gamma-(\alpha+\beta))+2c^2(1-\theta(\gamma+bc))\right]}{c(\gamma-\alpha)(2\gamma-(\alpha+\beta))}. \tag{15}$$

In these conditions, aggregate national outputs are equal to:

$$y^*_F = \hat{y} + \frac{(\alpha+bc)((\alpha+\beta)-2\theta\gamma(\gamma+bc))}{(\gamma-\alpha)(2\gamma-(\alpha+\beta))}\chi. \tag{16}$$

As they are equal, they are equal to the union's aggregate output level, denoted by $\overline{y}^*_F$.

2.2 Comparing outcomes.

The previous formulas allow us to compare the macroeconomic outcomes of the three fiscal variants of the monetary union.

2.2.1 Cooperation vs no cooperation.

We compare the outcome of the union characterized by cooperation among fiscal authorities with the outcome of the union without such a cooperation.

For inflation we get:

$$\pi_C^* - \pi_n^* = -c\theta \left[\frac{2c^2(\alpha - \beta)}{\left(\frac{1}{2}\theta(\alpha + \beta + 2bc)(\alpha + \beta) + 2c^2\right)\left(\theta(\alpha + bc)(\alpha + \beta) + 2c^2\right)} \right] \chi. \tag{17}$$

We notice that the various terms in brackets are all negative when $\alpha > \beta$. Thus the inflation bias (the coefficient associated with χ) is higher in the case of cooperation than in the case of no cooperation. This is consistent with the Barro-Gordon theory of the inflation bias: when cooperating, national authorities collude in attempting to trigger a surprise inflation taking into consideration the cross-border spillovers. These spillovers increase the benefits (for the national policymakers) of surprise inflation. Expecting this, the private agents form higher inflation expectations so as not to be surprised by inflation.

For the difference in national aggregate output, we get:

$$y_C^* - y_n^* = \theta \left[\frac{\frac{1}{2}\theta(\alpha + bc)(\alpha+\beta + 2bc)(\alpha + \beta)(\alpha + \beta - 1)}{\left(\frac{1}{2}\theta(\alpha + \beta + 2bc)(\alpha + \beta) + 2c^2\right)\left(\theta(\alpha + bc)(\alpha + \beta) + 2c^2\right)} \right] \chi. \tag{18}$$

It is positive as long as $\alpha + \beta > 1$. It can be negative. It does not have necessarily the same sign as the inflation bias difference because of the difference in objectives between the national policymakers and the federal one. This difference comes from the workings of fiscal spillovers on output.

2.2.2 Comparing cooperation and federalism.

We proceed similarly for the comparison between a union based on cooperation and one based on fiscal federalism.

For the difference in inflation we get:

$$\begin{aligned}\pi_C^* - \pi_F^* &= \left[\frac{\theta c(\alpha + \beta + 2bc)c(\gamma - \alpha) - \theta(\alpha + bc)(\gamma + bc)\left[\frac{\theta}{2}(\alpha + \beta + 2bc)(\alpha + \beta) + 2c^2\right]}{\left[\frac{\theta}{2}(\alpha + \beta + 2bc)(\alpha + \beta) + 2c^2\right]c(\gamma - \alpha)} \right] \chi \\ &= \left[\frac{\theta c(\alpha + \beta + 2bc)c(\alpha - \gamma) + \theta(\alpha + bc)(\gamma + bc)\left[\frac{\theta}{2}(\alpha + \beta + 2bc)(\alpha + \beta) + 2c^2\right]}{\left[\frac{\theta}{2}(\alpha + \beta + 2bc)(\alpha + \beta) + 2c^2\right]c(\alpha - \gamma)} \right] \chi. \end{aligned} \tag{19}$$

The term in brackets is positive for $\alpha > \gamma$. The inflation bias is bigger in a cooperating union than in a federation. The federal spillover effect is not sufficiently powerful to overcome the non-cooperative disadvantage.

For the difference in national aggregate output, we get:

$$y_C^* - y_F^* = \frac{(\alpha + bc)\left[\theta(\alpha + \beta)(2\alpha - \gamma + bc) - 2c^2\right]}{(\alpha - \gamma)} \chi. \tag{20}$$

It is not always positive as the numerator can be negative if c is sufficiently large. The output gap is not systematically higher under cooperation than in a federation.

2.3 Comparing losses.

In order to compare the three variants of a monetary union that we selected, we use as criteria the losses of the various policymakers. We concentrate on the comparison between the union based on fiscal cooperation and the union combined with a fiscal federation. The non-cooperative solution is solely developed as a benchmark. We focus our discussion on the losses of national fiscal players. We conjecture that the preferences of the national fiscal players are representative of the preferences of the national governments, or of the national political bodies because these players emanate from their government or their constituency. Thus at a constitutional stage (implicit here), the choice (whatever is the procedure used to choose an institutional setting for a monetary union) is likely to be based on a welfare criterion similar to the loss function used by a national fiscal authority. On the other hand, the federal Treasury can be seen as an agency, created by an agreement between the two member-countries of the union, and thus not directly and properly backed by a political constituency with constitutional powers.

2.3.1 Losses.

Losses for national fiscal authorities are based on the loss functions (4) and the various results given above for output and inflation under the three studied variants. The following table summarizes the results we obtain.

	Loss for fiscal authority i
No cooperation	$\frac{\theta(\theta(\alpha+bc))^2\left((1-(\alpha+\beta))^2+4c^4+(2c)^2(\alpha+\beta)\right)}{2(\theta(\alpha+bc)(\alpha+\beta)+2c^2)^2}\chi^2 = A_n^*\chi^2$
Fiscal cooperation	$\frac{\theta c^2\left(4c^2+\theta(\alpha+\beta+2bc)^2\right)}{2\left(\frac{1}{2}\theta(\alpha+\beta+2bc)(\alpha+\beta)+2c^2\right)^2}\chi^2 = A_C^*\chi^2$
Fiscal federalism	$\left\{\theta\left(\frac{(\gamma+bc)((\alpha+\beta)-2\theta\gamma(\alpha+bc))-2(\gamma-\alpha)\gamma}{2(\gamma-\alpha)(2\gamma-(\alpha+\beta))}\right)^2+\left(\frac{(\alpha+bc)\left[\theta(\gamma+bc)(2\gamma-(\alpha+\beta))+2c^2(1-\theta(\gamma+bc))\right]}{2c(\gamma-\alpha)(2\gamma-(\alpha+\beta))}\right)^2\right\}\chi^2$ $= A_F^*\chi^2$

Comparing losses incurred by cooperation and fiscal federalism for national fiscal players is equivalent to comparing A_F^* and A_C^*. The difference $A_F^* - A_C^*$ is of ambiguous sign. For example, if θ is arbitrarily close to 0, A_C^* is arbitrarily close to 0,[11] and

$$A_F^* \simeq \left(\frac{(\alpha+bc)\,c}{(\gamma-\alpha)\,(2\gamma-(\alpha+\beta))}\right)^2$$

which is positive. Cooperation achieves a better outcome than federalism. However, if $(\alpha+\beta)=1, \theta=c$, $\theta\,(\gamma+bc)=1$, we get:

$$A_F^* = \frac{c^2+\theta^2\,(\alpha+bc)^2}{(2\,(\gamma-\alpha)\,\theta c)^2}, A_C^* = 2c^2\left\{\frac{4c+(1+2bc)^2}{(4c+(1+2bc))^2}\right\}$$

[11] As the two fiscal policy makers solely care about inflation, they are able to reduce the inflation bias to 0 and achieve their goals.

and therefore $A^*_F < A^*_C$ if b is sufficiently small and c sufficiently large: fiscal federalism generates a lower loss than cooperation.

Where does this ambiguity result come from? The two institutional designs we compare have different advantages and inconvenients. On the one hand, cooperation allows the two national authorities to internalize the cross-border effects of their fiscal stance. This is beneficial for both of them. However this does not include an additional fiscal instrument to the array of available policy instruments. On the other hand, fiscal federalism presents the advantage of adding one policy instrument with respect to either no-cooperation or cooperation. Moreover it is reasonable (as done here) to assume that this authority cares about the functioning of the union as a whole. Yet it introduces an additional non-cooperating player and the non-intended effects of this additional layer of no-cooperation may actually generate negative effects, that is, higher losses, in particular for national policy authorities which overcome the benefits due to the action of the federal authority.

We can even prove that from the point of view of the federal Treasury itself, cooperation may be preferred to fiscal federalism. Denoting by L^*_{FC} the loss of the federal Treasury when it is inactive and countries cooperate and by L^*_{FF} its loss under fiscal federalism, and expressing L^*_{FC} as $A^*_{FC}\chi^2$ and L^*_{FC} as $A^*_{FF}\chi^2$, this amounts to prove that $A^*_{FC} < A^*_{FF}$. This is true for example when θ is sufficiently small.[12]

In sum, these two institutional designs have specific advantages. For a given structure of the union's economy – as described by the set of parameter values, one may be Pareto-superior to the other. It may not be so for another configuration, that is, for a different set of parameters: Fiscal federalism is not required in a monetary union in any circumstances.

3 Alternative models.

In this section we aim at proving that this result can be sustained when the structure of the union's economy is modified. We focus on the introduction of an active central bank and the introduction of shocks.[13]

3.1 Introducing monetary policy.

We introduce an active central bank in the model, controlling one monetary instrument, denoted by π_{cb}, and pursuing a discretionary monetary policy. Its loss function solely depends on inflation:

$$L_{cb} = \frac{1}{2}\pi^2 \tag{21}$$

Thus, from the first-order condition we get:

$$\pi = c\,(g_1 + g_2 + g_F) + \pi_{cb} = \pi^e = 0$$

which implies

$$-c\,(g_1 + g_2 + g_F) = \pi_{cb}.$$

[12] See Appendix 1.

[13] Other variants could of course be explored by relaxing other assumptions, such as the size equality of the two countries, or specifying other loss functions for the players.

Output is then:

$$y_i = \hat{y} + \alpha g_i + \beta g_j + \gamma g_F.$$

The variants are now 3- and 4-policymaker games. To simplify further the setting, we maintain the assumption that $\tilde{\chi}_F = \hat{y}$.[14] The following table summarizes the loss incurred by a national fiscal authority in the three variants we study.[15]

	Loss for fiscal authority i
No cooperation	$L_n^* = 2\theta \left(\frac{(\theta b(\alpha+bc)+c)c}{\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2} \right)^2 \chi^2$
Fiscal cooperation	$L_C^* = \frac{1}{2}\theta \left(\frac{(\alpha+\beta+4bc)\theta(\alpha+\beta+2bc)+4c^2}{2\left(\theta(\alpha+\beta+2bc)^2+2c^2\right)} \right)^2 \chi^2$
Fiscal federalism	$L_F^* = \frac{1}{2}\theta \left(\frac{\theta(\gamma+bc)[(\alpha+\beta)bc-2\gamma\theta(1+bc)(\alpha+bc)]+(\alpha+\beta)c^2}{(\alpha+\beta+2bc)[\theta c^2(\alpha-\gamma)]+2c^2\left(\theta(\gamma+bc)^2(1-\theta(\alpha+bc))+c^2(1-\theta(\gamma+bc))\right)} \right)^2 \chi^2$

Given these formulas, it is not always true that $L_C^* > L_F^*$. For example, suppose the following values for the various parameters of the problem: $\beta = 0, b = c = 1, \theta = 2$ and $\alpha = \gamma = 1/\theta = 1/2$. We get:

$$y_C^* - \hat{y} = -\frac{5}{108}\chi \text{ and } y_F^* - \hat{y} = \frac{13}{54}\chi.$$

Hence $|y_C^* - \hat{y}| < |y_F^* - \hat{y}|$ which implies $L_C^* < L_F^*$. Hence the result that fiscal federalism may be an inferior institutional design than cooperation in a monetary union is vindicated when the central bank pursues an active monetary policy.[16]

3.2 Introducing shocks.

We now introduce supply and demand shocks, in the model set up above and study the same three variants, in order to assess the superiority of fiscal federalism over cooperation in a stochastic environment. In this section the focus is on stabilizing issues and not on credibility issues. We return to the assumption of an inactive central bank.

The aggregate output level in country i, denoted by y_i, is given by the following equation:

$$y_i = \hat{y} + \alpha g_i + \beta g_j + \gamma g_F + b(\pi - \pi^e) + u_i \tag{22}$$

where u_i denotes the real (supply) shock affecting country i. Throughout the paper we consider $i, j = 1, 2; j \neq i$. u_i is an i.i.d. random variable of zero mean and variance σ_u^2. Real shocks are not correlated. It has the same distribution law in either country but realizations differ. The (average) union's aggregate output level is equal to:

$$\overline{y} = \hat{y} + \frac{1}{2}(\alpha + \beta)(g_1 + g_2) + \gamma g_F + b(\pi - \pi^e) + \overline{u} \tag{23}$$

where $\overline{u}$ is the mean of the real shocks u_i.

[14] Notice that by restricting the number of parameters we lose degrees of freedom and thus reduce the capacity of the model to generate ambiguous results.

[15] See Appendix 2 for the details of the proof.

[16] A more general loss function for the central bank (for example, introducing an output gap argument in (21)) would sustain this claim at the expenses of more cumbersome computations.

Inflation at the union's level is given by the following equation:

$$\pi = c\left(g_1 + g_2 + g_F\right) + \pi_M + \varepsilon \tag{24}$$

where the nominal (demand) shock ε is an i.i.d. random variable of mean zero and variance σ_ε^2. It is not correlated with the real shock. Again, we assume here that π_M ie equal to 0.

Policymakers now minimize an expected loss function. For fiscal authority of country i, it is given by:

$$E\left(L_i\right) = \frac{1}{2}E\left(\theta\left(y_i - \tilde{\chi}\right)^2 + \pi^2\right) \tag{25}$$

and for the federal fiscal authority:

$$E\left(L_F\right) = \frac{1}{2}E\left(\theta_F\left(\overline{y} - \tilde{\chi}\right)^2 + \pi^2\right). \tag{26}$$

For simplicity and so as to concentrate on shocks and not on constant terms we assume that all policymakers have the same output objective: $\tilde{\chi} = \hat{y}$.[17] Given this assumption, there is no incentive for any policymaker to generate unexpected inflation and thus no inflation bias. This is why the policy problem is solely on minimizing the impacts of shocks on output and inflation and can be referred to as a stabilization issue. Three stages constitute the economy:

1. Private agents form their expectations of inflation;
2. Shocks occur;
3. Policymakers make their decisions.

The following table summarizes the expected loss incurred by a national fiscal authority in the three variants we study.

	Expected Loss for fiscal authority i
No cooperation	$E\left(L_n^*\right)^2 = \frac{\theta c^2\left(c^2+\theta(\alpha+bc)^2\right)}{(\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2)^2}\sigma_u^2 + \frac{1}{2}\frac{\theta c^2\left((\alpha+\beta)^2+\theta(\alpha+bc)^2\right)}{(\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2)^2}\sigma_\varepsilon^2$
Fiscal cooperation	$E\left(L_C^*\right)^2 = \frac{c^2\left(\left(\frac{\theta}{2}\right)^2(\alpha+\beta+2bc)^2+\theta c^2\right)}{\left(\frac{\theta}{2}(\alpha+\beta+2bc)^2+2c^2\right)^2}\sigma_u^2 + \frac{1}{2}\frac{\left(\left(\frac{\theta}{2}\right)^2(\alpha+\beta+2bc)^2+\theta c^2\right)(\alpha+\beta)}{\left(\frac{\theta}{2}(\alpha+\beta+2bc)^2+2c^2\right)^2}\sigma_\varepsilon^2$
Fiscal federalism	$E\left(L_F^*\right)^2 = \theta\frac{2\gamma^2\alpha^2}{(2\gamma-\alpha-\beta)^2(\alpha-\beta)^2}\sigma_u^2$

The complexity of the formulas obtained above, despite the simplicity of the model we use, makes clear that the comparison is not immediate. More precisely, we get:

$$E\left(L_F^*\right)^2 - E\left(L_C^*\right)^2 = \left(B_F^* - B_C^*\right)\sigma_u^2 - C_C^*\sigma_\varepsilon^2$$

where $B_C^* = \frac{c^2\left(\left(\frac{\theta}{2}\right)^2(\alpha+\beta+2bc)^2+\theta c^2\right)}{\left(\frac{\theta}{2}(\alpha+\beta+2bc)^2+2c^2\right)^2}$, $B_F^* = \frac{2\theta\gamma^2\alpha^2}{(2\gamma-\alpha-\beta)^2(\alpha-\beta)^2}$ and $C_C^* = \frac{1}{2}\frac{\left(\left(\frac{\theta}{2}\right)^2(\alpha+\beta+2bc)^2+\theta c^2\right)(\alpha+\beta)^2}{\left(\frac{\theta}{2}(\alpha+\beta+2bc)^2+2c^2\right)^2}$.

It is a priori impossible to give the sign of this difference. Hence a fiscal federation is not necessarily Pareto-superior to a fiscal cooperation scheme for stabilization purposes. It is impossible to state that a monetary union is viable solely if complemented by a fiscal federation with an active federal Treasury

[17]Introducing the different objectives for policymakers studied in the previous section amounts to adding to the various expected losses the constants obtained above for each variant.

contributing to macroeconomic stabilization. Said differently, cooperation is preferable to a setting based on a fiscal federation for some configurations of shocks and objectives.

Given the impact of γ on the various coefficients in this equation (see above), this difference is an ambiguous function of the federal fiscal spillover coefficient. In other words, a more potent federal fiscal tool does not imply that the welfare of national fiscal authorities under fiscal federalism is increased. In sum, the "agnostic" result obtained in the previous section is sustained when we deal with stabilizing a stochastic monetary union's economy.

More precisely the condition for the superiority of fiscal federalism derived from the previous equation is:

$$E\left(L_F^*\right)^2 < E\left(L_C^*\right)^2 \Leftrightarrow \sigma_u^2 < \frac{C_C^*}{\left(B_F^* - B_C^*\right)}\sigma_\varepsilon^2. \tag{27}$$

As one achievement of a fiscal federation is eliminating the impact of the nominal shock thanks to the presence of an additional instrument, the higher is the variance of this shock, the more valuable is such a design. The two institutional frameworks are equivalent for national fiscal authorities (in the sense they generate the same expected loss) for pairs $\left(\sigma_u^2, \sigma_\varepsilon^2\right)$ satisfying the following equation:

$$\sigma_u^2 = \frac{C_C^*}{\left(B_F^* - B_C^*\right)}\sigma_\varepsilon^2.$$

This corresponds to a line splitting the positive orthant $\left(\sigma_u^2, \sigma_\varepsilon^2\right)$ in two regions. Its slope is equal to $\frac{C_C^*}{B_F^* - B_C^*}$, positive or negative. We get from above:

$$B_F^* - B_C^* = \left(\frac{2\theta\gamma^2\alpha^2}{\left(2\gamma - \alpha - \beta\right)^2\left(\alpha - \beta\right)^2} - \frac{c^2\left(\theta c^2 + \left(\frac{\theta}{2}\right)^2\left(\alpha + \beta + 2bc\right)^2\right)}{2\left(\frac{\theta}{2}\left(\alpha + \beta + 2bc\right)^2 + 2c^2\right)^2}\right).$$

This difference may be positive or negative.

For pairs posited above this line, cooperation is preferable to fiscal federalism. In other words, an increase in the variance of real shocks relative to the variance of nominal shock, makes cooperation relatively more enticing for each national policymaker. But it is not so much the nature of the shocks which explains this result as the fact that the real shocks are heterogenous (or "local") whereas the nominal shock is global, affecting the two countries in the same way. The interest of cooperation grows with the growing incidence of this heterogeneity. Formally, as the main interest of a federal scheme is to eliminate the incidence of the nominal shock, this interest vanishes relatively when the variance of the nominal shock decreases relatively to the variance of the real shocks.

Let us develop a numerical example, assuming the following values for the parameters of the model:

$$\alpha = 3, \beta = 1, \gamma = 1/2, \theta = 1, b = 1, c = 1.$$

Then

$$E\left(L_F^*\right) = \frac{1}{8}\sigma_u^2 \text{ and } E\left(L_C^*\right) = \frac{1}{40}\sigma_u^2 + \frac{1}{80}\sigma_\varepsilon^2.$$

Hence

$$E\left(L_F^*\right) = \frac{1}{8}\sigma_u^2 \geq E\left(L_C^*\right) = \frac{1}{40}\sigma_u^2 + \frac{1}{80}\sigma_\varepsilon^2 \Leftrightarrow \sigma_u^2 \geq \frac{1}{8}\sigma_\varepsilon^2.$$

Fiscal federalism is worse than cooperation for both national fiscal authorities for the set of couples $\left(\sigma_u^2, \sigma_\varepsilon^2\right)$ satisfying this last inequality.

Of course, cooperation generates lower expected losses than the absence of cooperation. When a fiscal federation is preferable to cooperation, it is thus preferable to non-cooperation. But the inverse is not true. If cooperation is preferable, but impossible to implement, it does not imply that a federal scheme should be put in place. It may be that the expected losses obtained through non-cooperation are weaker than the ones obtained with a federal scheme. The advantage of a supplementary instrument (thanks to a federal Treasury) may be lost because of the non-cooperative effects between three non-cooperating (and not two) players. From equations given the above table, we get:

$$E\left(L_n^*\right)^2 < E\left(L_F^*\right)^2 \Leftrightarrow \frac{C_n^*}{\left(B_F^* - B_n^*\right)}\sigma_\varepsilon^2 < \sigma_u^2. \tag{28}$$

Again, for given values of the structural parameters, non-cooperation may be more enticing than fiscal federalism.

4 Conclusion.

A monetary union between several countries reinforces the fiscal cross-border effects of fiscal decisions made by independent national fiscal authorities. It is thus necessary to find the best way to articulate these decisions. Given the absence of insulation from country shocks and fiscal decisions, a non-cooperating scheme is suboptimal and likely to amplify negative spillovers potentially damaging for the monetary union itself.

Two ways are possible to overcome the adverse consequences of the absence of cooperation between fiscal authorities within a monetary union: the formation of a fiscal federation and intergovernmental cooperation. They imply different institutional designs and sets of policy instruments. Each option has its merits and drawbacks. A cooperation scheme allows national fiscal authorities to internalize the cross-border spillover effects of fiscal decisions but does not add any new instruments. A fiscal federalism design, introducing a federal fiscal authority, adds new instruments but does not improve on the way authorities cooperate and may even worsen the non-cooperating features in the union. Fiscal federalism suffers from the cooperation pitfall. On the whole the comparative advantage of a given design over the other depends on the configuration of the union, particularly the cross-border effects of fiscal stances.

The analysis of a simple macroeconomic model which can be developed alternatively in the configurations of fiscal federalism or of cooperation leads to the following conclusion: it is a priori impossible to prefer either one or the other of these options. It depends on the structure of the union and in particular on the magnitude of cross-border fiscal spillovers, the variances of shocks and the (disagreement between the) objectives of the various policymakers.

A corollary of this result is that a precise analysis of the economic structure of the monetary union as well as the policy objectives of the governments (or the electorates in a political economy perspective) is needed when choosing its institutional design. The choice of the adequate institutional design of fiscal policy making in a monetary union is a pragmatic matter, not a matter of principles.

Other dimensions could be introduced in the analysis. As far as a fiscal federation is concerned, economic analysis as well as evidence show that it cannot be a panacea. Its success depends on the type of fiscal discipline imposed on national fiscal authorities. Cooperation, on the other hand, is made fragile by the difficulties encountered in the bargaining process, including the building of a consensus on the macroeconomic outcome and because its implementation has to be renewed year after year. These aspects reinforce the result obtained here: no option is clearly better than the other one.

This theoretical result sheds some light on the current travails of the European monetary union. We cannot be surprised to witness the hesitations in the building of a stronger union. They are clear evidence of a dilemma in choosing between (fiscal) federalism and intergovernmental cooperation. For the time being, maybe temporarily, the supremacy of the European council, in particular in dealing with the Greek debt drama, proves that the scheme chosen by the European authorities is de facto the cooperation option. Our results show that it is too simple to a priori condemn this choice without further analysis. Another lesson from our reasoning is that combining new ways to cooperate and additional instruments is likely to be the right strategy for any monetary union, including the European one.

The undetermined coefficients method we use to reach this result can be used to analyze more complex models than the one we study here. We set up simple models in order to reach the "agnostic" result stated above. More complex models, in particular including a dynamic dimension or different strategical settings such as the existence of a leading policymaker, could be studied. This opens many complex issues, including time-consistency and trust. Finally we abstract from the discussion on the relative difficulties of implementing these institutional design and then on the effectiveness of fiscal policy. These matters are crucial in the choice among institutional settings. These issues are left for further research.

In sum, once it is recognized that insulation within a monetary union is impossible and its fiscal side plays a crucial role in its functioning and achievements, no simple view can be held on the proper fiscal institutions of a monetary union.

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Appendix

Appendix 1.

From (4), (1) et (3), we get the following expression for output in country i:

$$y_i = \hat{y} + (\alpha + bc)\, g_i + (\beta + bc)\, g_j + (\gamma + bc)\, g_F - b\pi^e. \tag{29}$$

Non-cooperation.

In this variant there is no Federal Treasury, the central bank has no active policy and applies $\pi_M = 0$ in any circumstance. Moreover we assume $\theta_M = \theta$ et $\chi_M \equiv \hat{y} - \tilde{\chi}_M = 0$.

The optimization program of Treasury i is (using (29)):

$$max_{g_i} L = \frac{1}{2}\theta\left((\alpha + bc)\, g_i + (\beta + bc)\, g_j - b\pi^e - \chi\right)^2 + \frac{1}{2}\left(c\,(g_i + g_j)\right)^2 \tag{30}$$

with $\chi = \tilde{\chi} - \hat{y}$. The first-order condition is:

$$\frac{\partial L}{\partial g_i} = \theta\left(\alpha + bc\right)\left(\left(\alpha + bc\right) g_i + \left(\beta + bc\right) g_j - b\pi^e - \chi\right) + c^2\left(g_i + g_j\right) = 0 \tag{31}$$

ou equivalently:

$$\theta\left(\alpha + bc\right)\left(\left(\alpha + bc\right) g_i + \left(\beta + bc\right) g_j\right) + c^2\left(g_i + g_j\right) = \theta\left(\alpha + bc\right)\left(b\pi^e + \chi\right), i = 1, 2.$$

In equilibrium, using symmetry and $\pi^e = c\left(g_i + g_j\right)$, we get

$$g_n^* = \frac{\theta\left(\alpha + bc\right)}{\theta\left(\alpha + bc\right)\left(\alpha + \beta\right) + 2c^2}\chi.$$

Thus:

$$\pi_n^* = \frac{2c\theta\left(\alpha + bc\right)}{\theta\left(\alpha + bc\right)\left(\alpha + \beta\right) + 2c^2}\chi = \pi^e$$

This represents the inflation bias under no-cooperation.

We also get the output level:

$$y_n^* = \hat{y} + \left(\alpha + \beta + 2bc\right) g_n^* - b\pi^e = \hat{y} + \frac{\theta\left(\alpha + bc\right)\left(\alpha + \beta\right)}{\theta\left(\alpha + bc\right)\left(\alpha + \beta\right) + 2c^2}\chi.$$

Fiscal cooperation.

In this scenario we assume no federal Treasury, no active monetary policy $\pi_M = 0$, and cooperation between national fiscal authorities.

The program of the cooperating national fiscal authorities is:

$$\begin{aligned} max_{g_i,g_j}\frac{1}{2}\left(L_i + L_j\right) = {} & \frac{1}{2}\left(\frac{\theta}{2}\right)\left(\left(\alpha + bc\right) g_i + \left(\beta + bc\right) g_j - b\pi^e - \chi\right)^2 \\ & + \frac{1}{2}\left(\frac{\theta}{2}\right)\left(\left(\alpha + bc\right) g_j + \left(\beta + bc\right) g_i - b\pi^e - \chi\right)^2 + \frac{1}{2}\left(c\left(g_i + g_j\right)\right)^2 \end{aligned} \tag{32}$$

with $\chi \equiv \tilde{\chi} - \hat{y}$. The first-order condition for g_i implies:

$$\frac{1}{2}\theta\left(\alpha + \beta + 2bc\right)\left(\left(\alpha + bc\right) g_i + \left(\beta + bc\right) g_j\right) + c^2\left(g_i + g_j\right) = \frac{1}{2}\theta\left(\alpha + \beta + 2bc\right)\left(b\pi^e + \chi\right). \tag{33}$$

In equilibrium, using symmetry and $\pi^e = c\left(g_i + g_j\right) = 2cg_i$, we get:

$$\left(\frac{1}{2}\theta\left(\alpha + \beta + 2bc\right)\left(\alpha + \beta + 2bc - 2bc\right) + 2c^2\right) g_i = \frac{1}{2}\theta\left(\alpha + \beta + 2bc\right)\chi.$$

The solution is therefore:

$$g_c^* = \frac{\frac{1}{2}\theta\left(\alpha + \beta + 2bc\right)}{\frac{1}{2}\theta\left(\alpha + \beta + 2bc\right)\left(\alpha + \beta\right) + 2c^2}\chi.$$

Thus:

$$\pi_c^* = 2cg_c^* = \frac{\theta c\left(\alpha + \beta + 2bc\right)}{\frac{1}{2}\theta\left(\alpha + \beta + 2bc\right)\left(\alpha + \beta\right) + 2c^2}\chi.$$

This represents the inflation bias under cooperation. We also get the output level:

$$y_c^* = \hat{y} + \left(\alpha + \beta\right) g_c^* = \hat{y} + \frac{\frac{1}{2}\theta\left(\alpha + \beta + 2bc\right)\left(\alpha + \beta\right)}{\frac{1}{2}\theta\left(\alpha + \beta + 2bc\right)\left(\alpha + \beta\right) + 2c^2}\chi.$$

Fiscal federalism

In this scenario we assume the existence of federal Treasury, no active monetary policy $\pi_M = 0$, and no cooperation between national fiscal authorities. We assume that $\theta_F = \theta$ and $\chi_F = 0$.

The optimization program of the national fiscal authority i is:

$$max_{g_i} L = \frac{1}{2}\theta\left(\left(\alpha + bc\right) g_i + \left(\beta + bc\right) g_{-i} + \left(\gamma + bc\right) g_F - b\pi^e - \chi\right)^2 + \frac{1}{2}\left(c\left(g_1 + g_2 + g_F\right)\right)^2 \tag{34}$$

with $\chi = \tilde{\chi} - \hat{y}$. The first-order condition implies:

$$\begin{aligned} g_i = &-\frac{\left(\theta\left(\beta + bc\right)\left(\alpha + bc\right) + c^2\right)}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)} g_j - \frac{\left(\theta\left(\gamma + bc\right)\left(\alpha + bc\right) + c^2\right)}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)} g_F \\ &+ \frac{b\theta\left(\alpha + bc\right)}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)}\pi^e + \frac{\theta\left(\alpha + bc\right)}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)}\chi. \end{aligned} \tag{35}$$

The optimization program of the federal authority F is:

$$max_{g_i} L = \frac{1}{2}\theta\left(\left(\frac{1}{2}\left(\alpha + \beta\right) + bc\right)\left(g_i + g_j\right) + \gamma g_F - b\pi^e\right)^2 + \frac{1}{2}\left(c\left(g_1 + g_2 + g_F\right)\right)^2 \tag{36}$$

with $\chi = \tilde{\chi} - \hat{y}$. The first-order condition implies:

$$g_F = -\frac{\theta\left(\gamma + bc\right)\left(\frac{1}{2}\left(\alpha + \beta\right) + bc + c^2\right)}{\left(\theta\left(\gamma + bc\right)^2 + c^2\right)}\left(g_1 + g_2\right) + \frac{b\theta\left(\gamma + bc\right)}{\left(\theta\left(\gamma + bc\right)^2 + c^2\right)}\pi^e. \tag{37}$$

Denote by g^*_{iF} et g^*_{FF} the optimal solutions of this program and π^e_F the corresponding expected inflation. The reduced forms for these solutions are written as follows:

$$g^*_{iF} = f_{Fi\chi}\chi = f_{Fj\chi}\chi$$

$$g^*_{FF} = f_{FF\chi}\chi$$

$$\pi^e_F = f_{Fe\chi}\chi.$$

Given the symmetry between the two countries, implying $g^*_{iF} = g^*_F$, we can write:

$$f_{F1\chi} = f_{F2\chi}.$$

The equilibrium is characterized by:

$$\begin{aligned} f_{Fi\chi}\chi = &\frac{\theta\left(\alpha + bc\right)}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)}\chi + \frac{\theta b\left(\alpha + bc\right)}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)} f_{Fe\chi}\chi \\ &-\frac{\left(\theta\left(\beta + bc\right)\left(\alpha + bc\right) + c^2\right)}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)} f_{Fj\chi}\chi - \frac{\left(\theta\left(\gamma + bc\right)\left(\alpha + bc\right) + c^2\right)}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)} f_{FF\chi}\chi. \end{aligned}$$

Hence:

$$f_{Fi\chi} = \frac{\theta\left(\alpha + bc\right)}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)} + \frac{b\theta\left(\alpha + bc\right)}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)} f_{Fe\chi}$$

$$-\frac{\left(\theta\left(\beta+bc\right)\left(\alpha+bc\right)+c^2\right)}{\left(\theta\left(\alpha+bc\right)^2+c^2\right)}f_{Fj\chi}-\frac{\left(\theta\left(\gamma+bc\right)\left(\alpha+bc\right)+c^2\right)}{\left(\theta\left(\alpha+bc\right)^2+c^2\right)}f_{FF\chi}.$$

We also get:

$$f_{FF\chi}=-\frac{\theta\left(\gamma+bc\right)\left(\frac{1}{2}\left(\alpha+\beta\right)+bc+c^2\right)}{\left(\theta\left(\gamma+bc\right)^2+c^2\right)}\left(f_{F1\chi}+f_{F2\chi}\right)+\frac{b\theta\left(\gamma+bc\right)}{\left(\theta\left(\gamma+bc\right)^2+c^2\right)}f_{Fe\chi}$$

and

$$f_{Fe\chi}=c\left(f_{F1\chi}+f_{F2\chi}+f_{FF\chi}\right)f_{Fe\chi}=c\left(2f_{F1\chi}+f_{FF\chi}\right).$$

As

$$f_{FF\chi}=-\frac{\theta\left(\gamma+bc\right)\left(\frac{1}{2}\left(\alpha+\beta\right)+bc+c^2\right)}{\left(\theta\left(\gamma+bc\right)^2+c^2\right)}\left(2f_{F1\chi}\right)+\frac{b\theta\left(\gamma+bc\right)}{\left(\theta\left(\gamma+bc\right)^2+c^2\right)}f_{Fe\chi}$$

we get:

$$f_{FF\chi}=-\frac{\theta\left(\gamma+bc\right)\left(\frac{1}{2}\left(\alpha+\beta\right)+bc+c^2\right)}{\left(\theta\left(\gamma+bc\right)^2+c^2\right)}\left(2f_{F1\chi}\right)+\frac{b\theta\left(\gamma+bc\right)}{\left(\theta\left(\gamma+bc\right)^2+c^2\right)}\left(c\left(2f_{F1\chi}+f_{FF\chi}\right)\right)$$

hence:

$$f_{FF\chi}=-\frac{\theta\left(\gamma+bc\right)\left(\alpha+\beta+2c^2\right)}{\theta\gamma\left(\gamma+bc\right)+c^2}f_{F1\chi}$$

We also get:

$$f_{Fi\chi}=\frac{\theta\left(\alpha+bc\right)}{\left(\theta\left(\alpha+bc\right)^2+c^2\right)}+\frac{b\theta\left(\alpha+bc\right)}{\left(\theta\left(\alpha+bc\right)^2+c^2\right)}c\left(2f_{F1\chi}+f_{FF\chi}\right)$$
$$-\frac{\left(\theta\left(\beta+bc\right)\left(\alpha+bc\right)+c^2\right)}{\left(\theta\left(\alpha+bc\right)^2+c^2\right)}f_{Fi\chi}-\frac{\left(\theta\left(\gamma+bc\right)\left(\alpha+bc\right)+c^2\right)}{\left(\theta\left(\alpha+bc\right)^2+c^2\right)}f_{FF\chi}$$

hence:

$$f_{Fi\chi}=\frac{\theta\left(\alpha+bc\right)}{\theta\left(\alpha+bc\right)\left(\alpha+\beta\right)+2c^2}-\frac{\theta\gamma\left(\alpha+bc\right)+c^2}{\theta\left(\alpha+bc\right)\left(\alpha+\beta\right)+2c^2}f_{FF\chi}.$$

Combining these two equations we get:

$$f_{Fi\chi}=\frac{\left(\alpha+bc\right)\left(\theta\gamma\left(\gamma+bc\right)+c^2\right)}{c^2\left(\gamma-\alpha\right)\left(2\gamma-\left(\alpha+\beta\right)\right)}$$

and

$$f_{FF\chi}=-\frac{\theta\left(\gamma+bc\right)\left(\alpha+\beta+2c^2\right)}{\theta\gamma\left(\gamma+bc\right)+c^2}f_{F1\chi}=-\frac{\theta\left(\alpha+bc\right)\left(\gamma+bc\right)\left(\alpha+\beta+2c^2\right)}{c^2\left(\gamma-\alpha\right)\left(2\gamma-\left(\alpha+\beta\right)\right)}.$$

Thus:

$$\pi_{FF}=c\left(2f_{Fi\chi}+f_{FF\chi}\right)\chi=\frac{\left(\alpha+bc\right)\left[\theta\left(\gamma+bc\right)\left(2\gamma-\left(\alpha+\beta\right)\right)+2c^2\left(1-\theta\left(\gamma+bc\right)\right)\right]}{c\left(\gamma-\alpha\right)\left(2\gamma-\left(\alpha+\beta\right)\right)}\chi.$$

We also get the equilibrium output level under fiscal federalism:

$$\overline{y}_F^*=y_F^*=\hat{y}+\left(\frac{1}{2}\left(\alpha+\beta\right)\right)\left(2g_F^*\right)+\gamma g_F^*=\hat{y}+\frac{\left(\alpha+bc\right)\left(\left(\alpha+\beta\right)-2\theta\gamma\left(\gamma+bc\right)\right)}{\left(\gamma-\alpha\right)\left(2\gamma-\left(\alpha+\beta\right)\right)}\chi.$$

Cooperation vs no cooperation. We compare the outcome of the union characterized by cooperation among fiscal authorities with the outcome of the union with such a cooperation.

For inflation we get:

$$\pi_C^* - \pi_n^* = \left[\frac{\theta c\left(\alpha+\beta+2bc\right)}{\frac{1}{2}\theta\left(\alpha+\beta+2bc\right)\left(\alpha+\beta\right)+2c^2} - \frac{2c\theta\left(\alpha+bc\right)}{\theta\left(\alpha+bc\right)\left(\alpha+\beta\right)+2c^2}\right]\chi$$
$$= c\theta\left[\frac{2c^2\left(\beta-\alpha\right)}{\left(\frac{1}{2}\theta\left(\alpha+\beta+2bc\right)\left(\alpha+\beta\right)+2c^2\right)\left(\theta\left(\alpha+bc\right)\left(\alpha+\beta\right)+2c^2\right)}\right]\chi$$

which is always negative as $\beta<\alpha$.

For national aggregate output, we get:

$$y_C^* - y_n^* = \left[\frac{\frac{1}{2}\theta\left(\alpha+\beta+2bc\right)\left(\alpha+\beta\right)}{\frac{1}{2}\theta\left(\alpha+\beta+2bc\right)\left(\alpha+\beta\right)+2c^2} - \frac{\theta\left(\alpha+bc\right)}{\theta\left(\alpha+bc\right)\left(\alpha+\beta\right)+2c^2}\right]\chi$$
$$= \theta\left[\frac{\frac{1}{2}\theta\left(\alpha+bc\right)\left(\alpha+\beta+2bc\right)\left(\alpha+\beta\right)\left(\alpha+\beta-1\right)+c^2\left(\left(\alpha+\beta\right)^2+2bc\left(\alpha+\beta-1\right)-2\alpha\right)}{\left(\frac{1}{2}\theta\left(\alpha+\beta+2bc\right)\left(\alpha+\beta\right)+2c^2\right)\left(\theta\left(\alpha+bc\right)\left(\alpha+\beta\right)+2c^2\right)}\right]\chi$$

which is not always negative.

Comparing cooperation and federalism. We proceed similarly for the comparison between a union based on cooperation and one based on fiscal federalism.

For inflation we get:

$$\pi_C^* - \pi_F^* = \left[\frac{\theta c\left(\alpha+\beta+2bc\right)}{\frac{1}{2}\theta\left(\alpha+\beta+2bc\right)\left(\alpha+\beta\right)+2c^2} - \frac{\left(\alpha+bc\right)\left[\theta\left(\gamma+bc\right)\left(2\gamma-\left(\alpha+\beta\right)\right)+2c^2\left(1-\theta\left(\gamma+bc\right)\right)\right]}{c\left(\gamma-\alpha\right)\left(2\gamma-\left(\alpha+\beta\right)\right)}\right]\chi$$

of ambigous sign.

For the output gap, the difference is:

$$y_C^* - \overline{y}_F^* = \left[\frac{\frac{1}{2}\theta\left(\alpha+\beta+2bc\right)\left(\alpha+\beta\right)}{\frac{1}{2}\theta\left(\alpha+\beta+2bc\right)\left(\alpha+\beta\right)+2c^2} - \frac{\theta\left(\alpha+bc\right)}{\theta\left(\alpha+bc\right)\left(\alpha+\beta\right)+2c^2}\right]\chi$$
$$= \frac{\frac{1}{2}\theta\left(\alpha+\beta+2bc\right)\left(\alpha+\beta\right)\theta\left(\alpha+bc\right)\left(\alpha+\beta-1\right)+c^2\theta\left(\left(\alpha+\beta+2bc\right)\left(\alpha+\beta\right)-\left(2\alpha+2bc\right)\right)}{\left(\frac{1}{2}\theta\left(\alpha+\beta+2bc\right)\left(\alpha+\beta\right)+2c^2\right)\left(\theta\left(\alpha+bc\right)\left(\alpha+\beta\right)+2c^2\right)}\chi$$

It also is of ambigous sign.

Losses

When national fiscal authorities do not cooperate, their loss is egal to:

$$L_{in}^* = L_n^* = \frac{1}{2}\theta\left(y_n^*-\tilde{\chi}\right)^2+\frac{1}{2}\pi_n^{*2}.$$

Given the results given above, and using $\chi=\tilde{\chi}-\hat{y}$, we get:

$$L_n^* = \frac{1}{2}\theta\left(\hat{y}+\frac{\theta\left(\alpha+bc\right)}{\theta\left(\alpha+bc\right)\left(\alpha+\beta\right)+2c^2}\chi-\tilde{\chi}\right)^2+\frac{1}{2}\left(\frac{2c\theta\left(\alpha+bc\right)}{\theta\left(\alpha+bc\right)\left(\alpha+\beta\right)+2c^2}\chi\right)^2 = A_n^*\chi^2$$

with

$$A_n^* = \frac{1}{2}\frac{\theta\left(\theta\left(\alpha+bc\right)\right)^2\left(\left(1-\left(\alpha+\beta\right)\right)^2+4c^4+\left(2c\right)^2\left(\alpha+\beta\right)\right)}{\left(\theta\left(\alpha+bc\right)\left(\alpha+\beta\right)+2c^2\right)^2}.$$

On the other hand, when national fiscal authorities do cooperate, the loss for a national fiscal authority is equal to:

$$L_C^* = \frac{1}{2}\theta\left(y_C^* - \tilde{\chi}\right)^2 + \frac{1}{2}{\pi_C^*}^2 = A_C^*\chi^2.$$

Given the results given above we get:

$$L_C^* = \frac{1}{2}\theta\left(\frac{\frac{1}{2}\theta\left(\alpha+\beta+2bc\right)\left(\alpha+\beta\right)}{\frac{1}{2}\theta\left(\alpha+\beta+2bc\right)\left(\alpha+\beta\right)+2c^2} - 1\right)^2\chi^2 + \frac{1}{2}\left(\frac{\theta c\left(\alpha+\beta+2bc\right)}{\frac{1}{2}\theta\left(\alpha+\beta+2bc\right)\left(\alpha+\beta\right)+2c^2}\right)^2\chi^2 = A_C^*\chi^2$$

with

$$A_C^* = \frac{1}{2}\left\{\frac{\theta\left(2c^2\right)^2 + \left(\theta c\right)^2\left(\alpha+\beta+2bc\right)^2}{\left(\frac{1}{2}\theta\left(\alpha+\beta+2bc\right)\left(\alpha+\beta\right)+2c^2\right)^2}\right\} = 2\theta c^2\left\{\frac{4c^2+\theta\left(\alpha+\beta+2bc\right)^2}{\left(4c^2+\theta\left(\alpha+\beta+2bc\right)\left(\alpha+\beta\right)\right)^2}\right\}.$$

When a federal Treasury is active, the losses are equal to:

- for the national fiscal authorities, given the results given above:

$$L_F^* = \frac{1}{2}\theta\left(\hat{y} + \frac{\left(\alpha+bc\right)\left(\left(\alpha+\beta\right) - 2\theta\gamma\left(\gamma+bc\right)\right)}{\left(\gamma-\alpha\right)\left(2\gamma-\left(\alpha+\beta\right)\right)}\chi - \tilde{\chi}\right)^2 + \frac{1}{2}\left(\frac{\left(\alpha+bc\right)\left[\theta\left(\gamma+bc\right)\left(2\gamma-\left(\alpha+\beta\right)\right)+2c^2\left(1-\theta\left(\gamma+bc\right)\right)\right]}{c\left(\gamma-\alpha\right)\left(2\gamma-\left(\alpha+\beta\right)\right)}\chi\right)^2$$

$$= A_F^*\chi^2$$

with

$$A_F^* = \theta\left(\frac{\left(\gamma+bc\right)\left(\alpha+\beta-2\theta\gamma\left(\alpha+bc\right)\right) - 2\gamma\left(\gamma-\alpha\right)}{2\left(\gamma-\alpha\right)\left(2\gamma-\left(\alpha+\beta\right)\right)}\right)^2 + \left(\frac{\left(\alpha+bc\right)\left[\theta\left(\gamma+bc\right)\left(2\gamma-\left(\alpha+\beta\right)\right)+2c^2\left(1-\theta\left(\gamma+bc\right)\right)\right]}{2c\left(\gamma-\alpha\right)\left(2\gamma-\left(\alpha+\beta\right)\right)}\right)^2$$

$$A_F^* = \left[\theta\left(\frac{\left(\gamma+bc\right)\left(\alpha+\beta-2\theta\gamma\left(\alpha+bc\right)\right) - 2\gamma\left(\gamma-\alpha\right)}{2\left(\gamma-\alpha\right)\left(2\gamma-\left(\alpha+\beta\right)\right)}\right)^2 + \left(\frac{\left(\alpha+bc\right)\left[\theta\left(\gamma+bc\right)\left(2\gamma-\left(\alpha+\beta\right)\right)+2c^2\left(1-\theta\left(\gamma+bc\right)\right)\right]}{2c\left(\gamma-\alpha\right)\left(2\gamma-\left(\alpha+\beta\right)\right)}\right)^2\right]$$

- for the federal fiscal authority (remember that we assume $\tilde{\chi}_F = \hat{y}$), given the results given above:

$$L_{FF}^* = \frac{1}{2}\theta\left(\frac{\left(\alpha+bc\right)\left(\left(\alpha+\beta\right) - 2\theta\gamma\left(\gamma+bc\right)\right)}{\left(\gamma-\alpha\right)\left(2\gamma-\left(\alpha+\beta\right)\right)}\right)^2 + \frac{1}{2}\left(\frac{\left(\alpha+bc\right)\left[\theta\left(\gamma+bc\right)\left(2\gamma-\left(\alpha+\beta\right)\right)+2c^2\left(1-\theta\left(\gamma+bc\right)\right)\right]}{c\left(\gamma-\alpha\right)\left(2\gamma-\left(\alpha+\beta\right)\right)}\chi\right)^2$$

$$= A_{FF}^*\chi^2$$

with

$$A_{FF}^* = \frac{\left(\alpha+bc\right)^2}{2}\left\{\theta\left(\frac{\left(\alpha+\beta\right) - 2\theta\gamma\left(\gamma+bc\right)}{\left(\alpha-\gamma\right)\left(2\gamma-\left(\alpha+\beta\right)\right)}\right)^2 + \left(\frac{\left[\theta\left(\gamma+bc\right)\left(2\gamma-\left(\alpha+\beta\right)\right)+2c^2\left(1-\theta\left(\gamma+bc\right)\right)\right]}{c\left(\gamma-\alpha\right)\left(2\gamma-\left(\alpha+\beta\right)\right)}\right)^2\right\}$$

- Comparing losses for national fiscal players is equivalent to comparing A_F^* and A_C^*. The difference is of ambiguous sign. For example, if θ is arbitrarily close to 0, A_C^* is arbitrarily close to 0, and:

$$A_F^* \simeq \frac{\left(\alpha+bc\right)^2}{2}\left(\frac{2c}{\left(\gamma-\alpha\right)\left(2\gamma-\left(\alpha+\beta\right)\right)}\right)^2$$

which is positive. Cooperation achieves a better outcome than federalism. However, if $\theta\left(\gamma+bc\right) = 1$, we get:

$$A_F^* = \left(\frac{\alpha+\beta-2\theta\gamma\left(\alpha+bc\right) - 2\gamma\theta\left(\gamma-\alpha\right)}{2\left(\gamma-\alpha\right)\theta\left(2\gamma-\left(\alpha+\beta\right)\right)}\right)^2 + \left(\frac{\left(\alpha+bc\right)}{2c\left(\gamma-\alpha\right)}\right)^2$$

$$= \left(\frac{-1}{2\theta\left(\gamma-\alpha\right)}\right)^2 + \left(\frac{\left(\alpha+bc\right)}{2c\left(\gamma-\alpha\right)}\right)^2 = \frac{1}{\left(2\theta\left(\gamma-\alpha\right)\right)^2} + \frac{\left(\alpha+bc\right)^2}{\left(2c\left(\gamma-\alpha\right)\right)^2} = \frac{c^2+\theta^2\left(\alpha+bc\right)^2}{\left(2\left(\gamma-\alpha\right)\theta c\right)^2}$$

Plus if $c = \theta$, we get:

$$= c^2 \frac{1 + (\alpha + bc)^2}{(2(\gamma - \alpha)\theta c)^2} = \frac{1 + (\alpha + bc)^2}{4c^2 (\gamma - \alpha)^2}$$

If $(\alpha + \beta) = 1$,

$$A_C^* = \frac{1}{2} \left\{ \frac{\theta \left(2c^2\right)^2 + (\theta c)^2 (1 + 2bc)^2}{\left(\frac{1}{2}\theta (1 + 2bc) + 2c^2\right)^2} \right\} = 2c^2 \left\{ \frac{4c + (1 + 2bc)^2}{(4c + (1 + 2bc))^2} \right\}.$$

If c is sufficiently large and b sufficiently small, then $A_C^* > A_F^*$.

- We can also compute the loss of the "federal Treasury" when it is inactive and countries cooperate, which we denote by L_{FC}^*. Given the loss function (5) and the results given above, we get:

$$L_{FC}^* = \frac{1}{2}\theta_F \left(y_C^* - \tilde{\chi}_F\right)^2 + \frac{1}{2}\left(\pi_C^*\right)^2 = \frac{1}{2}\theta \left(y_C^* - \hat{y}\right)^2 + \frac{1}{2}\left(\pi_C^*\right)^2$$

$$= \frac{1}{2}\left\{ \theta \left(\frac{\frac{1}{2}\theta (\alpha + \beta + 2bc)(\alpha + \beta)}{\frac{1}{2}\theta (\alpha + \beta + 2bc)(\alpha + \beta) + 2c^2} \right)^2 + \frac{1}{2}\left(\frac{\theta c (\alpha + \beta + 2bc)}{\frac{1}{2}\theta (\alpha + \beta + 2bc)(\alpha + \beta) + 2c^2} \right)^2 \right\} \chi^2 = A_{FC}^* \chi^2$$

As

$$A_{FC}^* = \frac{1}{2} \frac{\theta \left(\frac{1}{2}\theta (\alpha + \beta + 2bc)(\alpha + \beta)\right)^2 + \left(\theta c (\alpha + \beta + 2bc)^2\right)}{\left(\frac{1}{2}\theta (\alpha + \beta + 2bc)(\alpha + \beta) + 2c^2\right)^2}$$

$$= \frac{\theta^2}{2} \frac{\theta \left(\frac{1}{2}(\alpha + \beta + 2bc)(\alpha + \beta)\right)^2 + c^2 (\alpha + \beta + 2bc)^2}{\left(\frac{1}{2}\theta (\alpha + \beta + 2bc)(\alpha + \beta) + 2c^2\right)^2} = \frac{\theta^2 (\alpha + \beta + 2bc)^2}{2} \frac{\frac{\theta}{4}(\alpha + \beta)^2 + c^2}{\left(\frac{1}{2}\theta (\alpha + \beta + 2bc)(\alpha + \beta) + 2c^2\right)^2}$$

If θ is arbitrarily close to 0, A_{FC}^* is arbitrarily close to 0, where we get:

$$A_{FF}^* \simeq \frac{(\alpha + bc)^2}{2} \left(\frac{2c}{(\gamma - \alpha)(2\gamma - (\alpha + \beta))} \right)^2$$

Thus: $A_{FC}^* < A_{FF}^*$.

Appendix 2. Introducing a central bank.

As the loss function of the central bank solely depends on inflation, we get from the first-order condition and under rational expectations:

$$\pi = c\left(g_1 + g_2 + g_F\right) + \pi_{cb} = \pi^e = 0$$

and

$$-c\left(g_1 + g_2 + g_F\right) = \pi_{cb}.$$

Output is then:

$$y_i = \hat{y} + \alpha g_i + \beta g_j + \gamma g_F.$$

No-cooperation.

Replacing $\pi^e = 0$ in conditions (31) obtained above, in equilibrium, we get:

$$\theta(\alpha+bc)\left((\alpha+bc)g_i + (\beta+bc)g_j - \chi\right) + c^2(g_i+g_j) = 0.$$

Using symmetry, we get:

$$g_n^* = \frac{\theta(\alpha+bc)}{\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2}\chi.$$

Output is then:

$$y_n^* = \hat{y} + (\alpha+\beta)g_n^* = \hat{y} + \frac{\theta(\alpha+bc)(\alpha+\beta)}{\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2}\chi$$

and the loss is:

$$L_n^* = \frac{1}{2}\theta\left(\frac{\theta(\alpha+bc)(\alpha+\beta)}{\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2} - 1\right)^2\chi^2 = 2\theta\left(\frac{(\theta b(\alpha+bc)+c)c}{\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2}\right)^2\chi^2.$$

Cooperation

Replacing $\pi^e = 0$ in condition (33), in equilibrium we get:

$$\frac{1}{2}\theta(\alpha+\beta+2bc)\left((\alpha+bc)g_i + (\beta+bc)g_j\right) + c^2(g_i+g_j) = \frac{1}{2}\theta(\alpha+\beta+2bc)\chi.$$

Using symmetry, we get:

$$g_C^* = \frac{\theta(\alpha+\beta+2bc)}{2\left(\theta(\alpha+\beta+2bc)^2+2c^2\right)}\chi.$$

Output is then:

$$y_C^* = \hat{y} + (\alpha+\beta)g_C^* = \hat{y} + \frac{\theta(\alpha+\beta)(\alpha+\beta+2bc)}{2\left(\theta(\alpha+\beta+2bc)^2+2c^2\right)}\chi$$

and the loss is:

$$L_{iC}^* = L_C^* = \frac{1}{2}\theta\left(\frac{\theta(\alpha+\beta)(\alpha+\beta+2bc)}{2\left(\theta(\alpha+\beta+2bc)^2+2c^2\right)}\chi - \chi\right)^2 = \frac{1}{2}\theta\left(\frac{(\alpha+\beta+4bc)\theta(\alpha+\beta+2bc)+4c^2}{2\left(\theta(\alpha+\beta+2bc)^2+2c^2\right)}\right)^2\chi^2.$$

Fiscal federalism

Replacing $\pi^e = 0$ in conditions (35) and (37), in equilibrium we get:

$$g_i = -\frac{\left(\theta(\beta+bc)(\alpha+bc)+c^2\right)}{\left(\theta(\alpha+bc)^2+c^2\right)}g_j - \frac{\left(\theta(\gamma+bc)(\alpha+bc)+c^2\right)}{\left(\theta(\alpha+bc)^2+c^2\right)}g_F + \frac{\theta(\alpha+bc)}{\left(\theta(\alpha+bc)^2+c^2\right)}\chi$$

and

$$g_F = -\frac{\theta(\gamma+bc)\left(\frac{1}{2}(\alpha+\beta)+bc+c^2\right)}{\left(\theta(\gamma+bc)^2+c^2\right)}(g_1+g_2).$$

Using symmetry, we get:

$$g_{Fi}\left(\frac{\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2}{\left(\theta(\alpha+bc)^2+c^2\right)}\right) = -\frac{\left(\theta(\gamma+bc)(\alpha+bc)+c^2\right)}{\left(\theta(\alpha+bc)^2+c^2\right)}g_{FF} + \frac{\theta(\alpha+bc)}{\left(\theta(\alpha+bc)^2+c^2\right)}\chi$$

and

$$g_{FF} = -\frac{\theta\left(\gamma+bc\right)\left(\alpha+\beta+2bc+2c^2\right)}{\left(\theta\left(\gamma+bc\right)^2+c^2\right)}g_i.$$

Therefore:

$$g_i\left[\frac{\left(\theta\left(\alpha+bc\right)\left(\alpha+\beta+2bc\right)+2c^2\right)\left(\theta\left(\gamma+bc\right)^2+c^2\right)-\left(\theta\left(\gamma+bc\right)\left(\alpha+bc\right)+c^2\right)\left(\alpha+\beta+2bc+2c^2\right)\theta\left(\gamma+bc\right)}{\left(\theta\left(\gamma+bc\right)^2+c^2\right)}\right]$$

$$=\theta\left(\alpha+bc\right)\chi.$$

Remark that:

$$\left(\theta\left(\alpha+bc\right)\left(\alpha+\beta+2bc\right)+2c^2\right)\left(\theta\left(\gamma+bc\right)^2+c^2\right)-\left(\theta\left(\gamma+bc\right)\left(\alpha+bc\right)+c^2\right)\theta\left(\gamma+bc\right)\left(\alpha+\beta+2bc+2c^2\right)$$

$$=\left(\alpha+\beta+2bc\right)\left[\theta c^2\left(\alpha-\gamma\right)\right]+2c^2\left(\theta\left(\gamma+bc\right)^2\left(1-\theta\left(\alpha+bc\right)\right)+c^2\left(1-\theta\left(\gamma+bc\right)\right)\right).$$

Hence:

$$g_F^* = \frac{\theta\left(\alpha+bc\right)\left(\theta\left(\gamma+bc\right)^2+c^2\right)}{\left(\alpha+\beta+2bc\right)\left[\theta c^2\left(\alpha-\gamma\right)\right]+2c^2\left(\theta\left(\gamma+bc\right)^2\left(1-\theta\left(\alpha+bc\right)\right)+c^2\left(1-\theta\left(\gamma+bc\right)\right)\right)}\chi$$

and:

$$g_{FF}^* = -\frac{\theta\left(\gamma+bc\right)\left(\alpha+\beta+2bc+2c^2\right)}{\left(\theta\left(\gamma+bc\right)^2+c^2\right)}g_F^*$$

$$= -\frac{\theta\left(\gamma+bc\right)\left(\alpha+\beta+2bc+2c^2\right)\theta\left(\alpha+bc\right)}{\left(\alpha+\beta+2bc\right)\left[\theta c^2\left(\alpha-\gamma\right)\right]+2c^2\left(\theta\left(\gamma+bc\right)^2\left(1-\theta\left(\alpha+bc\right)\right)+c^2\left(1-\theta\left(\gamma+bc\right)\right)\right)}.$$

Finally, output is equal to:

$$y_F^* = \hat{y}+\left(\alpha+\beta\right)g_F^*+\gamma g_{FF}^*$$

$$=\hat{y}+\left(\frac{\left(\alpha+\beta\right)\left(\theta\left(\gamma+bc\right)bc+c^2\right)-2\gamma\theta\left(\gamma+bc\right)\left(1+bc\right)}{\left(\theta\left(\gamma+bc\right)^2+c^2\right)}\right)g_F^*$$

$$=\hat{y}+\frac{\theta\left(\gamma+bc\right)\left[\left(\alpha+\beta\right)bc-2\gamma\theta\left(1+bc\right)\left(\alpha+bc\right)\right]+\left(\alpha+\beta\right)c^2}{\left(\alpha+\beta+2bc\right)\left[\theta c^2\left(\alpha-\gamma\right)\right]+2c^2\left(\theta\left(\gamma+bc\right)^2\left(1-\theta\left(\alpha+bc\right)\right)+c^2\left(1-\theta\left(\gamma+bc\right)\right)\right)}$$

The loss is:

$$L_{iF}^* = L_F^* = \frac{1}{2}\theta\left(\frac{\theta\left(\gamma+bc\right)\left[\left(\alpha+\beta\right)bc-2\gamma\theta\left(1+bc\right)\left(\alpha+bc\right)\right]+\left(\alpha+\beta\right)c^2}{\left(\alpha+\beta+2bc\right)\left[\theta c^2\left(\alpha-\gamma\right)\right]+2c^2\left(\theta\left(\gamma+bc\right)^2\left(1-\theta\left(\alpha+bc\right)\right)+c^2\left(1-\theta\left(\gamma+bc\right)\right)\right)}\right)^2\chi^2.$$

Appendix 3. The stochastic model.

From (4), (1) et (3), we get the following expression for output in country i:

$$y_i = \hat{y}+\left(\alpha+bc\right)g_i+\left(\beta+bc\right)g_{-i}+\left(\gamma+bc\right)g_F+b\left(\varepsilon-\pi^e\right)+u_i. \tag{38}$$

Non-cooperation.

Here there is no Federal Treasury, the central bank has no active policy and applies $\pi_M = 0$ in any circumstance. Given the definition of the objectives of policymaker and the fact that expectations are formed before the occurrence of shocks, we get $\pi^e = 0$ in each variant of the model.

The optimization program of Treasury i is (using (38)):

$$max_{g_i} L = \frac{1}{2}\theta\left((\alpha + bc)\, g_i + (\beta + bc)\, g_{-i} + b\varepsilon + u_i\right)^2 + \frac{1}{2}\left(c\left(g_1 + g_2\right) + \varepsilon\right)^2. \qquad (39)$$

The first-order condition implies:

$$g_i = \frac{-\left(\theta\left(\beta + bc\right)\left(\alpha + bc\right) + c^2\right) g_{-i} - \theta\left(\alpha + bc\right) u_i}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)} - \frac{\left(c + \theta b\left(\alpha + bc\right)\right)}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)}\varepsilon.$$

We denote g^*_{in} the optimal solutions of this game and π^e_n the corresponding expected inflation. To compute the equilibrium of the game, we use the undetermined coefficients method. We assume the following reduced forms:

$$g^*_n = f_{nii}u_i + f_{nij}u_j + f_{ni\varepsilon}\varepsilon$$

The equilibrium solution is thus characterized by:

$$f_{nii}u_i + f_{nij}u_j + f_{i\varepsilon}\varepsilon =$$

$$\frac{\theta\left(\alpha + bc\right)\chi - \left(\theta\left(\beta + bc\right)\left(\alpha + bc\right) + c^2\right)\left(f_{nji}u_i + f_{njj}u_j + f_{nj\varepsilon}\varepsilon\right)}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)} \frac{-\theta\left(\alpha + bc\right)u_i - \left(c + \theta b\left(\alpha + bc\right)\right)\varepsilon}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)}$$

which implies:

$$f_{n22} = \frac{-\left(\theta\left(\beta + bc\right)\left(\alpha + bc\right) + c^2\right) f_{n12} - \theta\left(\alpha + bc\right)}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)}$$

$$f_{n21} = \frac{-\left(\theta\left(\beta + bc\right)\left(\alpha + bc\right) + c^2\right) f_{n11}}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)}$$

$$f_{n2\varepsilon} = \frac{-\left(\theta\left(\beta + bc\right)\left(\alpha + bc\right) + c^2\right) f_{n1\varepsilon} - \left(c + \theta b\left(\alpha + bc\right)\right)}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)}.$$

Since countries are identical, we can write:

$$f_{n11} = f_{n22}, f_{n1\varepsilon} = f_{n2\varepsilon}, f_{n12} = f_{n21}.$$

After tedious computation, we get:

$$f_{n1\varepsilon} = f_{n2\varepsilon} = -\frac{c + \theta b\left(\alpha + bc\right)}{\theta\left(\alpha + bc\right)\left(\alpha + \beta + 2bc\right) + 2c^2}$$

$$f_{n11} = f_{n22} = -\frac{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)}{\left[\theta\left(\alpha + bc\right)\left(\alpha + \beta + 2bc\right) + 2c^2\right]\left(\alpha - \beta\right)}$$

$$f_{n21} = f_{n12} = \frac{\left(\theta\left(\beta + bc\right)\left(\alpha + bc\right) + c^2\right)}{\left[\theta\left(\alpha + bc\right)\left(\alpha + \beta + 2bc\right) + 2c^2\right]\left(\alpha - \beta\right)}.$$

This ends the identification of the coefficients of the reduced forms. To summarize the reduced forms are given by the following relations:

$$g_n^* = -\frac{(c+\theta b(\alpha+bc))}{\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2}\varepsilon$$

$$-\frac{\left(\theta(\alpha+bc)^2+c^2\right)}{\left[\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2\right](\alpha-\beta)}u_i+\frac{\left(\theta(\beta+bc)(\alpha+bc)+c^2\right)}{\left[\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2\right](\alpha-\beta)}u_j.$$

For inflation we get:

$$\pi_n^* = c(g_1+g_2)+\varepsilon = \frac{\theta(\alpha+bc)}{\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2}\left[-c(u_1+u_2)+(\alpha+\beta)\varepsilon\right].$$

The national aggregate outputs are equal to:

$$y_n^* = \hat{y}+\alpha g_{in}^*+\beta g_{jn}^*+b(\pi-\pi^e)+u_1$$

Hence using the expressions obtained above we get:

$$y_n^* = \hat{y}+\frac{c^2}{\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2}(u_1+u_2)-\frac{c(\alpha+\beta)}{\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2}\varepsilon$$

The union's aggregate output is equal to:

$$\overline{y}_n^* = \hat{y}+\frac{c^2}{\left[\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2\right]}\overline{u}-\frac{c(\alpha+\beta)}{\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2}\varepsilon$$

Expected losses.

The value of the actual loss for a national fiscal authority (taking into account that $\chi = \tilde{\chi}-\hat{y}$) is equal to:

$$L_n^* = \frac{1}{2}\theta\left\{\frac{c^2}{\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2}(u_1+u_2)-\frac{c(\alpha+\beta)}{\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2}\varepsilon\right\}^2$$
$$+\frac{1}{2}\left\{\frac{\theta(\alpha+bc)}{\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2}\left[(\alpha+\beta)\varepsilon-c(u_1+u_2)\right]\right\}^2.$$

Its expected loss is thus equal to:

$$E(L_n^*) = \frac{\theta c^2\left(c^2+\theta(\alpha+bc)^2\right)}{\left(\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2\right)^2}\sigma_u^2+\frac{1}{2}\frac{\theta c^2\left((\alpha+\beta)^2+\theta(\alpha+bc)^2\right)}{\left(\theta(\alpha+bc)(\alpha+\beta+2bc)+2c^2\right)^2}\sigma_\varepsilon^2.$$

Fiscal cooperation.

Identification of reduced forms

The program of the cooperating national fiscal authorities is:

$$max_{g_1,g_2}L = \frac{1}{2}\frac{\theta}{2}\left((\alpha+bc)g_1+(\beta+bc)g_2+b\varepsilon+u_1\right)^2 \tag{40}$$
$$+\frac{1}{2}\frac{\theta}{2}\left((\alpha+bc)g_2+(\beta+bc)g_1+b\varepsilon+u_2\right)^2+\frac{1}{2}\left(c(g_1+g_2)+\varepsilon\right)^2.$$

The first-order condition for g_i implies:

$$g_i = -\frac{\left[\theta(\alpha+bc)(\beta+bc)+c^2\right]}{\left[\frac{\theta}{2}\left((\alpha+bc)^2+(\beta+bc)^2\right)+c^2\right]}g_j$$

$$-\frac{\left[c+\frac{\theta}{2}b\left(\alpha+\beta+2bc\right)\right]}{\left[\frac{\theta}{2}\left(\left(\alpha+bc\right)^2+\left(\beta+bc\right)^2\right)+c^2\right]}\varepsilon-\frac{\frac{\theta}{2}\left(\left(\alpha+bc\right)u_i+\left(\beta+bc\right)u_j\right)}{\left[\frac{\theta}{2}\left(\left(\alpha+bc\right)^2+\left(\beta+bc\right)^2\right)+c^2\right]}.$$

Using symmetry we denote by g_C^* the optimal solutions of this game and π_C^e the corresponding expected inflation. The reduced forms for these solutions are written as follows:

$$g_C^*=f_{Cii}u_i+f_{Cij}u_j+f_{Ci\varepsilon}\varepsilon.$$

We then get:

$$f_{Cii}u_i+f_{Cij}u_j+f_{Ci\varepsilon}\varepsilon=$$

$$-\frac{\left[\theta\left(\alpha+bc\right)\left(\beta+bc\right)+c^2\right]f_{Cji}+\frac{\theta}{2}\left(\alpha+bc\right)}{\left[\frac{\theta}{2}\left(\left(\alpha+bc\right)^2+\left(\beta+bc\right)^2\right)+c^2\right]}u_i-\frac{\left[\theta\left(\alpha+bc\right)\left(\beta+bc\right)+c^2\right]f_{Cjj}+\frac{\theta}{2}\left(\left(\beta+bc\right)\right)}{\left[\frac{\theta}{2}\left(\left(\alpha+bc\right)^2+\left(\beta+bc\right)^2\right)+c^2\right]}u_j$$

$$-\frac{\left[c+\frac{\theta}{2}b\left(\alpha+\beta+2bc\right)\right]+\left[\theta\left(\alpha+bc\right)\left(\beta+bc\right)+c^2\right]f_{Cj\varepsilon}}{\left[\frac{\theta}{2}\left(\left(\alpha+bc\right)^2+\left(\beta+bc\right)^2\right)+c^2\right]}\varepsilon.$$

At the symmetrical equilibrium we get:

$$f_{C11}=f_{C22},f_{C1\varepsilon}=f_{C2\varepsilon},f_{C12}=f_{C21}.$$

Combining these equations we get:

$$f_{Ci\varepsilon}=-\frac{\left[c+\frac{\theta}{2}b\left(\alpha+\beta+2bc\right)\right]}{\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]}$$

$$f_{Cii}=-\frac{\frac{\theta}{2}\left(\alpha+\beta+2bc\right)\left(\alpha+bc\right)+c^2}{\left(\alpha-\beta\right)\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]}$$

$$f_{Cij}=f_{Cji}=\frac{\frac{\theta}{2}\left(\alpha+\beta+2bc\right)\left(\beta+bc\right)+c^2}{\left(\alpha-\beta\right)\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]}.$$

Replacing these expressions in the reduced forms we get:

$$g_{iC}^*=-\frac{\left[c+\frac{\theta}{2}b\left(\alpha+\beta+2bc\right)\right]}{\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]}\varepsilon$$

$$-\frac{\frac{\theta}{2}\left(\alpha+\beta+2bc\right)\left(\alpha+bc\right)+c^2}{\left(\alpha-\beta\right)\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]}u_i+\frac{\frac{\theta}{2}\left(\alpha+\beta+2bc\right)\left(\beta+bc\right)+c^2}{\left(\alpha-\beta\right)\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]}u_j.$$

We also obtain the following expression for inflation which we denote π_C^*:

$$\pi_C^*=-\frac{\frac{\theta}{2}c\left(\alpha+\beta+2bc\right)}{\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2}\left(u_1+u_2\right)+\frac{\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)\right]\left(\alpha+\beta\right)}{\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]}\varepsilon.$$

Using this expression, the national aggregate output are equal to:

$$y_{1C}^*=y_{2C}^*=\hat{y}+\left(\alpha g_{1C}^*+\beta g_{2C}^*\right)+b\pi_C^*+u_1=\hat{y}+\frac{c^2\left(u_1+u_2\right)-c\left(\alpha+\beta\right)\varepsilon}{\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]}.$$

Expected losses.

The actual loss for a national fiscal authority is equal to (taking into account that $\chi \equiv \tilde{\chi} - \hat{y} = 0$) :

$$L_{iC}^* = L_C^* = \frac{1}{2}\theta \left(y_{iC}^* - \tilde{\chi}\right)^2 + \frac{1}{2}\left(\pi\right)^2$$

$$= \frac{1}{2}\theta \left(\frac{c^2 \left(u_1 + u_2\right) - c\left(\alpha + \beta\right)\varepsilon}{\frac{\theta}{2}\left(\alpha + \beta + 2bc\right)^2 + 2c^2} \right)^2 + \frac{1}{2}\left(\frac{\frac{\theta}{2}\left(\alpha + \beta + 2bc\right)\left(\left(\alpha + \beta\right)\varepsilon - c\left(u_1 + u_2\right)\right)}{\frac{\theta}{2}\left(\alpha + \beta + 2bc\right)^2 + 2c^2} \right)^2$$

$$= \frac{1}{2}\frac{\theta\left(c^2\left(u_1 + u_2\right) - c\left(\alpha + \beta\right)\varepsilon\right)^2 + \left(\frac{\theta}{2}\left(\alpha + \beta + 2bc\right)\left(\left(\alpha + \beta\right)\varepsilon - c\left(u_1 + u_2\right)\right)\right)^2}{\left(\frac{\theta}{2}\left(\alpha + \beta + 2bc\right)^2 + 2c^2\right)^2}$$

and thus the expected loss is equal to:

$$E\left(L_C^*\right) = \frac{1}{2}\frac{\theta\left(2c^4\sigma_u^2 + c^2\left(\alpha + \beta\right)^2\sigma_\varepsilon^2\right) + \left(\left(\frac{\theta}{2}\right)^2\left(\alpha + \beta + 2bc\right)^2\left(\left(\alpha + \beta\right)^2\sigma_\varepsilon^2 + 2c^2\sigma_u^2\right)\right)}{\left(\frac{\theta}{2}\left(\alpha + \beta + 2bc\right)^2 + 2c^2\right)^2}$$

$$= \frac{c^2\left(\theta c^2 + \left(\frac{\theta}{2}\right)^2\left(\alpha + \beta + 2bc\right)^2\right)}{\left(\frac{\theta}{2}\left(\alpha + \beta + 2bc\right)^2 + 2c^2\right)^2}\sigma_u^2 + \frac{1}{2}\frac{\left(\left(\frac{\theta}{2}\right)^2\left(\alpha + \beta + 2bc\right)^2 + \theta c^2\right)\left(\alpha + \beta\right)^2}{\left(\frac{\theta}{2}\left(\alpha + \beta + 2bc\right)^2 + 2c^2\right)^2}\sigma_\varepsilon^2 = B_C^*\sigma_u^2 + C_C^*\sigma_\varepsilon^2. \quad (41)$$

Fiscal federalism.

The optimization program of the national fiscal authority i is:

$$max_{g_i} L = \frac{1}{2}\theta\left(\left(\alpha + bc\right)g_i + \left(\beta + bc\right)g_{-i} + \left(\gamma + bc\right)g_F + b\varepsilon + u_i\right)^2 + \frac{1}{2}\left(c\left(g_1 + g_2 + g_F\right) + \varepsilon\right)^2. \quad (42)$$

The first-order condition implies:

$$g_i = \frac{-\theta\left(\alpha + bc\right)u_i - \left(c + \theta b\left(\alpha + bc\right)\right)\varepsilon}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)}$$

$$-\frac{\left(\theta\left(\beta + bc\right)\left(\alpha + bc\right) + c^2\right)}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)}g_{-i} - \frac{\left(\theta\left(\gamma + bc\right)\left(\alpha + bc\right) + c^2\right)}{\left(\theta\left(\alpha + bc\right)^2 + c^2\right)}g_F.$$

The optimization program of the federal Treasury is:

$$max_{g_F} L_F = \frac{1}{2}\theta\left(\left(\frac{1}{2}\left(\alpha + \beta\right) + bc\right)\left(g_1 + g_2\right) + \left(\gamma + bc\right)g_F + b\varepsilon + \bar{u}\right)^2 + \frac{1}{2}\left(c\left(g_1 + g_2 + g_F\right) + \varepsilon\right)^2.$$

From the first-order condition we get:

$$g_F = -\frac{\left(\theta\left(\gamma + bc\right)\left(\frac{1}{2}\left(\alpha + \beta\right) + bc\right) + c^2\right)\left(g_1 + g_2\right)}{\left(\theta\left(\gamma + bc\right)^2 + c^2\right)}$$

$$-\frac{1}{2}\frac{\theta\left(\gamma + bc\right)}{\left(\theta\left(\gamma + bc\right)^2 + c^2\right)}\left(u_1 + u_2\right) - \frac{\left(b\theta\left(\gamma + bc\right) + c\right)}{\left(\theta\left(\gamma + bc\right)^2 + c^2\right)}\varepsilon.$$

Denote by g_F^* et g_{FF}^* the optimal solutions of this program and π_F^e the corresponding expected inflation. The reduced forms for these solutions are written as follows:

$$g_F^* = f_{Fii}u_i + f_{Fij}u_j + f_{Fi\varepsilon}\varepsilon$$

$$g_{FF}^{*}=f_{FF1}u_1+f_{FF2}u_2+f_{FF\varepsilon}\varepsilon$$

The equilibrium is characterized by:

$$f_{Fii}u_i+f_{Fij}u_j+f_{Fi\varepsilon}\varepsilon=\frac{-\theta\left(\alpha+bc\right)u_1-\left(c+\theta b\left(\alpha+bc\right)\right)\varepsilon}{\left(\theta\left(\alpha+bc\right)^2+c^2\right)}$$

$$-\frac{\left(\theta\left(\beta+bc\right)\left(\alpha+bc\right)+c^2\right)}{\left(\theta\left(\alpha+bc\right)^2+c^2\right)}\left(f_{Fji}u_i+f_{Fjj}u_j+f_{Fj\varepsilon}\varepsilon\right)-\frac{\left(\theta\left(\gamma+bc\right)\left(\alpha+bc\right)+c^2\right)}{\left(\theta\left(\alpha+bc\right)^2+c^2\right)}\left(f_{FFi}u_i+f_{FFj}u_j+f_{FF\varepsilon}\varepsilon\right).$$

Hence:

$$f_{Fii}=\frac{-\theta\left(\alpha+bc\right)}{\left(\theta\left(\alpha+bc\right)^2+c^2\right)}-\frac{\left(\theta\left(\beta+bc\right)\left(\alpha+bc\right)+c^2\right)}{\left(\theta\left(\alpha+bc\right)^2+c^2\right)}f_{Fji}-\frac{\left(\theta\left(\gamma+bc\right)\left(\alpha+bc\right)+c^2\right)}{\left(\theta\left(\alpha+bc\right)^2+c^2\right)}f_{FFi}$$

$$f_{Fij}=-\frac{\left(\theta\left(\beta+bc\right)\left(\alpha+bc\right)+c^2\right)}{\left(\theta\left(\alpha+bc\right)^2+c^2\right)}f_{Fjj}-\frac{\left(\theta\left(\gamma+bc\right)\left(\alpha+bc\right)+c^2\right)}{\left(\theta\left(\alpha+bc\right)^2+c^2\right)}f_{FFj}$$

$$f_{Fi\varepsilon}=-\frac{\left(c+\theta b\left(\alpha+bc\right)\right)}{\left(\theta\left(\alpha+bc\right)^2+c^2\right)}-\frac{\left(\theta\left(\beta+bc\right)\left(\alpha+bc\right)+c^2\right)}{\left(\theta\left(\alpha+bc\right)^2+c^2\right)}f_{Fj\varepsilon}-\frac{\left(\theta\left(\gamma+bc\right)\left(\alpha+bc\right)+c^2\right)}{\left(\theta\left(\alpha+bc\right)^2+c^2\right)}f_{FF\varepsilon}.$$

We also get:

$$\begin{aligned}&f_{FF1}u_1+f_{FF2}u_2+f_{FF\varepsilon}\varepsilon=\\&-\frac{\left(\theta\left(\gamma+bc\right)\left(\frac{1}{2}\left(\alpha+\beta\right)+bc\right)+c^2\right)\left(f_{F11}u_1+f_{F12}u_2+f_{F1\varepsilon}\varepsilon\right)}{\left(\theta\left(\gamma+bc\right)^2+c^2\right)}\\&-\frac{\theta\left(\gamma+bc\right)\left(\frac{1}{2}\left(\alpha+\beta\right)+bc+c^2\right)\left(f_{F21}u_1+f_{F22}u_2+f_{F2\varepsilon}\varepsilon\right)}{\left(\theta\left(\gamma+bc\right)^2+c^2\right)}\\&-\frac{1}{2}\frac{\theta\left(\gamma+bc\right)}{\left(\theta\left(\gamma+bc\right)^2+c^2\right)}\left(u_1+u_2\right)-\frac{\left(b\theta\left(\gamma+bc\right)+c\right)}{\left(\theta\left(\gamma+bc\right)^2+c^2\right)}\varepsilon.\end{aligned}$$

Hence:

$$f_{FFi}=-\frac{\left(\theta\left(\gamma+bc\right)\left(\frac{1}{2}\left(\alpha+\beta\right)+bc\right)+c^2\right)\left(f_{Fii}+f_{Fji}\right)+\frac{1}{2}\theta\left(\gamma+bc\right)}{\left(\theta\left(\gamma+bc\right)^2+c^2\right)}$$

$$f_{FF\varepsilon}=-\frac{\left(\theta\left(\gamma+bc\right)\left(\frac{1}{2}\left(\alpha+\beta\right)+bc\right)+c^2\right)\left(f_{F1\varepsilon}+f_{F2\varepsilon}\right)+b\theta\left(\gamma+bc\right)+c}{\left(\theta\left(\gamma+bc\right)^2+c^2\right)}$$

Given the symmetry between the two countries, we can write:

$$f_{F11}=f_{F22},f_{F1\varepsilon}=f_{F2\varepsilon},f_{F12}=f_{F21},f_{FF1}=f_{FF2}$$

Combining these equations we get:

$$f_{FF\varepsilon}=\frac{\left(\alpha+\beta\right)}{c\left[2\gamma-\alpha-\beta\right]}$$

$$f_{F11}=f_{F22}=\frac{1}{2\gamma-\alpha-\beta}\frac{\left[\alpha-\gamma\right]}{\left[\alpha-\beta\right]}$$

$$f_{F12}=f_{F21}=\frac{\left(\gamma-\beta\right)}{\left(2\gamma-\alpha-\beta\right)\left(\alpha-\beta\right)}$$

$$f_{FF1} = f_{FF2} = -\frac{1}{2\gamma - \alpha - \beta}.$$

This completes the identification of the coefficients of the reduced forms. The equilibrium solutions obtain:

$$g_F^* = -\frac{\gamma}{c\,[2\gamma - \alpha - \beta]}\varepsilon + \frac{(\alpha - \gamma)}{(2\gamma - \alpha - \beta)(\alpha - \beta)}u_i + \frac{(\gamma - \beta)}{(\alpha - \beta)(2\gamma - \alpha - \beta)}u_j$$

$$g_{FF}^* = \frac{(\alpha + \beta)}{c\,(2\gamma - \alpha - \beta)}\varepsilon - \frac{1}{2\gamma - \alpha - \beta}u_1 - \frac{1}{2\gamma - \alpha - \beta}u_2.$$

For inflation we get:

$$\pi_F^* = c\,(g_{F1}^* + g_{F2}^* + g_{FF}^*) + \varepsilon.$$

$$= \left(\frac{c\,[(\alpha + \beta) - 2\gamma]}{c\,[2\gamma - \alpha - \beta]} + 1\right)\varepsilon + c\left(\frac{(\alpha - \beta)}{(2\gamma - \alpha - \beta)(\alpha - \beta)} - \frac{1}{2\gamma - \alpha - \beta}\right)(u_1 + u_2) = 0$$

The country i's aggregate product is equal to:

$$y_{Fi}^* = \hat{y} + \alpha g_{Fi}^* + \beta g_{Fj}^* + \gamma g_{FF}^* + b\,(\pi_F^*) + u_i = \hat{y} + \alpha g_{Fi}^* + \beta g_{Fj}^* + \gamma g_{FF}^* + u_i$$

$$= \hat{y} + \left(\frac{\alpha^2 + 2\beta\gamma - \beta^2}{(2\gamma - \alpha - \beta)(\alpha - \beta)} + 1\right)u_i = \hat{y} + \left(\frac{\alpha^2 + 2\beta\gamma - \beta^2 + (2\gamma - \alpha - \beta)(\alpha - \beta)}{(2\gamma - \alpha - \beta)(\alpha - \beta)}\right)u_i$$

$$= \hat{y} + \left(\frac{2\gamma\alpha}{(2\gamma - \alpha - \beta)(\alpha - \beta)}\right)u_i.$$

Lastly, the union's aggregate output is equal to:

$$\overline{y}_F^* = \hat{y} - \frac{2\gamma\alpha}{(2\gamma - \alpha - \beta)(\alpha - \beta)}\overline{u}.$$

Expected losses.

We can derive the levels of loss for the various players (taking into account that $\chi \equiv \tilde{\chi} - \hat{y}$, $\chi_M \equiv \hat{y} - \tilde{\chi}_M = 0$ and $\chi_F \equiv \hat{y} - \tilde{\chi}_F = 0$):

- for the national fiscal authority in i, the loss is equal to:

$$L_F^* = -\frac{1}{2}\theta\left(\frac{2\gamma\alpha}{(2\gamma - \alpha - \beta)(\alpha - \beta)}u_i\right)^2$$

  and thus the expected loss for the national fiscal authority in i is:

$$E\,(L_F^*) = \theta\frac{2\gamma^2\alpha^2}{(2\gamma - \alpha - \beta)^2(\alpha - \beta)^2}\sigma_u^2.$$

- for the federal fiscal authority:

$$L_{FF}^* = -\frac{1}{2}\theta_F\left(\frac{2\gamma\alpha}{(2\gamma - \alpha - \beta)(\alpha - \beta)}\overline{u}\right)^2$$

  and thus the expected loss for the national fiscal authority in i is:

$$E\,(L_{FF}^*) = \theta\frac{2\gamma^2\alpha^2}{(2\gamma - \alpha - \beta)^2(\alpha - \beta)^2}\sigma_u^2 = B_F^*\sigma_u^2. \tag{43}$$

Comparing cooperation and non-cooperation.

The difference for inflation between cooperation and non-cooperation is given by the following equation:

$$\pi_C^* - \pi_n^* = -\left[\frac{\frac{\theta}{2}c\left(\alpha+\beta+2bc\right)}{\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2} - \frac{\theta c\left(\alpha+bc\right)}{\theta\left(\alpha+bc\right)\left(\alpha+\beta\right)+2c^2}\right]\left(u_1+u_2\right)$$

$$+\left[\frac{\left[\frac{\theta}{2}\left(\alpha+\beta+2\mathrm{bc}\right)\right]}{\left[\frac{\theta}{2}\left(\alpha+\beta+2\mathrm{bc}\right)^2+2\mathrm{c}^2\right]} - \frac{\theta\left(\alpha+\mathrm{bc}\right)}{\theta\left(\alpha+\mathrm{bc}\right)\left(\alpha+\beta\right)+2\mathrm{c}^2}\right]\left(\alpha+\beta\right)\varepsilon$$

$$= -\left[\frac{\frac{\theta}{2}c\left(\alpha+\beta+2bc\right)\left[\theta\left(\alpha+bc\right)\left(\alpha+\beta\right)+2c^2\right] - \left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]\theta c\left(\alpha+bc\right)}{\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]}\right]\left(u_1+u_2\right)$$

$$+\left[\frac{\left[\frac{\theta}{2}\left(\alpha+\beta+2\mathrm{bc}\right)\right]\left[\theta\left(\alpha+\mathrm{bc}\right)\left(\alpha+\beta\right)+2\mathrm{c}^2\right] - \theta\left(\alpha+\mathrm{bc}\right)\left[\frac{\theta}{2}\left(\alpha+\beta+2\mathrm{bc}\right)^2+2\mathrm{c}^2\right]}{\left[\frac{\theta}{2}\left(\alpha+\beta+2\mathrm{bc}\right)^2+2\mathrm{c}^2\right]\left[\theta\left(\alpha+\mathrm{bc}\right)\left(\alpha+\beta\right)+2\mathrm{c}^2\right]}\right]\left(\alpha+\beta\right)\varepsilon.$$

The difference in output between cooperation and non-cooperation is given by the following equation:

$$y_C^* - y_{in}^* = -\left[\frac{1}{\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]} - \frac{1}{\theta\left(\alpha+bc\right)\left(\alpha+\beta+2bc\right)+2c^2}\right]c^2\left(u_1+u_2\right)$$

$$+\left[\frac{1}{\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]} - \frac{1}{\theta\left(\alpha+bc\right)\left(\alpha+\beta+2bc\right)+2c^2}\right]\left(\alpha+\beta\right)c\varepsilon$$

$$= -\left[\frac{\frac{\theta}{2}\left(\alpha+\beta+2bc\right)\left[\alpha-\beta\right]}{\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]\left[\theta\left(\alpha+bc\right)\left(\alpha+\beta+2bc\right)+2c^2\right]}\right]\left(c^2\left(u_1+u_2\right)-\left(\alpha+\beta\right)c\varepsilon\right).$$

Comparing cooperation and federalism.

The difference for inflation between cooperation and federalism is given by the following equation:

$$\pi_C^* - \pi_F^* = -\left[\frac{\frac{\theta}{2}c\left(\alpha+\beta+2bc\right)}{\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2}\right]\left(u_1+u_2\right) + \left[\frac{\left[\frac{\theta}{2}\left(\alpha+\beta+2\mathrm{bc}\right)\right]\left(\alpha+\beta\right)}{\left[\frac{\theta}{2}\left(\alpha+\beta+2\mathrm{bc}\right)^2+2\mathrm{c}^2\right]}\right]\left(\alpha+\beta\right)\varepsilon$$

$$= -\left[\frac{\frac{\theta}{2}c\left(\alpha+\beta+2bc\right)}{\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2}\right]\left(u_1+u_2\right) + \left[\frac{\left[\frac{\theta}{2}\left(\alpha+\beta+2\mathrm{bc}\right)\right]\left(\alpha+\beta\right)}{\left[\frac{\theta}{2}\left(\alpha+\beta+2\mathrm{bc}\right)^2+2\mathrm{c}^2\right]}\right]\left(\alpha+\beta\right)\varepsilon.$$

The difference in output is:

$$y_{iC}^* - y_{iF}^* = \frac{\left(\alpha+\beta\right)c}{\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]}\varepsilon$$

$$-\left[\frac{c^2}{\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]} - \frac{2\gamma\beta}{\left(2\gamma-\alpha-\beta\right)\left(\alpha-\beta\right)}\right]u_i + \frac{2\gamma\beta c^2}{\left(2\gamma-\alpha-\beta\right)\left(\alpha-\beta\right)}u_j$$

$$= \frac{\left(\alpha+\beta\right)c}{\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]}\varepsilon$$

$$
-\left[\frac{c^2\left(2\gamma-\alpha-\beta\right)\left(\alpha-\beta\right)-\theta\gamma\beta\left(\alpha+\beta+2bc\right)^2-4\gamma\beta c^2}{\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]\left(2\gamma-\alpha-\beta\right)\left(\alpha-\beta\right)}\right]u_i+\frac{2\gamma\beta}{\left(2\gamma-\alpha-\beta\right)\left(\alpha-\beta\right)}c^2u_j.
$$

For the union's aggregate output, the difference is:

$$
y_C^*-\overline{y}_F^*=
$$

$$
-\left[\frac{c^2}{\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]}-\frac{\gamma\beta}{\left(2\gamma-\alpha-\beta\right)\left(\alpha-\beta\right)}\right]\left(u_1+u_2\right)+\frac{1}{\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]}\left(\alpha+\beta\right)c\varepsilon
$$

$$
=-\frac{c^2\left(2\gamma-\alpha-\beta\right)\left(\alpha-\beta\right)-\gamma\beta\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2-2\gamma\beta c^2}{\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]\left(2\gamma-\alpha-\beta\right)\left(\alpha-\beta\right)}\left(u_1+u_2\right)+\frac{1}{\left[\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right]}\left(\alpha+\beta\right)c\varepsilon.
$$

Impact of fiscal spillovers on expected losses.

Given the definition of B_F^* given by (43) we get:

$$
\frac{\partial B_F^*}{\partial\gamma}=\theta\frac{4\gamma\alpha^2\left(2\gamma-\alpha-\beta\right)^2\left(\alpha-\beta\right)^2-2\gamma^2\alpha^2.4\left(2\gamma-\alpha-\beta\right)\left(\alpha-\beta\right)^2}{\left(\left(2\gamma-\alpha-\beta\right)^2\left(\alpha-\beta\right)^2\right)^2}=-\frac{4\theta\gamma\alpha^2\left(\alpha+\beta\right)}{\left(2\gamma-\alpha-\beta\right)^2\left(\alpha-\beta\right)^2}<0.
$$

$$
\frac{\partial B_F^*}{\partial\beta}=\theta\left(2\gamma^2\alpha^2\right)\frac{-2\left(\alpha-\beta\right)^2\left(2\gamma-\alpha-\beta\right)-2\left(\alpha-\beta\right)\left(2\gamma-\alpha-\beta\right)^2}{\left(\left(2\gamma-\alpha-\beta\right)^2\left(\alpha-\beta\right)^2\right)^2}=-\frac{4\theta\gamma\left(2\gamma^2\alpha^2\right)\left(2\gamma-\alpha-\beta\right)\left(\alpha-\beta\right)}{\left(\left(2\gamma-\alpha-\beta\right)^2\left(\alpha-\beta\right)^2\right)^2}
$$

which is positive if $2\gamma<\alpha+\beta$ and negative otherwise.

Given the definition of B_C^* given by (41) we get:

$$
\frac{\partial B_C^*}{\partial\beta}=\frac{2c^2\left(\left(\frac{\theta}{2}\right)^2\left(\alpha+\beta+2bc\right)\right)\left(\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right)^2-2\theta c^2\left(\left(\frac{\theta}{2}\right)^2\left(\alpha+\beta+2bc\right)^2+\theta c^2\right)\left(\left(\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right)\left(\alpha+\beta+2bc\right)\right)}{\left(\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right)^4}
$$

$$
=\frac{2c^2\left(\frac{\theta}{2}\right)^2\left(\alpha+\beta+2bc\right)\left(1-\left(\alpha+\beta+2bc\right)\right)}{\left(\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right)^2}\lesseqgtr 0.
$$

Given the definition of C_C^* given by (41) we get $C_C^*=B_C^*\left(\alpha+\beta\right)^2/2$ and hence:

$$
\frac{\partial C_C^*}{\partial\beta}=\frac{1}{2}\left[\frac{2c^2\left(\frac{\theta}{2}\right)^2\left(\alpha+\beta+2bc\right)\left(1-\left(\alpha+\beta+2bc\right)\right)}{\left(\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right)^2}\left(\alpha+\beta\right)^2+2\left(\alpha+\beta\right)\frac{c^2\left(\left(\frac{\theta}{2}\right)^2\left(\alpha+\beta+2bc\right)^2+\theta c^2\right)}{\left(\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right)^2}\right]
$$

$$
=\left[\frac{c^2\left(\frac{\theta}{2}\right)^2\left(\alpha+\beta+2bc\right)\left(1-\left(\alpha+\beta+2bc\right)\right)}{\left(\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right)^2}\left(\alpha+\beta\right)^2+\left(\alpha+\beta\right)\frac{c^2\left(\left(\frac{\theta}{2}\right)^2\left(\alpha+\beta+2bc\right)^2+\theta c^2\right)}{\left(\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right)^2}\right]
$$

$$
=\frac{\left(\alpha+\beta\right)c^2}{\left(\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right)^2}\left[\left(\frac{\theta}{2}\right)^2\left(\alpha+\beta+2bc-\left(\alpha+\beta+2bc\right)^2\right)\left(\alpha+\beta-1\right)+\theta c^2\right]0\lesseqgtr 0.
$$

Comparing expected losses in cooperation and in federation.

Given the equations of expected losses for national fiscal authorities in the three configurations, we deduce the expected losses for national fiscal authorities in the three configurations of interest:

$$
E\left(L_n^*\right)=\frac{\theta c^2\left(c^2+\theta\left(\alpha+bc\right)^2\right)}{\left(\theta\left(\alpha+bc\right)\left(\alpha+\beta+2bc\right)+2c^2\right)^2}\sigma_u^2+\frac{1}{2}\frac{\theta c^2\left(\left(\alpha+\beta\right)^2+\theta\left(\alpha+bc\right)^2\right)}{\left(\theta\left(\alpha+bc\right)\left(\alpha+\beta+2bc\right)+2c^2\right)^2}\sigma_\varepsilon^2
$$

$$E\left(L_C^*\right)=\frac{c^2\left(\left(\frac{\theta}{2}\right)^2\left(\alpha+\beta+2bc\right)^2+\theta c^2\right)}{\left(\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right)^2}\sigma_u^2+\frac{1}{2}\frac{\left(\left(\frac{\theta}{2}\right)^2\left(\alpha+\beta+2bc\right)^2+\theta c^2\right)\left(\alpha+\beta\right)}{\left(\frac{\theta}{2}\left(\alpha+\beta+2bc\right)^2+2c^2\right)^2}\sigma_\varepsilon^2$$

and

$$E\left(L_F^*\right)=\frac{1}{2}\frac{\theta\left(\gamma+bc\right)^2\left(c^2+\theta\left(\alpha+bc\right)^2\right)}{c^2\left(\gamma-\alpha\right)^2}\chi^2+\theta\frac{2\gamma^2\beta^2}{\left(2\gamma-\alpha-\beta\right)^2\left(\alpha-\beta\right)^2}\sigma_u^2.$$