CESifo WORKING PAPERS

6857
2018
January 2018

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Andre Boik

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editors: Clemens Fuest, Oliver Falck, Jasmin Gröschl
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CESifo *Working Paper No. 6857*
Category 11: Industrial Organisation

Prediction and Identification in Two-Sided Markets

Abstract

I introduce a reduced form two-sided market model to study prediction and identification in two-sided markets. The model generates the hallmark features of two-sided markets: potentially below cost or even negative prices to one side of the market, and the “see-saw” or “waterbed” effect of a tendency for price movements across sides to be negatively correlated. I show that the standard “one-sided” model of complements is a special case of the two-sided model, and that it generates those same hallmark features of two-sided markets. The model of complements also performs well in predicting price effects even when the data is actually generated by the two-sided market model; the “wrong” model often delivers the right answers and can be used to estimate market power and pass through rates. I show that even a naive one-sided model that ignores any relationship across goods/sides can perform well when prices to one side of the market are censored at zero, a very common outcome in two-sided markets. The main cost to using a model of complements to estimate cross-group effects in a two-sided market is that it invites the use of invalid instruments. I show that these findings are consistent with the empirical regularities and identification strategies in the existing two-sided market and indirect network effects literatures. I conclude by discussing the general difficulty of separately identifying whether price differences across subgroups of users of a platform are driven by pricing of network effects or simple price discrimination based on price elasticity.

JEL-Codes: L000, L130.

Keywords: two-sided markets, complements, prediction, identification.

Andre Boik
Department of Economics
University of California
One Shields Avenue
USA – Davis CA 95616
aboik@ucdavis.edu

January 14, 2018
I would like to thank John Asker, Benjamin Dawson, Liran Einav, Joe Farrell, Katrina Jessoe, Hidenori Takahashi and Ralph Winter for helpful discussions and Anna Ignatenko and Katherine Liu for excellent research assistance.

1 Introduction

The two-sided market literature has grown very rapidly in the last ten to fifteen years. At least part of the reason is the rise of a number of prominent platforms (eg. Ride sharing apps, dating websites, internet search engines) that exist mostly because of reduced transaction costs introduced by the internet. The key driving force of two-sided market pricing is the existence of cross-group network effects and the literature has demonstrated many examples where profit-maximizing platform behavior may involve non-standard pricing strategies, such as below cost pricing. For example, credit cards offer consumers cash back on their purchases to draw consumers to their platform and then recoup those losses from higher fees charged to merchants that feel compelled to pay them because of how many consumers want to use that credit card. Advertising-supported platforms like internet search engines (eg. Google) and video hosting services (eg. YouTube) do not monetize searchers or viewers directly; instead, low or zero prices attract maximum usage that in turn draws in advertisers whom the platform monetizes. There has been a call to study the economics of two-sided market platforms because some standard predictions in industrial organization do not necessarily hold in these markets (Wright, 2004). This claim has been supported in the literature which has found that below cost pricing is not only possible but in some cases expected, and that more competition can actually lead to higher prices for consumers on one side of the platform (M. Armstrong, 2006).

In the markets mentioned above, the additional benefit obtained by the platform from a marginal decrease in price calls to mind the notion of complementarity. All else equal, a price cut will be more profitable for a product that complements other goods sold by the same seller compared to a price cut that will cannibalize the rest of the product line. It has long been known that changes in competition and pass through rates differ for complementary goods relative to the more commonly studied setting of substitute goods: mergers even absent inefficiencies are expected to lead to lower prices, and because pricing of complements involves a game of strategic substitutes, this implies offsetting price movements in response to demand and cost changes. It would seem then that the problem of pricing complements ought to share a relationship with the problem of how a platform should set prices to each side of a two-sided market.

In addition, while the internet has increased the prevalence of platform-based business models, the concept is not new and has been studied at least indirectly in the past. For example, credit cards pre-date the two-sided market literature as do advertising-supported media such as newspapers, television, and radio. The basic economics of such platforms, and their tendency not to price individual components only as a direct function of their own demand and cost characteristics, was noted at least as early as Kaldor (1950). He writes,

> "Subsidised commodities occur ... where the demands for different things are in complementary relation to each other, and where a reduction in the price of a 'minor' commodity or service leads to such an expansion of the demand for a 'major' commodity or service, sold by the same firm, that the total amount spent on both will be greater than if the price concession had been spread over both proportionately. Examples for this are innumerable. ... Standard Oil Co. in the nineteenth century sold oil-lamps at a nominal price, in order to increase the consumption of oil; ... department stores have 'loss-leaders' or deliberately incur losses on subsidiary services. ... The newspaper industry provides another example ... here the service provided to the public is subsidised (the papers are

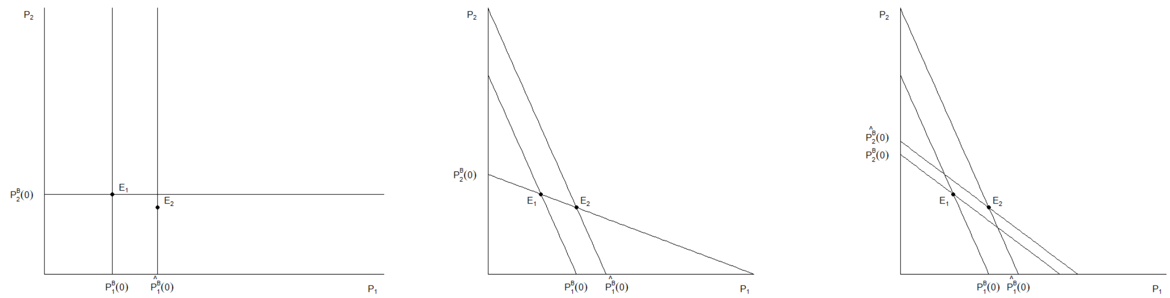
sold to the public below cost), in order to enhance the demand for advertising space, by the advertisers."

Later in the 1980s and 1990s, the indirect network effects literature studied hardware and software pricing decisions of computer and video game producers recognizing the additional benefit of lowering the price of hardware to induce software designers to design more games for their platform, or what that literature would call a system (eg. Church and Gandal (1993), Church and Gandal (2000)). In both advertising-supported markets and in markets for hardware and software, there is an additional benefit to lowering price aside from attracting marginal consumers to the given product: there is an indirect benefit through increased viewership and therefore increased advertising revenue, through increased varieties supplied to the system through third parties (Corts & Lederman, 2009), and so on. While the existing two-sided market literature has convincingly argued that using "one-sided logic in two-sided markets" can lead to serious errors in prediction (Wright, 2004), the alternative to a two-sided model is not necessarily an entirely naive model that ignores the existence of cross-group effects. A reasonable alternative would seem to be a model of complements.

In this paper, I ask: how well does a model of complements perform in answering standard comparative statics questions and in identifying key parameters of interest when the data is actually generated by a two-sided market model? To answer this question, I introduce a simple reduced form two-sided market model which, surprisingly, and at least to my knowledge has not appeared in the two-sided market literature.[1] This reduced form two-sided market model generates the hallmarks of two-sided market pricing behavior: the possibility of below cost or even negative pricing, and the "see-saw" or "waterbed" effect where prices tend to move in opposite directions in response to shocks (Rochet and Tirole (2006), Genakos and Valletti (2011)). In the first part of the paper, I show that a simple model of complements captures these qualitative features as well. I then test the predictive performance of the model of complements under the assumption that the data is generated by the two-sided market model. The model of complements performs well and can accurately predict price effects even though it is the "wrong" model. Although estimating the wrong model will generate estimates of the wrong parameters and cause proper identification to be virtually impossible, I show that for many practical applications (such as measuring market power or pass through), the distinction typically does not matter. For example, it is well known in the automated model selection literature that accurate prediction does not require a model that reflects causal relationships (Varian, 2014); I show that the same principle applies to prediction in two-sided markets. A model of complements that does not properly attribute indirect price effects to cross-group externalities is qualitatively useful as well as being useful for delivering estimates that can be used to make accurate counterfactual predictions. In other words, a two-sided market model is not always necessary to answer questions about two-sided markets.

A major benefit of the reduced form two-sided market model introduced is that virtually all of the results in the paper can be illustrated graphically. Consider the two data points illustrated Figure 1, E_1 and E_2, which correspond to two intersection points of reaction functions. The first data point represents an equilibrium pair of prices before a known demand shock, and the second data point represents a new equilibrium pair of prices after an observable, positive demand shock to good 1. It

[1] The existing literature tends to using a matching framework that begins by specifying users' utility from the number of users on the other side of the market, and is implemented through a Hotelling framework (eg. Caillaud and Jullien (2003), M. Armstrong (2006), Kaiser and Wright (2006), Chandra and Collard-Wexler (2009), Reisinger, Ressner, and Schmidtke (2009)).



(a) One-sided model (b) Model of complements (c) Two-sided market model

Figure 1: The same price data represented by points E_1 and E_2 rationalized by three models. The first model in (a) cannot rationalize the data, while the model of complements and the two-sided market model can.

turns out that while these two data points cannot be rationalized by the one-sided model in Figure 1(a) that ignores any link between the two goods, they can be rationalized either by the model of complements in Figure 1(b) or the two-sided market model in Figure 1(c): the reaction functions for good 1 are identical and respond identically to a demand shock for good 1. While the purely one-sided model that ignores any demand interdependency between the two goods (or two sides of the market) is clearly insufficient, it is not necessarily the relevant alternative to a two-sided market model: what about a model of complements that Kaldor (1950) might have had in mind?

Consider the relative performance of the three models under the assumption that the data is actually generated by the two-sided market model in Figure 1(c). The first thing to note is that the change to p_1 in response to a demand shock is correctly estimated using all three models, even the one-sided model that completely ignores any demand interdependency. All three models can be used to accurately estimate other counterfactual changes to p_1 in response to alternative demand shocks. Where the one-sided model and the model of complements go wrong are in understanding the nature of pricing on the other side of the market, for good 2. The one-sided model cannot rationalize why the demand shock to good 1 resulted in a price decrease for good 2 and therefore fails in predicting price changes to related products. However, the model of complements can rationalize that price decrease and in fact can be used to accurately estimate counterfactual changes to the price of good 2 in response to alternative demand shocks to good 1, despite underestimating the slope of the reaction function for good 2. This illustrates a theme that persists throughout the rest of this paper: a model of complements often predicts as well as the two-sided market model and is nearly identical qualitatively.

Of course, there is a cost to using the wrong model. An econometrician interested in estimating specific parameters of the model will be identifying something other than intended unless the two-sided market model is used. For example, an econometrician employing the model of complements will believe that shifts in the reaction function for the price of good 1 (induced by the exogenous demand shock) identify the slope of the reaction function for the price of good 2 when that is incorrect because in reality both best response functions shift in response to the demand shock. The resulting bias leads to an underestimate of the slope of the reaction function for the price of good 2. In the first part of this paper, I formalize these ideas and document the scenarios under which a model of complements can and cannot be used to answer questions about two-sided markets. The main cost to using the "wrong" model of complements is in identification and, in particular, the search for valid instruments. For example, if one uses the model of complements, demand shocks to side i would appear to be valid instruments to identify the slope of the reaction function for side j, but (as illustrated in Figure 1) such an instrument would not be valid because demand shocks to side i directly affect the reaction function for side j and not just indirectly through p_j. In the paper, I describe what data and conditions are necessary for identification and show that these requirements are consistent with the existing empirical strategies used in the indirect network effects literature.

In the second part of this paper, I consider two additional topics related to prediction and identification in two-sided markets. The first is the role played by a zero lower bound on pricing in two-sided markets and the second is the general inability to separately identify price differentials across groups of customers caused by cross-group effects versus those caused by simple price discrimination (i.e. differential pricing based solely on differences in underlying type).[2] It is well known that two-sided market pricing may involve pricing below cost, and therefore negative pricing if costs are low. However, negative prices are often infeasible because of opportunism. Farrell and Gallini (1988) noted the problem of opportunism under negative pricing when analyzing the optimal pricing strategy of a hardware manufacturer that also sells complementary software: negative hardware prices are infeasible to increase demand for software because "people could take computers and use them for landfill" (p.679). Jin and Rysman (2015) consider baseball card convention platforms to be constrained by a price at zero because while "conventions could use raffle tickets or door prizes to implement a sort of negative price, ... these are inefficient instruments by which to pay consumers relative to actual price, so we still view conventions with free admission as price constrained (p.721).". Although the literature has shown that tying can be used to implement de facto negative prices (eg. Amelio and Jullien (2012), Choi and Jeon (2016)), negative linear prices remain largely infeasible.

While there are limited examples of negative pricing[3], there are countless examples of zero pricing of complementary goods and in two-sided markets: many retail stores could charge for parking but do not, search engines could charge per search but do not, shopping malls could charge for entry but do not, etc. The prevalence of zero prices in markets with complementarities or network effects is economically significant. For example, in his survey paper of the two-sided market literature, Roson (2005) mostly cites examples of two-sided markets where one side is charged a zero price: the Yellow Pages, Adobe Acrobat Reader, Internet backbones and search engines, shopping malls, scientific journals, etc. Most of the same examples appear in the Rysman (2009) survey of the two-sided market literature. In all these examples, it is not difficult to imagine what would occur if consumers were paid a significant amount to flip through the Yellow Pages, open PDFs, park, search, or visit the mall. In an environment where firm pricing is censored at zero, the one-sided market and complements models often perform even better than in the environment described in Figure 1 above. In general, they not only can be used for prediction but can even produce unbiased estimates of the effects of various explanatory variables on prices. The major caveat is that any shock that arrives not be so large that it induces a (latent) negative price to one side of the market to suddenly turn

[2]Here, discrimination means charging different sides of the market different prices for use of the platform for reasons other than asymmetric cross-group externalities, and not discrimination among users on the same side of the market as in Liu and Serfes (2013).

[3]Credit card cashback is the main example that comes to mind, but for moderate cashback levels, there is a built-in defense against opportunism.

positive, but this is a caveat that also already applies to studying the pricing of complements.

The final results of the paper are related to challenges in identifying cross-group effects. The main challenge is in determining whether observed price differences across subgroups of consumers are the result of pricing off of cross-group effects or are the result of simple price discrimination. For example, night clubs are a canonical example of a two-sided market in the existing literature to illustrate differential pricing driven by cross-group effects (Wright, 2004). However, there is no obvious way to discern whether observed price differences across the subgroups of consumers (men and women) are due to the nightclub internalizing cross-group effects or simply discriminating between men and women because of type differences (eg. underlying differences in how they value the "standalone" benefits of the nightclub).[4]

To take an example where the incentive to discriminate based on "type" is more difficult to qualitatively separate from the incentive to discriminate because of cross-group effects, and where more data is available, consider the gym. There have been numerous lawsuits against gyms for gender-based discrimination, indicating gyms have an incentive and ability to discriminate.[5] I analyzed the publicly available data from DellaVigna and Malmendier (2006) and found that the average price per visit paid by women was roughly 6% lower than the average price per visit paid by men, despite men visiting the gym 21% more often than women. These facts are consistent both with the hypothesis that the discrimination occurs against men because they have a higher willingness to pay for the standalone value of the gym and the hypothesis that the gym subsidizes women to attract more men (from whom rent extraction would occur). As I describe later in the paper, the two possible effects turn out to be difficult if not impossible to identify separately.

In the rest of this paper I formalize these ideas and come to three major conclusions: (i) a model of complements often performs well in predicting price changes even when the correct model involves cross-group effects, (ii) even a naive model that ignores complementarity entirely can predict well when negative prices are infeasible, and (iii) whenever price differences emerge across two subgroups of consumers, it is in general impossible to determine to what extent those differences are driven by cross-group effects or simply traditional "textbook" price discrimination without appropriate estimation of those cross-group effects. Throughout the paper, I rationalize my findings through the lens of a number of empirical results from the existing indirect network effects and two-sided market literatures.

2 A reduced form two-sided market model and its relationship to a model of complements

In this section, I introduce a reduced form two-sided market model. By reduced form, I mean that there is not a formal treatment of the matching process across the two sides (eg. M. Armstrong

[4]While Wright (2004) notes that there are no apparent differences in the costs of serving men and women, and that the industry is likely perfectly competitive, that does not preclude the possibility of price discrimination based on gender in equilibrium. In fact, gender-based price discrimination is already occurring in the example, it is just a question of what the differences are between men and women that are driving it: differences in cross-group externalities or differences in underlying willingness to pay.

[5]For example, the Nevada Equal Rights Commission voted in 2008 that a Las Vegas athletic club offering women free memberships was discriminatory. Courts in California, Colorado, Florida, Iowa, Maryland and New Jersey have ruled that gender-based discrimination is unlawful while courts in Illinois, Michigan and Washington have ruled otherwise. See `http://www.nytimes.com/2008/08/13/us/13nevada.html`.

(2006), Chandra and Collard-Wexler (2009), Kaiser and Wright (2006), Reisinger et al. (2009)) nor explicit treatment of the economic agents on both sides having to solve a coordination or "chicken and egg" problem such as in Caillaud and Jullien (2003). I then compare its properties to a "model of complements" and derive conditions under which the model of complements generates accurate predictions of common economic relationships such as price changes in responsive to demand and cost shocks (i.e. pass through rates) and price elasticities when an interior solution obtains (i.e. when prices are non-negative).[6] I then proceed to consider how the simple model of complements performs when the equilibrium may feature negative prices or, when negative prices are infeasible, prices are censored from below at zero.

2.1 Prediction and identification in an equilibrium with positive prices

A monopolist platform in a two-sided market can potentially segment its users into two groups, 1 and 2, which each have their own individual demand for use of the platform which generates market-level demand functions described in equation (1). The positive A terms simply represent demand shifters (which may or may not be observed by the econometrician), the positive α's measure the responsiveness of demand to own price, and the γ's represent cross-group externalities. The assumptions that must be placed on the cross-group externality terms are reserved for discussion later because they determine the nature of equilibrium which in this paper is of special interest itself. While this specification permits heterogeneity across user groups in terms of the level of their demand and their responsiveness to price, there may also not be any differences across the groups.

$$q_i = A_i - \alpha_i p_i + \gamma_i q_j \quad \forall\, i, j = 1, 2, i \neq j \tag{1}$$

Because these users are segmented, they may potentially face different prices. To segment these users, the platform must observe some differentiating characteristic across the two types of users or have them self select into the appropriate group. Typically, self selection occurs. For example, men and women will self select into their respective categories on a heterosexual dating website even though the platform does not observe their gender, card users select into "cardholders" or retailers select into "merchants" for credit cards, and the same is true for riders and drivers in ride sharing, shoppers or retailers in shopping malls, and so on.

Substituting the two equations defined by (1) into each other yields reduced form demand equations defined by (2):

$$\hat{q}_i = \frac{1}{1 - \gamma_i \gamma_j} \left(A_i + \gamma_i A_j - \alpha_i p_i - \alpha_j \gamma_i p_j \right) \quad \forall\, i, j = 1, 2, i \neq j \tag{2}$$

where $1 - \gamma_i \gamma_j > 0$ and $A_i, \alpha_i > 0$ are required for a well-behaved demand function. A monopolist platform with weakly positive and constant marginal costs of serving each user group chooses prices

[6] To be specific about what I mean by a "model of complements", see equation (5).

to maximize $\pi_m = (p_1 - c_1)\hat{q}_1 + (p_2 - c_2)\hat{q}_2$ which results in the reaction functions defined by (3).[7]

$$p_i^B = \frac{1}{2\alpha_i}\left(A_i + \gamma_i A_j + \alpha_i c_i + \alpha_i \gamma_j c_j - (\alpha_j \gamma_i + \alpha_i \gamma_j) p_j\right) \quad \forall\, i, j = 1, 2, i \neq j \tag{3}$$

If an interior solution with positive prices exists, this system of equations leads to the following formula for the two equilibrium prices:

$$p_i^* = \frac{2\alpha_j(A_i + \gamma_i A_j + \alpha_i c_i + \alpha_i \gamma_j c_j) - (\alpha_j \gamma_i + \alpha_i \gamma_j)(A_j + \gamma_j A_i + \alpha_j c_j + \alpha_j \gamma_i c_i)}{4\alpha_i \alpha_j - (\alpha_j \gamma_i + \alpha_i \gamma_j)^2} \quad \forall\, i, j = 1, 2, i \neq j \tag{4}$$

At this point it is worth discussing the nature of equilibrium prices. If $\gamma_1 = \gamma_2 = 0$, then there are no cross-group effects and the profit-maximization problem collapses to two simple and independent monopoly pricing problems. If $(\alpha_1\gamma_2 + \alpha_2\gamma_1) > 0$, the reaction functions have negative slopes and we have a game of strategic substitutes while for $(\alpha_1\gamma_2 + \alpha_2\gamma_1) < 0$ we have a game of strategic complements. If, for example, γ_1 and γ_2 are both negative, then we are sure to have a game of strategic complements. This is worth noting because the resulting upward sloping reaction functions directly imply that the platform's optimal prices will not be below marginal cost (which also implies platform prices will be non-negative).[8] In contrast, if γ_1 and γ_2 are both positive, then both platform prices may be positive but it is possible that one will be zero or negative and the other positive.[9] Not surprisingly, a negative price for side i is more likely to occur when individuals on that side are relatively price elastic and generate relatively larger positive cross-group externalities (i.e. relatively high α_i and relatively low γ_i). Since positive γ's correspond to the usual case studied in the two-sided market literature of positive cross-group externalities, we quickly arrive at the result well known to the two-sided market literature that one group of users may be charged zero or even negative prices (M. Armstrong, 2006).

It is also worth noting the symmetry of the slopes of the reaction functions. This implies that even in two-sided markets where one side exhibits positive cross-group externalities and the other negative, such as for platforms supported by advertising which is disliked by viewers, that both reaction functions will have the same slope. Varying the strength of each cross-group externality may change the sign of the slopes, but the slopes will remain identical. In other words, it can never be the case that one reaction function slopes upwards while the other slopes downwards. This is particularly important for platforms supported by advertisers on one side because even if consumers dislike advertisers, a game of strategic substitutes will still obtain if advertisers like viewers more than they are disliked (ignoring the α terms representing price sensitivity).[10]

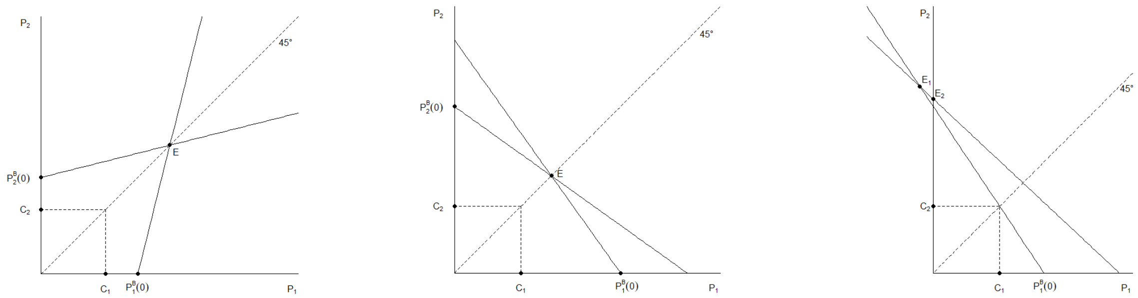
Figure 2 illustrates the nature of the different types of equilibria: in panel (a) the cross-group effects are negative which leads to a standard game of strategic complements reminiscent of Bertrand competition with differentiated products; in panel (b) the cross-group effects are positive and the two sides are sufficiently symmetric that prices to both sides are positive and above marginal cost;

[7] An additional but standard assumption here is that $A_i > c_i$ for $i, j = 1, 2$ with $i \neq j$.

[8] Both prices being negative violates profit maximization since pricing is below cost for both goods while sales are positive. One price below cost and the other above cost also violates profit maximization since an increase in the price below cost reduces losses and increases demand for the other good earning a positive margin.

[9] Direct substitution of specific parameters into the equilibrium values verifies this result, see Proposition 2.

[10] Anderson and Coate (2005) and Anderson and Gabszewicz (2006) are some of many studies examining media markets through the lens of two-sided market theory.



(a) Negative γ's (b) Positive γ's (symmetric) (c) Positive γ's (asymmetric)

Figure 2: With negative cross-group effects, prices are always positive and above cost. With positive cross-group effects, prices may be below cost or negative if there is sufficient asymmetry in demand across groups.

in panel (c) the cross-group effects are again positive but the two sides are asymmetric enough that profit-maximizing pricing involves a negative price at equilibrium point E_1 or, if a negative price is infeasible, a zero price at equilibrium point E_2.

While the nature of games of strategic substitutes and strategic complements are well known, it is worth emphasizing that this reduced form two-sided market model captures two of the well known predictions of existing two-sided market models, namely that prices may be below cost or even negative (M. Armstrong, 2006), which has already been illustrated in Figure 2(c), and that price changes tend to be in opposite directions. For example, if the platform is currently charging a large markup to one side of the market, that motivates charging lower prices to the other side to increase demand from the group facing the large markup; Rochet and Tirole (2006) refer to this dynamic as the "seesaw principle", which is essentially negatively correlated price movements in response to a shock.[11] Genakos and Valletti (2011) refer to this as the "waterbed effect" and find evidence of it in the European mobile telephony market: when regulators intervened in the market and reduced termination fees by 10%, mobile retail rates rose by 5%. The reduced form two-sided market model introduced here captures this phenomenon because with positive cross-group effects (the focus of the two-sided market literature), a game of strategic substitutes obtains. A pricing game of strategic substitutes implies the possibility of negatively correlated price movements in response to anything that shifts a reaction function; in contrast, price movements in a game of strategic complements are always positively correlated.[12]

Up to this point, the reduced form two-sided market model has generated several of the well known results from the two-sided market literature such as the possibility of below cost or negative prices in equilibrium and negatively correlated price movements. However, it is also well known that a multiproduct monopolist selling complements faces a similar environment: reaction functions slope downwards, below cost prices are possible, and price movements in response to shocks have potential

[11] The "seesaw principle: a factor that is conducive to a high price on one side, to the extent that it raises the platform's margin on that side, tends also to call for a low price on the other side as attracting members on that other side becomes more profitable." Rochet and Tirole (2006),p.659.

[12] Any time a shock shifts only one of the reaction functions in a game of strategic substitutes, negatively correlated price movements are guaranteed; however, if the shock shifts both reaction functions in the same direction then it is possible to have positively correlated price movements. Negatively correlated price movements can only occur in a game of strategic complements if the reaction functions shift in opposite directions in response to a shock.

"seesaw" effects. For example, it is well known that manufacturers of printers and compatible ink cartridges subsidize the printers and generate most of their profits through the sale of ink. If there is a positive demand shock for that manufacturer's ink, this will tend to cause the manufacturer to raise the price of ink but also to lower the price of printers to further drive sales of ink which are now generating even larger margins.

Throughout I have assumed that the linear demand system corresponds to a platform facing two distinct sets of groups that exhibit cross-group externalities. However, this linear demand system nests within it a simple reduced form set of linear demands for a model of complements. Re-considering equation (2) with a change of variables, we have a classic model of complements:

$$\tilde{q_i} = \tilde{A_i} - \tilde{\alpha_i}\tilde{p_i} - \tilde{\gamma_i}\tilde{p_j} \quad \forall\, i, j = 1, 2, i \neq j \tag{5}$$

where $\tilde{A_i} = \frac{A_i + \gamma_i A_j}{1-\gamma_i\gamma_j}$, $\tilde{\alpha_i} = \frac{\alpha_i}{1-\gamma_i\gamma_j}$, and $\tilde{\gamma_i} = \frac{\alpha_j\gamma_i}{1-\gamma_i\gamma_j}$. A multiproduct monopolist of complements facing this demand system and seeking to maximize profits $\tilde{\pi_m} = (\tilde{p_1} - \tilde{c_1})\tilde{q_1} + (\tilde{p_2} - \tilde{c_2})\tilde{q_2}$ yields the reaction functions defined as follows:

$$\tilde{p_i^B} = \frac{A_i + \alpha_i c_i + \gamma_j c_j - (\gamma_i + \gamma_j)p_j}{2\alpha_i} \quad \forall\, i, j = 1, 2, i \neq j \tag{6}$$

And in an interior solution, the equilibrium prices that are obtained are defined as follows:

$$\tilde{p_i^*} = \frac{2\alpha_j(A_i + \alpha_i c_i + \gamma_j c_j) - (\gamma_i + \gamma_j)(A_j + \alpha_j c_j + \gamma_i c_i)}{4\alpha_i\alpha_j - (\gamma_i + \gamma_j)^2} \quad \forall\, i, j = 1, 2, i \neq j \tag{7}$$

With this simple change of variables, several results become immediately apparent.

Proposition 1 *The demand system of complementary goods defined by equation (5) is a special case of the two-sided market demand system presented in equation (1). Proof: the two are equivalent whenever the parameters in equation (5) satisfy the change of variables equations that proceed it.*

Proposition 2 *A simple model of complements can generate below cost or negative prices in equilibrium and can feature the "seesaw" or "waterbed" effect where a shock to one good leads to negatively correlated price changes. Proof: By example, the following parameters* $\tilde{A_1} = 64, \tilde{A_2} = 12, \tilde{c_1} = 4, \tilde{c_2} = 4, \tilde{\alpha_1} = 2, \tilde{\alpha_2} = 8, \tilde{\gamma_1} = 0.5, \tilde{\gamma_2} = 0$ *lead to equilibrium prices* $(\tilde{p_1^*}, \tilde{p_2^*}) = (21, -1.5)$. *Given we have a game of strategic substitutes, reaction functions slope downwards; anything that shifts only one reaction function is guaranteed to generate negatively correlated price movements.*

Proposition 3 *Estimating the demand system for complementary goods in equation (5) generates the same predictors as would be obtained from estimating the demand system for a two-sided market described in equation (2). Proof: This can be seen from a quick inspection of equation (5) from the demand system for complements; the regression will return coefficients* $\tilde{A_i}$, $\tilde{\alpha_i}$, *and* $\tilde{\gamma_i}$ *as will a regression of equation (2) from the two-sided market model.*

The first two propositions illustrate the close, qualitative link between the simple model of complements and the reduced form two-sided market model. The third proposition implies that an econometrician who incorrectly models a two-sided market using a simple model of complements will still generate the same predictors. The econometrician using the simple model of complements believes p_j directly affects q_i when in fact p_j only influences q_i through its effect on q_j, but for several matters of prediction it does not matter: both models will accurately predict the effect of an exogenous one unit increase in p_j on q_i.[13] To take one example of an application, that estimate can be used to estimate own-price elasticity of demand which can in turn be used to invert the Lerner Index to obtain an estimate of marginal cost.

Consider an application to the estimation of pass-through rates. The monopolist platform operating in a two-sided market has the following pass through rate:

$$\frac{dp_1}{dc_1} = \frac{2\alpha_1\alpha_2 - \alpha_2\gamma_1(\alpha_2\gamma_1 + \alpha_1\gamma_2)}{4\alpha_1\alpha_2 - (\alpha_2\gamma_1 + \alpha_1\gamma_2)^2} \tag{8}$$

Meanwhile, a monopolist seller of complements whose demand functions are defined by equation (5) passes through costs according to the following rate.

$$\frac{d\tilde{p_1}}{dc_1} = \frac{2\tilde{\alpha_1}\tilde{\alpha_2} - \tilde{\gamma_1}(\tilde{\gamma_1} + \tilde{\gamma_2})}{4\tilde{\alpha_1}\tilde{\alpha_2}} \tag{9}$$

Since the pass through rates under both models are constant, they can be estimated through a linear regression of price on cost, assuming observed costs are uncorrelated with other costs observed only by the firm (MacKay, Miller, Remer, & Sheu, 2014). The econometrician will believe to have estimated equation (9) when in fact (8) has actually been estimated, but for purposes of predicting future pass through, the distinction does not matter. The simple model of complements can deliver correct qualitative and prediction-related answers to typical questions of interest despite a two-sided market model actually driving the data generating process.

Up to this point the simple model of complements has matched the qualitative features of the reduced form two-sided market model and it can also be used to generate accurate predictions, even though the data generating process is actually driven by the two-sided market model. Naturally, though, there is a cost to using the wrong model. That cost lies in the area of identification and comes in two forms: searching for instruments to handle the endogeneity of prices and estimating certain structural or conduct parameters.

Proposition 4 *In the model of complements, any observed demand shifter for good j that does not shift the demand for good i can be used to instrument for p_i in equation (5). Proof: From equation (7), p_i^* is correlated with observed demand shifters for good j such as A_j but from equation (5), A_j is uncorrelated with q_i.*

Proposition 5 *In the reduced form two-sided market model, no demand shifter for either side i or j can be used as an instrument for p_i or p_j in equation (2). Proof: From equation (1), q_i depends directly on q_j and so any factor that shifts demand for q_j shifts demand for q_i, violating the exclusion restriction. The direct dependency on own and cross demand shifters is clear in equation (3).*

These two propositions highlight a practical difficulty in using a simple model of complements to estimate a reduced form two-sided market model. A valid instrument for p_i in the simple model of complements is A_j but this is not a valid instrument in the two-sided market model; consequently, using the wrong model invites the use of an invalid instrument for price which will bias the predictor

[13] Of course, price endogeneity is a concern and is discussed later.

of own price and contaminate any subsequent calculations using that predictor, such as the own-price elasticity of demand used to calculate the Lerner Index.

The second type of cost to using the wrong model is in the estimation of structural conduct parameters. When estimating equation 5, the econometrician believes the predictors recovered directly correspond to structural parameters such as $\tilde{\alpha}_i$ and $\tilde{\gamma}_i$ when in fact those predictors are recovering combinations of parameters: $\tilde{\alpha}_i = \frac{\alpha_i}{1-\gamma_i\gamma_j}$, $\tilde{\gamma}_i = \frac{\alpha_j\gamma_i}{1-\gamma_i\gamma_j}$. Estimating counterfactuals in the context of a structural model is clearly going to suffer from the estimated parameters not corresponding to actual parameters.

Consider Figures 1(b) and 1(c) as an example that illustrates both of the difficulties of using the "wrong" model of complements when the correct model is the two-sided market model. To correctly identify the slope of the reaction function for good j requires an exogenous shift in the reaction function for good i. The simple model of complements considers it sufficient for a shift in A_j to identify the slope of the reaction function for p_i because A_j enters into the formula for the reaction function for good i but not the reaction function for good j. But, the two-sided market model recognizes that using A_j as an exogenous shifter to identify the slope of p_i is invalid because changes in A_j also directly affect the reaction function for p_i. The result is an underestimation of the slope of the reaction functions: the two prices appear to be more unrelated than they actually are. Qualitatively, this would cause the econometrician to underestimate the scope of cross-group effects. This leads to a result regarding appropriate instruments for estimating the two-sided market model.

Proposition 6 *In the reduced form two-sided market model described in equation (2), there do not exist any valid demand-related instruments for prices in a cross-sectional estimation of demand for either side. The only valid instruments for price in a cross-sectional estimation must be cost-based. Proof: The only demand-related options are contained in (A_i, A_j), both of which directly appear in each side's demand function. Cost-based instruments are potentially valid since they are correlated with prices and potentially uncorrelated with unobserved demand shocks.*

Now consider estimating the size of cross-group effects from equation (3), i.e. by estimating the slope of the reaction functions. Here there is little hope to obtaining a credible estimate of that slope. To estimate the slope of p_i^B requires exogenous variation in the reaction function p_j^B. But every potential demand or cost variable that shifts the reaction function for p_j^B also shifts the reaction function for p_i^B, leaving no potentially valid instruments. In a cross-sectional analysis, changes in the price of p_j must arise exogenously to estimate equation (3), perhaps as a result of a change in government regulation (eg. Knittel and Stango (2009)). This is stated formally below.

Proposition 7 *Estimating the strength of cross-group effects from equation (3) requires exogenous changes in a price to one side of the market because demand and cost instruments for price are invalid. Proof: If changes in p_j are exogenous, equation (3) can be estimated directly. If p_j is an endogenous, demand and cost instruments for p_j are invalid because not only are they correlated with p_j but they are also correlated with p_i^B, violating the exclusion restriction.*

For these reasons, it is advisable to estimate the strength of cross-group effects directly from equation (1). However, another endogeneity problem arises: any unobserved demand shifter for q_j that also shifts q_i will result in a biased estimate of the cross-group effect, γ_i. In two-sided markets, the overlap in demand shifters for side i with demand shifters for side j is expected to be significant:

for example, unobserved quality of a dating website is likely to affect both demand from men and demand from women. Nevertheless, demand-based instruments are potentially valid: any element of A_j is relevant because it is certain to be correlated with q_j, but to satisfy the exclusion restriction it must not appear as an element in A_i. This is stated formally below.

Proposition 8 *Estimating the strength of cross-group effects directly from equation (1) will typically require instrumenting for q_j. The only potentially valid instruments for q_j are contained in A_j and must not be correlated with unobserved elements in A_i. Proof: From equation (1) the only variables correlated with q_j are A_j or p_j, but p_j violates the exclusion restriction because from equation (4), p_j is correlated with unobserved elements in A_i. Any element in A_j, however, is correlated with q_j but potentially uncorrelated with any unobserved elements in A_i.*

The econometrician seeking to estimate the strength of cross-group effects has several options: equations (1),(2),(3) can all be used to establish the existence of non-zero cross-group effects. However, depending on the data available to the researcher, identification of cross-group effects may not be possible. The previous set of results characterized what the requirements are based upon the data that is available to the econometrician. If only price data is available, identification will be quite difficult. If quantity data for only one side of the market is available (along with price data), then cost data must be used to instrument for endogenous prices. Credibly estimating cross-group effects is most realistic when price and quantity data for both sides of the market are available, although even then it may not be possible, especially in media markets where quantities of advertisements may not vary across markets or across time (for example, 30 minute sitcoms in the U.S. most often contain 8 minutes worth of advertisements).

These theoretical results are consistent with the identification strategies used in the indirect network effects literature seeking to estimate the effects of installed base or software varieties on hardware demand, or the "waterbed" effect directly through prices. That literature has approached the problem either by estimating a dynamic model which allows lagged variables to be used as instruments, eg. Corts and Lederman (2009) or Lee (2013), or using cost-based instruments, eg. Gandal, Kende, and Rob (2000). For example, Corts and Lederman (2009) use price histories as an instrument for a gaming console's installed base and Gandal et al. (2000) use technological improvements in CD data compression as an instrument for CD variety. Clements and Ohashi (2005) use Japanese-American exchange rates as an instrument for American hardware adoption decisions (since most of that hardware was manufactured in Japan). Knittel and Stango (2009) use a plausibly exogenous government relaxation of a price restraint on ATM fees to study the response of debit card fees directly (i.e. directly from the reaction function in (3)).

Many of the canonical examples of two-sided market pricing appear qualitatively in a simple model of complements. One of the defining features of two-sided market pricing is that prices can appear below cost or even become negative; this is also an expected result from the pricing of complements. In this section, I showed formally that similarities between the simple model of complements and the two-sided market model extend beyond a qualitative similarity and propensity towards below cost pricing. For most matters of prediction, such as estimating pass through rates or recovering marginal cost estimates through inversion of the Lerner Index, a simple model of complements performs as well as the "correct" two-sided market model, with some caveats. The cost of using a simple model of complements is in potentially choosing an invalid (demand-based) instrument and in the identification of structural parameters: by definition, the wrong model will recover the wrong parameters, and this naturally poses problems for any structural econometric framework, particularly one developed to estimate counterfactuals. In the following section, I show that when negative prices are infeasible, the simple model of complements again predicts well and that, with some caveats, even a naive model that ignores any relationship between the two sides of the market may also generate accurate predictions.

2.2 Prediction and identification with negative prices or at a zero lower bound

The previous section showed that below cost pricing and even negative prices are a possible feature of equilibrium in either the reduced form two-sided market model or the model of complements. For example, a media platform such as a TV channel might in theory be willing to pay individuals to watch their channel if the additional advertising revenue it generated were more than enough to make up for the cost of paying viewers. In practice, however, negative prices may not be feasible. Significantly negative prices invite opportunism: viewers may leave their television set on the aforementioned channel while they are at work and not even at home. Paying individuals to take printers off warehouse shelves, to take a newspaper, or to join a dating website all invite opportunism without necessarily generating any additional revenue from ink sales, ad sales, or additional membership sales from the opposite sex. As a result, we may expect platform pricing and the pricing of complements at times to be censored at zero and for there to be clustering at a price of zero.

While not explicitly modeled here, there are clear benefits to zero prices for a platform or seller of complements. A zero price does not require paying the transaction costs associating with collecting revenue: there is no need for a cashier, a barrier to an entrance, or other infrastructure needed to collect payments. For example, consider the additional costs associated with a grocery store deciding to start charging for parking (a complement to all goods sold in the grocery store) such as from installing parking meters or hiring a parking attendant, etc. The existence of transaction costs involved in collecting revenues is another reason to expect clustering at zero prices. It is highly unlikely that a grocery store will ever charge a penny for parking or pay individuals to park. Because of the lack of infrastructure to collect zero prices, such prices easily go unnoticed. Few recognize that when they park, enter a mall, watch video online, or conduct an internet search that they have all been charged a zero price when in fact they could have been charged a strictly positive price.

Proposition 2 has already demonstrated that negative prices can theoretically occur in equilibrium, and which has been illustrated in Figure 2. The equilibrium occurs at E_1 if negative prices are feasible, and E_2 if not. When negative prices are infeasible, any parameter values that would have led to an equilibrium in the second quadrant will now result in an equilibrium on the vertical axis. Empirically, we should then expect to observe a clustering of zero prices in two-sided markets and in the sale of complementary goods, especially when the marginal cost of those goods are low or close to zero. While in some markets it is difficult to measure the extent of zero prices because in effect those prices do not exist, those that are known to be zero because in other settings they are strictly positive tend to have a low marginal cost: parking, admission to night clubs, and watching video online all have negligible marginal costs to the seller.

For the econometrician, the clustering of prices at zero poses a problem. The observed prices are on the vertical axis, which masks information about the underlying data generating process. Is the price of q_1 equal to zero because the seller has not considered charging for it, or is a model of complements or a two-sided market model appropriate, where the latent equilibrium in Figure 2(c) is at E_2 but the seller is constrained to setting non-negative prices? If the econometrician is aware and correct that q_2 is a complement to q_1 (complements terminology) or that q_2 creates positive cross-group externalities (two-sided market terminology) but that the seller is constrained to non-negative prices, how far is the "latent equilibrium" E_2 from the vertical axis? The answer to this question matters for prediction in two-sided markets and for models of complements.

Consider the equilibrium described in Figure 2c and how it would be affected by a tax on or an increase in the cost of supplying q_2. In either the model of complements or the two-sided market model, this shifts both reaction functions outwards and could lead to a new equilibrium in the first quadrant. The significance of that is the sudden existence of a positive price for good 1, which could come as a surprise to the econometrician or policymaker. The likelihood of the cost increase in supplying q_2 causing a strictly positive price for q_1 depends on how far the latent equilibrium at E_2 is from the vertical axis. If E_2 is far from the vertical axis, the cost shock is less likely to lead to a positive price for q_1. The same analysis applies for a demand shock. In the simple model of complements, demand shocks only shift the own reaction function and not the complementary good's reaction function, but in the two-sided market model, both shift and in the same direction.

In this environment, the model of complements compares to the two-sided market model in much the same way as in the previous subsection concerned with strictly positive prices for all goods. However, in this setting with censoring of prices at zero, even an entirely naive model (i.e. a "one-sided model") that ignores any link between q_1 and q_2 can perform well. When the seller is constrained to non-negative prices, the derived demand system for the two-sided market model becomes:

$$\hat{q}_i = \frac{1}{1-\gamma_i\gamma_j}\left(A_i + \gamma_i A_j - \alpha_i \max[p_i, 0] - \alpha_j\gamma_i \max[p_j, 0]\right) \quad \forall\, i, j = 1, 2, i \neq j \tag{10}$$

which leads to the following reaction functions that together characterize the equilibrium prices:

$$p_1^B = \frac{1}{2\alpha_1}\left(A_1 + \gamma_1 A_2 + \alpha_1 c_1 + \alpha_1\gamma_2 c_2 - (\alpha_2\gamma_1 + \alpha_1\gamma_2)\max[p_2, 0]\right) \tag{11}$$

$$p_2^B = \frac{1}{2\alpha_2}\left(A_2 + \gamma_2 A_1 + \alpha_2 c_2 + \alpha_2\gamma_1 c_1 - (\alpha_1\gamma_2 + \alpha_2\gamma_1)\max[p_1, 0]\right) \tag{12}$$

While the application to the pricing of complements and pricing in two-sided markets is new, these models have been studied extensively in the econometrics literature related to the Tobit model and, in particular, simultaneous equation Tobit models. Schmidt (1981) describes the constraints necessary to estimate a simultaneous equation Tobit model when truncated endogenous variables appear as explanatory variables; the econometric constraint described for this environment is the same as one of the primary constraints required in the theory above for demand to be well-behaved: $1 - \gamma_i\gamma_j > 0$.

For reasons described above, the econometrician may be tempted to think that if p_j is observed to be zero that it will always remain zero in response to any shock. Or, because p_j is zero, the

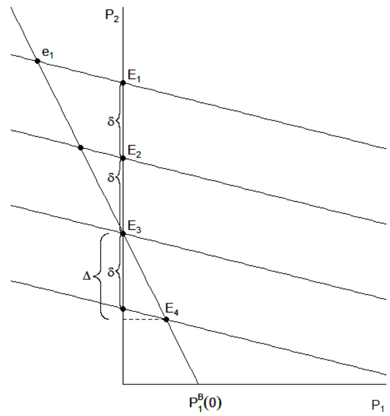


Figure 3: Unconstrained and constrained equilibria in response to a series of symmetric, negative demand shocks. When p_1 becomes positive, the effect of the demand shock on p_2 becomes larger ($\Delta > \delta$).

econometrician may not think about it at all because it will not appear in accounting records or other data. The cost of ignoring zero prices in this framework is the same as ignoring a censored variable in a simultaneous equations Tobit model: if the econometrician believes p_j is exogenously zero, and not just censored at zero, a linear estimation of equation (11) will generate biased estimators of predictors (such as how prices will change in response to demand or cost shifters). The reason is explained nicely by McDonald and Moffitt (1980) who showed that the change in a Tobit dependent variable p_i (to adopt my notation) with respect to a change of an independent variable, X_i, such as a demand or cost shifter, can be decomposed as follows:

$$\frac{dp_i^*}{dX_i} = Pr(p_j^* > 0)\frac{dp_i^B{}_{|p_j^*>0}}{dX_i} + Pr(\max[p_j^*, 0] = 0)\frac{\partial p_i^B{}_{|p_j=0}}{\partial X_i} \tag{13}$$

Suppose the change in X_i is a change in A_i, a demand shifter. The first term says the change in p_i in response to a demand shock is equal to the total change in p_i in response to the change in A_i including the indirect effects induced through changes in p_j (hence the total derivative), weighted by the probability of p_j being positive so that these indirect effects exist. The second term says the change in p_i in response to a demand shock is just equal to the direct effect on p_i because p_j is censored at zero so there are no indirect effects, weighted by the likelihood that p_j is indeed censored at zero. If the econometrician implicitly assumes $Pr(\max[p_j^*, 0] = 0$, either because prices for j are always zero or not even recorded, then the estimated predictor will be $\frac{\partial p_i^B{}_{|p_j=0}}{\partial X_i}$ when in fact it should be the entire right-hand-side of equation (13). The predictor will only be unbiased in expected value when the probability of p_j^* becoming positive remains zero.

Figure 3 illustrates the logic in the context of the model of complements in response to a sequence of negative shocks to the demand for good 2. If negative prices are feasible, the original equilibrium will be at e_1 but if negative prices are infeasible (as I will assume here) then the actual equilibrium will be at E_1. Now consider a negative demand shock to good 2 that shifts the reaction function for good 2 downward and leads to a new equilibrium at E_2. Since p_1 remains censored at zero, the change in the price of good 2 in response to the shock comes entirely from the direct effect $\frac{\partial p_i^B|_{p_j=0}}{\partial A_i}$ which is equal to δ. Another negative demand shock brings the equilibrium to E_3 and the direct effect remains δ because p_1 has still not quite been "activated". However, the final demand shock yields an interior solution and now the negative demand shock induces an indirect effect through changes in the price of good 1. The change in the price of good 2 in response to the shock is now $\Delta > \delta$. This intuition is summarized in the following propositions.

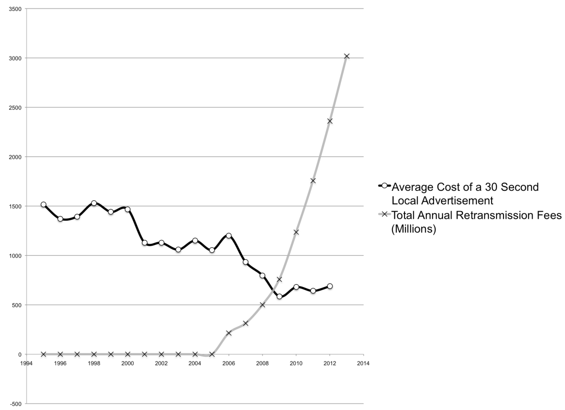
Proposition 9 *Estimated price effects in response to demand or cost shocks in the model of complements will be biased downwards if the econometrician treats zero prices as exogenous rather than endogenously censored at zero and there is a non-zero probability of that endogenously censored price becoming positive. Proof: From* $\tilde{p_i^B}$ *above,* $\frac{dp_i^B|_{p_j=0}}{dA_i} = \frac{1}{2\alpha_i}$ *while* $\frac{dp_i^B|_{p_j>0}}{dA_i} = \frac{2\alpha_j}{4\alpha_i\alpha_j-(\gamma_i+\gamma_j)^2}$ *where the latter is greater than the former. For cost shocks,* $\frac{dp_i^B|_{p_j=0}}{dc_i} = \frac{1}{2}$ *while* $\frac{dp_i^B|_{p_j>0}}{dc_i} = \frac{2\alpha_j\alpha_i-\gamma_i(\gamma_i+\gamma_j)}{4\alpha_i\alpha_j-(\gamma_i+\gamma_j)^2}$*.*

From McDonald and Moffitt (1980), $\frac{dp_i^*}{dX_i} = Pr(p_j^* > 0)\frac{dp_i^B|_{p_j^*>0}}{dX_i} + Pr(\max[p_j^*, 0] = 0)\frac{\partial p_i^B|_{p_j=0}}{\partial X_i}$*. If the econometrician treats* $p_j = 0$ *as exogenous, that is an implicit assumption that* $Pr(p_j^* > 0) = 0$*. Whenever this assumption is incorrect, the predictor will be biased downwards for* $X_i = A_i, c_i$*.*

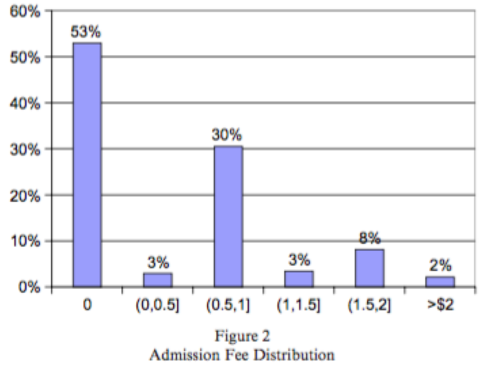
The intuition for the proposition is as follows. If the unconstrained equilibrium involving negative prices (at e_1) is very far from the vertical axis, then the probability of p_2 being activated is very low and the econometrician can comfortably estimate the effects of demand shocks to A_2 on p_2^* without worrying about good 1 at all. In other words, a one-sided model that completely ignores the existence of the other side of the market *can* deliver the right answer to certain questions so long as there is little chance of that other side ever being charged. For example, Google could charge for searches but does not. In fact, were it not for opportunism, it is possible Google would actually like to pay users to search to increase demand from advertisers. If we believe that Google is highly unlikely to consider charging for search, then that is a belief that e_1 is far from the vertical axis and that negative demand shocks for advertising on Google are unlikely to activate a price (p_1) for search. The implication is that a model of competition among search engines that ignores latent prices for search and only focuses on advertising prices will likely perform well.

However, there are documented examples where e_1 has been close to the vertical axis and p_1 has been activated. Consider Figure 4(a) from Boik (2016). Up until 2005, local U.S. television stations chose to charge a zero price to cable and satellite distributors for their content. From that point, however, stations began to charge distributors for content and those "retransmission fees" now account for billions of revenues for local television stations. In this industry, rising retransmission fees are largely attributed to the growth of internet advertising as a major competitor to television advertising and that stations looked for "alternative sources of revenue".[14] These data are consistent with the theory described in Figure 3 in response to a sequence of demand shocks: the original unconstrained equilibrium in the 1990s was at e_1, then at E_3 in the early to mid 2000s, and now at E_4 with positive prices being charged for content that was previously free. As the effect of competition from internet advertising affected Canada at the same time, local Canadian television stations lobbied the relevant federal regulator in 2006 to lift "must carry" rules that, while they forced distributors to carry their content, they also prevented stations from charging for it (R. Armstrong, 2010).

[14]This is based on my conversations with representatives of these stations.



(a) Boik (2016)



(b) Jin and Rysman (2015)

Figure 4: (a) A time series of censored and then uncensored retransmission fees for local television station content, from Boik (2016). (b) Admission fees for baseball card conventions (which also charge a table fee to sellers) from Jin and Rysman (2015)

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Additionally, consider Figure 4(b) from Jin and Rysman (2015) showing the distribution of admission fees to baseball card conventions that match collectors and sellers. There are strictly positive admission fees as well as clustering of fees at zero which the authors acknowledge is likely because of a constraint on non-negative prices. Across regions where baseball conventions are held, it appears that in the data the equilibria are all very close to the vertical axis, with some unconstrained equilibria falling to the left and some just to the right of the vertical axis which would generate exactly this Chi-squared(1)-like distribution of observed prices.[15]

The same idea applies to cost shocks in the model of complements and both cost and demand shocks in the two-sided market model, except the bias resulting from ignoring one side of the market can be positive or negative.

Proposition 10 *Estimated price effects in response to a demand shock to side i in the two-sided market model will be biased if (i) the econometrician treats zero prices as exogenous rather than endogenously censored at zero, (ii) there is a non-zero probability of that endogenously censored price becoming positive, and (iii) there is any asymmetry in own-price responsiveness or in the extent of cross-group externalities. The bias is downward when $\alpha_i\gamma_j < \alpha_j\gamma_i$, upward when $\alpha_i\gamma_j > \alpha_j\gamma_i$, and there is no bias without asymmetry in own-price responsiveness or in cross-group externalities across the two groups.*

Proof: From p_i^B above, $\frac{dp_i^B}{dA_i}_{|p_j=0} = \frac{1}{2\alpha_i}$ while $\frac{dp_i^B}{dA_i}_{|p_j>0} = \frac{2\alpha_j - \gamma_j(\alpha_j\gamma_i+\alpha_i\gamma_j)}{4\alpha_i\alpha_j-(\alpha_j\gamma_i+\alpha_i\gamma_j)^2}$ where the latter can easily be shown to be greater than the former whenever $\alpha_i\gamma_j > \alpha_j\gamma_i$.

From McDonald and Moffitt (1980), $\frac{dp_i^*}{dX_i} = Pr(p_j^* > 0)\frac{dp_i^B}{dX_i}_{|p_j^*>0} + Pr(\max[p_j^*, 0] = 0)\frac{\partial p_i^B}{\partial X_i}_{|p_j=0}$. If the econometrician treats $p_j = 0$ as exogenous, that is an implicit assumption that $Pr(p_j^* > 0) = 0$. Whenever this assumption is incorrect, it is direct to show that $\frac{dp_i^B}{dA_i}_{|p_j=0} \neq \frac{dp_i^B}{dA_i}_{|p_j>0}$ with a bias that is downward if $\alpha_i\gamma_j > \alpha_j\gamma_i$ and upward otherwise), with $X_i = A_i$. If $\alpha_i\gamma_j = \alpha_j\gamma_i$, then the two are equal so a weighted average of the two returns itself.

[15]Of course, this pattern could also be explained by the transaction cost theory described above that there are fixed cost savings from charging a zero price; baseball card conventions would not need to hire anyone to police entry, for example.

Proposition 11 *Estimated price effects in response to a cost shock in the two-sided market model will be biased downwards if the econometrician treats zero prices as exogenous rather than endogenously censored at zero and there is a non-zero probability of that endogenously censored price becoming positive. Proof: From* p_i^B *above,* $\frac{dp_i^B}{dc_i}_{|p_j=0} = \frac{1}{2}$ *while* $\frac{dp_i^B}{dc_i}_{|p_j>0} = \frac{2\alpha_i\alpha_j - \alpha_j\gamma_i(\alpha_j\gamma_i+\alpha_i\gamma_j)}{4\alpha_i\alpha_j-(\alpha_j\gamma_i+\alpha_i\gamma_j)^2}$ *where the latter is greater than the former.*

From McDonald and Moffitt (1980), $\frac{dp_i^*}{dX_i} = Pr(p_j^* > 0)\frac{dp_i^B}{dX_i}_{|p_j^*>0} + Pr(\max[p_j^*, 0] = 0)\frac{\partial p_i^B}{\partial X_i}_{|p_j=0}$*. If the econometrician treats* $p_j = 0$ *as exogenous, that is an implicit assumption that* $Pr(p_j^* > 0) = 0$*. Whenever this assumption is incorrect, it is direct to show that* $\frac{dp_i^B}{dc_i}_{|p_j=0} \neq \frac{dp_i^B}{dc_i}_{|p_j>0}$ *with a downward bias, for* $X_i = c_i$*.*

Proposition 12 *Conditional on* p_j^* *remaining negative so that it remains censored at zero, the naive or "one-sided" model that entirely ignores the existence of* j *will still correctly estimate how* p_i *changes in response to any demand or cost shock. Proof: Following the logic of the preceding proofs, conditional on* p_j^* *remaining below zero, then* $Pr(p_j > 0) = 0$ *by assumption, and indirect price effects need not be accounted for. The weighted average* $\frac{dp_i^*}{dX_i} = Pr(p_j^* > 0)\frac{dp_i^B{}_{|p_j^*>0}}{dX_i} + Pr(\max[p_j^*, 0] = 0)\frac{\partial p_i^B}{\partial X_i}_{|p_j=0}$ *from McDonald and Moffitt (1980) collapses to just an estimate of the direct effect* $\frac{\partial p_i^B}{\partial X_i}_{|p_j=0}$*.*

This analysis applies equally to models of complements and to two-sided market models, with the main difference being that demand shifters shift both reaction functions in two-sided markets but only the own reaction function in the model of complements, and comes with a clear warning for prediction: even though equilibrium prices may appear consistently at zero over time or across markets, it is possible a latent negative price is being masked that could be "activated" to become positive at some point.

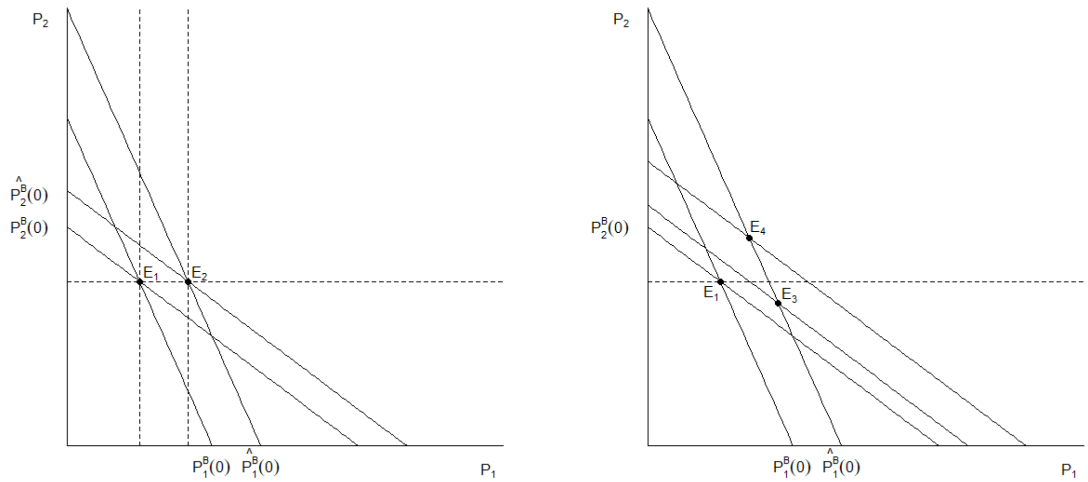
2.3 Application to Media Giants

Despite charging consumers zero prices, there is growing concern about the size of several media giants such as Google and Facebook. In the context of the model and figures presented in this paper, their current pricing strategies correspond to equilibrium points on the vertical axis: zero prices for p_1 (i.e. for Google searches or pages viewed on Facebook) and positive prices for p_2 (i.e. for advertisements or another form of monetizing usage). While these firms are large and exert market power in the advertising market, it is unclear whether there is a threat to consumer welfare (the relevant welfare standard in the United States) given consumers receive services from these firms for free. One theory of harm is that consumers are affected by the size of these firms because the markups charged in advertising markets raise the costs of other firms producing consumer products. These costs are then passed through to consumers in the form of higher product prices.[16] Under this theory of harm, one proposed remedy is to allow consumers and other firms access to the usage data accumulated by the incumbents, eroding their advantage in the advertising market that is derived from controlling such a commanding share of the market for good 1.[17] If this prevents Google and Facebook from exercising market power in advertising markets, this

which exists because it is priced as low as possible.

[16] See Zingales and Rolnik in the New York Times: https://www.nytimes.com/2017/06/30/opinion/social-data-google-facebook-europe.html. July 30, 2017.

[17] Google can offer advertisers virtually the entire population of people searching for certain items.



(a) (b)

Figure 5: The prices in panel (a) can be explained by either price discrimination (in a one-sided model) or pricing of cross-group effects (two-sided market model). The prices in panel (b) can only be explained by pricing of cross-group effects (two-sided market model).

2.4 Price discrimination and identification

Another challenge to identification in two-sided markets is whether observed price differences across groups are driven by cross-group effects in two-sided markets or driven simply by price discrimination. For example, firms that charge lower prices to women than to men could be doing so to increase the quantity demanded from women to generate increased demand from men (i.e. pricing because of cross-group effects) or it could be that there are no cross-group effects at all and women are simply more price elastic or have a lower willingness to pay (i.e. price discrimination).[18] To put it in the context of equation (4) that solves for the equilibrium price in a two-sided market, if $p_1^* < p_2^*$ is observed, it is clear to see that it could be driven by differences across groups in γ's (the parameter measuring cross-group effects) or differences across groups in A_i or a_i (parameters measuring willingness to pay and price elasticity). The identification problem is illustrated in Figure 5 which shows that a two-sided market model can rationalize the observed price difference in the data under the assumption that the two groups are identical except for their cross-group effects (i.e. $\gamma_i > 0$, $i, j = 1, 2$ and $i \neq j$); the associated reaction functions are drawn with solid lines. However, the data can also be rationalized with a simple model of price discrimination that assumes no cross-group effects but instead differences in demand parameters A_i, a_i. As expected, the associated reaction functions are flat. This is expected since the prices are unrelated whenever cross-group effects do not exist, which can be seen from equation (3).

Since the demand system defined in equation (1) applies to any arbitrary pair of groups of consumers, any observed price differences could always be driven by differences in demand parameters

[18] Technically, charging different prices to men and women exclusively because of differences in cross-group effects is already price discrimination. I use the term to mean charging different prices because of differences in underlying willingness to pay (A) and/or price responsiveness (α).

rather than cross-group effects. As Figure 5 suggests, differentiating the two is closely related to measuring the slopes of the reaction functions. If there is no slope, then the price differences are driven by discrimination and if there is a slope, then the price differences are at least partially explained by cross-group effects. From the equations defining the reaction functions in (3), the slope of the reaction function is given by $\frac{-(\alpha_j\gamma_i+\alpha_i\gamma_j)}{4\alpha_i\alpha_j-(\alpha_j\gamma_i+\alpha_i\gamma_j)^2}$, which is non-zero only when cross-group effects operating through γ_i are present. Naturally, testing to what extent price differences are driven by cross-group effects is closely related to measuring the slope of the reaction functions. Since any demand or cost factor unobserved by the econometrician will result in endogeneity of prices, an instrument for p_j is necessary to estimate equation (3). However, there does not exist any such valid instrument, a result formally described in Proposition 13 below.

Proposition 13 *There does not exist any valid demand or cost-based instrument for p_j in an estimation of equation (3). Proof: From equation (3), any demand or cost-based instruments for p_j are contained in (A_j, c_j) but all of those potential instruments are directly correlated with p_i, violating the exclusion restriction.*

Proposition 13 suggests that estimates of the slope of the reaction functions will be biased in any cross-sectional estimation with unobserved demand or cost factors, absent exogenous price variation. Since a non-zero estimate of the slope is the sought after evidence to support the hypothesis of the existence of cross-group effects, any cross-sectional estimation will be problematic. However, even though the slope of the reaction function cannot typically be estimated accurately, the alternative hypothesis of no cross-group effects can be rejected in the data. For example, under the hypothesis of cross-group effects being zero, any demand shock that only affects A_j (and not A_i) or any cost shock that only affects c_j (and not c_i) should only change the price of p_j. Therefore, a test for the existence of positive cross-group effects involves a simple linear regression of p_i on any elements contained in (A_j, c_j) which are believed not to be correlated with elements in (A_i, c_i) that are unobserved by the econometrician. If there is a significant positive or negative effect of (A_j, c_j) on p_i and (A_j, c_j) are in fact uncorrelated with any unobserved elements in (A_i, c_i) then that is only consistent with a model with non-zero cross-group effects. However, if there is no effect, that does not rule out the existence of non-zero cross-group effects. In other words, the existence of cross-group effects can be confirmed from this estimation but not always rejected. See Figure 5(b) where by assumptions there are positive cross-group effects, and consider a positive demand shock to q_i. Both reaction functions shift out and depending on the sizes of those shifts, p_j may increase, decrease, or stay the same. If cross-group effects are zero, then p_j must stay the same since the goods are entirely independent. Therefore, any positive or negative estimated effect on p_j is only consistent with a model featuring non-zero cross-group effects.

This test is useful for examining whether the goods are entirely independent and observed price differences are only driven by discriminatory incentives versus pricing of cross-group effects. It is worth emphasizing again that this test would not differentiate whether the data is driven by a two-sided market model with cross-group effects or a model of complements since both feature downward sloping reaction functions; only a model without sloped reaction functions, such as the "naive" one-sided model can be rejected.

3 Conclusion

The existence of two-sided markets have long preceded the economics literature studying two-sided markets, yet the basic pricing strategies have been known since at least Kaldor (1950). Indirect network effects, cross-group externalities and so on essentially convert users on both sides of a two-sided market into complements, such as in media markets where viewers have been subsidized for decades to attract viewers.The pricing strategies of multi-product firms selling complements have long been known to involve subsidizing one product to increase demand another; this is not surprising since the "within-firm" pricing game is one of strategic substitutes with downward sloping reaction functions. In this paper, I assessed the performance of a model of complements under the assumption that a two-sided market model is actually driving the data generating process. In general, the model of complements performs well. Qualitatively, it predicts many of the defining features of two-sided market platform pricing: zero or negative prices, pricing below cost, the "seesaw effect" of negatively correlated price movements, and in general the idea of "subsidizing" one component of demand to extract higher rents from another. In terms of prediction, the simple model of complements can accurately predict pass-through in two-sided markets and own-price elasticities which can be used to estimate market power through the Lerner Index or to estimate marginal costs. Where a model of complements can lead the econometrician astray is in the search for appropriate instruments for endogenous prices. Demand-based instruments are potentially valid when estimating a demand system of complements, but they are never viable when estimating a demand system corresponding to a two-sided market.

Many two-sided markets feature zero prices to one side of the market. In the paper, I showed that even a "one-sided" model that ignores the existence of cross-group effects entirely can still generate useful predictions, provided the side being charged a zero price remains at a price of zero. Only when there is a risk that a shock can activate positive prices for that side of the market can predictions be biased. For example, one of the most common examples of a platform in a two-sided market is a search engine. Despite agreement that search engines are platforms connecting searchers and advertisers, the price for search is zero. Consider a shock to the advertising side of the market, such as the introduction of a tax on internet advertising revenues. A "one-sided" model that ignores the possibility of users being charged for search can accurately predict the subsequent increase in advertising rates, provided users maintain access to free search. In this example, one would imagine that the tax would have to be substantial before search engines began to charge for search, though in other settings zero prices may become positive relatively easily, something which has occurred in the last decade in U.S. broadcast television in response to reduced demand for television advertising.

The final contribution of the paper noted the difficulty in separately identifying whether differential pricing across groups in a two-sided market is due to pricing of cross-group network effects or discrimination based on underlying willingness to pay for the standalone benefits of the platform. For example, women often receive lower prices than men to enter night clubs, but without more information it is impossible to identify whether the price difference is because women attract additional men more than men attract additional women, or because women are more price elastic. While most agree that cross-group effects are likely the cause of the price differentials in night clubs, it is less clear in other environments, such as the gym where there are many documented cases of men paying more, including in the data from DellaVigna and Malmendier (2006). It is far less clear whether women pay less because the gym is hoping to add women to attract men or because men enjoy exercising more; the DellaVigna and Malmendier (2006) data support the latter, since conditional on holding a membership, men visit the gym more than women. Of course, both effects can occur simultaneously. Unfortunately, the data requirements to separately identify the two effects are higher than usual because the slopes of the reaction functions contain both cross-group and price elasticity terms.

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