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A Model of Collateral

Abstract

This paper presents a simple equilibrium model in which collateralized credit emerges endogenously. Just like in repos, individuals cannot commit to the use of collateral as a guarantee of repayment, and both lenders and borrowers have incentives to renege. Our theory provides a micro-foundation to justify the borrowing constraints that are widely used in the existing macroeconomic models. We provide an explanation to the question of why assets are often used as collateral, rather than simply as a means of payment, why there is a tradeoff in assets between return and liquidity, and what kinds of assets are useful as collateral.

JEL-Codes: E300, E500, C730.

Keywords: collateral, search, medium of exchange, voluntary separable repeated game.

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1 Introduction

If you repay me not on such a day,
In such a place, such sum or sums as are
Express'd in the condition, let the forfeit
Be nominated for an equal pound
Of your fair flesh, to be cut off and taken
In what part of your body pleaseth me. *(1.3.156-163, The Merchant of Venice.)*

Collateral is the linchpin of credit and intertemporal resource allocations. This paper proposes a micro-founded model where secured credit emerges endogenously. Even in a frictional world, where commitment is limited, we show that collateral can serve as a credible device that prevents the participating parties from reneging.

It is commonly believed that assets with higher value are more valuable as collateral. Funds through loans could be more easily raised by pledging an asset with higher market value, e.g. a quality apartment in a rich neighbourhood, than a small mountain hut in a local village. However, credit can occur even with an object intrinsically worthless to lenders, e.g. a pound of Antonio's flesh to Shylock. Why are lenders willing to accept an object as collateral that has no private nor market value? In other words, what is the rationale for the practice that a bar manager waits for the payment of an anonymous first comer, e.g. a traveller, who offers her to keep one of his belongings, which is worthless for her, till he picks up cash and comes back?

Our starting point is to realize that credit is part of long-term relationships. In our theory, a borrower can choose to escape from a current lender and get into a relationship with a new one. However, if a borrower does not have pledgeable assets, it is indicated that he has reneged and his collateral has been confiscated in the past. Thus, current and future lenders can punish such a borrower—*collateral is memory.* The maximum payment a borrower can promise in equilibrium is determined not only by the return of the pledged asset, just like in the seminal work by Kiyotaki and Moore [16], but also by the value of future transactions.

Using this framework, we address the "payment puzzle" raised by Lagos [20]. Namely, why even to this day assets are used as collateral rather than simply as a means of payment. Our explanation is based on the role of collateral as memory. In our equilibrium with collateralized credit, the lender knows that the borrower will redeem his debt and get backhis pledged asset because it is a valuable pass for him to enter future transactions and he does not want to lose it. Hence, even with a poor means of payment, the borrower is able to make a credible promise, backed by future productivity, that he will never renege and so the lender agrees to trade with him.

While the above logic is driven solely by the borrower's incentive, we find that the lender's lack of commitment is also important for generalizing our insight especially for the following issues. First, a repurchase agreement (repo) is a short-term borrowing where a dealer (borrower) sells government securities to an investor (lender) and receives money, usually on an overnight basis, and buys the securities back the following day. Despite the similarities to collaterialized loans, repos are actual purchases. During the life of a repo, the investor holds legal title to the securities. The investor can renege and keep owning the securities, rather than returning them back to the original owner and getting their money back with some interests. Our theory captures this feature of repos with the setup that any individual cannot commit to the use of collateral as a guarantee of repayment, and so both dealers and investors have incentives to renege. Our setup allows for rehypothecation as well. Then, the payment puzzle still applies: why not settle the payment by using the securities, i.e, selling off the securities, rather than buying it back? In contrast to the borrower (dealer)'s deviation described above, which works in favor of repos over spot trade, the lender (investor)'s incentive makes repos harder to sustain, unless her action is observable to the future market, especially when the borrower's pledged asset has high returns.

Finally, many macroeconomic models consider cases in which some assets have high returns but low liquidity, and it is shown that such a case delivers interesting macroeconomic implications regarding market liquidity (see e.g., Matsuyama [22, 23], and Lagos and Rocheteau [21]). As a complementary effort to this line of works, we explore the very reason why this phenomenon can occur in the first place—why does an asset with high return, which should be attractive to many market participants, fail to deliver high liquidity? Our answer is that an asset with very high return is problematic because the lender has a very strong incentive to default and run away with it. Hence, an asset with high return can poorly back credit trades, only generating low liquidity.

Before closing this introductory section, it is worth comparing our paper with the existing literature. In their influential work, Kiyotaki and Moore [16] emphasize the importance of the borrowing constraint associated with collateralized credit. The debt limit is determined by the maximum payment the borrower can promise. In their model, given that the borrower cannot sell off his future labor to guarantee his debts, it is bounded by the value of the asset pledged as collateral, which is confiscated in case of default. We generalize their insight by allowing agents to form long-term relationships. In our model, with secured credit being part of long-term relationships, the punishment involves not only seizing assets pledged as collateral but also taking away defaulters' future credit.

The role of collateral as memory in our equilibrium is related to the role of money in monetary models (see Kiyotaki and Wright [17, 18]), in which an intrinsically worthless object—fiat money—has a positive equilibrium value because it provides partial information on whether an individual has worked in the past or not, i.e., *money is memory* (Kocherlakota [19]). Other related papers include Kehoe and Levine [15] and Gu, Mattesini, Monnet and Wright [12, 13], who study the issue of commitment, Albuquerque and Hopenhayn [1], who study some related issues using a dynamic contract approach, and Ferraris and Watanabe [6, 7], who study a monetary equilibrium with collateralized credit.

In terms of modeling technology, our model environment is a version of *voluntary separable repeated games*, originated in Datta [4] and Ghosh and Ray [10]. These papers consider environments in which players engage in long-term relationships. Players, unlike in standard repeated games, can choose to escape from current partners and find new ones. Limited commitment hence applies not only to actions within matches but also to partnerships themselves. Notice that the frictions are less severe than the environments considered by random matching models of money where players *have to* separate after a match (See, however, Corbae and Ritter [3]. In the literature of repeated games, community enforcement models—like random matching models of money—consider environments where players have to separate after a match. For these papers, see, for example, Kandori [14], Takahashi [24] and Awaya [2].) Among them, Datta [4], Ghosh and Ray [10] and Fujiwara-Greve and Okuno-Fujiwara [8] propose *strategies* that work under such frictions. Eeckhout [5] and Fujiwara-Greve, Okuno-Fujiwara and Suzuki [9] consider *institutions* that facilitate cooperation under such frictions. Eeckhout [5] considers *color*—a payoff irrelevant and history independent characteristic, and Fujiwara-Greve, Okuno-Fujiwara and Suzuki [9] consider *referral letters* as methods to convey partial information on past actions. In our paper, we show that another institution—collateral—works in the presence of such frictions.

The rest of the paper is organized as follows. Section 2 presents the basic setup and studies the equilibrium without collateral. Section 3 describes the equilibrium with collateralized credit. Section 4 derives the three macroeconomic implications. Section 5 considers extensions of the model. Section 6 concludes.

2 Baseline environment

We first examine a benchmark environment *without* any collateral object. The physical environment is based on a voluntarily separable repeated game of Datta [4] and Ghosh and Ray [10].

Time is discrete and lasts forever. It is indexed by $t = 1, 2, \cdots$. There is a continuum of individuals. Each individual is either a *borrower* or a *lender*.[1] The measures of both parties are unity. All individuals are long-lived and have a common discount factor $\delta \in (0, 1)$.

In this benchmark model, there are two kinds of goods—durable *production* and perishable *consumption* goods. In Section 3, we will introduce another kind of good into the economy, a good used as collateral. Both production and consumption goods are perfectly divisible. Each lender owns one unit of the production good. In each period, a lender (resp. a borrower) can costlessly produce one unit (resp. a units) of the consumption good by using one unit of the production good. Assume $a > 1$ so that borrowers have a better production technology. Individuals can only consume the consumption good. It is impossible to produce the production good.

In each period, a borrower and a lender engage in a pairwise trade. At the start of each period, some borrowers and lenders are unmatched. Each unmatched borrower (resp. lender) will find a lender (resp. borrower) from the set of unmatched lenders (resp. borrowers) at random. For simplicity, we assume that every unmatched individual can find his or her counterpart with probability one.[2]

Each period is divided into three subperiods in the following manner.

Subperiod 1 In a pair, the lender can lend the right to use of the production good to the borrower. She chooses what portion of it to lend. Let $q \in [0, 1]$ denote the portion lent

[1] In a repo, borrowers (resp. lenders) are also referred to as sellers (resp. buyers).

[2] Our results survive when there is a small chance that an unmatched individual cannot find a partner.

to borrower. The lender uses the rest $1-q$ by herself.

Subperiod 2 The borrower then produces aq units of the consumption good by using q units of the production good and chooses how much consumption good to give to the lender. Let $r \in [0, aq]$ denote the amount of consumption good given to lender.

Subperiod 3 Individuals simultaneously choose whether to continue or terminate the relationship. They separate if either individual chooses to terminate. Also, a fraction $1-\rho$ of the matches exogenously dissolve, even if they agree to continue.

The per-period payoff is linear in consumption. Given (q, r), the per-period payoff of a lender is $r + 1 - q$, while that of a borrower is $aq - r$. Obviously, the unique Nash equilibrium in the corresponding one-shot game is $(q, r) = (0, 0)$, i.e., no trade, while any efficient allocation must satisfy $q = 1$ (i.e., full lending). We assume that pairwise trades are the only possible opportunity to trade goods. In particular, there is no centralized market for the consumption good.

Importantly, the history of past actions is not public. Each individual only observes her partners' actions, but does not observe the past actions of any other individual. The equilibrium notion is *perfect Bayesian equilibrium* (simply, *equilibrium* henceforth). The lifetime payoff is

$$(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}c_t$$

where c_t is the consumption in period t.

The following proposition is a version of Datta's [4] result in our setup.

Proposition 1. *There is no equilibrium in which (almost) all lenders choose $q = 1$ on the equilibrium path.*

Proof. Suppose, by way of contradiction, that there were such an equilibrium. Then a positive fraction of borrowers must choose $r \geq 1$ with a positive probability because, otherwise, $q = 1$ could never be optimal for the lender. It would be profitable for such a borrower to choose $r = 0$ and then separate—a new lender would not know that the borrower had deviated, and so she would choose $q = 1$ against the borrower. □

3 Collateral

3.1 Durable good

We now introduce another kind of good into the economy. We will show that individuals can use it as collateral—in particular, they can arrange repurchase agreement using it,— and that it improves efficiency. The good—called *durable* good—is durable and indivisible. At start of the first period, each borrower has one unit of such a durable good. Importantly, we assume that the durable good cannot be produced.[3]

We keep the assumption that all trades must be made within each pair. In particular, it is impossible to transfer the durable good to outside the pair. Specifically, a pair can trade the durable good in the first and second subperiods. Let $g_b \in \{not\ give, give\}$ denote the borrower's action, where $g_b = give$ (resp. $g_b = not\ give$) denotes giving (resp. not giving) the durable good to the lender. And, $g_\ell \in \{not\ give, give\}$ is defined likewise as the lender's action.

In the third subperiod, the owner of the durable good can destroy it. If he or she does not destroy it, he or she obtains "utility" flow $y \in \mathbb{R}$ from holding it. We emphasize that we do *not* restrict y to be positive. If $y = 0$, the good is intrinsically useless, and if $y < 0$, the good is costly to hold. We call y the *value* of a durable good, because if an individual holds the durable good forever, he or she obtains lifetime utility $y = (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} y$.

With such a durable good, consider the following *repurchase agreement (repo) strategy*: In each period, in the first subperiod the borrower "sells" the durable good in exchange for the right to use of the production good, and in the second subperiod, he "buys back" the durable good by paying the consumption good.

3.2 Repurchase agreement strategy

Formally, define the repo strategy as follows:

Subperiod 1 If the borrower has the durable good, then trade it for the right to use of the production good, that is, $(q, g_b) = (1, give)$.

Otherwise, make no trade, that is, $(q, g_b) = (0, not\ give)$.

[3]Our results survive when the durable good can be costly produced and the cost of production is sufficiently large. On the other hand, if the cost of production is too small, our results fail to hold.

Subperiod 2 If the trade took place in the first subperiod, then trade the durable good for the consumption good, that is, $(r, g_\ell) = (r^*, give)$, where $r^* \in [0, a]$ will be specified later.

Otherwise, make no trade, that is, $(r, g_\ell) = (0, not\ give)$.

Subperiod 3 If the trades took place in the previous two subperiods, then keep the relationship. The borrower keeps the durable good.

Otherwise, separate. If the borrower has the durable good, then he keeps it. If the lender has it, then she keeps it if and only if $y > 0$.

Following the convention of exchange, the transfer of goods within a subperiod occurs at the same time. For example, in the first subperiod, once a borrower and a lender agree with the trade $(q, g_b) = (1, give)$, the lender cannot escape without giving the right to use of the production good after receiving the durable good from the borrower. The same in the second subperiod.

Notice, however, that a lender can, if she wants to, escape with collateral. In Section 5.1, we consider the case where lenders cannot, and we compare their results and implications.

By construction, clearly, the outcome of the repo strategy is efficient. In the following proposition, we provide a necessary and sufficient condition for which the repo strategy constitutes an equilibrium. In particular, we show that for any (δ, a), there is a non-empty open set of the value y of the durable good in which the repo strategy constitutes an equilibrium.

Theorem 1. *The repo strategy constitutes an equilibrium if and only if*

$$y \in [1 - \delta a, (1 - \delta)a] \tag{1}$$

Notice that $(1 - \delta)a - (1 - \delta a) = a - 1 > 0$. Thus, for any (δ, a), there is an open set of y in which the repo strategy constitutes an equilibrium. Notice also that it does not have to be the case that $1 - \delta a > 0$. When $\delta a > 1$, the repo strategy constitutes an equilibrium even when $y < 0$.

Proof. First, consider a borrower's incentives. Observe that the most profitable deviation is to refuse the trade in the second subperiod. To see this, notice that the borrower is indifferent between continuing and separating.[4] Also, in the first subperiod, it is not optimal for the

[4] If we introduce a small probability that an unmatched individual ends up with being unmatched, we can make this incentive strict.

borrower to refuse the trade. If he refuses, then there will be no trade and he will simply lose one period.

Here, we will check the deviation in the second subperiod. If he refuses the trade, he can go with all the consumption good he produced. However, he will lose the durable good. This has two consequences—he not only loses utility y from the durable good, but also loses all the possible future trades because all future lenders will refuse to trade with him. His continuation payoff is 0 in such a circumstance. On the other hand, the equilibrium payoff of a borrower is $a - r^* + y$. So, this deviation is not profitable if and only if $a - r^* + y \geq (1-\delta)a$, or

$$r^* \leq y + \delta a \tag{2}$$

Next, consider a lender's incentives. Obviously, it must be the case that $r^* \geq 1$, otherwise, she will produce the consumption good by herself. Also, consider a deviation to refuse the trade and escape with the durable good in the second subperiod. If she refuses and takes the durable good, she will receive the value of collateral from the period onward. However, she will lose the return r^* for this period. Hence, the payoff from such a deviation is $y + (1-\delta)r^*$. This deviation is not profitable if and only if $r^* \geq y + (1-\delta)r^*$ or

$$r^* \geq \frac{y}{1-\delta} \tag{3}$$

Finally the lender is indifferent between continuing and separating.[5]

Now, such $r^* \in [0, a]$ exists if and only if $\min\{a, y+\delta a\} - \max\left\{1, \frac{y}{1-\delta}\right\} \geq 0$. This inequality is satisfied if and only if (1) is satisfied. □

Figure 1 depicts $\min\{a, y+\delta a\} - \max\left\{1, \frac{y}{1-\delta}\right\}$ as a function of y. The repo strategy constitutes an equilibrium if and only if $\min\{a, y+\delta a\} - \max\left\{1, \frac{y}{1-\delta}\right\} \geq 0$.

3.3 Borrowing constraint

Equation (2) says that the maximum repayment that a borrower can credibly promise is $y + \delta a$. Notice that this is increasing in y, the value of the good used as collateral. This result is consistent with the argument by Kiyotaki and Moore [16].

Notice, also, that $y+\delta a$ exceeds the intrinsic value y of the durable good, and can be positive even when $y \leq 0$. The reason is somewhat similar to that of *money is memory* (Kocherlakotta [19]). There, an intrinsically useless object—fiat money—has a positive equilibrium value because it provides partial information on whether an individual has worked in the past or not. In the repo equilibrium, if a borrower does not have a durable good, it is indicated that

[5] Again, if we introduce a small probability that an unmatched individual ends up with being unmatched, we can make this incentive strict.

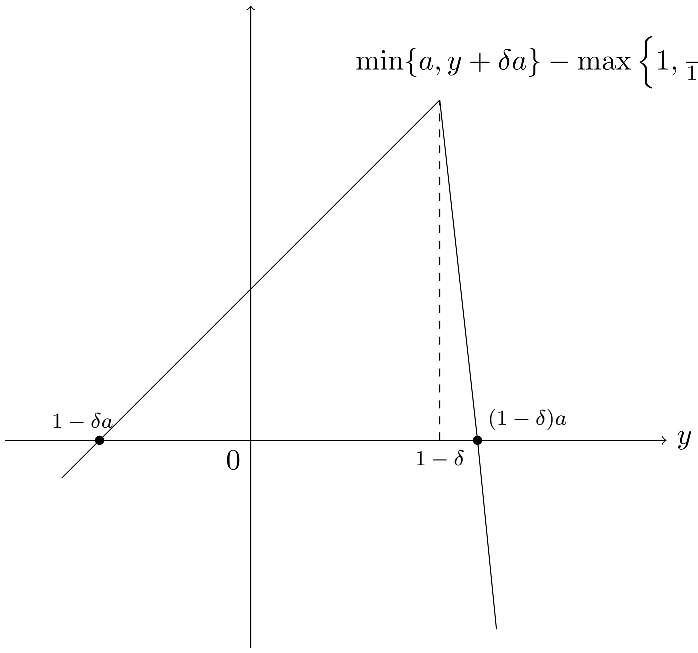

Figure 1: This figure depicts $\min\{a, y+\delta a\} - \max\left\{1, \frac{y}{1-\delta}\right\}$ as a function of y. The repo strategy constitutes an equilibrium if and only if $\min\{a, y+\delta a\} - \max\left\{1, \frac{y}{1-\delta}\right\} \geq 0$, or $y \in [1-\delta a, (1-\delta)a]$. Notice that y does not have to be positive for the repo strategy to constitute an equilibrium.

the borrower has escaped without giving the lender the consumption good, and so, future lenders can punish him. In other words, *collateral is also memory.*

4 Macroeconomic implications

4.1 Payment puzzle

Lagos [20] raised the "payment puzzle": Why even to this day assets are used as collateral rather than simply as a means of payment? Our result provides an explanation for it. To see this, consider a spot transaction using the durable good as a means of payment: in the first match, a borrower and a lender exchange the durable and production goods. From then on, no trade occurs. Now, notice that in autarky, the lender can produce one unit of the consumption good every period. Thus, if $y < 1$, the lender never agrees with such a transaction.

What about repo? Proposition 1 says that the repo strategy constitutes an equilibrium when

$$y \in [1 - \delta a, (1 - \delta)a]$$

Because $1 - \delta a < 1$, we have

Proposition 2. *For any (δ, a), there is a non-empty open set of y in which the durable good cannot be used as a method of payment while it can be used as collateral.*

This is an implication of collateral having a memory role. If a borrower does not pay back, he will lose collateral, and so, he will lose future lenders. Thus, borrowers are willing to pay back even when the intrinsic value of collateral is small. Given this, lenders are willing to accept the repo arrangement even when the intrinsic value of collateral is small.

We emphasize that the argument here does *not* depend on the assumption that lenders can escape with collateral. Section 5.1 provides further arguments for it.

4.2 Return vs. liquidity trade-off

Matsuyama [22, 23] and Lagos and Rocheteau [21]—among many others—consider cases in which some assets have high returns but low liquidity.

Our result explains why and when this can happen. Immediately from (1), we have

Proposition 3. *For any (δ, a), there exist y and $y' > y$ such that the repo strategy constitutes an equilibrium for y but not for y'.*

This is because when y is too big, the lender has an incentive to escape with the durable good (see (3)). In such a case, the asset always stays at the borrower and is hence not circulated.

In Sections 5.1 and 5.3, we will argue further about this return vs. liquidity trade-off.

5 Extensions

5.1 Inescapable lenders

There are some collateralized loans that do not look like repos. In the contract between Shylock and Antonio, Antonio did *not* give a pound of his flesh to Shylock at the moment when they agreed with a collateralized loan. Likewise, with a home equity loan, a lender gives money to a borrower, and the borrower promises to pay the loan in the future; if the borrower reneges, then at a promised time, the lender takes the house. But, the lender does *not* live in it between the periods. With this type of collateralized loans, lenders cannot escape with collateral goods. In this subsection, we compare this type of collateralized loans with repos.

For this purpose, we consider the case in which a pair can write the following *collateral contract* in the first subperiod: In the second subperiod, if the borrower pays some r^*, then he can keep the durable good. If he chooses not to pay, then the lender takes the durable good. If a pair agree with such a collateral contract in the first subperiod, the lender does not make any decision in the second subperiod—in particular, she cannot escape with the durable good. All the other assumptions are the same as those in Section 3. Obviously, we are now considering an environment with less frictions than the one we considered in the previous section.

In this environment, consider the following strategy (we call it the *collateral strategy with inescapable lenders*):

Subperiod 1 If the borrower has the durable good, then the lender chooses $q = 1$, and the lender and the borrower sign on a collateral contract.

Otherwise, make no trade.

Subperiod 2 If they signed on a collateral contract, the borrower pays r^* and keeps the durable good.

Otherwise, make no trade, i.e., the borrower chooses $r^* = 0$.

Subperiod 3 If the trades took place in the previous two subperiods, then keep the relationship. The borrower keeps the durable good.

Otherwise, separate. If the borrower has the durable good, then he keeps it. If the lender has it, then she keeps it if and only if $y > 0$.

We examine the condition for which this strategy constitutes an equilibrium. First, observe that because lenders' behavior is the same as in the repo strategy considered in Section 3.2, borrowers' incentives are also the same. Thus, (2) still holds. The maximum repayment that a borrower can credibly promise $y + \delta a$ exceeds the intrinsic value y of collateral.

From the lender side, borrowers' behavior is also the same as in the repo strategy considered in Section 3.2. Now, however, lenders cannot escape with collateral, and so (3) is not binding. The only binding constraint for a lender is now $r^* \geq 1$.

Thus, the collateral strategy with inescapable lenders constitutes an equilibrium if and only if $\min\{a, y + \delta a\} - 1 \geq 0$. Hence,

Proposition 4. *The collateral strategy with inescapable lenders constitutes an equilibrium if and only if*

$$y \in [1 - \delta a, \infty) \tag{4}$$

Compare (4) with (1). The range of value y expands in which the durable good can work as collateral. This is intuitive, because a pair can write a better contract. However, notice that the lower bound of the value y is the same as that in the case with repo arrangement, $1 - \delta a$. That is, the fact that lenders are impossible to escape with collateral does not allow individuals to use a less valuable good as collateral.

Because we have the same lower bound as in (1), Proposition 2 still holds. That is, even in this environment, there is a non-empty open set of y in which the durable good cannot be used as a method of payment while it can be used as collateral.

On the other hand, Proposition 3 fails. In contrast to the precious section, where the tighter incentive constraint is what makes the asset less liquid, high return and high liquidity always come together in the environment with inescapable lenders.

5.2 Observable lenders

Lending activities through repo contracts are in some cases operated by large financial institutions such as investment banks and commercial banks. Activities of such large institutions have often been paid careful attention to in the market, especially since the financial crisis of 2007-2008 (see, for example, Gorton and Metrick [11]). Motivated by this, we now consider the case where lenders can escape with the durable good, but it is observable by *all* (future) borrowers. All the other assumptions are the same as those in Section 3. In particular, we maintain the assumption that borrowers' histories are not observable by non-partner lenders.

In this case, a slightly modified version of the repo strategy is feasible and call it the repo strategy with observable lenders. The strategy on the equilibrium path is the same as the one defined in Section 3.2. However, if a lender has escaped with the durable good, all future borrowers refuse to trade with her and choose to separate.

Under this strategy, if a lender escapes with the durable good, her lifetime payoff is y. The lender's incentive constraint is now

$$r^* \geq \max\{1, y\}$$

The borrowers' incentives remain unchanged. Thus, the repo strategy with observable lenders constitutes an equilibrium if and only if $\min\{a, y + \delta a\} - \max\{1, y\} \geq 0$.

This gives us the following proposition.

Proposition 5. *The repo strategy with observable lenders constitutes an equilibrium if and only if*

$$y \in [1 - \delta a, a] \tag{5}$$

Comparing the upper bounds in (5) and (1), we have $a > (1 - \delta)a$, and so the range of value y expands in which the durable good can work as collateral. However, as in the case with inescapable lenders, the lower bound of the value y is unchanged, $1 - \delta a$. Like in the case with inescapable lenders, the fact that lenders are observable does not allow individuals to use a less valuable good as collateral.

Also, the counterpart of Proposition 2 holds because we have the same lower bound for y. The counterpart of Proposition 3 holds because the range of y is bounded from above.

Figure 2 depicts the cases with inescapable (Section 5.1) and observable lenders (this section).

5.3 Storable good

This section asks what kinds of goods are suitable as collateral—a house, wine or a pound of flesh? So far, we have assumed that an owner of the durable good obtains utility every period. Houses and brand bags are typical examples of such goods. Financial assets are another example, in the sense that it yields dividend every period. Wine, on the other hand, disappears once an owner of the good obtains utility. In this section, we examine which type of goods should be used as collateral.

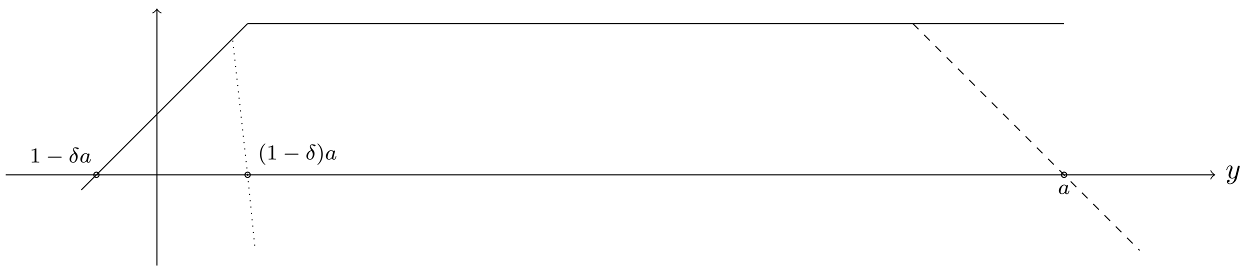

Figure 2: This figure depicts the three different cases—repo, inescapable lenders, and observable lenders. The real (resp. dotted, dashed) line is $\min\{a, y+\delta a\} - 1$ (res. $\min\{a, y+\delta a\} - \max\{1, \frac{y}{1-\delta}\}$, $\min\{a, y+\delta a\} - \max\{1, y\}$). They give the regions of the value y in which the durable good works as collateral.

Formally, let us introduce another kind of good—a *storable* good. The storable good is, like the durable good, durable and indivisible. However, one has to "liquidate" it to obtain utility, and once it is liquidated, the good disappears. This is in contrast to the durable good from which, *in each period*, its owner can enjoy utility. More precisely, in the third subperiod, an owner of the storable good determines whether to keep, trash, or liquidate it. If he or she liquidates it, he or she gets utility $z \in \mathbb{R}$. Again, we do *not* assume that $z > 0$.

Now consider an economy with three goods—production, consumption and storable goods. Except that the durable good is replaced by the storable good, everything else is the same as the one considered in Section 3.

With such a storable good, consider the repo strategy defined in Section 3.2. First, notice that unlike the durable good, when $z > 0$, *the repo strategy does not achieve the first best.* In any first best allocation, the storable good must be liquidated in the first period, but then it cannot be used as collateral. In the case of the durable good, one could use it as collateral and for its own purpose at the same time. This does not happen in the case of the storable good which one must liquidate to enjoy its value. Hence, from the efficiency point of view, the durable good is more suitable as collateral than the storable good is. Hereafter, we will show that when $y, z > 0$, the durable good is more suitable as collateral than the storable good is from the incentive point of view as well.

To see this, first, derive the region of z in which the repo strategy constitutes an equilibrium. The lender's incentives are the same as in the durable good case—she follows the strategy if and only if $r^* \geq \max\{1, \frac{z}{1-\delta}\}$.

The borrower's incentives are, however, different. First, his lifetime payoff is $a - r^*$, because he cannot enjoy the value of the good. His payoff, when he deviates in the second subperiod, is the same as in the case of the durable good, and is simply $(1-\delta)a$. This gives us one incentive constraint $r^* \leq \delta a$. This says that the maximum repayment that a borrower can credibly promise is δa, and it is independent of the intrinsic value z of the storable good. This is in contrast to the durable good case in which the maximum repayment that a borrower can credibly promise is increasing in its intrinsic value (see (2)).

Notice also that the borrower now has another possible deviation in the third subperiod, where he liquidates the storable good. In this case, he can enjoy the value of the storable good but will lose all future trades. His lifetime payoff from this deviation is z. This gives us another incentive constraint

$$r^* \leq a - \frac{z}{\delta} \tag{6}$$

For the repo strategy to constitute an equilibrium, there must exist an $r^* \in [0, a]$ where all the conditions above are satisfied. Thus, for the repo strategy to constitute an equilibrium, it must follow that

$$\min\left\{\delta a, a - \frac{z}{\delta}\right\} - \max\left\{1, \frac{z}{1-\delta}\right\} \geq 0$$

From this, we have

Proposition 6. *If $\delta a \geq 1$, then the repo strategy constitutes an equilibrium if and only if*

$$z \in (-\infty, \delta(1-\delta)a] \tag{7}$$

If $\delta a < 1$, then the repo strategy does not constitutes an equilibrium for any $z \in \mathbb{R}$.

Before comparing this case with the durable good case, first let us revisit the macro implications—payment puzzle in Section 4.1 and return vs. liquidity trade-off in Section 4.2—in this storable good case. First, consider the payment puzzle. As in the case with the durable good (see Section 4.1), lenders will refuse to exchange the production good for the storable good when $z < 1$. This gives us the counterpart of Proposition 2 as follows:

Proposition 7. *If $\delta a \geq 1$, there is a non-empty open set of z in which the storable good cannot be used as a method of payment while it can be used as collateral.*

Also, there is return vs. liquidity trade-off with the storable good. Indeed, from (7), the trade-off is even stronger than that with the durable good is, in the sense that

Proposition 8. *Suppose the repo strategy does not constitutes an equilibrium for z. Then it does not for any $z' > z$.*

It is also worth mentioning that the trade-off survives with the form of collaterized contracts considered in Section 5.1. This is a consequence of (6), one of the borrower's incentive constraints. This is in contrast to the durable good case (see Proposition 4).

Finally, compare the storable good with the durable good. To do so, let

$$y = z$$

so that the value that one gets from holding the durable good forever is the same as the value that one gets from liquidating the storable good today.

When $\delta a < 1$, from Proposition 6, the durable good is more suitable as collateral than the storable good is. Suppose $\delta a \geq 1$. Comparing the upper bounds in (7) and (1), we have $(1 - \delta)a - \delta(1 - \delta)a > 0$ (see Figure 3). This gives us that

Proposition 9. *Suppose that $y = z > 0$. Then, the repo strategy constitutes an equilibrium with the durable good if it does with the storable good, but not vice versa.*

Notice also that when $y = z < 0$, the result is reversed—when $\delta a > 1$, the storable good works as collateral no matter how small $z < 0$ is.

6 Conclusion

This paper presented a simple equilibrium model in which collateralized credit emerges endogenously. Even in a frictional world, where commitment is limited, we show that collateral can serve as a credible device that prevents the participating parties from reneging. Our theory provides a micro-foundation to justify the borrowing constraints that are widely used in the existing macroeconomic models. We provide an answer to the "payment puzzle" raised in the macroeconomic literature.

While our model captures the features of repos well, it would be interesting to investigate the dynamic implication of the model. We believe such an extension will offer a novel

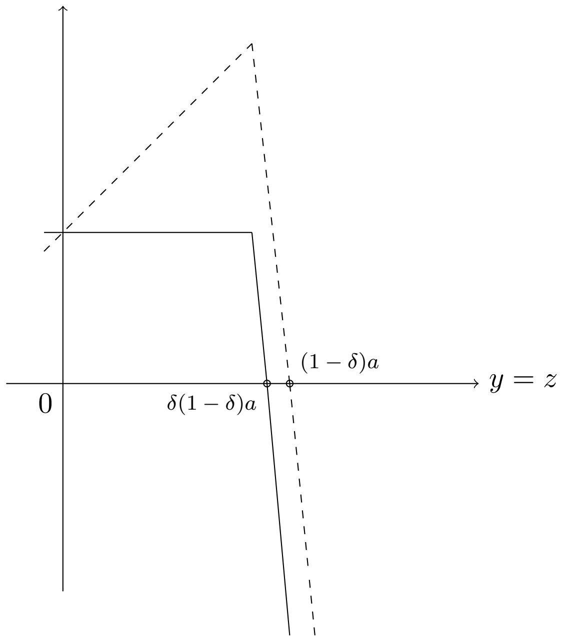

Figure 3: This figure compares the durable good and the storable good. The real (resp. dashed) line depicts $\min\{a, y + \delta a\} - \max\left\{1, \frac{y}{1-\delta}\right\}$ (resp. $\min\left\{\delta a, a - \frac{y}{\delta}\right\} - \max\left\{1, \frac{y}{1-\delta}\right\}$) as a function of y (normalized so that $y = z$). The range of y in which the repo strategy constitutes an equilibrium is wider with the durable good.

insight into the observed phenomena of run on repos. It would also be interesting to incorporate another media of exchange, fiat money, into the model to investigate the essentiality of collateralized credit to survive in monetary equilibrium.

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