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Majority Vote on Educational Standards

Abstract

The direct democratic choice of an examination standard, i.e., a performance level required to graduate, is evaluated against a utilitarian welfare function. It is shown that the median preferred standard is inefficiently low if the marginal cost of reaching a higher performance reacts more sensitively to ability for high than for low abilities, and if the right tail of the ability distribution is longer than the left tail. Moreover, a high number of agents who choose not to graduate may imply that the median preferred standard is inefficiently low even if these conditions fail.

JEL-Codes: I210, D720, I280.

Keywords: examination, school, drop-outs, democracy, median voter.

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1 Introduction

Improving educational achievements is a stated goal in most countries. In a democracy, however, education policies cannot simply be chosen by the government but need to find the support of a majority of voters. The present paper provides an analysis of such democratic decisions, focussing on a central feature of every education system: the choice of examination standard. I formulate and discuss sufficient conditions implying that the standard chosen by a majority of voters is less demanding than would be efficient. These conditions mean that there are few highly able students who would easily reach higher standards, but these are outvoted by a majority of less able students who prefer a moderate standard. Thus, the theory put forward in this paper explains why the educational policies actually implemented in democratic societies may sometimes work against raising achievement levels.

Examination standards are subject to intense policy debates in many countries. Possibly the most controversial discussions can be found concerning state high school exit exams in the U.S. (see McIntosh, 2012). Proponents of such tests argue that high targets create incentives for students to exert more effort at school and thus improve academic performance. Opponents complain that too many students fail the examination, and that students suffer from the pressure exerted by a high stakes examination.

This tension between boosting performance and mitigating pressure is mirrored in the demands of students, parents, or their organisations in education policy debates. Although opposition to high-stakes testing is widespread in the U.S., it is not uncommon to find commentators arguing in favour of raising standards. For example, surveys in the U.S. found that 37 percent of fourth-graders say that that their math work is too easy (Boser and Rosenthal, 2012, p. 15). Similarly, 69 per cent of high school students consider expectations to be moderate or low (Horatio Alger Association, 2005, p. 7).

On the other hand, the WHO reports substantial numbers of students who feel pressured by schoolwork (World Health Organization, 2016, p. 59-52). Corresponding to that, many parents feel that school is too tough for children, and that politics should aim at reducing the stress created by school. For example, in Germany, the Association of Catholic Parents bemoans excessive pressure and performance-orientation of schools (Katholische Elternschaft Deutschlands, 2013), and the Bavarian Parents' Association calls for abolishing marks (Merkur online, 2011). Moreover, when they are allowed to choose, most German parents prefer their children to spend a year longer until graduating from high school.

This paper shows how these conflicting goals shape a democratic decision on how rigorous an examination should be. For this purpose, I develop a model where students of differing abilities decide how much effort to put into schooling. The effort determines whether a student graduates, which requires to reach a certain performance at the examination called the standard. The standard determines the wage earned by graduates and effort is costly, but more able students find it easier to comply with any given standard. For this reason, more able students prefer higher standards than students with lower abilities.

The standard is determined by a majority vote among agents, say the students' parents, who care for the interest of students. In the main results it is analysed whether, starting from the standard preferred by agents with median ability, a marginal increase in the standard improves a utilitarian welfare criterion. Such a result obtains if two kinds of conditions are satisfied. The first type of condition requires that the marginal cost of satisfying a higher standard decreases more steeply in ability when ability is above the median than when it is below the median. The second kind of condition requires that the distribution of abilities is spread out more widely at the high end than at the low end. Intuitively, there is a lot to gain by tougher standards if there are a few students with very high abilities, and if for those students it is easy to satisfy more demanding standards.

In the model students are allowed to avoid the effort necessary to satisfy a standard they find too tough by not graduating. The existence of such 'drop-outs' has two consequences. First, it precludes the application of a standard median voter result. This is because a voter is indifferent between all standards which she will not satisfy, and hence political preferences are not single-peaked. To deal with this, I propose two modifications of the concept of a Condorcet winner. The first modification consists in requiring only that in every pairwise vote, a majority of agents weakly but not necessarily strictly prefer the Condorcet winner. I show that the standard preferred by the individual with median ability is a weak Condorcet winner in this sense, but not necessarily the only one. The second modification introduces a small probability of errors in the ensuing graduation decision. The median preferred standard is shown to be the only weak Condorcet winner which is, in the spirit of trembling hand perfection, robust against such errors.

As a second implication, the presence of drop-outs creates an independent force which pushes towards an inefficiently low standard. Drop-outs are not affected by a marginal increase in standard since they anyway do not bear any effort costs. Consequently, the standard should rise if the number of drop-outs is large. In an example, I illustrate that this effect may cause the democratically chosen standard to be too low even when the general condition on effort cost is not met.

The present paper contributes to the literature on education policy and, in particular, on the choice of graduation standards, which I will review in the following section. It is most closely linked to two contributions which address majority voting on standards. In a short section on this issue Costrell (1994, p. 963-964) concludes that the democratically chosen standard is excessively tough, based on an assumption on the ability distribution which is similar to the conditions which in the present model imply an inefficiently low standard. The difference between both approaches is that in Costrell (1994), voters, like schools, are only concerned with wages and educational outcomes but do not take students' effort into account. In the model presented here, such a disutility of learning is a major driver of voters' decisions, and consequently the chosen standard tends to be lower. In this sense, my model is tailored to parents who take great care to protect their children from stress, as in the statements from Germany cited above, whereas Costrell (1994) rather features an aspiring type of parents who push their children to highest performance, as in some of the comments from the U.S. mentioned earlier.

In the model by Brunello and Rocco (2008), a profit maximising private school competes with a public school whose standard is set by majority vote. These authors show that two regimes can arise in equilibrium: Either the public school sets the most demanding and the private school sets the most lenient among all admissible standards, or vice versa. At least for parameters calibrated to the U.S. and Italy, these standards maximise utilitarian welfare. While this contribution shares some features with the present paper, both provide different insights. Brunello and Rocco (2008) focus upon the interaction between private and public schools, which is absent in my paper. Thus, my model is tailored towards school systems, like the ones of most German states, where a unique examination standard is centrally chosen and applied to all students. Moreover, I relate general properties of the effort cost function and the distribution of abilities to the efficiency of the chosen standard, where Brunello and Rocco (2008) fix a log-normal distribution and liner cost of effort. My results suggest that an interior, uniform standard chosen by the median voter is less likely to be efficient than a menu of two extreme standards parents may choose from.

The paper continues in Section 2 with an overview of related literature. In Section 3, I describe the basic economic structure. Voting decisions are analysed in Section 4, and the assumptions required to establish the median preferred standard as a Condorcet winner are given in Section 5. Based on this, Section 6 provides the main welfare analysis, and Section 7 discusses the role of drop-outs. The final Section 8 offers some policy conclusions.

2 Related Literature

This paper is placed at the intersection of two strands of literature, the political economics of education and the analysis of examination standards. Research in the first strand is mainly concerned with the size of the budget available for public education or education subsidies, and the conflict of interest between different income classes when it comes to vote on the required taxes. In an early empirical study in this line, Romer et al. (1992) link local school expenditures to state grants and to the political rules governing referenda.

In theoretical work, voting behaviour of different income groups is often linked to market failures. Creedy and Francois (1990) provide a model where a human capital externality motivates low income individuals to support subsidies for higher education although they do not attend university. In the model by Fernandez and Rogerson (1995), middle and high income earners form a winning coalition which keeps education subsidies just low enough to exclude credit constrained poor individuals from higher education. Combining votes on admission criteria and tuition subsidies, De Fraja (2001) proves an ends-against-the middle result, where high income families and poor families with low ability children jointly oppose subsidising education. Ichino et al. (2011) show that the amount of public spending on education, and consequently the degree of intergenerational social mobility, depends on which percentile of the earnings distribution is decisive in voting.

Adding to the basic conflict of interest between income classes, Del Rey and Racionero (2012), Eckwert and Zilcha (2014), and Hatsor (2014) provide detailed analyses of the choice between different forms of subsidies and types of education systems. Moreover, research has addressed the political choice between education expenditures and other redistributive spending. Thus, Bearse et al. (2001) analyse the existence of voting equilibria in a model where taxes may be used for education or a lump sum transfer. Using three different empirical approaches, Bursztyn (2016) finds that poor individuals in Brazil prefer cash transfers to education subsidies. Poutvaara (2006) and Lancia and Russo (2016) show that there need not be an intergenerational conflict among students and pensioners over public spending as long as an increase in human capital raises pensions.

Further studies have added private schools and vouchers. Hoyt and Lee (1998) show that vouchers may attract support of a majority if the induced decline in public school enrolment allows to reduce taxes or to raise public school quality. This line of reasoning has been confirmed in a probabilistic voting model by De la Croix and Doepke (2009) and in a citizen candidate approach by Epple and Romano (2014). Distinguishing between universal and selective vouchers, which are granted only to families below an income limit, West and Chen (2000) show that the latter are more likely to prevail in a vote than the former.

As this overview shows, the political economics of education is mainly concerned with public spending on education and the implied redistribution, and has so far ignored the choice of graduation standards. In this part of the literature, the issue closest to the one analysed in this paper is voting on admission standards, as in De Fraja (2001). This author shows that tightening admission rules can improve equality of opportunity since it re-allocates university places from low-ability higher income students to poorer students with high ability. Admission tests and graduation standards, however, differ in that standards govern the incentives to learn. Extending previous knowledge, the present paper analyses how the interaction of such incentives and the costs and rewards of academic effort determine the outcome of a vote on standards.

The second strand of literature to which the present paper contributes is the theory of examination standards. In the seminal work by Costrell (1994, 1997) and Betts (1998), the standard determines students' academic effort, and the school chooses the standard by trading off a higher wage for graduates against a larger number of graduates. In line with the predictions from this theory, Figlio and Lucas (2004) show empirically that high school students with tough grading teachers perform better than those who are taught by lenient graders. Similarly, Babcock (2010) finds that study time at college is lower in courses with better grades.

Subsequent research has emphasised the information content of grades and degrees. Blankenau and Camera (2009) show that making grades more informative is a way to strengthen incentives to learn, and, hence, contributes to human capital formation. In the signaling model presented by Chan et al. (2007) schools always award better grades than deserved to at least some students. Such pooling arises since giving good grades is a costless way of raising the number of students who obtain high wages. Similarly, in the models by Ostrovsky and Schwarz (2010), Popov and Bernhardt (2013), and Zubrickas (2015), reducing the information content of transcripts or inflating grades allows schools to promote the job prospects of good or mediocre students at the expense of the truly excellent ones.

When grades do not fully reveal ability, other signals become relevant. Thus, employers may consider the reputation of a school (MacLeod and Urquiola, 2015) or a graduate's social origin (Schwager, 2012) in hiring decisions if individual grades are known to be noisy or biased. Intriguingly, as shown by Mechtenberg (2009), a gender bias in grading may distort the beliefs of female students about their own ability and hence lead to inefficient career choices.

A natural reaction to uninformative grades consists in disclosing information on grade distributions. Empirical studies typically find that such policies tend to improve grades. This may be due to increased grade inflation, since teachers emulate the behaviour of good graders (Lehr, 2016), or students self-select into easy courses (Bar et al., 2009). Alternatively, when tests are administered independently from the school, better informed parents are more likely to abandon a low quality school. From a randomised experiment involving private schools in Pakistan, Andrabi et al. (2017) conclude that through this mechanism, providing parents with information on initial test scores improves test scores later on.

Research has also studied factors which influence standard setting by schools, notably school competition and resources of schools. In the model by Brunello and Rocco (2008) a private and a public school differentiate their standards maximally and so cater to different segments of the ability distribution. In an empirical analysis of Swedish schools, Wikström and Wikström (2005) find that competition among public schools leads to somewhat inflated grades. According to Witte et al. (2014), an intervention in the Netherlands which provided schools with more resources had the same consequence. In contrast, in an experiment performed in Israeli schools investigated by Lavy (2009), introducing performance related pay for teachers improved academic results without changing grading ethics. Finally, various other determinants such as the social composition of a school's student body (Himmler and Schwager, 2013), students' opportunity to pester teachers (Franz, 2010), or even partisan affiliation of professors (Bar and Zussman, 2012) have been shown to affect grading standards.

To this literature, the present paper contributes by applying a political economy approach to the choice of graduation standard. As explained in the introduction, with the exception of Costrell (1994) and Brunello and Rocco (2008), this has not been done so far.

3 The Economic Model

There is a continuum of agents with mass one. Agents have two roles in the model, as students and as voters. One can interpret agents literally as adult individuals who still are in education, for example at a university, at an age where they have the right to vote. More broadly, agents can be seen as representing families composed of children in education and parents who use their right to vote to promote the interests of their children.

Agents are characterised by their ability $a \in A := [a_o, a_1)$, where $a_o \geq 0$ and a_1 may be infinite. Abilities are distributed according to the c.d.f. $F : A \to [0,1]$ which is continuous and strictly increasing on the support A. Thus, the density is strictly positive for $a \in A$. The mean and median abilities are denoted by $\overline{a} = \int_A a \, dF(a)$ and $a_m = F^{-1}(1/2)$.

In order to succeed at school, agents must exert effort denoted by $e \geq 0$. An agent with ability a who provides effort e incurs cost $c(e,a)$. The cost function $c : \mathbb{R}_{\geq 0} \times A \to \mathbb{R}_{\geq 0}$ is assumed to be three times continuously differentiable. I denote derivatives by subscripts; for example, $c_e(e,a)$ is the partial derivative of cost with respect to effort.

Assumption 1 $c(0,a) = 0$ *and* $\lim_{e\to\infty}[c(e,a)/e] > 1$ *for* $a \in A$.

Assumption 2 $c_e(0,a) = 0$ *for* $a \in A$, $c_e(e,a) > 0$ *and* $c_a(e,a) < 0$ *for* $(e,a) \in \mathbb{R}_{>0} \times A$

Assumption 3 $c_{ee}(e,a) > 0$, $c_{aa}(e,a) > 0$, *and* $c_{ea}(e,a) < 0$ *for* $(e,a) \in \mathbb{R}_{>0} \times A$

According to Assumption 1, a student who does not exert any effort does not incur any cost, and for increasing effort, the cost eventually exceeds the effort. Assumption 2 says that cost increases in effort, starting with a marginal cost of zero, but decreases in ability. Assumption 3 states that the marginal cost of effort is strictly increasing, that the cost-saving effect of ability becomes weaker (in absolute terms) as ability increases, and that higher ability decreases the marginal cost of effort. While effort is costly for all agents $a \in A$, an agent at the upper bound of the ability distribution may be able to learn without any cost. That is, I do not rule out that $c(e, a_1) := \lim_{a \to a_1} c(e, a) = 0$ for all e.

The standard $s \in \mathbb{R}_{\geq 0}$ defines the performance level required to pass the examination. Performance is entirely determined by, and measured in the same units as, effort, subsuming the influence of ability in the cost function $c(e, a)$. Students who exert effort $e \geq s$ graduate, while those with $e < s$ fail and will be referred to as drop-outs.

After leaving school, agents will be employed by firms which operate a constant returns to scale technology transforming one efficiency unit of labour into one unit of a numéraire output. The amount of efficiency units supplied by a worker is given by her examination performance, and hence by the effort level e deployed at school. Firms cannot observe the examination performance of an individual worker but know whether she graduated or not. Therefore, all graduates will obtain the same wage w_s, and all drop-outs will receive the same wage w_o. In a competitive equilibrium on the labour market, the graduate wage w_s (the drop-out wage w_o) must be equal to the expected productivity of all agents who exert effort $e \geq s$ ($e < s$). Ownership of firms is sufficiently widely distributed among the agents that price-taking behaviour is justified, but otherwise need not be specified. The reason is that, because of constant returns to scale, firms earn zero profits whatever the standard or the behaviour of students, so that firm ownership does not change the stakes any individual has in the choice of standards.

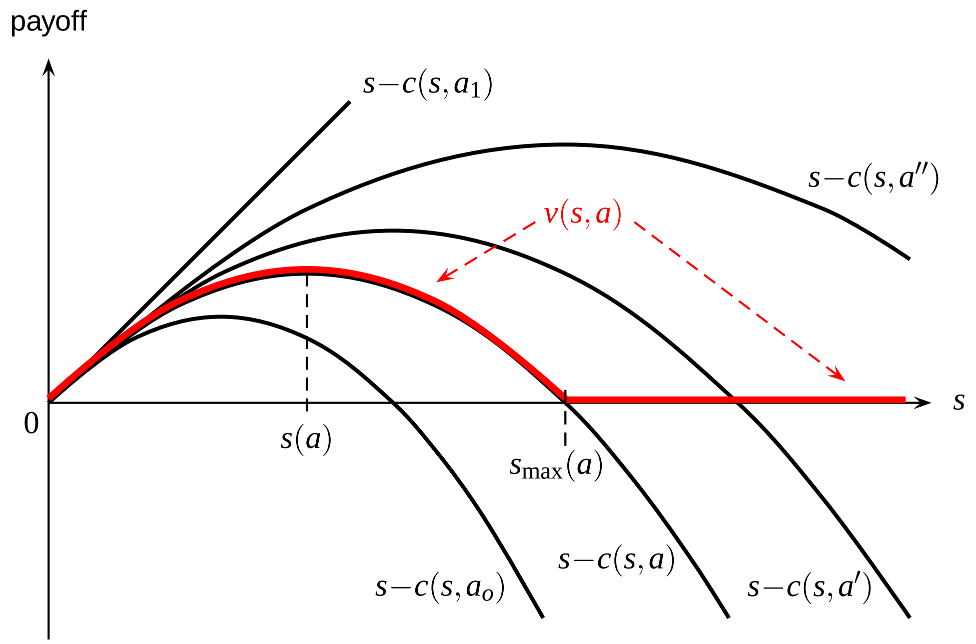
For given standard s, a student of ability $a \in A$ chooses effort so as to maximise the expected wage net of effort cost. Conditional on choosing an effort $e \geq s$ sufficient to graduate this payoff is $w_s - c(e, a)$. Since the wage does not depend on effort as long as the constraint $e \geq s$ is met, from $c_e > 0$ the minimal effort $e = s$ dominates all effort levels $e > s$. In the same way, conditional on not graduating ($e < s$), the payoff is $w_o - c(e, a)$ which is maximised by $e = 0$. Thus, students either just meet the standard and graduate, or they do not put in any effort at school and fail. Observing $c(0, a) = 0$ from Assumption 1, one sees that graduation (dropping out) is optimal if $w_s - c(s, a) \geq (<) \, w_o$.

With this behaviour, equilibrium wages will be $w_s = s$ and $w_o = 0$. Thus, in equilibrium graduation is optimal if

$$s - c(s, a) \geq 0 \,. \tag{1}$$

Figure 1 shows the payoff from graduating on the l.h.s. of (1) as a function of the standard s for several levels of ability. From Assumption 1, this payoff is zero at $s = 0$ and eventually becomes negative for high enough s. Moreover, from Assumption 2, the payoff's slope $1 - c_e(s, a)$ is positive at $s = 0$ implying that for all $a \in A$, at some (possibly low) standard, graduation is worthwhile and a positive payoff can be reached.

Figure 1: The payoff from graduating and indirect utility



The curves show the payoff from graduating $s - c(s, a)$ as a function of the standard s, for agents with varying abilities $a_o < a < a' < a'' < a_1$. The indirect utility $v(s, a)$ of an agent with ability a is displayed in bold red. For this agent, the optimal standard is $s(a)$, and the highest standard such that she will graduate is $s_{\max}(a)$.

Assumptions 1 to 3 imply that for each $a \in A$ there is a unique positive standard $s_{\max}(a)$, given by the solution to (1) as an equality, which yields a payoff of zero. As is apparent from Figure 1, an agent with ability a will graduate if $s \leq s_{\max}(a)$ and drop out if $s > s_{\max}(a)$. Thus, $s_{\max}(a)$ is the maximal standard which an agent of ability a is willing to satisfy.

Combining these observations, one finds the indirect utility function $v(s, a) = \max\{0; s - c(s, a)\}$ of an agent $a \in A$. This function, which in Figure 1 is illustrated by a bold red line, relates the standards about which agents vote to the individual agent's utility, anticipating her own effort and graduation choices and the equilibrium wages ensuing from the chosen standard. From Assumption 3, the payoff from graduating is strictly concave in s. Hence for each $a \in A$, there is a unique standard $s(a) = \text{argmax}_s\{v(s,a)|s \geq 0\} > 0$ which maximises the payoff of an agent with ability a.

Increasing ability shifts the payoff from graduating upward, since Assumption 2 implies that cost decreases in ability. Moreover, from Assumption 3, the marginal cost of effort decreases when ability rises. As illustrated in Figure 1, this means that both the utility maximising standard $s(a)$ and the highest standard $s_{\max}(a)$ which an agent will satisfy strictly increase in ability a.

Inverting the relationship $s_{\max}(a)$, one can express the decision to graduate or not by defining a minimal ability $a_{\min}(s)$ which an agent must have to be willing to satisfy a given standard s. This level is called the graduation threshold for standard s. To formalise this, one has to observe that for some s, the solution to (1) may not be in the support A of the ability distribution. In such a case, all agents (no agent) will graduate for the standard under consideration, and I define this threshold accordingly to be a_o or a_1. Formally:

Definition 1 *For all $s \in \mathbb{R}_{\geq 0}$:*

$$a_{\min}(s) = \begin{cases} a_o & \textit{if} \quad s - c(s,a) > 0 \quad \textit{for all } a \in A \\ \tilde{a} & \textit{if} \quad s - c(s,\tilde{a}) = 0 \quad \textit{for some } \tilde{a} \in A \\ a_1 & \textit{if} \quad s - c(s,a) < 0 \quad \textit{for all } a \in A \end{cases}$$

Notice that from $c_a < 0$ in Assumption 2, $\tilde{a}$ in the second line of Definition 1 must be unique if it exists and hence $a_{\min}(s)$ is well defined. Moreover, all agents with $a < a_{\min}(s)$ will fail and all agents with $a \geq a_{\min}(s)$ graduate. Finally, differentiating $s - c(s,a) = 0$ shows that

$$\frac{da_{\min}(s)}{ds} = \frac{1 - c_e(s, a_{\min}(s))}{c_a(s, a_{\min}(s))} > 0\,, \tag{2}$$

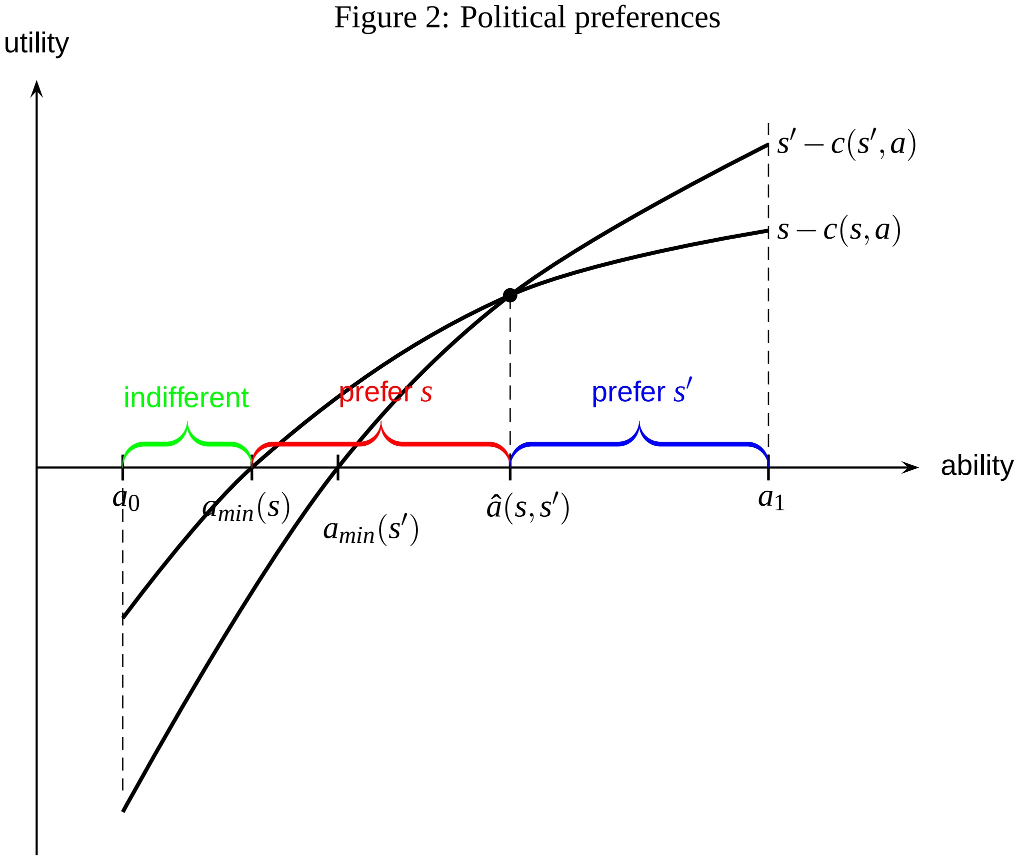
where the inequality follows on $c_a < 0$ and the fact that at $a_{\min}(s)$, the payoff from graduating must be decreasing. Therefore, the graduation threshold is weakly increasing in the standard s, and strictly so if the threshold is in the interior of A.

The graduation threshold and the indirect utility function guide the voting behaviour of agents to which I now turn.

4 Political Preferences

The upshot of this section is that more able individuals tend to prefer higher standards. To make this statement precise, consider two standards s and s' where s' is more demanding than s, that is, $0 < s < s'$.

Figure 2: Political preferences



The curves depict the payoff from graduating as a function of ability for two standards $s < s'$. At ability $\hat{a}(s,s')$ the payoff from graduating is equal under both standards. Agents with ability below $a_{\min(s)}$ drop out under both standards and are indifferent. Agents with ability between $a_{\min(s)}$ and $\hat{a}(s,s')$ strictly prefer the more lenient standard s. Agents with ability above $\hat{a}(s,s')$ strictly prefer the mode demanding standard s'.

Figure 2 displays the payoff from graduation as a function of ability for the two standards. Both curves are increasing and the curve representing the higher standard s' is steeper than the one representing the lower standards s. Thus, graduation is more rewarding for more able individuals, and an increase in ability procures a larger gain when the standard is higher. Analytically, we have from $c_a < 0$ and $c_{ea} < 0$ in Assumptions 2 and 3:

$$0 < \frac{\partial[s-c(s,a)]}{\partial a} < \frac{\partial[s'-c(s',a)]}{\partial a}. \qquad (3)$$

The curves intersect at the critical ability $\hat{a}(s,s')$, defined by the solution $\hat{a}$ to the equation $s-c(s,\hat{a}) = s'-c(s',\hat{a})$, where the payoffs from graduating under both standards s and s' are equal. Such a solution exists if $s-c(s,a_o) > s'-$

$c(s', a_o)$ and $s - c(s, a_1) < s' - c(s', a_1)$, that is, if an agent with the lowest ability obtains a higher payoff from the smaller standard, whereas an agent with an ability close to the upper bound gains more from the higher standard. Moreover, $\hat{a}(s, s')$ is unique from (3).

Since an agent with critical ability obtains the same payoff from graduating under two different standards, this payoff must be strictly positive (see Figure 1), i.e., $s - c(s, \hat{a}(s, s')) = s' - c(s', \hat{a}(s, s')) > 0$. Hence, the agent will graduate under both standards. Agents with even higher ability $a > \hat{a}(s, s')$ will also graduate under both standards, but obtain a strictly higher payoff when the more demanding standard is chosen, $v(s, a) = s - c(s, a) < s' - c(s', a) = v(s', a)$. In a pairwise vote, these individuals will strictly prefer s' over s.

Agents with low ability may choose to drop out under one or both of the standards considered. In Figure 2, it is assumed that this occurs even for the easier standard s, that is, $a_o < a_{\min}(s)$. In this case agents with low levels of ability $a \in [a_o, a_{\min}(s))$ will fail and obtain the same indirect utility $v(s, a) = v(s', a) = 0$ under both standards. Hence, in this case there is a positive mass of indifferent voters. In the opposite case, where $a_o = a_{\min}(s)$, the mass of indifferent individuals is zero.

For agents with intermediate ability $a \in (a_{\min}(s)), \hat{a}(s, s'))$ one has $v(s, a) = s - c(s, a) > v(s', a) = \max\{0; s' - c(s', a)\}$. These agents strictly prefer the lower standard s over the more demanding standard s', regardless of whether they would graduate under s' or not.

In two special cases there is no critical ability $\hat{a}$ in the interior of A. First, both standards may be so low that even for an agent with the lowest ability, the payoff from graduating is at least as large with the more demanding standard as with the more lenient standard, $s - c(s, a_o) \leq s' - c(s', a_o)$. In this case, I define the critical ability to be the lowest possible one, $\hat{a}(s, s') := a_o$. One can see in Figure 1 that this case can only occur if the lower standard s is on the increasing part of the payoff curve. Thus, in this case $s - c(s, a) > 0$ for all $a \in A$, or equivalently, $a_{\min}(s) = a_o$. All agents graduate under the standard s, yielding $v(s, a) = s - c(s, a) > 0$. Moreover, from (3), one has $s - c(s, a) < s' - c(s', a)$ for all $a > a_o$. It follows $0 < v(s, a) = s - c(s, a) < s' - c(s', a) = v(s', a)$ for all $a > a_o$. Hence in this case, all agents except possibly those with the lowest ability strictly prefer the higher standard.

The second special case obtains if $s - c(s, a_1) \geq s' - c(s', a_1)$. Here both standards are so high that even agents with the highest abilities reap a larger payoff from graduating under the lower standard than under the higher standard. In this case, the critical ability is set at the upper bound of the ability distribution, $\hat{a}(s, s') := a_1$. Agents with ability $a < a_{\min}(s)$ will again fail under both standards and are indifferent between them, $v(s, a) = v(s', a) = 0$. Agents with ability $a \in (a_{min}(s), a_1)$ will graduate under s, obtain utility $v(s, a) = s - c(s, a) >$

$v(s',a) = \max\{0; v(s',a)\}$, and strictly prefer the easier standard s. It is possible that all (no) agents are willing to satisfy the lower standard s, i.e., $a_{\min}(s) = a_o$ ($a_{\min}(s) = a_1$). If this holds, then the set of agents strictly preferring s over s' (the set of indifferent agents) covers the entire interval A, and the set of indifferent agents (the set of agents strictly preferring s over s') has measure zero.

For easy reference, the three cases of the critical ability are recapped in

Definition 2 *For all $s, s' \in \mathbb{R}_{>0}$ with $s < s'$:*

$$\hat{a}(s,s') = \begin{cases} a_o & \text{if} \quad s - c(s,a_o) \leq s' - c(s',a_o) \\ \hat{a} \text{ solving } s - c(s,\hat{a}) = s' - c(s',\hat{a}) & \text{if} \quad s - c(s,a_o) > s' - c(s',a_o) \\ & \quad \text{and } s - c(s,a_1) < s' - c(s',a_1) \\ a_1 & \text{if} \quad s - c(s,a_1) \geq s' - c(s',a_1)\,. \end{cases}$$

The following Lemma summarises the preceding discussion:

Lemma 1 *For any two standards $s, s' \in \mathbb{R}_{>0}$ with $s < s'$:*

a. For $a_o < a < a_1$:

- *(i) If $a \leq a_{\min}(s)$, then $v(s,a) = v(s',a) = 0$.*
- *(ii) If $a_{\min}(s) < a < \hat{a}(s,s')$, then $v(s,a) > v(s',a) \geq 0$.*
- *(iii) If $a = \hat{a}(s,s')$, then $v(s,a) = v(s',a) > 0$.*
- *(iv) If $\hat{a}(s,s') < a$, then $v(s',a) > v(s,a) > 0$.*

b.

- *(i) If $a_o < a_{\min}(s)$, then $v(s,a_o) = v(s',a_o) = 0$.*
- *(ii) If $a_o = a_{\min}(s) < \hat{a}(s,s')$, then $v(s,a_o) \geq v(s',a_o) \geq 0$.*
- *(iii) If $a_o = a_{\min}(s) = \hat{a}(s,s')$, then $v(s',a_o) \geq v(s,a_o) \geq 0$.*

This Lemma states that agents with low ability below the graduation threshold of the more lenient standard, if such agents exist, are indifferent between both standards since they do not plan to graduate under either of them. Agents with intermediate ability between the graduation threshold for the easier standard and the critical value strictly prefer the lower standard, and agents with ability above the critical value strictly prefer the higher standard.

5 Median Voter Results

The standard is determined by the agents in a series of pairwise votes. The voting equilibrium is described by a Condorcet winner, that is, a standard which collects a majority of votes against any other standard.

In the present context, the analysis of Condorcet winners is complicated by the opportunity to drop out. This option transforms the payoff from graduating $s - c(s,a)$, which is a single-peaked function, into the indirect utility $v(s,a)$, which stays equal to zero for all standards where the agent fails to graduate (see Figure 1). In this section, I present two modifications of the definition of a Condorcet winner which allow to establish a median voter equilibrium in spite of such flat parts of the indirect utility function.

The first modification is given by

Definition 3 *Standard $s \in \mathbb{R}_{\geq 0}$ is a weak Condorcet winner if for all standards $s' \in \mathbb{R}_{\geq 0}, s' \neq s$:*

$$\int_{\{a \in A | v(s,a) \geq v(s',a)\}} dF(a) > 1/2 .$$

According to this definition, in order to prevail, a standard must be weakly but not necessarily strictly preferred to any other standard by a majority of agents. Thus, agents behave optimally in each vote, but ties are broken in a way which supports the equilibrium.With this definition, one can allow flat parts in the indirect utility function (as, for example, in Persson and Tabellini, 2002, Definition 2, p. 22), and still obtain a median voter result:

Proposition 1 *The standard $s_m := s(a_m)$ preferred by agents with median ability is a weak Condorcet winner.*

Proof: See Appendix A.I. ■

Proposition 1 differs from a usual median voter result in that the median preferred standard s_m is only shown to be a *weak* Condorcet winner, and that the proposition does not claim uniqueness.

The tie-breaking assumption in Definition 3 becomes relevant in votes between s_m and somewhat higher standards. In such a vote, the majority for s_m must be secured by low ability agents. If some of these drop out under s_m, i.e., if $a_{\min}(s_m) > a_o$, they will also drop out under the alternative, higher standard. These agents are then indifferent and so might vote together with high ability agents in favour of the more demanding standard. Thus, with an arbitrary tie breaking rule, s_m might fail to win such a vote and hence might not be a Condorcet winner.

Definition 3 does not, however, ensure uniqueness: By attributing votes of indifferent agents in a different way, one can support other standards as weak Condorcet winners as well. This is shown in Appendix A.II. There, I give a sufficient condition implying that a standard $\tilde{s} > s_m$ is a weak Condorcet winner. This equilibrium is established by assuming that indifferent voters always support $\tilde{s}$, and the condition requires that in every vote involving $\tilde{s}$, the mass of indifferent agents is large enough.

The second way of establishing a median voter result consists in assuming a specific tie-breaking rule for indifferent agents. This rule stipulates that agents who drop out under both standards on the ballot support the lower one of these standards. Since such agents are located at the lower tail of the ability distribution, this kind of behaviour is rather plausible. For example, from earlier experiences these agents might have developed general reservations against a tough educational regime, or they might feel compelled to mimic the voting behaviour of their peers with slightly higher ability, who strictly prefer the lower standard.

This kind of tie-breaking rule can be supported by refining the Condorcet equilibrium in the spirit of trembling hand perfection. To make this precise, I define an ε-education model where for every standard $s \in \mathbb{R}_{\geq 0}$ each of the two options '$e = s$' and '$e = 0$' will be chosen with probability of at least $\varepsilon > 0$, where ε is a small number. Thus, with probability ε the agent will make an 'error' in her graduation decision. While such an error is conceivable in any strategic situation, modeling a deviation from planned behaviour is particularly appealing if one interprets agents as families where education decisions are taken by children. Here, ε measures the possibility that children do not follow the educational course which their parents deem optimal for them.

In an ε-education model, the payoff from standard s for an agent with ability a will be $v(s,a;\varepsilon) = (1-\varepsilon)[s - c(s,a)]$ if $s - c(s,a) \geq 0$ and $v(s,a;\varepsilon) = \varepsilon[s - c(s,a)]$ if $s - c(s,a) < 0$. Adapting Definition 3 and Proposition 1 to the ε-education model, one obtains

Definition 4 *Standard $s \in \mathbb{R}_{\geq 0}$ is a weak Condorcet winner in the ε-education model if for all standards $s' \in \mathbb{R}_{\geq 0}, s' \neq s$:*

$$\int_{\{a \in A | v(s,a;\varepsilon) \geq v(s',a;\varepsilon)\}} dF(a) > 1/2 \,.$$

Lemma 2 *For all $0 < \varepsilon < 1$, the standard s_m preferred by agents with median ability is the unique weak Condorcet winner in the ε-education model.*

Proof: From $1 - \varepsilon > 0$, $v(s,a;\varepsilon) = (1-\varepsilon)[s - c(s,a)]$ is strictly increasing (strictly decreasing) in s for $0 \leq s < s(a)$ (for $s(a) < s < s_{\max}(a)$). From $\varepsilon > 0$, $v(s,a;\varepsilon) = \varepsilon[s - c(s,a)]$ is still strictly decreasing in s for $s > s_{\max}(a)$. Therefore, in a vote among s_m and a lower (higher) alternative standard $s < s_m$ ($s' > s_m$), all agents with $a > \hat{a}(s, s_m)$ ($a < \hat{a}(s_m, s')$) strictly prefer s_m over s (over s'). As seen in the proof of Proposition 1, these agents constitute more than half of the electorate. Therefore, s_m beats every alternative standard, and no alternative standard can attract a majority against s_m. ■

The key difference between Proposition 1 and Lemma 2 is that in the ε-education model, the median preferred standard is the *unique* Condorcet winner.

This arises from the fact that the 'trembling hand' assumption blurs the decision to drop out from school. While the standard $s_{\max}(a)$ still defines the cut-off above which an agent of ability a does not plan to graduate anymore, she may still do so erroneously and thus experience the (negative) payoff from graduating with probability $\varepsilon > 0$. This breaks the indifference of low ability agents in favour of the less demanding standard, making the indirect utility function $v(s,a;\varepsilon)$ single-peaked.

According to the idea of trembling hand perfection, a Condorcet winner in the original model is only reasonable if it is robust against the possibility of small errors. This is captured by the following definition:

Definition 5 *A standard $s \in \mathbb{R}_{\geq 0}$ is a strong Condorcet winner if*

(i) s is a weak Condorcet winner, and

(ii) there is a sequence $\{s_n\}_{n=1,2,\ldots}$ such that $s_n \to s$ and for all n, s_n is a weak Condorcet winner in an ε_n-education model, where $\varepsilon_n \in (0,1)$ and $\varepsilon_n \to 0$.

Thus, a weak Condorcet winner is called 'strong' if it is the limit of a sequence of weak Condorcet winners in ε-education models the error probabilities of which converge to zero. One immediately concludes from Proposition 1 and Lemma 2:

Proposition 2 *The standard s_m preferred by agents with median ability is the unique strong Condorcet winner.*

To summarise, the median-preferred standard s_m is the unique Condorcet winner in the usual sense, i.e., it is strictly preferred by a majority of agents in any pairwise vote, if almost all agents graduate under this standard, $a_{\min}(s_m) = a_o$. If a positive mass of agents drop out under the median preferred standard, $a_{\min}(s_m) > a_o$, there are two ways to establish a median voter result. First, s_m is a weak Condorcet winner in the sense that a majority of agents weakly prefers this standard in any pairwise vote. Second, s_m is the only strong Condorcet winner, that is, it is the only voting outcome which is robust against small errors in the education decision.

In the following Sections 6 and 7, the welfare properties of the median preferred standard are examined.

6 Welfare Analysis

Welfare is defined by a utilitarian criterion, aggregating the indirect utility of all agents:

Definition 6 *For any given standard s, welfare is*

$$W(s) = \int_{a_o}^{a_1} v(s,a)\,dF(a)\,.$$

For agents who graduate, utility is the wage earned net of effort cost, and for drop-outs, utility is zero. Therefore, welfare is given by $W(s) = \int_{a_{\min}(s)}^{a_1} \big[s - c(s,a)\big]\,dF(a)$. Differentiating this equation w.r.t. s and using Definition 1, one finds that an increase in the standard changes welfare by

$$W'(s) = \int_{a_{\min}(s)}^{a_1} \big[1 - c_e(s,a)\big]\,dF(a)\,. \tag{4}$$

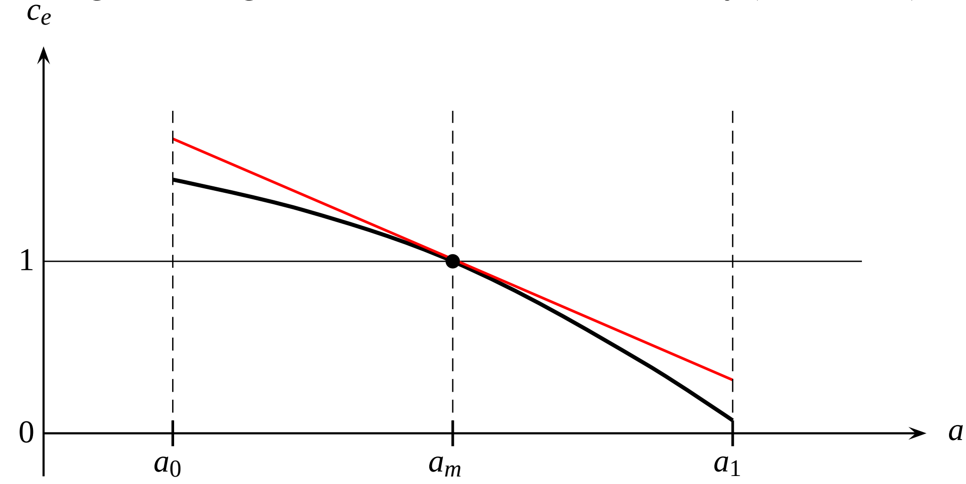
To understand (4), notice that a change in the standard affects both the graduation threshold $a_{\min}(s)$ and the utilities $v(s,a)$. The first effect cancels, however, since the utility of an agent at the threshold is zero by definition. Since drop-outs anyway receive a utility of zero, the second effect is relevant only for those agents who will graduate under the original standard. For these individuals, raising the standard by one unit increases the wage by one unit, since wage and standard are normalised to be equal. On the other hand, in order to satisfy the higher standard, students have to incur additional effort cost so that, for an agent with ability a, the net gain from increasing the standard is $1 - c_e(s,a)$.

In the following I will examine under what conditions welfare will increase if the standard is raised above the standard chosen by the majority. That is, I provide sufficient conditions for $W'(s_m) > 0$. Only a local welfare analysis is offered since any second order conditions ensuring a global maximum will necessarily require assumptions on the shape of the density $F'(a)$, which are likely to be either very strong or difficult to interpret.

The starting observation in this analysis is that, because s_m is optimal, agents with median ability are indifferent to an increase in standard, $1 - c_e(s_m, a_m) = 0$. Since marginal cost of effort is strictly decreasing in ability, agents with above-median ability will gain from an increase in standard, i.e., $1 - c_e(s_m, a) > 0$ for all $a > a_m$. Agents with below-median ability will lose, $1 - c_e(s_m, a_m) < 0$, as long as they still graduate. Therefore, the net welfare effect of an increase in the standard hinges on the relative sizes of aggregate gains and losses by high and low ability agents respectively. These aggregate amounts in turn are determined by three features: the shape of the marginal cost function c_e, the distribution function $F(\cdot)$, and the graduation threshold $a_{\min}(s_m)$. In this section, I provide two results highlighting the role of the first two features, whereas the importance of the graduation threshold is taken up in Section 7.

The properties of the cost and distribution functions used in these results are described by two pairs of conditions. The first pair are

Figure 3: Marginal cost of effort is concave in ability (Condition 1)



The bold black curve shows the marginal cost of effort at the median preferred standard $c_e(s_m, a)$ as a function of ability. The horizontal line represents the marginal benefit of an increase in the standard. Condition 1 requires that the marginal cost stays below the tangent, painted red.

Condition 1 $c_{eaa}(s_m, a) \leq 0$ *for* $a \in [a_o, a_1)$.

Condition 2 $\bar{a} \geq a_m$.

Condition 2 simply states that mean ability exceeds median ability. Condition 1, which is illustrated in Figure 3, requires that the marginal cost of effort c_e is a concave function of ability. In Figure 3, ability is depicted on the horizontal axis and marginal cost and benefits of an increase in standard are measured on the vertical axis. The marginal cost of effort evaluated at the median preferred standard, $c_e(s_m, a)$, decreases according to Assumption 3, and cuts the marginal benefit of 1 at the median ability a_m. As illustrated in this figure, if the cost function satisfies Condition 1, the marginal cost curve should become steeper as ability increases. Thus, the effort-enhancing effect of ability increases in ability.

Alternatively, I consider the following pair of conditions:

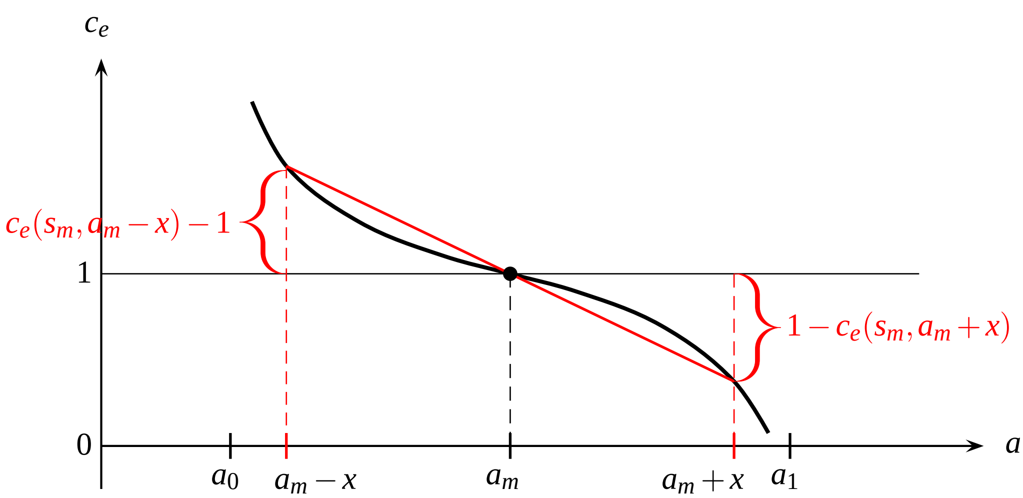
Condition 3 *For all* $x \in (0, \min\{a_m - a_o; a_1 - a_m\}]$*:*

$$\frac{1}{2}c_e(s_m, a_m - x) + \frac{1}{2}c_e(s_m, a_m + x) \leq 1 \,.$$

Condition 4 *For all* $x \in (0, \min\{a_m - a_o; a_1 - a_m\}]$*:*

$$\frac{1}{2} - F(a_m - x) \geq F(a_m + x) - \frac{1}{2} \,.$$

Figure 4: Marginal effort cost at abilities symmetric to the median (Condition 3)



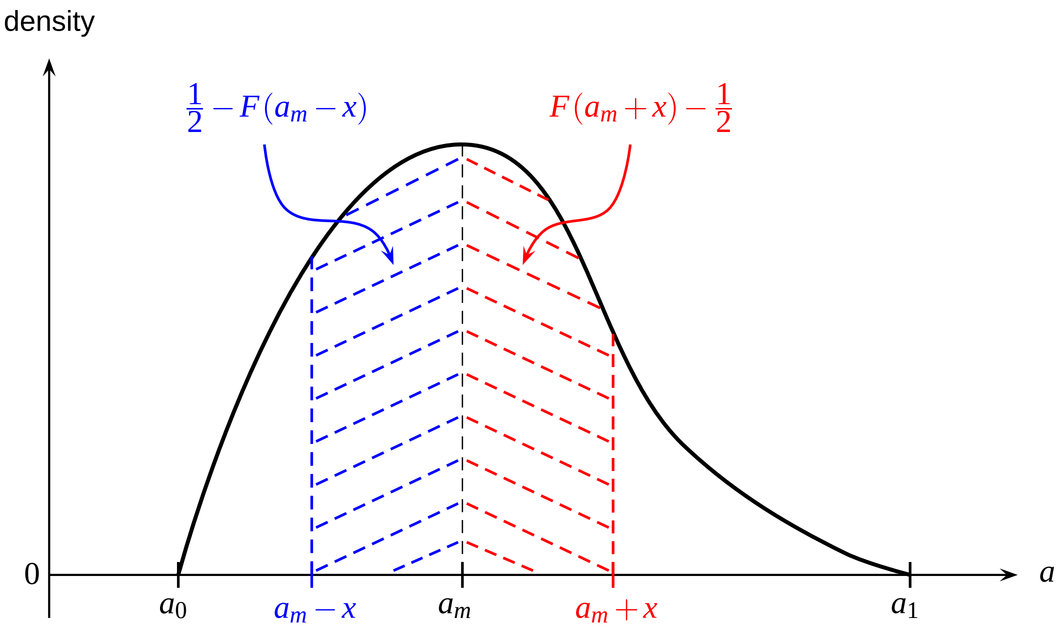
The bold black curve shows the marginal cost of effort at the median preferred standard $c_e(s_m, a)$ as a function of ability. The horizontal line represents the marginal benefit of an increase in the standard. The braces illustrate the loss and gain procured by such an increase to two individuals with abilities x units below and above the median. Condition 3 requires that the gain of the more able individual (right brace) is at least as large as the loss of the less able one (left brace).

In Condition 3, two agents are considered whose abilities exceed and, respectively, fall short of the median ability by the same amount x. The condition requires that the average marginal cost of these two individuals does not exceed the marginal benefit. Thus, on average, these two agents gain from raising the standard. Figure 4 gives a geometric intuition for this property, which is based on splitting the graph of the marginal cost curve c_e in the two parts corresponding to the domains of below and above median abilities. Condition 3 requires that, when one of these parts is mirrored at the point $(a_m, 1)$, the image should be located below the other part.

Also Condition 4 (see Figure 5) starts from considering two ability levels which are located symmetrically around the median. The condition requires that the mass of agents with abilities between the lower one of these values and the median is at least as large as the mass of agents with abilities between the median and the higher one of these values.

To summarise, Conditions 1 and 3 represent the idea that the impact of ability on marginal effort cost should be stronger on the high side of the ability distribution than on the low side. That is, academic performance is very sensitive to ability when one compares good and very good students, whereas below the median ability, differences in ability matter less. It is an empirical issue whether

Figure 5: Higher abilities are more spread out than lower abilities (Condition 4)



The curve illustrates an ability distribution which satisfies Condition 4. Starting from median ability, the probability mass covered by moving distance x to the left (shaded blue) is at least as large as the mass covered by moving the same distance to the right (shaded red).

such a property holds in reality. A priori, it seems plausible to me because, on the one hand, weak students mostly can reach a satisfactory performance with sufficient training, whereas, on the other hand, really excellent achievements are out of reach except for the very brightest.

According to Conditions 2 and 4, the distribution of abilities is more 'spread-out' at the upper end of the support than at the lower end. This can arise, for example, by the presence in the economy of a few agents with very high ability, who raise the mean, whereas a large mass of agents is concentrated at moderately low ability levels. This corresponds to the empirical fact that income distributions, which at least partially reflect distributions of productivity or ability, are typically right-skewed (for the U.S., see Proctor et al., 2016, p. 23).

Proposition 3 *If Conditions 1 and 2 hold, then* $W'(s_m) \geq 0$*. If in addition, an inequality in one of these conditions is strict or* $a_{\min}(s_m) > a_o$*, then* $W'(s_m) > 0$.

Proof. See Appendix A.III. ∎

Proposition 4 *If Conditions 3 and 4 hold, then* $W'(s_m) \geq 0$*. If in addition, an inequality in one of these conditions is strict or* $a_{\min}(s_m) > a_o$*, then* $W'(s_m) > 0$.

Proof. See Appendix A.IV. ■

Propositions 3 and 4 show that democratic choice leads to an inefficiently low examination standard if the cost function and the distribution function satisfy one of the pairs of Conditions 1 and 2, or 3 and 4. Intuitively, an increase in standard is beneficial if the gain conferred this way to agents whose abilities exceed the median by a certain amount outweighs the loss incurred by agents whose abilities fall short of the median by a similar amount. This is the case if the marginal cost of effort decreases fast once ability is raised above the median but rises only slowly when ability falls below the median, as required by Conditions 1 or 3. Moreover, the aggregate gain (loss) is large (small) if the mass of agents with very high (low) ability is relatively large (small), as postulated in Conditions 2 or 4.

Looking closer at the pairs of conditions required in each proposition, one notices that Condition 1 implies Condition 3, and that Condition 4 implies Condition 2. Therefore, there is a substitutive relationship between the properties of the cost function and the distribution function in the sense that it is possible to weaken one of them if one strengthens the other.

As mentioned in the introduction, Costrell (1994) proves a result which appears to be contrary to Propositions 3 and 4. In his model, the standard chosen by majority vote is inefficiently high if the distribution of preferred standards is symmetric unimodal. This contrasts with Conditions 2 and 4, both of which are satisfied with equality if the distribution of abilities is symmetric. The main difference in my set-up and the analysis by Costrell (1994) lies in the objective function of voters: In Costrell (1994), voters care only about academic performance and hence try to maximise the productivity of students, but do not take effort cost into account. In contrast, in the present analysis, it is assumed that parents will vote for reducing the standard if they feel that their children suffer too much from the effort required in school. Not surprisingly then, an education system where effort cost of students is politically important is likely to be less demanding than a system which only aims at raising educational outcomes.

In Propositions 3 and 4 the existence of agents who do not graduate under the median preferred standard figures only as a tie-breaking device in case both of the respective conditions are just satisfied as equalities. In the following Section 7, I show that the presence of a substantial number of drop-outs independently contributes to an insufficiently high median preferred standard. Hence, the pairs of conditions used in each proposition are sufficient for this result but not necessary.

7 The Role of Dropouts

By not graduating, students have the opportunity to avoid costly learning effort. Therefore, the possibility to drop out mitigates the negative welfare effect of a rising standard. As a result, in the presence of drop-outs, the median preferred standard may be too low even when Conditions 1 and 3 fail, that is, when marginal cost of effort is very high for low ability agents.

I will illustrate this effect by means of an example. In this example, ability is uniformly distributed on $A = [a_o, a_1] = [0, 2]$ with $a_m = \overline{a} = 1$. Effort cost is given by a family of functions

$$c(e, a; \gamma) = \frac{e^2}{2}\left[1 + (a_m - a) + \gamma(a_m - a)^2\right], \tag{5}$$

where the parameter is restricted to $0 \leq \gamma \leq 1/2$ to ensure that $c(e, a; \gamma)$ satisfies Assumptions 1 to 3. Computing $s_m = 1$, $c_e(s_m, a; \gamma) = 2 - a + \gamma(1 - a)^2$, $c_{ea}(s_m, a; \gamma) = -1 - 2\gamma(1 - a)$, and $c_{eaa}(s_m, a; \gamma) = 2\gamma$, one sees that γ determines the curvature of the marginal cost of effort. Specifically, for $\gamma > 0$ the example violates both Conditions 1 and 3.

The graduation threshold $a_{\min}(s_m; \gamma)$ solves the equation $c(s_m, a; \gamma) = s_m$, or equivalently, $1 + (a_m - a) + \gamma(a_m - a)^2 = 2/s_m$. With $a_m = s_m = 1$ it follows that for all admissible γ, the marginal cost of effort at the graduation threshold is $c_e(s_m, a_{\min}(s_m; \gamma); \gamma) = 2$. The change in welfare induced by a marginal increase in the standard can be computed from (4) as

$$\frac{\partial W(s_m; \gamma)}{\partial s_m} = -\frac{1}{2}\int_{a_{\min}(s_m; \gamma)}^{2} \left[(1 - a) + \gamma(1 - a)^2\right] da\,.$$

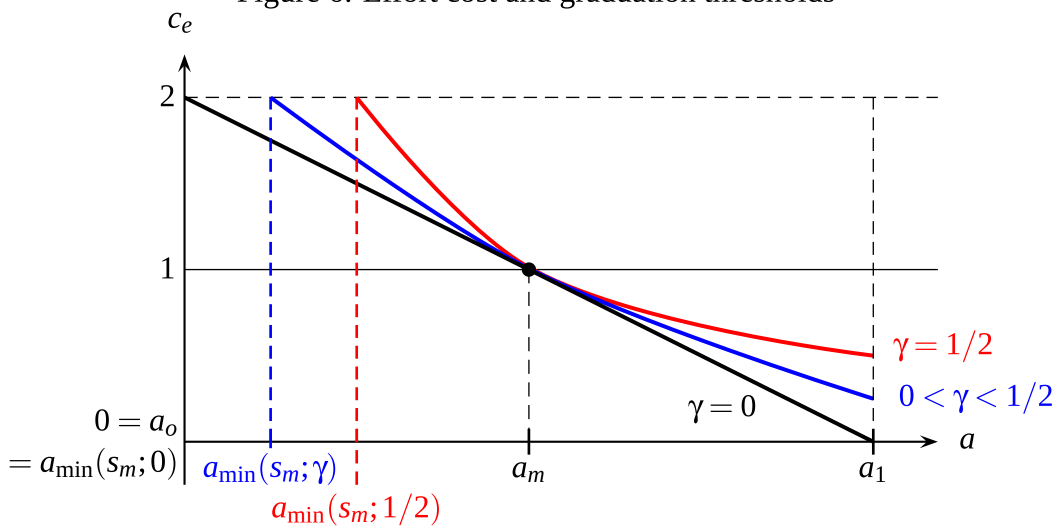
From these equations, one derives:

Proposition 5 *If the cost of effort is given by (5) and ability is distributed uniformly on* $[0, 2]$*, then* $\partial W(s_m; \gamma)/\partial s_m > 0$ *for all* $0 < \gamma < 1/2$.

Proof. Computations done with Mathematica, and available from the author upon request, reveal that the only two values of $\gamma \in [0, 1/2]$ with $\partial W(s_m; \gamma)/\partial s_m = 0$ are $\gamma = 0$ and $\gamma = 1/2$, and that $\partial W(s_m; \gamma)/\partial s_m$ is increasing in γ at $\gamma = 0$. From this, it follows $\partial W(s_m; \gamma)/\partial s_m > 0$ for $0 < \gamma < 1/2$. ■

The logic of Proposition 5 is illustrated in Figure 6. Here, the downward sloping straight line is the marginal cost of effort for the lowest admissible value $\gamma = 0$, which leads to the graduation threshold $a_{\min}(s_m; 0) = a_o = 0$. When γ rises above zero, the marginal cost of effort bends upwards and becomes strictly convex, as seen in the blue curve. The highest possible $\gamma = 1/2$ finally results in the highest curve, painted red. With uniform distribution of ability, the aggregate losses and gains of an increase in standard are directly measured by the areas between these curves and the marginal benefit of 1. It is apparent that the net gain would decrease in γ if the graduation threshold remained at $a_o = 0$. However, the threshold moves to the right as γ increases, so that the area representing the loss, which is bounded below by the vertical line at $a_{\min}(s_m; \gamma)$, shrinks.

Figure 6: Effort cost and graduation thresholds



The downward sloping lines display marginal cost of effort at the median preferred standard $c_e(s_m, a; \gamma) = 2 - a + \gamma(1 - a)^2$ as a function of ability for the lowest (black straight line), highest (higher curve, red), and an intermediate (lower curve, blue) curvature parameter γ. The horizontal line represents the marginal benefit of an increase in the standard. As marginal effort cost bend upwards, the difference between marginal benefit and marginal cost decreases for those agents who still graduate. At the same time, the graduation threshold rises. This mitigates the loss of low ability agents such that overall, increasing the standard above s_m raises welfare.

8 Conclusion

This paper investigates direct democratic votes on a graduation standard, that is, a performance level required from students to pass an examination. It is shown that the option not to graduate makes voters indifferent between two standards whenever these are both considered too tough. Median voter results nevertheless hold if one requires only that, in every pairwise vote, the median preferred standard is a weakly optimal choice for a majority, or if the voting equilibrium has to be robust against trembling-hand-like errors in the graduation decision.

Based on this, welfare properties of the median preferred standard are analysed. It is shown that the standard chosen by a majority of voters is less demanding than the standard which maximises a utilitarian welfare criterion if two conditions are satisfied. The first of these conditions requires that the marginal effort cost of learning decreases rapidly as one moves towards more able individuals. The second condition states that the distribution of abilities is right-skewed. When these two properties hold, there is much to gain from inducing more able individuals to exert more effort, and hence welfare increases if the standard is raised beyond the one preferred by the median voter. These results explain why democracies may find it hard to raise academic achievements when parents care for the effort cost their children experience at school.

It is worthwhile to discuss two effects not included in the model which may possibly counteract the tendency of democratic education policy towards overly lenient standards. The first such feature is the fact that turnout is generally lower among low income voters than in the general electorate. Inasmuch as income and ability are correlated, this otherwise deplorable fact tends to raise median ability among voters and hence works in favour of a higher standard. Second, a tax-transfer scheme may give low ability agents, who would be recipients of transfers, a stake in higher standards since these will raise wages and tax revenues. These examples illustrate that further research on the political economics of graduation standards is worthwhile.

Appendix

A.I Proof of Proposition 1

Consider s_m in a vote against some standard $s < s_m$. I first show that $\hat{a}(s,s_m) < a_m$. If the first line in Definition 2 applies, this is immediate from $\hat{a}(s,s_m) = a_o < a_m$. If the second line of Definition 2 applies, it exists $\hat{a}(s,s_m) \in A$ such that $s - c(s,\hat{a}(s,s_m)) = s_m - c(s_m,\hat{a}(s,s_m))$. Since the payoff from graduating $s - c(s,\cdot)$ is strictly concave in s, for the standard $s(\hat{a}(s,s_m))$ which maximises the utility of an agent with ability $\hat{a}(s,s_m)$, it must hold $s < s(\hat{a}(s,s_m)) < s_m$. Now $\hat{a}(s,s_m) < a_m$ follows from the second inequality and the fact that the optimal standard $s(a)$ is strictly increasing in ability. Finally, $\hat{a}(s,s_m) = a_1$ as in the third line of Definition 2 would imply $s - c(s,a_m) > s_m - c(s_m,a_m)$, contradicting the fact that s_m is optimal for the median. Hence this case is ruled out, establishing $\hat{a}(s,s_m) < a_m$. Therefore, the mass of agents with ability $a \in (\hat{a}(s,s_m),a_1)$ exceeds $1/2$. From Lemma 1a(iv), these agents strictly prefer s_m over s so that $\int_{\{a\in A|v(s_m,a)>v(s,a)\}} dF(a) > 1/2$ follows.

Consider now s_m in a vote against some standard $s' > s_m$. By an argument analogous to the one laid out in the previous paragraph, one derives $a_m < \hat{a}(s_m,s')$. From Lemma 1a(i), all agents with ability a such that $a_o \leq a < a_{\min}(s_m)$ are indifferent between both standards. From Lemma 1a(ii), all agents with ability a such that $a_{\min}(s_m) < a < \hat{a}(s_m,s')$ strictly prefer the lower standard s_m. From $a_m < \hat{a}(s_m,s')$, these subsets of agents together make up more than half of the electorate so that $\int_{\{a\in A|v(s_m,a)\geq v(s',a)\}} dF(a) > 1/2$ is proved. ∎

A.II Non-uniqueness of weak Condorcet winner

In this Appendix I provide a sufficient condition for a standard $\tilde{s} \neq s_m$ to be a weak Condorcet winner. Such a standard cannot be less demanding than the median preferred standard or so high that the median will not satisfy it. To see why, observe first that all agents with ability $a > a_m$ would graduate under both $\tilde{s} < s_m$ and s_m, and hence would necessarily support s_m in a vote between these two standards. Second, a standard $\tilde{s} \geq s_{\max}(a_m)$ would lead to zero utility for the median and all agents with lower ability $a < a_m$. However, with a small enough positive standard, positive utility is feasible for all agents, and hence $\tilde{s}$ would lose a vote against such an alternative.

Consider then $s_m < \tilde{s} < s_{\max}(a_m)$, and denote by $\tilde{a} := s^{-1}(\tilde{s})$ the ability of agents for whom $\tilde{s}$ is the optimal standard. Moreover, denote by s_o the standard which yields the same indirect utility for the median as $\tilde{s}$, given by $\hat{a}(s_o, \tilde{s}) = a_m$.

The key element in the argument is that in all votes opposing $\tilde{s}$ to some other standard s, the tie of indifferent agents is broken in favour of $\tilde{s}$. There are three cases. (1) If $\tilde{s} < s$, then all agents with ability $a < \tilde{a}$ weakly or strongly prefer $\tilde{s}$. Since $a_m < \tilde{a}$, this is the majority. (2) If $0 \leq s < s_o$, all agents with $a \geq a_m$ strictly prefer $\tilde{s}$, and hence $\tilde{s}$ obtains a majority.

The critical case is (3): $s_o \leq s < \tilde{s}$. (Note that this interval contains s_m.) All agents with ability $a \leq a_{\min}(s)$ drop out under both standards and hence are indifferent according to Lemma 1a(i). From (2), $a_{\min}(s_o) \leq a_{\min}(s)$, and therefore the mass of indifferent agents is at least $F(a_{\min}(s_o))$. All agents with $a > \hat{a}(s, \tilde{s})$ strictly prefer $\tilde{s}$ from Lemma 1a(iv). Since $\hat{a}(s, \tilde{s}) < \tilde{a}$, the mass of these agents is at least $1 - F(\tilde{a})$. Both groups of agents together ensure a majority for $\tilde{s}$ if $F(a_{\min}(s_o)) + 1 - F(\tilde{a}) > 1/2$, or equivalently

$$F\left(a_{\min}(s_o)\right) > F\left(\tilde{a}\right) - \frac{1}{2}. \tag{A.1}$$

Inequality (A.1) implies that $\tilde{s}$ is a weak Condorcet winner. Its left-hand-side is the (lower bound of the) mass of indifferent low ability agents who join the high-ability agents in supporting the high standard $\tilde{s}$. The right-hand-side is the (upper bound of the) mass of agents with ability above the median who might vote against $\tilde{s}$.

The following numerical example shows that the sufficient condition (A.1) is consistent with the assumptions of the model. Let ability follow a triangular distribution on the support $[a_o, a_1) = [0, 2\sqrt{2})$ with c.d.f. $F(a) = a^2/8$. Median ability is $a_m = 2$. The learning cost function is assumed to be $c(e, a) = \left(e^2/2\right) \cdot (3 - a)$. This yields optimal standards $s(a) = 1/(3 - a)$ and graduation thresholds $a_{\min}(s) = 3 - (2/s)$. It follows $s_m = 1$, $a_{\min}(s_m) = 1$, and $\tilde{a} = 3 - (1/\tilde{s})$. Solving $s_o - \left(s_o^2/2\right) \cdot (3 - a_m) = \tilde{s} - \left(\tilde{s}^2/2\right) \cdot (3 - a_m)$ one finds $s_o = 2 - \tilde{s}$, and hence $a_{\min}(s_o) = 3 - [2/(2-\tilde{s})]$. After inserting into $F(\cdot)$, condition (A.1) becomes

$$\frac{1}{8} \cdot \left(3 - \frac{2}{2-\tilde{s}}\right)^2 > \frac{1}{8} \cdot \left(3 - \frac{1}{\tilde{s}}\right)^2 - \frac{1}{2}.$$

It can be verified numerically that this inequality is satisfied for $s_m < \tilde{s} < 1.1326$.

A.III Proof of Proposition 3

(i) For brevity, define $\beta := -c_{ea}(s_m, a_m) > 0$. With this, from Condition 1, one has

$$c_e(s_m, a) \leq c_e(s_m, a_m) + \beta(a_m - a) \tag{A.2}$$

for all $a \in A$ (see Figure 3), and, since $c_e(s_m, a_m) = 1$, it follows $c_e(s_m, a) \leq 1 + \beta(a_m - a)$ for all $a \in A$. Inserting into (4), one obtains

$$\begin{aligned} W'(s_m) &\geq \beta \int_{a_{\min}(s_m)}^{a_1} (a - a_m)\, dF(a) \qquad \text{(A.3)} \\ &= \beta\left[1 - F(a_{\min}(s_m))\right] \cdot \left[E\left(a|a > a_{\min}(s_m)\right) - a_m\right], \end{aligned}$$

where $E\left(a|a > a_{\min}(s_m)\right) = \int_{a_{\min}(s_m)}^{a_1} a\, dF(a) / \left[1 - F(a_{\min}(s_m))\right]$ is the expected ability of graduates, which is well defined since under the median preferred standard, a positive mass of agents will graduate. Clearly, $E\left(a|a > a_{\min}(s_m)\right) \geq \overline{a}$, and hence (A.3) implies

$$W'(s_m) \geq \beta\left[1 - F(a_{\min}(s_m))\right] \cdot [\overline{a} - a_m]. \tag{A.4}$$

From this and Condition 2, it follows $W'(s_m) \geq 0$.

(ii) If the inequality in Condition 1 is strict, then (A.2), and by consequence (A.3) hold with strict inequality. If the inequality in Condition 2 is strict, then the right-hand-side of (A.4) is strictly positive. If $a_{\min}(s_m) > a_o$, then $E\left(a|a > a_{\min}(s_m)\right) > \overline{a}$ so that in (A.3), the right-hand-side is strictly positive. In all three cases, it follows $W'(s_m) > 0$. ∎

A.IV Proof of Proposition 4

(i) Splitting the integral in (4) at the median and writing $a_m - x = a$ for $a < a_m$ and $a_m + x = a$ for $a > a_m$, one obtains

$$\begin{aligned} W'(s_m) = &- \int_{a_{\min}(s_m)}^{a_m} \left[c_e(s_m, a_m - x) - 1\right] dF(a_m - x) \qquad \text{(A.5)} \\ &+ \int_{a_m}^{a_1} \left[1 - c_e(s_m, a_m + x)\right] dF(a_m + x). \end{aligned}$$

From Condition 4, it must hold $a_m - a_o \leq a_1 - a_m$ so that Condition 3 implies

$$c_e(s_m, a_m - x) - 1 \leq 1 - c_e(s_m, a_m + x) \tag{A.6}$$

for all $x \in (0, a_m - a_o]$. Using (A.6) in (A.5), one concludes

$$\begin{aligned} W'(s_m) \geq & - \int_{a_{\min}(s_m)}^{a_m} \left[1 - c_e(s_m, a_m + x)\right] dF(a_m - x) \\ & + \int_{a_m}^{a_1} \left[1 - c_e(s_m, a_m + x)\right] dF(a_m + x) . \end{aligned} \tag{A.7}$$

Define the two distribution functions $G(x) := 1 - 2F(a_m - x)$ for $x \in [0, a_m - a_o]$ and $H(x) := 2F(a_m + x) - 1$ for $x \in [0, a_1 - a_m]$. ($G(x)$ resp. $H(x)$ is the probability that ability is at most a distance x away from the median, conditional on being below resp. above the median.) One has $dG(x) = -2dF(a_m - x)$ and $dH(x) = 2dF(a_m + x)$. Using G and H in (A.7), adjusting the integration bounds appropriately and reversing the order of integration in the first integral, one arrives at

$$\begin{aligned} W'(s_m) \geq & -\frac{1}{2} \int_0^{a_m - a_{\min}(s_m)} \left[1 - c_e(s_m, a_m + x)\right] dG(x) \\ & + \frac{1}{2} \int_0^{a_1 - a_m} \left[1 - c_e(s_m, a_m + x)\right] dH(x) . \end{aligned}$$

Since $a_{\min(s_m)} \geq a_o$ and $1 - c_e(s_m, a_m + x) > 0$ for $x > 0$, it follows furthermore

$$\begin{aligned} W'(s_m) \geq & -\frac{1}{2} \int_0^{a_m - a_o} \left[1 - c_e(s_m, a_m + x)\right] dG(x) \\ & + \frac{1}{2} \int_0^{a_1 - a_m} \left[1 - c_e(s_m, a_m + x)\right] dH(x) . \end{aligned} \tag{A.8}$$

Now observe that the two integrals in (A.8) have the form of expected utilities, with $1 - c_e(s_m, a_m + x)$ as the utility function which strictly increases in the random variable x because of Assumption 3. Moreover, Condition 4 implies that the distribution $H(x)$ first order stochastically dominates the distribution $G(x)$. Since a decision-maker with monotonic preferences will prefer the dominating to the dominated lottery (see Yildiz, 2015, Theorem 4.1), the second integral must be at least as large as the first one. This implies $W'(s_m) \geq 0$.

(ii) If the inequality in Condition 3 is strict, then (A.6) and hence (A.7) hold as strict inequalities. If the inequality in Condition 4 is strict, the dominance of the second over the first integral in (A.8) is strict, implying that the right-hand-side of this inequality is strictly positive. Finally, if $a_{\min}(s_m) > a_o$, extending the integration in (A.8) to the values $x \in (a_m - a_{\min}(s_m), a_m - a_o]$ adds a positive mass of strictly negative values, so that (A.8) holds as a strict inequality. In all three cases, it follows $W'(s_m) > 0$. ∎

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