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Abstract

This paper investigates the incentives that may induce workers to supplement income from unemployment benefits by engaging in temporary informal work. Using a dynamic model of job-search with moral hazard that incorporates a stylised schedule of benefit payments, we describe how informal sector participation changes over the duration of unemployment, in turn affecting the incentive to search for formal employment. We find that increasing benefit generosity makes job seekers less reliant on informal work, enabling them to search more intensively. At the same time, when detection rates are low, informal work participation may decline as benefit exhaustion approaches, reinforcing this effect. From a policy perspective, the analysis identifies scope for reallocation of resources towards less generous programmes within unemployment protection, which would reduce the size of the informal sector and unemployment in the economy.

JEL-Codes: J460, J640, J650, K420.

Keywords: job-search, informal sector, unemployment insurance, moral hazard.

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1 Introduction

Social security nets against the risk of unemployment are important components of labour market policy in advanced economies. In Europe, for example, workers typically may be eligible for unemployment protection through two main programmes, unemployment insurance (UI) and unemployment assistance (UA), that differ in the duration and generosity of benefit payments. Social assistance (SA) programmes further provide minimum income support to those ineligible for unemployment protection (OECD 2014).

Despite the widespread provision of unemployment protection and income support, a significant proportion of workers in advanced economies still engage in informal labour market activities to supplement their income during unemployment spells.[1] According to the Eurobarometer survey on undeclared work, the highest proportion of individuals declaring informal work is by far that of the unemployed, averaging around 12 percent. This is more than double that of both workers and those who are not actively seeking formal jobs, like students and pensioners (European Commission 2014). Similarly, the 2015 Survey of Informal Work Participation provides evidence of large engagement in informal labour market activities among the unemployed in the United States (Federal Reserve Bank of New York 2015). Data from both these two surveys also suggest that informal activities in advanced economies predominantly involve temporary side-jobs undertaken for friends and neighbours, rather than undeclared waged- and self-employment prevalent in developing countries (Albrecht et al. 2009; Bennett and Rablen 2015; Bosch and Esteban-Pretel 2012).

In light of these observations, this paper evaluates how the current design of unemployment protection in advanced economies influences the incentive of unemployed workers to engage in temporary informal labour market activities while searching for formal jobs, and whether it would be possible to implement policy changes that may reduce this incentive without negatively affecting formal job search.

As we are interested in the design of the unemployment protection system, we extend a model of dynamic job-search where unemployed workers are eligible for benefit payments (as in Mortensen 1977 and van den Berg 1990), by allowing them to engage in informal labour market activities over the duration of unemployment. In this environment, job-search dynamics are non-stationary due to the finite duration of unemployment protection. Moral hazard arises because informal activity and job-search effort are unobservable. Working in the informal sector provides income opportunities that helps smooth consumption during unemployment spells. At the same time, it distracts unemployed workers from finding jobs in the formal sector. As working informally whilst receiving unemployment benefits constitutes fraudulent behaviour, it also carries the risk of being caught and sanctioned.

The provision of unemployment protection has given rise to two long-standing theoretical predictions. First, since search effort is unobservable, unemployed workers search less intensively when they receive more generous unemployment protection. This is the traditional moral hazard argument. Second, as unemployed workers near the expiry date

[1] For the purpose of this paper, informal work is defined as any paid market-based economic activity that is legal but concealed from public authorities to avoid taxes and social security contributions (Schneider and Enste 2000).

of benefits, remaining unemployed becomes much less appealing relative to finding work. For this reason, the incentive to search for formal-sector jobs increases over time. This is referred to as the re-employment spike (Mortensen 1977; Feldstein 2005).

We find that there is another force determining job-search dynamics over the duration of unemployment: the incentive to accept informal work. This brings three main non-stationary effects.

Across programmes we identify a *payment effect*. When benefits reduce as unemployed workers switch to less generous programmes, the instantaneous utility gain that they enjoy from accepting a given informal opportunity increases. They become more likely to accept informal work as a result. This tends to counteract the re-employment spike highlighted by the standard theory, by distracting them from formal job-search.

Within unemployment protection, there are two opposing and time-varying effects. The first – the *exhaustion effect* – is entirely due to the exhaustion of benefit payments. In the early stages of unemployment, individuals have many payments to look forward to, thus relatively little incentive to search for formal employment. This enables them to accept informal sector work. However, as the expiry date of unemployment protection approaches, they understand that there are fewer payments remaining. This provides a greater incentive to search for formal work, thus curtailing their incentive to accept informal work over time. The second effect – the *deterrence effect* – derives from the sanctions workers face if they are caught engaging in informal work whilst receiving unemployment protection. In advanced economies, sanctions typically include the loss of eligibility for unemployment protection, returning any overpayment and fines. In the early stages of unemployment, these sanctions provide a significant deterrent, as those caught engaging in the informal sector risk losing entitlement to many future payments. As the duration of unemployment increases, there are fewer remaining payments to loose and their incentive to engage in the informal sector increases. To the extent that accepting informal work distracts from formal job search, the re-employment spike will be compounded by the exhaustion effect and offset by the deterrence effect.

The relative importance of the payment versus the moral-hazard effect is mainly driven by the generosity of unemployment benefits. Whether the exhaustion or the deterrence effect dominate ultimately depends on the perceived probability of been caught while engaging in informal activities and sanctioned: the lower (higher) is this probability the more likely is that the exhaustion (deterrence) effect will dominate and the incentive to engage in informal sector work should decline (increase) over time.

To quantify how these effects impact on the formal job search and the informal sector participation of unemployed workers, we employ a numerical version of the model calibrated to match aggregate moments of the labour market in Spain. We choose Spain because its unemployment protection system is similar to that of a typical advanced economy and, in contrast to many other European countries, detailed information with regard to sanctions is readily available. As pointed out by Bosch and Esteban-Pretel (2015), quantitative analyses of the informal labour market are bound to give results that vary across countries and over time. This also applies to our model. Nevertheless, the analysis identifies a number of new results and policy implications that hold beyond the specifics of the calibration.

We build an algorithm for the numerical solution that traces the states in which a

worker can be over the duration of unemployment and whilst in employment, depending on whether or not she has been sanctioned. This allows us to break down the unemployed workforce, the informal sector participation rate and the earning gap between formal and informal employment into those receiving UI, UA and SA. Consequently, a further contribution of our analysis is that, unlike existing studies of the informal sector based on general equilibrium search models, we can evaluate changes in labour market policy that target specific components of the unemployment protection system.

Under the baseline calibration, we quantify a number of key characteristics of the informal labour market, for which data is typically not available. In particular, we find that informal sector participation crowds-out, rather than provides a stepping-stone to, formal employment. Around 40 percent of unemployed individuals accept temporary informal work in a given month earning about 30 percent less on average relative to the formal sector. The probability of being caught doing informal work is just under 4 percent, within the range of existing estimates.

The quantitative analysis identifies large discontinuities in informal sector participation and formal job-search over the duration of unemployment. We find that with an effective replacement ratio close to 60 percent, benefit payments during UI are sufficiently large to deter any informal sector activity by unemployed. Informal sector participation is thus concentrated among those who are not eligible for UI. In particular, the informal sector participation rate declines within the UA, thus suggesting that the measured detection rate is low enough so that the exhaustion effect dominates. For those outside unemployment protection, informal sector participation is higher and stationary. The formal job-finding rate declines on average over the duration of unemployment, but increases within the UI and the UA as a result of the reduction in the continuation value of unemployment and the dominance of the exhaustion effect.

We carry out two types of policy experiments. The first considers ceteris paribus changes in specific components of the unemployment protection system, by varying the effective replacement ratios and durations of either UI or UA, and in the sanctions system, through variation of the detection rate and fines. We find that increasing the generosity of unemployment protection can reduce informal sector participation and increase formal job search, thus leading to reduction of the size of the informal sector relative to GDP and of the unemployment rate at the aggregate level. These effects are larger when increasing benefit payments and/or duration of UA, the less generous programme. With regard to sanctions, a similar result is found when increasing the detection rate, while increasing fines has little impact when detection rates are low.

The second type of policy experiments quantifies self-financing changes in the structure of the unemployment protection system. We find that reallocation of resources from UI to UA would result in reduction of informal sector participation, thus leading to a lower unemployment rate and smaller informal sector relative to GDP. We also find that changes in sanctions are more effective if concentrated among those who have the largest incentive to engage in informal sector activities. From a policy perspective, this suggests that monitoring should concentrate on those in the early stage of UA and/or those who are not eligible for unemployment protection.

It is important to relate these quantitative findings to those in the literature. Tatsiramos and van Ours (2014) review a large body of empirical evidence suggesting that

increase in UI benefit payments can have adverse effects on the labour market, in that it increases the unemployment rate. We also find this to be true in our model: an increase in UI benefits, reduces job-search among UI recipients. The additional contribution of our analysis stems from looking at the effects of increasing the generosity of payments in supplementary programmes with replacement ratios lower than UI. There we find that a different mechanism is driving job-search dynamics. When replacement ratios are low, unemployed workers have a greater incentive to supplement their income with informal work. This distracts them from formal job-search. Contrary to the prescriptions of the moral hazard argument, raising the generosity of benefit payments in these programmes can actually increase job-search.

There is a growing literature revisiting labour market policy in the context of economies with informal sectors: the contribution of the present paper differs from the existing results in several dimensions. Fugazza and Jacques (2004); Albrecht et al. (2009); Bosch and Esteban-Pretel (2012, 2015); Meghir et al. (2015) and Di Porto et al. (2017) employ general equilibrium search models to study labour market flows and policy intervention in developing countries with large informal sectors. These models account for the whole informal sector in the economy, but can only assess the effect of changing the overall level of unemployment benefits on the labour market. Our paper focuses only on a segment of the informal sector, that determined by the participation of unemployed workers, but complements these works by highlighting that the distribution of benefit payments across unemployment protection programmes, as well as their level, is an important determinant of labour market outcomes.

In terms of its methodology, our paper belongs to the literature employing partial equilibrium search models to study labour market dynamics over the duration of unemployment under either the actual (Lalive et al. 2006; Chetty 2008) or the optimal (Shavell and Weiss 1979; Hopenhayn and Nicolini 1997; Pavoni and Violante 2007) design of unemployment protection. Álvarez-Parra and Sánchez (2009) and Espino and Sánchez (2015) incorporate the informal labour market in this type of economic environments to study labour-market dynamics under the optimal design of unemployment protection. Our paper instead considers labour-market dynamics under the actual design of unemployment protection. Bardey et al. (2015) also study the interplay between job-search dynamics and informal work using a partial equilibrium search model. As their interest is on the long-term effects of labour market policy, their analysis covers only the stationary implications of policy, while we highlight the significance of non-stationary effects during unemployment spells and how these could be altered by policy intervention.

The economic effects and policy implications of informality have been studied in a wide range of areas, other than the labour market. Recent contributions include taxation (Cuff et al. 2011), health insurance (Bosch and Campos-Vazquez 2014), macroeconomic volatility (Restrepo-Echavarria 2014), microfinance (Demont 2016), pension system design (Joubert 2015) and inequality (Chong and Gradstein 2007). The paper is also related to an emerging literature on the so-called *gig economy*, which refers to irregular temporary work such as selling goods or services online, freelancing and on-call work (Bracha et al. 2015; Katz and Krueger 2016; Robles and McGee 2016; Bracha and Burke 2016), since an alternative interpretation of our model considers the decisions of

unemployed workers with access to these alternative employment opportunities.

The paper proceeds as follows. Section 2 presents the main characteristics of unemployment protection programmes in advanced economies and reviews some of the findings about engagement in the informal sector by unemployed in these countries. Section 3 describes the economic environment. Section 4 outlines the evolution of the agent's job-search and informal work decisions over the duration of unemployment. Section 5 describes the quantitative model and its calibration. Sections 6 and 7 present the results of the quantitative analysis. Section 8 concludes by summarising the main results of the paper and its implications for policy. Appendix A includes proofs of the main propositions. Appendix B describes in details the algorithm for the solution of the numerical model.

2 Unemployment protection and informal work in advanced economies

To motivate our analysis, we provide some background information on the structure of unemployment protection systems in advanced economies and the nature of informal sector activities carried out by unemployed workers. This is based on thirteen Western European countries and the United States. All data refers to 2013, since this is the most recent year for which information on informal sector engagement is available in all of these countries. With regard to unemployment protection, we summarise information about effective benefit payments and maximum durations of UI and UA in each country. This highlights that the typical system provides payments of limited duration, that decline as recipients transition from UI to UA. With regard to unemployed workers engagement in the informal sector, we summarise evidence on what activities they do, who their customers are, and how frequently they undertake these jobs. This suggests that a significant proportion of these activities includes small, temporary jobs carried out for family and friends, rather than firms.

2.1 Unemployment protection

Unemployment protection in advanced economies typically consists of two programmes. UI provides benefits to individuals upon becoming unemployed, conditional on their employment and contribution history. In most countries, payments are subject to income tax and social security contributions. UA is means-tested and provided to individuals who have either exhausted, or are otherwise ineligible for, UI. Actual payments under each programme may also depend on age, industry and family composition.

Table 1 shows the effective replacement ratios and maximum duration of UI and UA in our sample of advanced economies. Effective replacement ratios are calculated by dividing the total monthly expenditure on each programme by the stock of recipients and the average monthly wage in each country. These therefore provide summary measures of the generosity of each programme across countries, capturing differences in payments

Table 1: Unemployment protection in advanced economies

	UI		UA	
Country	Effective Replacement Ratio (%)	Maximum Duration (months)	Effective Replacement Ratio (%)	Maximum Duration (months)
Austria	35.8	5	30.2	Unlimited
Belgium	20.2	Unlimited	–	–
Denmark	44.3	24	–	–
Finland	41.3	25	21.1	Unlimited
France	39.3	24	25.7	Renewable
Germany	37.2	12	11.1	Unlimited
Ireland	30.2	12	30.2	Unlimited
Italy	59.0	8	–	–
Luxembourg	66.5	12	NA	12
Netherlands	32.4	22	–	–
Spain	56.8	24	22.1	18
Sweden	NA	15	NA	15
United Kingdom	10.5	6	10.5	Unlimited
United States	40.6	6	40.6	3

Sources: Authors' calculations based on data from European Commission (2016); Employment and Training Administration (2017); OECD (2014).

Notes: For Sweden, only combined UI and UA data is available. This suggests an average effective replacement ratio across the two programmes of 26.1 percent.

due to eligibility criteria, taxation and other factors specified in national legislation.[2]

The table highlights two common features of unemployment protection systems in advanced economies. First, the generosity of benefits declines over time, as unemployed workers move between programmes. Second, both UI and UA are typically of limited duration.

Most countries also have a SA programme. This is means-tested and is granted to individuals that have exhausted unemployment protection payments, as the last resort of social security support. The duration of SA is unlimited, so long as eligibility criteria are met.[3]

[2]This approach to calculating effective replacement ratios is equivalent to that of effective tax rates in the fiscal policy literature (Mendoza et al. 1994) and effective pension replacement ratios in the public finance literature (OECD 2015).

[3]A detailed description of the structure of unemployment protection and SA in each of these countries is provided in OECD (2014).

2.2 Informal activities of the unemployed

To characterise the main features of informal work undertaken by the unemployed in advanced economies, we use data from two large surveys of undeclared work. Special Eurobarometer 402 surveyed around 27,000 individuals across 27 EU countries regarding undeclared work they had done, or work they had paid for which they had reason to believe was undeclared (European Commission 2014). The Survey of Informal Work Participation asked similar questions of around 1200 individuals in the United States

Table 2: Informal sector by employment status in advanced economies

Country	Informal Sector (% GDP)	Proportion declaring informal work (%)			Share of Informal Sector Income (%)			Informal Income (% GDP)		
		E	U	I	E	U	I	E	U	I
Austria	7.5	3.9	21.7	6.6	43.2	8.2	48.6	3.2	0.6	3.6
Belgium	16.4	3.9	12.7	2.3	50.4	17.2	32.4	8.3	2.8	5.3
Denmark	13.0	8.1	9.6	9.2	67.8	5.0	27.2	8.8	0.7	3.5
Finland	13.0	2.6	4.1	2.6	47.8	4.4	47.8	6.2	0.6	6.2
France	9.9	4.8	8.0	5.0	57.0	14.7	28.3	5.6	1.5	2.8
Germany	13.0	2.5	2.8	1.8	73.6	0.7	25.7	9.6	0.1	3.3
Ireland	12.2	1.9	2.3	2.0	30.8	59.4	9.7	3.8	7.2	1.2
Italy	21.1	1.1	10.0	0.7	45.9	42.9	11.3	9.7	9.0	2.4
Luxembourg	8.0	4.4	20.0	4.2	86.7	0.0	13.3	6.9	0.0	1.1
Netherlands	9.1	11.0	14.8	9.5	63.2	1.1	35.6	5.8	0.1	3.2
Spain	18.6	4.9	9.7	3.1	59.4	27.5	13.1	11.1	5.1	2.4
Sweden	13.9	7.1	29.6	4.0	86.1	7.7	6.2	12.0	1.1	0.9
United Kingdom	9.7	2.8	6.3	1.3	44.5	24.5	31.0	4.3	2.4	3.0
United States	6.6	42.4	37.3	24.3	93.4	3.4	3.1	6.2	0.2	0.2
Average (EU14)	12.3	4.5	11.7	4.0	60.7	15.5	23.8	7.5	1.9	2.9

Sources: Authors' calculations based on data from Schneider (2013); European Commission (2014); Federal Reserve Bank of New York (2015).

Notes: E, U and I denote Employed, Unemployed and Inactive respectively. Proportions declaring informal work refer to the previous twelve months, except for the United States which refers to the previous twenty-four months. Responding to surveys and renting out property are excluded from informal work in the United States data, following Bracha and Burke (2014).

(Federal Reserve Bank of New York 2015).[4]

Table 2 presents data on the size of the informal sector and the extent of engagement in informal activities, distinguishing between the employed, unemployed and inactive.[5] The first column reports the informal sector as a proportion of GDP. The next three columns show the proportion of individuals that have undertaken informal work over the last twelve months, based upon declarations in the Eurobarometer and the Survey of Informal Work Participation, given their employment status at the time of the survey. The next three columns give estimates of the share of informal sector income, calculated as the total informal income reported by individuals in each category as a proportion to the total informal sector income across the three employment statuses. The final three columns multiply the size of the informal sector by these shares, to estimate the total informal income earned by each category of worker.

The informal sector covers a sizeable share of production in these economies, on average 12.3 percent of their GDP, with largest shares (above average) in Belgium, Germany, Italy, Spain and the Scandinavian countries.

The highest proportion of individuals declaring informal work is by far that of the unemployed, averaging 12 percent. The spread across countries is very large, ranging between 2 to 30 percent. Since the data comes from surveys, these figures are likely to underestimate the true proportions due to under-reporting. The bias is thought to be particularly large for the unemployed, who are deemed to be less likely to report their informal work for fear of losing welfare payments (Williams and Windebank 2001).

On average, unemployed workers contribute about 16 percent of income deriving from the informal economy. This reflects both a smaller population of unemployed individuals relative to the employed, and also lower reservation wages in the informal sector. Cross-country differences are significant, with largest shares (above average) in Belgium, Ireland, Italy, Spain and the United Kingdom.

The average estimated value of informal income earned by the unemployed is approximately two percent of GDP. Highest values are found in Italy and Spain, due to their larger than average informal sectors, and in Ireland, due to the large share of informal income earned by the unemployed.

Table 3 shows the types of informal activities carried out by unemployed workers in advanced economies. Over 60 percent of these consists of small jobs including babysitting, cleaning, repairs, gardening, helping to moving house, assisting a dependent and on-line activities such as selling goods on eBay. Waged- and self-employment, the two most prevalent types of informal activities in developing countries, appear to have a less significant role in advanced economies. This finding is not limited to the small group of advanced economies presented here, but extends across the majority of countries in the

[4]The data is made available on condition of the following declaration: "*Source:* Survey of Consumer Expectations, © 2013-2015 Federal Reserve Bank of New York (FRBNY), and SCE Survey of Informal Work Participation sponsored by the Federal Reserve Bank of Boston (FRBB), as part of the SCE. The SCE data are available without charge at /microeconomics/sceIndex and may be used subject to license terms posted there. FRBNY disclaims any responsibility for this analysis and interpretation of Survey of Consumer Expectations data."

[5]'Employed' includes the self-employed, managers, other white collar workers and manual workers. 'Inactive' includes house persons, the retired and students.

Table 3: Informal activities undertaken by unemployed workers

	% of Activities		
Country	Small jobs	Self-employment	Waged Employment
Austria	83.3	8.3	8.3
Belgium	90.9	9.1	0.0
Denmark	76.9	0.0	23.1
Finland	33.3	0.0	66.7
France	50.0	20.0	30.0
Germany	100.0	0.0	0.0
Ireland	100.0	0.0	0.0
Italy	33.3	33.3	33.3
Luxembourg	38.5	30.8	30.8
Netherlands	81.8	0.0	18.2
Spain	61.5	23.1	15.4
Sweden	30.8	0.0	69.2
United Kingdom	16.7	16.7	66.7
United States	90.6	7.2	2.2
Average	63.4	10.6	26.0

Sources: European Commission (2014); Federal Reserve Bank of New York (2015).

European Union (Williams and Horodnic 2017).

Table 4 shows for whom informal sector activities are carried out by unemployed workers in advanced economies. The vast majority was undertaken for friends or family, rather than businesses or strangers. Combined with the types of activities performed informally presented in Table 3, it has been argued that most informal work consists of 'paid favours' offered by close relations wishing to help out the unemployed person (Williams 2001; Williams and Nadin 2014). This contrasts sharply with the way the informal sector has traditionally been modelled in the context of the developing world, whereby workers and firms are matched and then jointly decide whether to make the match formal. According to this data, this accounts for only 14 percent of informal work undertaken by the formally unemployed in advanced economies.

Table 5 provides evidence on the frequency of informal work carried out by unemployed workers in advanced economies. The data suggest that a significant proportion of these small jobs are undertaken on a sporadic basis. The first three columns report data on how frequently the respondent's main informal activity was undertaken. The majority of unemployed workers undertook informal work only once or a few times over a period of twelve months. The penultimate column reports the average reported earnings from informal activities by unemployed respondents. For reference, the last column shows the average annual income in each country during the same year. This result

Table 4: Recipients of informal activities undertaken by unemployed workers

	% Admitting to working for each category of recipient					
Country	Friends	Relatives	Neighbours	Other Households	Firms	Other
Austria	82	0	44	63	27	0
Belgium	13	9	31	68	0	10
Denmark	64	0	0	48	0	0
Finland	86	81	0	0	0	0
France	85	34	0	15	20	0
Germany	54	26	0	56	15	0
Ireland	100	0	33	0	0	0
Italy	14	29	0	34	51	18
Luxembourg	100	69	62	62	0	0
Netherlands	48	46	14	14	0	13
Spain	25	12	18	52	6	5
Sweden	32	31	32	69	31	0
United Kingdom	49	29	0	0	29	0
Average	58	28	18	37	14	4

Source: European Commission (2014).

Notes: Rows do not sum to 100 percent, as respondents admitted to working for multiple recipients. Data for the United States is not available.

thus suggests than unemployed workers earn significantly less from informal activities relative to gaining formal employment.

Overall, the data from these surveys suggest that unemployed workers engage in non-negligible amounts of informal work. This tends to involve small jobs, undertaken on a sporadic basis from family and friends. Earlier studies, employing data from the English Localities Survey (Williams and Windebank 2001; Williams 2014), give a similar picture as they find that 90 percent of informal activities were carried out by the unemployed in low-paid sectors on a temporary basis. Examples cited in these studies include: doing bar work for two weeks, working early mornings in a small bakery, working for three weeks in a canteen or on a building site, cleaning trucks for three hours, and refurbishing a pub for one week.

The nature of informal work by unemployed individuals in advanced economies emerging from these data is very different to that of developing countries, where it largely involves waged- or self-employment on a more permanent basis (Albrecht et al. 2009; Bosch and Esteban-Pretel 2012; Meghir et al. 2015).

This may reflect the monitoring technology employed in these countries. Most make extensive use of information reported by third parties, e.g. banks and firms, on income

Table 5: Frequency and earnings of informal activities undertaken by unemployed workers

	Frequency over 12 months (%)			Annual earnings (€)	
Country	Once	A few times	Regularly	Informal	Formal
Austria	11.1	44.4	44.4	800.00	41,693
Belgium	0.0	60.0	40.0	644.10	46,810
Denmark	44.4	44.4	11.1	548.70	51,444
Finland	0.0	100.0	0.0	159.40	42,493
France	0.0	60.0	40.0	553.90	36,980
Germany	25.0	50.0	25.0	50.00	45,170
Ireland	0.0	0.0	100.0	2711.90	32,381
Italy	0.0	66.7	33.3	1715.80	29,704
Luxembourg	0.0	0.0	100.0	–	52,902
Netherlands	20.0	20.0	60.0	127.30	48,109
Spain	0.0	83.3	16.7	702.50	26,027
Sweden	0.0	0.0	100.0	842.30	47,039
United Kingdom	0.0	100.0	0.0	581.40	46,039
Average	7.7	48.4	43.9	725.95	42,074

Sources: European Commission (2007, 2014); OECD (2014).

Notes: Data for the United States is not available.

paid to employees or received from clients (see, for example, Employment and Training Administration 2009). Cross-referencing of these reports with unemployment protection databases makes both self- and, in particular, waged-employment more difficult to conceal. In contrast, small jobs undertaken for friends and family are more likely to be paid in cash, thus much easier to conceal. Experimental evidence reported by Kleven et al. (2011) lends credence to this argument. Whilst they found very little evidence of under-reporting of income to tax authorities that was subject to third-party reporting (only around 0.23 percent of income), under-reporting of other income was much higher (41.6 percent).

Alternatively, the sporadic nature of informal work may be an equilibrium phenomenon. In several studies, Williams and co-authors report that individuals in neighbourhoods suffering from high unemployment effectively operated a secondary insurance scheme within their community (Williams 2001; Williams and Windebank 2001; Williams and Horodnic 2017). Individuals in work offered informal opportunities to those who were currently unemployed on the understanding that, if the roles were reversed in the future, the favour would be returned.

3 Economic environment

The environment consists of a discrete-time search model with temporary unemployment protection and two labour markets: formal and informal. An infinitely-lived, risk averse worker becomes unemployed in period $t = 1$. For $t = 1, ..., T$, she is covered by UI, and receives payments of $b_t = b^{ui} > 0$. We refer to T as the date on which her UI is *exhausted*. If the worker is still unemployed after exhaustion, she receives no further payments, $b_t = 0$. We abstract from UA and SA only to ease explanation, without any loss of generality. The literature on unemployment protection and informal work cited in the introduction typically assumes that benefit recipients have no access to credit markets, on the grounds that they come from low-income households. Following this, we assume that the worker has no savings and cannot borrow.

The worker can work in either the formal or informal sectors. All formal jobs are identical, and provide wage w. We assume that w is sufficiently high that offers of formal employment are always accepted. Formal employment is not permanent, and an exogenous proportion, λ, of jobs is destroyed every period.

While unemployed,[6] the worker receives an offer of informal work – an *informal opportunity* – providing her with income χ_t. Informal opportunities last for one period, and are independent and identically distributed according to $F(\chi)$, capturing the temporary nature of informal work in advanced economies, as described in Section 2.2.

Since our aim is to characterise the behaviour of unemployed agents as they pass through unemployment protection, we make formal and informal work mutually exclusive (as in Albrecht et al. 2009; Álvarez-Parra and Sánchez 2009; Espino and Sánchez 2015). Allowing employed agents to also work in the informal sector would not qualitatively affect our results, as explained below. Our specification of informal opportunities also includes a constant informal sector wage as a special case, when $F(\chi)$ is degenerate.

The unemployed worker makes three decisions each period. First, she chooses whether to accept or reject her current informal opportunity (denoted by $a_t = 1$ and $a_t = 0$ respectively). Accepting informal work provides her with higher income, $b_t + \chi_t$, but requires costly effort described below.

Second, the worker chooses her formal job-finding probability, p_t. Doing so also incurs an effort cost. Endogenising the worker's job-finding rate captures the same decision as allowing the worker to draw offers from a formal labour market wage distribution. An increase in p_t is equivalent to a reduction in her reservation wage. We will refer to p_t and search effort interchangeably.

Third, if the worker is part of the UI programme, she decides whether to remain in ($x_t = 0$) or voluntarily exit ($x_t = 1$). Accepting informal work whilst receiving UI constitutes fraudulent behaviour. If the worker remains in the programme, she risks her informal work being detected and thus facing sanctions. Her perceived probability of detection is denoted by $\delta > 0$. Whilst the exact structure of sanctions is specified in the quantitative analysis of Section 5, for the theoretical results it is sufficient that it involves exclusion from UI.

[6] Throughout, we use *unemployed* to describe a worker without a formal job, even though she may occasionally work in the informal sector. Similarly, we use *employed* to refer to a worker with a formal sector job.

The worker chooses $\{a_t, p_t, x_t\}_{t=1}^{\infty}$ to maximise her lifetime utility:

$$\mathbb{E}\left\{\sum_{t=1}^{\infty}\beta^{t-1}\left[u(y_t)-c(a_t,p_t)\right]\middle|\,\chi_t\right\}, \tag{1}$$

where $y_t = b_t + a_t\chi_t$ if unemployed, and $y_t = w$ if employed.

Each period, the worker derives utility from consuming her income, $u(y_t)$. Utility exhibits positive but diminishing marginal returns. She also incurs an additively separable effort cost, $c(a_t, p_t)$, jointly dependent upon her informal work and formal job-finding rate. $c(a_t, p_t)$ is assumed to be both increasing and convex in p_t (as, for example, in Chetty 2008) and increasing in a_t.

Working in the informal sector crowds out job-search, as in Álvarez-Parra and Sánchez 2009. [7] The marginal cost of p_t increases when $a_t = 1$, introducing a trade-off between formal job-search and informal work for the worker. In principle, working in the informal sector could be a stepping-stone towards finding formal employment. Whilst our quantitative analysis does not preclude this possibility, our calibration suggests that crowding-out effects dominate.

No search effort is required to generate informal opportunities, as in Álvarez-Parra and Sánchez 2009. This also seems consistent with the evidence presented in Table 4, which suggests that the majority of informal opportunities are offered to unemployed workers by friends and family.

The worker's problem, (1), can be represented recursively by the following system of Bellman equations:

$$V^e = u(w) + \beta\left[(1-\lambda)V^e + \lambda\mathbb{E}V_1^u\right], \tag{2}$$

$$V_t^{ui}(\chi_t) = \max_{a_t\in\{0,1\},p_t\in[0,1]}\left\{u(b^{ui}+a_t\chi_t) - c(a_t,p_t) + \beta(1-a_t\delta)[p_tV^e + (1-p_t)\mathbb{E}V_{t+1}^u] + \beta a_t\delta[p_tV^{es} + (1-p_t)V^{us}]\right\}, \tag{3}$$

$$V^{out}(\chi_t) = \max_{a_t\in\{0,1\},p_t\in[0,1]}\left\{u(a_t\chi_t) - c(a_t,p_t) + \beta[p_tV^e + (1-p_t)\mathbb{E}V^{out}]\right\}, \tag{4}$$

where:

$$\mathbb{E}V_{t+1}^u = \begin{cases}\mathbb{E}\left[\max_{x_{t+1}\in\{0,1\}}\left\{(1-x_{t+1})V_{t+1}^{ui}(\chi_{t+1}) + x_{t+1}V^{out}(\chi_{t+1})\right\}\right], & t+1\le T\\ \mathbb{E}V^{out}, & t+1>T,\end{cases} \tag{5}$$

and $\mathbb{E}(\cdot)$ denotes the expectation over χ_{t+1}.

If employed in the formal sector, the worker receives V^e given by (2). Each period, she derives utility from consuming her wage. With probability $1-\lambda$, she keeps her job in the following period. With probability λ, the job is destroyed and she enters the first period of unemployment. The value of employment is stationary, and would remain so if the model were extended to allow employed workers to accept informal work.

If she is eligible for, and chooses to remain in, the UI programme then she receives (3). She gains utility from consuming her income. She also exerts effort searching for

[7] Albrecht et al. 2009 go further by assuming that informal work entirely precludes job-search.

a formal job, receiving an offer with probability p_t. Otherwise she remains unemployed and can continue accept informal work receiving χ_{t+1}.

Accepting informal work ($a_t = 1$) whilst in receipt of UI may trigger a sanction with probability $a_t\delta$. Being sanctioned does not, however, prevent the worker from simultaneously accepting a formal job offer. We thus condition her continuation value from being sanctioned on her employment state. If she receives a job offer and is detected at the same time, she receives V^{es}. Otherwise, detection provides her with V^{us}. *Ex post*, the worker would always have preferred to either reject an opportunity or exit UI to accept it: $V^{es} < V^{e}$ and $V^{us} < \mathbb{E}V^{out}$. In line with the assumptions underpinning many dynamic models of the informal sector (Fugazza and Jacques 2004; Boeri and Garibaldi 2007; Albrecht et al. 2009), she is either caught immediately or her informal work goes undetected.

Outside of the UI programme, the worker receives (4). This has a similar interpretation to (3), but the worker receives no UI payments. Clearly, V^{out} is stationary.

The temporary nature of UI payments is captured by $\mathbb{E}V^{u}_{t+1}$ in (5). Suppose the worker chooses to remain in the UI in period $t < T$. In $t + 1 \leq T$, she receives informal opportunity χ_{t+1} and then decides whether to exit, choosing between $V^{ui}_{t+1}(\chi_{t+1})$ and $V^{out}(\chi_{t+1})$. If $t = T$ then her UI payments are exhausted and she automatically leaves the programme.

Summarising, the timing is as follows. Each period of unemployment begins with the worker receiving an informal opportunity that yields income $\chi_t \sim i.i.d.F(\chi)$. The worker decides whether to accept the opportunity (a_t), her formal job-finding probability (p_t) and whether to exit the UI programme (x_t). She then transitions to the next period, learning whether she has received a formal employment offer and whether she faces any sanctions for having accepted informal work.

4 Worker decisions

We describe here the decisions of the worker over the duration of unemployment. We first consider choices whilst receiving UI. We next describe decisions once benefits are exhausted. Finally, we focus on decisions at the point of leaving UI.

4.1 Decisions under UI

Whilst receiving UI, the worker makes three decisions every period: whether to accept or reject her current informal income opportunity, her job-finding rate, and whether to remain in the programme or to exit in favour of accepting informal work without facing sanctions. Since the latter decision requires an understanding of the worker's indirect utility outside the programme, its discussion is left until Section 4.3. We will instead proceed under the assumption that the worker chooses to remain in the UI.

That UI can be exhausted introduces non-stationarity into the worker's decision-making. We first outline the worker's choices within a given period of UI receipt. We then characterise the effect of this non-stationarity on the evolution of her choices.

4.1.1 Period-t job-finding and informal work

The worker's choices of p_t and a_t are interdependent. We will initially take the latter as given, deriving conditions for the worker's job-finding rate conditional on accepting or rejecting her current informal opportunity. This will enable us to then calculate the payoffs necessary to determine which opportunities the worker accepts. Finally, we will combine these finding to assess the worker's overall, unconditional job-finding rate.

Taking a_t as given, the worker's optimal *conditional job-finding rate*, $p_t^{ui}(a_t)$, satisfies:

$$\beta\left[(1-a_t\delta)\left(V^e-\mathbb{E}V_{t+1}^u\right)+a_t\delta\left(V^{es}-V^{us}\right)\right]=c_p\left[a_t,p_t^{ui}(a_t)\right], \tag{6}$$

where $c_p(a,p)=\partial c/\partial p$ is the marginal cost of search. By searching more intensively, the worker increases the probability that she gets a formal job offer. If she is not detected working in the informal sector (with probability $1-a_t\delta$), she starts work in the following period, increasing her lifetime utility from $\mathbb{E}V_{t+1}^u$ to V^e. If she is detected and sanctioned (with probability $a_t\delta$) she still starts her job, increasing her lifetime utility from V^{us} to V^{es}. Note that, if $V^{es}-V^{us}>V^e-\mathbb{E}V_{t+1}^u$, the marginal benefit of search is higher when the worker accepts informal work, as the worker wishes to insure themselves against harsher sanctions.

Searching more intensively requires additional effort. If the worker accepts informal work then her marginal cost is higher still, consistent with the crowding out of job-search.

That both the marginal benefit and marginal cost of search potentially increase when the worker accepts informal work leads to ambiguous theoretical results. To avoid this, we will occasionally bound the difference between sanctions in each employment state:

Assumption 1 *The increase in the marginal cost of search caused by accepting χ_t always exceeds the marginal benefit deriving from differences in sanctions between employment and unemployment:*

$$\beta\delta\left(V^{es}-V^{us}\right)<c_p(1,0)-(1-\delta)c_p(0,0).$$

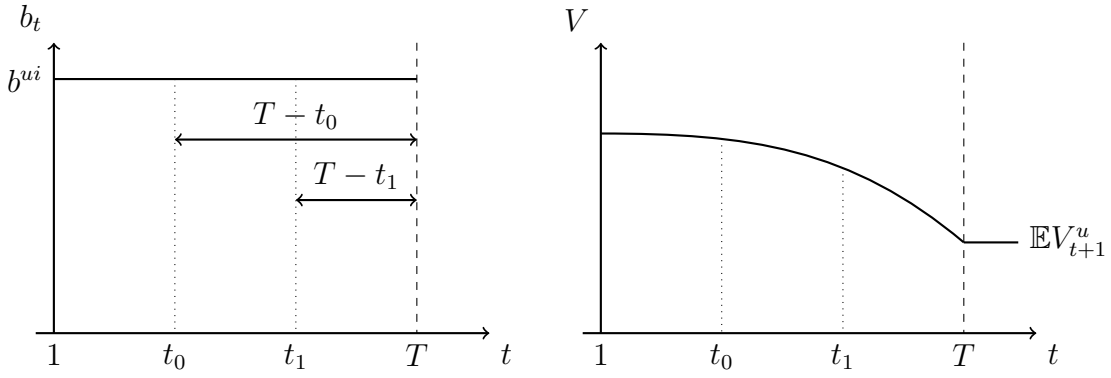
Intuitively, Assumption 1 states that the incentive to increase search effort to avoid facing harsher sanctions is always dominated by the increase in the cost of effort. As a result, the worker's conditional job-finding rate is lower whenever she accepts informal work. Whilst our quantitative analysis in Section 5 does not impose this assumption, we find that $p_t^{ui}(1)<p_t^{ui}(0)$ in the calibrated model.

Turning to informal sector work, the worker can increase her instantaneous utility if she accepts χ_t whilst inside the UI programme. But she could also be detected and sanctioned. She will accept informal work if and only if:

$$\begin{aligned}
u(b^{ui}+\chi_t)-c\left[1,p_t^{ui}(1)\right]+\beta(1-\delta)\left\{p_t^{ui}(1)V^e+\left[1-p_t^{ui}(1)\right]\mathbb{E}V_{t+1}^u\right\}& \\
+\beta\delta\left\{p_t^{ui}(1)V^{es}+\left[1-p_t^{ui}(1)\right]V^{us}\right\}& \\
\geq u(b^{ui})-c\left[0,p_t^{ui}(0)\right]+\beta\left\{p_t^{ui}(0)V^e+\left[1-p_t^{ui}(0)\right]\mathbb{E}V_{t+1}^u\right\}& \qquad (7)\\
\iff \chi_t\geq\chi_t^{ui}.& \qquad (8)
\end{aligned}$$

Whilst accepting informal work enables the worker to enjoy higher instantaneous utility

Figure 1: The continuation value of unemployment under temporary insurance



(a) Payment

(b) Continuation value

from consumption, it increases her marginal cost of effort. She generates a formal job offer with a lower probability, reducing her expected future payoff. She also faces the risk of sanctions. She therefore only accepts informal income opportunities that provide a sufficiently large increase in instantaneous consumption to compensate for these adverse effects. There exists a *reservation informal opportunity* in each period, χ_t^{ui}, such that she accepts χ_t if and only if $\chi_t \geq \chi_t^{ui}$.

Equation (6) and inequality (8) jointly solve the worker's problem inside the UI programme. In each period, t, she accepts a proportion $1-F(\chi_t^{ui})$ of informal opportunities. If she accepts informal work, she generates a formal job offer with probability $p_t^{ui}(1)$. Otherwise, she generates an offer with probability $p_t^{ui}(0)$. Her overall, *unconditional job-finding rate* is:

$$p_t^{ui} = [1 - F(\chi_t^{ui})]p_t^{ui}(1) + F(\chi_t^{ui})p_t^{ui}(0). \tag{9}$$

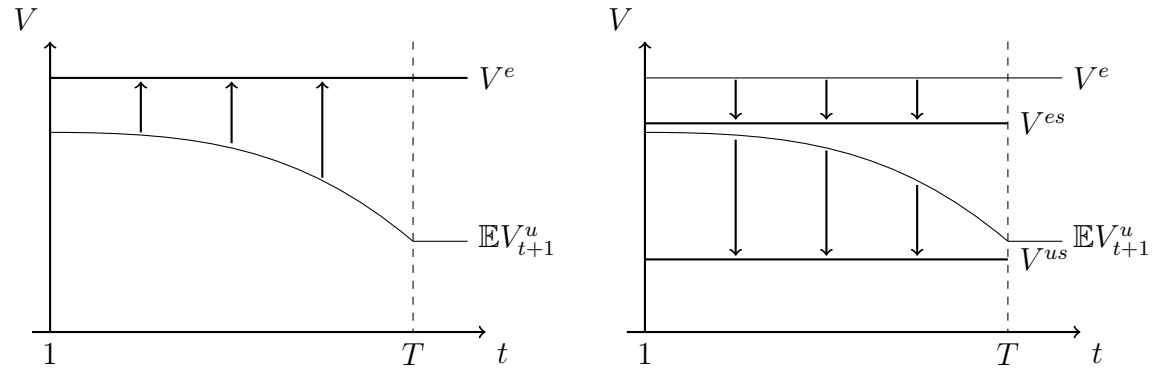
A greater willingness to accept informal work (a reduction in χ_t^{ui}) lowers the probability that the worker will find formal work, by increasing the weighting on $p_t^{ui}(1)$.

4.1.2 Evolution of job-finding and informal work

We now consider how the worker's job-search and informal sector participation vary over the duration of the UI programme. Her problem is non-stationary, as illustrated in Figure 1. Panel (a) shows the worker's payment schedule. For the first T periods of unemployment, she receives b^{ui}. If she is still unemployed thereafter, she receives nothing. Consider the worker's situation when she has been unemployed for t_0 periods. She receives b^{ui} from the UI programme, and still has $T - t_0$ future payments of b^{ui} to look forward to if she is unable to secure a job. After remaining unemployed until period t_1, the worker has consumed a further $t_1 - t_0$ UI payments. She now has only $T - t_1$ remaining before exhaustion.

Clearly, the prospect of remaining unemployed is less appealing in period t_1 than in period t_0. More generally, with each passing period of unemployment, she has one fewer UI payment remaining before exhaustion. As Panel (b) in Figure 1 illustrates, the continuation value of remaining unemployed, $\mathbb{E}V^u_{t+1}$, is declining, even though the

Figure 2: The effects of non-stationarity



(a) Gain from employment

(b) Loss from detection

worker's instantaneous UI payment remains constant.

This feature of temporary UI introduces non-stationarity into the worker's decision-making. To analyse its effects we adopt a similar approach to the previous section. First, we consider the worker's conditional job-finding rates, i.e. her choice of $p^{ui}_t(a_t)$ given $a_t = 0$ and $a_t = 1$. Second, we evaluate which informal opportunities she accepts. Finally, we combine these results to draw conclusions about her unconditional job-finding rate.

The declining continuation value of unemployment implies a *re-employment spike* in conditional job-finding rates:[8]

Lemma 1 (Re-employment spike) *For each $a_t \in \{0, 1\}$, the worker's conditional job-finding rate, $p^{ui}_t(a_t)$, is strictly increasing over the duration of the UI programme.*

The lemma is stated without proof. It is illustrated by Panel (a) of Figure 2, which shows both the stationary continuation value of employment, V^e, and the non-stationary continuation value of unemployment. Receiving a formal job offer causes the worker's lifetime utility to immediately increase from $\mathbb{E}V^u_{t+1}$ to V^e. Her gain from finding formal employment is thus $V^e - \mathbb{E}V^u_{t+1}$ (the gap between the two lines). As the continuation value of unemployment declines, this gain increases. In (6), the marginal benefit of job-search gets larger over time, resulting in higher conditional job-finding rates.

This non-stationarity also has implications for how the worker's participation in the informal sector evolves over the duration of UI. $\mathbb{E}V^u_{t+1}$ features in both sides of inequality (8), which determines the worker's reservation informal opportunity. There are consequently two opposing effects.

We refer to the first as the *exhaustion effect*. It is entirely due to the exhaustion of UI payments, and can also be seen with reference to Panel (a) of Figure 2. When the worker accepts informal work, her conditional job-finding rate falls from $p^{ui}_t(0)$ to $p^{ui}_t(1)$. Part of the opportunity cost of informal work is thus foregoing the greater chance of enjoying

[8]The re-employment spike results was first stated by Mortensen (1977). A large literature has followed since. See, for example, Lentz and Tranæs (2005); Card et al. (2007b,a); Chetty (2008); Kroft et al. (2013).

the gain from formal employment. During the early stages of UI, this opportunity cost is relatively small as continuation value of unemployment is high. The worker is thus willing to accept relatively small informal opportunities. As the duration of her unemployment increases, however, the worker has fewer future UI payments remaining. The opportunity cost of not searching intensively for a formal job (the gap between V^e and $\mathbb{E}V^u_{t+1}$) increases. This makes her less willing to accept informal work over time.

The second, the *deterrence effect*, derives from the nature of sanctions, and is illustrated by Panel (b) Figure 2. If the worker accepts informal work and is subsequently detected, her continuation value jumps either from V^e to V^{es} or from $\mathbb{E}V^u_{t+1}$ to V^{us} depending on whether her job-search is successful. The utility loss from detection when the worker simultaneously receives a formal offer is constant (the gap between V^e and V^{es}). However, the loss when the worker remains formally unemployed declines over the duration of UI receipt (the gap between $\mathbb{E}V^u_{t+1}$ and V^{us}). Intuitively, during the first few periods of unemployment, the worker risks a lot by accepting informal work. She has a lot of future UI payments to look forward to, and would lose them all if she were detected and failed to secure a formal job. She only accepts very profitable opportunities, as they sufficiently compensate her for this risk. As her duration of unemployment increases, she has less to lose. This makes her more willing to accept informal work over time.

The overall evolution of the worker's informal sector participation depends upon whether the exhaustion or deterrence effect dominates. Whilst the exhaustion effect is determined purely by the structure of the unemployment protection programme, the deterrence effect also depends upon the probability the worker assigns to detection, δ. To fully characterise the dynamics of informal work and formal job-finding, it is therefore necessary to distinguish between three cases. First suppose that the perceived detection probability is relatively low:

Proposition 1 (Low probability of detection) *If the perceived probability of detection is small, $\delta < \underline{\delta}$, then the worker's informal sector participation decreases over the duration of UI. For all $t \leq T$:*

$$1 - F\left(\chi^{ui}_t\right) > 1 - F\left(\chi^{ui}_{t+1}\right).$$

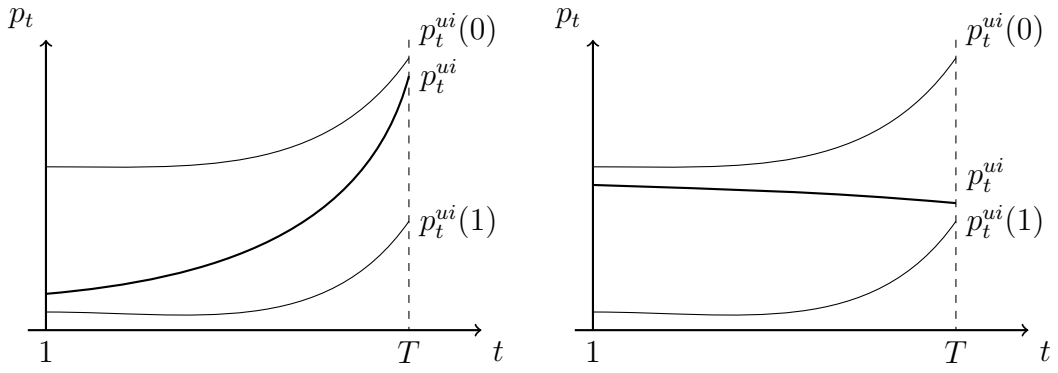
With a low perceived probability of detection, the exhaustion effect dominates. The worker is unconcerned by sanctions, and thus the deterrence effect is muted. As the worker approaches period T, she becomes more concerned about finding employment. Her informal sector participation declines.

Under this scenario, the worker's informal sector participation interacts with the usual incentive to search for a formal job to create an even more pronounced re-employment spike:

Corollary 1 *Under Assumption 1, if the perceived probability of detection is small then the worker's unconditional job-finding rate increases over the duration of UI.*

The corollary is illustrated in Panel (a) of Figure 3. This shows the worker's conditional job-finding rates. Following Lemma 1, both exhibit a re-employment spike: as exhaustion approaches, the worker searches more intensively conditional on either always accepting or always rejecting her informal opportunity.

Figure 3: Unconditional job-finding rate under temporary insurance



(a) Exhaustion effect dominates

(b) Deterrence effect dominates

If the exhaustion effect dominates, the worker's informal sector participation declines over time. The weights in (9) shift away from $p_t^{ui}(1)$ towards $p_t^{ui}(0)$, causing her unconditional job-finding rate, p_t^{ui}, to increase more rapidly as exhaustion approaches.

Suppose instead that the worker's perceived probability of detection is relatively large:

Proposition 2 (High probability of detection) *If the perceived probability of detection is large, $\delta > \overline{\delta}$, then the worker's informal sector participation increases over the duration of UI. For all $t \leq T$:*

$$1 - F\left(\chi_t^{ui}\right) < 1 - F\left(\chi_{t+1}^{ui}\right).$$

The worker believes that she is likely to lose all future UI payments as a result of being detected engaging in informal work. The deterrence effect dominates. As the worker approaches period T, she becomes less concerned about sanctions. Her informal sector participation increases.

This time, the worker's informal sector participation counteracts the usual non-stationary incentive to search for a job. The re-employment spike is less pronounced, and may even disappear entirely:

Corollary 2 *Under Assumption 1, if the perceived probability of detection is large then the worker's unconditional job-finding rate need not increase over time.*

The corollary is illustrated in Panel (b) of Figure 3. If the deterrence effect dominates, the worker's informal sector participation increases over time. The weights in (9) shift towards $p_t^{ui}(1)$ and away from $p_t^{ui}(0)$, causing her unconditional job-finding rate to increase less rapidly than the conditional rates, or even fall, as exhaustion approaches.

Finally, for intermediate perceived probabilities of detection ($\underline{\delta} < \delta < \overline{\delta}$) it is not generally possible to describe the evolution of informal sector participation without making restrictive assumptions on the marginal cost of search. The size of both the exhaustion and deterrence effects vary over the duration of UI and, depending on which

dominates, the worker could accept more or fewer informal opportunities at any given point in time. More precisely, the exhaustion effect dominates in period t if and only if:

$$\begin{aligned}(1-\delta)\left[1-p_t^{ui}(1)\right] &> 1-p_t^{ui}(0) \\ &\iff \delta < 1-\frac{1-p_t^{ui}(0)}{1-p_t^{ui}(1)}. \end{aligned} \tag{10}$$

If the worker accepts her informal opportunity, she remains in the UI programme with probability $(1-\delta)[1-p_t^{ui}(1)]$. If she rejects, she remains in the programme with probability $1-p_t^{ui}(0)$. If (10) holds, the worker is more likely to remain in if she accepts her informal opportunity. Rejecting makes it likely that she will transition to formal employment. The declining continuation value of unemployment, $\mathbb{E}V_{t+1}^u$, therefore causes her payoff from accepting a given informal opportunity in (8) to fall relative to rejecting it. She becomes less willing to accept informal work over time.

Conversely, if (10) does not hold, she is more likely to remain in the UI if she rejects her informal opportunity. Accepting makes it likely that she will be detected and excluded from the programme. The declining continuation value of unemployment, $\mathbb{E}V_{t+1}^u$, causes her payoff from rejecting a given informal opportunity in (8) to fall relative to accepting it. She becomes more willing to accept informal work over time.

4.2 Decisions outside UI

If the worker exhausts UI, or chooses to leave voluntarily, she no longer receives benefit payments. Her problem is stationary. She has two decisions: her job-finding probability in the formal sector and whether she accepts her current informal income opportunity. Proceeding as before by taking a_t as given, her optimal conditional job-finding rate, $p^{out}(a_t)$, satisfies:

$$\beta\left(V^e - \mathbb{E}V^{out}\right) = c_p\left[a_t, p^{out}(a_t)\right]. \tag{11}$$

The intuition is very similar to that of (6). With no threat of sanctions, the marginal benefit of search derives solely from the utility gain associated with formal employment, $V^e - \mathbb{E}V^{out}$.

The worker's decision to undertake informal work similarly depends upon her likelihood of receiving a formal job offer. She will accept χ_t if and only if it offers a higher discounted utility:

$$\begin{aligned} u(\chi_t) - c\left[1, p^{out}(1)\right] + \beta\left\{p^{out}(1)V^e + \left[1-p^{out}(1)\right]\mathbb{E}V^{out}\right\} & \\ \geq u(0) - c\left[0, p^{out}(0)\right] + \beta\left\{p^{out}(0)V^e + \left[1-p^{out}(0)\right]\mathbb{E}V^{out}\right\} & \quad (12) \\ \iff \chi_t \geq \chi^{out}. & \quad (13) \end{aligned}$$

As in (8), the worker only accepts informal opportunities above a reservation level, χ^{out}. In this case, however, the opportunity must only compensate the worker for the lower conditional job-finding probability associated with informal work.

Equation (11) and inequality (13) jointly solve the worker's problem outside the UI programme. She accepts a proportion $1-F(\chi^{out})$ informal opportunities, and finds a

job with an average, unconditional job-finding rate:

$$p^{out} = \left[1 - F(\chi^{out})\right] p^{out}(1) + F(\chi^{out}) p^{out}(0). \tag{14}$$

4.3 Leaving unemployment insurance

The worker can leave UI for three reasons. She might have reached benefit exhaustion, be detected working in the informal sector, or choose to exit voluntarily. Regardless of the reason, leaving significantly alters the worker's incentive to accept informal work. She now no longer receives UI payments. This makes informal opportunities more valuable:

Proposition 3 (Payment effect) *If the worker is still formally unemployed after her UI payments are exhausted, her informal sector participation strictly increases:*

$$1 - F\left(\chi_T^{ui}\right) < 1 - F\left(\chi^{out}\right).$$

The worker is subject to diminishing marginal utility: $u''(y_t) < 0$. As she no longer receives UI payments, her marginal utility of consumption thus increases. A given informal opportunity provides her with greater utility, helping to compensate for a lower conditional job-finding rate. Her willingness to accept informal work strictly increases immediately after leaving the UI programme. As this result has important implications for the quantitative analysis of Section 7, we call it the *payment effect*.

Accepting more informal opportunities, of course, comes at a cost. It is more difficult to search for a formal job:

Corollary 3 (Job-finding after leaving) *The worker's unconditional job-finding rate may decrease after leaving the UI programme.*

Conditional on informal work, the worker's job-finding rates are higher outside the programme than inside: $p^{out}(0) \geq p_t^{ui}(0)$ and $p^{out}(1) > p_t^{ui}(1)$ for all $t \leq T$. Without UI payments, the marginal benefit of search has increased in (11) relative to (6). However, the worker undertakes more informal work. She is thus more likely to choose the job-finding rate $p^{out}(1) < p^{out}(0)$. As (14) makes clear, this has the potential to lower her unconditional job-finding rate outside of the UI scheme.

Of course, there is no guarantee that the worker exhausts her UI payments. She may find a formal job. In each period, with probability $\delta[1 - F(\chi_t^{ui})]$, she is detected accepting informal work and expelled from the scheme. She may also voluntarily leave UI to avoid the possibility of sanctions. In a given period, t, and for a given informal opportunity, χ_t, she will voluntarily exit ($x_t = 1$) if and only if:

$$V^{out}(\chi_t) > V_t^{ui}(\chi_t).$$

Whilst V^{out} is stationary, the declining continuation payoff from remaining in the UI programme causes $V_t^{ui}(\chi_t)$ to fall over time. Once again, the worker's decision to exit is non-stationary:

Proposition 4 (Voluntary exit from UI) *As unemployment duration increases, the probability that the worker voluntarily exits the UI programme weakly rises, and is strictly positive by period T.*

To illustrate, consider the worker's period-T decision, given a particular informal opportunity, χ_T. As she faces immediate UI exhaustion, she knows that she will not receive future payments: $\mathbb{E}V^u_{T+1} = \mathbb{E}V^{out}$. If she accepts her current opportunity, then her payoffs from remaining in the programme and voluntarily exiting are, respectively:

$$u(b^{ui} + \chi_T) - c[1, p^{ui}_T(1)] + \beta(1-\delta)\{p^{ui}_T(1)V^e + [1 - p^{ui}_T(1)]\mathbb{E}V^{out}\} \\ +\beta\delta\{p^{ui}_T(1)V^{es} + [1 - p^{ui}_T(1)]V^{us}\}, \tag{15}$$

and

$$u(\chi_T) - c[1, p^{out}(1)] + \beta\{p^{out}(1)V^e + [1 - p^{out}(1)]\mathbb{E}V^{out}\}. \tag{16}$$

It is straightforward to show that, excluding instantaneous utility from consumption, the worker is better off exiting the programme. She no longer faces sanctions, and is more likely to find a job. The only benefit to remaining in the UI is receiving b^{ui} today. If χ_T is sufficiently large, then the extra utility from receiving b^{ui}, $u(b^{ui} + \chi_T) - u(\chi_T)$, does not compensate her for the risk of detection because her marginal utility of consumption is too small. She prefers to voluntarily exit the programme early.

5 Quantitative analysis

The model presented in the previous sections describes the decisions of a worker as she passes through the unemployment protection system. We now provide a quantitative analysis of the model, calibrated to Spain in 2013. The Spanish unemployment protection system has a structure similar to that of a typical advanced economy, as is the extent of engagement in temporary informal work by unemployed workers. The case of Spain is especially interesting for policy analysis, however, due to the higher than average unemployment rate and informal sector size. Unlike many other European countries, information on sanctions is also readily available through the Spanish Public Employment Service (2017a,b).

5.1 Functional forms

Like most other advanced economies, the Spanish unemployment protection system consists of two temporary programmes: UI and UA (see Table 1). We model this as follows. For the first T^{ui} periods of unemployment, the worker receives payments of b^{ui}. She then receives UA for the next T^{ua} periods of unemployment, paying b^{ua}. Spain also has a SA programme for long-term unemployed known as Minimum Insertion Income, which aims to provide recipients with a subsistence income (OECD 2014). If she exhausts unemployment protection, the worker in our model then moves into SA, receiving b^{sa} until she finds a formal job.

Instantaneous utility from consumption is CRRA, with coefficient of relative risk aversion, σ: $u(y_t) = y_t^{1-\sigma}/(1-\sigma)$.

The derivation of the effort cost of job-search, $c(a_t, p_t)$ is done into two stages, employing commonly used functional forms (see, for example, Hopenhayn and Nicolini 1997; Álvarez-Parra and Sánchez 2009). First, suppose that the worker chooses search effort, $s_t \geq 0$. This comes at an instantaneous cost of $\hat{c}(a_t, s_t) = \frac{1+a_t\alpha}{\gamma} s_t^{\gamma}$. α measures how accepting informal work affects the total and marginal cost of search effort. If the worker rejects her informal opportunity (i.e. $a_t = 0$) then job-search costs her $\hat{c}(0, s_t) = \frac{1}{\gamma} s_t^{\gamma}$. If, instead, she accepts (i.e. $a_t = 1$) then it costs $\hat{c}(1, s_t) = \frac{1+\alpha}{\gamma} s_t^{\gamma}$. Accepting informal work increases the cost of job-search if and only if $\alpha > 0$. This effort is then transformed into a job-finding probability through the search technology, $p(s_t) = 1 - \exp(-\rho s_t)$. Taking the inverse of $p(s_t)$ and substituting into $\hat{c}(a_t, s_t)$ yields our effort cost of search: $c(a_t, p_t) = \hat{c}[a_t, s(p_t)]$.

Informal income opportunities are assumed to be log-normally distributed: $\chi_t \sim \ln N(\mu, \nu^2)$. As no empirical evidence exists on the distribution of income from sporadic work in the informal sector in advanced economies, we also calibrated the model using both uniform and exponential distributions. Whilst the former placed too much weight on high-value opportunities, the latter yielded very similar results.

Sanctions are modelled using information provided to recipients by the Spanish Public Employment Service (2017a,b). Agents caught accepting informal work whilst receiving UI or UA lose their eligibility for unemployment protection payments. They are also required to repay any payment they received when their informal work was detected. Although Spain does not impose fines, our specification allows for the introduction of a proportional fine, ϕb_t. The total repayment is thus: $(1+\phi)b_t$. Since the worker has no savings and cannot borrow, the repayment is made when the worker becomes employed. Having repaid, she re-qualifies for unemployment protection with probability ζ each period.[9]

5.2 Steady-state solution

To derive the model's stationary solution, we built a new numerical algorithm for solving dynamic job-search models with temporary stochastic income opportunities from the informal sector. Whilst a detailed description is provided in Appendix B, we give a brief

[9]The worker's discounted payoffs from employment and unemployment respectively, following detection in the unemployment protection system are:

$$V^{es,up} = u\left[w - (1+\phi)b^{up}\right] + \beta\zeta[\lambda\mathbb{E}V_1^u + (1-\lambda)V^e] + \beta(1-\zeta)[\lambda\mathbb{E}V^{u,out} + (1-\lambda)V^{e,out}],$$
$$V^{us,up}(\chi_t) = \max_{a_t,p_t}\left\{u(b^{sa} + a_t\chi_t) - c(a_t,p_t) + \beta\left[p_t V^{es,up} + (1-p_t)\mathbb{E}V^{us,up}\right]\right\},$$

where $up \in \{ui, ua\}$ denotes UI and UA respectively and:

$$V^{e,out} = u(w) + \beta\zeta[\lambda\mathbb{E}V_1^u + (1-\lambda)V^e] + \beta(1-\zeta)[\lambda\mathbb{E}V^{u,out} + (1-\lambda)V^{e,out}],$$
$$V^{u,out}(\chi_t) = \max_{a_t,p_t}\left(u(b^{sa} + a_t\chi_t) - c(a_t,p_t) + \beta\left\{p_t\left[\zeta V^e + (1-\zeta)V^{e,out}\right] + (1-p_t)\mathbb{E}V^{u,out}\right\}\right),$$

represent the continuation payoffs from employment and unemployment whilst ineligible for unemployment protection.

outline here. The algorithm consists of two parts. The first solves the problem of a representative worker, as outlined in Section 3, extended to include UA and SA, and to account for the structure of sanctions. This is then used in the second part to construct a stationary distribution of agents across states, from which aggregate moments are calculated.

The main loop iterates until convergence on the continuation value of the first period of unemployment protection, $\mathbb{E}V_1^u$. Taking this as given, the value of employment, V^e, is first calculated. We next solve the worker's problem under SA and sanctions, deriving the expected value of being in each state. We then work backwards through the non-stationary part of the model, beginning with the final period of unemployment protection.

The conditional job-finding rates, reservation informal opportunities and exit decisions from each period are used to populate the Markov transition matrix for the worker's progression through the unemployment protection system. The aggregate moments used for the calibration and subsequent policy analysis are derived from the stationary distribution of workers across employment states.

In total, the model captures 220 states: each period of UI or UA receipt; SA receipt; exclusion from UI or UA without a formal job; exiting unemployment protection having received each possible informal opportunity; employed; employed and paying fines following exclusion from UI or UA; employed but still excluded from unemployment protection following payment of fines. This rich description of workers' states allows us to break down the unemployed workforce, the informal sector participation rate and the earning gap between formal and informal employment into those receiving UI, UA and SA. This enables us to evaluate changes in labour market policy that target specific components of the unemployment protection system, as illustrated in Section 7.

5.3 Calibration

The model is calibrated on a monthly basis. We use data where available. Parameters for which data is not available are measured endogenously.

The three behavioural parameters are calibrated following Álvarez-Parra and Sánchez (2009). Thus, the discount factor is $\beta = 0.994$, the coefficient of relative risk aversion is $\sigma = 2$ and the effort cost parameter is $\gamma = 2$.

The formal wage is normalised to $w = 100$. We calibrate the exogenous formal job destruction rate to be $\lambda = 0.0139$. This yields an average employment duration of six years, in line with Eurostat data for Spain (European Commission 2017).

As we normalise the formal sector wage to 100, we set $b^{ui} = 56.76$ and $b^{ua} = 22.08$, as in Table 1. We calibrate the replacement ratio for SA in a similar manner, using data from OECD (2014). This yields $b^{sa} = 19.30$.

The maximum duration of UI and UA in Spain depend jointly upon continuing to meet eligibility criteria and the worker's history of contributions whilst employed. Although the statutory maximum durations reported in Table 1 are 24 months and 18 months for UI and UA respectively, these do not represent the average recipient's experience. We consequently derive the effective durations, T^{ui} and T^{ua}, endogenously, discussed in more detail below.

Table 6: Calibrated parameter values

Parameter	Value	Definition	Source
α	24	Effort cost parameter when $a_t = 1$	Endogenous
β	0.994	Discount factor	APS (2009)
γ	2	Effort elasticity	APS (2009)
δ	0.037	Detection probability	Endogenous
ζ	0.0833	Eligibility hazard rate	SPES (2017)
λ	0.0139	Job separation rate	EC (2017)
μ	3.23	χ distribution parameter	Endogenous
ν	1.58	χ distribution parameter	Endogenous
ρ	0.40	Search effort parameter	Endogenous
σ	2	Coefficient of relative risk aversion	APS (2009)
ϕ	0	Fine rate	SPES (2017)
b^{ui}	56.76	UI payment	EC (2016)
b^{ua}	22.08	UA payment	EC (2016)
b^{sa}	19.30	SA payment	EC (2016)
T^{ui}	5	Duration of UI	Endogenous
T^{ua}	8	Duration of UA	Endogenous
w	100	Formal after-tax wage	Normalisation

Notes: APS refers to Álvarez-Parra and Sánchez (2009), EC to European Commission (2016, 2017) and SPES to Spanish Public Employment Service (2017a,b).

Sanctions are calibrated using information from Spanish Public Employment Service (2017a,b). Those caught accepting informal work lose eligibility for unemployment protection for a maximum of twelve months. We therefore set $\zeta = 0.0833$. They are required to repay benefits to which they were not entitled, but are not subject to additional fines ($\phi = 0$). Whilst the Spanish system allows for partial repayments, we assume that the unemployment protection payment is always returned in full for tractability of the numerical algorithm.[10]

Parameter values are summarised in Table 6. T^{ui} and T^{ua}, along with five other endogenous parameters (α, δ, μ, ν and ρ) are chosen to minimise the percentage distance between the following simulated and empirical moments: the unemployment rate; the informal sector income generated by the unemployed as a proportion of GDP; the cost of the unemployment protection system; the shares of unemployed agents receiving UI and UA. All empirical moments were drawn from Eurostat, apart from informal sector

[10]Partial UI and UA – reductions in benefits following declaration of paid work – is designed to encourage reporting of waged- or self-employment. Excluding these from the quantitative model is equivalent to assuming that the bureaucratic cost of declaring small jobs for friends and family while unemployed exceeds the benefit for the worker.

Table 7: Estimated and empirical moments

Moment	Model	Data
Unemployment rate	22.55	26.09
Unemployed receiving:		
UI	19.42	21.17
UA	19.20	19.51
SA	61.38	59.32
Cost of unemployment protection:		
Total	2.84	2.69
UI	2.05	1.84
UA	0.79	0.85
Informal sector income of unemployed	5.11	5.11

Notes: Rates are reported as a percentage. Incomes and costs are reported as a percentage of GDP.

income generated by the unemployed (see Table 2).

In calibrating the endogenous parameters, we allowed for $\alpha < 0$. This reflects the possibility that informal work could enhance formal job-finding, the so-called stepping stone effect. Our baseline parametrisation finds $\alpha = 24$, suggesting that engaging in informal work significantly increases the cost of search. We also find a perceived detection rate of 3.7 percent, consistent with existing estimates (Fugazza and Jacques 2004; Boeri and Garibaldi 2007; Bosch and Esteban-Pretel 2012, 2015; Di Porto et al. 2017). The average effective durations of UI and UA are found to be below the statutory maxima ($T^{ui} = 5$ and $T^{ua} = 8$ respectively), reflecting agents' heterogeneous employment histories and family circumstances. We note that the Spanish UI also has two tiers, whereby payments may reduce after six months (OECD 2014). The calibrated T^{ui} suggests that the average unemployed worker leaves the UI before this switch.

6 Baseline results

Table 7 shows the model's performance in replicating the targeted empirical moments under the baseline calibration. The model matches the informal sector income of the unemployed, the total cost of unemployment protection and the shares of individuals receiving UI and UA payments quite closely. The unemployment rate is slightly underestimated. The model is also able to reproduce the decomposition of the cost of unemployment protection between UI and UA, even though this was not targeted explicitly.

Table 8 reports the model's predictions regarding unemployed agents' participation in, and earnings from, the informal sector, for which data is not available. The informal

Table 8: Predicted informal sector moments

Moment	Model
Participation rate of the unemployed	40.53
Share of workforce in:	
UI	0.00
UA	18.34
SA	81.66
Informal/formal wage ratio:	
Average	67.64
UI recipients	–
UA recipients	77.60
SA recipients	65.40

Notes: All moments are reported as a percentages.

sector participation rate suggests that around 40 percent of unemployed agents accept informal work in a given month. Perhaps not surprisingly, this is lower than participation rates found in developing countries (for example, Meghir et al. (2015) report that 80 percent of the Brazilian workforce who are not formally employed work in the informal sector) and closer to estimates of unemployed participation in advanced economies (for example, see Lemieux et al. 1994).[11]

The vast majority of unemployed workers participating in the informal sector do not receive unemployment protection.[12] Those who receive protection payments account for around 20 percent of the informal workforce. These are all recipients of the less-generous UA programme. The dynamics of informal sector participation are discussed in more detail below.

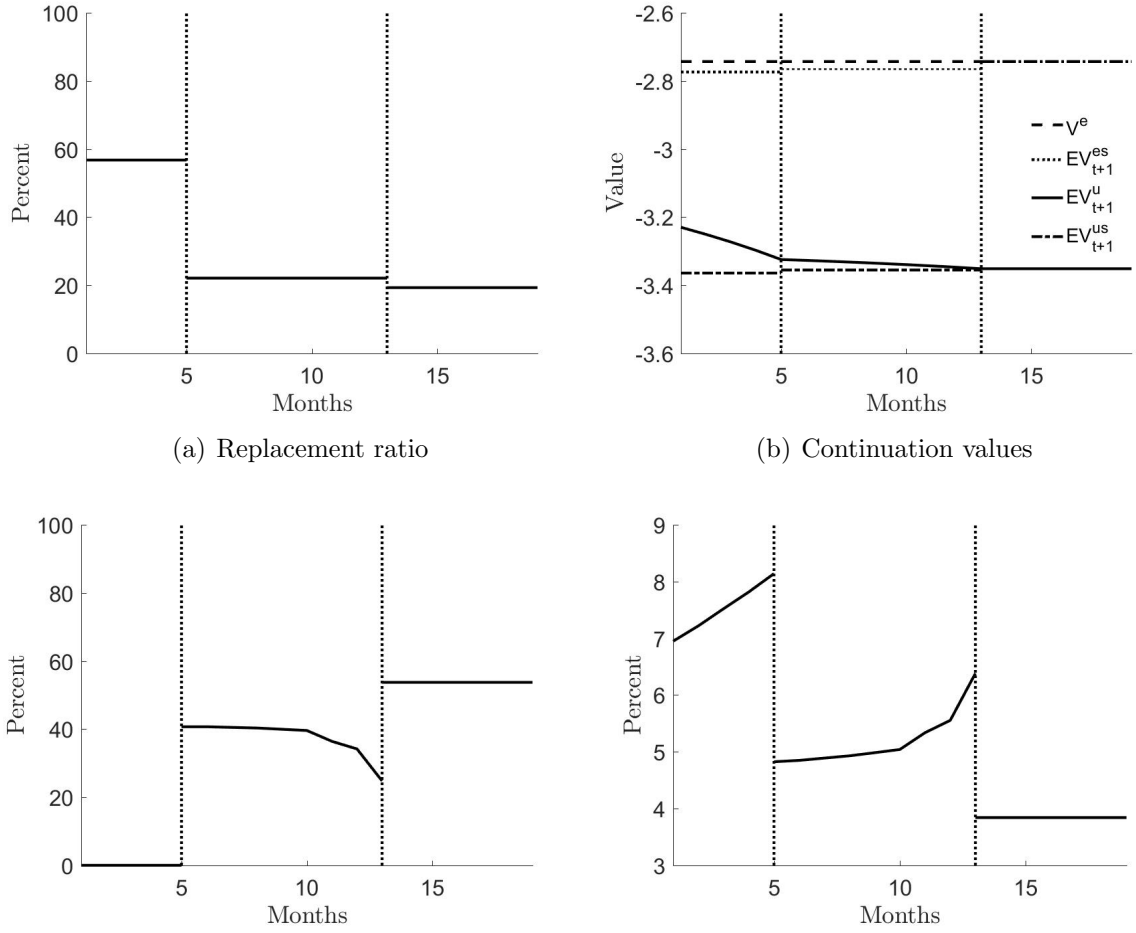
The informal/formal wage ratio, representing the average accepted informal opportunity, as a proportion of the formal sector wage, is 67.44 percent. In other words, under the baseline calibration, unemployed workers earn about 30 percent less on average from the informal labour market, relative to the formal sector. This is within the range of existing estimates (for developing countries, see Albrecht et al. 2009; Meghir et al. 2015), and is consistent with informal work being small jobs undertaken for friends and relatives.

The informal/formal wage ratio for those outside unemployment protection is lower

[11]The participation rate calculated in the model is not directly comparable with the proportion of individuals declaring informal work presented in Table 2. The latter measures the proportion of people who were unemployed at the time of the survey and declared undertaking informal work in the previous twelve months. No information is available regarding their employment history over this period.

[12]Of these, around 12 percent were previously detected working in the informal sector and excluded from the protection system. The remainder either exhausted payments or chose to leave. In particular, around 0.7 percent of unemployment protection recipients exit voluntarily in each period to accept an informal opportunity, in line with Proposition 4.

Figure 4: Payoffs and decisions over the duration of unemployment



(a) Replacement ratio

(b) Continuation values

(c) Informal sector participation rate

(d) Formal sector job-finding rate

on average than for those receiving payments. This reflects the fact that the incentive to undertake informal work increases when payments are reduced, as outlined in Proposition 3.

Figure 4 shows how the payoffs and decisions of a representative unemployed worker evolve over the duration of her unemployment spell under the baseline calibration. Panel (a) gives her replacement ratio as she moves through the unemployment protection system, and each programme's effective duration. She receives UI for the first five months, providing her with 57 percent of the formal wage. Should she exhaust this, she then enters in UA for a further eight months, receiving 22 percent of the formal wage. Finally, if she is still unemployed after 13 months, she become eligible for SA, at a replacement ratio of 19 percent.

When deciding her job-finding rate and informal participation, the worker considers the continuation values of the various employment states she may find herself in. These are shown in Panel (b). Employment provides stationary utility (V^e). The continuation value of unemployment ($\mathbb{E}V^u_{t+1}$) is the main source of non-stationarity in the model. As

the worker receives unemployment protection, the prospect of remaining unemployed becomes gradually less appealing because benefit payments decline over time.

If she engages in informal work and is detected, her continuation value depends upon whether she finds a job or not. If she finds formal work, her continuation value ($\mathbb{E}V^{es}_{t+1}$) is lower than V^e, but increases over the duration of her unemployment. This is because the fine the worker is charged is related to her overpaid benefits. Since benefit payments decline over time, her fine become smaller. Thus $\mathbb{E}V^{es}_{t+1}$ approaches V^e. If she does not find formal work, the worker is excluded from unemployment protection and no longer faces benefit exhaustion in the future. Her continuation value ($\mathbb{E}V^{us}_{t+1}$) is still non-stationary, but lower than $\mathbb{E}V^{u}_{t+1}$ and increasing over time, reflecting changes in the fine.

Panel (b) also shows the drivers of the exhaustion and deterrence effects. As the worker's continuation value of unemployment falls, the gain from formal employment – the gap between V^e and $\mathbb{E}V^{u}_{t+1}$ – increases . Since informal work makes job-search more costly, this provides an incentive to accept fewer informal opportunities over time (the exhaustion effect). Simultaneously, the loss associated with being excluded from unemployment protection – the gap between $\mathbb{E}V^{u}_{t+1}$ and $\mathbb{E}V^{us}_{t+1}$ – declines. The worker has fewer remaining payments to lose, and faces smaller fines when she moves onto UA. This provides an incentive to accept more informal opportunities over time (the deterrence effect).

Panel (c) shows how informal sector participation evolves over the duration of unemployment. Within UI, the worker rejects all informal opportunities. The intuition behind this result is straightforward. The generosity of unemployment protection is sufficient to keep her away from the informal sector in the early stages of unemployment. At the point of exhaustion, she starts to receive less generous UA payments. This increases her marginal utility of consumption, causing a discrete increase in her informal participation to around 40 percent, as in Proposition 3. Since the perceived detection rate is only 3.7 percent, the exhaustion effect dominates throughout. The worker accepts fewer opportunities as UA exhaustion approaches, as in Proposition 1. At the point of exhaustion, her participation rate has declined to around 25 percent. Finally, with no risk of sanctions and relatively small benefit payments, the worker accepts 54 percent of informal work whilst on SA.

Panel (d) shows the evolution of the unconditional job-finding rate. Under UI, the worker does not accept informal work. Her unconditional job-finding rate is driven entirely by the increasing gap between the continuation values of unemployment and employment. Since the worker receives relatively generous payments, the continuation value of unemployment falls rapidly as exhaustion approaches, causing the job-finding rate to increase monotonically from 6.95 to 8.14 percent, as per the standard theory.

As the worker enters UA, the evolution of the unconditional job-finding rate changes significantly, becoming discontinuous due to her engagement in the informal labour market. Upon entering UA, the job-finding rate drops to 4.85 percent, as she starts accepting informal work. Over the duration of UA, her unconditional job-finding rate increases mirroring the fall in informal participation due to the exhaustion effect, reaching 6.38 percent by month thirteen. Upon exiting unemployment protection, the job-finding rate drops again to a stationary 3.84 percent, consistent with the worker's increased informal

sector participation from then on.

We studied the sensitivity of these results to changes the three behavioural parameters. Ceteris paribus, a decrease in patience makes workers more willing to accept income in the current period, thus increasing informal sector participation. A reduction in risk aversion raises workers' marginal utility of consumption. Since this has a greater effect on larger income, the value of employment increases relative to that of unemployment. For this reason, participation in the informal sector declines. A reduction in the cost of search parameter, γ, increases the incentive to search for a formal job, again lowering participation. A full recalibration using different behavioural parameters, however, has little effect on the formal/informal wage ratio and informal sector participation rate, since these reflect the unemployment rate and the total informal income of the unemployed, both of which are explicitly targeted.

We conclude this section by observing that the results in Figure 4 also show that negative duration dependence in unemployment arises naturally in a job-search model, once accounting for the interaction between the finite duration of unemployment benefits and the availability of informal sector opportunities. Negative duration dependence refers to the stylised fact that the longer a worker is unemployed, the higher the probability that the worker will still be unemployed in the next period. Various reasons for negative duration dependence have been given in the labour economics literature, including despondency at a lack of job offers, social stigma or human capital depreciation (see Kroft et al. 2013 for a recent discussion). The increasing incentive to accept informal work the longer an individual is unemployed adds a new facet to understanding negative duration dependence. As the incentive to participate in the informal sector increases on average across UI, UA and SA (Panel c), the job-finding rate declines on average across the three programmes (Panel d).

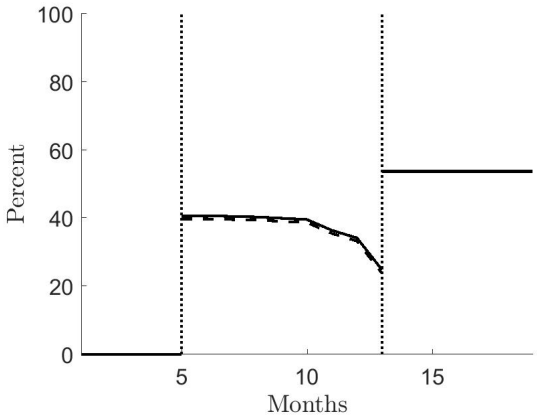
7 Policy experiments

The results of the previous section highlight the importance of accounting for informal labour market participation when quantifying (formal) job-search incentives in countries that provide unemployment protection. Both informal participation and job-search crucially depend on the design of unemployment protection and sanctions, as captured in the model by the effective replacement ratio (b) and duration (T) of each programme, along with the detection rate (δ) and the level of fines (ϕ). We therefore performed a series of experiments to quantify the implications of changes in these four key policy parameters.

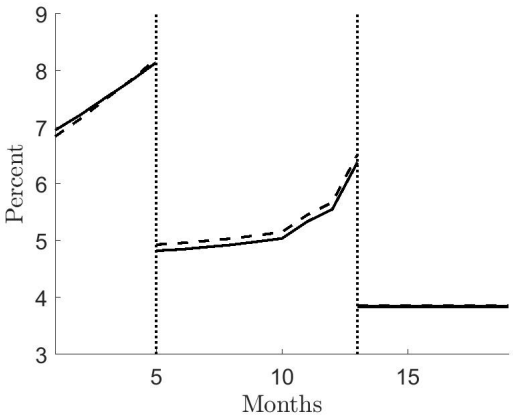
We conduct two types of experiment. First, we perform comparative statics exercises to illustrate the dynamic effects of changing each parameter qualitatively and to provide intuition. We then quantify the likely impact of self-financing policy changes, i.e. policies that involve reallocating resources within unemployment protection, whilst leaving its overall cost unchanged.

Figure 5: A 25 percent increase in unemployment protection payments

Panel 5.1: Unemployment insurance

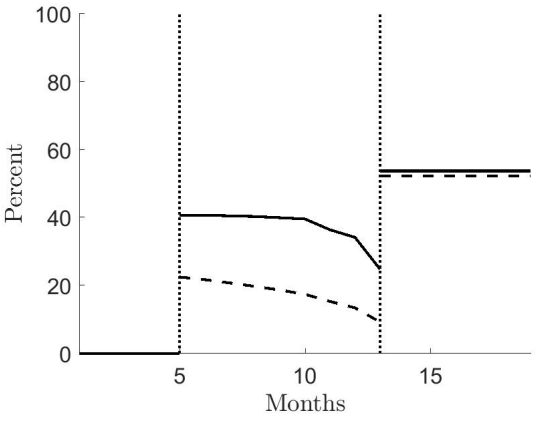


(a) Informal sector participation rate

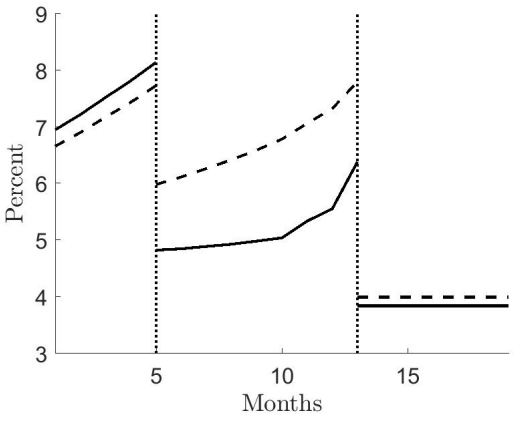


(b) Formal sector job-finding rate

Panel 5.2: Unemployment assistance



(c) Informal sector participation rate



(d) Formal sector job-finding rate

7.1 Unemployment protection

Figure 5 illustrates the effect of increasing the generosity of unemployment protection payments by 25 percent. Decreases are symmetric. In each case, solid lines replicates the dynamics under the baseline calibration (the bottom panels of Figure 4), whereas dashed lines represents the worker's response to the new policy environment. Increasing the generosity of payments affects the worker's incentives through three main channels. First, they reduce her marginal utility of consumption. Each informal opportunity provides a smaller utility gain, deterring the worker from accepting it. Second, the continuation value of unemployment increases before the payments are exhausted. Third, the value of employment increases. If she finds a formal job but subsequently loses it, she will be entitled to the new, more generous payments again.

The upper two panels focus on increases in UI payments. Panel (a) displays the worker's informal sector participation. Since she rejects all informal opportunities whilst receiving UI, reductions in her marginal utility ensure that she continues to do so under after the policy change. Whilst receiving UA, her marginal utility of consumption is unaffected. Similarly, since she has already exhausted her UI payments, her continuation value of unemployment remains relatively unchanged. The increase in the value of employment increases the opportunity cost of accepting informal work, as it distracts the worker from job-search. Her informal sector participation declines modestly. Once unemployment protection payments are exhausted, her informal sector participation is unaffected.

Panel (b) displays the effect on the worker's unconditional job-finding rate. Whilst receiving UI, the worker is less likely to find a job. This reflects the standard moral hazard argument, as the continuation value of unemployment is higher. Under UA, increases in job-finding mirror the fall in the worker's informal sector participation. After unemployment protection has been withdrawn, there is no change in the job-finding rate.

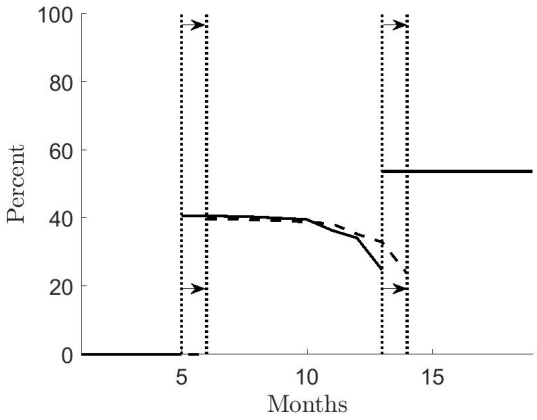
The bottom two panels focus on increases in UA payments. Panel (c) again shows the worker's informal sector participation. The reduction in the worker's marginal utility of consumption whilst in receipt of UA makes her much less willing to accept informal opportunities. After the worker moves onto SA, her informal sector participation remains lower. This reflects the higher value of employment.

Panel (d) shows the effect on the worker's unconditional job-finding rate. Whilst increases in the continuation value of unemployment lower job-finding during UI, this is dominated by declines in informal sector participation during UA. Under the latter programme, the worker accepts far fewer opportunities, enabling her to devote more effort to job-search. Similarly, lower informal sector participation raises her job-finding rate under SA.

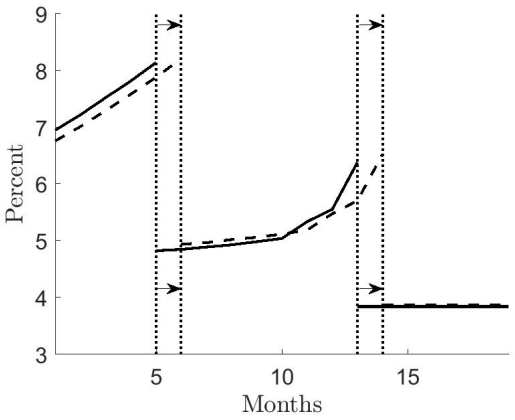
Comparing the two policies, two things stand out. First, the standard moral hazard story – that more generous payments lower search by making unemployment more comfortable – has a relatively modest impact. The biggest driver of job-search is the worker's informal sector participation. This counteracts the moral hazard effect, as more generous payments make the worker less dependent on informal work to supplement her consumption. Second, the impact of varying the less generous, UA programme is much more pronounced, because of its effect on informal sector participation.

Figure 6: A one month increase in unemployment protection duration

Panel 6.1: Unemployment insurance

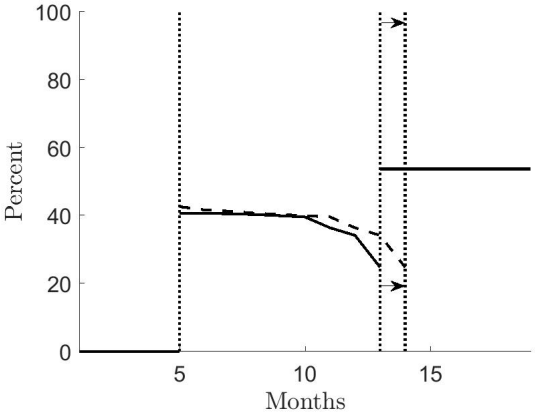


(a) Informal sector participation rate

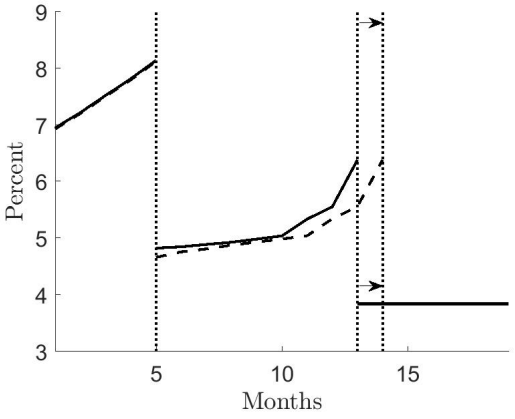


(b) Formal sector job-finding rate

Panel 6.2: Unemployment assistance



(c) Informal sector participation rate



(d) Formal sector job-finding rate

Increases in the duration of each unemployment protection programme by one month are illustrated in Figure 6. Such policies have two primary effects. They raise the continuation value of unemployment in every period of unemployment protection receipt, as the worker now has more benefit payments to consume. They also raise the value of employment, as any worker who loses a job will be entitled to unemployment protection for longer. In addition, during the extra period in which higher payments are made, the worker's marginal utility of consumption declines, deterring her from accepting informal opportunities.

The upper two panels of Figure 6 show the effect of extending UI by one period. During the additional period of UI receipt, the worker rejects all informal opportunities (Panel (a)). After exhausting UI, she accepts slightly fewer informal opportunities, reflecting the increase in the value of employment. Since her UA payments are now exhausted in month 14 (rather than month 13) the impact of the exhaustion effect is delayed by one month.

Panel (b) shows the effect on her unconditional job-finding rate. Job-finding falls in every period of original UI receipt, as the worker's continuation value of remaining unemployed has increased (the moral hazard effect). Since she postpones participating in the informal sector, however, it remains high for one extra period. Under UA, the worker's job-finding mirrors her informal sector participation. It is slightly higher during the early stages of the programme, but the spike driven by the exhaustion effect is postponed by one month. After unemployment protection is exhausted, the higher value of employment leads to slightly higher job-finding whilst the worker receives SA.

Panels (c) and (d) consider the impact of extending UA for one month. The most notable effect is that the worker still rejects all informal opportunities whilst receiving UI. The higher continuation value of unemployment reduces the opportunity cost of searching less intensively under UA, causing a modest increase in participation. The decline at exhaustion is postponed by one period. The effect on the worker's unconditional job-finding rate is again driven primarily by informal sector participation.

Table 9 shows how self-financing policy changes of the structure of the unemployment protection system impact on the aggregate moments predicted by the model under the baseline calibration (see Tables 7 and 8). These represent reallocations of resources within the unemployment protection system that leave the overall cost as a proportion of GDP (2.84 percent) unchanged. Columns one and two report the effect of increasing the generosity of UI and UA benefits by one percent respectively, financed by decreasing the benefits of the other programme. Columns three and four (five and six) report the effects of increasing the duration of UI (UA) financed by reducing the generosity of UA and UI payments respectively. Thus, the changes in columns one, three and five are equivalent to further front-loading unemployment protection payments, whilst those in the other three columns are equivalent to further back-loading of payments.

Two factors drive the results in the first two columns. First, as noted above, informal sector participation has a large effect on job-finding. Second, participation is much more sensitive to changes in the generosity of UA than to changes in the generosity of UI. Since, in the first column, UA payments are reduced in order to fund an increase in UI payments, informal sector participation by UA recipients increases. They become willing to accept smaller informal opportunities, lowering their informal/formal wage ratio. This

Table 9: Self-financing policy changes (difference from baseline)

	Policy Experiment					
	(1)	(2)	(3)	(4)	(5)	(6)
Increase in	$b^{ui}+1\%$	$b^{ua}+1\%$	$T^{ui}+1$	$T^{ui}+1$	$T^{ua}+1$	$T^{ua}+1$
Financed by reducing	b^{ua}	b^{ui}	b^{ua}	b^{ui}	b^{ua}	b^{ui}
Formal sector:						
Unemployment rate	0.15	-0.04	0.15	-0.44	0.53	-0.03
Unemployed receiving:						
UI	-0.20	0.06	2.89	3.80	-0.65	0.08
UA	-0.22	0.07	-2.16	-0.86	0.88	1.93
SA	0.42	-0.12	-0.73	-2.95	-0.24	-2.00
Informal sector:						
Income of unemployed	0.09	-0.02	-0.04	-0.37	0.29	-0.02
Participation rate	0.60	-0.18	0.71	-1.96	2.05	-0.18
Share of workforce in:						
UI	0.00	0.00	0.00	0.00	0.00	0.00
UA	0.59	-0.18	1.61	-0.02	3.83	2.34
SA	-0.59	0.18	-1.61	0.02	-3.83	-2.34
Informal/formal wage for:						
Average	-0.47	0.14	-2.18	0.05	-1.64	0.17
UI recipients	0.00	0.00	0.00	0.00	0.00	0.00
UA recipients	-2.94	0.94	-10.18	0.27	-9.07	-0.48
SA recipients	0.02	-0.01	-0.44	0.00	-0.12	-0.02

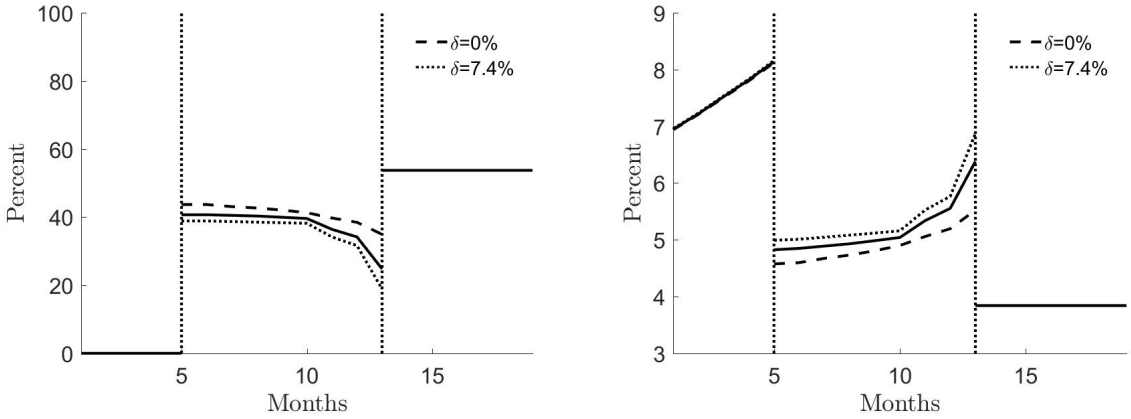
Notes: Experiments are as follows: (1) Increasing b^{ui} by one percent, financed by reducing b^{ua}, (2) Increasing b^{ua} by one percent, financed by reducing b^{ui}, (3) Increasing T^{ui} by one period, financed by reducing b^{ua}, (4) Increasing T^{ui} by one period, financed by reducing b^{ui}, (5) Increasing T^{ua} by one period, financed by reducing b^{ua}, (6) Increasing T^{ua} by one period, financed by reducing b^{ui}.

not only increases the size of the informal sector, but also reduces their unconditional job-finding rate. Average unemployment duration increases (a larger proportion of the unemployed receive SA), and the aggregate unemployment rate increases.

In contrast, increasing the generosity of UA, paid for by a reduction in UI reduces both the unemployment rate and the size of the informal sector. UI recipients still receive large enough payments to deter any participation in the informal sector. However, with a lower marginal utility of consumption afforded by larger benefit payments, participation by UA recipients falls. They devote more effort to searching for a formal job, lowering the share of the unemployed who enter SA.

The impact of extending the effective duration of each programme also crucially depends upon how it is financed. Any gain from this policy is dominated by reductions

Figure 7: Changes in the detection rate



(a) Informal sector participation rate

(b) Formal sector job-finding rate

in UA payments, leading to higher unemployment and informal sector participation rates. In contrast, funding the extensions by reducing the generosity of UI payments has beneficial effects. Insurance recipients still receive enough to deter informal work. However, extending UI duration keeps unemployed workers out of the informal sector (and hence searching for a job) for one extra period. Extending either programme also postpones agents' entry into SA. This keeps their informal sector participation relatively low for one additional month, raising their average search effort. In either case, both the amount of income they earn informally and the unemployment rate decline.

With regard to the validity of the results in Table 9, it could be argued that partial equilibrium models are subject to the critique that the formal labour market wage is not invariant to policy changes. The empirical support for this observation is, however, weak. The detrended series of the nominal average wage reported by the OECD for Spain (OECD 2014) is almost entirely constant between 2002 and 2013, despite several changes in the structure of unemployment protection over the same period. We also note that the policy changes in Table 9 involve only small deviations from the baseline calibration.

The finding that increasing benefit generosity in supplementary programmes increases job-search contrasts with the literature on optimal UI, which advocates the front-loading of benefits to reduce the problems associated with moral hazard (see Shavell and Weiss 1979; Hopenhayn and Nicolini 1997). It further suggests that informal sector participation may play a significant role in determining job-finding rates. By making payments more generous in the later stages of unemployment, or by extending the duration of larger payments, informal sector participation is deterred, thereby increasing formal job-search.

7.2 Sanctions

The effect of altering the perceived detection rate is shown in Figure 7. We pursue two experiments: removing monitoring of unemployment protection recipients ($\delta = 0$); and

doubling the detection rate relative to the baseline case ($\delta = 7.4$ percent). As in the previous two figures, solid lines replicate the dynamics under the baseline calibration.

Under either scenario, UI recipients do not accept informal opportunities (Panel (a)). This suggests that payments are sufficiently generous to make the opportunity cost of informal work (lower search effort) dominate any instantaneous utility gain deriving from higher consumption, even in the absence of monitoring. Under UA, increasing or decreasing the detection rate has the expected effect. A greater likelihood of being detected and sanctioned reduces the worker's willingness to accept informal opportunities, lowering participation.

Panel (b) shows the effect on job-finding. As ever, changes in the worker's unconditional job-finding rate reflect changes in informal participation. By reducing the worker's willingness to accept informal work, higher detection rates increase job-search.

Varying sanctions (ϕ) have no substantial effect on incentives. Since the perceived detection rate is relatively small, the worker attaches a very small probability to facing the new fines. The results are consequently not reported.

Changing the perceived probability of detection is potentially very expensive. Unemployment protection agencies use a variety of techniques to monitor recipient activity, from spot checks to cross-referencing with income tax records. Whilst we do not model this cost explicitly, Figure 7 suggests that there is scope for a self-financing policy involving the detection rate that would reduce both unemployment and recipients' informal sector participation through the reallocation of monitoring resources from UI to UA recipients. The former receive sufficiently generous payments to deter them from participating in the informal sector. Monitoring these could be redundant. In contrast, increasing the perceived detection probability for the latter has the potential to substantially reduce informal sector participation and increase unconditional job-finding. This is similar to the finding of the optimal welfare-to-work literature (Pavoni and Violante 2007), which suggests that less intensive monitoring is necessary during the early stages of unemployment.

8 Conclusion

Many people in advanced economies still look for work in the shadow sector while unemployed, despite protection against the risk of unemployment and income support being widely provided by social security systems in all these countries. Motivated by this observation, this paper studies how the design of unemployment protection in advance economies influences the incentives of individuals to participate in informal labour markets and search for formal work during their unemployment spells.

We began with a review of unemployment protection programmes and recent evidence on engagement in the informal sector across western European countries and the United States. The typical unemployment protection programme includes a finite two-tier system of UI followed by, less generous, UA benefit payments. Minimum income support is provided through SA programmes to those who are ineligible or have exhausted UI and UA benefits. With regard to the engagement in the informal sector of workers during unemployment, we review recent survey evidence for advanced economies suggesting

that this is non-negligible and it predominantly entails undertaking temporary low-paid jobs for friends and neighbours, rather than wage-employment or self-employment as often found in developing economies.

We therefore extended a standard model of job search with moral hazard and a stylised unemployment protection programme by giving unemployed workers access to temporary job opportunities from a separate informal labour market. In the absence of the informal sector, the model would predict that the incentive to search for work should increase over the duration of unemployment (re-employment spike) and that augmenting the generosity of benefit payments should reduce the job-finding rate (the moral hazard argument).

Our theoretical contribution is to highlight that the presence of the informal labour market significantly changes these standard predictions in three main dimensions. The first operates across programmes. As benefit payments reduce when unemployed workers switch to less generous programmes, the instantaneous utility gain from a given informal opportunity increases, thus increasing the likelihood of undertaking informal work. This *payment effect* tends to counteract the re-employment spike at the point in which benefits are reduced highlighted by the standard theory. The other two dynamic mechanisms operate with opposing effects within each programme (either UI or UA). As the expiry date of unemployment protection approaches, benefit recipients understand that there are fewer payments remaining. This *exhaustion effect* curtails the incentive to accept informal work thus providing greater incentive to search for formal work. At the same time, as the duration of unemployment increases, the cost of being sanctioned for working informally (exclusion from unemployment protection) declines as there are fewer remaining payments to loose. This *deterrence effect* increases the incentive to engage in the informal sector increases and reduces that of searching for formal work.

We investigate the quantitative relevance of these effects using a computational version of the model calibrated to Spain. This identifies a number of new results and implications for policy.

First, the level of unemployment benefits can significantly influence unemployed workers' incentive to accept informal work. From a policy perspective, this implies that replacement ratios for unemployment benefits should be set considering not only the insurance-incentive trade-off identified by the standard theory, but also their potential effects on informal sector participation.

Second, the payment effect resulting from accepting informal work can dominate quantitatively the traditional moral hazard argument when benefit payments are low. If at the time of switch from UI to UA benefit payments reduce too much, the marginal utility from accepting informal work could rise so that the job-finding rate falls instead of remaining high as prescribed by the standard theory. The policy implication of this result is that increasing the benefit payments for the less generous components of the unemployment protection system may actually increase, rather than reduce, the job-finding rate.

Third, the quantitative analysis suggests that informal sector participation is likely to be high among those unemployed workers who are either ineligible for unemployment protection or are receiving benefit payments at relatively low replacement ratios. These individuals are more likely to respond to changes in the generosity of benefit payments.

This brings implications also for monitoring policy, as it suggests that it can be used more effectively if concentrated on those receiving low or without unemployment protection.

Further, quantitative analyses on the effects of increasing unemployment benefits based on general equilibrium search models typically incorporate an increase in the cost of formality as a financing mechanism. Our analysis instead highlights the role of a different policy option: the self-financing reallocation of benefit payments across unemployment protection programmes. In particular, we find that it would be possible to engineer reallocations of benefit payments from UI to UA that would reduce the informal-sector share in the economy and the unemployment rate, while leaving the overall cost of unemployment protection at the current level.

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Appendices

A Proofs

Proof of Proposition 1

Implicitly differentiating (8) with respect to $\mathbb{E}V_{t+1}^u$ yields:

$$\frac{\partial \chi_t^{ui}}{\partial \mathbb{E}V_{t+1}^u} = \frac{[1-p_t^{ui}(0)]-(1-\delta)[1-p_t^{ui}(1)]}{u'(b_t+\chi_t^{ui})}.$$

In any period, $\partial\chi_t^{ui}/\partial\mathbb{E}V_{t+1}^u < 0$ if and only if $\delta < 1-[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)]$. Over time, however, $[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)]$ varies. So if $\delta < \min\{1-[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)] : t \leq T\}$ then $\partial\chi_t^{ui}/\partial\mathbb{E}V_{t+1}^u < 0$ in every period. Since $\mathbb{E}V_{t+1}^u$ is declining, χ_t^{ui} increases over time. The worker will accept fewer informal opportunities over time.

Now, $\min\{1-[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)] : t \leq T\}$ also depends on δ through (6). In order to guarantee that χ_t^{ui} increases over time, we restrict δ by showing that there exists some $\underline{\delta} > 0$ such that for all $\delta < \underline{\delta}$, $\delta < \min\{1-[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)] : t \leq T\}$.

Begin by defining the following set of fixed points:

$$\underline{D} \equiv \left\{\delta \in [0,1] : \delta = \min\left\{1-\frac{1-p_t^{ui}(0)}{1-p_t^{ui}(1)} : t \leq T\right\}\right\}$$

Note that $1-[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)]$ is bounded between zero and one, and is continuous in δ. Consequently $\min\{1-[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)] : t \leq T\}$ is also continuous and maps

any $\delta \in [0,1]$ back into $[0,1]$. Brouwer's Fixed Point Theorem applies, and guarantees that $\underline{D}$ is non-empty.

Our next step is to show that $0 \notin \underline{D}$. If $\delta = 0$, it is still the case that $p_t^{ui}(1) < p_t^{ui}(0)$ as the marginal cost of effort is higher when the worker accepts informal work in (6). In this case:

$$\min\left\{1 - \frac{1-p_t^{ui}(0)}{1-p_t^{ui}(1)} : t \leq T\right\} > 0$$

So not only is $0 \notin \underline{D}$, but when $\delta = 0$, $\delta < \min\{1-[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)] : t \leq T\}$. Let $\underline{\delta} \equiv \inf \underline{D}$. Since $\min\{1-[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)] : t \leq T\}$ is continuous in δ, it must also be the case that $\delta < \min\{1-[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)] : t \leq T\}$ for all $\delta \in [0, \underline{\delta})$, as required. Thus, for all $\delta < \underline{\delta}$ the worker accepts fewer informal opportunities as her duration of unemployment increases. This completes the proof. ■

Proof of Proposition 2

Implicitly differentiating (8) with respect to $\mathbb{E}V_{t+1}^u$ yields:

$$\frac{\partial \chi_t^{ui}}{\partial \mathbb{E}V_{t+1}^u} = \frac{[1-p_t^{ui}(0)] - (1-\delta)[1-p_t^{ui}(1)]}{u'(b_t + \chi_t^{ui})}.$$

In any period, $\partial \chi_t^{ui}/\partial \mathbb{E}V_{t+1}^u > 0$ if and only if $\delta > 1-[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)]$. Over time, however, $[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)]$ varies. So if $\delta > \max\{1-[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)] : t \leq T\}$ then $\partial \chi_t^{ui}/\partial \mathbb{E}V_{t+1}^u > 0$ in every period. Since $\mathbb{E}V_{t+1}^u$ is declining, χ_t^{ui} decreases over time. The worker will accept more cash-in-hand opportunities over time.

Now, $\max\{1-[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)] : t \leq T\}$ also depends on δ through (6). In order to guarantee that χ_t^{ui} declines over time, we restrict δ by showing that there exists some $\overline{\delta} < 1$ such that for all $\delta > \overline{\delta}$, $\delta > \max\{1-[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)] : t \leq T\}$.

Begin by defining the following set of fixed points:

$$\overline{D} \equiv \left\{\delta \in [0,1] : \delta = \max\left\{1 - \frac{1-p_t^{ui}(0)}{1-p_t^{ui}(1)} : t \leq T\right\}\right\}$$

Note that $1-[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)]$ is bounded between zero and one, and is continuous in δ. Consequently $\max\{1-[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)] : t \leq T\}$ is also continuous and maps any $\delta \in [0,1]$ back into $[0,1]$. Brouwer's Fixed Point Theorem applies, and guarantees that $\overline{D}$ is non-empty.

Our next step is to show that $1 \notin \overline{D}$. Since $p_t^{ui}(0), p_t^{ui}(1) \in (0,1)$, it must be that:

$$\max\left\{1 - \frac{1-p_t^{ui}(0)}{1-p_t^{ui}(1)} : t \leq T\right\} < 1$$

So not only is $1 \notin \overline{D}$, but when $\delta = 1$, $\delta > \max\{1-[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)] : t \leq T\}$. Let $\overline{\delta} \equiv \sup \overline{D}$. Since $\max\{1-[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)] : t \leq T\}$ is continuous in δ, it must also be the case that $\delta > \max\{1-[1-p_t^{ui}(0)]/[1-p_t^{ui}(1)] : t \leq T\}$ for all $\delta \in (\overline{\delta}, 1]$, as required. Thus, for all $\delta > \overline{\delta}$ the worker accepts a greater range of informal

opportunities as her duration of unemployment increases. This completes the proof. ■

Proof of Proposition 3

From (5), the continuation payoff from remaining unemployed in period T is simply $\mathbb{E}V^{out}$. Compare (13) to (8) in period T. If $b^{ui} = \delta = 0$, then $\chi_T^{ui} = \chi^{out}$. Applying the Implicit Function Theorem to (8):

$$\frac{\partial \chi_T^{ui}}{\partial b} = \frac{u'(b) - u'(b + \chi_T^{ui})}{u'(b + \chi_T^{ui})} > 0,$$
$$\frac{\partial \chi_T^{ui}}{\partial \delta} = \frac{\beta\{p_t^{ui}(1)(V^e - V^{es}) + [1 - p_t^{ui}(1)](\mathbb{E}V^{out} - V^{us})\}}{u'(b + \chi_T^{ui})} > 0.$$

With $b^{ui} > 0$ and $\delta > 0$, $\chi_T^{ui} > \chi^{out}$. This completes the proof. ■

Proof of Proposition 4

We first need to show that, in period T, condition (**??**) holds. Compare:

$$\begin{aligned} -c[1, p_T^{ui}(1)] + \beta(1-\delta)\{p_t^{ui}(1)V^e + [1 - p_t^{ui}(1)]\mathbb{E}V^{out}\} \\ +\beta\delta\{p_t^{ui}(1)V^{es} + [1 - p_t^{ui}(1)]V^{us}\}, \end{aligned} \tag{17}$$

and

$$-c[1, p^{out}(1)] + \beta\{p^{out}(1)V^e + [1 - p^{out}(1)]\mathbb{E}V^{out}\}. \tag{18}$$

Note that if $\delta = 0$, (11) and (6) are equivalent and so (17) and (18) are equal.

Now, consider the effect of increasing δ on (17):

$$\begin{aligned} \frac{d(17)}{d\delta} &= \frac{\partial(17)}{\partial p_t^{ui}(1)} \frac{\partial p_t^{ui}(1)}{\partial \delta} + \frac{\partial(17)}{\partial \delta} \\ &= -\beta\{p_t^{ui}(1)(V^e - V^{es}) + [1 - p_t^{ui}(1)](\mathbb{E}V^{out} - V^{us})\} \\ &< 0, \end{aligned}$$

since $V^{es} < V^e$ and $V^{us} < \mathbb{E}V^{out}$. So, for any $\delta > 0$, (17) $<$ (18). Call the difference, (18) $-$ (17) $\equiv \Delta$. This is, importantly, independent of χ_T.

It remains to show that, for some range of χ_T, $V^{out}(\chi_T) > V_T^{ui}(\chi_T)$. We have:

$$V^{out}(\chi_T) - V_T^{ui}(\chi_T) = u(\chi_T) - u(b^{ui} + \chi_T) + \Delta.$$

The worker will prefer to leave if and only if:

$$u(b^{ui} + \chi_T) - u(\chi_T) < \Delta.$$

Since $u(y_t)$ satisfies diminishing marginal utility and the Inada conditions, $u(b^{ui} + \chi_T) - u(\chi_T)$ is strictly decreasing with $u(b^{ui} + \chi_T) - u(\chi_T) \to 0$ as $\chi_T \to \infty$. There exists some reservation opportunity, $\tilde{c}_T$ such that the worker prefers to leave whenever she receives

an opportunity $\chi_T > \tilde{\chi}_T$. The probability that she leaves the programme in period T is $1 - F(\tilde{\chi}_T)$.

In periods prior to T, a similar argument holds. Since $\mathbb{E}V^u_{t+1}$ is declining over time, $\tilde{\chi}_t \geq \tilde{\chi}_{t+1}$. The probability that the worker leaves is increasing from one period to the next. This completes the proof ■

B Algorithm

This appendix gives a detailed description of the algorithm used to solve the model numerically. It is based upon a discrete grid of informal opportunities, X, to approximate the continuous distribution assumed in the theoretical analysis. We define the probability that the received opportunity takes value $\chi_i \in X$ to be $f_i = \mathbb{P}(\chi = \chi_i)$. Full MATLAB code is available upon request from the authors.

The algorithm proceeds as follows:

1. Take an initial $\mathbb{E}V^u_1$ as given.
2. Compute the stationary value of formal employment from (2).
3. Calculate the worker's solution from SA:
   (a) Take an initial $\mathbb{E}V^{sa}$ as given.
   (b) Compute the stationary value of formal employment when the worker is only eligible for SA.
   (c) Using the results from the previous steps, compute the worker's conditional job-finding rates.
   (d) Using the conditional job-finding rates, compute the informal opportunities that the worker accepts, and the implied utility.
   (e) Derive a new $\mathbb{E}V^{sa}$:
   (f) Iterate steps 3b to 3e until $\mathbb{E}V^{sa}$ converges.
4. Calculate the value of sanctions under UA:
   (a) Compute the stationary value of sanctions when simultaneous employed in the formal sector having previously been on UA.
   (b) Compute the stationary value of sanctions when the worker remains unemployed, having previously been receiving UA:
      i. Take an initial $\mathbb{E}V^{us,ua}$ as given.
      ii. Using the results from the previous steps, compute the worker's conditional job-finding rates.
      iii. Using the conditional job-finding rates, compute the informal opportunities that the worker accepts, and the implied utility.
      iv. Derive a new $\mathbb{E}V^{us,ua}$.

v. Iterate steps 4(b)ii to 4(b)iv until $\mathbb{E}V^{us,ua}$ converges.

5. Calculate the solution under UA using backwards induction:

   (a) Take $\mathbb{E}V^{ua}_{T-k+1}$ from the previous iteration. If $k = 0$, note that $\mathbb{E}V^{ua}_{T^{ua}+1} = \mathbb{E}V^{sa}$, as computed in step 3f.

   (b) Using the results from the previous steps, compute the worker's conditional job-finding rates.

   (c) Using the conditional job-finding rates, compute the informal opportunities that the worker accepts, and the implied utility.

   (d) Derive $\mathbb{E}V^{ua}_{T^{ua}-k}$.

   (e) If $k < T^{ua} - 1$, return to step 5a with $k' = k - 1$. Otherwise move on.

6. Calculate the value of sanction under UI in an identical manner to step 4.

7. Calculate the solution under UI using backwards induction in an identical manner to step 5.

8. Iterate steps 2 to 7 until $\mathbb{E}V^{u}_{1}$ converges.

9. Create a $(T^{ui} + T^{ua} + N + 7) \times (T^{ui} + T^{ua} + N + 7)$ matrix of zeros, denoted by Ω. Each row represents a state in period τ, call it ω_τ. Each column represents a state in period $\tau + 1$, call it $\omega_{\tau+1}$. For example, ui_t represents the state in which the worker is in the tth period of the UI programme. We consider transitions between states in which job-finding rates are chosen. So, for example, if a worker was in receipt of her first UI payment in period τ, was subsequently unsuccessful but received an informal opportunity in $\tau + 1$ that caused her to voluntarily leave the unemployment protection programme, we would model this as a transition directly from ui_1 to sa:

   Table 10 outlines how states are allocated to the rows and columns of Ω.

10. Populate Ω with transition probabilities.

11. We will denote values under the stationary distribution with a superscript $\sim$. Derive $\tilde{\omega}$, the normalised eigenvector of Ω with associated eigenvalue equal to one.

12. Calculate the stationary formal unemployment rate:

$$\tilde{\mu} = 1 - \tilde{\omega}_{T^{ui}+T^{ua}+1} - \tilde{\omega}_{T^{ui}+T^{ua}+2} - \tilde{\omega}_{T^{ui}+T^{ua}+4} - \tilde{\omega}_{T^{ui}+T^{ua}+6}. \tag{19}$$

13. Calculate labour's income from the formal sector:

$$(1 - \tilde{\mu})w. \tag{20}$$

Table 10: Rows and Columns of the Transition Matrix, Ω

Row/Column	ω	Description	Notes
t	ui_t	tth period of UI	$t = 1, ..., T^{ui}$
$T^{ui} + t$	ua_t	tth period of UA	$t = 1, ..., T^{ua}$
$T^{ui} + T^{ua} + 1$	e	Employed and eligible for unemployment protection	
$T^{ui} + T^{ua} + 2$	e, sa	Employed and ineligible for unemployment protection	
$T^{ui} + T^{ua} + 3$	sa	SA	
$T^{ui} + T^{ua} + 4$	es, ui	Employed and sanctioned from UI	
$T^{ui} + T^{ua} + 5$	us, ui	Unemployed and sanctioned from UI	
$T^{ui} + T^{ua} + 6$	es, ua	Employed and sanctioned from UA	
$T^{ui} + T^{ua} + 7$	us, ua	Unemployed and sanctioned from UA	
$T^{ui} + T^{ua} + 7 + j$	$exit(\chi_j)$	Exited unemployment protection to accept χ_j	$j = 1, ..., N$

14. Calculate officially measured GDP:

$$\widetilde{GDP} = \frac{(1 - \tilde{\mu})w}{\psi}. \tag{21}$$

15. Calculate the stationary size of the informal sector using the following decomposition:

$$\begin{aligned}\tilde{\iota}^u =& \tilde{\iota}^{ui} + \tilde{\iota}^{ua} + \tilde{\iota}^{u,out} + \tilde{\iota}^{us,ui} + \tilde{\iota}^{us,ua} + \tilde{\iota}^{exit} \\ =& \overbrace{\sum_{t=1}^{T^{ui}} \left(\tilde{\omega}_t \sum_i a_{t,i}^{ui} f_i \chi_i \right)}^{ui} + \overbrace{\sum_{t=1}^{T^{ua}} \left(\tilde{\omega}_{T^{ui}+t} \sum_i a_{t,i}^{ua} f_i \chi_i \right)}^{ua} + \overbrace{\tilde{\omega}_{T^{ui}+T^{ua}+3} \sum_i a_i^{sa} f_i \chi_i}^{u,out} \\ &+ \underbrace{\tilde{\omega}_{T^{ui}+T^{ua}+5} \sum_i a_i^{us,ui} f_i \chi_i}_{us,ui} + \underbrace{\tilde{\omega}_{T^{ui}+T^{ua}+7} \sum_i a_i^{us,ua} f_i \chi_i}_{us,ua} + \underbrace{\sum_{j=1}^{N} \tilde{\omega}_{T^{ui}+T^{ua}+7+j} \chi_j}_{exit}.\end{aligned} \tag{22}$$

16. Unemployed individuals in Spain are estimated to contribute 27 percent of the

total value of the informal sector (European Commission 2014). Assuming no changes in the value of informal sector activities engaged in by formally employed agents, the total size of the informal sector is:

$$\begin{aligned}\tilde{\iota} &= \tilde{\iota}^u + \tilde{\iota}^e \\ &= \tilde{\iota}^u + \frac{0.73\tilde{\iota}^u}{0.27}.\end{aligned} \tag{23}$$

Note that, when conducting policy experiments, we will assume that $\tilde{\iota}^e$ is fixed at its original value.

17. Calculate the stationary size of the informal sector as a proportion of GDP:

$$\tilde{I} = \frac{\tilde{\iota}}{\widetilde{GDP}}. \tag{24}$$

18. Calculate the stationary mass of unemployed agents participating in the informal sector using the following decomposition:

$$\begin{aligned}\tilde{\pi}^u =& \tilde{\pi}^{ui} + \tilde{\pi}^{ua} + \tilde{\pi}^{u,out} + \tilde{\pi}^{us,ui} + \tilde{\pi}^{us,ua} + \tilde{\pi}^{exit} \\ =& \sum_{t=1}^{T^{ui}} \left(\tilde{\omega}_t \sum_i a_{t,i}^{ui} f_i \right) + \sum_{t=1}^{T^{ua}} \left(\tilde{\omega}_{T^{ui}+t} \sum_i a_{t,i}^{ua} f_i \right) + \tilde{\omega}_{T^{ui}+T^{ua}+3} \sum_i a_i^{sa} f_i \\ &+ \tilde{\omega}_{T^{ui}+T^{ua}+5} \sum_i a_i^{us,ui} f_i + \tilde{\omega}_{T^{ui}+T^{ua}+7} \sum_i a_i^{us,ua} f_i + \sum_{j=1}^{N} \tilde{\omega}_{T^{ui}+T^{ua}+7+j}.\end{aligned} \tag{25}$$

19. Calculate the informal sector participation rate amongst unemployed agents:

$$\tilde{\Pi}^u = \frac{\tilde{\pi}^u}{\tilde{\mu}}. \tag{26}$$

20. The average informal sector wage is given by:

$$w^\iota = \frac{\tilde{\iota}^u}{\tilde{\pi}^u}. \tag{27}$$

21. Calculate the wage gap:

$$w - w^\iota. \tag{28}$$

22. Calculate the cost of the unemployment protection programme:

$$\tilde{C}^{up} = \overbrace{b^{ui} \sum_{t=1}^{T^{ui}} \tilde{\omega}_t}^{ui} + \overbrace{b^{ua} \sum_{t=1}^{T^{ua}} \tilde{\omega}_{T^{ui}+t}}^{ua}. \tag{29}$$

23. Calculate the cost of SA:

$$\tilde{C}^{sa} = b^{sa}\left[\overbrace{\tilde{\omega}_{T^{ui}+T^{ua}+3}}^{u,out} + \overbrace{\tilde{\omega}_{T^{ui}+T^{ua}+5}}^{us,ui} + \overbrace{\tilde{\omega}_{T^{ui}+T^{ua}+7}}^{us,ua} + \overbrace{\sum_{j=1}^{N}\tilde{\omega}_{T^{ui}+T^{ua}+7+j}}^{exit}\right]. \quad (30)$$

With regard to the baseline calibration, the grid of informal sector opportunities includes 200 equally-distant points, $\chi_t \in (1, 200)$ in each period. The search through the endogenous parameters started using the following grids: $\alpha \in [-0.99, 0.99]$, $\delta \in [0, 0.15]$, $\mu \in [2.8, 3.6]$, $\upsilon \in [0.8, 1.5]$, $T^{ui} \in [3, 8]$ and $T^{ua} \in [3, 8]$. We set the labour market share of GDP, ψ, in (21) to 0.64, which is standard in the macroeconomics literature.

The distance is computed by summing up the squared proportional deviations between the following moments predicted by the model and their empirical counterparts: the unemployment rate; the informal sector income generated by the unemployed as a proportion of GDP; the cost of the unemployment protection system; the shares of unemployed agents receiving UI and UA. Under this measure, the baseline calibration returns a final distance of 0.0289.

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