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Fabrice Defever, Alejandro Riaño

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Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
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Twin Peaks

Abstract

Received wisdom suggests that most exporters sell the majority of their output domestically. In this paper, however, we show that the distribution of export intensity not only varies substantially across countries, but in a large number of cases is also bimodal, displaying what we refer to as twin peaks. We reconcile this new stylized fact with an otherwise standard, two-country model of trade in which firms are heterogeneous in terms of the demand they face in each market. We show that when firm-destination-specific revenue shifters are distributed lognormal, gamma, or Fréchet with sufficiently high dispersion, the distribution of export intensity has two modes in the boundaries of the support and their height is determined by a country's size relative to the rest of the world. We estimate the deep parameters characterizing the distribution of export intensity. Our results show that when the conditions for the existence of twin peaks are met, differences in relative market size can explain most of the observed variation in the distribution of export intensity across the world.

JEL-Codes: F120, F140, C120, O500.

Keywords: exports, export intensity distribution, bimodality, firm heterogeneity.

Fabrice Defever
Department of Economics
City, University of London
United Kingdom - EC1V 0HB, London
fabrice.defever@city.ac.uk

Alejandro Riaño
School of Economics
University of Nottingham
United Kingdom - NG7 2RD Nottingham
alejandro.riano@nottingham.ac.uk

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1 Introduction

Received wisdom suggests that most exporters sell the majority of their output domestically (Bernard et al., 2003; Brooks, 2006; Arkolakis, 2010; Eaton et al., 2011). This is considered one of the key empirical regularities that characterize the behavior of firms engaged in international trade, summarized in the reviews by Bernard et al. (2007) and Melitz and Redding (2014).[1]

In this paper we challenge this notion. Using harmonized cross-country firm-level data from the World Bank Enterprise Surveys (WBES), we show that the distribution of export intensity —the share of sales accounted for by exports conditional on exporting— varies tremendously across countries.[2] This is vividly illustrated in Figure 1. Countries like Argentina, Russia and South Africa exhibit the same pattern identified in previous studies; firms with export intensity below 0.2 constitute more than half of exporters, while firms with an export intensity above 0.8 account for less than 10% of exporters. In Bangladesh, Madagascar and the Philippines, we observe the opposite pattern —the average share of exporters with intensity below 0.2 and above 0.8 are, respectively, 11% and 75%. A large number of countries (China, Ireland and Uruguay, to name a few) also display 'twin peaks' —a high concentration of firms on both ends of the distribution. In fact, we find that unimodality is rejected for two-thirds of the countries in our data. To the best of our knowledge, we are the first in identifying this novel stylized fact.

The workhorse models of international trade in which firms only differ in their productivity, such as Melitz (2003), are at odds with the wide range of patterns presented in Figure 1. In a two-country model with CES preferences, all exporters sell the same share of their revenues abroad; i.e. the distribution of export intensity is degenerate. When there are more than two countries, more productive firms have a higher export intensity than less productive ones, because the former serve more markets than the latter. If the model is to be consistent with the well-established fact that a small number of large firms coexist with a large number of small firms (see Simon and Bonini, 1958; Axtell, 2001), then it would not be able to generate right-skewed nor bimodal export intensity distributions.[3]

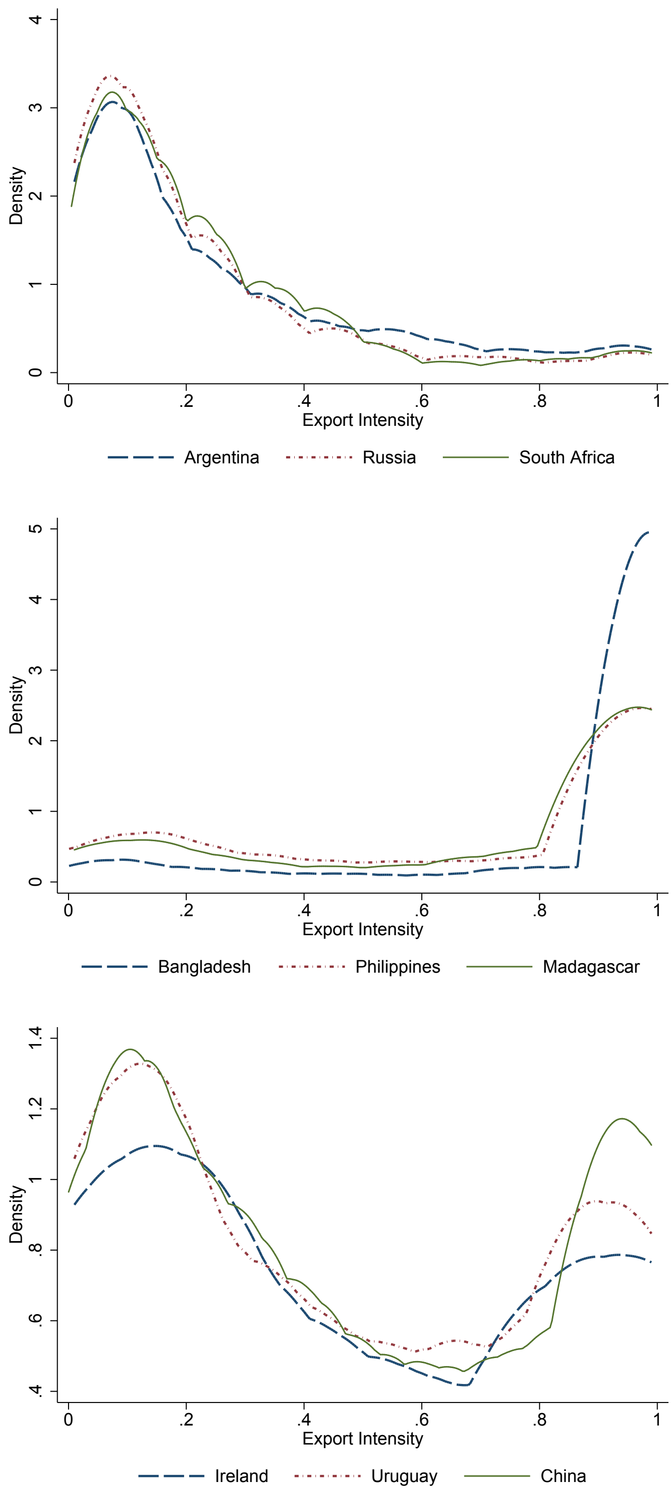
We show, however, that with one single adjustment, the standard two-country model of trade with CES preferences can reproduce the wide variety of shapes depicted in Figure 1 very successfully. Namely, we require large idiosyncratic differences in the demand that firms face domestically and abroad. When the firm-destination-specific revenue shifters that generate these differences are distributed lognormal, gamma or Fréchet —three of the distributions most frequently used to model firm heterogeneity— with sufficiently high dispersion, then the probability density function of export intensity is bimodal. The modes of the distribution are located near 0 and 1 and their

[1]The other two stylized facts identified by the literature are that only a minority of firms engage in exporting and that exporters perform better than domestic firms. These two results have been verified for almost every country in the world.

[2]In Section 2 we show that these patterns also arise when we use more representative data from national manufacturing surveys.

[3]The positive correlation between productivity and export intensity also arises in the Melitz and Ottaviano (2008) two-country model with quasi-linear utility. This again implies that the distribution of export intensity inherits the properties of the productivity distribution.

Figure 1: Export Intensity Distribution Across Countries: Selected Examples



The figure depicts the kernel density of export intensity, the share of total sales accounted for by exports among exporters. The data, which is described in detail in Section 2, comes from the World Bank Enterprise Surveys.

'height' is determined by a country's relative market size with respect to the rest of the world. Thus, the interaction between large heterogeneity in firms' performance across markets and differences in relative market size can explain the observed variation in the distribution of export intensity across countries.

Using firm-level data for 72 countries drawn from several waves of the WBES over the period 2002-2016, we estimate the deep parameters that characterize the distribution of export intensity. These are the shape parameter of the distribution of firm-destination-specific revenue shifters and a country's relative market size with respect to the rest of the world. We tease out the latter directly from the data by exploiting the fact that for all the underlying distributions of revenue shifters we consider, there is a one-to-one relationship between relative market size and the median export intensity which is independent of the shape parameter. Conditional on this inferred measure of relative market size, we estimate the shape parameter by maximum likelihood. The identification of the parameters is very transparent: if the conditions for twin peaks are satisfied, greater dispersion of revenue shifters (which is determined by the shape parameter) increases the distribution's mass in the extremes of the support. For a given level of dispersion, changes in market size shift mass from one extreme of the support to other. Conversely, if the dispersion of revenue shifters is not sufficiently high, then the distribution of export intensity would be unimodal and its mass would be tightly concentrated around its median in the interior of the support.

The results provide strong support for the existence of twin peaks. When we estimate country-specific scale and shape parameters, the conditions for bimodality are satisfied in all but a handful of cases. We then estimate a model in which all cross-country variation in the distribution of export intensity is accounted for by differences in relative market size, since firms in every country draw revenue shifters from a distribution with the same shape parameter. Our results are striking. Conditional on relative market size, we are are able to fit the distribution of export intensity across the wide range of countries in our data with *only one* shape parameter. This provides the main takeaway message from the paper: conditional on firm-destination-specific factors exhibiting sufficiently high dispersion, differences in relative market size explain most of the observed variation in the distribution of export intensity across the world.

We carry out a thorough robustness analysis of our main result. More specifically, we explore the possibility that the bimodality of the export intensity distribution is the result of a composition effect —i.e. if most exporters were low-intensity ones, but specific subsets were particularly inclined to export most of their output. We find that the dispersion of firm-destination-specific revenue shifters remains sufficiently high to generate bimodality when we exclude multinational affiliates, firms engaged in processing activities, and even firms exporting all their output. Similarly, excluding countries that provide subsidies conditioned on firms' export performance or splitting our sample according to countries' level of development leaves our results intact. We also show that bimodality is not a product of a country's sectoral composition of exports. In our last robustness exercise, we work directly with domestic and export sales instead of export intensity data. Doing so allows us to validate the relative market size estimates based on the median export intensity and to carry out a variance decomposition of sales in each market. The latter reveals that —consistent with the empirical literature relying on customs data (Kee and Krishna, 2008; Munch and Nguyen, 2014; Lawless and Whelan, 2014, e.g.)— firm-destination-specific factors account for a large share of the variation in firms' sales in a given destination.

Understanding export intensity is important on its own right. In the paper we show that the existence of twin peaks affects the response of the export intensity distribution to reductions in trade costs as well as the sales diversification benefits of exporting. Characterizing accurately the distribution of export intensity in an undistorted economy is also a key input to infer the magnitude of distortions that affect firms' access to foreign markets, following the approach pioneered by Hsieh and Klenow (2009). This strategy has been used by Brooks and Wang (2016) to quantify the effect of idiosyncratic trade taxes and preferential access to foreign exchange, and by Defever and Riaño (2017a) to evaluate the welfare cost of incentives subject to export performance requirements. Alessandria and Avila (2017) rely on changes in the distribution of export intensity over time to discipline a model that quantifies the contribution of technology and trade policy in Colombia's process of trade opening. Export intensity has also been show to be strongly correlated with a wide variety of outcomes such as female employment (Ozler, 2000), lobbying activity on trade policy (Osgood et al., 2017) and financial constraints (Egger and Erhardt, 2014). Kohn et al. (2017) show that the response of aggregate exports to large devaluations in emerging markets, where exporters often borrow in foreign currency, is crucially shaped by the distribution of export intensity. Alfaro et al. (2017) show that differences in average export intensity explain why East Asian firms' productivity and R&D investment are more responsive to real exchange rate depreciations than their counterparts in Europe and Latin America.

Our paper is related to several strands in the literature. First, we add to the recent work that investigates the implications of using different probability distributions to model firm heterogeneity in international trade models (Head et al., 2014; Mrázová et al., 2015; Nigai, 2017). While most of this literature focuses on productivity, we highlight the importance of the idiosyncratic interaction between individual firms and markets. The importance of firm-destination factors in explaining export sales variation has been identified by Eaton et al. (2011), Crozet et al. (2012) and Fernandes et al. (2015). We show that this feature of the data can also explain the variation of the export intensity distribution across countries. Our work is also related to Lu (2010) and Defever and Riaño (2017a), who first documented the fact that the export intensity distribution in China is markedly bimodal. Our work shows that this pattern is not specific to China, and is, in fact, quite common across the world.

The rest of the paper is organized as follows. Section 2 presents the data used in our analysis, documents the prevalence of bimodality in the distribution of export intensity across countries and provides a comparison with more representative, national firm-level surveys for a selected subset of countries. Section 3 presents our theoretical framework. We show the conditions under which the distribution of export intensity is bimodal and discuss implications of the existence of twin peaks. Section 4 presents the identification strategy we follow in our estimation and our benchmark results.

Section 5 reports the results of an extensive battery of robustness checks to our main specification. Section 6 concludes.

2 Data

Our data comes from several waves of the World Bank Enterprise Surveys (WBES) spanning the period 2002-2016. These surveys are carried out by the World Bank's Enterprise Analysis Unit (usually every 3-4 years) using a uniform methodology and questionnaire, and are intended to be representative of a country's non-agricultural private economy.[4] The unit of observation in the surveys is the establishment, i.e. a physical location where business is carried out or industrial operations take place, which should have its own management and control over its own workforce. Since the vast majority of establishments surveyed report to be single-establishment firms, hereafter we refer to them as 'firms'. We use the data for the manufacturing sector only (i.e. firms that belong to ISIC Rev. 3.1 sectors 15-37).

The WBES provides information on firms' main sector of operation, total sales, export intensity, ownership status (whether the firm is domestic or foreign-owned), labor productivity and the share of material inputs accounted for by imports. The main variable of interest is export intensity —the share of sales that a firm exported, both directly or indirectly through an intermediary, in a fiscal year— and is therefore defined on the interval $(0, 1]$.[5]

Our sample consists of 72 developing and transition countries (with the exception of Ireland and Sweden), for which we observe at least 97 exporting firms when we pool the data across all available survey waves. Table 1 lists the countries in our sample and provides information on the number of exporters and survey waves. The number of exporters per country ranges from 97 in Ireland to 2,112 in India, accounting, on average, for 40% of firms surveyed in a country. In terms of geographic coverage, the countries in our data are evenly distributed across Eastern Europe, Latin America and the Caribbean, Asia and the Middle East and Africa; Ireland and Sweden are the only two countries in our sample from Western Europe.

Table 1 also indicates whether a country provides incentives that are directly conditioned on firms' export intensity.[6] Defever and Riaño (2017a) investigate the effects that these subsidies subject to export share requirements (ESR) have on the intensity of competition and welfare in the context of China. We rely on information from the "Performance Requirements and Incentives" and "Foreign Trade Zones/Free Trade Zones" sections of the Investment Climate Statements produced by the U.S. State Department to identify countries offering these incentives. Table 1 reveals that

[4]More specifically, the survey targets formal (registered) firms with more than 5 employees that are not 100% state-owned.

[5]Since the survey asks firms directly about the percentage of their sales exported, the response is bounded at 100%, and therefore does not capture 'carry along' trade —a phenomenon first identified by Bernard et al. (2012)— in which firms export goods that they do not produce.

[6]Examples include direct cash transfers, tax holidays and deductions, and the provision of utilities at below-market rates. These incentives require recipient firms to export more than a certain share of their output. They are frequently used in special economic zones and duty-drawback regimes.

Table 1: Summary Statistics

Country	Code	Survey Waves	# Exporters	ESR	Country	Code	Survey Waves	# Exporters	ESR
Albania	ALB	2002, 05, 07, 13	116	No	Lithuania	LTU	2002, 04, 05, 09, 13	258	No
Argentina	ARG	2006, 10	1,140	No	Madagascar	MDG	2005, 09, 13	256	Yes
Armenia	ARM	2002, 05, 09, 13	170	No	Malaysia	MYS	2002, 15	735	Yes
Bangladesh	BGD	2002, 07, 13	1,255	Yes	Mauritius	MUS	2005, 09	183	No
Belarus	BLR	2002, 05, 08, 13	146	Yes	Mexico	MEX	2006, 10	727	No
Bolivia	BOL	2006, 10	242	No	Moldova	MDA	2002, 03, 05, 09, 13	221	No
Bosnia & Herzegovina	BIH	2002, 05, 09, 13	200	Yes	Morocco	MAR	2004, 13	585	Yes
Brazil	BRA	2003, 09	832	Yes	Namibia	NAM	2006, 14	109	Yes
Bulgaria	BGR	2002, 04, 05, 07, 09, 13	538	No	Nicaragua	NIC	2003, 06, 10	283	Yes
Chile	CHL	2004, 06, 10	1,001	No	Nigeria	NGA	2007, 14	316	Yes
China	CHN	2002, 03, 12	1,439	Yes	Pakistan	PAK	2002, 07, 13	534	Yes
Colombia	COL	2006, 10	703	No	Panama	PAN	2006, 10	134	No
Costa Rica	CRI	2005, 10	238	Yes	Paraguay	PRY	2006, 10	246	Yes
Croatia	HRV	2002, 05, 07, 13	335	No	Peru	PER	2006, 10	701	Yes
Czech Rep.	CZE	2002, 05, 09, 13	232	No	Philippines	PHL	2003, 09, 15	914	Yes
Ecuador	ECU	2003, 06, 10	385	No	Poland	POL	2002, 03, 05, 09, 13	439	No
Egypt	EGY	2004, 13	676	Yes	Romania	ROU	2002, 05, 09, 13	290	No
El Salvador	SLV	2003, 06, 10, 16	840	No	Russian Fed.	RUS	2002, 05, 09, 12	468	No
Estonia	EST	2002, 05, 09, 13	175	No	Senegal	SEN	2003, 07, 14	148	Yes
Ethiophia	ETH	2002, 11, 15	118	Yes	Serbia	SRB	2002, 03, 05, 09, 13	330	No
FYR Macedonia	MKD	2002, 05, 09, 13	191	No	Slovak Rep.	SVK	2002, 05, 09, 13	164	No
Ghana	GHA	2007, 13	159	Yes	Slovenia	SVN	2002, 05, 09, 13	245	No
Guatemala	GTM	2003, 06, 10	580	Yes	South Africa	ZAF	2003, 07	558	No
Honduras	HND	2003, 06, 10	345	Yes	Sri Lanka	LKA	2004, 11	403	Yes
Hungary	HUN	2002, 05, 09, 13	300	No	Sweden	SWE	2014	286	No
India	IND	2002, 06, 14	2,212	Yes	Syrian Arab Rep.	SYR	2003	251	No
Indonesia	IDN	2003, 09, 15	720	Yes	Tanzania	TZA	2003, 06, 13	222	Yes
Ireland	IRL	2005	97	No	Thailand	THA	2004, 16	1,066	Yes
Jordan	JOR	2006, 13	377	No	Tunisia	TUN	2013	213	Yes
Kazakhstan	KAZ	2002, 05, 09, 13	116	No	Turkey	TUR	2002, 04, 05, 08, 13	2,152	Yes
Kenya	KEN	2003, 07, 13	511	No	Uganda	UGA	2003, 06, 13	221	Yes
Korea, Rep.	KOR	2005	99	No	Ukraine	UKR	2002, 05, 08, 13	435	No
Kyrgyz Rep.	KGZ	2002, 03, 05, 09, 13	126	No	Uruguay	URY	2006, 10	467	No
Lao PDR	LAO	2006, 12, 16	128	No	Uzbekistan	UZB	2002, 03, 05, 08, 13	112	Yes
Latvia	LVA	2002, 05, 09, 13	161	No	Vietnam	VNM	2005, 09, 15	1,251	Yes
Lebanon	LBN	2006, 13	252	No	Zambia	ZMB	2002, 07, 13	146	Yes

subsidies subject to ESR are pervasive in developing countries, with half of the countries in our data employing them.

Table 2: Firm Characteristics by Export Status and Export Intensity

	Employment (1)	Output (2)	Output per worker (3)	% Foreign-owned (4)	% Imported inputs (5)
Non-Exporters	0.5	0.5	0.8	6.0	23.7
Exporters					
Export intensity:					
$\in (0.0, 0.2]$	1.7	2.1	1.4	17.4	37.7
$\in (0.2, 0.4]$	1.5	1.8	1.3	18.8	35.6
$\in (0.4, 0.6]$	1.7	1.9	1.3	21.0	35.2
$\in (0.6, 0.8]$	2.3	2.3	1.4	23.0	35.4
$\in (0.8, 1.0]$	2.2	2.3	1.6	31.1	41.0

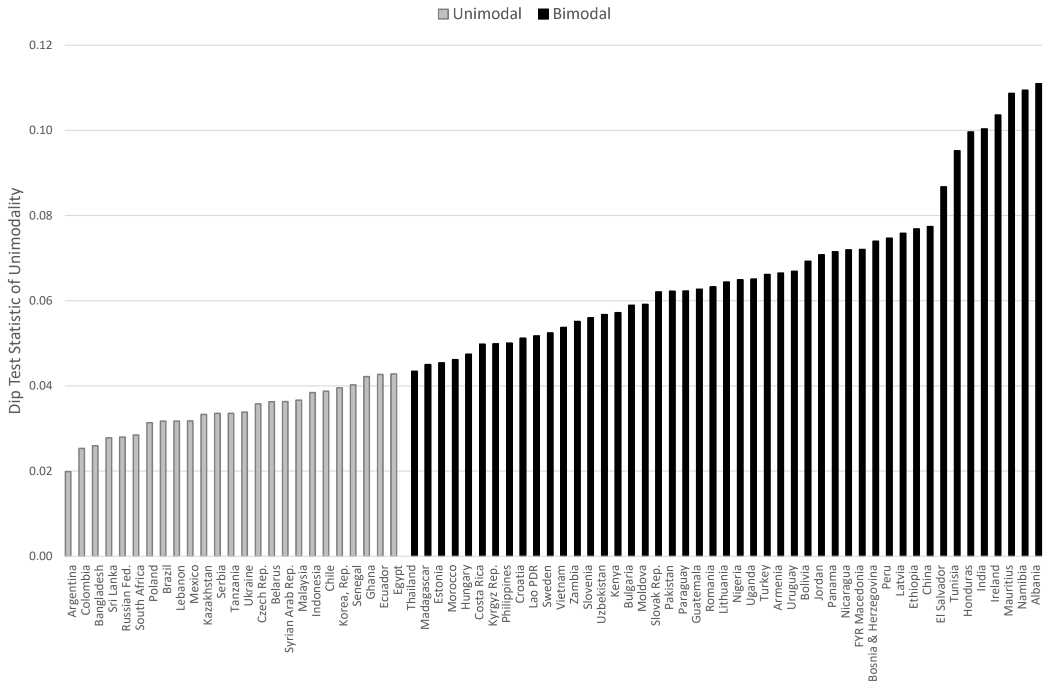
Columns (1)-(3) report the average across countries of the relative size and labor productivity of exporters and domestic firms relative to the mean value of each variable calculated in each country-survey year cell. Thus, for instance, domestic firms across all countries in our sample are 50% smaller (in terms of employment) than the average firm, while exporters with an export intensity lower than 20% are 70% larger than the average firm. Column (4) reports the percentage of foreign-owned firms (firms with a share of foreign equity at least 10% or greater) and column (5) presents the percentage of imported inputs in total intermediate inputs in each export intensity bin.

Table 2 provides a first pass at the data comparing exporters (along the distribution of export intensity) with domestic firms across various performance measures. Columns (1)-(3) provide information on firm size and productivity relative to the average value of the respective statistic in each country-survey year pair. Columns (4) and (5) report the percentage of foreign-owned firms and the use of imported inputs. Table 2 reveals that —consistent with the evidence summarized by Bernard et al. (2007) and Melitz and Redding (2014)— exporters are larger (both in terms of employment and output) and more productive than domestic firms in our sample. Looking across the export intensity distribution, we find that although there is a positive correlation between firm size and export intensity, there is not a clear relationship between labor productivity and export intensity.[7] Columns (4) and (5) show that exporters — and high-intensity ones in particular— are more likely to be foreign-owned (Antràs and Yeaple, 2014) and to use imported intermediate inputs more intensively than domestic firms (Amiti and Konings, 2007).

While Figure 1 provides suggestive evidence about export intensity distributions exhibiting twin peaks, we now carry out a systematic statistical test of unimodality in our data, using the so-called dip statistic proposed by Hartigan and Hartigan (1985). This test measures departures from unimodality in the empirical cumulative distribution function (cdf) by relying on the fact that

[7] Díaz de Astarloa et al. (2013) and Heid et al. (2013) find that high-intensity exporters are larger than domestic firms and other exporters in Bangladesh and Mexico respectively. Defever and Riaño (2017b), however, document that firms that export all their output in China, although larger and more productive than domestic firms, are smaller and less productive than low-intensity exporters.

Figure 2: Dip Test of Unimodality of Export Intensity



The figure reports the value of the Hartigan and Hartigan (1985) dip test statistic of unimodality. Countries are identified as having having a unimodal export intensity distribution if their dip statistic does not reject the null hypothesis of unimodality at the 1% confidence level; otherwise, they are identified as bimodal. The reported value of the dip statistic is calculated as the mean across 1,000 bootstrapped samples of 200 exporters drawn for each country. The algorithm used to calculate the dip test statistic is adjusted to take into account the discreteness of the export intensity data.

a unimodal distribution has a unique inflection point.[8] As Henderson et al. (2008) note, the dip measures the amount of 'stretching' needed to render the empirical cdf of a multi-modal distribution unimodal. Thus, a higher value for the dip leads to a rejection of the null hypothesis of unimodality. Hartigan and Hartigan (1985) choose the uniform distribution as the null distribution because its dip is the largest among all unimodal distributions. The dip test has been widely used in economics to, among other things, identify convergence clusters in the distribution of GDP per capita, total factor productivity and other indicators of economic growth (Henderson et al., 2008), characterize the degree of price stickiness (Cavallo and Rigobon, 2011) and to assess the identification of hazard function estimates (Heckman and Singer, 1984).

Figure 2 presents the dip statistic for all countries in our sample. We identify the distribution of export intensity in a country as unimodal if the null hypothesis of the dip statistic is not rejected at the 1% significance level at least; otherwise, we consider it to be bimodal.[9] Based on this criterion, we find that the distribution of export intensity is bimodal in 47 out of 72 countries. As we noted in the introduction, this result stands in sharp contrast with previous work suggesting that this

[8]More precisely, the cdf is convex on the interval $(-\infty, x_m)$ and concave between $(x_m, +\infty)$, where x_m denotes the unique mode of the distribution.

[9]Although the dip test only tells us whether a distribution is unimodal or not, visual inspection of kernel densities does not suggest the existence of more than two modes in the distribution of export intensity in any country in our data.

distribution was generally unimodal, with a majority of exporters selling a small share of their output abroad.

There are two important points that need to be made regarding the calculation of our dip test. Firstly, since the WBES survey questionnaire asks directly 'what percentage of the establishment's sales were exported', the response to this question tends to cluster in figures that are multiples of 5%. Therefore, we adjust the dip statistic to take into account the discreteness of the data, since not doing so would lead to over-rejecting the null hypothesis of unimodality. Secondly, since the number of exporters varies substantially across countries, the dip statistic reported in Figure 2 is calculated as the mean across 1,000 bootstrapped samples of 200 exporters in each country, which allows us to directly compare the dip across countries.

Is the high prevalence of twin peaks in the distribution of export intensity a figment of the WBES data? Asker et al. (2014), for instance, note that the stratification procedure across sectors, size categories and geographic locations used in the construction of the WBES leads to an oversampling of larger firms relative to a random sample of all firms in the economy. Although it is not clear that this would necessarily increase the likelihood of observing bimodal export intensity distributions, it is crucial for our purposes to show that twin peaks also appear in more representative datasets.

To this end, we asked fellow researchers to calculate for us the share of exporters across export intensity bins using well-known firm-level manufacturing surveys for a sub-sample of 13 countries in our data.[10] The countries that we consider include 5 that we classify as unimodal based on the dip statistic (Argentina, Chile, Colombia, Indonesia and South Africa) and 8 bimodal (China, Hungary, India, Ireland, Pakistan, Thailand, Uruguay and Vietnam). Most surveys used in our analysis include all manufacturing firms with more than 10 to 20 employees, although the data for China and Ireland is based on a survey of larger firms. It is also important to note that the distribution of export intensity based on manufacturing surveys is calculated with data for a single year, whereas —as we have noted above— the data we use pools exporters from all survey waves available for a given country in the WBES.

Table 3 presents the comparison between the distribution of export intensity based on manufacturing surveys and the one based on WBES data. The key message is that the WBES data provides an accurate picture of the distribution export intensity. Both data sources yield similar results with regards to the existence or not of twin peaks —even in the cases in which there are significant differences in the share of exporters in individual bins (e.g. in Hungary and India). Crucially, note that in all countries but one, the share of exporters with export intensity in the middle bins (where export intensity is between 0.2 and 0.8) is higher in the WBES than in the more representative data. Thus, if anything, it seems that we are underestimating the prevalence of twin peaks by relying on the WBES data.

[10] The choice of these countries was driven by data availability. The manufacturing survey data that we rely upon for the comparison has been used in a large number of prominent papers in international trade, including —but not limited to— Bustos (2011) (Argentina), Alvarez and López (2005) (Chile), Feenstra and Hanson (2005) (China), Roberts and Tybout (1997) (Colombia), Békés and Muraközy (2012) (Hungary), Goldberg et al. (2010) (India), Javorcik and Poelhekke (2017) (Indonesia), Ali et al. (2017) (Pakistan), Mamburu (2017) (South Africa), Cole et al. (2010) (Thailand), Casacuberta and Gandelman (2012) (Uruguay) and Ha and Kiyota (2014) (Vietnam).

Table 3: Comparison of Export Intensity Distributions —WBES vs Other Surveys

Country	Survey	# Exporters	Share of Exporters with Export Intensity ∈:				
			$(0.0, 0.2]$	$(0.2, 0.4]$	$(0.4, 0.6]$	$(0.6, 0.8]$	$(0.8, 1.0]$
Argentina	ENIT	830	0.660	0.123	0.071	0.065	0.081
	WBES	1,140	0.535	0.225	0.102	0.067	0.071
Chile	ENIA	900	0.582	0.116	0.113	0.093	0.096
	WBES	1,001	0.490	0.186	0.098	0.065	0.162
China	NBS	50,902	0.221	0.101	0.091	0.101	0.486
	WBES	1,439	0.282	0.198	0.116	0.093	0.311
Colombia	EAM	1,332	0.643	0.157	0.068	0.038	0.095
	WBES	703	0.459	0.273	0.128	0.067	0.073
Hungary	APEH	7,143	0.488	0.127	0.081	0.079	0.225
	WBES	300	0.243	0.207	0.160	0.123	0.267
India	Prowess	3,133	0.576	0.136	0.088	0.071	0.129
	WBES	2,212	0.260	0.214	0.116	0.072	0.338
Indonesia	Census	3,949	0.124	0.097	0.089	0.127	0.563
	WBES	720	0.125	0.172	0.144	0.139	0.419
Ireland	FAME	151	0.371	0.093	0.079	0.099	0.358
	WBES	173	0.330	0.155	0.113	0.082	0.320
Pakistan	FRBP	6,043	0.207	0.056	0.041	0.043	0.652
	WBES	534	0.148	0.146	0.109	0.062	0.536
South Africa	SARS-NT	7,530	0.844	0.076	0.036	0.025	0.019
	WBES	558	0.529	0.279	0.113	0.020	0.060
Thailand	OIE	1,591	0.302	0.136	0.111	0.121	0.331
	WBES	1,066	0.147	0.151	0.134	0.141	0.427
Uruguay	EAAE	389	0.424	0.131	0.090	0.095	0.260
	WBES	467	0.328	0.173	0.105	0.103	0.291
Vietnam	ASE	4,946	0.292	0.108	0.094	0.115	0.391
	WBES	1,251	0.173	0.128	0.065	0.104	0.530

Argentina: National Survey on Innovation and Technological Behavior of Industrial Argentinean Firms (ENIT) for the year 2001; this is a representative sample of establishments with more than 10 employees. Chile: Annual National Industrial Survey (ENIA) for the year 2000, covering the universe of Chilean manufacturing plants with 10 or more workers. Colombia: Annual Manufacturing Survey (EIA) for they year 1991, covering the universe of Colombian manufacturing plants with 10 or more workers. China: National Bureau of Statistics (NBS) Manufacturing Survey for the year 2003, which includes state-owned enterprises and private firms with sales above 5 million Chinese Yuan. Hungary: universe of manufacturing firms for the year 2014 drawn from APEH, the Hungarian tax authority. India: Prowess database collected by the Centre for Monitoring the Indian Economy (CMIE) for the year 2001. Indonesia: Indonesian Census of Manufacturing Firms for the year 2009, which surveys all registered manufacturing plants with more than 20 employees. Ireland: FAME database collected by Bureau van Dijk for the year 2007. Pakistan: export intensity is constructed from two administrative datasets from the Federal Board of Revenue of Pakistan (FBRP); export figures are from customs records and domestic Sales from VAT data records. The data records all firms with annual turnover above 2.5 million Pakistani rupees for the year 2013. South Africa: South African Revenue Service and National Treasury Firm-Level Panel (SARS-NT) for the year 2010, which covers the universe of all tax registered firms. Thailand: Annual Survey of Thailand's manufacturing industries by the Office of Industrial Economics (OIE) for the year 2014. The questionnaire's response rate is about 60% of all firms accounting for about 95% of total manufacturing. Uruguay: Annual Survey of Economic Activity (EAAE), which covers all manufacturing plants with 10 or more employees for the year 2005. Vietnam: Annual Survey on Enterprises (ASE) for the year 2010, which covers all state-owned enterprises and foreign-owned firms and domestic private firms with more than 10 employees.

3 Theoretical Framework

Consider a monopolistically-competitive industry in which each firm produces a unique, differentiated good indexed by ω (which will also denote a firm's identity hereafter). Firms can sell their output domestically (d), or export it to the rest of the world (x). Firm ω's sales in destination market $i \in \{d, x\}$, $r_i(\omega)$, are given by:

$$r_i(\omega) = A_i \cdot z_i(\omega) \cdot p_i(\omega)^{1-\sigma}, \tag{1}$$

where $p_i(\omega)$ is the price charged by firm ω in market i, $\sigma > 1$ is the elasticity of demand, A_i is a measure of market i's size —which is common to all firms selling there— and $z_i(\omega)$ is a firm-destination-specific revenue shifter.[11] Our choice for the underlying distribution of revenue shifters (lognormal, gamma or Fréchet) is driven both by their wide usage to model economic heterogeneity and, as noted below, the need for a closed-form expression for the pdf of the ratio of revenue shifters.[12]

Recent work has shown that idiosyncratic, destination-specific factors, such as cross-country differences in tastes, the extent of a firm's network of customers or its participation in global value chains are important drivers of firms' export decisions (Eaton et al., 2011; Demidova et al., 2012; Crozet et al., 2012). In fact, empirical evidence for Bangladeshi, Danish, French and Irish exporters has consistently found that firm-destination fixed effects account for a similar share of the variation in export sales as firm-specific factors such as productivity (see respectively, Kee and Krishna (2008), Munch and Nguyen (2014), Eaton et al. (2011), and Lawless and Whelan (2014)).

As is standard in international trade models with heterogenous firms, producers pay a sunk cost f_e to draw their idiosyncratic productivity, φ, from a distribution with probability density function (pdf), $g(\varphi)$.[13] Firms draw their idiosyncratic revenue shifter in market i, $z_i(\omega)$, from a distribution with pdf $f(z_i)$. We assume that the distribution $f(\cdot)$ is the same in both markets, although its parameters can, in some cases, differ across destinations; furthermore, revenue shifters and productivity are assumed to be orthogonal with respect to each other. Firms incur a fixed cost f_d to set up a plant and produce their respective good using a linear technology, with labor being the only input. Thus, the marginal cost of production for a firm of productivity φ, is w/φ, where

[11] The revenue function (1) obtains when there is a representative consumer in each country with CES preferences of the form: $\mathcal{U} = \left[\sum_{i\in\{d,x\}} \left(\int_{\omega\in\Omega_i} [z_i(\omega)^{\frac{1}{\sigma-1}} q_i(\omega)]^{\frac{\sigma-1}{\sigma}} \mathrm{d}\omega\right)\right]^{\frac{\sigma}{\sigma-1}}$, where $q_i(\omega)$ denotes the quantity of good ω from country i consumed, and Ω_i is the set of varieties produced in market i available to consume. Under this interpretation, market i's size is given by $A_i \equiv R_i P_i^{\sigma-1}$, where R_i denotes country i's aggregate expenditure and P_i is the ideal price index prevailing in the same country.

[12] The lognormal distribution has recently become a popular alternative to model the distribution of firm-level productivity in international trade (see e.g. Head et al., 2014; Mrázová et al., 2015; Nigai, 2017); Hanson et al. (2016) find that the distribution of export capabilities at the industry level is also well approximated by a lognormal distribution. Both Eaton et al. (2011) and Fernandes et al. (2015) use the lognormal distribution to model firm-destination-specific revenue shifters in models with more than two destination markets. Luttmer (2007) uses the gamma distribution to model the size distribution of firms, while Eaton and Kortum (2002) is the seminal reference on the use of the Fréchet distribution to model cross-country differences in productivity.

[13] All fixed costs are denominated in units of labor.

w is the wage prevailing in the domestic market. A firm that chooses to export needs to incur an additional fixed cost, f_x, and an iceberg transport cost, $\tau \geqslant 1$.

Remark. We assume that firms choose whether to operate or not and what markets to serve after observing their productivity, but *before* knowing the realization of revenue shifters.

We get a lot of mileage from this assumption.[14] As will become clearer below, it allow us to derive the pdf of export intensity only requiring knowledge of the distribution of the ratio of relative revenue shifters. Moreover, we will also show that when revenue shifters are distributed lognormal, gamma or Fréchet, we can back out the relative market size s_d/s_x directly from the data without having to solve the full model in general equilibrium. If, on the other hand, firms made the decision to export after observing both their productivity and revenue shifters, then the distribution of export intensity would depend on the joint distribution of these variables and the truncation caused by the fixed cost of exporting;[15] Defever and Riaño (2017a) calibrate the distribution of export intensity in the latter type of model.

A firm that is productive enough to export,[16] sets prices $p_d(\omega) = \frac{\sigma}{\sigma-1}\frac{w}{\varphi}$ and $p_x(\omega) = \tau p_d(\omega)$ at home and abroad respectively (since revenue shifters are multiplicative, they do not affect prices). Sales in market $i \in \{d, x\}$ are given by:

$$r_i(\omega) = \Phi(\omega) \cdot s_i \cdot z_i(\omega), \tag{2}$$

where $s_d \equiv \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} w^{1-\sigma} A_d$, $s_x \equiv \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} w^{1-\sigma}\tau^{1-\sigma} A_x$, and $\Phi(\omega) \equiv \varphi^{\sigma-1}$. Thus, sales in a given destination i are composed of three terms: s_i encompasses all variables that are common across all firms selling in i (i.e. market i's size, home's wage, transport costs), $\Phi(\omega)$ (productivity), varies across firms but is the same across destinations, and $z_i(\omega)$ represents the factors that determine the appeal of firm ω's product specifically in market i.

Define an exporting firm's export intensity as the share of total sales accounted for by exports. Let $E(\omega)$ be the random variable denoting the export intensity of firm ω, while lowercase e denotes a realization of this random variable. Thus, the export intensity for firm ω is given by:

$$E(\omega) \equiv \frac{r_x(\omega)}{r_d(\omega) + r_x(\omega)} = \frac{s_x z_x(\omega)}{s_d z_d(\omega) + s_x z_x(\omega)} = \frac{Z_x(\omega)}{Z_d(\omega) + Z_x(\omega)}. \tag{3}$$

Since firms can only sell their output in two destinations, and charge the same constant markup in both, it follows that export intensity is independent of productivity. Thus, in the absence of firm-

[14] This timing assumption has also been used by Cherkashin et al. (2015); it is consistent with the fact that while firm-level productivity strongly predicts export entry, it explains much less of the variation in sales across destinations conditional on entry (Eaton et al., 2011).

[15] Under this assumption the decision to export is characterized by a downward-sloping relationship in the productivity-export revenue shifter (φ, z_x) space instead of a standard productivity cutoff. That is, for a given level of productivity, only firms with sufficiently high foreign demand choose to export; similarly, for a given value of the export revenue shifter, only the most productive firms sell some of their output abroad.

[16] That is, a firm for which the expected profit of exporting exceeds the fixed cost wf_x.

destination-specific revenue shifters, all exporters would have the same export intensity —namely, $\frac{\tau^{1-\sigma}A_x}{A_d+\tau^{1-\sigma}A_x}$.[17] Therefore, heterogeneity in sales at the firm-destination level is a necessary feature for our model to be able to reproduce the within-country variation in export intensity that we observe in the data.

We now derive expressions for the pdf of export intensity when revenue shifters are distributed lognormal, gamma and Fréchet. We do so by relying on the method of transformations for random variables. This approach requires two conditions: firstly, the distribution of revenue shifters has to be closed under scalar multiplication. This implies that the 'total' revenue shifters, $Z_d(\omega) \equiv s_d z_d(\omega)$ and $Z_x(\omega) \equiv s_x z_x(\omega)$, follow the same distribution as the firm-destination-specific components $\{z_i(\omega)\}_{i\in\{d,x\}}$. Secondly, since E can be expressed as a strictly increasing function of the ratio of export to domestic revenue shifters, $Z \equiv Z_x/Z_d$, we need this random variable to have a closed-form pdf. With these conditions at hand, we can establish our first result:

Proposition 1. *Assume that firm-destination-specific revenue shifters $\{z_i(\omega)\}_{i\in\{d,x\}}$ are drawn from the same distribution independently across destinations.*

(i) When revenue shifters are distributed lognormal ($\mathcal{LN}$) with underlying mean 0 and variance σ^2_{zi}, i.e. when $z_i(\omega) \sim \mathcal{LN}\left(0, \sigma^2_{zi}\right)$, and therefore, $Z_i(\omega) \equiv s_i z_i(\omega) \sim \mathcal{LN}\left(\ln(s_i), \sigma^2_{zi}\right)$, then the probability density function of export intensity is given by:

$$h^{\mathcal{LN}}(e) = \frac{1}{[e(1-e)]\sqrt{2\pi(\sigma^2_{zd}+\sigma^2_{zx})}} \times \exp\left[-\frac{\left(\ln\left(\frac{e}{1-e}\right) - \ln\left(\frac{s_x}{s_d}\right)\right)^2}{2(\sigma^2_{zd}+\sigma^2_{zx})}\right], \quad e \in (0,1). \tag{4}$$

(ii) When revenue shifters are distributed gamma (Γ) with scale parameter 1 and shape parameter $\alpha > 0$, i.e. when $z_i(\omega) \sim \Gamma(1,\alpha)$, and therefore, $Z_i(\omega) \equiv s_i z_i(\omega) \sim \Gamma(s_i,\alpha)$, then, the probability density function of export intensity is given by:

$$h^{\Gamma}(e) = \frac{\left(\frac{s_d}{s_x}\right)^{\alpha}}{\mathcal{B}(\alpha,\alpha)} \times \frac{e^{\alpha-1}(1-e)^{-(1+\alpha)}}{\left[1+\left(\frac{s_d}{s_x}\right)\left(\frac{e}{1-e}\right)\right]^{2\alpha}}, \quad e \in (0,1), \tag{5}$$

where $\mathcal{B}(\cdot,\cdot)$ denotes the Beta function.

(iii) When revenue shifters are distributed Fréchet with scale parameter 1 and shape parameter $\alpha > 0$, i.e. when $z_i(\omega) \sim$ Fréchet$(1,\alpha)$, and therefore, $Z_i(\omega) \equiv s_i z_i(\omega) \sim$ Fréchet(s_i,α), then,

[17] If firms could sell their output in more than two countries (paying a fixed cost per destination), then the more productive producers would export to more markets and would have a higher export intensity than less productive ones —even without firm-destination-specific revenue shifters. The prediction that firms enter export markets according to a strict, productivity-driven hierarchy, however, is strongly rejected by the data (Eaton et al., 2011; Fernandes et al., 2015). Also, as we noted in the introduction, this model would not be able to generate neither right-skewed nor bimodal export intensity distributions if it were to be consistent with the stylized facts that characterize the size distribution of firms.

the probability density function of export intensity is given by:

$$h^{Fréchet}(e) = \alpha \left(\frac{s_d}{s_x}\right)^{\alpha} \times \frac{e^{\alpha-1}(1-e)^{-(1+\alpha)}}{\left[1+\left(\frac{s_d}{s_x}\right)^{\alpha}\left(\frac{e}{1-e}\right)^{\alpha}\right]^2}, \quad e \in (0,1). \tag{6}$$

Proof. See Appendix A.1.

Figures A.1-A.3 in Appendix A.1 provide examples of the pdf of export intensity for each underlying distribution of revenue shifters and different values of their shape and scale parameters.

Two remarks are in order in regards to the distributions (4)-(6). Firstly, note that when revenue shifters are distributed gamma or Fréchet, we have assumed that the shape parameter α is the same in the domestic and export market, whereas in the lognormal case the variance of revenue shifters can vary across markets. Imposing this assumption allows us to prove Proposition 3 for gamma-distributed shifters, while for the case of Fréchet, we need it to obtain a closed-form expression for the cdf of the ratio of revenue shifters (Nadarajah and Kotz, 2006). Secondly, when revenue shifters are lognormal, export intensity follows a so-called logit-normal distribution with scale parameter $\ln(s_x/s_d)$ and variance $\sigma^2_{zd}+\sigma^2_{zx}$; Johnson (1949) derives the main properties of this distribution.[18]

We now move to describe the conditions under which the distribution of export intensity is bimodal. These are spelled out in our second proposition:

Proposition 2. *The distribution of export intensity is bimodal if:*

- *Revenue shifters are distributed lognormal, and the following two conditions are satisfied:*

$$\sigma^2_{zd} + \sigma^2_{zx} > 2, \tag{7}$$

and

$$|\ln(s_d/s_x)| < \left(\sigma^2_{zd}+\sigma^2_{zx}\right)\sqrt{1-\frac{2}{\sigma^2_{zd}+\sigma^2_{zx}}} - 2\tanh^{-1}\left(\sqrt{1-\frac{2}{\sigma^2_{zd}+\sigma^2_{zx}}}\right). \tag{8}$$

The two modes lie in the interior of the support but do not have a closed-form solution. The major mode is located near 0 when $s_d/s_x > 1$, and near 1 in the converse case; if $s_d/s_x = 1$, then the distribution is symmetric around 0.5.

- *Revenue shifters are distributed gamma or Fréchet and $\alpha < 1$. In this case, the modes are located at 0 and 1. The distribution of export intensity is unimodal when $\alpha \geqslant 1$. The major mode occurs at 0 when $s_d/s_x > 1$, and at 1 in the converse case; if $s_d/s_x = 1$, then the distribution is symmetric around 0.5.*

Proof. See Appendix A.2.

[18] X is a logit-normal random variable if $Y = \text{logit}(X) = X/(1-X)$ is normally distributed. We thank Chris Jones for pointing this fact to us.

The dispersion of sales in a market increases when $\sigma_{zd}^2 + \sigma_{zx}^2$ increases in the case of lognormally-distributed revenue shifters, or when the shape parameter α falls in the case of the gamma or Fréchet distributions. Since revenue shifters are independent across destinations, the likelihood that firms face a very high demand in only one of the two markets they serve also increases. When the dispersion of revenue shifters is sufficiently high —i.e. when the conditions in Proposition 2 are satisfied— the distribution of export intensity exhibits twin peaks. That is, the majority of exporters sell either a very small share or most of their output abroad. When the dispersion in revenue shifters increases, twin peaks become more prominent as the mass of the distribution shifts towards the boundaries of the support. While there is no closed-form solution for the modes when revenue shifters are lognormal, Figure A.4 in Appendix A.2.1 shows that they are near 0 and 1, and that they move closer to the extremes as the variance of revenue shifters increases.

Although the critical value of the shape parameter necessary to produce bimodality is the same in the gamma and Fréchet cases, there is a crucial difference between these two distributions. Bimodality obtains with Fréchet shifters only when their dispersion is so large that their expected value tends to infinite.[19] This is problematic in monopolistic competition because in these circumstances the integrals defining the price index and the free-entry condition do not converge. This is not an issue for lognormal and gamma-distributed shifters because all the moments of these random variables are finite.[20]

It is important to note that the bimodality of export intensity can also arise when revenue shifters follow distributions other than the three we consider in this paper. A notable example is the Pareto distribution —perhaps the most ubiquitous distribution in the international trade literature. Using simulations, we find that when revenue shifters are distributed Pareto with a shape parameter lower than 1, the distribution of export intensity is multi-modal (it can have 2 or 3 modes); unfortunately, Ali et al. (2010) show that the distribution of the ratio of two Pareto-distributed random variables does not admit a closed-form expression.[21]

Implications of Twin Peaks

In this section we discuss some instances in which twin peaks affect economic outcomes. First, we show that when the distribution of export intensity exhibits twin peaks, trade liberalization leads to the rise of high-intensity exporters. Then, we show that twin peaks also reduce the diversification benefits of exporting.

Reduction in Trade Costs. A reduction in the iceberg cost faced by home exporters directly

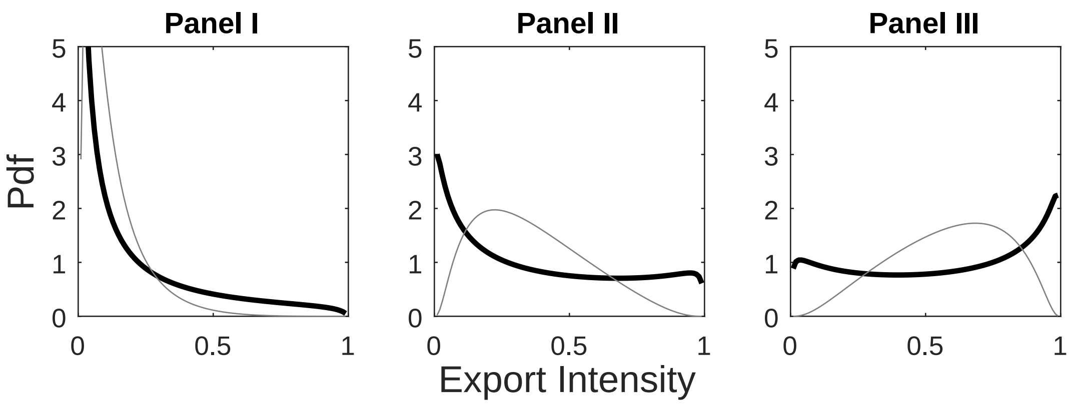
[19] This is the case because the moment of order q for Fréchet-distributed random variables is finite if and only if $\alpha > q$.

[20] Since a gamma random variable with shape parameter 1 is distributed exponentially, it follows that twin peaks arise if the gamma-distributed revenue shifters have dispersion greater than that of the exponential distribution.

[21] Other examples of distributions that generate a bimodal export intensity distribution are the beta, chi-squared and F distributions. Just as in the case of Pareto, there is no closed-form expression for the pdf of the ratio of these random variables.

lowers the ratio s_d/s_x.[22] Figure 3 shows how the distribution of export intensity changes as the relative size of the foreign market increases (when moving from Panel I on the left to Panel III on the right). The darker line shows the distribution of export intensity when revenue shifters are lognormal and the sum of their variances is equal to 4 —thus, producing a bimodal distribution— while the lighter line represents a unimodal distribution (i.e. when $\sigma^2_{zd} + \sigma^2_{zd} = 1$).

Figure 3: Reduction in Export Costs with and without Twin Peaks



The figure plots the probability density function of export intensity when revenue shifters are lognormal with $\sigma^2_{zd} + \sigma^2_{zd} = 4$ (darker line) and $\sigma^2_{zd} + \sigma^2_{zd} = 1$ (lighter line) for different values of the ratio of scale parameters; namely, $s_d/s_x = 0.1$ in Panel I, 0.5 in Panel II and 1.5 in Panel III.

In panel I of Figure 3, trade costs are so high that most exporters sell only a small share of their output abroad regardless of the level of dispersion in revenue shifters, and therefore, the two export intensity distributions look quite similar.[23] As trade costs fall, the intensity of all exporters increases and the distribution of export intensity shifts to the right. However, the difference in the distribution with and without twin peaks also becomes starker. When the variance of revenue shifters is sufficiently large to generate twin peaks, greater access to foreign markets increases the prevalence of high-intensity exporters; conversely, when the dispersion is unimodal, the increase in the intensity of exporters following liberalization is gradual.

Exports and Firm-level Volatility. A recent literature investigates how exporting affects firm-level volatility (see e.g. Riaño, 2011; Vannoorenberghe, 2012; Kurz and Senses, 2016; Girma et al., 2016). In principle, exporting allows firms to diversify demand shocks that are not perfectly correlated across markets, thereby lowering the volatility of sales.

We now show that the existence of twin peaks in the distribution of export intensity reduces the

[22] If both countries are symmetric in size and the reduction in trade costs is bilateral, this direct effect coincides with the full general equilibrium change in relative market size. More generally, a lower trade cost also affects wages and price indices in both countries. Demidova and Rodríguez-Clare (2013) show that is not possible to unambiguously sign the effect of trade liberalization on wages and price indices in the Melitz (2003) model when countries are asymmetric in terms of size unless the model is parameterized and fully solved.

[23] To fix ideas, assume that the foreign country is twice as large as home, i.e. $A_x/A_d = 2$. Then, a s_d/s_x ratio of 0.1 (Panel I) implies an iceberg cost of 4.47 (given $\sigma = 3$). Based on the same parametrization, the iceberg cost in Panels II and III are 2 and 1.15 respectively.

diversification benefits of exporting. To do so, we introduce a minor modification to the revenue functions defined in (2). We assume that aggregate variables, productivity and firm-destination-specific revenue shifters are time-invariant, but sales vary over time in response to idiosyncratic i.i.d. demand shocks, $\epsilon_{it}(\omega)$, which are drawn from a distribution with mean 1 and variance ν_i^2 —the same set up considered by Vannoorenberghe (2012). Thus, firm ω's sales in market $i \in \{d, x\}$ in period t are given by:

$$r_{it}(\omega) = \Phi(\omega) \cdot s_i \cdot z_i(\omega) \cdot \epsilon_{it}(\omega). \tag{9}$$

Since shocks are multiplicative and transitory, the firm chooses the same optimal prices as in the static model described before, and therefore, $E(\omega) = \frac{Z_x(\omega)}{Z_d(\omega)+Z_x(\omega)}$ —the same expression as (3)— now denotes firm ω's long-run export intensity.

Assuming that demand shocks are equally volatile across markets, it can be shown that firms' volatility is minimized when the firm's long-run export intensity equal 0.5.[24] If the distribution of long-run export intensity displays twin peaks, a greater dispersion of revenue shifters increases the share of firms with export intensity close to 0 and 1. As a result, the aggregate volatility-reduction effect of exporting is weakened because most exporters sell the majority of their output either domestically or abroad.

4 Estimation

In the previous section we showed that the distribution of export intensity is fully characterized by the shape parameter governing the distribution of firm-destination-specific revenue shifters and a country's relative size with respect to the rest of the world. We now discuss how we estimate these parameters using the cross-country firm-level data from the WBES.

Identification Strategy. We first present a result that greatly facilitates the identification and estimation of the parameters governing the distribution of revenue shifters. Namely,

Proposition 3. *If revenue shifters are independent across firms and destinations and follow either lognormal, gamma, or Fréchet distributions as specified in Proposition 1, then the median export intensity, e^{med}, is given by:*

$$e^{med} = \frac{s_x}{s_d + s_x}, \tag{10}$$

which is independent of the shape parameter of revenue shifters.

Proof. See Appendix A.3.

[24] In Appendix B we show that the volatility of the growth rate of total sales for firm ω, $\mathrm{Vol}(\omega)$, is given by:

$$\mathrm{Vol}(\omega) \approx 2\left[(1 - E(\omega))^2 \cdot \nu_d^2 + (E(\omega))^2 \cdot \nu_x^2\right].$$

The volatility of sales is therefore minimized when the firm's long-run export intensity is equal to $\frac{\nu_x^2}{\nu_d^2+\nu_x^2}$. Assuming agnostically that demand shocks are equally volatile across markets, the latter expression implies that the diversification effect of exporting is maximized when a firm's long-run export intensity is 0.5.

Thus, Proposition 3 allows us to recover the relative market size for each country directly from the export intensity data by inverting equation (10):

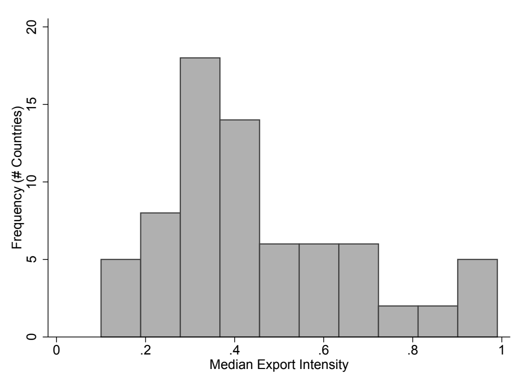
$$\left(\frac{s_d}{s_x}\right) = \frac{1 - e^{med}}{e^{med}}. \tag{11}$$

It also follows from (11) that our measure of relative market size is invariant to the underlying distribution of revenue shifters. Crucially, Proposition 3 allows us to recover relative market sizes without having to estimate or calibrate other parameters of the model such as trade costs, or the parameters characterizing the productivity distribution, all of which are necessary to compute the value of s_i when solving the model explicitly.

Conditional on relative market size, the identification of the shape parameter is straightforward. The value of this parameter is determined by whether the mass of the export intensity distribution is concentrated in the interior or near the boundaries of the support. If the dispersion of revenue shifters is low, then the distribution of export intensity is unimodal, with most exporters exhibiting an intensity close to $\frac{s_x}{s_d+s_x}$. On the other hand, if there large clusters of exporters with intensities near 0 and 1, then the dispersion of revenue shifters will be sufficiently high so that the shape parameter satisfies the conditions spelled out in Proposition 2.

Scale Parameters. Figure 4 displays the distribution of the median export intensity in our data. Although there is large variation across countries, the median export intensity ranges between 0.2 and 0.8 for most of them. In the lower end of the distribution, Brazil and Russia have a median export intensity of 0.1, while at the other extreme, Bangladesh, Madagascar, Morocco, Pakistan, Philippines and Sri Lanka all have a median export intensity greater than 0.9. Summary statistics for the relative country size implied by countries' median export intensity are reported in Table 4. For the median country in our sample, the domestic market is 50% larger than the foreign market.

Figure 4: Distribution of Median Export Intensity Across Countries



Source: World Bank Enterprise Surveys (WBES), 2002-2016.

Table 4: Inferred Relative Domestic Market Size

	Mean	Percentile				
		5	25	50	75	95
$\widehat{\frac{s_d}{s_x}}$	1.967	0.010	0.667	1.500	2.333	5.667

Country-Specific Shape Parameters. We first estimate country-specific shape parameters for each of the three revenue-shifter distributions that we consider. These are estimated by maximum likelihood conditional on relative market size being given by (11). Our objective is to determine whether the conditions for bimodality of the export intensity distribution that we identified in Proposition 2 bear out in the data. With our estimates at hand we then use the Vuong (1989) test to compare the resulting distributions in terms of their fit to the data. Vuong (1989) proposes a likelihood-ratio (LR) based statistic based on the Kullback-Leibler information criterion to measure the closeness of a model to the true data generating process.[25] This test has two desirable properties for our purposes: first, it can be used to compare non-nested econometric models —in particular those that are obtained from different families of distributions, as in our case. Second, the test is directional —i.e. it indicates which competing model is better when the null hypothesis that two models are indistinguishable from each other is rejected.

It is important to note that the export intensity pdfs we derive are not defined at an export intensity of 1 —in other words, they do not admit firms exporting all their output. Since 'pure exporters' are ubiquitous in the data, we need to censor their export intensity, and we do so at a conservative value of 0.99. Increasing the censoring cutoff biases the shape parameter in the direction of bimodality (i.e. lowers the shape parameter for gamma and Fréchet and increases the sum of variances in the lognormal case). This is the case because the distribution of revenue shifters would need to generate large shares of extremely low and high realizations to be able to reproduce a significant number of exporters with intensity at or above the censoring threshold.[26] To ensure that our results are not driven by our chosen censoring threshold, we re-estimate the shape parameter dropping all pure exporters in the robustness analysis in the next section.

The country-specific shape parameter estimates are reported in Table C.1 in Appendix C. In all but a handful of cases they imply export intensity distributions that are bimodal. The mean estimates across countries are 6.489 for lognormal and 0.672 and 0.740 for gamma and Fréchet-distributed revenue shifters respectively. In the lognormal case, all shape parameters satisfy conditions (7) and (8) for bimodality (see Figure A.5 in Appendix A.2.1). When revenue shifters are distributed gamma or Fréchet, we estimate shape parameters greater than 1 for 6 and 7 countries respectively —all of which are identified as unimodal by the dip test reported in Figure 2.

[25] Mrázová et al. (2015) also use the Kullback-Leibler divergence to evaluate how different combinations of demand functions and productivity distributions fit the size distribution of French firms exporting to Germany.

[26] For instance, for a firm to have an export intensity of 0.99, its export revenue shifter has to be 99 times larger than its domestic one; increasing the censoring threshold to 0.999 would shift this ratio up by an order of magnitude.

The results of the Vuong (1989) test reported in columns (1)-(3) and (7)-(9) of Table D.1 in Appendix D do not suggest that one of the underlying distributions of revenue shifters clearly dominates the others in terms of fitting the data. Comparing the lognormal-based export intensity with the gamma and Fréchet distributions reveals that in 20 countries we cannot discriminate between the two competing models, while the remaining ones are evenly split between the lognormal and the competing distribution. Among the set of countries for which lognormal performs worse than the alternative, gamma fits the data better than Fréchet in all but two countries.[27]

Single Shape Parameter. The results discussed above show that large heterogeneity in firms' performance across domestic and export markets can successfully explain the occurrence of twin peaks. We now investigate if our model can account for the variation we observe in the distribution of export intensity across countries. Putting it differently, can our model explain the patterns depicted in Figure 1?

To answer this question we consider a scenario in which firms in all countries draw revenue shifters in each destination they serve from a distribution with the same shape parameter. This implies that the variation in the distribution of export intensity across countries is entirely due to differences in relative market size. In order to estimate this 'restricted' model, we pool data across all countries and weight each observation by the inverse of the number of observations available for each country.

The estimates for the single-shape parameter models are reported in Table 5. The point-estimates are very similar to the average across the country-specific estimates reported above and all imply a bimodal distribution of export intensity. Notably, the Vuong test reveals that for approximately two-thirds of the countries we cannot discriminate between the restricted model and the one where shape parameters are country-specific (see columns (4)-(6) and (10)-(12) of Table D.1 in Appendix D), even though the latter fits the data better by definition. The restricted model has the additional advantage that it greatly facilitates conducting the robustness analysis below in which we investigate whether the existence of a second mode near 1 is driven by specific subsets of firms being particularly export intensive.

Using customs-level data from France and the World Bank's Export Dynamics Database, Eaton et al. (2011) and Fernandes et al. (2015) estimate structural models of trade that incorporate lognormally-distributed firm-destination-specific revenue shifters. Although their models are richer than ours —including, for instance, shocks to productivity and fixed costs and convex marketing costs— they both find that the variance of revenue shifters is large enough to generate bimodality in our model.[28]

[27] When the Vuong test indicates that lognormal dominates gamma it also suggests that the former provides a better fit to the data than Fréchet. The only exception is in the case of Uganda in which the test rejects gamma in favor of lognormal but cannot discriminate between lognormal and Fréchet.

[28] Eaton et al. estimate this parameter at 2.856, while Fernandes et al. find it to be 6.60. In comparison, if we assume agnostically that the variance of domestic and export shifters is the same, the point estimate reported in column (1) of Table 5 implies a value of 3.24.

Table 5: Single Shape Parameter Estimate

Distribution:	Lognormal	Gamma	Fréchet
Parameter:	$\sigma_{zd}^2 + \sigma_{zx}^2$	α	α
	(1)	(2)	(3)
Estimate:	6.489	0.623	0.702
	(0.050)	(0.004)	(0.003)
Countries	72	72	72
Obs.	33,224	33,224	33,224

The table reports the maximum likelihood estimate of the shape parameter governing firm-destination-specific revenue shifters for different underlying distributions, conditional on s_x/s_d being given by (11). The pdf used in the estimation are given by equations (4), (5), and (6), for lognormal, gamma and Fréchet distributed revenue shifters respectively. Standard errors are reported in parentheses.

Figure 5 presents the fit of the three restricted models to the data, with countries sorted according to their estimated relative market size. Relying only on variation in countries' relative market size and a unique shape parameter governing firm-destination revenue shifters, our model reproduces closely the wide range of shapes observed in the distribution of export intensity across the world: unimodal distributions where the majority of exporters exhibit either very low or very high export intensity, just as well as those displaying twin peaks. Figure 5 also echoes the results of the Vuong test discussed above —all three distributions of revenue shifters, lognormal, gamma and Fréchet, fit the export intensity data quite well.

Relative Country Size and Bimodality

We have established that when the conditions in Proposition 2 are satisfied, the distribution of export intensity is bimodal. Nevertheless, Figure 2 shows that one-third of the countries in our sample have a unimodal distribution. These two are reconciled in our model by noting that as relative market size either becomes too small or too large, the height of the minor mode shrinks enough for the distribution to appear unimodal. In these circumstances, a statistical test of unimodality, such as the dip test, is likely to not reject the null hypothesis of unimodality. That is, the probability that the test produces a Type-II error increases.

Figure 6 plots the dip statistic calculated for data simulated using the shape parameters reported in Table 5 for different values of the median export intensity. Figure 6 shows that there is an inverse-U relationship between the dip statistic (recall that a larger number means that the distribution is less likely to be unimodal) and the median export intensity. Countries that are either very small or very large relative to the rest of the world —which have very high and low median export intensities respectively— are likely to be classified as unimodal by the dip test (dashed lines in Figure 6); conversely, countries for which the domestic and export markets are more similar in size display more prominent twin peaks.

Figure 5: Model Fit Export Intensity Distribution

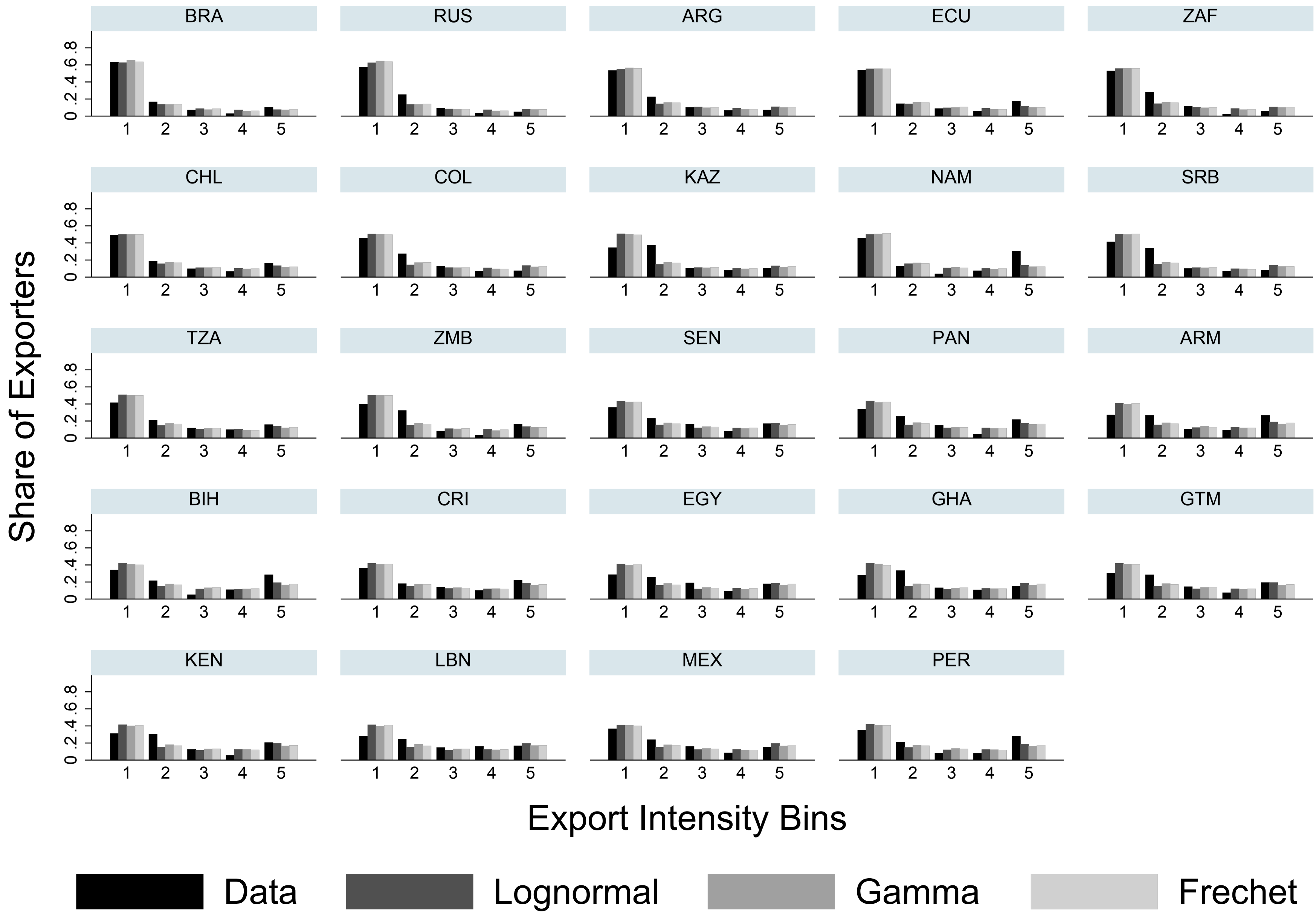


Figure 5: Model Fit Export Intensity Distribution, Continued

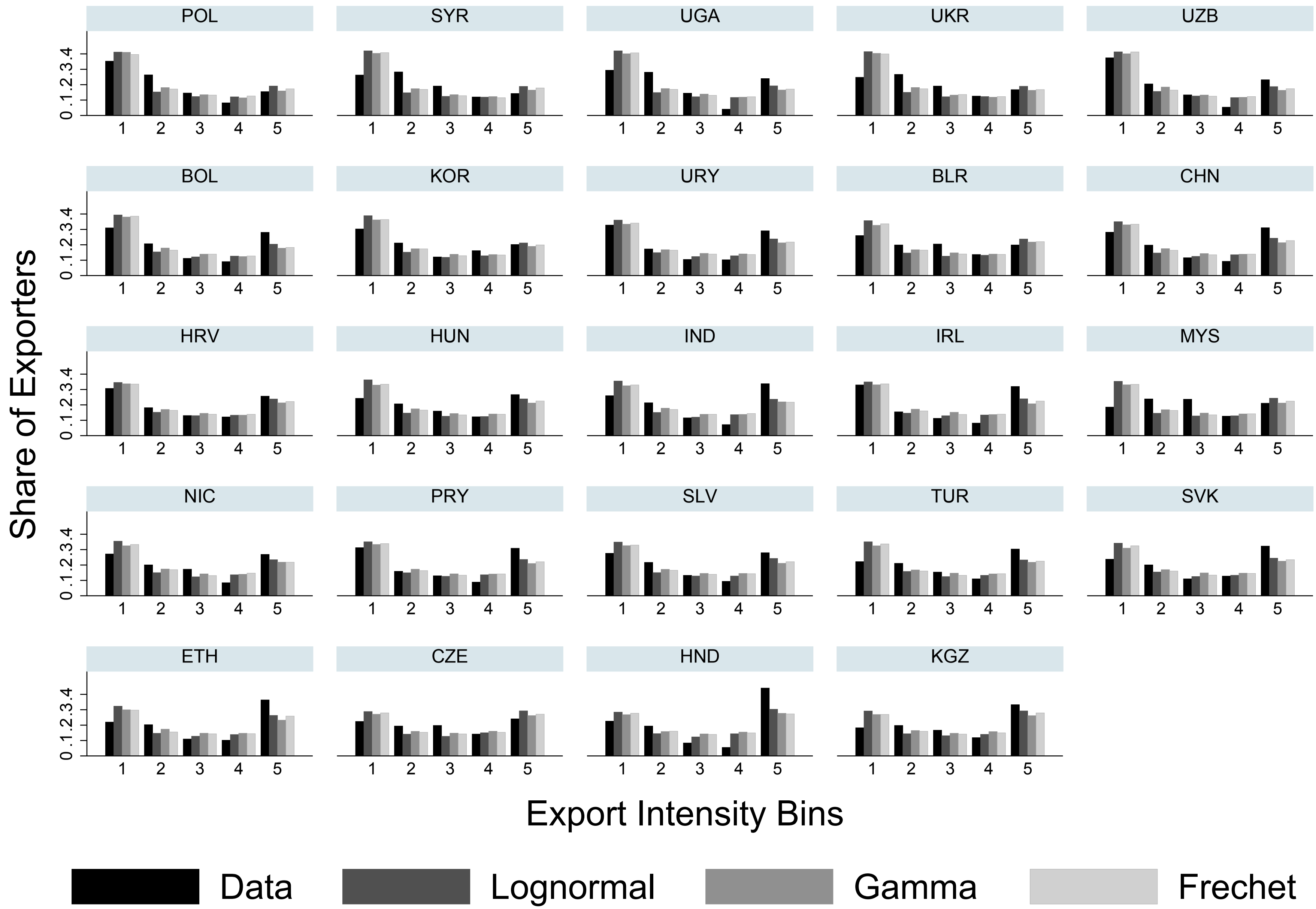


Figure 5: Model Fit Export Intensity Distribution, Continued

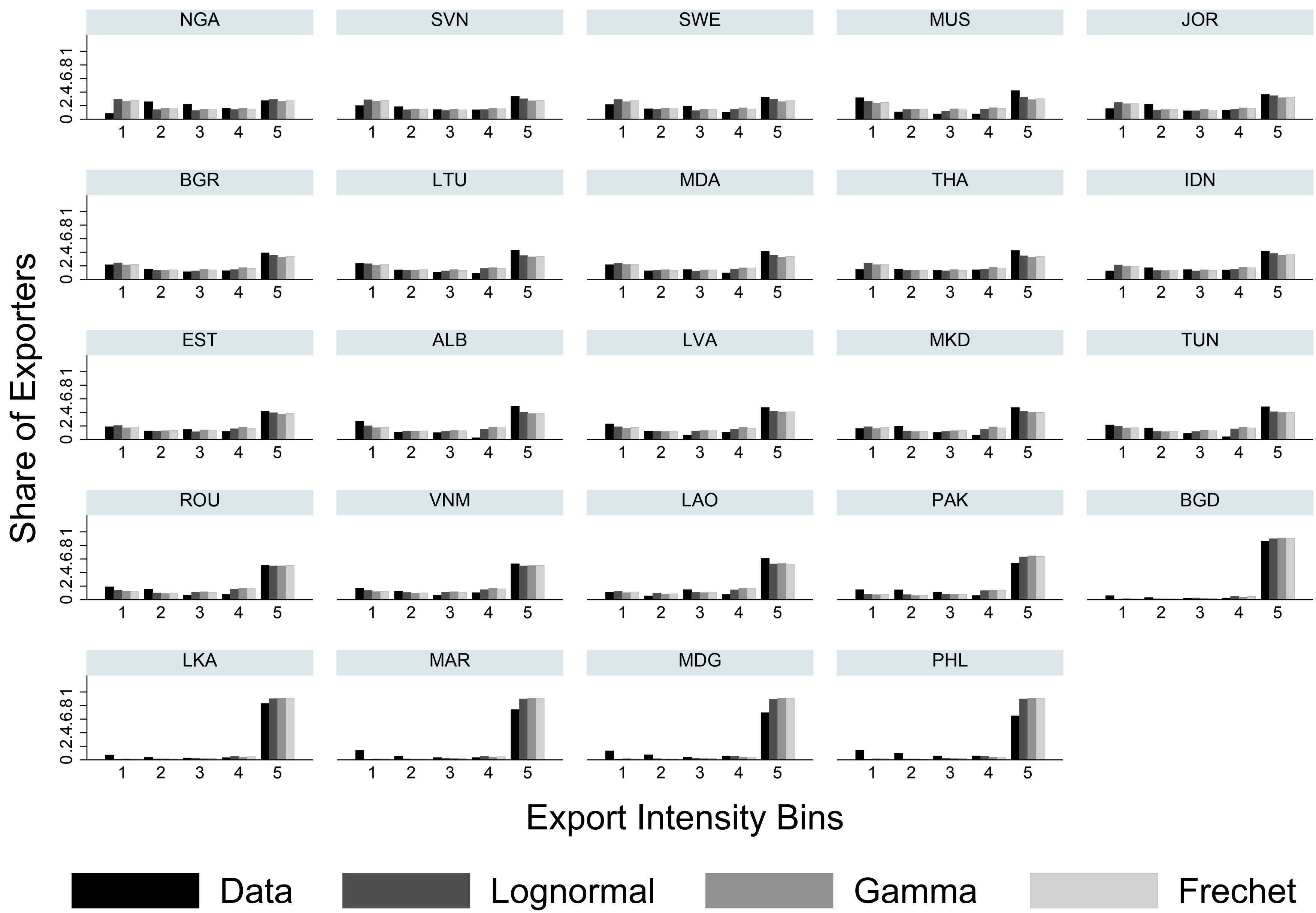
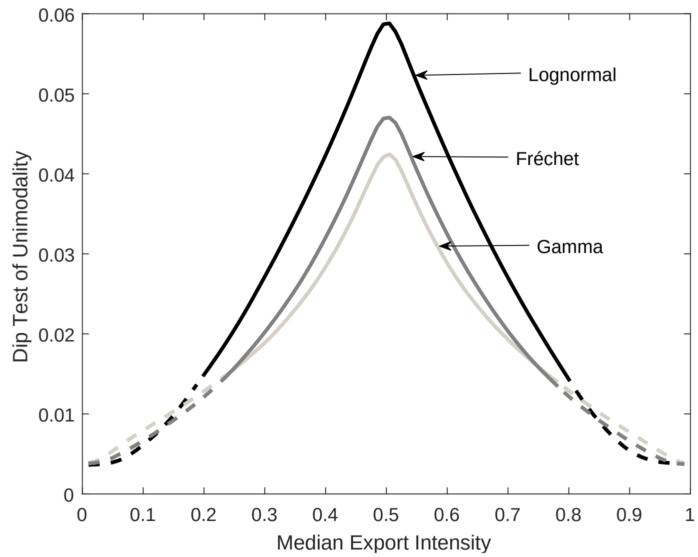
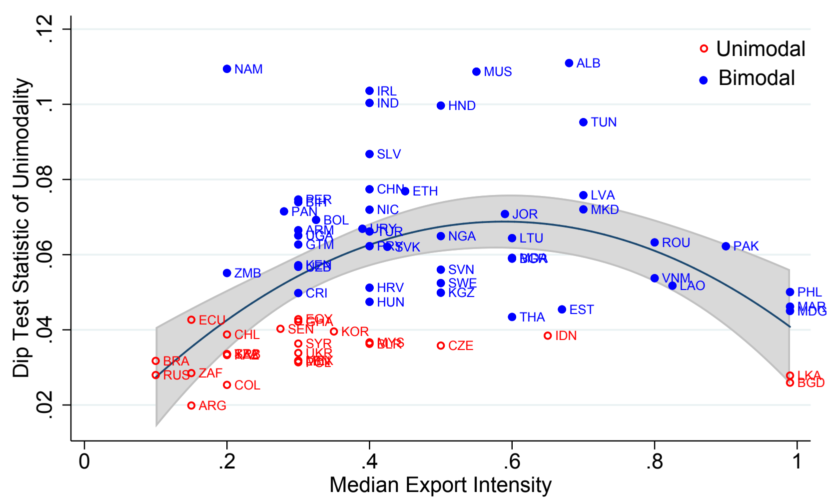


Figure 6: Relative Market Size and Bimodality of the Export Intensity Distribution



The figure reports the value of the dip test statistic calculated on simulated export intensity draws using the estimated shape parameters reported in Table 5 for different values of the median export intensity. Solid lines represent sets of draws in which the null hypothesis of unimodality is rejected at the 1% significance level, while dashed lines show the realizations for which unimodality is not rejected.

Figure 7: Prevalence of Bimodality and Median Export Intensity across Countries



The figure plots the fitted values obtained after regressing each country's dip test statistic (reported in Table 2) on a country's median export intensity and median export intensity squared. The estimated equation is $dip = \underset{(0.0096)}{0.0086} + \underset{(0.0326)}{0.2044} median - \underset{(0.0397)}{0.1736} median^2$, and the shaded area denotes the 95% confidence interval. Hollow circles denote countries for which the dip test does not reject the null hypothesis of unimodality at the 1% level (unimodal) and filled circles indicate countries with a bimodal export intensity distribution (those for which the p-value of the dip test is below 1%).

Figure 7 reveals that the inverted-U pattern is also clearly borne in the data. Thus, the high dispersion in firm-destination-specific revenue shifters is able to explain why the distribution of export intensity is bimodal in some countries but appears unimodal in others. Figure 5 shows that relative country size clearly determines whether a country has uni- or bimodal distribution.

5 Robustness

An alternative explanation for the existence of twin peaks is that they are the product of a composition effect arising from certain groups of firms having markedly different export intensities. Thus, in this section we probe the robustness of our single-shape-parameter estimates by excluding different subsets of firms that the literature has identified as being more export-oriented than average. These results are reported in columns (2)-(7) of Table 6 (column (1) reproduces our benchmark single-shape parameters estimates for convenience). Notice that as we restrict the sample across different specifications, the median export intensity changes for each country relative to the estimates presented in Figure 4.

Foreign Ownership. We start our analysis by excluding firms that are foreign-owned —i.e. those with a share of foreign equity of at least 10%— from the estimation. As Antràs and Yeaple (2014) note, multinational firm affiliates tend to be more export intensive than non-multinational firms because of intra-firm vertical specialization taking place between parent companies and affiliates. Arnold and Javorcik (2009) find that the export intensity of Indonesian plants that are acquired by foreign investors increases substantially after acquisition.

Export Processing. Firms engaged in export processing activities have also been identified as being highly export intensive (see e.g. Brandt and Morrow, 2017). Export processing occurs when a producer ships an unfinished product to a foreign country where some value-added is incorporated into it before being re-exported again. Firms undertaking processing activities have high export intensity because the tariff concessions available in this customs regime require firms to export all goods that incorporate duty-free imported inputs. Although the WBES data does not identify firms that export through a processing custom regime explicitly, we use the share of intermediate inputs accounted for by imports to construct a proxy for export processing. Therefore, we classify firms as processing exporters if their share of imported inputs exceeds 90% of their total expenditure in intermediate inputs.

Pure Exporters. In our most stringent exercise, we exclude firms that export all their output —'pure exporters'— from the estimation. In the model we proposed in Section 3, we assumed that all firms pay a fixed cost f_d to set up a plant, and then, additionally, a fixed cost f_x if they choose to export. Therefore, all operating firms would sell a positive quantity of output —no matter how small— in the domestic market. Alternatively, firms could face destination-specific fixed costs, as in Eaton et al. (2011) for instance, that incorporate production and market access costs. In

Table 6: Robustness Checks

	Full	Excluding			Countries		
	Sample	Foreign	Processing	Pure Exp.	without ESR	OECD	non-OECD
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
				Lognormal			
$\sigma_{zd}^2 + \sigma_{zx}^2$	6.489	6.044	6.056	3.81	5.762	5.497	6.670
	(0.050)	(0.053)	(0.051)	(0.033)	(0.069)	(0.111)	(0.056)
				Gamma			
α	0.623	0.656	0.657	0.886	0.660	0.680	0.614
	(0.004)	(0.005)	(0.004)	(0.006)	(0.006)	(0.011)	(0.004)
				Fréchet			
α	0.701	0.732	0.732	0.913	0.730	0.748	0.694
	(0.003)	(0.004)	(0.004)	(0.005)	(0.005)	(0.009)	(0.003)
Countries	72	72	72	72	39	11	61
Obs.	33,224	26,217	28,375	26,647	13,691	4,916	28,308

The table reports the maximum likelihood estimate of a single shape parameter for the three distributions of firm-destination-specific revenue shifters conditional on s_x/s_d being given by (11). Each firm-level export intensity observation is weighted so that each country receives an equal weight in the estimation. Column (1) reproduces the single shape parameter estimates reported in Table 5. Column (2) excludes foreign-owned firms (i.e. those with a share of foreign equity of at least 10%) from the estimation. Column (3) excludes 'processing' exporters, i.e. firms for which imports account for more than 90% of their total expenditure in intermediate inputs. Column (4) excludes firms with export intensity exactly equal to 1 (pure exporters). Column (5) excludes countries that provide subsidies subject to export share requirements (ESR), i.e. incentives that require firms to export more than a certain share of their output to receive them. These countries are identified using information collected from the Investment Climate Statements produced by the U.S. State Department, following Defever and Riaño (2017a). Standard errors are reported in parentheses.

the latter model, firms with relatively low productivity but facing sufficiently high demand abroad would choose to operate as pure exporters.

The results reported in columns (2)-(4) of Table 6 show that the estimated shape parameters are extremely robust, and they all imply a bimodal distribution of export intensity. Excluding pure exporters produces the largest reduction in the dispersion of revenue shifters as it directly affects the export intensity distribution. The important thing, however, is that even when we exclude these firms, which account for a substantial share of high-intensity exporters, our main result remains unchanged. Having examined firm-level characteristics, we now turn to country-level factors that could potentially generate bimodal export intensity distributions through a composition effect.

Subsidies with Export Share Requirements. The use of incentives subject to export share requirements distorts a country's 'natural' export intensity distribution because firms are induced to operate at a higher export intensity than the one they would have chosen otherwise. Defever and Riaño (2017a) show that this can produce large negative welfare effects in countries enacting them. As we have noted in Section 2, approximately half of the countries in our data provide this class of incentives. ESR are frequently imposed in special economic zones —geographically-bounded areas in which customs, tax and investment regulations are more liberal than in the rest of the country (Farole and Akinci, 2011; Defever et al., 2016). By excluding countries that offer subsidies with ESR from our estimation we seek to allay the concern that the high prevalence of twin peaks across the world is due to the use of these incentives.

Level of Development. Although our sample consists primarily of developing countries, we also investigate if there are significant differences in the estimated shape parameter between OECD and non-OECD countries. If, for instance, the high prevalence of high-intensity exporters is caused by vertical specialization driven by cross-country wage differentials, then it could be the case that the value of the shape parameter is influenced by the composition of countries in our sample.

The results presented in columns (5)-(7) of Table 6 show, once again, that the estimated shape parameters imply bimodality. Excluding countries that provide subsidies with ESR, which are also countries where high-intensity exporters are ubiquitous, has a similar effect on the shape parameter as excluding foreign-owned and export processing firms. Consistent with the intuition outlined above, the dispersion of revenue shifters is higher for developing countries than for developed ones.

Sectoral Differences. Bernard et al. (2007) document large differences in average export intensity across manufacturing industries in the U.S. Thus, we now explore the possibility that a country's bimodal export intensity distribution is the result of a mixture of sectoral distributions, which may differ substantially due to technological differences or comparative advantage. For this exercise we include countries in which there are more than 50 exporters operating in a given sector. Since there

Table 7: Estimated Shape Parameter —By Sector

	Leather & Textiles (1)	Metals & Machinery (2)	Food & Beverages (3)	Non-metal Products (4)	Electric Products (5)	Paper & Furniture (6)	Chemicals & Pharma (7)	Other Mfg. (8)
	Lognormal							
$\sigma^2_{zd} + \sigma^2_{zx}$	6.406	4.476	6.393	6.372	6.762	7.053	3.547	3.841
	(0.137)	(0.174)	(0.253)	(0.440)	(0.377)	(0.503)	(0.236)	(0.258)
	Gamma							
α	0.650	0.810	0.610	0.615	0.610	0.579	0.971	0.893
	(0.011)	(0.026)	(0.019)	(0.034)	(0.027)	(0.033)	(0.054)	(0.050)
	Fréchet							
α	0.731	0.862	0.686	0.692	0.688	0.660	0.987	0.925
	(0.009)	(0.020)	(0.016)	(0.028)	(0.023)	(0.028)	(0.038)	(0.037)
Countries	29	14	14	4	4	5	5	7
Obs.	4,368	1,327	1,277	420	643	393	453	443

The table reports the maximum likelihood estimate of a single shape parameter for the three distributions of firm-destination-specific revenue shifters conditional on s_x/s_d being given by (11). Each firm-level export intensity observation is weighted so that each country receives an equal weight in the estimation. The shape parameter is estimated separately for each manufacturing sector. Standard errors are reported in parentheses.

is a significant degree of concentration of exporters across sectors and countries, these estimates need to be considered with caution.[29]

Table 7 shows that the distribution of export intensity is bimodal even *within* industries. The industry-specific parameters are similar in magnitude to estimates presented in Table 5. Only in chemical & pharmaceutical products is the shape parameter near the limit necessary to generate bimodality with gamma and Fréchet shifters.

The key takeaway message from our robustness analysis is that, regardless of how you slice it, the data strongly supports the bimodality of the distribution of export intensity.

Estimating Parameters using Sales Data

We now assume that firms' productivity and revenue shifters —and therefore, their sales— are log-normal and estimate the mean and variance of sales for each country-destination pair in our data.[30] Letting $z_i(\omega) \sim \mathcal{LN}\left(0, \sigma_{zi}^2\right)$, $i \in \{d, x\}$, and $\Phi(\omega) \sim \mathcal{LN}\left(0, \sigma_{\Phi}^2\right)$ and assuming that productivity and revenue shifters are all independent from each other, it follows that,

$$r_i(\omega) \sim \mathcal{LN}\left(\ln(s_i), \sigma_{\Phi}^2 + \sigma_{zi}^2\right), \qquad i \in \{d, x\}. \tag{12}$$

Estimating the parameters for the distribution of sales allows us to carry out two exercises: first, we use the domestic and export scale parameters to calculate $\frac{s_x}{s_d+s_x}$, and compare it to the relative market size obtained from the median export intensity. Second, we decompose the variance of sales between firm- and firm-destination-specific factors utilizing the estimated variance of sales and the sum of the variances of revenue shifters reported in Table C.1 in Appendix C.

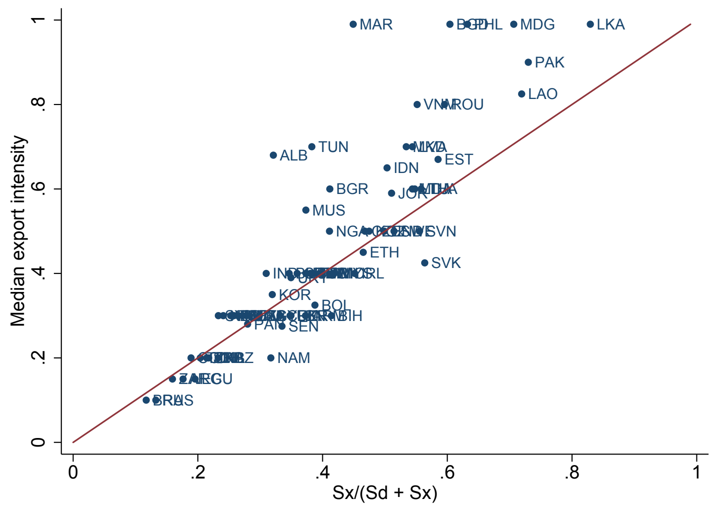
Scale Parameters. Figure 8 plots the ratio $\frac{s_x}{s_d+s_x}$ based on the estimated scale parameters for domestic and export sales against each country's median export intensity. With the exception of a few countries —namely, those with median export intensity very close to 1— all countries lie close to the 45 degree line. Thus, the transformation of median export intensity (11) provided by Proposition 3 delivers sensible estimates of a country's size relative to the rest of the world. Using the relative market size based on sales data we obtain estimates of the shape parameter that are almost identical to those reported in Table 5.

Variance Decomposition. Combining the estimates of $\sigma_{\Phi}^2 + \sigma_{zi}^2$ with those of $\sigma_{zd}^2 + \sigma_{zx}^2$ obtained from the export intensity data, we carry out a variance decomposition of sales. This entails solving

[29] One third of exporters in our sample sell textile garments & leather products, while food & beverages and metal & machinery industries account for approximately 15% of exporters each. About half of Eastern European exporters operate in the metals & machinery industry, while leather & textiles exporters account for more than 40% of exporters in several Latin American and South and East Asian countries. As expected, richer countries have a more diversified export base.

[30] These are estimated by maximum likelihood. To be consistent with the results based on export intensity data reported before, we only include exporting firms in the estimation. We express both domestic and export sales of firms across all countries in our sample in 2007 US dollars.

Figure 8: Scale Parameters Obtained by Fitting Domestic and Export Sales



The figure plots the ratio $\frac{s_x}{s_d+s_x}$, which is calculated based on the estimated scale parameters obtained when fit a lognormal distribution to the domestic and export sales in each country, against each country's median export intensity. The red line denotes the 45 degree line.

a system of three equations (one for the variance of export intensity and two for the variance of sales in each market) and three unknowns, σ^2_{zd}, σ^2_{zx}, and σ^2_{Φ}. The share of the variance of sales in each market accounted for by firm-destination-specific revenue shifters is given by:

$$\text{Share}_{zi} = \frac{\sigma^2_{zi}}{\sigma^2_{\Phi} + \sigma^2_{zi}}, \qquad i \in \{d, x\}. \tag{13}$$

Table 8 reports the average of (13) across all countries. In our benchmark specification, firm-destination-specific revenue shifters account for between 55 to 61 percent of the variation of sales. These figures are consistent with high dispersion in firm-destination-specific generating twin peaks,

Table 8: Share of the Variance of Sales Accounted for Revenue Shifters

	Full	Excluding			Countries		
	Sample	Foreign	Processing	Pure Exp.	without ESR	OECD	non-OECD
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Share_{zd}	0.553	0.548	0.519	0.291	0.501	0.376	0.585
Share_{zx}	0.611	0.613	0.588	0.381	0.586	0.504	0.630

The table reports the average across all countries of the share of the variance of sales in each market (domestic and export) which is accounted for by firm-destination-specific factors (defined in equation (13)). The different subsamples used in columns (2)-(7) are defined in Section 5.

and are also in line the empirical evidence based on customs data (Kee and Krishna, 2008; Eaton et al., 2011; Munch and Nguyen, 2014; Lawless and Whelan, 2014). Columns (2)-(7) of Table 8 report the variance decomposition for the different subsamples considered in our robustness analysis above. Our main message remains unchanged: firm-destination-specific factors account for a substantial share of the variance of domestic and export sales.

6 Conclusions

In this paper we have shown that the distribution of export intensity varies considerably across the world. While previous work suggested that this distribution was consistently unimodal, with a majority of exporters selling most of their output domestically, we show that bimodal export intensity distributions appear to be the rule rather than the exception.

We then show that with a single modification, the workhorse two-country model of trade with CES preferences and heterogenous firms can very successfully reproduce the wide range of patterns for the export intensity distribution that we observe in the data. The key is to incorporate idiosyncratic firm-destination-specific revenue shifters drawn from a distribution with sufficiently high dispersion into the model. If this is the case, we find that differences in relative market size explain most of the observed cross-country variation in the distribution of export intensity.

Our findings open up exciting avenues for future research. It would be interesting to investigate the evolution of the distribution of export intensity over time, and how this process responds to changes in trade policy and technology. Our model suggests that the dynamics will depend crucially on whether there are twin peaks or not. Another exciting line of inquiry would be to unpack the underlying causes that explain the high level dispersion in the demand that the same firm faces across different destinations.

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Twin Peaks
Appendix

A Proofs

A.1 Proof of Proposition 1.

We use the method of transformations for random variables (stated below) to derive the probability density function of export intensity. Since export intensity is a monotone transformation of the ratio of export to domestic revenue shifters in our model, we can use the method of transformations to obtain the pdf of export intensity whenever there is a closed-form solution for the pdf of the ratio of revenue shifters.

Theorem 1. *Let Z be a continuous random variable with pdf $f(z)$, and let $\mathcal{Z}$ denote the support of Z. Consider the random variable $E = j(Z)$, where $e = j(z)$ defines a one-to-one transformation that maps the set $\mathcal{Z}$ onto the set $\mathcal{E}$. Then the pdf of the random variable $E = j(Z)$ is given by:*

$$h(e) = \begin{cases} f[j^{-1}(e)] \cdot \frac{d}{de}\left(j^{-1}(e)\right), & e \in \mathcal{E}, \\ 0 \quad \textit{elsewhere.} \end{cases} \tag{A.1}$$

Let E denote export intensity —the share of total revenues accounted for by exports. Then, we have:

$$E = \frac{Z_x}{Z_d + Z_x} = \frac{\left(\frac{Z_x}{Z_d}\right)}{1 + \left(\frac{Z_x}{Z_d}\right)} = \frac{Z}{1+Z} \quad \rightarrow j(Z) = \frac{Z}{1+Z}. \tag{A.2}$$

$$Z = j^{-1}(E) = \frac{E}{1-E}. \tag{A.3}$$

$$\frac{\mathrm{d}}{\mathrm{d}E}\left(j^{-1}(E)\right) = \frac{1}{(1-E)^2} > 0. \tag{A.4}$$

Thus, the pdf of export intensity is:

$$h(e) = \frac{1}{(1-e)^2} \cdot \widetilde{f}\left(\frac{e}{1-e}\right), \quad e \in (0,1), \tag{A.5}$$

where $\widetilde{f}\left(\frac{z_x}{z_d}\right)$ denotes the pdf of the ratio of export to domestic revenue shifters.

A.1.1 Lognormal

Let the firm-destination-specific revenue shifter in market i in (1) be given by $z_i(\omega) = \exp[\theta_i(\omega)]$, with $\theta_i(\omega) \sim \mathcal{N}(0, \sigma_{zi}^2)$. This implies that $z_i(\omega) \sim \mathcal{LN}(0, \sigma_{zi}^2)$, or in other words, that $\ln z_i(\omega)$ is normally-distributed with mean 0 and variance σ_{zi}^2, and therefore, that,

$$Z_i(\omega) = s_i \cdot \exp[\theta_i(\omega)] \sim \mathcal{LN}\left(\ln s_i, \sigma_{zi}^2\right), \quad i \in \{d, x\}. \tag{A.6}$$

Since the ratio of two lognormal random variables is also distributed lognormal, it follows that

$Z \equiv Z_x/Z_d \sim \mathcal{LN}\left(\ln\left(s_x/s_d\right), \sigma_{zd}^2 + \sigma_{zx}^2\right)$. Thus, the pdf of Z is:

$$\widetilde{f}(z) = \frac{1}{z\sqrt{2\pi(\sigma_{zd}^2 + \sigma_{zx}^2)}} \exp\left[-\frac{(\ln z - \ln(s_x/s_d))^2}{2(\sigma_{zd}^2 + \sigma_{zx}^2)}\right], \quad z > 0. \tag{A.7}$$

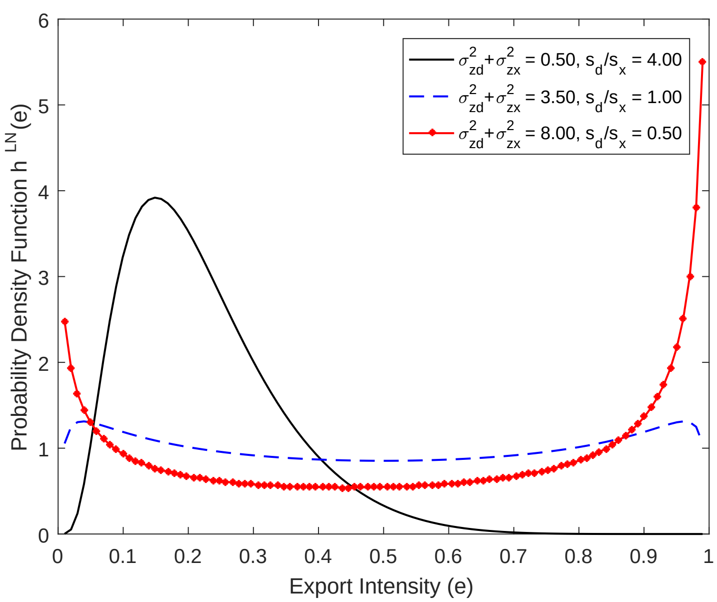
Applying Theorem 1 and using (A.7), we obtain the pdf for export intensity:

$$h^{\mathcal{LN}}(e) = \frac{1}{[e(1-e)]\sqrt{2\pi(\sigma_{zd}^2 + \sigma_{zx}^2)}} \exp\left[-\frac{\left(\ln\left(\frac{e}{1-e}\right) - \ln\left(\frac{s_x}{s_d}\right)\right)^2}{2(\sigma_{zd}^2 + \sigma_{zx}^2)}\right], \quad e \in (0,1). \tag{A.8}$$

The distribution (A.8) is known as the logit-normal distribution (Johnson, 1949).

Figure A.1 presents some examples of the pdf of export intensity when revenue shifters are distributed lognormal for different values of the relative scale parameter s_d/s_x and the sum of the variance of revenue shifters $\sigma_{zd}^2 + \sigma_{zx}^2$.

Figure A.1: Pdf Export Intensity Distribution —Lognormal-Distributed Revenue Shifters



A.1.2 Gamma

Coelho and Mexia (2007) show that the pdf of the ratio of two gamma-distributed random variables, $Z \equiv Z_x/Z_d$ with parameters (α, s_x) and (α, s_d) respectively, is given by:

$$\widetilde{f}(z) = \frac{\left(\frac{s_d}{s_x}\right)^\alpha}{\mathcal{B}(\alpha,\alpha)} \left[1 + \left(\frac{s_d}{s_x}\right) z\right]^{-2\alpha} z^{\alpha-1}, \quad z > 0, \tag{A.9}$$

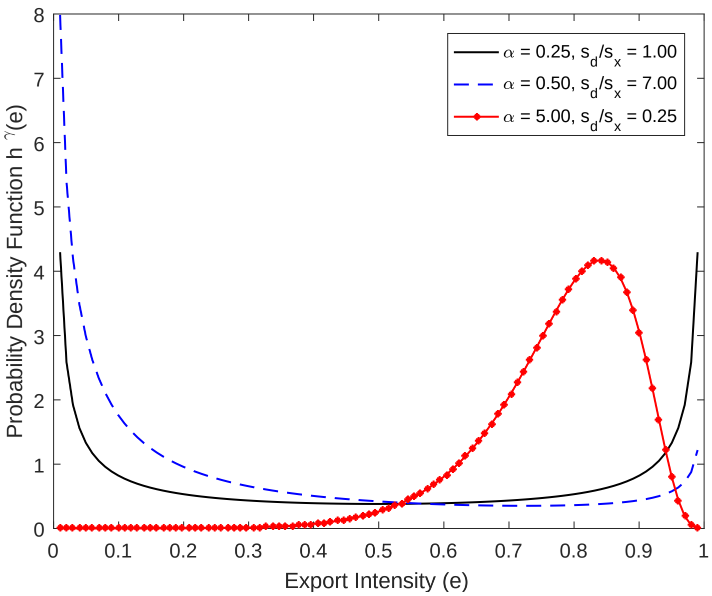
where $\alpha > 0$ denotes the shape parameter, $s_d > 0$ and $s_x > 0$ are respectively, the scale parameters for the domestic and export market, and $\mathcal{B}(\cdot,\cdot)$ is the Beta function.

Substituting (A.9) into (A.5), we obtain (after some simplification):

$$h^{\gamma}(e) = \frac{\left(\frac{s_d}{s_x}\right)^{\alpha}}{\mathcal{B}(\alpha,\alpha)} \times \frac{e^{\alpha-1}(1-e)^{-(1+\alpha)}}{\left[1+\left(\frac{s_d}{s_x}\right)\left(\frac{e}{1-e}\right)\right]^{2\alpha}}, \quad e \in (0,1). \tag{A.10}$$

Figure A.2 presents some examples of the pdf of export intensity when revenue shifters are distributed gamma for different values of the relative scale parameter, s_d/s_x, and the shape parameter of revenue shifters, α.

Figure A.2: Pdf Export Intensity Distribution —Gamma-Distributed Revenue Shifters



A.1.3 Fréchet

Nadarajah and Kotz (2006) show that the cdf of the ratio $Z \equiv Z_x/Z_d$ of two Fréchet distributions with parameters (α, s_x) and (α, s_d), is given by:

$$\widetilde{F}(z) = \frac{\left[\left(\frac{s_d}{s_x}\right)z\right]^{\alpha}}{1+\left[\left(\frac{s_d}{s_x}\right)z\right]^{\alpha}}, \quad z > 0. \tag{A.11}$$

where, again, $\alpha > 0$ denotes the shape parameter and $s_d > 0$ and $s_x > 0$ are the scale parameters for the domestic and export market.

We can derive the cdf for export intensity as follows:

$$
\begin{aligned}
H^{\text{Fréchet}}(e) \equiv \text{Prob}[E \leqslant e] &= \text{Prob}\left[\frac{Z_x}{Z_d + Z_x} \leqslant e\right], \\
&= \text{Prob}\left[\frac{Z_x}{Z_d} \leqslant \frac{e}{1-e}\right], \\
&= \widetilde{F}\left(\frac{e}{1-e}\right), \\
&= \frac{\left[\left(\frac{s_d}{s_x}\right)\left(\frac{e}{1-e}\right)\right]^{\alpha}}{1 + \left[\left(\frac{s_d}{s_x}\right)\left(\frac{e}{1-e}\right)\right]^{\alpha}}, \quad e \in (0,1).
\end{aligned} \tag{A.12}
$$

Taking the derivative of (A.12) yields the pdf of export intensity:

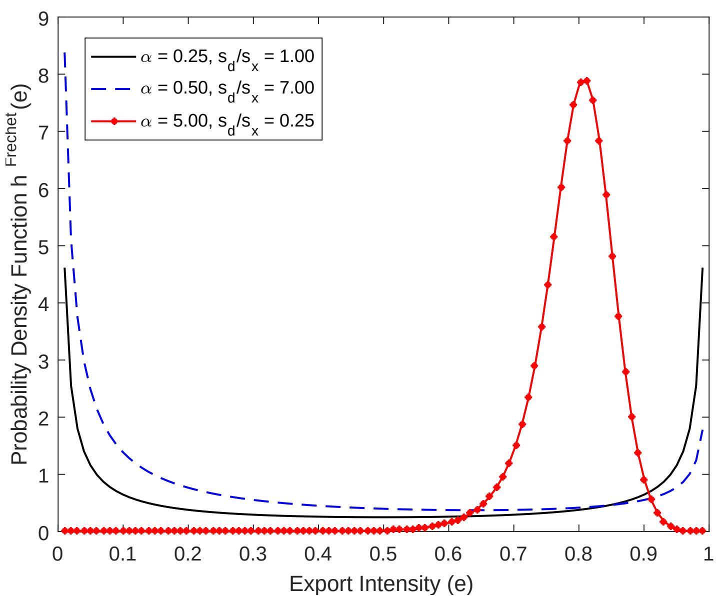
$$
h^{\text{Fréchet}}(e) \equiv \frac{\mathrm{d}H^{\text{Fréchet}}(e)}{\mathrm{d}e} = \frac{\alpha\left(\frac{s_d}{s_x}\right)^{\alpha} \frac{e^{\alpha-1}}{(1-e)^{\alpha+1}}\left[1 + \left[\left(\frac{s_d}{s_x}\right)\left(\frac{e}{1-e}\right)\right]^{\alpha}\right] - \alpha\left(\frac{s_d}{s_x}\right)^{2\alpha} \frac{e^{2\alpha-1}}{(1-e)^{2\alpha+1}}}{\left[1 + \left[\left(\frac{s_d}{s_x}\right)\left(\frac{e}{1-e}\right)\right]^{\alpha}\right]^2},
$$

which, after some simplification, becomes:

$$
h^{\text{Fréchet}}(e) = \alpha\left(\frac{s_d}{s_x}\right)^{\alpha} \times \frac{e^{\alpha-1}(1-e)^{-(1+\alpha)}}{\left[1 + \left(\frac{s_d}{s_x}\right)^{\alpha}\left(\frac{e}{1-e}\right)^{\alpha}\right]^2}, \quad e \in (0,1), \tag{A.13}
$$

Figure A.3 presents some examples of the pdf of export intensity when revenue shifters are distributed Fréchet for different values of the relative scale parameter, s_d/s_x, and the shape parameter of revenue shifters, α.

Figure A.3: Pdf Export Intensity Distribution —Fréchet-Distributed Revenue Shifters



A.2 Proof of Proposition 2.

A.2.1 Lognormal

As shown above, the export intensity follows a Logit-normal distribution when revenue shifters are distributed lognormal. Johnson (1949) characterized the properties of the Logit-normal distribution, which is referred to in the paper as the System S_B of frequency curves (see equation (23), page 158).

The conditions for bimodality of the Logit-normal distribution are stated in equation (26) in page 159. Translating the terms in our equation for the pdf of export intensity (4) to the notation used by Johnson (1949), yields:

$$\delta \equiv \frac{1}{\sqrt{\sigma_{zd}^2 + \sigma_{zx}^2}}, \tag{A.14}$$

$$\gamma \equiv \frac{\ln(s_d/s_x)}{\sqrt{\sigma_{zd}^2 + \sigma_{zx}^2}}. \tag{A.15}$$

Substituting (A.14) and (A.15) in equation (26) of Johnson (1949), yields equations (7) and (8).

The modes of the distribution can be found by taking the derivative of the log of the pdf (4) with respect to e and set it equal to zero. Doing so, reveals that the modes of the export intensity distribution solve:

$$\ln\left(\frac{e}{1-e}\right) = -\ln(s_d/s_x) + \left(\sigma_{zd}^2 + \sigma_{zx}^2\right)(2e-1), \tag{A.16}$$

which does not have a closed-form solution. Figure A.4 plots the two modes of the export intensity distribution as a function of the median export intensity, e^{med}, and $\sigma_{zd}^2 + \sigma_{zx}^2$. The modes are very close to 0 and 1. For a given relative country size, a higher sum of the variance of revenue shifters pushes the modes towards the extremes of the support; taking the dispersion of shifters as given, increasing the median export intensity increases both modes.

Figure A.5 plots the solution to the equation

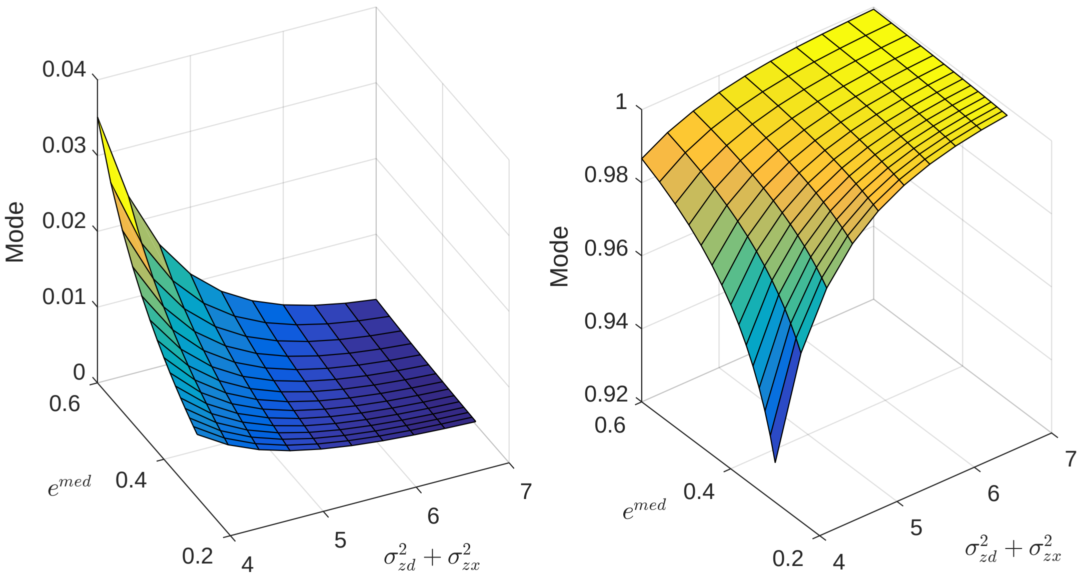
$$\left|\ln(1-e^{med}) - \ln(e^{med})\right| = \left(\sigma_{zd}^2 + \sigma_{zx}^2\right)\sqrt{1 - \frac{2}{\sigma_{zd}^2 + \sigma_{zx}^2}} - 2\tanh^{-1}\left(\sqrt{1 - \frac{2}{\sigma_{zd}^2 + \sigma_{zx}^2}}\right), \tag{A.17}$$

which describes the combination of median export intensity and the sum of the variance of revenue shifters for which the distribution of export intensity is bimodal. Note that we have used Proposition 3 to express (A.17) in terms of the median export intensity rather than in terms of the ratio of scale parameters.

What condition (A.17) tells us is that the minimum sum of variances necessary to induce bimodality in the distribution of export intensity increases when a country's median export intensity gets closer to either 0 or 1. Figure A.5 also includes the country-specific estimated sum of revenue shifter variances reported in columns (1) and (4) of Table C.1. Thus, it follows that all the estimated shape parameters under lognormally-distributed revenue shifters imply a bimodal export intensity distribution.

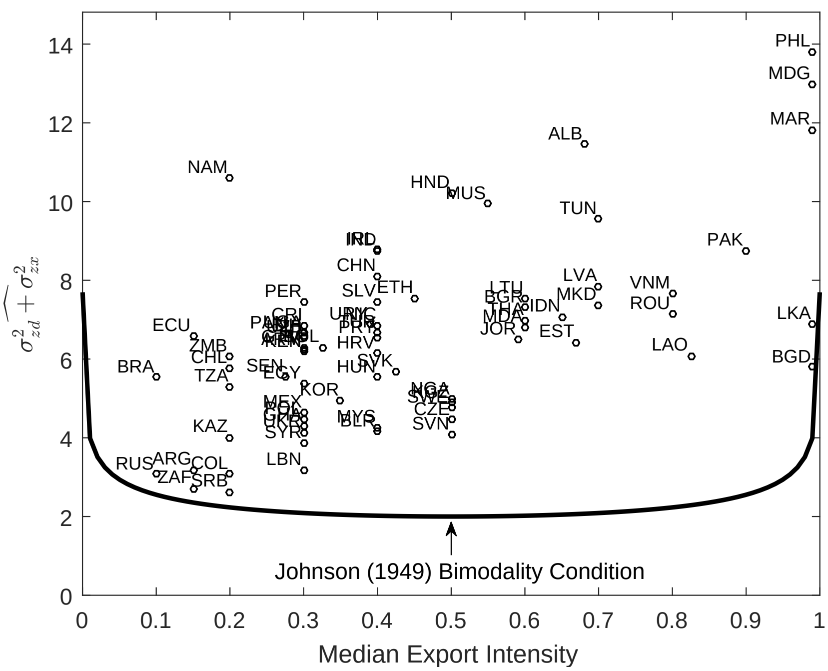
To establish how is the distribution of export intensity affected by changes in relative country

Figure A.4: Export Intensity Modes —Lognormal-distributed Revenue Shifters



The figure plots the two modes of the export intensity distribution (i.e. the solutions to equation (A.16) as a function of the median export intensity (recall that $s_d/s_x = (1 - e^{med})/e^{med}$) and $\sigma^2_{zd} + \sigma^2_{zx}$ when revenue shifters are distributed lognormal.

Figure A.5: Bimodality Condition for Lognormal-Distributed Revenue Shifters



The figure plots the lower bound of the condition for bimodality of the distribution of export intensity stated in equation (8) when revenue shifters are distributed lognormal along with the country-specific estimates of $\sigma^2_{zd} + \sigma^2_{zx}$.

size, we calculate $\frac{\partial h^{\mathcal{LN}}(e)}{\partial(s_d/s_x)}$:

$$\frac{\partial h^{\mathcal{LN}}(e)}{\partial\left(s_d/s_x\right)} = \underbrace{\exp\left[-\frac{\left(\ln\left(\frac{e}{1-e}\right)+\ln(s_d/s_x)\right)^2}{2(\sigma_{zd}^2+\sigma_{zx}^2)}\right]}_{>0} \cdot \frac{-\left[\ln\left(\frac{e}{1-e}\right)+\ln(s_d/s_x)\right]}{\sigma_{zd}^2+\sigma_{zx}^2} \cdot \underbrace{(s_d/s_x)^{-1}}_{>0}. \tag{A.18}$$

The sign of (A.18) is therefore determined by the sign of the second term. Thus, it follows that $\frac{\partial h^{\mathcal{LN}}(e)}{\partial(s_d/s_x)} > 0$ if $e < e^{med}$, and vice versa when $e > e^{med}$. This means that when the size of the domestic market increases relative to the foreign market, the mass of firms with export intensity below the median increases, while the share of high-intensity exporters falls.

A.2.2 Gamma

Rewrite (A.10) as:

$$h^{\gamma}(e) = \frac{(s_d s_x)^{\alpha}}{\mathcal{B}(\alpha,\alpha)} \times \frac{e^{\alpha-1}(1-e)^{\alpha-1}}{\left[s_x(1-e)+s_d e\right]^{2\alpha}}. \tag{A.19}$$

Thus, it follows that when $\alpha < 1$,

$$\lim_{e\to 0} h^{\gamma}(e) = \lim_{e\to 1} h^{\gamma}(e) \to +\infty, \tag{A.20}$$

which proves that the distribution (A.19) has modes at 0 and 1. We then need to verify that when $\alpha < 1$, $h^{\gamma}(e)$ has no additional modes; in other words, we need to show that $h^{\gamma}(e)$ does not have any local maxima in the interior of the support.

We can find the critical points of (A.19) by taking the derivative with respect to e and setting it equal to zero:

$$\frac{\mathrm{d}h^{\gamma}(e)}{\mathrm{d}e} = \frac{e^{\alpha-2}\left[\left(\frac{s_d}{s_x}\right)e+(1-e)\right]^{-(2\alpha+1)}\left[\frac{s_d}{s_x}(1-e)\right]^{\alpha}}{(e-1)^2\mathcal{B}(\alpha,\alpha)} \times \left(Ae^2+Be+C\right) = 0, \tag{A.21}$$

where $A = 2\left(s_d/s_x - 1\right)$, $B = (3-\alpha) - (s_d/s_x)(1+\alpha)$ and $C = \alpha - 1$.

We use the intermediate value theorem to show that only one of the roots of the quadratic polynomial $\mathcal{P}(e) = Ae^2 + Be + C$ lies in the interval $(0,1)$, by showing that $\mathcal{P}(0)\cdot\mathcal{P}(1) < 0$ when $\alpha < 1$:

$$\mathcal{P}(0) = C = \alpha - 1. \tag{A.22}$$

$$\mathcal{P}(1) = A+B+C = 2\left(s_d/s_x - 1\right) + (3-\alpha) - (s_d/s_x)(1+\alpha) + \alpha - 1 = (s_d/s_x)(1-\alpha). \tag{A.23}$$

Since $h^{\gamma}(e)$ is continuous and has two asymptotes at 0 and 1 when $\alpha < 1$, it follows that the critical value in the interior of the interval $(0,1)$ has to be a minimum. Otherwise, if a local maximum (i.e. a third mode) were to exist in the interior of the support, it would be necessary for $h^{\gamma}(e)$ to have at least two critical points in the support. This shows that when $\alpha < 1$, the distribution of export intensity is bimodal.

When $\alpha > 1$, we have $h^{\gamma}(0) = h^{\gamma}(1) = 0$, and still only one critical point in the interior of the support, which shows that the distribution of export intensity is unimodal.

When $\alpha = 1$, $h^\gamma(0) = s_d/s_x$ and $h^\gamma(1) = s_x/s_d$, since $\mathcal{B}(1,1) = 1$. Moreover, since

$$\frac{\mathrm{d}h^\gamma(e)}{\mathrm{d}e} = \frac{(s_d s_x)(s_x - s_d)}{[s_x(1-e) + s_d e]^3}, \tag{A.24}$$

there is a mode at 1 when $s_d/s_x < 1$ because the pdf is strictly increasing; conversely, when $s_d/s_x > 1$ the unique mode is at 0. When $s_d/s_x = 1$, then the distribution of export intensity becomes the uniform distribution, which is considered unimodal.

It is straightforward to see that the pdf $h^\gamma(e)$ is skewed to the left when $s_d/s_x > 1$ (i.e. that for any $e \in (0,1)$, $h^\gamma(e) > h^\gamma(1-e)$), and therefore, that when $\alpha < 1$, the major mode is located at 0. Conversely, when $s_d/s_x < 1$, the pdf is right-skewed, which means that the major mode is located at 1. When $s_d/s_x = 1$, then $h^\gamma(e) = h^\gamma(1-e)$, which means that the pdf is symmetric around 0.5.

A.2.3 Fréchet

We first rewrite (A.13) as:

$$h^{\text{Fréchet}}(e) = \alpha\,(s_d s_x)^\alpha \times \frac{e^{\alpha-1}(1-e)^{\alpha-1}}{\left[s_x^\alpha(1-e)^\alpha + s_d^\alpha e^\alpha\right]^2}, \quad e \in (0,1). \tag{A.25}$$

Thus, just as in the case of the gamma-distributed revenue shifters above, it follows that

$$\lim_{e\to 0} h^{\text{Fréchet}}(e) = \lim_{e\to 1} h^{\text{Fréchet}}(e) \to +\infty, \tag{A.26}$$

when $\alpha < 1$. Which proves that the distribution (A.25) has modes at 0 and 1.

Unlike in the case of gamma-distributed revenue shifters, we cannot prove analytically that there is a unique critical point for the pdf (A.13) in the interior of the support. Instead, we solve numerically the non-linear equation $\frac{dh^{\text{Fréchet}}(e)}{de} = 0$ over a 100×100 grid for the shape and scale parameter. For each pair of parameters we solve the first-order condition using 500 different starting values in the interval $(0,1)$. We find that when $\alpha < 1$, we always converge to the same solution (a minimum), regardless of the starting value. This exercise suggests that $h^{\text{Fréchet}}(e)$ has no additional modes other than 0 and 1.

A.3 Proof of Proposition 3.

A.3.1 Lognormal

Since the ratio of two independent lognormal random variables is itself a lognormal random variable, it follows that the median of the ratio Z_x/Z_d is $z^{med} = \exp[\ln(s_x/s_d)] = s_x/s_d$.

We then use the fact that the median of a monotone transformation of a random variable is equal to the transformation of the median. That is, if E and Z are random variables and $E = j(Z)$, with $j(\cdot)$ being a monotone function, then $e^{med} = j\left(z^{med}\right)$. Since $E = j(Z) = \frac{Z}{1+Z}$, it follows that:

$$e^{med} = \frac{\frac{s_x}{s_d}}{1 + \frac{s_x}{s_d}} = \frac{s_x}{s_d + s_x}. \tag{A.27}$$

A.3.2 Gamma

We use the result that the ratio of two independent gamma-distributed random variables $Z_d \sim \Gamma(\alpha, s_d)$ and $Z_x \sim \Gamma(\alpha, s_x)$ can be expressed in terms of the F distribution (Johnson et al., 1995). Namely,

$$\frac{\alpha s_d}{\alpha s_x} \cdot \frac{Z_x}{Z_d} \sim F(2\alpha, 2\alpha). \tag{A.28}$$

Since the median of a random variable distributed F with the same number of degrees of freedom in the numerator and denominator is 1, then it follows from (A.28) that the median of the ratio Z_x/Z_d, z^{med}, is equal to s_d/s_x. Using again the result that the median of a monotone transformation of a random variable is equal to the transformation of the median again, we obtain $e^{med} = \frac{s_x}{s_d+s_x}$.

A.3.3 Fréchet

Using (A.12), we can easily find the median export intensity by solving the equation $H^{\text{Fréchet}}(e^{med}) = 0.5$:

$$\frac{\left[\left(\frac{s_d}{s_x}\right)\left(\frac{e^{med}}{1-e^{med}}\right)\right]^{\alpha}}{1 + \left[\left(\frac{s_d}{s_x}\right)\left(\frac{e^{med}}{1-e^{med}}\right)\right]^{\alpha}} = 0.5, \tag{A.29}$$

which results in $e^{med} = \frac{s_x}{s_d+s_x}$.

B Calculating the Volatility of Sales among Exporters

Let $r_t(\omega)$ denote total sales for firm ω, and $g(r_t(\omega)) \equiv \ln r_t(\omega) - \ln r_{t-1}(\omega) = \Delta \ln r_t(\omega)$ its growth rate. More specifically,

$$g(r_t) = \Delta \ln \left[\sum_{i \in \{d,x\}} \Phi(\omega) \cdot s_i \cdot z_i(\omega) \cdot \epsilon_{it} \right]. \tag{B.1}$$

Recall that we have assumed that temporary demand shocks, ϵ_{it}, are such that $\mathbb{E}(\epsilon_{it}) = 1$ and $\text{Var}(\epsilon_{it}) = \nu_i^2$. Thus, let $\epsilon \equiv [\epsilon_{dt} \;\; \epsilon_{dt-1} \;\; \epsilon_{xt} \;\; \epsilon_{xt-1}]$, and $\bar{\epsilon} = [1 \;\; 1 \;\; 1 \;\; 1]'$.

Following Vannoorenberghe (2012), we use the Delta method to approximate the volatility of the growth rate of total sales by:

$$\text{Vol}(g_t) \approx [\nabla g_t(\bar{\epsilon})]' \cdot V(\epsilon) \cdot [\nabla g_t(\bar{\epsilon})], \tag{B.2}$$

where $\nabla g_t(\bar{\epsilon}) \equiv \left[\frac{\partial g_t}{\partial \epsilon_{dt}}\Big|_{\epsilon=\bar{\epsilon}} \;\; \frac{\partial g_t}{\partial \epsilon_{dt-1}}\Big|_{\epsilon=\bar{\epsilon}} \;\; \frac{\partial g_t}{\partial \epsilon_{xt}}\Big|_{\epsilon=\bar{\epsilon}} \;\; \frac{\partial g_t}{\partial \epsilon_{xt-1}}\Big|_{\epsilon=\bar{\epsilon}} \right]'$ and $V(\epsilon) = \begin{pmatrix} \nu_d^2 & 0 & 0 & 0 \\ 0 & \nu_d^2 & 0 & 0 \\ 0 & 0 & \nu_x^2 & 0 \\ 0 & 0 & 0 & \nu_x^2 \end{pmatrix}$.

Replacing

$$\begin{aligned}
\frac{\partial g_t}{\partial \epsilon_{dt}} &= \frac{A_d z_d \epsilon_{dt}}{A_d z_d \epsilon_{dt} + \tau^{1-\sigma} A_x z_x \epsilon_{xt}}\Bigg|_{\epsilon=\bar{\epsilon}} = \frac{Z_d}{Z_d + Z_x} = 1 - E, \\
\frac{\partial g_t}{\partial \epsilon_{dt-1}} &= -\frac{A_d z_d \epsilon_{dt-1}}{A_d z_d \epsilon_{dt-1} + \tau^{1-\sigma} A_x z_x \epsilon_{xt-1}}\Bigg|_{\epsilon=\bar{\epsilon}} = -\frac{Z_d}{Z_d + Z_x} = -(1 - E), \\
\frac{\partial g_t}{\partial \epsilon_{xt}} &= \frac{A_x z_x \epsilon_{xt}}{A_d z_d \epsilon_{dt} + \tau^{1-\sigma} A_x z_x \epsilon_{xt}}\Bigg|_{\epsilon=\bar{\epsilon}} = \frac{Z_x}{Z_d + Z_x} = E, \\
\frac{\partial g_t}{\partial \epsilon_{xt-1}} &= -\frac{A_x z_x \epsilon_{xt-1}}{A_d z_d \epsilon_{dt-1} + \tau^{1-\sigma} A_x z_x \epsilon_{xt-1}}\Bigg|_{\epsilon=\bar{\epsilon}} = -\frac{Z_x}{Z_d + Z_x} = -E.
\end{aligned}$$

in (B.2) yields:

$$\text{Vol}(g_t) \approx 2 \left[(1 - E)^2 \nu_d^2 + E^2 \nu_x^2 \right], \tag{B.3}$$

which is minimized at the level of export intensity $E^* = \frac{\nu_d^2}{\nu_d^2 + \nu_x^2}$.

C Country-Specific Estimates of Shape Parameters

Table C.1: Country-Specific Estimates of Shape Parameter

Distribution:	Lognormal	Gamma	Fréchet	Distribution:	Lognormal	Gamma	Fréchet
Parameter:	$\sigma_{zd}^2 + \sigma_{zx}^2$	α	α	Parameter:	$\sigma_{zd}^2 + \sigma_{zx}^2$	α	α
Country:	(1)	(2)	(3)	Country:	(4)	(5)	(6)
Albania	11.444	0.375	0.469	Lithuania	7.541	0.521	0.603
Argentina	3.191	1.047	1.042	Madagascar	12.988	0.441	0.527
Armenia	6.226	0.648	0.727	Malaysia	4.264	0.843	0.893
Bangladesh	5.788	0.878	1.062	Mauritius	9.933	0.426	0.515
Belarus	4.163	0.825	0.865	Mexico	4.643	0.798	0.855
Bolivia	6.299	0.622	0.698	Moldova	6.806	0.561	0.639
Bosnia & Herzegovina	6.584	0.616	0.695	Morocco	11.823	0.501	0.597
Brazil	5.531	0.765	0.847	Namibia	10.602	0.447	0.535
Bulgaria	7.301	0.539	0.619	Nicaragua	6.861	0.599	0.680
Chile	5.765	0.673	0.744	Nigeria	4.970	0.766	0.834
China	8.102	0.524	0.609	Pakistan	8.750	0.457	0.548
Colombia	3.079	1.057	1.041	Panama	6.647	0.625	0.707
Costa Rica	6.847	0.598	0.678	Paraguay	6.536	0.595	0.672
Croatia	6.142	0.631	0.706	Peru	7.447	0.561	0.644
Czech Rep.	4.451	0.790	0.838	Philippines	13.811	0.415	0.497
Ecuador	6.580	0.638	0.721	Poland	4.470	0.815	0.867
Egypt	5.380	0.731	0.804	Romania	7.148	0.541	0.622
El Salvador	7.465	0.561	0.644	Russian Fed.	3.067	1.121	1.108
Estonia	6.427	0.588	0.663	Senegal	5.555	0.712	0.786
Ethiopia	7.553	0.541	0.623	Serbia	2.630	1.194	1.148
FYR Macedonia	7.369	0.516	0.598	Slovak Rep.	5.684	0.662	0.732
Ghana	4.307	0.878	0.937	Slovenia	4.082	0.810	0.843
Guatemala	6.285	0.663	0.746	South Africa	2.716	1.211	1.169
Honduras	10.209	0.428	0.515	Sri Lanka	6.892	0.732	0.860
Hungary	5.535	0.678	0.746	Sweden	4.765	0.735	0.789
India	8.760	0.498	0.584	Syrian Arab Rep.	3.871	0.929	0.966
Indonesia	7.072	0.553	0.632	Tanzania	5.299	0.751	0.824
Ireland	8.778	0.485	0.571	Thailand	6.985	0.562	0.641
Jordan	6.490	0.589	0.665	Tunisia	9.552	0.424	0.515
Kazakhstan	4.013	0.920	0.970	Turkey	6.668	0.607	0.686
Kenya	6.181	0.671	0.753	Uganda	6.661	0.630	0.713
Korea, Rep.	4.948	0.728	0.786	Ukraine	4.145	0.881	0.929
Kyrgyz Rep.	4.881	0.742	0.800	Uruguay	6.874	0.575	0.654
Lao PDR	6.087	0.604	0.676	Uzbekistan	6.548	0.620	0.699
Latvia	7.852	0.505	0.589	Vietnam	7.663	0.507	0.591
Lebanon	3.175	0.997	0.986	Zambia	6.066	0.717	0.808

Coefficients estimated by maximum likelihood, conditional on s_d/s_x being given by equation (10). All estimated shape parameters are statistically different from 0 at the 1% level.

D Vuong (1989) Test for Selection of Non-Nested Models

In this section we briefly describe how the Vuong (1989) test statistic is calculated and we present our results in Table D.1. Below we briefly describe how the test statistic is

Consider two competing parametric models of the distribution of export intensity indexed by $k = 1, 2$. Let $h^k\left(e|\theta^k, s_d/s_x\right)$ denote the density of model k conditional on the shape parameter θ^k (estimated by maximum likelihood) and s_d/s_x respectively, where the notation reflects the fact that the scale parameter is the same across all models, and equal to $1/e^{med} - 1$, as shown in Proposition 3.

Vuong (1989) shows that the LR test statistic

$$\Psi = \frac{\sum_{i=1}^{n}\left[\ln\left(h^1\left(e_i|\theta^1, s_d/s_x\right)\right) - \ln\left(h^2\left(e_i|\theta^2, s_d/s_x\right)\right)\right]}{\sqrt{n\widehat{\omega}^2}} \xrightarrow{d} \mathcal{N}(0,1), \tag{D.1}$$

where $\widehat{\omega}^2 \equiv \frac{1}{n}\sum_{i=1}^{n}\left[\ln\frac{h^1\left(e_i|\theta^1,s_d/s_x\right)}{h^2\left(e_i|\theta^2,s_d/s_x\right)}\right]^2 - \left[\frac{1}{n}\sum_{i=1}^{n}\ln\frac{h^1\left(e_i|\theta^1,s_d/s_x\right)}{h^2\left(e_i|\theta^2,s_d/s_x\right)}\right]^2$, is the variance of the limiting normal distribution of the LR statistic, and n denotes the sample size.

Thus, for a given critical value c for the standard normal distribution (e.g. $c = 1.96$ at the 5% significance level, or, $c = 2.58$ at the 1% level), the directional test for model selection recommends:

- If $\Psi > c$, reject the null hypothesis that models 1 and 2 are equivalent in favor of $h^1\left(e|\theta^1, s_d/s_x\right)$ being better than $h^2\left(e|\theta^2, s_d/s_x\right)$.
- If $\Psi < -c$, reject the null hypothesis that models 1 and 2 are equivalent in favor of $h^2\left(e|\theta^2, s_d/s_x\right)$ being better than $h^2\left(e|\theta^2, s_d/s_x\right)$.
- If $|\Psi| \leqslant c$, then one cannot discriminate between the two competing models given the data.

We use the Vuong test statistic in two ways:

1. We compare the performance of the three different parametric models that we propose in the paper (lognormal, gamma, Fréchet) to fit the export intensity data across countries.
2. For each distribution of revenue shifters, we compare the performance a model in which the shape parameter is estimated for each country individually against a 'restricted' model with a single shape parameter estimated by pooling data for all countries. The latter model is indexed by the subscript r in Table D.1.

Table D.1: Vuong (1989) Test Results

Model 1:	$\mathcal{LN}$	$\mathcal{LN}$	Γ	$\mathcal{LN}$	Γ	Fréchet	Model 1:	$\mathcal{LN}$	$\mathcal{LN}$	Γ	$\mathcal{LN}$	Γ	Fréchet
Model 2:	Γ	Fréchet	Fréchet	$\mathcal{LN}_r$	Γ_r	Fréchet$_r$	Model 2:	Γ	Fréchet	Fréchet	$\mathcal{LN}_r$	Γ_r	Fréchet$_r$
Country:	(1)	(2)	(3)	(4)	(5)	(6)	Country:	(7)	(8)	(9)	(10)	(11)	(12)
Albania	**15.87**	**27.67**	**-11.29**	**4.78**	**5.69**	**5.93**	Lithuania	**9.46**	**13.47**	**-6.42**	1.26	1.99	2.15
Argentina	**-5.48**	**-5.44**	-1.29	**6.83**	**7.58**	**7.94**	Madagascar	**-3.97**	-2.54	**6.54**	**3.01**	2.14	1.93
Armenia	-0.95	-0.87	1.12	0.18	0.25	0.25	Malaysia	**-4.34**	**-4.04**	**3.48**	**3.77**	**3.71**	**3.70**
Bangladesh	**-21.44**	**-11.62**	0.47	0.76	**3.16**	**3.68**	Mauritius	**11.59**	**18.73**	**-8.08**	**3.43**	**4.03**	**4.17**
Belarus	0.79	0.76	-0.89	2.06	1.77	1.70	Mexico	**-4.47**	**-4.51**	**2.73**	**2.86**	**2.99**	**3.05**
Bolivia	1.95	1.93	-1.98	0.17	0.01	0.06	Moldova	**7.47**	**10.35**	**-4.93**	0.36	1.01	1.16
Bosnia & Herzegovina	0.25	0.10	-0.58	0.07	0.09	0.09	Morocco	**-9.96**	**-5.96**	**18.95**	**3.42**	1.76	1.34
Brazil	**-14.05**	**-12.56**	**4.13**	1.12	2.19	2.49	Namibia	1.74	1.95	-1.44	1.89	1.81	1.79
Bulgaria	**10.49**	**14.41**	**-7.03**	1.35	2.16	2.33	Nicaragua	0.36	0.40	-0.25	0.33	0.33	0.33
Chile	-0.13	-0.76	-1.74	1.26	1.18	1.17	Nigeria	**-3.45**	**-2.72**	**5.32**	1.58	1.61	1.62
China	**7.07**	**7.96**	**-5.66**	**3.32**	**3.52**	**3.59**	Pakistan	**27.39**	**21.37**	**-27.51**	**3.63**	**6.12**	**6.64**
Colombia	**-2.88**	**-2.80**	-0.36	**6.01**	**6.41**	**6.65**	Panama	-1.11	-0.92	1.58	0.09	0.02	0.04
Costa Rica	0.68	0.60	-0.83	0.29	0.33	0.33	Paraguay	**4.11**	**4.72**	**-3.25**	0.05	0.41	0.50
Croatia	2.52	**2.67**	-2.19	0.39	0.13	0.05	Peru	**2.71**	**2.88**	-2.37	1.35	1.41	1.43
Czech Rep.	0.87	0.81	-1.07	2.18	1.86	1.80	Philippines	**-5.03**	**-2.58**	**8.72**	**6.68**	**5.02**	**4.65**
Ecuador	**-3.43**	**-3.44**	2.52	0.08	0.22	0.27	Poland	**-2.88**	**-2.95**	1.77	2.52	2.56	2.59
Egypt	**-5.54**	**-4.86**	**5.45**	1.52	1.80	1.87	Romania	**8.90**	**10.55**	**-6.69**	0.80	1.61	1.82
El Salvador	**2.61**	**3.15**	-1.57	1.54	1.54	1.54	Russian Fed.	**-4.87**	**-4.92**	-2.00	**3.88**	**4.85**	**5.30**
Estonia	**5.53**	**7.01**	**-3.80**	0.06	0.48	0.59	Senegal	-2.24	-2.10	1.96	0.59	0.72	0.74
Ethiopia	**2.95**	**3.60**	-2.17	0.70	0.86	0.90	Serbia	**-2.65**	**-2.60**	-1.56	**5.09**	**5.20**	**5.29**
FYR Macedonia	**11.97**	**19.47**	**-7.63**	1.09	2.10	2.31	Slovak Rep.	2.08	2.12	-1.98	0.66	0.42	0.36
Ghana	**-5.12**	**-4.27**	1.00	1.44	1.70	1.78	Slovenia	**5.02**	**5.68**	**-2.82**	**3.29**	2.50	2.32
Guatemala	**-5.36**	**-4.22**	**7.37**	0.24	0.65	0.74	South Africa	**-5.23**	**-5.21**	**-2.94**	**5.24**	**6.12**	**6.61**
Honduras	**11.12**	**16.85**	**-7.77**	**4.45**	**4.81**	**4.90**	Sri Lanka	**-11.22**	**-6.56**	**3.20**	0.26	0.96	1.26
Hungary	1.85	1.93	-1.65	1.06	0.77	0.70	Sweden	**3.93**	**4.40**	**-2.79**	2.24	1.61	1.46
India	**9.07**	**11.11**	**-6.31**	**5.70**	**5.63**	**5.66**	Syrian Arab Rep.	**-5.13**	**-4.93**	0.70	2.33	**2.60**	**2.73**
Indonesia	**10.55**	**14.13**	**-6.98**	1.12	2.02	2.21	Tanzania	**-4.53**	**-4.27**	2.36	0.86	1.16	1.24
Ireland	**3.69**	**4.72**	**-2.75**	1.32	1.48	1.52	Thailand	**10.94**	**13.77**	**-7.55**	1.11	2.01	2.22
Jordan	**7.03**	**8.97**	**-4.79**	0.00	0.64	0.78	Tunisia	**19.36**	**33.15**	**-13.18**	**4.24**	**5.57**	**5.86**
Kazakhstan	**-3.45**	**-3.17**	0.47	1.41	1.65	1.72	Turkey	2.31	**2.80**	-1.17	0.47	0.60	0.62
Kenya	**-4.90**	**-3.96**	**6.40**	0.34	0.71	0.79	Uganda	-2.11	-1.71	**3.07**	0.13	0.07	0.12
Korea, Rep.	1.25	1.18	-1.37	1.04	0.83	0.78	Ukraine	**-6.04**	**-5.66**	1.74	**2.74**	**3.03**	**3.14**
Kyrgyz Rep.	0.77	0.75	-0.85	1.22	1.01	0.96	Uruguay	**6.11**	**7.07**	**-4.85**	0.52	1.01	1.12
Lao PDR	**5.25**	**6.14**	**-3.72**	0.35	0.23	0.37	Uzbekistan	-0.02	-0.03	-0.02	0.03	0.03	0.03
Latvia	**7.76**	**10.60**	**-5.44**	1.27	1.88	2.02	Vietnam	**25.02**	**30.95**	**-18.15**	**3.18**	**5.44**	**5.97**
Lebanon	0.68	0.67	-0.17	**4.25**	**3.96**	**3.92**	Zambia	**-5.29**	**-3.61**	**3.21**	0.22	0.62	0.73

The table reports the Vuong (1989) test statistic Ψ defined in (D.1) for different pairs of models: lognormal ($\mathcal{LN}$), gamma (Γ) and Fréchet. Entries in bold are those for which $|\Psi| > 2.58$, the critical value for the test at the 1% significance level. A positive value for Ψ greater than 2.58 implies that Model 1 fits the data better than Model 2. A negative value of Ψ below 2.58 implies the opposite. We cannot discriminate between the two models when $|\Psi| \leqslant 2.58$. The subscript r indicates a model in which a single shape parameter is estimated pooling data for all countries in our sample.