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Charles F. Mason, Rémi Morin Chassé

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Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
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The Transition to Renewable Energy

Abstract

The existing economics literature neglects the important role of capacity in the production of renewable energy. To fiill this gap, we construct a model in which renewable energy production is tied to renewable energy capacity, which then becomes a form of capital. This capacity capital can be increased through investment, which we interpret as arising from the allocation of energy, and which therefore comes at the cost of reduced general production. Requiring societal well-being to never decline, we describe how society could optimally elect to split energy in this fashion, the use of non-renewable energy resources, the use of renewable energy resources, and the implied time path of societal well-being. Our model delivers an empirically satisfactory explanation for simultaneous use of non-renewable and renewable energy. We also discuss the optimality of ceasing use of non-renewable energy before the non-renewable resource stock is fully exhausted.

JEL-Codes: C610, Q420, Q560.

Keywords: sustainability, energy, resource use.

*Charles F. Mason**
Department of Economics
University of Wyoming / Laramie / USA
bambuzlr@uwyo.edu

Rémi Morin Chassé
Department of Economics
University of Prince Edward Island
Charlottetown / Canada
rmorinchasse@upei.ca

*corresponding author

The Transition to Renewable Energy

1. Introduction

For roughly 50 years, economists have debated the concept of sustainability Pezzey and Toman (2002). Much of this literature interprets sustainability as non-decreasing well-being of a typical member of society Solow (1991). To operationalize this concept, much of the existing economic literature employs economic growth models, and adapts the associated results on capital accumulation and resource use to "real-world" data Moe et al. (2013). A general finding is that for future generations to be at least as well off as current generations, society must invest the rent from non-renewable resource use to increase the stock of physical capital Hartwick (1977). When consumption of a non-renewable resource is associated with pollution, as with fossil fuels, society is motivated to transition to an alternative, more sustainable, resource.

There is a conceptual link between sustainability and the use of a non-renewable resource whose usage generates pollution (Jevons, 1865; Hartwick, 1977; Forster, 1980). When consumption of a non-renewable resource is associated with pollution, society is motivated to lower its use of that resource (Withagen, 1994). This motivates a transition to an alternative, more sustainable, resource.

In general, economists have modeled this sort of transition by contrasting resource use from a non-renewable source against the use of a "backstop" technology. The backstop is usually assumed to be able to deliver any amount of energy at a constant marginal cost, which implies the resource use can be expanded to the extent society desires without increasing its marginal cost. For many renewable resources however, the associated marginal cost of production is zero, or close to zero. Most of the costs associated with renewable energies are sunk; it is expensive to build the capacity to generate energy from renewable resources Energy Information Administration (2015).[1] This generating capacity then constrains the

[1]The US Energy Information Administration Energy Information Administration (2015) lists the levelized cost of electricity (LCOE) for new energy sources to come online in 2020. The LCOE breaks down into

amount of renewable energy that is available; to increase renewable resource use, capacity must be expanded.

In this paper, we address this inconsistency in the existing literature by presenting a model that more satisfactorily characterizes the role of renewable energy. Our model focuses on the role energy plays in society's potential to produce goods and services. As with much of the existing literature, energy can be allocated to output production or pollution abatement; our extension allows energy to be invested in the development of renewable resource capacity National Renewable Energy Laboratory (2004); Knapp and Jester (2001). We also allow for capacity changes to depend on the stock level, which could reflect learning-by-doing. Working against any increases in capacity is wear and tear from the use of the renewable energy Staffel and Green (2014); Jordan and Kurtz (2013). In this way, our model provides a more satisfactory characterization of the role played by renewable energy in the time path of the typical individual's well-being, and hence the implications for sustainability, than can be found in the extant literature.

When pollution is linked to the non-renewable energy use, the associated social costs are accounted for; this can lead to a phase where both resources are used simultaneously, even though non-renewable energy and the clean alternative (the "backstop") are perfectly substitutable Tahvonen (1997). This feature is also observed by Jouvet and Schumacher (2012), who link the time of the switch to the economy's level of man-made capital. At that moment, society has depleted its natural resource stock, with no potential for simultaneous use. Switching too early would imply forgoing low-cost energy, which helps boost the economy and increases consumption.

Simultaneous use can also occur when the marginal cost of the renewable backstop is increasing. When both the energy demand and the cost of renewable energies are low, society

capital costs, fixed and variable operations and maintenance (O&M) costs and transmission costs. For wind and solar energies, variable O&M costs are zero while for hydroelectricity, they only account for 8.4% of the total LCOE. When sunk initial capital costs and fixed O&M costs are combined, they represent from 89% to 98% of the LCOE for renewables (wind, solar and hydro electricity). This compares to less than 35% for the multiple technologies of gas-fired plants, and up to about 74% for newest coal-fired plants.

may use renewables only first, then switch to simultaneous resource use before switching again to renewables in either finite or infinite time. Some non-renewable energy stock may be left in the ground, implying a phase in which the economy only uses renewable energies Tahvonen and Sallo (2001).

van der Ploeg and Withagen (2014) link the initial levels of capital, pollution and non-renewable resource stock to the type of energy used and the order in which they are used. They find that simultaneous energy usage always follows an oil-only economy and only happens as man-made capital is above its carbon-free steady-state level. Oil is never phased out and man-made capital has to be reduced to reach its long-run level; the oil-only economy "overshoots." According to this view, simultaneous use occurs as the man-made capital stock is drawn down. As society currently is using both renewable and non-renewable energy, their model would require society to be in the process of lowering man-made capital to reach the carbon-free steady-state level, which seems at odds with the real-world.

As we noted above, generating energy from renewable sources is typically constrained by the installed capacity at any given time. In the presence of such constraints, it is entirely possible that there will be a period when both types of resource are used simultaneously. Expanding the renewable capacity would ease this constraint, leading to a period in which the share of renewable resource use rises over time, as society transitions away from the non-renewable resource base. Such a pattern is fully in line with reality: for example, over the past decade the role of coal in producing electricity in the United States has been steadily shrinking, while the share attributable to wind and solar energy has been rising. Moreover, the presence of capacity constraints implies the need to introduce a second state variable, measuring capacity; this feature is absent from all the papers discussed above. A key innovation in this paper is the introduction of this second form of capital.

The paper is organized as follows. We start in section 2 by discussing the main features of our model, emphasizing natural resources, the production of energy and its different uses. In section 3, we describe the main results from our growth model and the optimal choices of consumption and energy production and allocation. We present a numerical simulation of the model in section 4, and offer some concluding thoughts in section 5.

2. Modeling Preliminaries

In this paper, we adopt the usual economic interpretation of sustainability: the requirement that future generations be made at least as well off as current generations, *i.e.*, societal well-being may not fall over time. Employing a neoclassical growth model, we use a "Capital Approach" to sustainability (Moe et al., 2013). Our point of departure regards the role energy plays in society's potential to produce goods and services, and the manner in which that energy is itself produced.

We envision a world with two types of energy producing resources. One type of resource is non-renewable; one can think of conventional fossil fuel as a good example. The other type of resource is renewable, for example wind and solar power, or biofuels. While our conceptualization of a renewable resource is related to the concept of a "backstop technology" in the extant literature, these papers typically assume the backstop will be fully available – *i.e.*, society has access to whatever quantity of the resource as it wishes to use at a given time. We believe this fails to capture a key element: that society must first invest in renewable natural capital before it can access renewable energy.

Both non-renewable and renewable resources are capable of producing energy, but one is relatively abundant (at least initially) and relatively dirty, namely the non-renewable source. Production of energy from this resource generates an unattractive side effect, which we refer to as pollution; one might here think of carbon emissions, leading to an atmospheric carbon stock that may inflict damages on future members of society. The production of goods and services also opens up the potential to increase the stock of what we refer to as physical capital, itself an important input into the production process. By adding to physical capital, current generations can create a potential offset to accumulated pollution, which might then allow future generations to enjoy a level of well-being at least as large as that of the present.

But as the non-renewable natural capital stock is drawn down, actions must be taken to facilitate its replacement, by building up the renewable natural capital stock.

Felicity depends on the flow of aggregate consumption, C, as well as two state variables: the effective capacity of renewable resources to produce energy, which we denote by X and often refer to as "renewable natural capital" in the pursuant discussion, and environmental pollution, which we denote by P. In a conventional growth model, society allocates output to consumption or investment. In our interpretation, energy – a key input into the production function that generates output – is directed to productive purposes or to increase the effective renewable natural capital stock. One can think of energy as directed towards maintenance, which serves to offset some of the depreciation that arises from use of windmills or solar panels (as discussed in footnote 1), or as investment into new structures. These efforts can either be helped or hindered by the level of existing stock of renewable resources.[2] Pollution is generated by the extraction of what we call non-renewable natural capital, denoted by S. The stock of pollution P is undesirable for humans. For instance, petroleum extraction is associated with high levels of carbon emissions, and the use of petroleum-derived products can lead to air and water pollution. Another example is the presence of floating patches of discarded plastic in northern and southern Atlantic and Pacific oceans and in the Indian ocean; they pose threats to wildlife and marine ecosystems.[3]

Non-renewable natural capital can be thought as a measure of the aggregated stock levels of fossil fuels, or other resources that can be used to produce energy, for example uranium. The extraction of minerals for energy production also entails environmental degradation. Tailings piles, the accumulated residue from mineral extraction, are a prime example. These stockpiled residues decrease utility by their mere presence and may also introduce hazardous mineral elements into the environment.[4] In addition, environmental pollution from certain

[2]One could also think of the renewable resource as bio-energy, whose stock naturally grows in relation to the stock level, but is diminished by harvesting (Zilberman et al., 2013). In that setting, one can think of energy as being used to increase the capability of a facility to generate bio-fuels.

[3]See http://www.garbagepatchstate.org. We note that most plastics are made from petrochemicals.

[4]For example, oil sands extraction in Alberta, Canada leaves significant tailings, in particular containment ponds retaining water with fine or toxic residues (Heyes et al., 2017). Environmental regulations require industry to restore the extraction site to its pre-extraction state.

chemicals used in resource extraction can accumulate in the environment and cause disutility; mitigating this pollution requires the expenditure of effort and energy.

We assume that society prefers larger levels of consumption or the renewable resource stock, but at a declining rate. In addition, we assume an increase in the renewable resource stock raises the marginal felicity from consumption. Pollution diminishes felicity at an increasing rate, and lowers marginal felicity from consumption (Michel and Rotillon, 1995) and renewable natural capital. These characteristics impact decision making at the margin and affect the respective evolution and long-run values of consumption, pollution and the renewable resource stock are linked together. Denoting partial derivatives by subscripts, these assumptions may be succinctly written as:

$$\begin{aligned} U_C > 0,\ \ U_X > 0,\ \ U_P < 0 \\ U_{CC} < 0, U_{XX} < 0, U_{PP} > 0 \\ U_{CX} > 0, U_{CP} < 0, U_{XP} < 0 \end{aligned} \tag{1}$$

To facilitate consumption, society must first generate an amount of aggregate production, Y. The generation of production is linked to two types of inputs: physical capital K and energy E. Physical capital is an abstract component which can be thought of as reflecting, for example, infrastructure (roads, pipelines, communication networks and the like) and physical machinery. Energy can be produced using renewable and non-renewable resources.[5] We describe the relation between these two inputs and aggregate output by means of a production function: $Y = F(K, E)$.[6] This function has some important mathematical properties: larger values of either K or E yield increased production, but at a decreasing rate (reflecting

[5]In the numerical simulation we discuss in section 4 below, we assume the two inputs are perfectly substitutable in the energy transformation function. We discuss the implications of relaxing this assumption in subsection 4.4.

[6]In the next section, we describe a simplified version of the model where there is no role for energy in production; in section 4 we return to the full version of the model. The production function does not explicitly account for labor as our focus is on capital and energy. We thus make the implicit assumption that labor is constant through time and normalized to 1 (as do many earlier authors). This is consistent with our numerical simulation, as we have decreasing returns to scale in K, E and overall.

diminishing returns to scale). These assumptions may be succinctly written as:[7]

$$F_E > 0, F_K > 0, F_{EE} < 0, F_{KK} < 0.$$

Society can generate energy through the rate of use of non-renewable natural capital and renewable natural capital. Henceforth, we will refer to the flow of non-renewable natural capital used as "extraction," and denote it by q; we refer to the flow of renewable natural capital used as "harvest," and denote it by h. The quantity of energy produced is described by the energy transformation function

$$E = E(q, h).$$

This energy, which can also be viewed as effort, can be allocated to multiple uses, including pollution abatement, output production and renewable resource development.

We envision a central decision-maker, or "social planner," who is charged with promoting the well-being of society for all time. The social planner evaluates the level of felicity at every moment, weighting each contribution depending on the time at which the felicity is generated. In this regard, she uses an inter-temporal social discount rate, $\tilde{r}$.[8] The social planner's task is to select a time path of consumption level, energy allocation and resource uses so as to maximize the discounted flow of felicity over an infinite horizon. The planner's decision problem is constrained by equations that describe the evolution of the capital stocks.

3. Sustainability

We now turn to a discussion of the key aspects of sustainability in our model. We start our discussion in subsection 3.1 with a simplified variant of the problem, so as to enhance intuition; in this setting sustainability can be achieved provided consumption never

[7]For technical reasons, we also require $F_{EE} + F_{KK} - 2F_{EK} > 0$.

[8]This discount rate was the source of considerable controversy in the context of climate policy. For example, Stern (2007) proposes a very low discount rate, barely positive, while Nordhaus (2007) suggests a rate closer to 3%. The ultimate choice of discount rate implies an ethical judgment, which is beyond the scope of our paper and discussion. We merely point out the nature of the dynamic optimization scheme our mythical social planner must decide upon, and relate it to a notion of sustainability.

falls. We then expand the discussion to the full model in subsection 3.2. We relegate the technical details of these analyses to the Appendices; in the main text we will focus on the interpretation.

3.1. **Sustainabilty With Physical Capital Only.** In this version, renewable natural capital X does not play a role in the planner's thought process, so felicity depends only on C; in addition, the evolution of K is not influenced by S. Physical capital is prone to depreciation, at the rate $\tilde{\delta}$, so that the evolution of the physical capital stock is described by the differential equation:[9]

$$\dot{K} = -\tilde{\delta}K + F - C, \tag{2}$$

The amount society invests in physical capital equals the difference between output, $F(K)$, and consumption, C. Because all output will be consumed or invested, we need only discuss one of the two; in this section, focus on the optimal time path of consumption. The social planner's goal is to maximize the discounted flow of felicity, subject to the equation of motion governing physical capital, eq.(2); the initial level of physical capital, $K(0)$; and the stipulation that consumption not exceed aggregate production.

In this simple setting, where felicity depends only on aggregate consumption, sustainability corresponds to the constraint that consumption never fall. As we show in Appendix A, this criterion is equivalent to the stipulation that $K_0 < \bar{K}$, where

$$F_K(\bar{K}) = \tilde{r} + \tilde{\delta}. \tag{3}$$

That is, sustainability requires physical capital be small enough that its marginal product never falls below the sum of the discount rate and the depreciation rate.

3.2. **Sustainability With Energy Input and Natural Capital.** We now proceed to a consideration of the initial framework with renewable and non-renewable natural capital, pollution and energy production and allocation. In this setting, the social planner can

[9]We adopt the notation $\dot{x} = \frac{\mathrm{d}x(t)}{\mathrm{d}t}$ to denote the time derivative of a variable.

elect to extract resources in order to boost output production, until resources run out. The actual quantity of non-renewable natural capital that remains at each moment in time does not directly influence utility, which implies that total exhaustion of these resources is possible. The same is not true for renewable resource stock, however, as we assume society derives utility from the presence of such resources (which may provide amenities or ecological services).

We call renewable (respectively, non-renewable) energy the amount of energy derived from renewable (respectively, non-renewable) natural capital. For later use, we denote the fraction of energy allocated to use i by s^i and the corresponding quantity of energy allocated to use i by the product of s and energy: $e^i \equiv s^i E(q, h)$. Part of the social planner's decision problem entails the allocation of energy between the various potential uses at each point in time.

Extraction subtracts from the stock of the non-renewable natural capital:

$$\dot{S} = -q. \tag{4}$$

The renewable resource stock, or capacity, X increases according to the total energy that is allocated to it; one can think of this process as investing in new renewable energy capacity.[10] Although the marginal cost of renewable energies is zero, the cost of the investment existing capacity is the lost benefits associated with allocating energy to other valuable uses.

We also allow that change in capacity to depend itself on the stock level X: This could reflect society's choice to first develop the best suited sites on which to build that capacity on. Alternatively, if we interpret X as reflecting the productive capacity of the current stock of renewable capital, the dual role of e^X and X in promoting growth of X could be thought of as capturing learning-by-doing. We summarize these effects in the function $G(X, e_X)$. Working against an increase in capacity is wear and tear from actually using renewable

[10]This approach is similar to the concept of energy payback from lifecycle analysis. For instance, the National Renewable Energy Laboratory lists payback times – how long a unit has to operate to offset the total energy used to manufacture the system – for photovoltaics to be between 1 and 3.3 years (National Renewable Energy Laboratory, 2004). Knapp and Jester (2001) estimate that, depending on the technology, between 3 MWh to 16.5 MWh are necessary to produce a 1 kW solar panel.

energies.[11] We include that feature by explicitly having the capacity depreciating with the use of the renewable resource, as measured by h. Accordingly, the rate of change in X is summarized by the following state equation:

$$\dot{X} = G(X, e_X) - h. \tag{5}$$

The planner can take steps to reduce the stock of pollution by allocating a fraction s^A of total energy to "abatement." The corresponding level of energy directed towards abatement is $e^A = s^A E(q,h)$; this results in a level of abatement activities $A(s^A E(q,h))$. The rate of change in the pollution stock equals the difference between the flow of emissions, which is a function $J(q)$ of the rate of extraction of the non-renewable energy source, less abatement:

$$\dot{P} = J(q) - A(s^A E(q,h)). \tag{6}$$

In addition to its ability to foster renewable resources regeneration and pollution abatement, we assume energy is a key ingredient in aggregate production. Noting that the share of energy allocated to production is $s^Y = (1 - s^A - s^X)$, we define the amount of energy allocated to production as $e^Y = s^Y E(q,h)$; we assume the production function is $F(K, e^Y)$. Aggregate production can be used for consumption or saved to increase the stock of capital, which otherwise depreciates at rate $\tilde{\delta}$; this is characterized by the differential equation:

$$\dot{K} = -\tilde{\delta} K + F(K, e^Y) - C. \tag{7}$$

The optimization problem here is similar to that of the previous subsection – the social planner seeks to maximize the discounted flow of felicity – although in this context felicity depends on renewable natural capital and pollution as well as consumption. The key distinction to the optimization problem here is that the planner has multiple instruments with which to accomplish this task: the rates of consumption, C, extraction, q, harvest, h, and

[11]This depreciation can be substantial. For example, Staffel and Green (2014) find that for wind farms in the United Kingdom, load factors – the percentage of electricity actually produced, compared to the theoretical maximum – falls by roughly 1.5% per year. While less dramatic, depreciation rates for solar photovoltaics are on the order of 0.5% per year (Jordan and Kurtz, 2013).

the shares of energy allocated to output production, s^Y, abatement, s^A, and regeneration of renewable natural capital, s^X. There are also multiple state variables of relevance, and the optimization problem is constrained by the evolution of these variables; the relevant equations of motion are given in equations (4), (5), (6) and (7).

The solution to the social planner's problem is described by several conditions. The first condition is that the marginal impact of consumption upon felicity is equal to the shadow price of capital:

$$U_C = \lambda. \tag{8}$$

Intuitively, the marginal benefit of a small increase in consumption are weighed against the marginal cost of a small increase in consumption. As output is either consumed or invested, the implicit cost from a small increase in consumption is the induced accumulation of physical capital, the value of which is the shadow price λ.

The condition governing the rate of extraction is similar in spirit: marginal benefits are balanced against marginal costs. The marginal benefits from incremental extraction q derive from the increase in non-renewable energy and the gains that arise from its allocation to the three possible uses. Specifically, the total marginal benefits are, *ceteris paribus*, the sum of the marginal benefits of additional energy in output, abatement and renewable energy growth. On the other side of the ledger, the marginal costs are the sum of the shadow price of the non-renewable resource – that is not having the resource available for later use – and the increase in pollution from additional extraction. Thus, the optimal level of extraction satisfies

$$\lambda F_{e^Y} s^Y E_q - \phi A_{e^A} s^A E_q + \mu G_{e^X} s^X E_q = \psi - \phi J_q. \tag{9}$$

The condition for the contemporaneous use of renewable-resource is similar to the non-renewable one; the increased harvest increases the quantity of renewable, sustainable, energy in the global energy mix. Hence the marginal benefits from additional renewable resource use is, again, increased energy to allocate to each end use and the associated increase in output, abatement and resource regeneration. The associated costs are simply the costs of not having more renewable resource for later use. Thus, the optimal level of harvest satisfies

$$\lambda F_{e^Y} s^Y E_h - \phi A_{e^A} s^A E_h + \mu G_{e^X} s^X E_h = \mu. \tag{10}$$

The planner chooses the optimal shares of energy to allocate to output and abatement, respectively s^Y and s^A. By construction, the share of energy allocated to renewable resource is the fraction not allocated to the other uses, $s^X = 1 - s^Y - s^A$. The optimal share s^Y will be such that the marginal benefits from additional energy allocated toward production, the value of the marginal increase in output from higher energy, equal the associated marginal costs, that is the foregone value of the marginal increase in the stock of renewable resource:

$$\lambda F_{e^Y} = \mu G_{e^X}. \tag{11}$$

Similarly for the share of energy allocated to abatement, in equilibrium, the marginal benefits from an incremental s^A, the value of an increase abatement, equal the marginal costs, which are the same as for s^Y, that is the foregone value of the marginal increase in the stock of renewable resource. We have

$$-\phi A_{e^A} = \mu G_{e^X}. \tag{12}$$

We interpret sustainability as the requirement that future generations have the same (or higher) level of well-being as that enjoyed by the current generation – *i.e.*, that felicity never declines (Solow, 1991). In the simple version of the model of subsection 3.1, which includes only consumption and physical capital, sustainability requires that consumption never fall. In the full model, the condition is more complex. In our specification, both renewable and non-renewable resources enter in energy production, which is necessary to produce output. Should these inputs be perfect substitutes, the current generation would need to leave enough of at least one resource to future generations so that they can enjoy the same level of well-being. If they are not perfect substitutes, the current generation needs to manage both resources efficiently through time so that future generations have enough to achieve the same level of felicity.

3.3. **Regime Shifts.** To discuss the qualitative features of the energy switch, we use equations (9) and (10). Following Tahvonen (1997), define the switching function

$$\sigma = \psi - \phi J_q - \mu. \tag{13}$$

This condition compares the marginal cost of each energy source. It accounts for the non-renewable resource rent ψ, the social cost of pollution ϕ and shadow value of renewable resources μ. We observe that whenever $\sigma < 0$, the marginal cost of non-renewable energies is lower than that of renewable energies and society relies only on non-renewables. For $\sigma = 0$, both energy types are used simultaneously and shadow value of the renewable resource is equal to the non-renewable resource rent augmented by the marginal social cost of pollution. For this phase to last for some time, it must also be that σ is zero and the change in σ over that period of time is also zero. We need

$$\dot{\sigma} = \dot{\psi} - (\dot{\phi} J_q + \phi J_{qq} \dot{q}) - \dot{\mu} = 0,$$

*i.e.*the relative costs move together along the optimal path. Finally, when $\sigma > 0$, renewable energies are, at the margin, cheaper than the non-renewable alternative and society relies solely on the former. Because it is always optimal to allocate energy to output, $s_Y > 0$, then σ and equation (11) indirectly specify a level man-made capital a the time of the switch. This is similar to Jouvet and Schumacher (2012), however their model's specification rules out simultaneous resource use. In our description of the economy, a complete energy switch to renewable energies in finite time, while the non-renewable resource is still plentiful, can happen endogenously.[12]

We note that the switch to renewable energies observed in the literature is generally a function of the marginal cost of renewable energies,; as we noted above, this is zero for many renewable energy technologies. As such, our approach allows us to restate the switching conditions in terms of the shadow prices and and distaste for pollution.

[12] If a candidate solution is such that the non-renewable resource resource stock is exhausted in finite or infinite time, we have that either (i) $q(t), S(t) = 0$ for $t \geq \tau$ with $\tau < \infty$ or (ii) $q(t), S(t) \to 0$ as $t \to \infty$, respectively.

4. Numerical simulation

As the dynamics in the system we study are rather complicated, we focus on broader features, by numerically simulating the model; this simulation produces some interesting results. To facilitate this simulation, we must first specify the various functional forms in the model.

Given the chosen parameters and functional forms, society gradually shifts from using non-renewable to renewable energies, eventually abandoning the use of the former. This transformation occurs even though the non-renewable resource is still plentiful. We then analyze the post non-renewable resource phase, its long term equilibrium and stability. Modelling the effective capacity of renewable resources to produce energy as a state variable, and capturing capacity constraints, impacts this long run equilibrium. For the steady-state level of renewable resource to be positive, society has to be relatively averse to intergenerational inequity. We also simulate the effects of a short run moratorium of renewable natural capital accumulation and discuss the long-run effects and the impacts on sustainability.

Following Xepapadeas (2005), we assume the utility function is non-separable in consumption C, the renewable resource stock X and pollution stock P.[13] First note that all exogenous parameter are marked with a tilde '$\sim$'. We assume the inter-temporal elasticity

[13]Felicity can be either separable non-separable in X and P. Separable utility would imply that an increase in pollution would have no effect on the marginal felicity from consumption or from the stock of renewable resource (because the cross-partial derivatives between consumption and both X and P would then be zero). While this facilitates analytical tractability, it precludes some important features in terms of how renewable resource capital and pollution affect felicity and marginal utility from consumption. Because our analysis relies on simulations, as opposed to seeking analytical closed-form solutions, we prefer to use a non-separable utility formulation.

of substitution $\tilde{\sigma} = \frac{1}{\tilde{a}} = 2$ so that $\tilde{a} = 0.5$.[14] We also set $\tilde{b} = 0.1$ and $\tilde{c} = 1.1$.[15]

$$U(C, X, P) = \frac{C^{1-\tilde{a}}}{1-\tilde{a}} \frac{X^{\tilde{b}}}{P^{\tilde{c}}}$$

The ratio of X over P can be thought of as warm glow, positively impacted by the stock of renewable capacity X and negatively by the stock of pollution P. Although these levels of stock impact the marginal utility from consumption, they do not have an effect on society's elasticity of inter-temporal substitution.

The energy-generating function $E(q, h) = q + h$ is assumed additive and linear in both renewable and non-renewable resources, so that both are perfect substitutes in total generation. The production function $F(K, e^Y) = K^{\tilde{\gamma}}[e^Y]^{\tilde{\eta}}$ is Cobb-Douglas. Hence, capital and energy are imperfect substitutes; there can be no output without some energy as an input. Following van der Ploeg and Withagen (2014), we chose $\tilde{\gamma} = 0.2$ and $\tilde{\eta} = 0.1$.[16]

Pollution is an increasing, convex function of non-renewable resources $J(q)$. We set $J(q) = \frac{q^2}{2}$, so that pollution is quadratic in extraction q. Abatement is increasing in energy allocated, $\partial A/\partial e^A > 0$, but at a decreasing rate, $\partial^2 A/\partial e^A \partial e^A < 0$. To achieve this, we use the form $A(e^A) = [e^A]^{\tilde{e}}$ with $\tilde{e} = 0.5$.

To facilitate the numerical computation and analysis, we assume the renewable resource stock grows according to $G(e^X)$, a function of energy alone. In this application, the growth of renewable resources is not affected by the level of stock, nor can the stock regenerate by

[14]We make this assumption to be in line with results from section 3.2. The magnitude of $\tilde{a}$ indicates how willing society is to substitute current consumption for consumption in the future: The lower $\tilde{\alpha}$, the greater is society's willingness to smooth consumption over time. This is consistent with Tahvonen and Sallo (2001), who require $\tilde{a}$ to be between zero and one. In Hansen and Singleton (1982), $\tilde{a}$ ranges from 0.68 and 0.95, while van der Ploeg and Withagen (2014) assumes $\tilde{a} = 2$.

[15]While the utility function we use in the numerical simulations is not well defined at zero pollution stock, and that the pollution stock tends to zero in the infinite limit, this poses no difficulties since we simulate for a finite number of periods. To avoid the conceptual problem, one could add an arbitrarily small positive value to stock (which might be interpreted as a "pre industrial" level). For a suitably small value, such a refinement would not change the simulation results, though it would complicate the presentation.

[16]We also use a Cobb-Douglas production that is not homogeneous of degree one.

itself.[17] We operationalize these features by setting

$$G(e^X) = [e^X]^{\tilde{f}}.$$

We choose $\tilde{f} = 0.5$, so that $G_{e^X} > 0$, but $G_{e^X e^X} < 0$. Capital K depreciates at rate $\tilde{\delta} = 5\%$ and the discount rate $\tilde{r}$ is equal to 0.014.

We shall focus on one sequence of interest. Initially, society simultaneously uses both resources; after time, society may switch completely into green energy.[18]

4.1. **Simultaneous use.** First we study a possible scenario in which society uses both resources. The initial value in $t = 0$ for K is assumed to be half its steady-state value in the post non-renewables phase, while $X(0)$ is assumed to be 10% of its steady-state value. Similar to van der Ploeg and Withagen (2014), we set $P(0) = 24$ and $S(0) = 20$. Also we use $s^A(0) = 0.083$, $s^Y(0) = .5$, $s^X(0) = 1 - s^Y(0) - s^A(0) = 0.417$, $h(0) = 0.1$, $q(0) = 0.5$, $C(0) = 0.618$.

The simulation results are summarized in Figures 1–5. Figure 1 shows the evolution of the flow of renewable h and non-renewable q resources as energy inputs. Over time, the use of non-renewable energies decreases as society gradually switches to renewable energies. In this particular simulation, non-renewable energies become virtually irrelevant by $t = 1.87$. This represents a corner solution which is an interesting feature in economic growth models: it may become optimal to completely stop using non-renewable energy and in favor of sustainable energy from renewable resources. The fact that this path involves such a corner solution is a direct consequence of the perfect substitution between the two natural resources.

Figure 2 plots the evolution of the four state variables over time. The stocks of physical capital and of renewable natural capital are both plotted from the beginning. Since extraction

[17] In light of our discussion above, this specification abstracts from learning-by-doing.

[18] While some renewable energies are perfect substitutes to non-renewable energies, there are some potential difficulties in substitution. For example, renewable energy production is subject to intermittency due to meteorological uncertainties, while energy production from coal is not. By assuming the resources are perfect substitutes, we are implicitly assuming that these issues can be overcome.

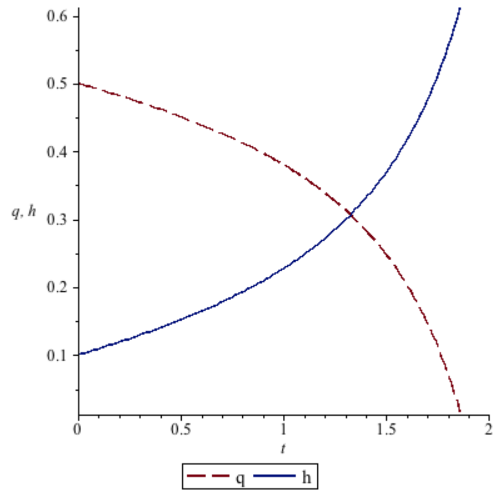

FIGURE 1. Non-renewable and renewable resource use

of non-renewable physical capital is initially positive, the associated stock S decreases over time up to the point where q falls to zero.

Figure 3 plots the evolution of the four respective shadow values. We note that the shadow values for physical capital and both the renewable and the non-renewable natural capital are all positive, while the shadow value of pollution is negative, as anticipated. Let τ be the time of the switch, at $\tau = 1.87$, where q falls to zero, the shadow values of the two types of natural capital become equal.[19]

Figure 4 plots the shares of energy allocated to the different uses, s^Y, s^A and s^X. Related to these paths, Figure 5 displays the actual levels of energy allocated to each use. We see that the shares of energy allocated to renewable natural capital and abatement decrease over time, implying that the share allocated to output increases. These effects are mirrored in the paths of total energy allocation.

While Figure 1 shows a monotonic decrease in the use of non-renewable energies, this path need not preclude growing felicity. Figure 6 illustrates. Not only is switching to renewable energies optimal, it also is associated, in our simulation, with continuously expanding felicity and consumption over time.

[19]We interpret a period as one year, so 1.87 periods is roughly 20 months.

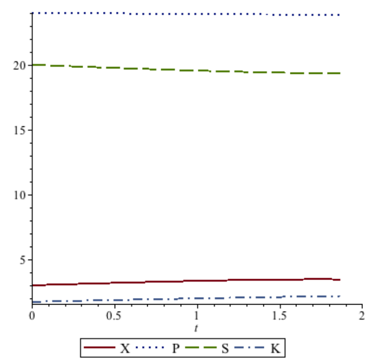

FIGURE 2. State variables

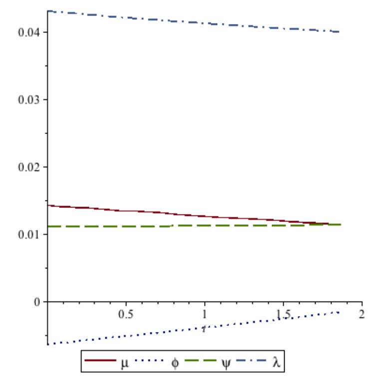

FIGURE 3. Shadow values

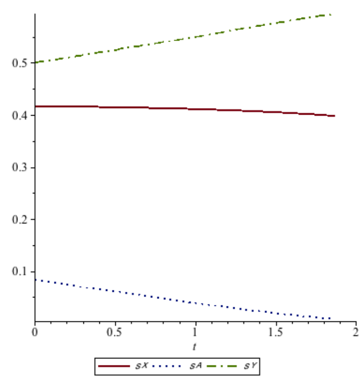

FIGURE 4. Energy shares

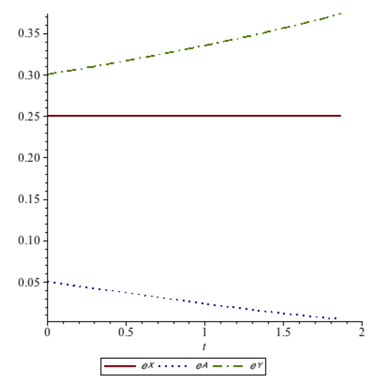

FIGURE 5. Energy allocation

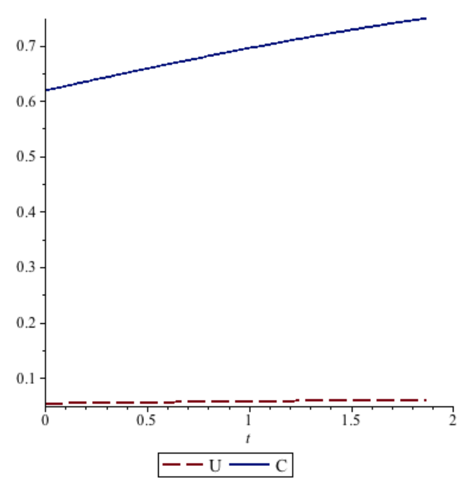

FIGURE 6. Felicity and Consumption

At time $\tau = 1.87$, the values for each state and controls are

$$C = .789, \quad h = 0.629, \quad K = 2.197, \quad X = 3.471, \quad s^Y = 0.595.$$

In the next section, we use values similar to these to illustrate how society can achieve a long-run stable outcome.

We now analyze the effect of variations in the discount and capital depreciation rates, as well as variations in the initial values of pollution and renewable resource stock, on the time of the switch τ toward renewable resources. As in van der Ploeg and Withagen (2014), we find that increasing the discount rate $\tilde{r}$ (here, from 0.014 to 0.05) postpones the time of the switch τ (from 1.87 to 2.02). A higher discount rate implies that the future is discounted more, and hence is less important in today's decision. This induces a postponement of the switch to renewables, since the discounted costs from higher future pollution are given smaller weight.

An increase in the rate of depreciation of capital also leads to a postponement of the switch. When $\tilde{\delta}$ increases from 0.05 to 0.10, τ increases from 1.87 to 1.96. Since capital depreciates faster, society must increase its use of non-renewable resources to compensate for lower future capital stocks.

Altering the initial values of P and X also lead to different scenarios. A lower initial level of pollution (e.g., $P(0) = 20$ instead of 24) would also postpone the time of the switch to renewables (from $\tau = 1.87$ to 1.89). As society has a distaste for pollution, lower levels of P permit longer extraction of non-renewables.

An increase in the initial stock of renewables, e.g. raising $X(0)$ from 10% to 50% of the steady-state value of X, would induce society to suspend its use of renewables altogether at $\tau = 0.489$. But this suspension can only be temporary, as the disutility from pollution will increase ever-more rapidly, while the stock of non-renewables will decrease more rapidly. Accordingly, at some point in the future, society will start resume its use of renewable energies, possibly alongside non-renewables before phasing the latter out (Tahvonen, 1997)). Given the exhaustible nature of the non-renewable natural capital, society must ultimately fully switch to using renewable resources.

4.2. **The Post-Non-Renewables Phase.** We now turn to the phase in which society is no longer using non-renewables (a situation that some might regard as a sustainable outcome). During this phase, society could still be abating pollution from previous non-renewable resource use, with $s^A > 0$; this will depend upon society's distaste of pollution.

Given our functional forms, it is possible to find analytical solutions for the steady-state values in this phase. We denote the steady-state values of the key variable with an overbar. Since $q = s^A = 0$ here, we have[20]

$$\dot{K} = 0 \Rightarrow \bar{C} = \tilde{\delta}\bar{K} + F \tag{14}$$

$$\dot{X} = 0 \Rightarrow \bar{h} = [1 - \bar{s}^Y]^{\frac{\tilde{f}}{1-\tilde{f}}} \tag{15}$$

$$\bar{K} = \left[\frac{\tilde{\gamma}}{\tilde{\delta} + \tilde{r}}[\bar{s}^Y \bar{h}]^{\tilde{\eta}}\right]^{\frac{1}{1-\tilde{\gamma}}} \tag{16}$$

$$\bar{X} = \frac{\tilde{b}\tilde{f}\bar{s}^Y \bar{C} G}{\tilde{\eta}\tilde{r}F[1 - \bar{s}^Y][1 - \tilde{a}]} \tag{17}$$

$$\bar{s}^Y = 1 - \frac{\tilde{f}^{\frac{1}{\tilde{f}}}}{\bar{h}}. \tag{18}$$

From (17), we see that $\bar{X}$ has the same sign as $[1 - \bar{s}^Y][1 - \tilde{a}]$. For a positive stock of renewable resource, we need $[1 - \bar{s}^Y][1 - \tilde{a}]$ to be positive, which can only happen if $\tilde{a} < 1$.[21] The parameter $\tilde{a}$ corresponds to the representative agent's elasticity of intergenerational inequality aversion in consumption, with $\tilde{a} = \frac{1}{\tilde{\sigma}}$. Requiring $\tilde{a} < 1$ imposes a restriction on $\tilde{\sigma}$, the representative agent's elasticity of intertemporal substitution, namely that $\tilde{\sigma} > 1$. That is, for the steady-state value of the renewable resource stock to be positive, the representative

[20]In a steady-state, the different state (man-made and renewable capital) and control (renewable resource use, consumption and energy shares) variables stop changing, *i.e.*, $\dot{C} = \dot{K} = \dot{X} = \dot{h} = \dot{s^Y} = 0$. Several other variables also stop changing once a steady-state has been reached: $\dot{P} = \dot{q} = q = \dot{s^A} = s^A = 0$. Using equations derived from (B.2), (B.4) and (B.5), evaluated at $q = s^A = 0$, combined with the dynamic efficiency conditions found in Appendix B, yields equations (14)–(18). We also found that in this last phase, the shadow values of man-made and renewable capital will have the same sign only if $\frac{h}{G} > \tilde{f}$.

[21]This follows from the fact that $1 - \bar{s}^Y \in (0, 1)$. The inequality could be satisfied for negative values of $\tilde{a}$, but this would implies an increasing marginal felicity from consumption, which is ruled out by the assumptions from equation (1).

agent must be averse to intergenerational inequality. Based on the chosen parameters, one may determine the steady-state values of each relevant state and control variables:

$$\bar{C} = .94, \quad \bar{h} = 0.5, \quad \bar{K} = 3.49, \quad \bar{X} = 30.13, \quad \bar{s}^Y = 0.5.$$

As the system of equations is non-linear, we proceed by linearizing it around the steady-state; this allows us to study the behavior of the system close to the equilibrium point. We find that the system is saddle-point.[22] For society to reach that steady-state, the path it selects must follow the stable branch after society stops using the non-renewable resource.

Following van der Ploeg and Withagen (2014), we set the initial level of physical capital equal to one half its steady-state level. In figures 7 through 10, we plot the time trajectory of each state and control variable, with the top horizontal line in each figure representing their respective steady-state level. With the exception of s^Y, all variables approach the steady-state monotonically from below.[23]

Figure 11 shows the stable trajectories for felicity U, capital K and the renewable resource stock X for five scenarios; the steady-state is represented as the black dot in the centre. The trajectory identified by circles shows the case for which both $X(0)$ and $K(0)$ are each half their steady-state value, which we refer to as "scenario (i)" in the discussion below. Figure 11 also includes trajectories corresponding to the following scenarios: scenario (ii) sets $K(0) = 0.5\bar{K}$ and $X(0) = 1.5\bar{X}$; scenario (iii) sets $K(0) = 1.5\bar{K}$ and $X(0) = 0.5\bar{X}$; scenario (iv) sets $K(0) = 1.5\bar{K}$ and $X(0) = 1.5\bar{X}$. Trajectory (v) starts at the off-center diamond, which corresponds to values of the state and control variables close to those obtained at $\tau = 1.87$ in the simulation from Section 4.1.[24] This point shows an unsustainable level of utility, as consumption C, harvest h and the share of energy to output are all too high when

[22]See Appendix D for details.

[23]In this linearized representation, $h = s^Y$, and so we do not plot the time path for h. These graphs are based on initial levels of capital and renewable resource stock that are smaller than their steady-state levels; there are other stable paths, starting from initial levels that are not both smaller than the steady-state levels.

[24]We assume $s^A = 0$, whereas $s^A(1.87) = 0.0075$ in the simulation. Because renewable capital is procured via investment, one would expect the initial level to lie below the steady-state value. Accordingly, scenarios (iii) and (iv) are unlikely to be empirically plausible; we include them solely for the sake of completeness.

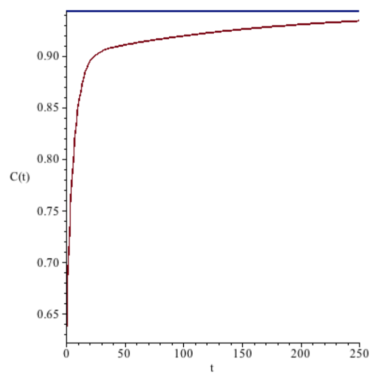

FIGURE 7. Consumption

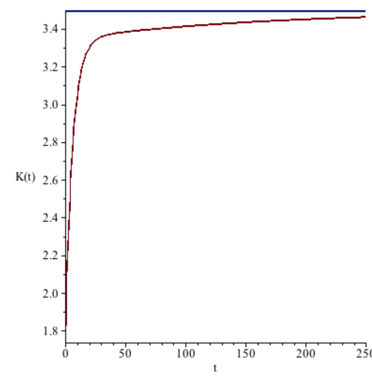

FIGURE 8. Capital

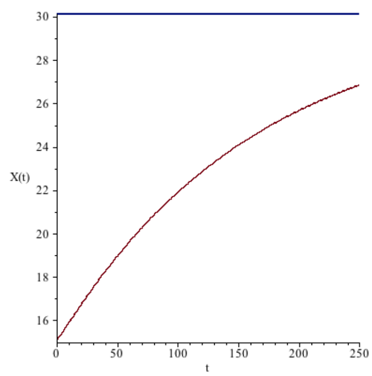

FIGURE 9. Renewable Resource

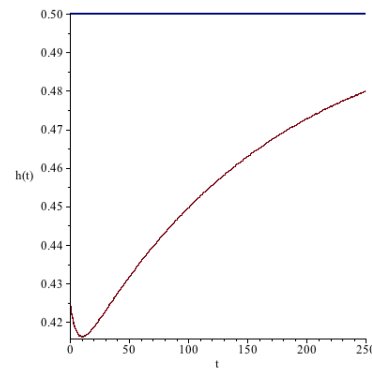

FIGURE 10. Energy Share to Output

compared to the their values should society be on the stable arm leading to the steady-state. Because the levels of X and K cannot be instantaneously adjusted, society has to lower consumption C (from 0.75 to 0.69) and renewable resource use h (from 0.63 to 0.35). This adjustment allows society to redirect some energy to increasing renewable natural capital, by reducing s^Y (from 0.59 to 0.35). But this shift causes a one-time loss in felicity, as represented by the arrow on the graph.[25]

As Figure 11 illustrates, any trajectory starting from initial values of X and K that are larger than their steady-state value would imply that felicity declines at some point. Thus, only scenario (i) is sustainable; the other scenarios represented in Figure 11 are not

[25] Interestingly, as soon as this policy is adopted, it takes just about 2 years to achieve the same level of felicity as before.

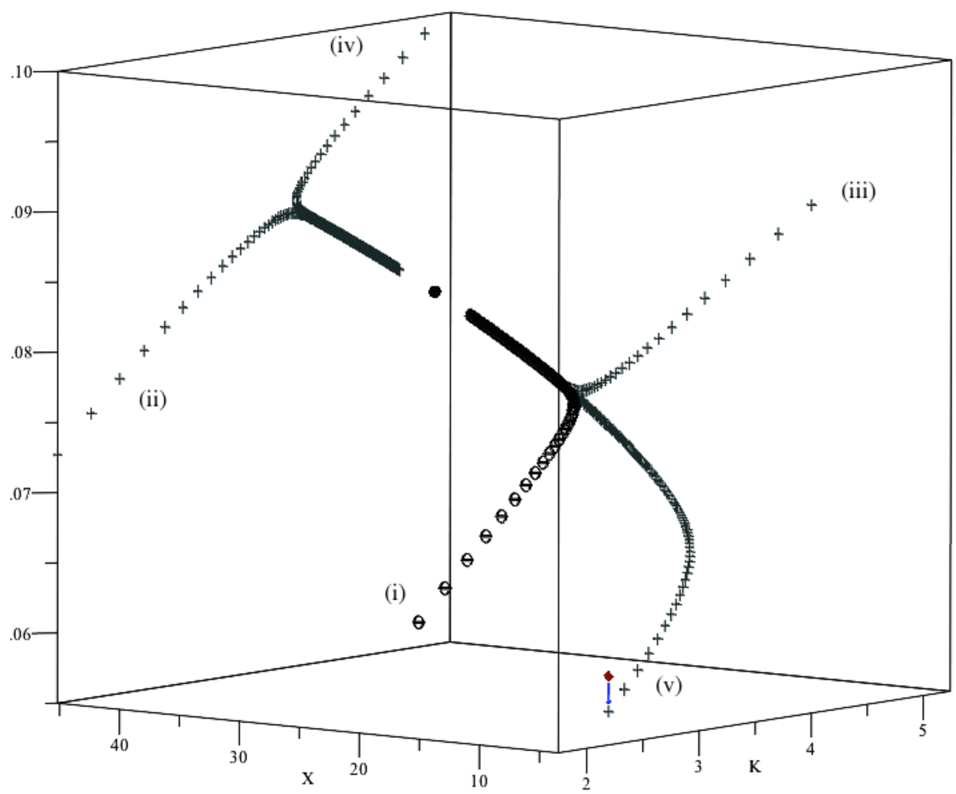

FIGURE 11. Stable paths

sustainable. Thus, a necessary condition for sustainability is that the initial values of X and K are smaller than the corresponding steady-states. But there are trajectories with a small initial value of K that is not sustainable, so the condition is not sufficient. We illustrate the point in Figure 12: there, we see that for some values of K below the steady-state level $\bar{K}$ felicity decreases for a period (though it ultimately would increase as we moved closer to the steady-state).

4.3. **Policy Simulation.** In this sub-section, we provide a numerical simulation to illustrate a potential policy-relevant application of our analysis. This simulation investigates the long-run effects of a moratorium on renewable natural capital expansion. The obvious benchmark against which this simulation should be compared is scenario (v) in Figure 11. In the benchmark, it takes 69 years to be within 5% of the steady-value of capital $\bar{K}$. Hence, at

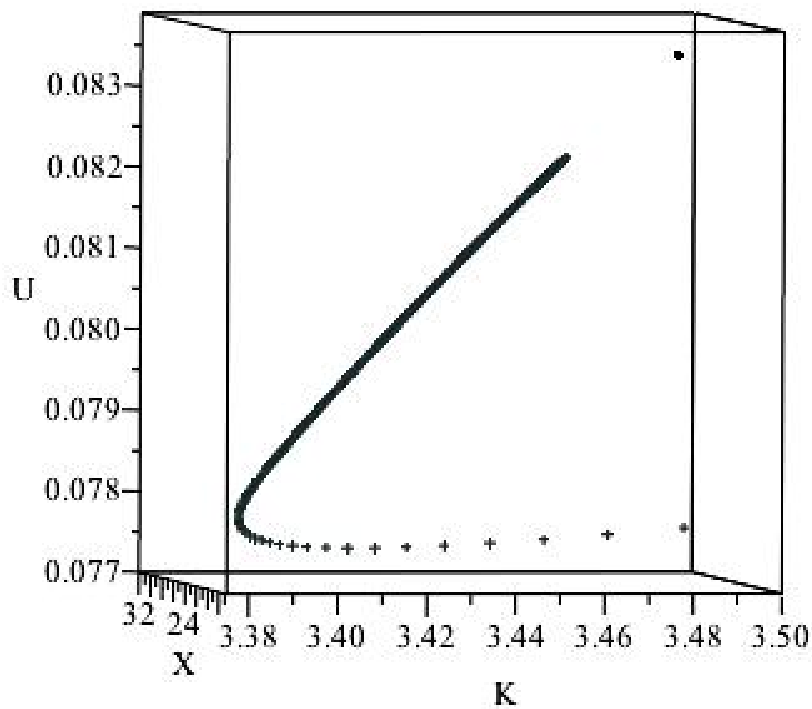

FIGURE 12. An unsustainable path starting from a small level of capital

$t = 69$,

$$K = 3.32, \quad X = 12.66, \quad U = 0.07.$$

Imagine that for ten years, society reinvests just enough energy s^X to maintain the initial stock of X. This could be interpreted, for instance, as a moratorium put in place by a newly elected government, perhaps as a result of a decision to favor conventional energy use. The benefits of this policy are short-run increases in the levels of man-made capital and welfare; for example, the level of K reached after 24 years in the benchmark scenario is reached after only 10 years under the moratorium scenario. Welfare is higher under the moratorium than in the benchmark scenario for the first 10 years. As the moratorium ends, society resumes the expansion of X by increasing s^X. This has the effect of lowering s^Y and lowers output generation Y, which in turn *lowers* consumption and felicity. But this result is inconsistent with our concept of sustainability. To reach the values of K, X and U found at the 5% threshold of $\bar{K}$, it takes an overall of 79 years in the moratorium case, as compared to 10 years in the benchmark. The long-run effect of the moratorium is clear: A 10 year moratorium delays the time it takes to reach $\bar{K}$ by 10 years. Because society derives welfare from renewable natural capital – it is a valuable asset to society – it is better for society to start investing in it straightaway, as opposed to delaying for a period of time.

4.4. **Imperfect substitution between energy inputs.** The model we employed in our numerical simulations assumed perfect substitution between renewable and non-renewable resources in energy generation. In practice, renewable energies are subject to intermittency and storage constraints which can inhibit substitution. That said, one would expect that it is possible to produce some energy even if $q = 0$.[26] For it to be possible to generate energy if society no longer relies on non-renewable resources, *i.e.* $E(q = 0, h) > 0$, one of two scenarios would obtain: Either non-renewable capital would be exhausted in finite time, in which case society would reach a post non-renewable resource phase similar to the one described in Section 4.2; or society would have to manage its use of both types of natural capital so as to asymptotically converge to the post non-renewable resource steady-state. In the second situation, non-renewable resources would be gradually phased out in favor of their renewable alternative. Thus, the steady-state values of physical and renewable natural capital, consumption and energy shares correspond to post non-renewable regime we discussed above, whether society switches to renewable energies in finite time or infinite time (Vardar, 2013).

[26]Note that $E(q, h) = 0$ at $q = 0$ would imply the elasticity of substitution between renewable and non-renewable resources could not equal unity; in turn, this would preclude the use of a Cobb-Douglas function to model energy generation.

5. Concluding remarks

In this paper, we investigated the role of energy in sustainability in a model where non-renewable energy contributes to a pollution stock, and where the use of renewable energy requires the endogenous development of a stock of renewable capital. Societal well-being, or "felicity," is increasing in both consumption and the remaining renewable resources, but is decreasing in the level of pollution: people like to consume and enjoy the environment for all the different kinds of ecological services it supplies, but dislike pollution. Felicity is also assumed to be non-separable in consumption and the remaining renewable resources. Renewable and non-renewable resources are used to generate energy, which can be used to increase the regeneration process of the renewable natural capital, to abate pollution that comes with the use of non-renewable energies or to increase output. Interpreting sustainability as the restriction that felicity never decrease as time goes by, we find that consumption and the stock of renewable natural capital grow faster than the stock of pollution, each weighted by their respective marginal effect on utility. As we observed in section 4.2, a necessary condition for sustainability is that the initial levels of X and K are smaller than their respective steady-state values. Any trajectory that requires a reduction in one of the state variables is unsustainable, as is any scenario that requires society accept a reduction in output, as when it ends a moratorium on renewable capital accumulation.

We illustrate some important properties of the model by use of a numerical simulation. In the example we analyze, it is optimal to completely switch from non-renewable to renewable resources in energy production. This transformation allows energy to be directed away from abatement, which facilitates a perpetual increase in consumption, thereby allowing continual growth in felicity. In the post non-renewables phase, we find that the system dynamics are saddle-path stable in a neighborhood of the steady-state. We also find that the steady-state level of renewable natural capital depends on the society's elasticity of inter-temporal substitution in consumption. We also note that it takes a relatively long time to get close to the renewable natural capital steady-state in our simulation, suggesting that society must take a long view of the resource allocation problem. Finally, we observe that a moratorium on investment in renewable resource capital accumulation is unsustainable, as there will be a reduction in output – followed by a decrease in consumption and felicity – when the moratorium is removed.

The existing economics literature on sustainability and energy use typically assumes that energy from renewable natural capital can be bought at a fixed price, with no other constraint. This assumption is quite strong, particularly from the social planner's standpoint, as it ignores the dynamic aspects of renewable natural capital and its accumulation. It also commonly leads to an empirical prediction that is at odds with reality: that society will use the non-renewable energy resource exclusively up to a certain time, at which point it will switch to using renewable energy exclusively. To this empirical awkwardness, some authors have adapted their modeling framework, for example by having the incremental cost of renewables rise with its use, but again that is inconsistent with reality. An alternative approach, which in our view is the most natural way to extend the analysis, is to impose limits on the magnitude of renewable energy use in accordance with a capacity constraint. This constraint arises from a capital stock that facilitates the exploitation of renewable energy. With this interpretation, for society to reap the benefits from the renewable resource it has to continually invest: While delaying investment yields a short increase in welfare, this increase dissipates quickly. In addition to producing a more empirically satisfactory explanation for simultaneous use of non-renewable and renewable energy, we also find that it can be optimal for society to cease use of non-renewable energy, switching to the exclusive use of renewable energy, even though the non-renewable resource stock is not fully exhausted.[27]

Important policy implications emerge from this framework. Although we did not discuss the manner in which society determines the allocation of resource bases to energy production, we can envision a number of approaches. Society could impose standards that require a certain level of usage of renewables, as with Renewable Performance Standards – popular

[27]This calls to mind the quote from Sheik Yamani, former oil minister for OPEC: "[t]he stone age did not end for lack of rocks, and the oil age will not end for lack of oil."

in some US states. Or society could adopt a tradable permit scheme for carbon emissions, as with the EU's emission trading system. A third option is to invoke a carbon tax – as in the Canadian province of British Columbia – which raises the cost of (non-renewable) fossil fuels. The presence of a capacity constraint on the use of renewables would suggest that none of these policies can induce increased renewable production in the moment (as the capacity constraint would preclude such expansion), though it seems likely to encourage increased investment in renewable capacity – and perhaps research into new innovations (Acemoglu et al., 2012)). In this regard, the financial rewards associated with avoiding the the carbon tax would seem pose an attractive incentive in a decentralized economy, such as that found in most Western countries. In addition, we find that a short period in which renewable energy development is put on hold – as one might envision occurring under U.S. President Trump – can cause a dramatic delay in the time it takes to reach long-run equilibrium.

Throughout the paper, we have assumed that felicity is not separable: additional felicity from marginal consumption is directly affected by the stocks of renewable natural capital and pollution. The more society cares about renewable natural capital, the higher is its steady-state value.[28] Moreover, for a non-zero steady-state level of renewable natural capital to exist, society must be sufficiently willing to trade today's consumption for tomorrow's consumption; this can be interpreted as the need for society to be averse to intergenerational inequity. This is because society can only increase the stock of renewable natural capital by reducing energy used in current output production, which lowers both output and consumption today. The non-linear relationship between consumption, pollution and the renewable resource stock imply that none can be analyzed independently, their evolution through time and their long-run values are linked. The time when society starts using renewable energy is determined by the relative magnitude of the shadow price of pollution and the shadow value of each resource stock. The evolution of each of these components is dictated by society's preferences and the initial levels pollution as well as man-made, non-renewable and renewable capital.

[28] This is a consequence of a increase in parameter $\tilde{b}$ in the utility function.

Appendix A. Sustainabilty With Physical Capital Only

In this appendix, we provide the technical details for the model in subsection 3.1, where only physical capital matters. The optimization problem here is to:

$$\max_{C} W = \int_0^\infty e^{-\tilde{r}t} U(C) dt$$

$$\text{s.t. } \dot{K} = -\tilde{\delta} K + F(K) - C$$

$$0 \leq C \leq F(K), \ \ K(0) \text{ given.}$$

Because all output will be consumed or invested, only one variable need be analyzed; in this section, we focus on the optimal time path of consumption. We now omit the time subscript for clarity. The current-value Hamiltonian associated with the problem is[29]

$$\mathscr{H} = U\big(C\big) + \lambda\big[F(K) - C - \tilde{\delta}K\big], \tag{A.1}$$

where λ is the co-state variable, or "shadow value," associated with physical capital. To enhance clarity of the derivations we omit the each function's argument. The solution to the optimization problem satisfies Pontryagin's maximum principle:

$$0 = \frac{\partial \mathscr{H}}{\partial C} = U_C - \lambda \Leftrightarrow$$

$$U_C = \lambda \tag{A.2}$$

$$\dot{\lambda} = \tilde{r}\lambda - \frac{\partial \mathscr{H}}{\partial K} \Leftrightarrow$$

$$\dot{\lambda} = \big[\tilde{r} + \tilde{\delta} - F_K\big]\lambda. \tag{A.3}$$

[29]One could append Lagrangian multipliers for the non-negativity constraint on s_K, and the implicit non-negativity constraint on C; we ignore these appendages in the interest of expositional clarity. Nevertheless, as we shall see the optimal solution generally honors these constraints, so nothing of consequence is lost by their omission from the discussion.

In this simple setting, sustainability corresponds to the restriction that felicity never declines; in turn, this requires consumption never falls. To better understand this constraint, we differentiate both sides of eq. (A.2)

$$
\begin{aligned}
U_{CC}\dot{C} &= \dot{\lambda} \\
&= \big[\tilde{r} + \tilde{\delta} - F_K\big]\lambda \\
&= \big[\tilde{r} + \tilde{\delta} - F_K\big]U_C,
\end{aligned}
$$

which implies

$$\dot{C} = -\frac{U_C}{U_{CC}}\big[F_K - \tilde{r} - \tilde{\delta}\big]. \tag{A.4}$$

Evidently, consumption increases only so long as $F_K \leq \tilde{r} + \tilde{\delta}$, *i.e.* $K_0 < \bar{K}$, where $\bar{K}$ is defined in eq. (3) in the text. One can also think of $\bar{K}$ as the long run equilibrium level of physical capital. For society to be able to reach that state with an increasing level of consumption, the initial level of capital must be smaller than $\bar{K}$. In this simple setting, it can be shown that in the first best outcome, society will adopt a strategy which will lead directly and smoothly toward $\bar{K}$ over time. Slight deviations from that strategy would inevitably lead to an undesirable outcome, with either consumption or capital reaching zero.[30]

[30] The interested reader can find a thorough analysis of this simple model in Kamien and Schwartz Kamien and Schwartz (1991)

Appendix B. Sustainability With Energy Input and Natural Capital

In this appendix, we provide the technical details for the full model (from subsection 3.2). The discount rate $\tilde{r}$ and depreciation rate $\tilde{\delta}$ are exogenous parameters. Variable s^i denotes the share of energy allocated to end-use $i \in \{Y, X, A\}$, . Similarly, the energy quantity allocated to end-use i is $e^i \equiv s^i \cdot E(q, h)$. The social planner's problem is to:

$$\max_{C,q,h,s^Y,s^A} W = \int_0^\infty e^{-\tilde{r}t} U(C, X, P) dt$$

$$\begin{aligned} \text{s.t. } \dot{K} =& F(K, e^Y) - \tilde{\delta} K - C \\ \dot{X} =& G(X, e^X) - h \\ \dot{S} =& -q \\ \dot{P} =& J(q) - A(e^A) \end{aligned}$$

$$K(0) = K_0, \;\; P(0) = P_0, \;\; S(0) = S_0, \;\; X(0) = X_0$$

$$s^X = 1 - s^Y - s^A \quad s^Y, s^A \in (0, 1).$$

Let λ, μ, ψ and ϕ be the costate variables associated with K, X, S and P respectively. The associated current-value Hamiltonian is

$$\begin{aligned} \mathscr{H} =& U(C, X, P) + \lambda\big[F(K, s^Y E(q, h)) - \tilde{\delta} K - C)\big] \\ &+ \mu\big[G\big(X, [1 - s^Y - s^A]E(q, h)\big) - h\big] \\ &+ \psi\big(-q\big) + \phi\big[J(q) - A\big(s^A E(q, h)\big)\big] \end{aligned} \tag{B.1}$$

We again omit each function's argument. The optimality conditions governing the control variables are

$$\frac{\partial \mathscr{H}}{\partial C} = U_C - \lambda = 0; \tag{B.2}$$

$$\frac{\partial \mathscr{H}}{\partial q} = \lambda[F_{e^Y} s^Y E_q] + \phi[J_q - A_{e^A} s^A E_q] - \psi + \mu[G_{e^X}[1 - s^Y - s^A]E_q] = 0; \tag{B.3}$$

$$\frac{\partial \mathscr{H}}{\partial h} = \lambda[F_{e^Y} s^Y E_h] + \phi[-A_{e^A} s^A E_h] + \mu[G_{e^X}[1 - s^Y - s^A]E_h - 1] = 0; \tag{B.4}$$

$$\frac{\partial \mathscr{H}}{\partial s^Y} = \lambda[F_{e^Y} E] + \mu[-G_{e^X} E] = 0; \tag{B.5}$$

$$\frac{\partial \mathscr{H}}{\partial s^A} = \phi[-A_{e^A} E] + \mu[-G_{e^X} E] = 0; \tag{B.6}$$

In addition, there are equations of motion governing the four co-state variables:

$$\dot{\lambda} = \lambda[\tilde{r} + \tilde{\delta} - F_K], \tag{B.7}$$

$$\dot{\mu} = \mu[\tilde{r} - G_X] - U_X. \tag{B.8}$$

$$\dot{\psi} = \psi \tilde{r}, \tag{B.9}$$

$$\dot{\phi} = \phi \tilde{r} - U_P, \tag{B.10}$$

By substituting (B.5) and (B.6) into (B.4), we get

$$\mu G_{e^X} E_h = \mu,$$

which for a non-zero μ is equal to

$$G_{e^X} E_h = 1, \tag{B.11}$$

and when they are substituted into (B.3), we have:

$$\psi = \mu \left[\frac{E_q}{E_h} - \frac{G_{e^X}}{A_{e^A}} J_q \right]. \tag{B.12}$$

Appendix C. The System of differential equations

Using the optimality condition for C, we have that

$$U_C = \lambda.$$

By logarithmic time differentiation, we have:

$$\frac{\dot{U_C}}{U_C} = \frac{\dot{\lambda}}{\lambda}.$$

Eq. (B.7), the dynamic efficiency condition for K, then implies

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{U_C}}{U_C} = \tilde{r} + \tilde{\delta} - F_K.$$

The first differential equation of the system is then

$$\frac{U_{CC}}{U_C}\dot{C} + \frac{U_{CX}}{U_C}\dot{X} + \frac{U_{CP}}{U_C}\dot{P} = \tilde{r} + \tilde{\delta} - F_K. \tag{C.1}$$

We define $\alpha = \frac{E_q}{E_h}$, $\beta = -\frac{G_{eX}}{A_{eA}}$ and $\nu = \frac{G_{eX}}{F_{eY}}$. From eq. (B.5), we have $\lambda = \frac{G_{eX}}{F_{eY}}\mu$, so $\lambda = \nu\mu$. Using logarithmic time differentiation, we have that:

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\nu}}{\nu} + \frac{\dot{\mu}}{\mu}.$$

Since $\lambda = U_C$, we can express $\frac{\dot{\mu}}{\mu}$ as a function of marginal utility from consumption:

$$\frac{\dot{\mu}}{\mu} = \frac{\dot{U_C}}{U_C} - \frac{\dot{\nu}}{\nu},$$

with $\frac{\dot{\lambda}}{\lambda} = \frac{\dot{U_C}}{U_C} = \tilde{r} + \tilde{\delta} - F_K$. Then, using eq. (B.8), we get the second differential equation of our system:

$$\frac{\dot{\nu}}{\nu} - \nu\frac{U_X}{U_C} = \tilde{\delta} + G_X - F_K \tag{C.2}$$

From (B.12), $\psi = \mu\left[\frac{E_q}{E_h} - \frac{G_{eX}}{A_{eA}}J_q\right]$. Defining $\omega = \alpha + \beta J_q = \frac{E_q}{E_h} - \frac{G_{eX}}{A_{eA}}J_q$,

$$\psi = \omega\mu$$
$$\Rightarrow \frac{\dot{\psi}}{\psi} = \frac{\dot{\omega}}{\omega} + \frac{\dot{\mu}}{\mu}$$

By equation (B.9), the rate of change in the shadow value of non-renewable natural capital equals the discount rate $\tilde{r}$, hence $\frac{\dot{\psi}}{\psi} = \tilde{r}$. Hence we have

$$\frac{\dot{\omega}}{\omega} = \tilde{r} - \frac{\dot{\mu}}{\mu}, \tag{C.3}$$

consequently, we get equation

$$\frac{\dot{\nu}}{\nu} - \frac{\dot{\omega}}{\omega} = \tilde{\delta} - F_K, \tag{C.4}$$

which is the third differential equation of the system.

Using logarithmic time differentiation on equation (B.6) we have

$$\frac{\dot{\phi}}{\phi} = \frac{\dot{\beta}}{\beta} + \frac{\dot{\mu}}{\mu}.$$

Combining this with (B.10), we get

$$\frac{\dot{\phi}}{\phi} = \frac{\dot{\beta}}{\beta} + \frac{\dot{\mu}}{\mu} = \tilde{r} - \frac{U_P}{\phi}$$

Since ϕ is also equal to $\frac{\beta}{\nu}U_C$, then have

$$\frac{\dot{\nu}}{\nu} - \frac{\dot{\beta}}{\beta} - \frac{\nu}{\beta}\frac{U_P}{U_C} = \tilde{\delta} - F_K, \tag{C.5}$$

the fourth differential equation of our system.

Equation (B.11) also tells us that $G_{e^X} E_h = 1$. This serves as an algebraic constraint in what is a differential algebraic system. Taking the time derivative of this equation will give a condition on starting values of the controls in the initial-value problem

$$\begin{aligned} \Rightarrow \frac{d}{dt} G_{e^X} E_h =& 0 \\ \Rightarrow E_h [G_{e^X X} \dot{X} + G_{e^X e^X} \dot{e^X}] =& - G_{e^X} \dot{E}_h. \end{aligned}$$

Expanding this leads to

$$\dot{s}^X E + \frac{G_{e^X X}}{G_{e^X e^X}} \dot{X} + \dot{q} \left[\frac{G_{e^X}}{G_{e^X e^X}} \frac{E_{hq}}{E_h} + s^X E_q \right] + \dot{h} \left[\frac{G_{e^X}}{G_{e^X e^X}} \frac{E_{hh}}{E_h} + s^X E_h \right] = 0$$

If, as in the numerical simulation, we assume that clean and dirty energy are perfect substitute in energy generation and they both enter linearly in E, then $E_q = E_h = 1$ and $E_{qq} = E_{hh} = E_{hq} = 0$, the relationship above simplifies to

$$\dot{s}^X E(q,h) + \frac{G_{e^X X}}{G_{e^X e^X}} \dot{X} + s^X [\dot{q} + \dot{h}] = 0.$$

Appendix D. The Linearized System

The linearized system can be expressed in the form

$$\mathbf{B}\dot{\mathbf{\Theta}} = \mathbf{A}\mathbf{\Theta},$$

in which A, B are 5×5 matrices of coefficients and $\mathbf{\Theta} = [\mathbf{C}, \mathbf{K}, \mathbf{X}, \mathbf{h}, \mathbf{s^Y}]'$, a 5×1 vector. Suppose $\mathbf{B}$ is invertible, the solution to the linearized system of differential equation is

$$\dot{\mathbf{\Theta}} = \mathbf{B^{-1}}\mathbf{A}\mathbf{\Theta};$$

we rename $\mathbf{B^{-1}A} = \mathbf{\Omega}$. The behavior of the system around the steady-state is dictated by the type and signs of the eigenvalues of matrix $\mathbf{\Omega}$. In our case, given the parameter values we chose, these were all real-valued, but 2 were greater than zero. This implies an explosive behavior unless the states and controls are exactly on a stable arm leading to the steady-state. The solution takes the form of a sum of exponentials, with the arbitrary constants set equal to zero for the terms associated with the 2 positive eigenvalues. The 3 other constants are calibrated given initial values of the state variables X and K.

References

Acemoglu, D., Aghion, P., Bursztyn, L. and Hemous, D. (2012). The environment and directed technical change, *American Economic Review* **102(1)**: 131–166.

Energy Information Administration (2015). Annual energy outlook 2015, *Technical Report EIA-0383*, U.S. Energy Information Administration, Washington, DC.
URL: *http://www.eia.gov/forecasts/aeo/pdf/0383(2015).pdf*

Forster, B. A. (1980). Optimal energy use in a polluted environment, *Journal of Environmental Economics and Management* **7(4)**: 321–333.

Hansen, L. P. and Singleton, K. J. (1982). Generalized instrumental variables estimation of nonlinear rational expectations models, *Econometrica* **50(5)**: 1269–1286.

Hartwick, J. (1977). Intergenerational equity and the investing of rents from exhaustible resources, *American Economic Review* **67(5)**: 972–974.

Heyes, A., Leach, A. and Mason, C. F. (2017). The economics of the Canadian oil sands, *Review of Environmental Economics and Policy* . in press.

Jevons, W. S. (1865). *The Coal Question: An Inquiry Concerning the Progress of the Nation and the Probable Exhaustion of Our Coal Mines*, MacMillan, London.

Jordan, D. C. and Kurtz, S. R. (2013). Photovoltaic degradation rates: An analytical review, *Progress in Photovoltaics: Research and Applications* **21(1)**: 19–29.

Jouvet, P.-A. and Schumacher, I. (2012). Learning-by-doing and the costs of a backstop for energy transition and sustainability, *Ecological Economics* **73**: 122–132.

Kamien, M. and Schwartz, N. (1991). *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*, North Holland.

Knapp, K. E. and Jester, T. L. (2001). An empirical perspective on the energy payback time for photovoltaic modules, *Solar Energy* **71(3)**: 165–172.

Michel, P. and Rotillon, G. (1995). Disutility of pollution and endogenous growth, *Environmental and Resource Economics* **6(3)**: 279–300.

Moe, T., Alfsen, K. and Greaker, M. (2013). Sustaining welfare for future generations: A review note on the capital approach to the measurement of sustainable development, *Challenges in Sustainability* **1(1)**: 16–26.

National Renewable Energy Laboratory (2004). What is the energy payback for PV?, *Technical Report GO-102004-1847*, National Renewable Energy Laboratory, Washington, DC.
URL: *www.nrel.gov/docs/fy04osti/35489.pdf*

Nordhaus, W. D. (2007). Critical assumptions in the stern review on climate change, *Science* **317**: 201–202.

Pezzey, J. C. and Toman, M. (2002). The economics of sustainability: A review of journal articles, *Technical report*, Resources for the Future. Discussion Paper 02-03.

Solow, R. M. (1991). *Sustainability: an Economist's Perspective*, J. Seward Johnson lecture in marine policy, Marine Policy Center.

Staffel, I. and Green, R. (2014). How does wind farm performance decline with age?, *Renewable Energy* **66**: 775–786.

Stern, N. (2007). *The Economics of Climate Change: The Stern Review*, Cambridge University Press.

Tahvonen, O. (1997). Fossil fuels, stock externalities, and backstop technology, *Canadian Journal of Economics* **30(4a)**: 855–874.

Tahvonen, O. and Sallo, S. (2001). Economic growth and transitions between renewable and nonrenewable energy resources, *European Economic Review* **45(8)**: 1379–1398.

van der Ploeg, F. and Withagen, C. (2014). Growth, renewables and the optimal carbon tax, *International Economic Review* **55(1)**: 283–311.

Vardar, N. B. (2013). Imperfect resource substitution and optimal transition to clean technologies, *Technical report*, Université Catholique de Louvain. CORE discussion paper 2013/72.

Withagen, C. (1994). Pollution and exhaustibility of fossil fuels, *Resource and Energy Economics* **16**: 235–242.

Xepapadeas, A. (2005). Economic growth and the environment, *in* K. G. Mäler and J. R. Vincent (eds), *Handbook of Environmental Economics*, Vol. 3 of *Handbook of Environmental Economics*, Elsevier, chapter 23, pp. 1219–1271.

Zilberman, D., Barrows, G., Hochman, G. and Rajagopal, D. (2013). On the indirect effect of biofuel, *American Journal of Agricultural Economics* **95**: 1332–1337.