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Social Interactions, Mechanisms, and Equilibrium:
Evidence from a Model of Study Time and Academic Achievement

Abstract

We develop and estimate a model of student study time choices on a social network. The model is designed to exploit unique data collected in the Berea Panel Study. Study time data allow us to quantify an intuitive mechanism for academic social interactions: own study time may depend on friend study time in a heterogeneous manner. Social network data allow us to embed study time and resulting academic achievement in an estimable equilibrium framework. We develop a specification test that exploits the equilibrium nature of social interactions and use it to show that novel study propensity measures mitigate econometric endogeneity concerns.

JEL-Codes: C520, C540, I200.

Keywords: social networks, peer effects, homophily, time-use.

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1 Introduction

A large body of research has recognized the importance of understanding the mechanisms through which peer effects arise and how the impacts of these mechanisms depend on one's social network. In this paper we exploit unique data from the Berea Panel Study (BPS) to provide an improved understanding of these issues in a higher-education setting. Motivated by research hypothesizing that student effort is likely to be an input that is readily influenced by peers in the short run (Stinebrickner and Stinebrickner, 2006; Calvó-Armengol et al., 2009; Cooley Fruehwirth, 2013; De Giorgi and Pellizzari, 2014), we focus on study time as an explicit mechanism through which peer effects could arise in college. To explicitly account for the role of social networks we estimate an equilibrium model of study time choice and resulting grade determination on a given social network. We focus on what is likely the most relevant set of peers in our context, a student's friends.

The BPS was designed to address the substantial data challenges that arise when trying to improve our understanding of peer effects in higher education. One prominent challenge is that, because equilibrium outcomes depend on the entire social network, it is necessary to characterize the set of peer connections. This is possible with the BPS because the design involved surveying full cohorts of students and included questions characterizing friendships. A second prominent challenge is that it is necessary to have access to student-level data on study time. While collecting reliable time-use information is very difficult in annual surveys (and, therefore, such information is typically not available), the BPS took advantage of its high frequency of contact with respondents to collect eight time-diaries each year. To stress the unique nature of the BPS data we note that, among existing sources of social network data, perhaps only one, the National Longitudinal Survey of Adolescent Health (Add-Health), could potentially provide a full view of a social network in an educational setting where academic outcomes and student characteristics are also observed. However, because the Add-Health dataset has a primary focus on adolescent health and risk-related behaviors, it does not contain information about time spent studying. Thus, to the best of our knowledge, there is no other data source that is able to both fully characterize a social network of students and provide direct evidence about a central input in the grade production function that has been hypothesized to generate social interactions.

We develop our model to exploit the unique BPS data. The social network is known at the beginning of a model time period. Subsequently, all students in the social network simultaneously choose their study time to maximize their own achievement, net of studying costs. Achievement depends on a student's own study time, and a student's studying cost depends on her own study time and friend study time, e.g., students may conform to their friends, or it may be more fun to study if your friends are studying. Cost functions are allowed to be heterogeneous across students. This specification allows us to provide some of the first empirical evidence about the importance of heterogeneity in how responsive students are to their friends' study time choices, which previous research suggests matters (see, e.g., Hoxby and Weingarth, 2005; Epple and Romano, 2011).

The social interactions literature has paid close attention to the endogeneity problem that is present if there exist correlated unobserved variables, that is, unobserved information related to both peer group membership (in our context, friendship links) and outcomes of interest (Manski, 1993; Moffitt, 2001; Epple and Romano, 2011). In our case, where we focus on a social interaction in study time choices, a relationship between friends' study times could arise because friends influence each other ("endogenous social interactions", or "peer effects") or because students with similar unobserved determinants of study time become friends ("correlated unobserved variables"). Institutional details, together with empirical checks we conduct, suggest that correlated shocks arising through some typically considered channels, e.g., coursework and dormitories, are not the most salient type of correlated unobserved variables. The most relevant type of correlated unobserved variable would seem to be an unobserved individual characteristic, which could be thought of as a student's propensity to study.

We adopt a novel two-step approach for dealing with this endogeneity problem. First, we take advantage of the fact that our role as BPS administrators afforded us with a unique opportunity to directly measure the typically unobserved variable of relevance—students' propensities to study. Specifically, the day before freshman classes began, we collected information about how much a student actually studied in high school and how much the student expected to study in college. We find that both high school study time and expected college study time have strong correlations with study time in college and are also strongly related to friendship patterns in our data.

Our second step is to develop a specification test that can detect endogeneity problems caused by unobserved determinants of study time. We exploit the fact that the equilibrium nature of social interactions in our model implies that such unobserved determinants would generate cross-sectional dependence in residuals.[1] We use a measure of cross-sectional correlation in residuals as our test statistic. Crucially, we demonstrate that our test can have power to detect cross-sectional correlation in residuals caused by unobserved determinants even if they produce an endogeneity bias that leads to inconsistently estimated model

[1] Individual characteristics (observed or unobserved) typically influence equilibrium choices of many (if not all) others in a large class of social interactions models (see, e.g., Calvó-Armengol et al., 2009; Blume et al., 2015). A very similar situation occurs in spatial autoregressive econometric models (see, e.g., Pinkse et al., 2002; Lee, 2004).

parameters.

We estimate the model using data from students' freshman year. The study of the freshman year is appealing because it allows us to characterize the network as completely as possible—freshmen survey response rates were close to 90 percent and most friends of freshmen are also freshmen—and because institutional details of the school imply that course difficulty, number of classes taken, and hours of paid employment tend to be quite similar for freshmen.

Under the baseline specification, in which we use our study propensity data to estimate the model, we find no evidence of the cross-sectional residual correlation described above. However, we do find significant cross-sectional residual correlations when we re-estimate the model excluding our study propensity data, i.e., using only measures of student characteristics that are typically available to researchers. This shows that our specification test has the power to detect unobserved determinants of study time. Therefore, these findings provide evidence that our study propensity measures play an important role in addressing endogeneity concerns.

Our estimates provide strong evidence that friend study time has a substantial effect on one's own study time. We also find that one's own study time is an important determinant of one's own achievement. We estimate students to have different best response functions, i.e., they react differently to changes in friend study time. Hereafter, we will often refer to this as heterogeneity in *reactiveness*. This heterogeneity has equilibrium implications: the extent to which heterogeneity in reactiveness affects total achievement depends on the relationship between own and friend reactiveness, among all friendship links. Therefore, it is important to take into account the joint distribution of friendship links and student characteristics to understand social interactions.[2]

We use our estimated model to perform two counterfactual exercises. First, we examine how the network structure, combined with the homophilous sorting into friendships observed in the data, affects the response to changes in friend study time. As Golub and Jackson (2012) note, despite a large amount of work documenting the existence of homophily and a smaller literature examining its origins, the literature modeling the effect of homophily is in its infancy.[3] We exogenously increase (shock) the study time of each student and

[2]Kline and Tamer (2011) discuss the importance of distinguishing between estimates of technological parameters and the equilibrium effects of social interactions. Carrell et al. (2013) discuss the differences between reduced-form estimates of social interactions estimated using experimental variation of administrative units and those resulting from the interactions of students given a social network. Richards-Shubik (2015) separates supply and demand mechanisms in a model of sexual initiation.

[3]Jackson (2008) provides a discussion of work documenting the existence of homophily; see Camargo et al. (2010) for a specific example. For theoretical models of homophily's origins see Currarini et al. (2009), Currarini et al. (2010), and Bramoullé et al. (2012). Badev (2021) allows for homophily in his empirical

assess how study times and achievement change for other students in the social network. There is substantial heterogeneity in study time responses depending on which student is shocked, with larger impacts associated with more central students and students connected to more reactive peers. The specific manner in which students with different characteristics are arranged on the network is important for responses. This exercise also provides a natural framework for quantifying the importance of equilibrium interactions.

In our second counterfactual, we examine how achievement would differ if friend characteristics were identically distributed across students, instead of being strongly correlated with one's own characteristics, or homophilous, as in the data. On average, women, black students, and students with above-median high school GPAs have high propensities to study and tend, in the data, to sort into friendships with students similar to themselves. Therefore, these groups tend to see declines in their friends' propensities to study in the counterfactual. However, these groups' losses are not offset by the gains of their complements. Intuitively, the estimated heterogeneity in best response functions means that total study time (and hence, achievement) is highest when students with high propensities to study are friends with others with high propensities to study, as is on average the case in the data. In contrast, there is a lack of such assortative matching in the counterfactual networks.

The remainder of this paper is organized as follows. Section 2 discusses related literature. Section 3 contains a description of the BPS data. Section 4 presents our model. Section 5 presents our empirical specification. Section 6 discusses our specification test and Section 7 presents estimation results. Section 8 presents the results from our counterfactual exercises and Section 9 concludes.

2 Related Literature

There is an extensive literature on academic peer effects, which has been surveyed by Epple and Romano (2011) and Sacerdote (2011). As discussed in Sacerdote (2011), papers in this literature typically do not directly examine mechanisms through which peer effects are generated. Cooley Fruehwirth (2013), Calvó-Armengol et al. (2009), De Giorgi and Pellizzari (2014), and Tincani (2018) all stress the importance of equilibrium models of students' effort choices, but lack direct data on student effort. Cooley Fruehwirth (2013) and Calvó-Armengol et al. (2009) estimate parameters of their respective models, identifying effort through residual variation in peer outcomes. De Giorgi and Pellizzari (2014) and Tincani (2018) test the implications of different theoretical models of social interactions using student achievement data.

study of friendship formation and smoking behavior.

Much of our contribution to the literature stems from access to unique data on an input, study effort, that, as discussed earlier, is not available in the other data that contain social network information. Our work is perhaps most complementary to that of Calvó-Armengol et al. (2009), who also seek to contribute to the understanding of network-mediated peer effects in an education setting, taking a social network as given. However, the papers have different methodological and substantive contributions. Given that the Add-Health data used by Calvó-Armengol et al. (2009) do not contain study effort information, they derive a theoretical result: in an environment with an input subject to endogenous, network-mediated social interactions, where best response functions have a common slope and are linear in the input choices of first-order neighbors, students' equilibrium choices of inputs are proportional to Katz-Bonacich centrality, a commonly used statistical measure of network topology. This result allows them to empirically relate network structure to achievement, without direct data on the latent social-interaction input.

In contrast, having direct data on study effort allows our paper to provide direct evidence about the relationship between this input and achievement and also allows us to relax other assumptions that are needed to obtain the theoretical result of Calvó-Armengol et al. (2009). For example, we can directly estimate heterogeneous best response functions and can also allow for nonlinearity in best response functions, which breaks the connection between network topology and equilibrium outcomes required in Calvó-Armengol et al. (2009). We find heterogeneity across students in reactiveness to be important in our application, and we estimate that the variation in the characteristics of students and their specific arrangement in the network matter a great deal for outcomes. We are also able to examine the relevance of production complementarities in generating endogenous social interactions, as well as the role of contextual effects. Our work is the first to include all of these features in a common empirical framework.

Our unique study propensity data also distinguish our paper from Calvó-Armengol et al. (2009), with respect to how we deal with a standard endogeneity concern that has received much attention in the peer effects literature. Due to data limitations, it is not possible for Calvó-Armengol et al. (2009) to distinguish between cross-student outcome correlations due to actual peer effects (e.g., endogenous social interactions) and cross-student outcome correlations due to sorting based on unobserved determinants of inputs (e.g., unobserved propensities to study). In contrast, our propensity to study measures provide a natural way to potentially address the endogeneity concern directly, with our desire to gauge the success of these measures serving as a motivation for our specification test.

Finally, there is a separate, but related, literature that has focused on modeling the formation of social networks, an important and notoriously difficult problem (see Christakis et al., 2010; Badev, 2013; Hsieh and Lee, 2016; Mele, 2017; Paula et al., 2018; Sheng, 2020; Badev, 2021).[4] Because we do not model how friendships are formed, in our counterfactuals we examine fully-specified networks of interest. In particular, we consider the observed networks and randomly generated networks.

3 Data

The BPS is a longitudinal survey that was designed by Todd Stinebrickner and Ralph Stinebrickner to provide detailed information about educational outcomes in college and labor market outcomes in the early post-college period. The BPS survey design involved collecting information about all students who entered Berea College in the fall of 2000 and the fall of 2001. Baseline surveys were conducted immediately before the start of first-year classes and students were subsequently surveyed 10-12 times each year during school. As has been discussed in previous work that uses the BPS, caution is appropriate when considering exactly how results from the BPS would generalize to other specific institutions (e.g., Stinebrickner and Stinebrickner, 2006, 2013). At the same time, from an academic standpoint, Berea has much in common with many four-year colleges. It operates under a standard liberal arts curriculum and the students at Berea (which is in central Kentucky) are similar in academic quality to, for example, students at the University of Kentucky (Stinebrickner and Stinebrickner, 2008b).

Our study is made possible by three types of information that are available in the BPS. First, the BPS elicited each student's closest friends. Our analysis utilizes friendship observations from the end of the first semester and the end of the second semester. The survey question for the end of the first semester is shown in Appendix A.1. The survey question for the end of the second semester is identical (except for the date). Our friendship survey questions have a full-semester flavor to them, as they asked students to list the four people who had been their best friends that semester. Second, the BPS collected detailed time-use information eight times each year, using the twenty-four hour time diary shown in Appendix A.1. Finally, questions on the baseline survey reveal the number of hours that a student studied per week in high school and how much the student expects to study per week in college. Because these variables were collected just before the start of the first semester of freshman year, we refer to these variables as our study propensity measures. The survey data

[4]Goldsmith-Pinkham and Imbens (2013) posit a model of network formation and, within this model, derive a testable implication of endogenous network formation (for further discussion, see Boucher and Fortin, 2016). We do not use our specification test to test for a specific model of network formation; rather, the goal of our specification test is to detect unobserved determinants of study time that we believe to be relevant to our context, taking as given the network. Therefore, we view our work as complementary to that of Goldsmith-Pinkham and Imbens (2013).

are merged with detailed administrative data on race, sex, high school grade point average (GPA), college entrance exam scores, and college GPA in each semester.

This paper focuses on the freshman year for students in the 2001 entering cohort.[5] There exists a practical data-related reason for focusing on freshmen; we are able to characterize the network most completely in the first year both because survey response rates are very high in the first year and because over 80% of friends reported by students in their freshman year are themselves freshmen.[6] However, our focus on freshmen, when combined with important institutional details at Berea, is also of relevance for whether it is reasonable to avoid the modeling complications that would result from taking into account variation across students in course difficulty, the number of classes taken, or hours of paid employment. With respect to course difficulty, it seems particularly relevant that students at Berea are all enrolled in a general, liberal arts curriculum. Importantly, the specifics of this curriculum imply that the majority of classes in the freshman year fall under the heading of "general studies," with many of these general studies classes being identical across students and the remainder being similar in nature across students.[7] With respect to the number of classes taken, there is less variation at Berea than other schools. Students are not allowed to study part-time, and there exists a strong tradition of students taking four classes in each semester to complete the required number of courses, 32, in an eight semester (four year) period.[8] With respect to hours of paid employment, students at Berea are all assigned work-study jobs, and students are not allowed to work in off-campus jobs. Then, a large degree of uniformity is present in hours worked across our sample because the work-study jobs have 10-hour weekly minimums, which students are not allowed to appreciably exceed in their freshman year. The mean and standard deviation of work hours for our sample are 10.9 and 1.4 hours per week, respectively.

3.1 Sample Construction

Our focus is on students who stayed in school for the full first year. There were a total of 331 students who fit this description. Our estimation sample consists of the 307 students

[5] We focus on this cohort because the survey contains more comprehensive time-use and friendship information for them. Information about time use was collected using time diaries for the 2001 cohort, while, for the 2000 cohort, this information was collected using questions that asked respondents to "think carefully about how much time was spent studying" in the last twenty-four hours. First-semester friendship information was collected at the end of the first semester for the 2001 cohort, while, for the 2000 cohort, first-semester friendship information was collected retrospectively during the second semester.

[6]Approximately 88% of all entering students in the 2001 cohort completed our baseline survey, and response rates remained high for the eleven subsequent surveys that were administered during the freshman year.

[7]On average, students take about one course in their area of specialization per semester in their freshman year.

[8]This tradition has persisted from past institutional details, which required degrees to be completed in four years, except in exceptional circumstances.

(i.e., 93% of the 331) with friends in each semester. A student j is deemed to be a friend of student i if either i lists j as a friend or j lists i as a friend. This means that a student can have friends in a particular semester even if the student did not complete the friendship question in that semester. Pooling the two semesters, we find that about 85% of the students in our final sample reported friendship information directly, via the friendship survey.

3.2 Descriptive Statistics

This section presents descriptive statistics for the sample.[9] We start by describing student characteristics (this discussion is based on Table A1). Forty-four percent of students are male, 18% of students are black, the mean high school grade point average (high school GPA) for the sample is 3.39, the mean combined score on the American College Test (ACT) is 23.26, and, on average, students studied 11.24 hours per week in high school and expect to study 24.96 hours per week in college. Examining the characteristics of certain subgroups of interest, on average, males have lower high school GPAs than females (3.24 vs. 3.51) and black students have lower high school GPAs than nonblack students (3.14 vs. 3.45). Black students studied more, on average, in high school than other students (15.29 vs. 10.36).[10]

Focusing on descriptive statistics of outcomes during the first year (this discussion is based on Table A2), on average, students study 3.49 hours per day in the first semester and 3.50 hours per day in the second semester.[11] On average, males study less than females, black students study more than nonblack students, and students with above-median high school GPAs study more than students with below-median high school GPAs.[12] The average first semester GPA is 2.89 and the average second semester GPA is 2.93, and males, black students, and students with below-median high school GPAs all have lower average GPAs than their counterparts.[13]

Turning to friend data for our sample (this discussion is based on Table A3), as described at the beginning of this section, we define friendship as the union of reported links between

[9]The first three tables pertaining to these statistics were a bit cumbersome, so they are in Appendix A.2.

[10]The first two differences in means are significantly different at the 0.001 level. The averages of high school study time for black students and nonblack students are significantly different at the 0.01 level.

[11]Descriptive statistics about study time outcomes presented in Table A2 are computed at the level of individual study time reports, of which there may be up to four in each semester, for each student. When computing other descriptive statistics (including regressions), we use the semester-specific average (over the study time reports) for each student The two measures are very similar, other than the larger variance of the individual-report-based measure. As we make clear when we describe our estimation procedure, we use individual study time reports when estimating the structural parameters of our model.

[12]Pooling observations from both semesters, the first and last differences in means are significantly different at a 0.05 level and, given the relatively small number of black students, the middle difference in means is significant at a 0.10 level.

[13]Pooling observations from both semesters, all of these differences are significant at a 0.05 level.

Table 1: Network characteristics

Friendship transitions	
Prob. friendship reported first semester but not second	0.51
Prob. second semester friendship is new	0.51

Note: Top row is computed according to $\Pr\{A_2(i,j) = 0|A_1(i,j) = 1\}$ and bottom row is computed according to $\Pr\{A_2(i,j) = 0|A_1(i,j) = 1\}$, where A_t is the adjacency matrix in semester t.

Correlations between own and avg. of friends	
Black	0.74
Male	0.71
HS GPA	0.23
Combined ACT	0.31
HS study time	0.23
Expected study time	0.14

Note: Each row is presents the correlation for a student's own measure and the average of their friends' measures, pooled over both semesters.

two students that semester.[14] Students have 3.3 friends on average, and there is considerable variation in the number of friends: the minimum number of friends is one, while the maximum number of friends is 10. Male and black students (and, therefore, female and nonblack students) sort strongly towards students with the same characteristics. For example, 74% of the friends of male students are male, while only 18% of the friends of female students are male. Similarly, 69% of the friends of black students are black, while only 7% of the friends of nonblack students are black. While male and black students have friends with lower incoming GPAs and lower combined ACT scores, males have friends who studied less in high school and expect to study less in college (compared to females), while black students have friends who studied more in high school and expect to study more in college (compared to nonblack students). Finally, we describe friend study time. Consistent with own study time, on average, friend study time is 3.5 hours per day. Also, average friend study time is much lower for males than for females (3.16 vs. 3.76 hours per day).

Table 1 shows other network characteristics. The network evolves over time: both the probability that a first-semester friendship no longer exists in the second semester and the probability that a second-semester friendship was not present in the first semester are 0.51 (the similarity of these values is coincidental). Consistent with the findings from Table A3, the correlations on the right side of the table show substantial sorting on the basis of observable characteristics.

Table 2 presents descriptive OLS regression results predicting own study time (left column) and GPA (right column), pooling observations over both semesters. The study time regression shows evidence of significant partial correlations of one's own study time (com-

[14]Therefore, the number of friends may exceed that elicited in the survey in Appendix A.1.

Table 2: Study time and GPA OLS regressions

	Dependent variable:	
	Own study	GPA
	(1)	(2)
Male	-0.369^{**}	-0.131^{*}
	(0.171)	(0.076)
Black	0.116	-0.225^{**}
	(0.214)	(0.109)
HS GPA	0.413^{**}	0.437^{***}
	(0.188)	(0.081)
ACT	-0.032	0.040^{***}
	(0.023)	(0.013)
HS study	0.043^{***}	0.001
	(0.008)	(0.004)
Expected study	-0.002	-0.006
	(0.009)	(0.003)
Friend study	0.166^{***}	
	(0.039)	
Own study		0.090^{***}
		(0.022)
Constant	1.915^{**}	0.417
	(0.759)	(0.362)
Observations	574	571
R^2	0.169	0.259

Note: *p<0.1; **p<0.05; ***p<0.01. Standard errors clustered at the student level are in parentheses. GPA is measured in GPA points (0-4). Own and friend HS study and expected study are measured in hours/week. Own and friend study are measured in hours/day. The variable "Friend z" for student i in period t is the average of the variable z across i's friends in period t.

puted as the average amount the student reports studying in the time diaries within a semester) with own sex and own high school GPA. As for our study propensity measures, we estimate a positive, significant partial correlation between own study time and own high school study time. We do not estimate a significant correlation between own study time and expected study time when both propensity measures are included. However, in results not shown here, when expected study time is the only study propensity measure included, we find that it has a positive, significant partial correlation with own study time (t-statistic of 2.2). The overall contribution of these two variables is substantial, with their omission reducing R-squared from 0.169 to 0.087 (see Table A8 in the appendix). Our novel measures of the propensity to study clearly have content. One's own study time also has a significant positive partial correlation with friend study time (computed as the average over friends of their own study times). The GPA regression shows that own GPA has a significant positive partial correlation with being female, being nonblack, and having above-median high school GPA. Own GPA also has a significant partial correlation with own study time.

4 Model

Students are indexed by $i = 1, \ldots, N$ and time periods (semesters) are indexed by $t = 1, 2$. We denote the study time of student i in time period t as s_{it} and let S_t define a column vector collecting all students' study times during that period. Study time represents a student's average daily study time over the course of a semester.[15] We treat the adjacency matrix representing the network of friendships as pre-determined. This matrix in period t, denoted A_t, has a main diagonal of zeros and an (i, j) entry of one if student i has j as a friend and zero otherwise.[16] The average study time of i's friends during period t is

$$s_{-it} = \frac{\sum_{j=1}^{N} A_t(i,j) s_{jt}}{\sum_{j=1}^{N} A_t(i,j)}. \tag{1}$$

[15]This is isomorphic to the natural input of total study time that period.

[16]Other than its being a square matrix of full rank (i.e., we exclude students with no friends, as their friend study time is not defined), we impose no restrictions on A_t. Though we use the union of reported links (i.e., $A_t(i, j) = 1$ if either i reports being friends with j, or vice versa), the model could also accommodate non-reciprocal links (i.e., i may link to j without j linking to i). Our model has the potential to analyze any network configuration because we have a well-defined, observed, input that has a natural upper bound (one cannot study more than 24 hours per day), which allows us to show there exists a unique equilibrium under conventional parameter restrictions (e.g., weak concavity). This is contrast to other work, e.g., the latent input structure in Calvó-Armengol et al. (2009) means that the existence of an equilibrium depends on a stability condition: their (homogeneous) best response slope must be less than the inverse of the largest eigenvalue of the adjacency matrix. Because this largest eigenvalue tends to grow with the size of the largest connected component of a school's network, their model will tend not have an equilibrium for large, yet realistic, networks; for example, at their pooled point estimate, the networks in both freshman year semesters at Berea would be inadmissible.

Taking into account their friends, students make decisions about how much to study in a particular semester by considering the costs and benefits of studying.[17]

The benefits of studying come from academic achievement. The (deterministic) production function for achievement, $y(\cdot)$, depends on how much a student studies, s_{it}. However, it is also desirable to recognize that performance in school will vary across students with different observable characteristics x_i. For example, students with higher college entrance exam scores may enter college better prepared or "smarter" than other students at the time of entrance. We allow the effect of these observable characteristics to enter through a function $\mu_{yi}(x_i)$, and we refer to this part of a student's initial endowment as one's "human capital type." Noting that this function will be specified in Section 5 and suppressing the x_i portion of this notation, the production function is

$$y(s_{it}, \mu_{yi}) = \beta_1 + \beta_2 s_{it} + \mu_{yi}, \tag{2}$$

where the human capital type is assumed to be a sufficient statistic for the history of prior inputs.

Social interactions may arise because the cost of studying, $c(\cdot)$, is allowed to depend on friends' study time choices. We focus on a specification wherein a student's cost of studying can depend on how much their study time deviates from a "target" based on friends' study time choices. (see, e.g., Brock and Durlauf, 2001; Moffitt, 2001; Blume et al., 2015).[18] The costs of studying are allowed to vary across students with different observable characteristics. For example, students with higher college entrance exam scores may feel differently about the deviation of their own study time from friend study time, or may find studying more (or less) enjoyable than other students. Analogous to our introduction of a human capital type in equation (2), we allow the effect of these observable characteristics to enter through a function $\mu_{si}(x_i)$, and we refer to this part of a student's initial endowment as one's "study type." Again noting that this function will be specified in Section 5 and suppressing the x_i portion of this notation, the cost function is

$$c(s_{it}, s_{-it}, \mu_{si}) = (\chi_1 + \chi_2 \gamma(\mu_{si}))\, s_{it} + \frac{\chi_3}{2} s_{it}^2 + \frac{\chi_4}{2} \left(s_{it} - (1 + \chi_5 \gamma(\mu_{si}))\, s_{-it}^{\tau_s}\right)^2. \tag{3}$$

The exponent τ_s allows for curvature in how friend study time affects the costs of one's own

[17]We define friend study time as the average study times of one's friends. Our framework could also accommodate specifications where friend study time was defined to be the total study time of one's friends.

[18]We discuss below (in Section 4.2.1) how this specification of the cost function, which is commonly referred to as featuring a "conformity" effect, is observationally equivalent with one in which friends' study choices directly influence the marginal costs of studying (i.e., in which there is no "target input").

studying. The function $\gamma(\cdot)$ shows how the study type enters the cost function. We define

$$\gamma(\mu_{si}) = \frac{1}{\exp(\tau_{\mu,1}\mu_{si} + \tau_{\mu,2}\mu_{si}^2)}, \tag{4}$$

which allows the cost function to vary across people of different study types. (We do not include a fixed cost of studying because only 7% of all individual study time reports are zero, and less than 2% of students report zero study time in all of their reports in a semester.) Since it is always the function $\gamma(\mu_{si})$ (rather than μ_{si} itself) that enters our specification, we refer to $\gamma(\mu_{si})$ as the "effective study type". The effective study type potentially enters via two channels: by directly affecting the marginal cost of studying (via $\chi_2\gamma(\mu_{si})$) and by affecting how much the student's marginal cost is affected by her friends' study effort (via $\chi_5\gamma(\mu_{si})$). The latter term allows the "target" to which a student seeks to "conform" to vary with the student's characteristics. As we show just below when solving the model, the effective study type may affect both the level of a student's study effort and how responsive the student is to her friends' study effort. We conform to Blume et al. (2015) and assume χ_3 is positive and normalize it to one.[19]

With knowledge of $\{A_1, A_2\}$, all students' human capital types $\{\mu_{yi}\}_{i=1}^N$ and all students' study time types $\{\mu_{si}\}_{i=1}^N$, students simultaneously choose study times to maximize utility, which is separable across periods:[20]

$$u(s_{i1}, s_{i2}) = \left\{ \sum_{t=1}^{2} y(s_{it}, \mu_{yi}) - c(s_{it}, s_{-it}, \mu_{si}) \right\}. \tag{5}$$

Remark 1. *Before solving the model, it may be useful to include a brief discussion of what may seem to be the somewhat spare specification laid out thus far. When developing our model, we appealed to a subset of the peer effects literature that had specifically considered what would be most important for generating social interactions in academic achievement in the first-year college context that we study, our proposed mechanism being that friends' study time choices affect one's own choice of study time and, thus, achievement (Stinebrickner and Stinebrickner, 2006; Foster, 2006). That being said, one strength of our unique data is that we are able to test this specification against others that have received attention in the academic context, e.g., those including direct effects of peer characteristics in the production*

[19] This creates convexity in the cost function. Similar assumptions are used in papers positing a utility function with a component in own effort that is quadratic with a fixed coefficient; see, for example, Blume et al. (2015), p. 449 or 452. Papers that work directly with the best response function (see, e.g., equations (1)-(2) of Moffitt, 2001 or equation (1) of Bramoullé et al., 2009) also implicitly impose similar assumptions.

[20] The alternative assumption, where students know only the current adjacency matrix when choosing their study times and calculate expectations over the future adjacency matrix, would have identical predictions in our model. See Section 4.1.

of achievement (contextual effects) and those including production complementarities. We discuss alternative specifications in Sections 4.2 and 7.2. As described in Section 7.2, our extensive testing of other specifications supports the parsimonious one we present here.

4.1 Model Solution

Each student's decision problem is additively separable across time periods, meaning each student can solve each period's problem separately.[21] Student i's best response to friend study time in t is given by

$$s_{it} = \underset{s \in [0,24]}{\arg\max} \; \{y(s, \mu_{yi}) - c(s, s_{-it}, \mu_{si})\}, \tag{6}$$

with the natural constraints that study time is nonnegative and cannot exceed 24 hours per day. The first order condition of (6) with respect to own study time yields $\frac{\partial y}{\partial s} = \frac{\partial c}{\partial s}$, i.e., the utility-maximizing study time equates the marginal return for increasing study time with the marginal cost. Expanding the first order condition and solving for own study time yields the best response function, which expresses student i's study time as a function of friend study time, at an interior solution:

$$s_{it} = \frac{\beta_2 - \chi_1}{1+\chi_4} + \frac{-\chi_2}{1+\chi_4}\gamma(\mu_{si}) + \frac{\chi_4}{1+\chi_4} s_{-it}^{\tau_s} + \frac{\chi_4 \cdot \chi_5}{1+\chi_4}\gamma(\mu_{si}) s_{-it}^{\tau_s} \equiv \psi(s_{-it}, \mu_{si}). \tag{7}$$

Equation (7) clearly shows that we allow for the possibility of finding no evidence of endogenous social interactions, which would occur if we estimated that $\frac{\chi_4}{1+\chi 4} = 0$. Note that while best response functions depend on study type μ_{si}, it is sometimes notationally convenient to suppress the study type and write the best response function as $\psi_i(s_{-it})$. We restrict parameters so that own study time has a strictly positive intercept and is a weakly increasing and weakly concave function of friend study time.[22]

As shown in Section 4.1.1, concave best response functions ensure existence of a unique equilibrium for the study time game. As shown in equation (7), the separable form we adopt for the cost function has the benefit of producing a closed-form solution for the student best response function. We show in Appendix D.1 that concavity of the best response function

[21] If utility were nonlinear in semester achievement or the argument of the cost function were study time over the whole year, the problem would no longer be separable across time periods. We assume student utility is linear in achievement because non-linearity of utility in achievement would be difficult to separate from non-linearity in the cost function without relying on functional form restrictions.

[22] The strictly positive intercept restriction corresponds to $\min_{i \in N}\{\frac{\beta_2 - \chi_1}{1+\chi_4} + \frac{-\chi_2}{1+\chi_4}\gamma(\mu_{si})\} > 0$. The weakly increasing restriction corresponds to $\min_{i \in N}\{\frac{\chi_4}{1+\chi_4} + \frac{\chi_4 \cdot \chi_5}{1+\chi_4}\gamma(\mu_{si})\} \geq 0$. Weak concavity corresponds to further requiring $\tau_s \leq 1$. These restrictions, combined with $s_{it} < 24$, are sufficient to have the well-behaved equilibrium described in Section 4.1.1. In practice, however, we are able to estimate the model using weaker restrictions, described in Section 5.

would result from any cost function possessing the natural properties of being strictly convex in s_{it} and weakly concave in s_{-it}.

4.1.1 Equilibrium

Definition 1 (Period Nash equilibrium)**.** *A pure strategy Nash equilibrium in study times* $S^* = [s_1^*, s_2^*, \cdots, s_N^*]'$ *satisfies* $s_i^* = \psi(s_{-i}^*, \mu_{si})$*, for* $i \in N$*, given adjacency matrix* A*.*

Claim 1. *Let* $k = 24$*. There exists a unique pure strategy Nash equilibrium if* $\psi_i : R^N \mapsto R$ *are weakly concave and weakly increasing,* $\psi_i(0) > 0$*, and* $\psi_i(k) < k$ *for* $i \in N$*.*

Proof. See Appendix D.2. □

We compute the equilibrium by iterating best responses.[23]

4.2 Model Discussion

4.2.1 Other Mechanisms Generating Endogenous Social Interactions

Cost Reduction Our specification of the cost function allows social interactions to emerge from a force promoting conformity to a "target".[24] An alternative specification would allow friend study effort to directly reduce one's own cost of studying. We show in Appendix D.3.1 that such a specification can be observationally equivalent to the one we presented above.

Production Complementarities Another proposed mechanism is that social interactions arise through production complementarities, where increases in peer inputs increase the marginal product of one's own input (e.g., Calvó-Armengol et al., 2009). From a conceptual standpoint, the decision to specify our model without production complementarities was informed by the notion that friends in the first year of college may spend relatively little time talking about coursework, with some empirical support for this provided by Stinebrickner and Stinebrickner (2006).[25]

That being said, we show in Appendix D.3.2 that in the typical case, where one only had data on either the input (e.g., study effort) or output (e.g., achievement), our conformity-based specification (or, equivalently, a cost-reduction-based specification) would be observa-

[23]Convergence to the equilibrium is extremely fast in practice.

[24] Note that a conformity mechanism could also naturally capture a "social learning" mechanism, wherein students studied more or less, due to their friends' studying more or less due to their updating about the "true" productivity of study time. See Conley and Udry (2010) for an example disentangling social learning about productivity from conformity (or mimicry) forces, using data on the timing of information arrival and input decisions.

[25]Stinebrickner and Stinebrickner (2006) find that students spend very little time talking about coursework with their roommates; it is not a big leap to imagine the same would be true of students and their friends.

tionally equivalent to a specification exhibiting production complementarities. (This point is also made by Blume et al., 2015.) Because we measure both inputs and outcomes, we are in the unique position to examine the potential roles played by cost-based mechanisms and production complementarities. As we discuss in Section 7.5, we do not find evidence for such a mechanism in our context.

4.2.2 Dynamic Behavior

We assume the human capital type is constant between the periods. Though it would be feasible to extend our static framework to a dynamic framework allowing the human capital type to evolve between periods, the benefits of doing this are mitigated by two facts: (1) we study students during their freshman year, which, under the liberal-arts curriculum at Berea, is typically before they start taking a substantial amount of specialized course material (meaning second semester coursework does not build heavily on first semester coursework), and (2) each model period corresponds to a semester, which is shorter than the period typically considered when estimating value-added production functions in an educational context (see Hanushek, 1979; Todd and Wolpin, 2003, for discussions of issues related to the estimation of education production functions). Consistent with these facts, as we discuss in Section 7.6, we find that out-of-sample outcomes, simulated from parameters estimated on only first-semester data, fit second-semester data quite well.

5 Estimation

The model provides a mapping from the adjacency matrix A_t and all the students' types $\{(\mu_{si}, \mu_{yi})\}_{i=1}^{N}$ to a unique equilibrium in study times for all students, S_t^*. The equilibrium study times S_t^* generate achievement in equilibrium y_{it}^*, via the production function $y(s_{it}, \mu_{yi})$. The model is operationalized by parameterizing a student's types as linear combinations of observable characteristics collected in a vector x_i. That is, $\mu_{si} = x_i'\omega_s$ and $\mu_{yi} = x_i'\omega_y$, where the parameter vectors ω_s and ω_y respectively determine study and human capital types.[26] The vector x_i includes indicators for being black and being male, along with high school GPA, combined ACT score, average hours per week of study time in high school, and expected hours per week of study time in college. This allows us to express each student's equilibrium study time and achievement as a function of A_t and all students' characteristics, which we collect in a matrix X. Given the full set of model parameters

[26]We set the coefficient on high school GPA in the study type $\omega_{s,\text{HS GPA}} = 1$ to identify $\tau_{\mu,1}$.

$\Gamma = (\beta_1, \beta_2, \chi_1, \chi_2, \chi_3, \chi_4, \chi_5, \omega_s, \omega_y, \tau_{\mu,1}, \tau_{\mu,2}, \tau_s)'$, we write these outcomes for individual i as

$$s^*_{it} = \psi(s^*_{-it}, \mu_{si}) = \delta_{si}(A_t, X; \Gamma) \tag{8}$$

and

$$y^*_{it} = y(s^*_{it}, \mu_{yi}) = \delta_{yi}(A_t, X; \Gamma), \tag{9}$$

where s^*_{-it} is defined by applying equation (1) to S^*_t and A_t.

Our measure of y^*_{it}, a student's semester-level achievement, is the student's semester grade point average (GPA), denoted $\widetilde{y}_{it}$.[27] We treat GPA as a noisy measure of equilibrium achievement, so with our parameterization: $\widetilde{y}_{it} = \delta_{yi}(A_t, X; \Gamma) + \eta_{yit}$. The measurement error η_{yit} is assumed to be mean independent of A and X, independent across students, but allowed to be potentially correlated within student across time.

Our measures of s^*_{it}, a student's average daily study time over all days in a semester, come from up to four 24-hour time diaries completed by each student i in semester t. Each time diary provides a report of hours studied during the previous 24 hours. We use R_{it} to denote the set of reports for student i in semester t, and study time report r for student i in semester t is denoted $\widetilde{s}_{rit}$. We view each report as a noisy measure of s^*_{it}, because on any given day a student may study more or less than their semester-long average.[28] Thus, with our parameterization: $\widetilde{s}_{rit} = \delta_{si}(A_t, X; \Gamma) + \eta_{srit}$, with η_{srit} denoting a measurement error that is assumed to be mean independent of A and X, independent across students, and allowed to be potentially correlated within student across time.

We estimate our model by Non-Linear Least Squares (NLLS)[29] combining the sum of squared errors for both achievement and study time data.[30] We obtain parameter estimates

[27] Course grades are measured on a four-point scale.

[28] Indeed, in previous research that specifically focused on measurement issues we found that substantial within-person variation exists in daily study effort, even within a particular semester (Stinebrickner and Stinebrickner, 2004).

[29] Only a small number of our measurements are at their boundaries: 7% of student-semester GPA observations are equal to four, 1% of student-semester GPA observations are zero, and 7% of individual study time reports are zero. Parameter estimates (and their associated quantitative results) obtained when using Tobit specifications for GPA and reported study time are virtually identical to those obtained from the specification presented here. We focus on results from the least-squares estimation criterion, which does not explicitly account for boundary values of GPA or reported study times, because it is tightly linked with the specification test statistic discussed in the next section.

[30] Although we estimate our parameters jointly, our achievement equation estimation procedure can be viewed as utilizing a constructed "first stage" estimate of study time, which is ultimately based on our estimates of δ_{si}, not observed study time.

as the minimizers[31] of:

$$\sum_i \sum_t (\eta_{yit})^2 + \sum_i \sum_t \sum_{r \in R(i,t)} (\eta_{srit})^2. \tag{10}$$

Our estimator is consistent under standard regularity conditions for NLLS estimators, augmented with rank conditions for identification involving A_t discussed in Appendix F. In order to demonstrate that identification is not dependent on an absence of contextual effects, we also discuss in that appendix identification of an augmented model with contextual effects, where friends' characteristics enters a student's best response function. The key identification conditions are analogous to those in Proposition 1 from Bramoullé et al. (2009) or Theorem 4(iii) from Blume et al. (2015).

6 Specification Test Rationale

We propose a specification test based on the cross-sectional correlation in study time residuals. In addition to serving as an overall test of proper specification, it will in general have power to detect unobserved determinants of study time best responses because they induce cross-sectional correlation in residuals. Potential unobservables include, for example, an exogenous structural error or even unobservables that are correlated with other determinants of study time. In this section we discuss an alternative data generating process (DGP) with an omitted unobserved determinant of study time to illustrate how, in general, our social interactions model implies that such alternative DGPs would induce cross-sectional correlations in residuals across students because they would enter students' best responses in equilibrium. In contrast, under the null hypothesis of a properly specified model, study time residuals would have zero cross-sectional correlation.

Letting $\widehat{\Gamma}$ denote the vector of estimated parameters, the study time residual for student i, report r, in semester t is

$$\widehat{\eta}_{srit} = \widetilde{s}_{rit} - \delta_{si}(A_t, X; \widehat{\Gamma}). \tag{11}$$

Any test for a zero cross-sectional correlation in these residuals could be used. We present our test statistic in Section 7.3, which pools these residuals across multiple reports r and semesters t.

[31] Recall that we assume that best response functions are strictly positive, nondecreasing, and weakly concave (and that best responses are strictly less than 24 hours/day). These restrictions are difficult to directly impose in terms of restrictions on the parameter space when there is heterogeneity in best response functions. Therefore, we adopt an indirect approach, of verifying whether some best response functions derived from posited parameters satisfy the restrictions. Specifically, when estimating the model, we use the weaker restrictions that the 75th percentile effective study type's best response function is nonnegative and that equilibrium study times are strictly positive. As we show in Section 7, none of any of the stronger (or, consequently, weaker) restrictions are close to binding at our estimated parameters.

Alternative DGP with an omitted unobserved determinant of study time We proceed by considering how the study time component of our model would be altered by the presence of an unobserved determinant of study time. For ease of exposition in the remainder of this section, we assume there is one study time report and one period, which allows us to simplify notation by dropping the r and t subscripts. We examine the special case with $\tau_s = 1$. This case is consistent with our baseline empirical results, where we find that best response functions are linear (see Section 7).

Using a composite parameter $\chi_0 \equiv [\frac{\beta_2 - \chi_1}{1+\chi_4}]$, define the subset of parameters identified by just equation (7), the student's best response function, as $\Gamma_2 = (\chi_0, \chi_1, \chi_2, \chi_4, \chi_5, \omega_s, \tau_{\mu,1}, \tau_{\mu,2})'$. To simplify notation, we refer to the terms in (7), $[\chi_0 + \frac{-\chi_2}{1+\chi_4}\gamma(\mu_{si})]$ and $[\frac{\chi_4}{1+\chi_4} + \frac{\chi_4 \cdot \chi_5}{1+\chi_4}\gamma(\mu_{si})]$, as $f_1(x_i; \Gamma_2)$ and $f_2(x_i; \Gamma_2)$, respectively. As in Section 5, x_i contains student i's observed characteristics, which enter the best response function through effective study type $\gamma(\mu_{si})$. The best response equation for an individual student is

$$s_i = f_1(x_i; \Gamma_2) + f_2(x_i; \Gamma_2) s_{-i}. \tag{12}$$

In order to represent the system of equations for all students in a vector S, use $F_1(X; \Gamma_2)$ to denote a column vector stacking the $f_1(x_i; \Gamma_2)$ for all i. We use the notation $W(X; \Gamma_2)$ for a matrix that has zeros in the same positions as the zeros in A and nonzero entries in locations where A has ones. In place of the ones in row i of A, $W(X; \Gamma_2)$ contains

$$\frac{1}{\sum_{j=1}^{N} A(i,j)} [f_2(x_i; \Gamma_2)]. \tag{13}$$

The system of equations is thus

$$S = F_1(X; \Gamma_2) + W(X; \Gamma_2) S. \tag{14}$$

Note that (14) is simply a re-written version of the model we developed in Section 4, but with linear best response functions. Solving for S, we obtain the equilibrium vector of study times

$$S^* = (I - W(X; \Gamma_2))^{-1} [F_1(X; \Gamma_2)], \tag{15}$$

where the right side corresponds to the vector stacking $\delta_{si}(A, X; \Gamma)$ for all students.

Incorporating our idiosyncratic error η_{si}, we obtain the DGP for observed study time $\widetilde{S}$ under the *null hypothesis* of correct specification:

$$\widetilde{S} = (I - W(X; \Gamma_2))^{-1} [F_1(X; \Gamma_2)] + \eta_s, \tag{16}$$

where η_s stacks the η_{si} for all the students.

Now consider an alternative scenario in which our model was misspecified. In particular, suppose a vector of characteristics V was omitted by the econometrician but was observed by all students, entering the best response system in the following manner:

$$S = F_1(X; \Gamma_2) + W(X; \Gamma_2)S + V. \tag{17}$$

Again solving for S, the equilibrium system of equations has the following form:

$$S^* = (I - W(X; \Gamma_2))^{-1}[F_1(X; \Gamma_2) + V]. \tag{18}$$

In general, the matrix $(I - W(X; \Gamma_2))^{-1}$ will have many non-zero entries because students will typically be directly or indirectly connected to many other students. Therefore, many, if not all, elements of V will influence a given student's equilibrium study time in this alternative.[32]

Decompose V into two components according to:

$$V = \Pi(X) + u, \tag{19}$$

where we assume that u is mean zero conditional on X and A. We are agnostic about correlation patterns in u across students; in particular, friends may have correlated u. Consider, for example, a scenario where male and female students have the same expected value of u, but where a (mean-zero) sex-specific shock induces correlations among students of the same sex, who are likely to be friends with each other. Substituting this expression for V into (18) and incorporating our error η_s gives the data generating process for observed study time under the *alternative hypothesis*:

$$\begin{aligned}\widetilde{S} =& (I - W(X; \Gamma_2))^{-1}[F_1(X; \Gamma_2) + \Pi(X) + u] + \eta_s \\ =& (I - W(X; \Gamma_2))^{-1}[F_1(X; \Gamma_2) + \Pi(X)] + (I - W(X; \Gamma_2))^{-1}u + \eta_s. \end{aligned} \tag{20}$$

It is convenient to re-write (20) with a composite error ϵ:

$$\begin{aligned}\widetilde{S} =& (I - W(X; \Gamma_2))^{-1}[F_1(X; \Gamma_2)] + \epsilon, \qquad (21)\\ \text{where } \ \epsilon =& (I - W(X; \Gamma_2))^{-1}\Pi(X) + (I - W(X; \Gamma_2))^{-1}u + \eta_s. \end{aligned}$$

Of course, residuals must be computed using estimates of Γ_2, rather than the true value. To derive the residuals $\widetilde{\epsilon}$, consider the least squares estimator of Γ_2 in the study time regression, $\widehat{\Gamma}_2$, i.e., the estimate of Γ_2 that minimizes $\epsilon'\epsilon$ in (21). The fitted values for $\widetilde{S}$ using $\widehat{\Gamma}_2$ are $(I - W(X; \widehat{\Gamma}_2))^{-1}[F_1(X; \widehat{\Gamma}_2)]$. Let $\widetilde{\Gamma}_2$ denote the probability limit of $\widehat{\Gamma}_2$. In large samples,

[32] One implication of this is that, even if a subset of students had randomly assigned friends, the V of non-randomly assigned, yet connected, students would affect their equilibrium outcomes.

the fitted values of $\widetilde{S}$ based on our estimator would then be $(I - W(X;\widetilde{\Gamma}_2))^{-1}[F_1(X;\widetilde{\Gamma}_2)]$, which we can add and subtract from (21), resulting in

$$\widetilde{S} = (I - W(X;\widetilde{\Gamma}_2))^{-1}[F_1(X;\widetilde{\Gamma}_2)] + \widetilde{\epsilon}, \tag{22}$$

where

$$\begin{aligned}\widetilde{\epsilon} =& \underbrace{\{(I - W(X;\Gamma_2))^{-1}[F_1(X;\Gamma_2) + \Pi(X)] - (I - W(X;\widetilde{\Gamma}_2))^{-1}[F_1(X;\widetilde{\Gamma}_2)]\}}_{\text{“prediction bias”}} \\ &+ \underbrace{(I - W(X;\Gamma_2))^{-1}u}_{\text{equilibrium propagation of } u} + \eta_s.\end{aligned} \tag{23}$$

The first term (“prediction bias”) in $\widetilde{\epsilon}$ is due to the omission of $\Pi(\cdot)$, i.e., it represents a mean misspecification in (22). $\widehat{\Gamma}_2$ will likely be inconsistent for Γ_2 if $\Pi(\cdot) \neq 0$. The second term is due to the influence of u upon equilibrium study effort. This second term is what our specification test is designed to detect.

In general, our test will have power against the type of alternative DGP implied by (17), because the error $\widetilde{\epsilon}$ will exhibit cross-sectional correlation when $V \neq 0$. We show this by considering cases (i) with zero prediction bias and (ii) with non-zero prediction bias.

In large samples, there would be zero prediction bias if there exists a $\ddot{\Gamma}_2$ such that $F_1(X;\ddot{\Gamma}_2)$ nests $F_1(X;\Gamma_2) + \Pi(X)$. This nesting could potentially be accomplished by adopting a sufficiently flexible functional form for $F_1(\cdot;\cdot)$. In such a case, although elements of Γ_2 could be inconsistently estimated (i.e., plim $\widehat{\Gamma}_2 \neq \Gamma_2$), the conditional mean of the outcome would still be properly specified. Thus, bias in $\widehat{\Gamma}_2$ would not pervade to the residuals.[33] In practice, however, a non-zero prediction bias is possible, as it may be necessary to impose restrictions (based on the model or motivated by computational reasons) on $F_1(X;\cdot)$. We discuss this below in case (ii).

Case (i): Zero prediction bias: Consider first the case with no prediction bias, leaving us to focus on the u component of $\widetilde{\epsilon}$ in (23). In general, the term $(I - W(X;\Gamma_2))^{-1}u$ will exhibit cross-sectional dependence because its elements are linear combinations of many of the components of u.

In order for there to be no cross-sectional covariance in $(I - W(X;\Gamma_2))^{-1}u$, the shocks u would need to have a covariance matrix that was orthogonalized by $(I - W(X;\Gamma_2))^{-1}$. For

[33] For example, this would be true in the commonly considered case where we can write $F_1(X;\Delta_1) = X\Delta_1$ and $\Pi(X) = X\Delta_2$, where Δ_1 and Δ_2 are matrices of parameters, in which case the estimated $\widehat{\Delta}_1$ would have plim $\widehat{\Delta}_1 = \Delta_1 + \Delta_2$.

example, consider the case where u was generated according to

$$u = (I - W(X;\Gamma_2))e, \tag{24}$$

with e IID and $\mathrm{E}[ee'] = I$. For reasonable ranges of $W(X;\Gamma_2)$ in our application, such a u process would possess strong negative correlations among closely linked students. For example, consider our point estimate for Γ_2, which we present in Section 7, and our adjacency matrix for the first semester, A_1. For the process in (24), in order for u to be orthogonalized by $(I - W(X;\Gamma_2))^{-1}$, the ratio of the average covariance of u between friends to the average variance across students would have to be -0.31.

The main focus of the peer effects literature is on the case of positive assortative matching.[34] Therefore, we believe such a negative correlation is not the most salient one. Further note that the necessary orthogonalization could not occur when u are independent in the cross section. Moreover, even in the presence of negative cross-sectional correlations in u, only specific correlation structures could produce the necessary orthogonalization.

Case (ii): Non-zero prediction bias: In the case where there is a prediction bias, our test would not have power if the prediction bias term exactly offset cross-sectional correlations in $(I - W(X;\Gamma_2))^{-1}u$. For example, negative covariances in the bias term could, in principle, exactly cancel with the positive covariances that we anticipate in $(I - W(X;\Gamma_2))^{-1}u$. Our strong prior is that this scenario is implausible, due to the positive covariances across friends in their values of x_i and a prior that $\Pi(\cdot)$ is a reasonably smooth function of x_i. Intuitively, because friends have similar observed characteristics (x_i), the "prediction bias" error components of students and their friends will likely be positively correlated. This also implies that even if the u component were negligible, prediction bias could generate cross-sectional residual correlations. Most importantly, *prediction bias would have to exactly cancel out the u component* to result in zero residual correlation under the alternative DGP. Such a problematic scenario would be a knife-edge case.

[34] Epple and Romano (2011) contains a thorough discussion of sorting in the presence of peer effects. Zeitlin (2011) studies peer effects in a social learning context, finding that own and friend information shocks are negatively correlated. This finding is unsurprising in a learning environment, where one may gain more when one's friends have different information.

7 Estimation Results

7.1 Parameter Estimates

Table A4, in Appendix B, contains parameter estimates.[35] The key parameter from the achievement production function (presented in the top panel of the table) is the marginal product of own study time on achievement, β_2. The point estimate of 0.245 implies that increasing own study time by one hour per day increases achievement by about a quarter of a GPA point, ceteris paribus.[36] Students with high GPAs in high school and high ACT scores have significantly higher human capital, and black students have significantly lower human capital.

As can be seen in equation (7), the curvature in the best response function is given by τ_s, the exponent on s_{-it}. We estimated the model allowing τ_s to be in the set [0,1], nesting the assumption of a linear best response function (i.e., that τ_s=1). However, because our initial estimation provided evidence that τ_s is indistinguishable from 1, we re-estimated the model, fixing τ_s=1. We refer to the specification of our model specified in Section 4, with $\tau_s = 1$, as our baseline specification.

To ease the interpretation of the parameters governing the study cost function (which appear in the second panel of Table A4), we substitute them into the best response function (equation (7)), yielding

$$\widehat{\psi}(s_{-it}, \widehat{\mu}_{si}) = \{\underbrace{0.945}_{\widehat{\frac{\beta_2-\chi_1}{1+\chi_4}}} + \underbrace{-0.085}_{\widehat{\frac{-\chi_2}{1+\chi_4}}}\widehat{\gamma}(\widehat{\mu}_{si})\} + \{\underbrace{1.304}_{\widehat{\frac{\chi_4}{1+\chi_4}}} + \underbrace{-0.874}_{\widehat{\frac{\chi_4 \cdot \chi_5}{1+\chi_4}}}\widehat{\gamma}(\widehat{\mu}_{si})\}s_{-it}. \tag{25}$$

The first bracketed term in equation (25) represents the intercept of the best response function for student i, i.e., how much this student would study even if her friends did not study at all. This term consists of 0.945, the common component of the intercept across students, and $-0.085\widehat{\gamma}(\widehat{\mu}_{si})$, the component characterizing variation in the intercept across students. Likewise, the second bracketed term in equation (25) reveals the slope, or *reactiveness*, of the best response function, that is, how a student's choice of study time depends on the study time of her friends. This term consists of 1.304, the common component of the slope across students (our estimate of the common component of the slope has a standard error of 0.2198), and $-0.874\widehat{\gamma}(\widehat{\mu}_{si})$, the component characterizing variation in the slope across

[35]The parameter estimates are in the appendix because it is much more convenient to directly examine the best response functions formed from the structural parameters.

[36] It is reassuring that this result is quantitatively similar to that from Stinebrickner and Stinebrickner (2008a), who estimate that, for freshman at Berea, an extra hour per day of studying would increase GPA by 0.36 points (with a standard error of 0.183 points), using whether a randomly assigned roommate brought a video game as a shifter for one's own study time.

students. With $\gamma(\mu_{si}) = \frac{1}{\exp(\tau_{\mu,1}\mu_{si}+\tau_{\mu,2}\mu_{si}^2)}$, the latter component in both the first and second bracketed terms depends on the estimated values of $\widehat{\tau}_{\mu,1} = 0.105$ and $\widehat{\tau}_{\mu,2} = -0.003$, which indicate that $\gamma(\cdot)$ is decreasing and convex in one's study type, μ_s. In turn, the value of one's study type, μ_s, is determined by the cost function parameters ω_s; study type is increasing in high school GPA and high school study time, but is smaller for males.[37]

To provide a better sense of the total effect of peer study effort in the best response functions, Table 3 calculates equation (25) for the lowest, 25th percentile, median, 75th percentile, and the highest effective study types, presenting the type-specific intercept (i.e., the first bracketed term in equation (25)) in the top row and the coefficient on friend study time (i.e., the second bracketed term in equation (25)) in the bottom row. The first row shows that there is little heterogeneity in the intercepts of best response functions. The second row shows how reactiveness to peer study time increases in effective study type. The effect of peer study time is significantly positive for all effective study types, including the lowest effective study type (first column of the table).[38] Combining the slope and intercept terms, one's optimal study choice is increasing in effective study type. Moreover, the best response is always increasing in s_{-it} and is often substantial.[39]

Table 3: Estimated study best response functions for different effective study types $\widehat{\gamma}(\widehat{\mu}_s)$

Effective study type $\widehat{\gamma}(\widehat{\mu}_s)$:	Lowest	25th pctile	Median	75th pctile	Highest
Intercept	0.86	0.88	0.89	0.90	0.91
Coefficient on s_{-it}	0.47	0.65	0.73	0.81	0.94

Note: Own and friend study times are measured in hours/day. Each column represents the estimated best response function for an effective study type. For example, the middle column indicates that the median effective study type has the estimated best response function $s_{it} = 0.89 + 0.73s_{-it}$.

To get a sense of whether the estimated heterogeneity in reactiveness is significant, in Table 4 we present 95% confidence intervals for differences in best response slopes for different groups of students. Females have significantly steeper best response functions than males, students with above-median high school GPAs have significantly steeper best response functions than those with below-median high school GPAs, and students with above-median

[37] Though black students study considerably more than nonblack students, the coefficient on being black is negative. Black students have much higher high school study levels, which we find to be an important determinant of study type.

[38] The flexible specification we have developed to allow for heterogeneity in best response functions makes it difficult to discern whether the slopes of best response functions are significantly positive via direct examination of parameters in Table A4. Therefore, we adopted a conservative approach to assess statistical significance. We computed the 95% confidence interval for the best response slope for each student and then examined whether any of these confidence intervals contained zero; the lower bound on the union of these confidence intervals is 0.163.

[39] As noted in Section 5, we did not need to impose that best response functions are increasing in estimation.

high school study time have significantly steeper best response functions than those with below-median high school study time. Our estimates indicate that the game exhibits a potential complementarity. If matched by study type, students may study more in total, and therefore, have higher total achievement. However, whether students will take advantage of this complementarity depends on how they sort into friendships.

Table 4: Means and 95% confidence intervals for difference in slope of best response function, by group

Comparison	Mean	2.5%	97.5%
Female-Male	0.050	0.014	0.101
Black-Nonblack	0.015	−.010	0.042
High HS GPA-Low HS GPA	0.032	0.015	0.052
High Study HS-Low Study HS	0.169	0.074	0.288

"High-" and "Low HS GPA" respectively refer to above- and below-median high school GPA. "High-" and "Low Study HS" respectively refer to above- and below-median high school study time.

Figure A3, in Appendix C, shows that the model closely fits mean observed study time (left panel) and GPA (right panel), both in total and by student characteristics.[40] Figure A4, in Appendix C, plots own versus friend study time, for both the data (solid red line) and simulated outcomes (dashed blue line). Even though the relationship between own and friend study time is not explicitly targeted (i.e., friend study time outcomes do not enter the likelihood), the model also closely captures this relationship.

In the remainder of this section we discuss potential endogeneity problems, present the results from our specification test, and present evidence about the robustness of our estimates to changes in model specification.

7.2 Endogeneity

Our primary endogeneity concerns arise from the potential for the relationship between a student's study effort and that of her peers to be due, in part, to friendships being formed on the basis of potentially unobserved determinants of study time. One possible concern is that the relationship between own and friend study time is driven by institutional factors. One prominent example is that if students in science courses tend to study more and befriend students in their courses, there may be a spurious relationship between own and friend study time. We find that a version of the descriptive regression in Table 2, including both own and friend fraction of courses which are science, does not appreciably change the partial

[40]Model outcomes are simulated by first solving for equilibrium outcomes given $\widehat{\Gamma}$ and then applying measurement errors, using the specification in Section 5.

correlation between own and friend study time (0.166 vs. 0.160).[41] This is not surprising given that students may make friends outside their classes and that, as discussed previously, students' freshman years are largely standardized in terms of curriculum. In the same vein, dormitories are not specialized at Berea (e.g., there are not "study" dormitories or separate dormitories for student athletes).

Perhaps a more important concern is that students arrive at school with differing propensities to study, which affects how they sort into friendships. We address this concern by taking advantage of our survey collection to obtain direct measures of students' propensities to study. Our baseline survey elicited information about: 1) how much a student expected to study in college and 2) how much a student studied in high school. As we discussed in Section 3.2, these measures of the propensity to study clearly have content, as they are strongly correlated with how much a student studies. We stress a crucial feature of this information on study propensity is that our survey design allowed this information to be collected immediately after students arrived on campus, before students could be influenced by their friendships at Berea.

As is always the case, it is difficult to know *a priori* whether observable characteristics can address potential endogeneity concerns. Therefore, we next present results from our specification test, which was designed to detect a wide variety of unobserved determinants of study time, in particular, those underlying endogeneity concerns.

7.3 Specification Test Results

This section begins by showing how we implement a cross-sectional correlation specification test using our data for two periods (semesters) and multiple study time reports. Recall that predicted equilibrium study time for student i in semester t is $\delta_{si}(A_t, X; \widehat{\Gamma})$. We define i's semester-t study time residual as the average residual over i's semester-t study time reports, $\widetilde{s}_{rit}$:

$$\widehat{\eta}_{sit} \equiv \frac{1}{\sum_{r \in R_{it}} 1} \sum_{r \in R_{it}} \left(\widetilde{s}_{rit} - \delta_{si}(A_t, X; \widehat{\Gamma}) \right). \tag{26}$$

To implement the test, we average students' residuals over both semesters, i.e., $\widehat{\eta}_{si} = \frac{\widehat{\eta}_{si1} + \widehat{\eta}_{si2}}{2}$. We then compute the average of friends' average residuals for each student in each semester according to $\widehat{\eta}_{s,-it} = \frac{\sum_{j=1}^{N} A_t(i,j) \widehat{\eta}_{sj}}{\sum_{j=1}^{N} A_t(i,j)}$. Our test statistic is the cross-sectional correlation between $\widehat{\eta}_{si}$ and $\widehat{\eta}_{s,-it}$, pooled across semesters. Under the null of proper specification this correlation is zero.

In our baseline specification, in which our new measures of study propensity (high school study time and expected study time) enter students' study types, our specification test

[41] See Table A9 in the appendix.

statistic has a p-value of 0.6694, corresponding to a correlation between own and friend study time residuals of 0.021.[42] Thus, our test results suggest that our model is well-specified.[43] In particular, there is no evidence of an endogeneity problem arising from students positively sorting into friendships based on unobserved determinants of study time.

In the presence of an omitted characteristic, which may generate an endogeneity problem, our test should indicate a relationship in own and friends' residuals. To demonstrate that our test can detect a relationship in such a scenario, we construct an example where there is likely an endogeneity problem, by estimating a restricted version of our model in which we purposefully omit our novel measures of study propensity. Notably, this restricted specification uses only measures of student characteristics that are typically available to researchers. Because our empirical results show that these measures are both determinants of study time and also related to our measures of incoming human capital and friendship choices, their omission should generate a correlation across friends' residuals. The estimated correlation test statistic in this scenario is 0.2111, with a p-value of 0.000022, providing strong evidence against the null hypothesis of zero correlation.

Taking these two residual correlations together, our test results show that our new measures of study propensity play a crucial role in addressing endogeneity concerns in our context.[44] Our specification test results also suggest that exogenous, structural errors of the type typically considered in the spatial econometrics literature (see, e.g., Anselin, 1988) do not likely play an important role in our context. Intuitively, structural errors would have induced correlations in friends' residuals, which would be detectable by our specification test.

7.4 Human Capital Spillovers ("Contextual Effects")

We have focused on a mechanism wherein friend study time may affect one's own study time, which in turn may affect one's achievement via a production function. An alternative mechanism often considered in the literature involves peer characteristics directly entering

[42] We use a distribution approximation for this test that allows correlation within students.

[43] While the results from our specification test suggest that the model is well-specified, a researcher may nevertheless wish to include exogenous structural errors to best response functions. To accomplish this, one could modify the criterion function used to estimate our model to accommodate the resulting error structure. However, it is important to note that the estimated best response functions we obtain from using our (least-squares) estimation criterion would be robust to the presence of the type of error described just above. Therefore, estimates of average treatment effects from either counterfactual exercise (unconditional or conditioning on, e.g., student race), which represent our primary quantitative findings, would not be affected.

[44] While our study propensity measure captures salient determinants of freshman study effort, our approach treats this variable as exogenous at the beginning of university. Therefore, while we may (perhaps) reasonably ascribe causal interpretations to our estimates, we do not have a model for how high school study time itself is determined. While outside the scope of the current paper, examining this would be an interesting area for future research.

the achievement production technology ("contextual effects"). For example, friends with high human capital may provide quick and reliable answers to questions or may know more about specific course requirements, generating human capital spillovers.

Our *a priori* belief that a model without such spillovers may be quite natural is directly related to the mechanism for social interactions that we examine. In the short run that we study, it seems reasonable to believe that the primary reason a student's academic performance would be related to a particular observable characteristic of her friends is that the student's time-use is influenced by the good (or bad) study habits of friends with these characteristics.[45] Models estimated without study time information would label this relationship as "contextual effects". In contrast, in our approach, which is made possible by the collection of time-use information, this relationship would be explicitly accounted for by our proposed mechanism in which one's study time is influenced by the study time of one's peers, thus removing a channel that would otherwise be labeled as "contextual effects".

Nevertheless, because, in theory, there could be spillovers not captured by our proposed mechanism, it is prudent to thoroughly examine whether friend characteristics explain achievement, even after accounting for our mechanism of interest. To this end, we re-estimated the model using two alternative specifications. First, since a human capital spillover would most naturally emerge from friend human capital types, we extend the technology (2) to allow for direct achievement transmission via human capital types:

$$y(s_{it}, \mu_{yi}) = \beta_1 + \beta_2 s_{it} + \beta_{3,\text{cont}} \mu_{y,-i,t} + \mu_{yi}, \tag{27}$$

where $\mu_{y,-i,t} \equiv \frac{\sum_{j=1}^{N} A_t(i,j)\mu_{yj}}{\sum_{j=1}^{N} A_t(i,j)}$, i.e., the average of period-t-friend human capital types. This specification's parsimoniousness makes it attractive from a practical level, but it is also conceptually attractive, as one would naturally expect friends with higher-than-predicted achievement (i.e., those with higher own human capital types, μ_{yj}) to be those who would also transmit more achievement to their friends. In this specification, a human capital spillover in the production of student achievement would correspond to $\beta_{3,\text{cont}} \neq 0$. As discussed in detail in Appendix E.1, we fail to reject that $\beta_{3,\text{cont}}$ is zero, with a point estimate of $\widehat{\beta}_{3,\text{cont}} = 0.111$ that has an accompanying standard error of 0.138.[46] In our second specification, also shown in Appendix E.1, we show that the restriction that the determinants of one's own human capital type and one's human capital spillovers on one's friends are the same (to scale) does not drive the lack of significance in the estimates of direct human capital spillovers.

[45] Foster (2006) and Stinebrickner and Stinebrickner (2006), which are especially relevant because they focus on first-year grade performance, suggest (and provide evidence) that peers may be most likely to influence first-year grades by affecting time use.

[46] These results are based on Tobit specifications for GPA and reported study times, which are essentially equivalent to those from the least-squares specification.

Because we have data on both inputs and outcomes, there is more than one place in which contextual effects could enter our model. We chose the extensions described above instead of, say, including a direct effect of friend characteristics in the cost function (3), because the results from our specification test do not provide strong evidence of omitted characteristics in the determination of study time choices.[47]

Altogether, these findings lead us to conclude that mechanisms involving a direct role of friends characteristics in explaining achievement (either directly, or via study effort) are not motivated in our application. Given our *a priori* belief that this would be the case, we have therefore chosen to retain the specification without human capital spillovers as our baseline specification.

7.5 Production Complementarities

As we show in Appendix D.3.2, our data on study time inputs and achievement outcomes allow us to separately identify production complementarities from cost-based mechanisms (which, as we show in Appendix D.3.1, are observationally equivalent to conformity-based mechanisms). As we discussed in Section 4.2.1, prior research suggests that production complementarities may not be very large, because students are not obliged to talk about coursework with their friends. However, given our unique ability to separately identify them from other proposed mechanisms, it is prudent to examine the potential role they play in determining study time and achievement. To this end, we also re-estimated the model using a specification that extends the technology (2) to be

$$y(s_{it}, \mu_{yi}, s_{-it}) = \beta_1 + \beta_2 s_{it} + \beta_{3,\text{comp}} \frac{s_{it}}{s_{-it}} + \mu_{yi}, \tag{28}$$

where s_{it} is own study effort and s_{-it} is friend study effort. If $\beta_{3,comp.} < 0$, then increases in peer effort increase the marginal product of one's own effort. As we discuss in Appendix E.2, we fail to reject that $\beta_{3,\text{comp}}$ is zero, with a point estimate of $\widehat{\beta}_{3,comp} = 0.904$ that has an accompanying standard error of 0.633.[48] This estimation result, combined with our *a priori* belief that production complementarities would likely not play an important role in our context, led us to retain the specification without production complementarities in the technology as our baseline.

[47] Moreover, from a conceptual standpoint, it is *a priori* not obvious why friend characteristics, such as their high school GPA, would relate to one's own study time choices, after taking into account how much friends study. That being said, as we discuss in Appendix F, the negative result of Manski (1993) does not apply in our setting, meaning such contextual effects would be identified under the linear model.

[48] This result is based on Tobit specifications for GPA and reported study times, which are essentially equivalent to those from the least-squares specification.

7.6 Dynamic Behavior and Model Validation

Our framework assumes that the human capital type is constant across semesters—that is, first semester achievement does not increase students' human capital at the beginning of the second semester. While the discussion in Section 4.2.2 suggests that this is reasonable from a conceptual standpoint, it is worth checking this assumption's validity. In Appendix E.3, we perform an out-of-sample test based on the intuition that, if the static assumption were violated, a model estimated using only first-semester data might have difficulty fitting second-semester outcomes. Indeed, such an exercise could also be useful in discerning, more generally, whether the assumed micro-structure of our model (e.g., functional form assumptions, etc.) does a reasonably good job of capturing our context. We find that the second-semester fit is good, which provides some support for the notion that the static model represents a reasonable approximation to reality.

8 Quantitative Findings

How much does it matter who your friends are? To help answer this question, we use our estimated model to conduct two counterfactual exercises. First, we characterize how students respond to changes in friend study time by exogenously increasing (shocking) the study time of each student and measuring how outcomes would change for other students in the network. In addition to providing evidence about how network structure and student characteristics jointly determine how students are affected by their peers, this exercise provides a natural framework for quantifying the importance of equilibrium effects as well as the importance of heterogeneity in the effect of peers. Second, because peer effects are a function of not only how students respond to changes in peer inputs but also who is friends with whom, we examine how outcomes would differ if, instead of sorting into friendships as summarized in Table 1, students were randomly assigned friends. This exercise provides a natural comparison point from which we can assess the importance of homophily in friendships.

Throughout this section, we compare outcomes between baseline and counterfactual scenarios for achievement, own study time, and friend study time. We use s_{it}^{cf} and s_{it}^{baseline} to denote student i's study time in the counterfactual and baseline scenarios, respectively. We define the treatment effect on achievement for student i in period t as $\Delta_{it}^{y} \equiv y(s_{it}^{\text{cf}}, \mu_{yi}) - y(s_{it}^{\text{baseline}}, \mu_{yi})$. Treatment effects for own and friend study time are defined analogously.

8.1 Network Structure, Student Characteristics, and the Response to Peer Input Changes

To provide quantitative evidence about how students respond to changes in peer study time, we estimate the impulse response to an impulse of increasing study effort. Specifically, we increase (shock) the study time of a single student by one hour per day over that student's baseline equilibrium study time in a particular semester and examine the responses of all other students in the network in that semester. We summarize our findings when we perform this exercise 614 times (once for each of the 307 students in each of the two semesters).

The averages in the first row of Table 5 show how the mean effect of the study shock evaluated at the new equilibrium, i.e., taking into account the full set of feedback effects in the network, varies with a student's distance from the shocked student. For example, to obtain the number in the second column we first compute, for each student j in each of the two semesters t, the mean response in achievement for all students who are one link away from j when j is shocked in semester t. Averaging this mean response over all shocked students j and both semesters shows that students who are one link away from the shocked student have an average achievement gain of 0.074 GPA points. Similarly, the third, fourth, and fifth columns, respectively, show that students who are two links, three links, and four links away from the shocked student, respectively, have average achievement gains of 0.021, 0.005, and 0.001 GPA points, respectively. The final column involves first computing, for each student j in each of the two semesters t, the *total* response in achievement, $\sum_{i \neq j} \Delta_{it}^{y}$, for all students (other than j) who are in the network when j is shocked in semester t. Averaging this total response over all students and semesters shows that, on average, the total effect of the shock is 0.48 GPA points.

Effects evaluated at the new equilibrium will be larger than partial equilibrium effects, which only take into account how the shock to a student influences students who are directly linked to her (i.e., iterating best response functions once). To quantify the importance of this difference, the second row of Table 5 shows the partial equilibrium effects. The average effect on students who are one link away from the shocked student is about 1/4 smaller under partial equilibrium than when than under the new equilibrium (0.056 vs. 0.074 GPA points), while, by definition, the effect on the (typically) large number of students who are two or more links away from the shocked student is zero in the partial equilibrium case. The last column shows that, on average, the total response of the shock is only 0.18 GPA points. Therefore, if we considered only partial equilibrium effects we would, on average, understate the achievement response by 63%.

We next examine how much the total response $\sum_{i \neq j} \Delta_{it}^{y}$ varies, depending on which student j is shocked in t. We find that the total response in achievement varies substantially

Table 5: Average change in achievement (GPA points)

	Avg. response, by distance from shocked node					Total
Dist. from shocked stud.:	0	1	2	3	4	response
New equilibrium	0.245	0.074	0.021	0.005	0.001	0.481
Partial equilibrium	0.245	0.056	0.000	0.000	0.000	0.180

Note: The top row presents the mean effect on achievement (averaging over shocked students and semesters) at the new equilibrium, by distance from shocked student, where the shocked student has distance 0. The bottom row presents the mean effect on GPA immediately due to the impulse, by distance from shocked student. The mean total response, in the last column, is the average GPA response to shocking students j over periods t, excluding the effect on the shocked student, i.e., $\frac{1}{\sum_{j,t} 1} \sum_{j,t} \left(\sum_{i \neq j} \Delta_{it}^{y} \right)$.

depending on which student is shocked. For example, the first quartile, median, and third quartile of the total increase in achievement at the new equilibrium are 0.31, 0.46, and 0.62 GPA points, respectively. Heterogeneity in effects will depend both on the location of the shocked student in the network and the responsiveness of students that are close to the shocked student. The importance of the former is explored in Appendix G, where we explore the relationship between a node's centrality and the effect of a shocked node. The importance of the latter was the focus of Section 7.1. Here, we illustrate how the structure of the social network interacts with the distribution of best response functions to produce variation in achievement gains, by examining the subgraphs of the students within three degrees of i) the student whose shock creates the largest total achievement response and ii) the student whose shock creates the smallest total achievement response.

The left panel of Figure 1a shows the subgraph containing students within three degrees of the student whose shock creates the largest total achievement response (1.20 GPA points). The right panel shows the subgraph containing students within three degrees of the student whose shock creates the smallest total achievement response (0.084 GPA points). In each case, the shocked student is denoted by a red star. Squares represent males and circles represent females. Shapes corresponding to black students are shaded and those corresponding to nonblack students are unshaded. The area of the circle or square representing a student other than the shocked student is proportional to the slope of that student's best response function, where larger shapes correspond to more reactive students. Both subgraphs show homophilous sorting: black students tend to be friends with other black students (and nonblack students with nonblack students), males tend to be friends with males (and females with females). In general, students with steeper best response functions tend to be friends with each other.

Differences in the total response can be due to differences in link structure and how heterogeneous students are arranged on the network. The link structures of the subgraphs are very different. The shocked student in the left panel has more friends (7 vs. 1) and more students within two and three degrees (respectively, 17 vs. 5 and 39 vs. 23).[49] In addition to the structure of links, how the heterogeneous students are arranged on the network matters. Although the average slope of best response functions is roughly similar between the subgraphs, 0.752 in the left vs. 0.681 in the right, the friends of the shocked student in the left panel have steeper best response functions than the friend of the shocked student in the right panel. In the right panel, the shock is immediately dampened by being passed through the student's only, relatively nonreactive friend.

Figure 1b shows the analogous plot, where the area of the shape is now proportional to the achievement gain for that student. The effect of the shock dies off in the same pattern illustrated by the first row of Table 5, that is, shapes further from the star tend to be smaller. Friends of the shocked student in the left subgraph gain much more than the friend of the shocked student in the right subgraph. Due to the much steeper best response functions of the shocked student's friends, the impulse dies out much less quickly in the left subgraph. Indeed, the gains for students who are two links from the shocked student in the left subgraph are about as large as the gain for the student directly connected to the shocked student in the right subgraph. This persistence comes from both the steeper best response functions of direct friends of the shocked student and the fact that many of them are also connected to each other, further augmenting the effects of the shock through feedback. This implies the effectiveness of policies targeting students may depend critically on how they fit into the arrangement of the social network.[50]

8.2 The Effect of Sorting into Friendships

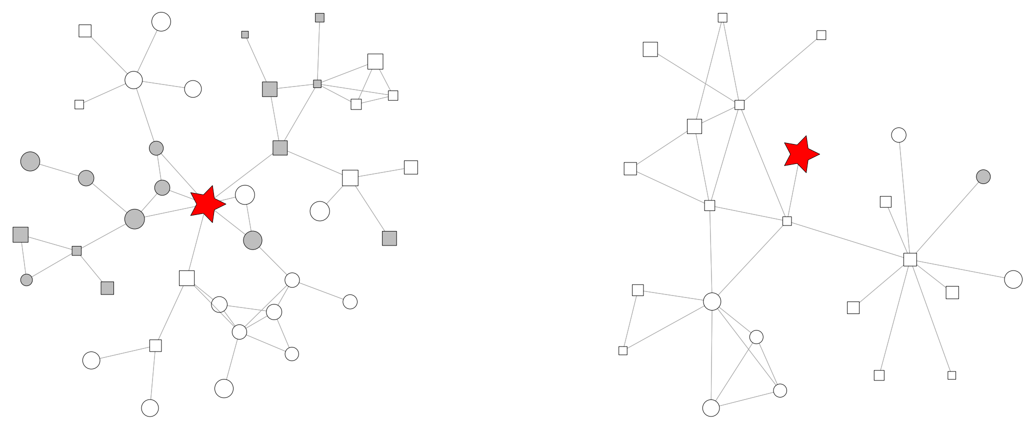
Section 8.1 studied how students respond to the input choices of others, taking into account the baseline network, which exhibits homophily. To directly examine homophily and, therefore, provide further evidence about the importance of peers, we compare achievement under the baseline social network with achievement under a counterfactual where friends are homogeneously distributed across students. In this counterfactual, for each semester, we maintain the marginal distribution of friends per student observed in the data, but replace reported links with random draws from the entire sample of students. We then form a counterfactual symmetrized A matrix in the same manner as it was formed for the actual data, as described in Section 3. Repeating this process 300 times for each of the two semesters produces 300

[49] We limit this illustration to students within three degrees, based on the first row of Table 5 (which shows the total impact dies off quite quickly in distance from the shocked student).

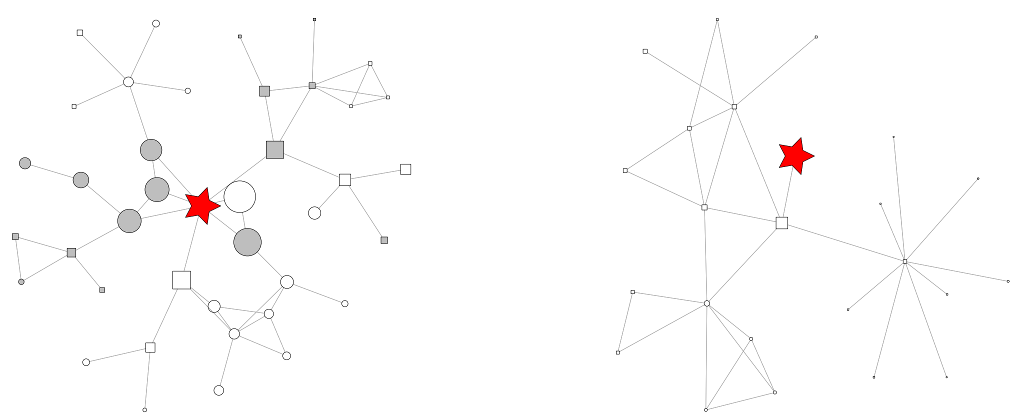
[50] See Fryer (2011) for an example in which students are incentivized based on inputs to achievement.

Figure 1: Subgraphs corresponding to students producing the largest and smallest total achievement responses

(a) Slope of best response functions for students within three degrees of the student producing largest total response when shocked (left) and smallest total response when shocked (right)



(b) Gain in achievement for students within three degrees of the student producing largest total response when shocked (left) and smallest total response when shocked (right)



Note: Red star indicates shocked student, males are square (females are circles), black students are shaded (nonblack students are unshaded), and area of squares and circles is proportional to outcome of interest for corresponding students (i.e., (a) slope of best response function or (b) gain in achievement from shocking starred student) pairs of simulated adjacency matrices.[51]

Table 6 summarizes changes in model outcomes between the baseline and counterfactual, averaged over all simulations. Achievement is measured in GPA points and study times are in hours per day. The first column shows the average change in study time, across all students and all simulated networks, that results from moving to homogeneous (i.e., randomly assigned) friends. The first row shows that, on average, moving to this counterfactual would reduce own study time by 0.09 hours. Intuitively, students who under the baseline have friends with high study types are most harmed by the move to a homogeneous distribution, which makes them much more likely to have lower study type friends. This explains why females, black students, and students with above-median high school GPAs, who tend to be high study types and are seen in Table 1 to often have friends with high-study-type characteristics under the baseline, see own study time fall by 0.18, 0.24, and 0.14 hours, respectively. Conversely, males, who have less studious peers under the baseline, tend to study more when friends are homogenized. Importantly, the estimated complementarities, which arise due to the heterogeneity in best response functions combined with sorting into friendships based on effective study type, imply that the gains of lower study types are smaller than the losses of the higher study types. Accordingly, the standard deviation of own study time drops by 30%. This explains the overall decrease in own study time. Removing the sorting in the manner of our experiment does not merely re-allocate output, but also lowers total output. A similar story drives both the overall results and the stratified results associated with changes in friend study time in the second column of Table 6.

The third column of Table 6 shows the average change in achievement across all students and all simulated networks that result from the changes in study time found in the first column. The first row shows that, on average, moving to the counterfactual would reduce achievement by 0.02 GPA points. However, as expected given the findings of study time, the declines are largest for black students, female students, and students with above-median high school GPAs. As before, the losses to these groups are not offset by the gains to other groups.

Homogenizing the distribution of friends' characteristics would increase the baseline GPA gap between nonblack and black students of 0.5 GPA points by 11% (black students study more than white students in the baseline), reduce the baseline GPA gap between female and male students of 0.31 GPA points by almost 20% (female students study more than male students in the baseline), and reduce the baseline GPA gap between students with

[51] For example, in the first semester the algorithm starts with IID draws of counterfactual "friends per student" from the empirical marginal distribution of friends per student in A_1, divided by two and rounded to the nearest integer, because A_1 has been union-symmetrized. The number of directed links per student is set to the student's "friends per student" draw. Directed links are IID draws from the whole set of other students.

above-median and below-median high school GPAs of 0.60 GPA points by 5% (students with above-median high school GPAs study more than those with below-median high school GPAs in the baseline). Overall, homogenizing friends would reduce the standard deviation of achievement by 4%. However, while reducing homophily could reduce inequality, it would do so at the expense of reducing total achievement.

Table 6: Average changes for study time (hours/day) and achievement (GPA points) resulting from counterfactual homogeneous distribution of friend characteristics, across simulated networks

	Own study time (1)	Friend study time (2)	Achievement (3)
Total	-0.086	-0.080	-0.019
Nonblack	-0.056	-0.025	-0.009
Black	-0.238	-0.344	-0.063
Female	-0.179	-0.221	-0.043
Male	0.045	0.108	0.013
Below-med. HS GPA	-0.026	0.021	-0.003
Above-med. HS GPA	-0.136	-0.177	-0.034

Note: Means are computed over simulated networks.

Finally, we provide a back-of-the-envelope calculation to anchor our findings about the effects of this counterfactual to graduation rates, an outcome of particular interest to policymakers. We do this by estimating the mapping between first year achievement and the probability that a student graduates from college within ten years of starting university.[52] The share of students graduating would fall slightly (about half a percentage point, from a baseline graduation rate of 75%) under the counterfactual assignment of friends. However, as expected given our previous results, there are non-trivial differences in the effects across groups. For example, while the share of female students graduating would decrease by 1.5 percentage points (from a baseline graduation rate of 84%), the share of male students graduating would increase by almost one percentage point (from a baseline graduation rate of 62%). While the share of black students graduating would decrease by 2.8 percentage points (from a baseline graduation rate of 69%), the share of nonblack students graduating would essentially remain the same (i.e., at the baseline graduation rate of 76%). These changes are not trivial when compared to the size of other effects in the literature. For example, Belley and Lochner (2007) find that moving from the lowest to highest income quartile would increase college graduation rates by 10 percentage points.

[52] Details are in Appendix H.1.

9 Conclusion

This paper presents an equilibrium model of student study time choices and the production of achievement. Social interactions are present because costs of study time for a student depend on the study times of that student's peers. We estimate this model and provide evidence that this mechanism is important in the production of academic achievement. Our approach was made possible by three key features of the BPS: direct measurements of study time, measurements of a social network for a cohort of Berea students, and measures of student propensities to study. We develop a specification test that can detect unobserved determinants of study time. The results of our specification test suggest that our study propensity measures play a crucial role in addressing endogeneity concerns.

We use the structural model to examine counterfactuals that are informative about the role of network feedback effects and sorting in peer characteristics. Heterogeneity in student characteristics and how students are interconnected determine the distribution of responses to changes in a student's study time. Our structural approach provides a very clear and intuitive interpretation for quantities of policy interest. For example, we estimate substantial best response heterogeneity, wherein the most reactive student has a best response function slope that is twice as steep as that of the least reactive student. Our results indicate that equilibrium effects, mediated by the whole social network, are quantitatively important in determining the responses of network-wide study time and achievement to shocks in study time. In addition, our results indicate that homophily, or sorting in peers' characteristics, plays an important role in the production of achievement. For example, homogenizing friends would reduce average achievement and the standard deviation of achievement.

A Data

A.1 Survey Questions

Figure A1: Time diary question

Survey #5 (Please complete both sides of this sheet) CPO 1971 (3)

Question A.

Reminders: Be sure to put an arrow (-->) next to the time that it is right now. And label this arrow with the words **YESTERDAY** and **START**.

Beginning with the **What were you doing** box next to the arrow, fill in your activities starting 24 hours ago (yesterday) and ending right before you began completing this survey.

Please use the 13 words listed in BOLD on the right of this page to describe your activities.

Time Period	What were you doing?	Time Period	What were you doing?
MORNING		EVENING	
6:00 AM		6:00 PM	
6:20 AM		6:20 PM	
6:40 AM		6:40 PM	
7:00 AM		7:00 PM	
7:20 AM		7:20 PM	
7:40 AM		7:40 PM	
8:00 AM		8:00 PM	
8:20 AM		8:20 PM	
8:40 AM		8:40 PM	
9:00 AM		9:00 PM	
9:20 AM		9:20 PM	
9:40 AM		9:40 PM	
10:00 AM		10:00 PM	
10:20 AM		10:20 PM	
10:40 AM		10:40 PM	
11:00 AM		11:00 PM	
11:20 AM		11:20 PM	
11:40 AM		11:40 PM	
AFTERNOON		NIGHT	
12:00 noon		12:00 midnight	
12:20 PM		12:20 AM	
12:40 PM		12:40 AM	
1:00 PM		1:00 AM	
1:20 PM		1:20 AM	
1:40 PM		1:40 AM	
2:00 PM		2:00 AM	
2:20 PM		2:20 AM	
2:40 PM		2:40 AM	
3:00 PM		3:00 AM	
3:20 PM		3:20 AM	
3:40 PM		3:40 AM	
4:00 PM		4:00 AM	
4:20 PM		4:20 AM	
4:40 PM		4:40 AM	
5:00 PM		5:00 AM	
5:20 PM		5:20 AM	
5:40 PM		5:40 AM	

LIST OF WORDS in bold

In Class
Attending class, attending labs, attending required class sessions

Studying (Outside of class time)
(refer to **pg 2** for more details)

Athletics
(Intercollegiate or Intramural - games or practice)

Clubs

Exercising

Recreation
(reading which is unrelated to courses, listening to music, watching movie, spending time with friends, etc.)

Shopping

Eating

Sleeping

Partying

Personal

Working (in Labor position)

Other
(Please describe on your sheet)

Figure A2: Friends question

(6

Question K. Please write down the **first and last names** of the four people that have been your best friends at Berea College during **this fall term (2001)** . That is, write down the names of the four people with whom you have been spending the most time during the fall term. Also please mention how many hours per week you spend with each person and how many hours you spend studying or talking about classes with each person.
Please include your boyfriend/girlfriend or husband/wife if they are among your four best friends.
Also include your roommate if he/she is among your four best friends.
Place a check next to the name of your boyfriend/girlfriend or husband/wife.

Four best friends	Hours spent with this person in a typical week	Hours spent with this person studying /talking about classes in a typical week
1. ____________	________	________
2. ____________	________	________
3. ____________	________	________
4. ____________	________	________

Current Roommate: (Your roommate should be listed above also if he/she is one of four best friends.)

______________ ________ ________

Question L. How would you describe your relationship with your roommate during **Fall term**? **Circle one.**
1. Good friends, spent a lot of time together.
2. Got along OK, but didn't spend much time together.
3. Didn't get along very well.
4. Had significant conflicts.
5. Did not have a roommate.

Question M. 1) Since the start of the fall term, did your father lose his job without being able to find a similarly paying replacement job? **Note**: Please answer NO if your father did not lose his job, lost his job but found a similarly paying new job, did not work at a job for pay, you do not know the status of your father , or your father is deceased.

Yes **No** **Not applicable**

2) Since the start of the fall term did your mother lose her job without being able to find a similarly paying replacement job? Note: Please refer to the **Note** in part 1) of this question.

Yes **No** **Not applicable**

Question N. Have you encountered any academic difficulties during the **fall term**? Circle one. **Yes** **No**

If you have encountered academic difficulties during the **fall term**, please circle the people that you discussed these problems with? Also indicate whether each circled item was helpful or not in providing encouragement.

1. Parents	helpful	not helpful
2. Family members other than parents	helpful	not helpful
3. Friends at Berea	helpful	not helpful
4. Friends not at Berea	helpful	not helpful
5. Counselors, Advisers, or Teachers at Berea	helpful	not helpful
6. Former high school or elementary teachers	helpful	not helpful
7. Other (describe) ____________	helpful	not helpful

A.2 Summary Statistics Tables

Table A1 shows descriptive statistics of student characteristics. The first row in each of the six panels shows overall descriptive statistics for the variable of interest described in the first column. Forty-four percent of students are male, 18% of students are black, the mean high school grade point average for the sample is 3.39, the mean combined score on the American College Test (ACT) is 23.26, and, on average, students studied 11.24 hours per week in high school and expect to study 24.96 hours per week in college. The subsequent rows in each panel show descriptive statistics for the variable of interest in the first column for different subgroups. For example, the third panel shows that, on average, males have lower high school grade point averages than females (3.24 vs. 3.51) and black students have lower high school grade point averages than nonblack students (3.14 vs. 3.45). The fifth panel shows that black students studied more, on average, in high school than other students (15.29 vs. 10.36).[53]

Table A2 shows descriptive statistics of outcomes during the first year. The first rows of panels 1 and 2, respectively, show that, on average, students study 3.49 hours per day in the first semester and 3.50 hours per day in the second semester.[54] The subsequent rows of the first two panels show that, on average, males study less than females, black students study more than nonblack students, and students with above-median high school GPAs study more than students with below-median high school GPAs.[55] The first rows of panels 3 and 4, respectively, show that the average first semester GPA is 2.89 and the average second semester GPA is 2.93. The subsequent rows of the third and fourth panels show that males, black students, and students with below-median high school GPAs all have lower average GPAs than their counterparts.[56]

[53]The first two differences in means are significantly different at the 0.001 level. The averages of high school study time for black students and nonblack students are significantly different at the 0.01 level.

[54]Descriptive statistics about study time outcomes presented in Table A2 are computed at the level of individual study time reports, of which there may be up to four in each semester, for each student. When computing other descriptive statistics (including regressions), we use the semester-specific average (over the study time reports) for each student The two measures are very similar, other than the larger variance of the individual-report-based measure. As we make clear when we describe our estimation procedure, we use individual study time reports when estimating the structural parameters of our model.

[55]Pooling observations from both semesters, the first and last differences in means are significantly different at a 0.05 level and, given the relatively small number of black students, the middle difference in means is significant at a 0.10 level.

[56]Pooling observations from both semesters, all of these differences are significant at a 0.05 level.

Table A1: Own summary statistics

Variable	Group	N	Mean	SD	Min	q1	q2	q3	Max
(1) Male indicator	all	307	0.44	0.5	0	0	0	1	1
	black	55	0.45	0.5	0	0	0	1	1
	nonblack	252	0.43	0.5	0	0	0	1	1
	above-med. HS GPA	155	0.33	0.47	0	0	0	1	1
	below-med. HS GPA	152	0.55	0.5	0	0	1	1	1
(2) Black indicator	all	307	0.18	0.38	0	0	0	0	1
	male	134	0.19	0.39	0	0	0	0	1
	female	173	0.17	0.38	0	0	0	0	1
	above-med. HS GPA	155	0.10	0.31	0	0	0	0	1
	below-med. HS GPA	152	0.26	0.44	0	0	0	1	1
(3) HS GPA	all	307	3.39	0.47	1.68	3.09	3.50	3.80	4.00
	male	134	3.24	0.51	1.68	2.9	3.21	3.7	4.00
	female	173	3.51	0.40	2.13	3.30	3.60	3.85	4.00
	black	55	3.14	0.46	2.24	2.78	3.1	3.52	4.00
	nonblack	252	3.45	0.46	1.68	3.19	3.53	3.8	4.00
	above-med. HS GPA	155	3.77	0.17	3.5	3.6	3.8	3.9	4
	below-med. HS GPA	152	3.00	0.35	1.68	2.8	3.08	3.29	3.47
(4) ACT	all	307	23.26	3.61	14	21	23	26	33
	male	134	22.54	3.77	14	20	23	25	31
	female	173	23.82	3.39	17	21	24	26	33
	black	55	19.91	2.51	14	18	20	21	25
	nonblack	252	23.99	3.4	14	22	24	26	33
	above-med. HS GPA	155	24.45	3.53	17	22	25	27	33
	below-med. HS GPA	152	22.04	3.28	14	20	22	24	31
(5) HS study	all	307	11.24	11.35	0	4	8	15	70
	male	134	11.43	11.94	0	3.12	8	15	70
	female	173	11.10	10.9	0	4	9	15	70
	black	55	15.29	14	0	5	10.5	20	70
	nonblack	252	10.36	10.51	0	3	7	14	70
	above-med. HS GPA	155	10.66	10.44	0	4	8	14.5	70
	below-med. HS GPA	152	11.84	12.21	0	3.38	8.25	15	70
(6) Expected study	all	307	24.96	11.61	0	17	23	31	64
	male	134	22.72	11.08	0.97	16	20.75	27.38	64
	female	173	26.68	11.74	0	19	25.5	33	57.5
	black	55	28.56	13.56	0	19	25	38.5	57.5
	nonblack	252	24.17	11.01	0	17	22.5	30.62	64
	above-med. HS GPA	155	25.18	10.47	0	18	23.5	32	56
	below-med. HS GPA	152	24.72	12.69	0	16	22.25	30.12	64

Note: The rows in each panel show descriptive statistics for the variable of interest in the first column, for the group in the second column. GPA is measured in GPA points (0-4). HS study and expected study are measured in hours/week.

Table A2: Own summary statistics for outcomes, by semester

Variable	Group	N	Mean	SD	Min	q1	q2	q3	Max
(1) Sem. 1 Own study	all	955	3.49	2.23	0	2	3.25	4.67	16
	male	401	3.23	2.38	0	1.67	3	4.33	14.67
	female	554	3.68	2.1	0	2	3.33	5	16
	black	158	3.83	2.23	0	2.33	3.41	5.33	11.67
	nonblack	797	3.43	2.23	0	2	3	4.67	16
	above-med. HS GPA	518	3.62	2.27	0	2	3.33	5	16
	below-med. HS GPA	437	3.34	2.17	0	2	3	4.67	14.67
(2) Sem. 2 Own study	all	945	3.5	2.12	0	2	3.33	4.67	14.33
	male	384	3.22	2.11	0	2	3	4.33	12
	female	561	3.7	2.11	0	2	3.33	5	14.33
	black	169	3.75	1.98	0	2.33	3.33	5	9.67
	nonblack	776	3.45	2.15	0	2	3.31	4.67	14.33
	above-med. HS GPA	513	3.66	2.06	0	2	3.33	5	12
	below-med. HS GPA	432	3.32	2.18	0	2	3	4.67	14.33
(3) Sem. 1 GPA	all	307	2.89	0.78	0	2.49	3.06	3.46	4.00
	male	134	2.72	0.80	0.30	2.17	2.80	3.29	4.00
	female	173	3.02	0.74	0	2.66	3.13	3.55	4.00
	black	55	2.42	0.78	0	1.82	2.57	2.84	4.00
	nonblack	252	3.00	0.74	0.3	2.58	3.11	3.55	4.00
	above-med. HS GPA	155	3.19	0.62	0.52	2.81	3.29	3.69	4.00
	below-med. HS GPA	152	2.59	0.8	0	2.00	2.66	3.12	4.00
(4) Sem. 2 GPA	all	301	2.93	0.78	0	2.53	3.05	3.46	4.00
	male	131	2.74	0.84	0	2.38	2.82	3.33	4.00
	female	170	3.07	0.71	0.44	2.66	3.20	3.54	4.00
	black	53	2.58	0.86	0.44	2.22	2.62	3.33	3.78
	nonblack	248	3	0.75	0.00	2.58	3.08	3.5	4.00
	above-med. HS GPA	155	3.21	0.66	0	2.82	3.36	3.74	4.00
	below-med. HS GPA	146	2.63	0.79	0.26	2.15	2.66	3.24	4.00

Note: The rows in each panel show descriptive statistics for the variable of interest in the first column, for the group in the second column. GPA is measured in GPA points (0-4). Own study is measured in hours/day and in this table is reported at the individual study report level.

Table A3: Average friend summary statistics, pooled over both semesters

Variable	Group	N	Mean	SD	Min	q1	q2	q3	Max
(1) Num. friends	all	614	3.31	1.58	1	2	3	4	10
	male	268	3.22	1.59	1	2	3	4	10
	female	346	3.38	1.57	1	2	3	4	9
	black	110	3.21	1.35	1	2	3	4	7
	nonblack	504	3.33	1.62	1	2	3	4	10
	above-med. HS GPA	310	3.34	1.62	1	2	3	4	10
	below-med. HS GPA	304	3.28	1.53	1	2	3	4	8
(2) Frac. male friends	all	614	0.43	0.39	0	0	0.33	0.75	1
	male	268	0.74	0.31	0	0.5	0.82	1	1
	not male	346	0.18	0.25	0	0	0	0.33	1
	black	110	0.43	0.4	0	0	0.33	0.83	1
	not black	504	0.42	0.39	0	0	0.33	0.75	1
	above-med. HS GPA	310	0.35	0.38	0	0	0.25	0.67	1
	below-med. HS GPA	304	0.5	0.39	0	0	0.5	1	1
(3) Frac. black friends	all	614	0.18	0.32	0	0	0	0.25	1
	male	268	0.18	0.32	0	0	0	0.25	1
	not male	346	0.17	0.33	0	0	0	0.2	1
	black	110	0.69	0.38	0	0.43	1	1	1
	not black	504	0.07	0.16	0	0	0	0	1
	above-med. HS GPA	310	0.10	0.22	0	0	0	0	1
	below-med. HS GPA	304	0.26	0.39	0	0	0	0.45	1
(4) Friend HS GPA	all	614	3.37	0.32	2.24	3.2	3.41	3.62	4
	male	268	3.29	0.33	2.25	3.07	3.34	3.53	4
	not male	346	3.44	0.29	2.24	3.29	3.46	3.64	4
	black	110	3.18	0.34	2.25	2.96	3.19	3.41	4
	not black	504	3.42	0.30	2.24	3.25	3.45	3.63	4
	above-med. HS GPA	310	3.46	0.27	2.65	3.29	3.46	3.63	4
	below-med. HS GPA	304	3.29	0.35	2.24	3.08	3.35	3.55	3.92
(5) Friend ACT	all	614	23.29	2.63	16	21.67	23.33	25	32
	male	268	22.72	2.64	16.33	21	23	24.64	31
	not male	346	23.74	2.54	16	22	23.67	25.5	32
	black	110	21.2	2.53	16	19.33	21	22.5	29
	not black	504	23.75	2.43	16.33	22.25	23.67	25.33	32
	above-med. HS GPA	310	23.79	2.42	17.5	22.23	23.67	25.33	32
	below-med. HS GPA	304	22.78	2.74	16	21	23	25	30
(6) Friend HS study	all	614	11.03	7.64	0	6	9.5	14.47	70
	male	268	10.53	7.37	0.5	5.17	9	14	37.33
	not male	346	11.41	7.83	0	6.5	9.79	14.6	70
	black	110	14.62	7.31	2.5	9.18	13.92	18.75	37
	not black	504	10.24	7.49	0	5.5	8.68	13.19	70
	above-med. HS GPA	310	11.48	8.44	0.5	6	9.7	14	70
	below-med. HS GPA	304	10.57	6.7	0	6	9.17	14.64	37.33
(7) Friend expected study	all	614	24.82	7.4	0	19.75	23.55	29.62	55
	male	268	22.89	6.97	4.06	18.23	21.65	27.05	55
	not male	346	26.33	7.38	0	21.02	25.06	31.38	52
	black	110	28.05	8.53	12	21.35	28.9	33.79	51
	not black	504	24.12	6.94	0	19.5	23	28.2	55
	above-med. HS GPA	310	24.72	7.42	0	20	23.55	29.48	55
	below-med. HS GPA	304	24.93	7.39	10.5	19.31	23.61	29.81	52
(8) Friend study	all	614	3.5	1.72	0	2.47	3.26	4.28	11.93
	male	268	3.16	1.49	0.5	2.21	3	3.88	8.46
	not male	346	3.76	1.83	0	2.65	3.51	4.5	11.93
	black	110	3.78	1.77	0.5	2.7	3.52	4.47	10.81
	not black	504	3.44	1.7	0	2.4	3.2	4.24	11.93
	above-med. HS GPA	310	3.64	1.79	0	2.56	3.36	4.41	11.93
	below-med. HS GPA	304	3.36	1.64	0.5	2.36	3.17	4.13	10.81

Note: The rows in each panel show descriptive statistics for the variable of interest in the first column, for the group in the second column. GPA is measured in GPA points (0-4). Own and friend HS study and expected study (top panel) are measured in hours/week. Friend study (bottom panel) is measured in hours/day. The variable "Friend z" for student i in period t is the average of the variable z across i's friends in period t.

B Parameter Estimates

Table A4: Parameter Estimates

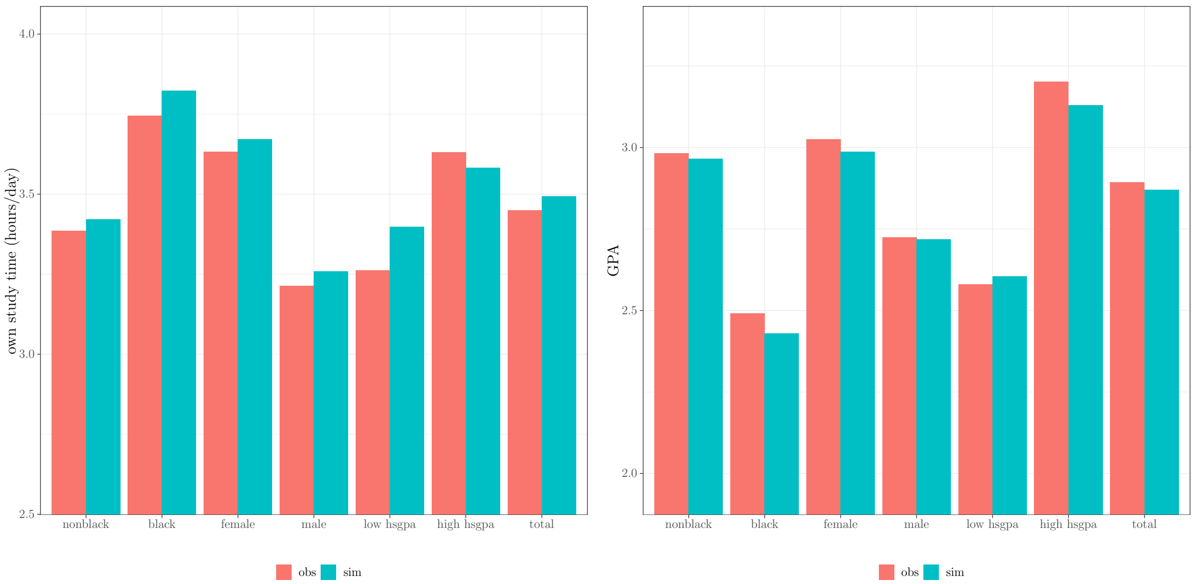
Parameter	Estimate	SE	Description
Production function*			
β_1	-0.073	0.4257	intercept
β_2	0.245	0.0683	marginal product of own study time
$\omega_{y,\text{HS GPA}}$	0.445	0.0775	coefficient on HS GPA in human capital type
$\omega_{y,\text{ACT}}$	0.039	0.0108	coefficient on ACT in human capital type
$\omega_{y,\text{Black}}$	-0.214	0.0978	coefficient on Black in human capital type
$\omega_{y,\text{Male}}$	-0.056	0.0779	coefficient on Male in human capital type
$\omega_{y,\text{HS study}}$	-0.007	0.0039	coefficient on HS study in human capital type
$\omega_{y,\text{expected study}}$	-0.005	0.0032	coefficient on expected study in human capital type
Study cost function / Best response function**			
χ_1	3.353	3.2751	affects common best response intercept
χ_2	-0.281	4.5767	affects heterogeneity in best response intercept
χ_3	1.000	–	normalization
χ_4	-4.288	2.3765	affects common best response slope
χ_5	-0.670	0.2696	affects heterogeneity in best response slope
$\tau_{\mu,1}$	0.105	0.0358	linear term for study type
$\tau_{\mu,2}$	-0.003	0.0026	quadratic term for study type
$\omega_{s,\text{HS GPA}}$	1.000	–	coefficient on HS GPA in study type, fixed to 1
$\omega_{s,\text{ACT}}$	-0.054	0.0859	coefficient on ACT in study type
$\omega_{s,\text{Black}}$	-0.646	0.8193	coefficient on Black in study type
$\omega_{s,\text{Male}}$	-0.925	0.8490	coefficient on Male in study type
$\omega_{s,\text{HS study}}$	0.344	0.2080	coefficient on HS study in study type
$\omega_{s,\text{expected study}}$	0.007	0.0309	coefficient on expected study in study type
Shocks			
σ_{η_y}	0.675	0.0181	sd measurement error for human capital
σ_{η_s}	2.039	0.0363	sd measurement error for observed study time

* Production function: $y_{it} = \beta_1 + \beta_2 s_{it} + \mu_{yi}$, where $\mu_{yi} = x_i'\omega_y$.

** Best response function: $s_{it} = \frac{\beta_2 - \chi_1}{1+\chi_4} + \frac{-\chi_2}{1+\chi_4}\gamma(\mu_{si}) + \frac{\chi_4}{1+\chi_4} s_{-it} + \frac{\chi_4 \cdot \chi_5}{1+\chi_4}\gamma(\mu_{si}) s_{-it}$, where $\mu_{si} = x_i'\omega_s$. Recall that we allowed for $\tau_s \in [0,1]$ in our estimation, but, finding it to be indistinguishable from 1, we fixed $\tau_s = 1$ and re-estimated.

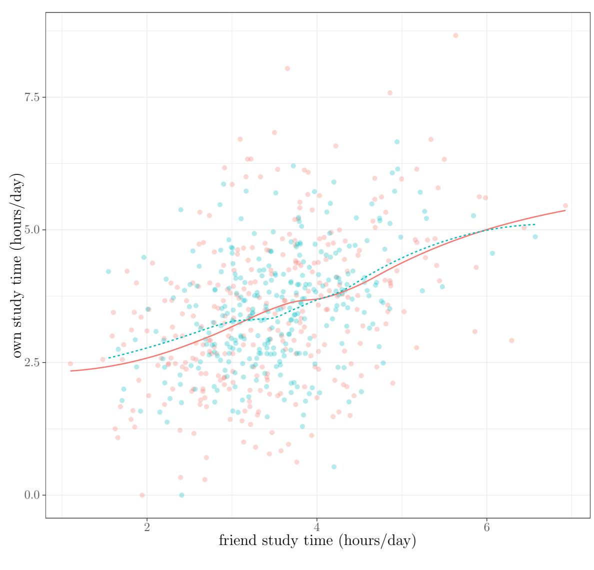
C Model Fit

Figure A3: Fit of mean study time (left) and GPA (right), by group



Note: “obs” are means computed using the data and “sim” are means of outcomes simulated from the model. “High-” and “Low HS GPA” respectively refer to above- and below-median high school GPA.

Figure A4: Fit of own study time against friend study time



Note: “obs” correspond to data and “sim” correspond to model simulations. Each point corresponds to a pair of own and friend study time (both are measured in hours/day). The lines are fitted values from a local quadratic regression. For each value of friend study time the fit is computed using the closest 75% of the observations via weighted least squares, with weights proportional to $(1-(\text{distance}/\text{max. distance})^3)^3$). See stat_smooth in the R package ggplot2 for details (Wickham, 2009; R Core Team, 2019).

D Additional Model Material

D.1 Concavity of Best Response Function

The optimal choice of study time for the period game solves the function $G(s, s_{-i}) = \frac{\partial c}{\partial s} - \beta_2 = 0$. To find how s varies with friend study time, use the Implicit Function Theorem:

$$\frac{\partial s}{\partial s_{-i}} = -\frac{\frac{\partial G}{\partial s_{-i}}}{\frac{\partial G}{\partial s}} = -\frac{\frac{\partial^2 c}{\partial s \partial s_{-i}}}{\frac{\partial^2 c}{\partial s^2}}.$$

If friend study time decreases the cost of increasing one's own study time, the numerator is positive. If the cost of studying is convex in own study time, the denominator is negative, meaning the overall sign is positive. Moreover, if friend study time enters $c(\cdot)$ in a weakly concave manner, e.g., $\tau_s \leq 1$, the numerator is weakly smaller in absolute value for larger values of s_{-i}, i.e., study time is weakly concave in friend study time.

D.2 Proof of Existence and Uniqueness of Equilibrium

Claim 2. *Let $k = 24$. There exists a unique pure strategy Nash equilibrium if $\psi_i : R^N \mapsto R$ are weakly concave and weakly increasing, $\psi_i(0) > 0$, and $\psi_i(k) < k$ for $i \in N$.*

Proof. Define $\mathbf{S} = [0, k]^N$, i.e., a compact and convex set. Define a function Ψ:

$$\Psi : \mathbf{S} \mapsto \mathbf{S} = \begin{bmatrix} \psi_1(x_{-1}) \\ \psi_2(x_{-2}) \\ \vdots \\ \psi_N(x_{-N}) \end{bmatrix}.$$

Existence: $\Psi(\cdot)$ is a continuous self map on the compact set $\mathbf{S}$, so an equilibrium exists by Brouwer's Fixed Point Theorem.

Uniqueness: If $\Psi(\cdot)$ is strictly concave and weakly increasing we can apply Kennan (2001). Next, consider the case where $\Psi(\cdot)$ is linear (i.e., weakly, but not strictly, concave), in which case we can prove $\Psi(\cdot)$ is a contraction. Write the linear form of $\Psi(\cdot)$ as

$$\Psi(X) = \begin{bmatrix} \alpha_{11} + \alpha_{21} x_{-1} \\ \alpha_{12} + \alpha_{22} x_{-2} \\ \vdots \\ \alpha_{1N} + \alpha_{2N} x_{-N} \end{bmatrix},$$

where, by assumption, $\max_{i \in N}\{\alpha_{2i}\} < 1$. Let distance be calculated according to the taxicab distance, i.e., $d(X_1, X_2) = \sum_{g \in N} |X_{1g} - X_{2g}|$ for $X_1, X_2 \in \mathbf{S}$. The Contraction Mapping

Theorem holds if $d(\Psi(X_1),\Psi(X_2)) \leq bd(X_1,X_2)$, for $b \in (0,1)$. Calculating this for the special case where Ψ is a linear map, we have

$$d(\Psi(X_1),\Psi(X_2)) = \sum_{i\in N}\alpha_{2i}|X_1 - X_2| \leq \max_{i\in N}\{\alpha_{2i}\}|X_1 - X_2| < d(X_1,X_2),$$

i.e., the condition for the Contraction Mapping Theorem is satisfied, where $b = \max_{i\in N}\{\alpha_{2i}\} \in (0,1)$. □

D.3 Other Mechanisms for Social Interactions

D.3.1 Cost-Reduction Specification of Cost Function

We refer to the cost function specification in (3) as the "conformity model". Consider the alternative effort cost function, which we refer to as the "cost-reduction model":

$$c(s_{it}, s_{-it}, \mu_{si}) = \theta_1 s_{it} + \theta_2\gamma(\mu_{si})s_{it} + \frac{\theta_3 s_{it}}{s_{-it}^{\tau_s}} + \frac{\theta_4\gamma(\mu_{si})s_{it}}{s_{-it}^{\tau_s}} + \frac{\theta_5 s_{it}^2}{2s_{-it}^{\tau_s}}. \tag{A1}$$

In this specification, the cost of studying is allowed to depend on friend study time to take into account that studying may be less arduous when one's friends are studying.

Solving the student's problem results in the best response function

$$s_{it} = -\frac{\theta_3}{\theta_5} - \frac{\theta_4}{\theta_5}\gamma(\mu_{si}) + \frac{(\beta_2-\theta_1)}{\theta_5}s_{-it}^{\tau_s} - \frac{\theta_2}{\theta_5}\gamma(\mu_{si})s_{-it}^{\tau_s}. \tag{A2}$$

Equation (A1) shows that the term associated with θ_5 allows the student's cost function to be convex, a common assumption in this literature, and equation (A2) shows that one of the preference parameters θ must be normalized. If we make a similar normalization as was performed for the conformity model, by setting $\theta_5 = 1$,[57] we obtain

$$s_{it} = \underbrace{-\theta_3}_{\frac{\beta_2-\chi_1}{1+\chi_4}} + \underbrace{-\theta_4}_{\frac{-\chi_2}{1+\chi_4}}\gamma(\mu_{si}) + \underbrace{(\beta_2-\theta_1)}_{\frac{\chi_4}{1+\chi_4}}s_{-it}^{\tau_s} + \underbrace{-\theta_2}_{\frac{\chi_4\cdot\chi_5}{1+\chi_4}}\gamma(\mu_{si})s_{-it}^{\tau_s}. \tag{A3}$$

That is, we can represent the parameters in equation (A1) above in terms of parameters in (3), which are in braces beneath their counterparts in the cost-reduction model in (A3). Both cost-function specifications result in the same reduced-form policy function identifying four reduced-form coefficients, which map to different sets of four structural cost parameters, depending on the cost-function specification. Thus, the distinction between the different for-

[57] As was also the case with our normalization of $\chi_3 = 1$ in the conformity based specification, this normalization clearly shows that we allow for the possibility of finding no evidence of endogenous social interactions; this would occur if we estimated that both $(\beta_2 - \theta_1) = 0$ and $\theta_2 = 0$.

mulations of the cost function—a cost of deviating from friend behavior vs. a cost (reduction) from studying with ones friends—has no empirical content.

D.3.2 Production Complementarities

Suppose we did not have achievement data. For simplicity, consider the homogeneous, linear, best response specification (i.e., $\theta_2, \theta_4 = 0$); the following result also obtains when using the more general specification of the cost function.[58] Consider the following specification of our achievement equation:

$$y(s_{it}, \mu_{yi}, s_{-it}) = \beta_1 + \beta_2 s_{it} + \beta_{3,\text{comp}} \frac{s_{it}}{s_{-it}} + \mu_{yi}, \tag{A4}$$

where s_{it} is own study effort and s_{-it} is friend study effort. If $\beta_{3,\text{comp}} < 0$, then increases in peer effort increase the marginal product of one's own effort.[59] The student's problem would still be separable across periods, resulting in the best response function

$$s_{it} = (\beta_{3,\text{comp}} - \theta_3) + (\beta_2 - \theta_1) s_{-it}. \tag{A5}$$

It is obvious from (A5) that we cannot separately identify $\beta_{3,\text{comp}}$ and θ_3 without having data on the marginal product of inputs (i.e., data on achievement outcomes). Indeed, this is the same argument that, without data on achievement, we could not identify the extent to which students study because it is enjoyable (θ_1) versus doing so because it affects their achievement (β_2). On the other hand, having both study time and achievement data would clearly allow one to identify the extent to which production complementarities underlie social interactions.

E Additional Estimation Results

E.1 Human Capital Spillovers ("Contextual Effects")

Table A5 presents the estimation results of our specifications allowing for contextual effects in achievement, or human capital spillovers, described in Section 7.4.[60] Specification (1)

[58] Due to its relative parsimony, we use the cost-reduction based specification for the cost function, which we show is observationally equivalent to the conformity based specification in Appendix D.3.1.

[59] Note, one could instead have defined the production complementarity according to $\beta_{3,\text{comp}} s_{it} s_{-it}$, where $\beta_{3,\text{comp}} > 0$ would correspond to production complementarities. Though such a functional form would technically produce identification in the cost-reduction model we consider, it would not in the conformity model (where the interaction above, $\beta_{3,\text{comp}} s_{it}/s_{-it}$, for similar reasons, would produce identification via functional form).

[60] We use the cost-reduction based specification for the cost function, which we show is observationally equivalent to the conformity based specification in Appendix D.3.1. Also, we present results based on Tobit specifications for GPA and reported study times, which are essentially equivalent to those from the least-squares specification.

presents the baseline estimates (i.e. those where friend characteristics do not directly affect achievement), specification (2) presents results obtained when we re-estimated parameters allowing for contextual effects generated by human capital type, as in (27), and specification (3) presents results obtained when we re-estimated parameters allowing for achievement contextual effects generated by a more flexible specification based on a new "contextual human capital type". Specifically, we define student i's contextual human capital type according to $\mu_{y,\text{cont},i} = x_i'\omega_{y,\text{cont}}$, where $\omega_{y,\text{cont}}$ is a vector containing six new parameters (one for each characteristic entering human capital and study types), and extend the technology (equation (2)) to be

$$y(s_{it}, \mu_{yi}) = \beta_1 + \beta_2 s_{it} + \mu_{yi} + \mu_{y,\text{cont},-i,t}, \tag{A6}$$

where $\mu_{y,\text{cont},-i,t} \equiv \frac{\sum_{j=1}^{N} A_t(i,j)\mu_{y,\text{cont},j}}{\sum_{j=1}^{N} A_t(i,j)}$, i.e., the average of period-t-friend contextual human capital types. In this more flexible specification, a human capital spillover in the production of student achievement would correspond to one of the parameters in $\omega_{y,\text{cont}} \neq 0$.

In specification (2), we obtain a point estimate on the achievement contextual effect parameter of $\widehat{\beta}_{3,\text{cont}} = 0.111$, which has a standard error of 0.138. Similarly, in specification (3), none of the estimated coefficients in $\widehat{\omega}_{y,\text{cont}}$, reported in the bottom six rows of the top panel, are significantly different from zero. The coefficient on friend HS GPA ($\omega_{y,\text{HS GPA}}$), which would seem to be the most likely source of direct achievement spillovers, is one-tenth the value of the (significant) coefficient on HS GPA in one's own human capital type and not significantly different from zero. The contextual characteristic with the most explanatory power for achievement is the share of male friends ($\omega_{y,\text{Male}}$), although this too is not a significant determinant of achievement. Based on a likelihood-ratio test, we would not reject the baseline model for that in specification (3) at any conventional significance level (the likelihood ratio test statistic has a p-value of 0.3673). We further note that there is a striking similarity between the point estimates and statistical significance of common parameters estimated under the baseline and under both specifications allowing for achievement contextual effects. This means the inclusion of such effects would not appreciably change our quantitative (or qualitative) results.

Our *a priori* belief was that our direct collection of study time data would diminish the potential role played by contextual effects. Because we do not find evidence supporting the direct transmission of peer characteristics in academic achievement, we have retained our baseline specification for the exposition of our results.

Table A5: Estimates for Contextual Effects Specifications

Parameter	Baseline Estimate	Baseline SE	Human Capital Type Contextual Estimate	Human Capital Type Contextual SE	Flexible Contextual Estimate	Flexible Contextual SE
	(1)		(2)		(3)	
Production function*						
β_1	-0.350	0.4185	-0.591	0.5419	-0.895	0.6453
β_2	0.254	0.0651	0.258	0.0681	0.332	0.0946
$\beta_{3,\text{cont}}$			0.111	0.1384		
$\omega_{y,\text{HS GPA}}$	0.470	0.0808	0.462	0.0817	0.449	0.0868
$\omega_{y,\text{ACT}}$	0.047	0.0112	0.046	0.0114	0.047	0.0119
$\omega_{y,\text{Black}}$	-0.213	0.1074	-0.182	0.1141	-0.125	0.1401
$\omega_{y,\text{Male}}$	-0.037	0.0849	-0.021	0.0822	-0.080	0.0991
$\omega_{y,\text{HS study}}$	-0.007	0.0042	-0.007	0.0044	-0.009	0.0053
$\omega_{y,\text{expected study}}$	-0.005	0.0035	-0.005	0.0035	-0.005	0.0037
$\omega_{y,\text{cont,HS GPA}}$					0.044	0.1251
$\omega_{y,\text{cont,ACT}}$					0.005	0.0157
$\omega_{y,\text{cont,Black}}$					-0.119	0.1747
$\omega_{y,\text{cont,Male}}$					0.189	0.1231
$\omega_{y,\text{cont,HS study}}$					-0.010	0.0073
$\omega_{y,\text{cont,expected study}}$					0.006	0.0050
Study cost function / Best response function**						
θ_1	-1.074	0.1551	-1.068	0.1569	-1.051	0.1563
θ_2	0.874	0.2351	0.870	0.2355	0.899	0.2063
θ_3	-0.907	0.8097	-0.918	0.8084	-0.981	0.8805
θ_4	0.096	1.2800	0.098	1.2713	0.183	1.2996
$\tau_{\mu,1}$	0.105	0.0601	0.105	0.0604	0.089	0.0479
$\tau_{\mu,2}$	-0.003	0.0028	-0.003	0.0028	-0.003	0.0023
$\omega_{s,\text{HS GPA}}$***	1.000	–	1.000	–	1.000	–
$\omega_{s,\text{ACT}}$	-0.063	0.0870	-0.065	0.0890	-0.063	0.0930
$\omega_{s,\text{Black}}$	-0.735	0.7459	-0.720	0.7516	-0.710	0.7782
$\omega_{s,\text{Male}}$	-1.065	0.7892	-1.081	0.8004	-1.188	0.8562
$\omega_{s,\text{HS study}}$	0.344	0.1554	0.347	0.1578	0.350	0.1651
$\omega_{s,\text{expected study}}$	0.005	0.0309	0.004	0.0311	-0.0003	0.0324
Shocks						
σ_{η_y}	0.721	0.0185	0.721	0.0185	0.717	0.0187
σ_{η_s}	2.159	0.0377	2.159	0.0377	2.159	0.0376
Log Likelihood	-4696.361		-4695.932		-4693.100	

* Production function in specifications (1)-(2) is $y_{it} = \beta_1 + \beta_2 s_{it} + \beta_{3,\text{cont}}\mu_{y,-i,t} + \mu_{yi}$, where $\mu_{yi} = x_i'\omega_y$ and $\mu_{y,-i,t} \equiv \frac{\sum_{j=1}^{N} A_t(i,j)\mu_{yj}}{\sum_{j=1}^{N} A_t(i,j)}$. Production function in specification (3) is $y_{it} = \beta_1 + \beta_2 s_{it} + \mu_{yi} + \mu_{y,\text{cont},-i,t}$, where $\mu_{yi} = x_i'\omega_y$, $\mu_{y,\text{cont},-i,t} \equiv \frac{\sum_{j=1}^{N} A_t(i,j)\mu_{y,\text{cont},j}}{\sum_{j=1}^{N} A_t(i,j)}$, and $\mu_{y,\text{cont},j} = x_j'\omega_{y,\text{cont}}$.

** Best response function: $s_{it} = -\theta_3 - \theta_4\gamma(\mu_{si}) + (\beta_2 - \theta_1)s_{-it} - \theta_2\gamma(\mu_{si})s_{-it}$, where $\mu_{si} = x_i'\omega_s$. As in the baseline estimates, we have set $\tau_s = 1$.

***: Normalized to 1.

E.2 Production Complementarities

Table A6 presents the estimation results of our specification allowing for production complementarities, as described in Section 7.5.[61] Specification (1) is the baseline estimates (i.e. those without production complementarities) and specification (2) allows for production complementarities, as in (28). Unlike the extensions including achievement contextual effects, extending the model to allow for production complementarities results in a different best response function:[62]

$$s_{it} = (\beta_{3,\text{comp}} - \theta_3) - \theta_4\gamma(\mu_{si}) + (\beta_2 - \theta_1)s_{-it} - \theta_2\gamma(\mu_{si})s_{-it}. \tag{A7}$$

We obtain a point estimate on the production complementarity parameter of $\widehat{\beta}_{3,\text{comp}} = 0.904$, which has a standard error of 0.633. Although this estimate may have a surprising sign, wherein increases in friend study time *reduce* the marginal product of one's own study time (potentially due to friends goofing off when studying together), the statistically insignificant estimate does not provide evidence supporting production complementarities in achievement as the source generating social interactions in our application; rather a cost-based (or, equivalently, conformity-based) mechanism seems to generate the data. As suggested by inspection of (A7), the parameter most affected by this extension is θ_3, the intercept of the best response function. Notably, parameters governing best response function slopes (θ_1, θ_2, and the study type parameters ω_s), which determine the level and distribution of the effects of social interactions in study time, are relatively unaffected.

[61] We use the cost-reduction based specification for the cost function, which we show is observationally equivalent to the conformity based specification in Appendix D.3.1. Also, we present results based on Tobit specifications for GPA and reported study times, which are essentially equivalent to those from the least-squares specification.

[62] As in the baseline specification, we have set the best response function to be linear.

Table A6: Estimates for Production Complementarities Specification

Parameter	Baseline Estimate	Baseline SE	Prod. Complementarities Estimate	Prod. Complementarities SE
	(1)		(2)	
Production function*				
β_1	-0.350	0.4185	-1.099	0.6830
β_2	0.254	0.0651	0.267	0.0693
$\beta_{3,\text{comp}}$			0.904	0.6334
$\omega_{y,\text{HS GPA}}$	0.470	0.0808	0.430	0.0956
$\omega_{y,\text{ACT}}$	0.047	0.0112	0.048	0.0129
$\omega_{y,\text{Black}}$	-0.213	0.1074	-0.191	0.1195
$\omega_{y,\text{Male}}$	-0.037	0.0849	-0.017	0.0928
$\omega_{y,\text{HS study}}$	-0.007	0.0042	-0.013	0.0061
$\omega_{y,\text{expected study}}$	-0.005	0.0035	-0.006	0.0040
Study cost function / Best response function**				
θ_1	-1.074	0.1551	-1.056	0.1495
θ_2	0.874	0.2351	0.839	0.2256
θ_3	-0.907	0.8097	-0.251	1.2293
θ_4	0.096	1.2800	0.561	1.4312
$\tau_{\mu,1}$	0.105	0.0601	0.108	0.0595
$\tau_{\mu,2}$	-0.003	0.0028	-0.004	0.0028
$\omega_{s,\text{HS GPA}}$***	1.000	–	1.000	–
$\omega_{s,\text{ACT}}$	-0.063	0.0870	-0.055	0.0760
$\omega_{s,\text{Black}}$	-0.735	0.7459	-0.585	0.6456
$\omega_{s,\text{Male}}$	-1.065	0.7892	-0.886	0.6592
$\omega_{s,\text{HS study}}$	0.344	0.1554	0.297	0.1284
$\omega_{s,\text{expected study}}$	0.005	0.0309	0.009	0.0273
Shocks				
σ_{η_y}	0.721	0.0185	0.719	0.0183
σ_{η_s}	2.159	0.0377	2.160	0.0377
Log Likelihood:	-4696.361		-4694.625	

* Production function: $y_{it} = \beta_1 + \beta_2 s_{it} + \beta_{3,\text{comp}} \frac{s_{it}}{s_{-it}} + \mu_{yi}$, where $\mu_{yi} = x_i'\omega_y$.

** Best response function: $s_{it} = (\beta_{3,\text{comp}} - \theta_3) - \theta_4\gamma(\mu_{si}) + (\beta_2 - \theta_1)s_{-it} - \theta_2\gamma(\mu_{si})s_{-it}$, where $\mu_{si} = x_i'\omega_s$. As in the baseline estimates, we have set $\tau_s = 1$.

***: Normalized to 1.

E.3 Out-of-Sample Validation

Table A7 presents the baseline parameter estimates (Col. (1)) and those obtained when using only first-semester data (Col. (2)).[63] The sets of parameters are strikingly similar between the two columns; this is confirmed by their having very similar (first-semester-only) log likelihoods, which are presented at the bottom of each column. This suggests that the out-of-sample fit of second-semester outcomes may be reasonable when based on parameters estimated using only first-semester data.

To further investigate, we compare model fit for the first-semester data, which was used to estimate model parameters, and the validation data from the second semester. We can see that the out-of-sample fit for study time (Figure A5), GPA (Figure A6), and own-vs.-friend study time (Figure A7) all seem quite good.

F Identification

F.1 Identification of Model with Linear Best Response Functions

Our baseline empirical specification has $\tau_s = 1$, i.e., best response functions are linear in friend study time. The identification conditions we discuss below are closely related to those in Proposition 1 from Bramoullé et al. (2009) or Theorem 4(iii) from Blume et al. (2015). Both papers note that the negative result of Manski (1993) is not robust to many deviations that are commonly found in practical applications, especially those using social network data (as we do).

We start by discussing identification of the different subsets of parameters in our baseline model, which does not have "contextual effects" (i.e., friends' characteristics do not enter a students' best response function directly). Next, in order to demonstrate that identification in our baseline model is not dependent on an absence of contextual effects, we discuss identification of an augmented baseline model with contextual effects, where friends' characteristics enters a student's best response function.

F.1.1 Identification of Baseline Model (No Contextual Effects)

Consider identification of our baseline model's parameters when $\tau_s = 1$. We have three subsets of parameters in our estimating equations. The first subset collects those in our achievement equation:

$$y_{it} = \beta_1 + \beta_2 s^*_{it} + x'_i \omega_y + \eta_{yit}, \tag{A8}$$

[63]Due to its relative parsimony, we use the cost-reduction based specification for the cost function, which we show is observationally equivalent to the conformity based specification in Appendix D.3.1. Also, we present results based on Tobit specifications for GPA and reported study times, which are essentially equivalent to those from the least-squares specification.

Table A7: Parameters Under Baseline and Only-First-Semester Data

	Baseline (1)	Only First Semester (2)
Production function*		
β_1	-0.350	-0.154
β_2	0.254	0.230
$\omega_{y,\text{HS GPA}}$	0.470	0.400
$\omega_{y,\text{ACT}}$	0.047	0.053
$\omega_{y,\text{Black}}$	-0.213	-0.278
$\omega_{y,\text{Male}}$	-0.037	-0.035
$\omega_{y,\text{HS study}}$	-0.007	-0.009
$\omega_{y,\text{expected study}}$	-0.005	-0.005
Study cost function / Best response function**		
θ_1	-1.074	-1.103
θ_2	0.874	0.848
θ_3	-0.907	-1.138
θ_4	0.096	0.144
$\tau_{\mu,1}$	0.105	0.127
$\tau_{\mu,2}$	-0.003	-0.005
$\omega_{s,\text{HS GPA}}$***	1.000	1.000
$\omega_{s,\text{ACT}}$	-0.063	-0.121
$\omega_{s,\text{Black}}$	-0.735	-0.765
$\omega_{s,\text{Male}}$	-1.065	-0.873
$\omega_{s,\text{HS study}}$	0.344	0.271
$\omega_{s,\text{expected study}}$	0.005	0.000
Shocks		
σ_{η_y}	0.721	0.709
σ_{η_s}	2.159	2.203
Log Likelihood: (first-semester)	-2375.58255	-2373.61342

Note: * Production function: $y_{it} = \beta_1 + \beta_2 s_{it} + \mu_{yi}$, where $\mu_{yi} = x_i'\omega_y$. ** Best response function: $s_{it} = -\theta_3 - \theta_4\gamma(\mu_{si}) + (\beta_2 - \theta_1)s_{-it} - \theta_2\gamma(\mu_{si})s_{-it}$, where $\mu_{si} = x_i'\omega_s$. As in the baseline estimates, we have set $\tau_s = 1$. ***: Normalized to 1. Col. (1) presents estimates from baseline model and the log likelihood in the first semester. Col. (2) presents results when parameters were estimated using only first-semester data and the log likelihood in the first semester.

Figure A5: In and-Out-of-sample fit; study time

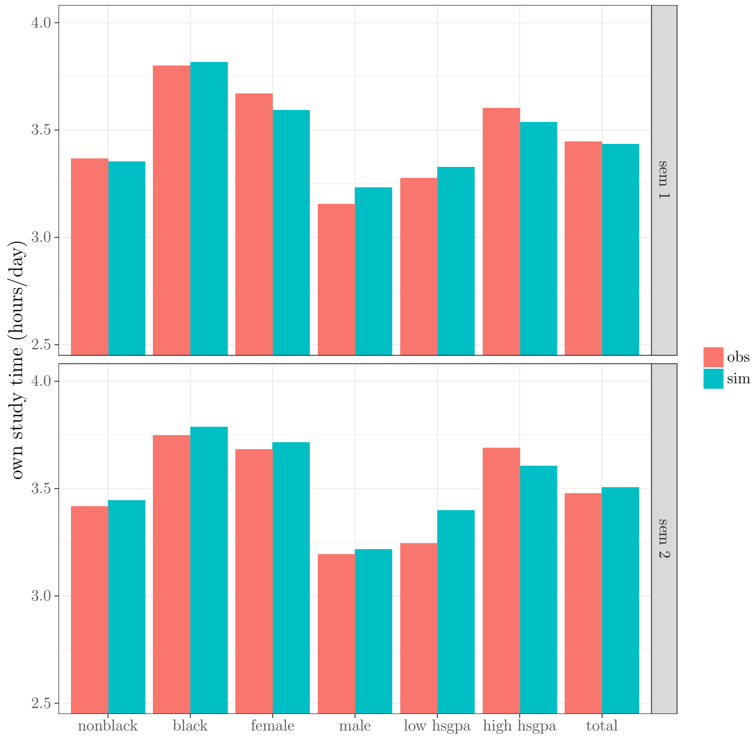


Figure A6: In- and-Out-of-sample fit; GPA

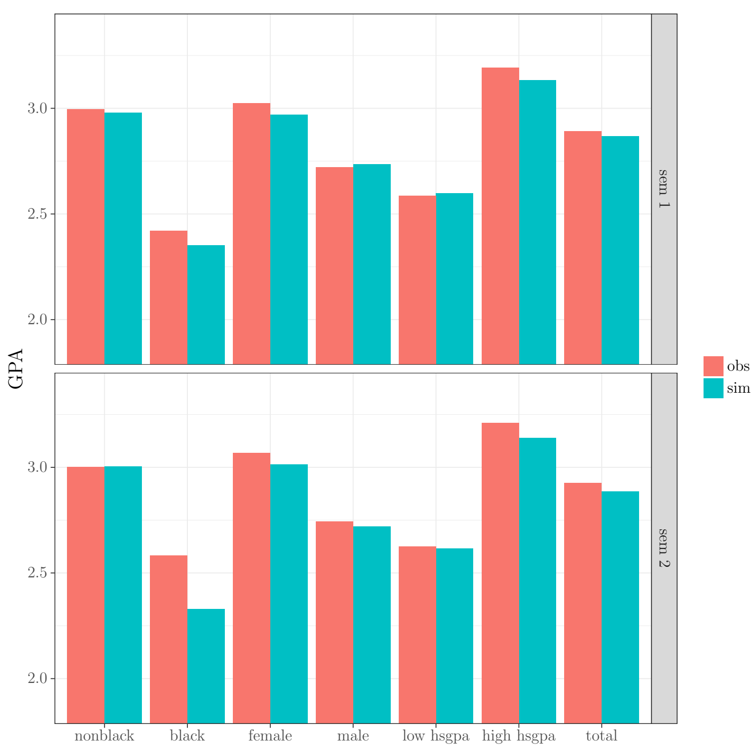
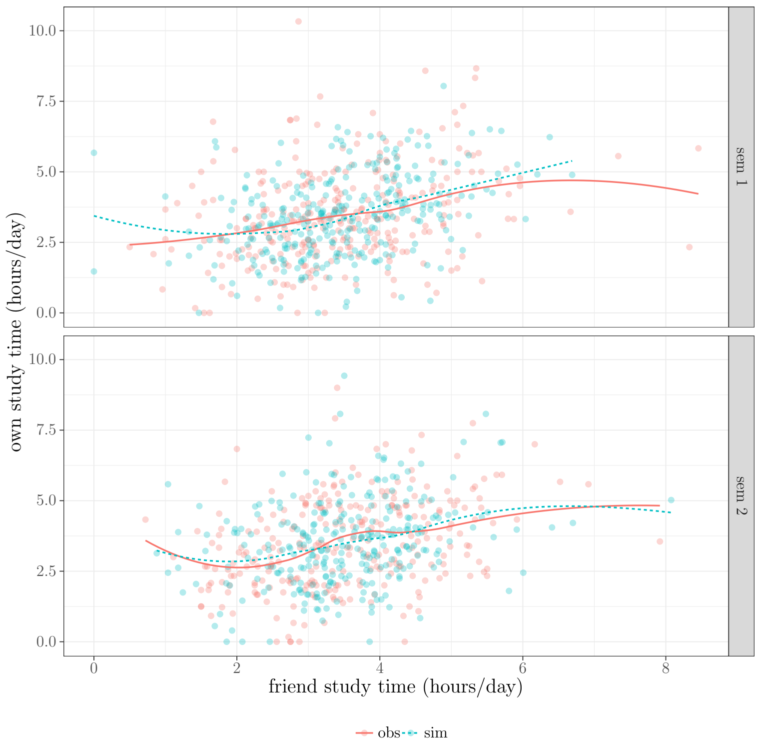


Figure A7: In- and-Out-of-sample fit; own vs. friend study time



where s_{it}^* is our estimate of model study time, which depends on all students' characteristics, the network in semester t, and parameters that are identified in the reduced-form study time regressions below. The vector $(\beta_1, \beta_2, \omega_y)$ is identified given linear independence of a constant and stacked versions of s_{it}^*, and x_i, a standard condition (which also holds in our data).

The remaining parameters are estimated using the specification for equilibrium study time. Imposing that $\tau_s = 1$, the best response function in our baseline model, equation (7), is

$$s_{it} = \frac{\beta_2 - \chi_1}{1+\chi_4} + \frac{-\chi_2}{1+\chi_4}\gamma(\mu_{si}) + \frac{\chi_4}{1+\chi_4}s_{-it} + \frac{\chi_4 \cdot \chi_5}{1+\chi_4}\gamma(\mu_{si})s_{-it}. \tag{A9}$$

There are two sets of parameters in the best response function above, the composite parameters $\pi_0 \equiv \frac{\beta_2-\chi_1}{1+\chi_4}$, $\pi_1 \equiv \frac{-\chi_2}{1+\chi_4}$, $\pi_2 \equiv \frac{\chi_4}{1+\chi_4}$, $\pi_3 \equiv \frac{\chi_4 \cdot \chi_5}{1+\chi_4}$, and those implicit in $\gamma(\mu_{si})$. We first treat $\gamma(\mu_{si})$ as though it were observable data, to understand identification of the composite π parameters, and then discuss identification of the parameterization of $\gamma(\mu_{si})$. Our model parameters $(\chi_1, \chi_2, \chi_4, \chi_5)$ are trivially identified from π using β_2 (identified in our achievement regression above) and the assumption that $\chi_3 = 1$.

Student i's best response function is

$$s_{it} = \pi_0 + \pi_1 \gamma(\mu_{si}) + (\pi_2 + \pi_3 \gamma(\mu_{si}))\, s_{-it}.$$

This results in the stacked best response function

$$S_t = \pi_0 1_N + \pi_1 \gamma + (\pi_2 + \pi_3 diag(\gamma))\, W_t S_t,$$

where S_t is the vector of study times for all N students, 1_N is an $N-$vector of ones, γ is an N-vector stacking the $\gamma(\mu_{si})$, $diag(\gamma)$ is a diagonal matrix with its (i,i) entry equal to $\gamma(\mu_{si})$, and W_t is the row-normalized adjacency matrix A_t (i.e., if $A_t(i,j) = 0$ then $W_t(i,j) = 0$ and if $A_t(i,j) = 1$ then $W_t(i,j) = 1/[\sum_{j=1}^{N} A(i,j)]$).

Consider the following reduced-form estimating equation, which, to ease exposition, uses only one study time report per student in each semester t:

$$\widetilde{S}_t = [I - (\pi_2 + \pi_3 diag(\gamma)) W_t]^{-1} [\pi_0 1_N + \pi_1 \gamma] + \eta_t, \tag{A10}$$

where $\widetilde{S}_t$ is the vector of observed study reports.[64] We follow Bramoullé et al. (2009) and address identification by examining whether there are multiple parameter sets that would generate the same reduced form.

An argument very similar to that in Proposition 1 of Bramoullé et al. (2009) can be applied for our baseline model, which allows for heterogeneous reactiveness.

Claim 3 (Identification with $\tau_s = 1$)**.** *If $\{I, W_t, diag(\gamma)W_t\}$ are linearly independent and either $\pi_0 \neq 0$ or $\pi_1 \neq 0$, then $(\pi_0, \pi_1, \pi_2, \pi_3)$ are identified.*

Proof. If two sets of parameters $(\pi_0, \pi_1, \pi_2, \pi_3)$ and $(\pi_0', \pi_1', \pi_2', \pi_3')$ have the same reduced form, then the following must hold:

$$[I - (\pi_2 + \pi_3 diag(\gamma)) W_t]^{-1} \pi_0 = [I - (\pi_2' + \pi_3' diag(\gamma)) W_t]^{-1} \pi_0' \tag{A11}$$

$$[I - (\pi_2 + \pi_3 diag(\gamma)) W_t]^{-1} \pi_1 = [I - (\pi_2' + \pi_3' diag(\gamma)) W_t]^{-1} \pi_1'. \tag{A12}$$

First, suppose $\pi_1 \neq 0$. After some manipulation, (A12) can be re-written as

$$(\pi_1 - \pi_1') I + (\pi_1' \pi_2 - \pi_1 \pi_2')\, W_t + (\pi_1' \pi_3 - \pi_1 \pi_3')\, diag(\gamma) W_t = 0.$$

[64] Our actual estimating equation, with multiple reports per student i, predicts each report with student i's row of $[I - (\pi_2 + \pi_3 diag(\gamma)) W_t]^{-1} [\pi_0 1_N + \pi_1 \gamma]$, where each report has its own idiosyncratic η error.

If $\{I, W_t, diag(\gamma)W_t\}$ are linearly independent, this equation implies

$$\pi_1 = \pi_1' \tag{A13}$$

$$\pi_1'\pi_2 = \pi_1\pi_2' \tag{A14}$$

$$\pi_1'\pi_3 = \pi_1\pi_3'. \tag{A15}$$

Since $\pi_1 \neq 0$, equations (A13), (A14), and (A15) imply $(\pi_1, \pi_2, \pi_3) = (\pi_1', \pi_2', \pi_3')$ and plugging these values into equation (A11) implies $\pi_0 = \pi_0'$. If $\pi_0 \neq 0$, the same algebra applied to (A11) demonstrates identification. □

Linear independence of $\{I, W_t, diag(\gamma)W_t\}$ will occur with probability approaching one for a large sample, as long as the distribution of heterogeneity $\gamma(\mu_{si})$ is non-degenerate and weakly dependent across individuals. Intuitively, students need friends, and enough students need differing relevant characteristics.

To complete the parameterization of our model, we specify $\gamma(\mu_{si})$ as a scale-normalized, nonlinear function of student characteristics, x_i. We were initially motivated to use a nonlinear functional form for $\gamma(\mu_{si})$ to help ensure stability of our general solution algorithm (in which τ_s is a free parameter). Because $\gamma(\mu_{si})$ is specified to be nonlinear, we do not have a global identification proof for its parameters, as is standard. However, in practice, we obtained stable estimates for the $\gamma(\mu_{si})$ parameters across a wide variety of starting values, so we are confident that these parameters are identified. We note that if our functional form for $\gamma(\mu_{si})$ were linear in characteristics $\{x_{i1}, x_{i2}, \ldots, x_{iK}\}$, then identification conditions would be straightforward extensions of those above, which treat $\gamma(\mu_{si})$ as known. Specifically, identification of $(\pi_0, \pi_1, \pi_2, \pi_3)$ and the additional coefficients in the linear $\gamma(\mu_{si})$ would follow from $\pi_0 \neq 0$ or $\pi_1 \neq 0$ and linear independence of a set of matrices: $\{I, W_t, diag(X_1)W_t, diag(X_2)W_t, \ldots, diag(X_K)W_t\}$, where X_k is the N-vector stacking x_{ik} for all students.

The rank conditions in Claim 3 are satisfied in our data on student characteristics and W_t for each semester, at our estimates of $\gamma(\mu_i)$. The implied estimates of study times satisfy the rank condition for identification of the parameters in equation (A8).

F.1.2 Identification of Baseline Model, Augmented with Contextual Effects

Identification for our model parameters is not dependent upon a lack of contextual effects in best responses. Consider a modification of our baseline model that allows best responses to also depend on the average of friends' effective study types $\gamma(\mu_{sj})$. Again for ease of exposition, first treat $\gamma(\mu_{si})$ as observed to the researcher.

Student i's best response function can be written as

$$s_{it} = \pi_0 + \pi_1\gamma(\mu_{si}) + (\pi_2 + \pi_3\gamma(\mu_{si}))\, s_{-it} + \pi_4 \frac{\sum_{j=1}^{N} A(i,j)\gamma(\mu_{sj})}{\sum_{j=1}^{N} A(i,j)},$$

where the new composite parameter π_4 embodies contextual effects in the study time equation. The composite π parameters map to our underlying structural parameters in the same way as above, in our baseline model.

Using the same notation as in the baseline model,[65] the stacked best response function is

$$S_t = \pi_0 1_N + \pi_1\gamma + (\pi_2 + \pi_3 diag(\gamma))\, W_t S_t + \pi_4 W_t \gamma.$$

Reduced-form estimating equations with equilibrium study times in this augmented model would then be

$$\widetilde{S}_t = [I - (\pi_2 + \pi_3 diag(\gamma))W_t]^{-1} [\pi_0 1_N + \pi_1\gamma + \pi_4 W_t\gamma] + \eta_t, \tag{A16}$$

where $\widetilde{S}_t$ is the vector of observed study reports with idiosyncratic error term η_t.

Claim 4 (Identification with $\tau_s = 1$ and with contextual effects)**.** *If $\pi_1\pi_2 + \pi_4 \neq 0$ and $\{I, W_t, W_t^2, diag(\gamma)W_t, diag(\gamma)W_t^2\}$ are linearly independent, then $(\pi_0, \pi_1, \pi_2, \pi_3, \pi_4)$ are identified.*

Proof. From equation (A16) above, if two sets of parameters $(\pi_0, \pi_1, \pi_2, \pi_3, \pi_4)$ and $(\pi_0', \pi_1', \pi_2', \pi_3', \pi_4')$ have the same reduced form, then the following must hold:

$$[I - (\pi_2 + \pi_3 \cdot diag(\gamma))W_t]^{-1}\pi_0 = [I - (\pi_2' + \pi_3' \cdot diag(\gamma))W_t]^{-1}\pi_0' \tag{A17}$$

$$[I - (\pi_2 + \pi_3 \cdot diag(\gamma))W_t]^{-1}[\pi_1 + \pi_4 W_t] = [I - (\pi_2' + \pi_3' \cdot diag(\gamma))W_t]^{-1}[\pi_1' + \pi_4' W_t]. \tag{A18}$$

After some manipulation, (A18) can be re-written as

$$(\pi_1 - \pi_1')I + (\pi_1'\pi_2 - \pi_1\pi_2' + \pi_4 - \pi_4')\, W_t + (\pi_1'\pi_3 - \pi_1\pi_3')\, diag(\gamma)W_t + \\ (\pi_4'\pi_2 - \pi_4\pi_2')\, W_t^2 + (\pi_4'\pi_3 - \pi_4\pi_3')\, diag(\gamma)W_t^2 = 0.$$

[65] We continue to use one study time observation per student in each semester t.

If $\{I, W_t, W_t^2, diag(\gamma)W_t, diag(\gamma)W_t^2\}$ are linearly independent, this equation implies

$$\pi_1 = \pi_1' \tag{A19}$$

$$\pi_1'\pi_2 + \pi_4 = \pi_1\pi_2' + \pi_4' \tag{A20}$$

$$\pi_1'\pi_3 = \pi_1\pi_3' \tag{A21}$$

$$\pi_4'\pi_2 = \pi_4\pi_2' \tag{A22}$$

$$\pi_4'\pi_3 = \pi_4\pi_3'. \tag{A23}$$

In addition, both parameterizations must satisfy our parameter restriction, which we call Assumption A: $\pi_1\pi_2 + \pi_4 \neq 0$ and $\pi_1'\pi_2' + \pi_4' \neq 0$. This assumption essentially means that either endogenous or exogenous social interactions are present, and also rules out the knife-edge case where they cancel each other out.

We will show that $(\pi_1, \pi_2, \pi_3, \pi_4) = (\pi_1', \pi_2', \pi_3', \pi_4')$, which, when combined with equation (A17), implies $\pi_0 = \pi_0'$ and, thus, $(\pi_0, \pi_1, \pi_2, \pi_3, \pi_4) = (\pi_0', \pi_1', \pi_2', \pi_3', \pi_4')$.

First, suppose that $\pi_1 = 0$. Then equation (A20) implies $\pi_4 = \pi_4'$, where $\pi_4 \neq 0$ because $\pi_1\pi_2 + \pi_4 \neq 0$ (by Assumption A). Substituting $\pi_4 = \pi_4'$ into equations (A22) and (A23) and dividing by π_4', we respectively have $\pi_2 = \pi_2'$, and $\pi_3 = \pi_3'$. Therefore, we have $(\pi_1, \pi_2, \pi_3, \pi_4) = (\pi_1', \pi_2', \pi_3', \pi_4')$.

Now suppose that $\pi_1 \neq 0$ and consider the subset where $\pi_3 \neq 0$ also holds. From equation (A19) $\pi_1 = \pi_1'$, so equation (A21) implies $\pi_3 = \pi_3' \neq 0$. With $\pi_3 = \pi_3' \neq 0$, equation (A23) implies $\pi_4 = \pi_4'$. With $\pi_4 = \pi_4'$ and $\pi_1 = \pi_1' \neq 0$, equation (A20) implies $\pi_2 = \pi_2'$, therefore $(\pi_1, \pi_2, \pi_3, \pi_4) = (\pi_1', \pi_2', \pi_3', \pi_4')$.

Now consider the remaining cases, where $\pi_1 \neq 0$ and $\pi_3 = 0$, and split these into two sets. For cases where equation (A22) is not zero on both sides, i.e., $\pi_4'\pi_2 = \pi_4\pi_2' \neq 0$, we can define $\lambda \equiv \pi_2'/\pi_2 \neq 0$, such that we have $\pi_2' = \lambda\pi_2$ and $\pi_4' = \lambda\pi_4$. Substituting for π_2' and π_4' in equation (A20), we have $\pi_1'\pi_2 + \pi_4 = \lambda(\pi_1\pi_2 + \pi_4)$. Substituting $\pi_1 = \pi_1'$ (from equation (A19)), the expression from the previous sentence becomes $\pi_1\pi_2 + \pi_4 = \lambda(\pi_1\pi_2 + \pi_4)$. Because by Assumption A we have $\pi_1\pi_2 + \pi_4 \neq 0$, we can divide the expression through by $\pi_1\pi_2 + \pi_4$, yielding $\lambda = 1$. Thus, both $\pi_2' = \pi_2$ and $\pi_4' = \pi_4$, giving $(\pi_1, \pi_2, \pi_3, \pi_4) = (\pi_1', \pi_2', \pi_3', \pi_4')$.

The final cases are where $\pi_1 \neq 0$, $\pi_3 = 0$ and $\pi_4'\pi_2 = \pi_4\pi_2' = 0$. At least one parameter in each of $\pi_4'\pi_2$ and $\pi_4\pi_2'$ is zero, meaning there are nine possible cases, corresponding to different arrangements of zeros across these two pairs of parameters. The cases with any three of the four parameters being zero and that with all four being zero are ruled out by Assumption A. Assumption A also rules out $\pi_2 = \pi_4 = 0$ and $\pi_2' = \pi_4' = 0$. The remaining case with $\pi_2 = \pi_2' = 0$ implies via equation (A20) that $\pi_4 = \pi_4'$. Finally, the remaining case with $\pi_4 = \pi_4' = 0$, when combined with equation (A20), implies $\pi_2 = \pi_2'$ because

$\pi_1 = \pi_1' \neq 0$. Thus, under these last two cases either $\pi_2 = \pi_2' = 0$ or $\pi_4 = \pi_4' = 0$, and we have $(\pi_1, \pi_2, \pi_3, \pi_4) = (\pi_1', \pi_2', \pi_3', \pi_4')$. $\square$

In our application, the matrices $\{I, W_t, W_t^2\}$ are linearly independent and the whole set $\{I, W_t, W_t^2, diag(\gamma)W_t, diag(\gamma)W_t^2\}$ will be linearly independent with probability approaching one in large samples, if the distribution of $\gamma(\mu_{si})$ is non-degenerate and weakly dependent across students. The issues with identifying $\gamma(\mu_{si})$ in this model are identical to those in our baseline model.

F.2 Identification of Model with Nonlinear Best Response Functions

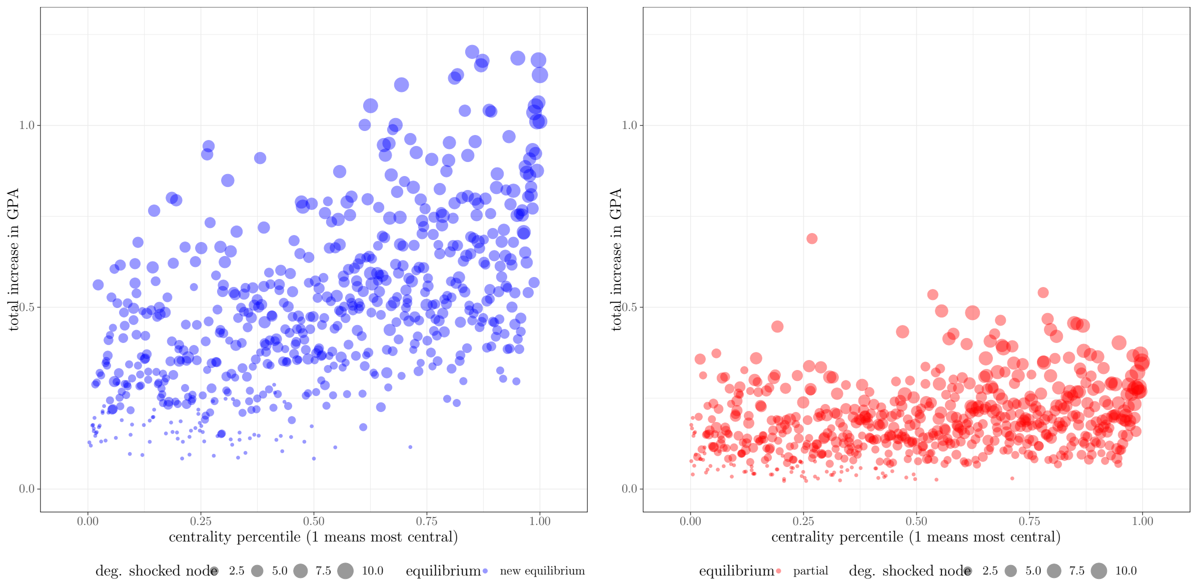
While we did not find evidence that $\tau_s < 1$, it seems worthwhile for completeness to briefly discuss this nonlinear case. There is no generally applicable (global) identification result for nonlinear social interactions models. Brock and Durlauf (2001) discuss why identification of nonlinear social interactions models is generic and how nonidentification is a special case. They develop a theorem (Theorem 7, on page 3328) showing (local) identification for the "nonlinear-in-means" social interactions model, which like the typical linear-in-means model features non-overlapping peer groups, but unlike the typical linear-in-means model has endogenous agent's actions that are linear in a nonlinear function of peer actions. Therefore, while we do not formally prove identification of contextual effects for the nonlinear model, we are confident that the model parameters would be identified in this case as well.

G Variation in Achievement Response, by Centrality of Shocked Student

To get a sense of why shocking different students can produce such different gains, the left panel of Figure A8 shows the relationship between the centrality of the shocked student and the total response at the new equilibrium.[66] As before, this calculation excludes the mechanical gain in achievement experienced by the shocked student. Each dot records the total achievement response (y-axis) by the percentile centrality that semester, i.e., by how central the shocked student is (x-axis). The size of each (blue) dot shows the degree (i.e., number of friends) of the shocked student. Larger dots are concentrated at the top-right, and smaller ones at the bottom-left. That is, students with more friends tend to have higher centrality indices and larger achievement gains. Intuitively, because the effects of effort changes are stronger the closer students are, the total response is higher when the shocked

[66] We use what is called a "closeness" centrality measure, given by the reciprocal of the sum of shortest distances between that student and every other student in the graph. Average distance to others for unconnected students is set to the number of students (Csardi and Nepusz (2006), Freeman (1979)).

Figure A8: Total achievement response (GPA points), by centrality of shocked student



Note: The vertical location of each dot represents the total achievement response to shocking a different student; the left panel presents the total gain at the new equilibrium and the right panel presents the partial equilibrium total gain. The x-axis indicates the shocked student's centrality to other students and dot size denotes the degree of the shocked student.

student is more centrally located.[67] The right panel of Figure A8 plots partial equilibrium effects (red dots). We can see here that, though shocked students have the same degree (dot sizes), the average response is not as strongly increasing in centrality of the shocked student. This is the case because the equilibrium effects play a larger role the more densely connected the shocked student is to the rest of the network.

H Additional Tables

[67]The notion that certain students may disproportionately affect other students is related to the concept of a "key player", studied in Ballester et al. (2006).

Table A8: Study time regressions controlling for different sets of characteristics, pooled over both semesters

	Dependent variable: Own study			
	(1)	(2)	(3)	(4)
Male	-0.369^{***}	-0.328^{**}	-0.391^{***}	
	(0.136)	(0.140)	(0.135)	
Black	0.116	0.333^{*}	0.324^{*}	
	(0.186)	(0.192)	(0.172)	
HS GPA	0.413^{***}	0.392^{**}		
	(0.149)	(0.156)		
ACT	-0.032	-0.029		
	(0.021)	(0.022)		
HS study	0.043^{***}			
	(0.006)			
Expected study	-0.002			
	(0.006)			
Friends study	0.166^{***}	0.198^{***}	0.202^{***}	0.228^{***}
	(0.037)	(0.039)	(0.039)	(0.038)
Constant	1.915^{***}	2.167^{***}	2.850^{***}	2.648^{***}
	(0.671)	(0.679)	(0.172)	(0.152)
Observations	574	574	574	574
R^2	0.169	0.087	0.076	0.058

Note: *p<0.1; **p<0.05; ***p<0.01 GPA is measured in GPA points (0-4). HS study and expected study are measured in hours/week. Own and friend study are measured in hours/day. The variable "Friend z" for student i in period t is the average of the variable z across i's friends in period t.

H.1 Effects on 10-Year Graduation

Table A10 shows the results of a probit of graduating within ten years of starting college on first year achievement. Column (1) shows the results of a probit of graduating on the estimated human capital type $\widehat{\mu}_y$ and average model achievement during the first two semesters

Table A9: Study time regressions, pooled over both semesters

	Dependent variable: Own study	
	(1)	(2)
Male	-0.369^{***}	-0.365^{***}
	(0.136)	(0.137)
Black	0.116	0.115
	(0.186)	(0.187)
HS GPA	0.413^{***}	0.389^{***}
	(0.149)	(0.150)
ACT	-0.032	-0.034
	(0.021)	(0.021)
HS study	0.043^{***}	0.041^{***}
	(0.006)	(0.006)
Expected study	-0.002	-0.002
	(0.006)	(0.006)
Own share science courses		0.349
		(0.390)
Friend study	0.166^{***}	0.157^{***}
	(0.037)	(0.038)
Avg. friend share science courses		0.880
		(0.562)
Constant	1.915^{***}	1.873^{***}
	(0.671)	(0.684)
Observations	574	574
R^2	0.169	0.176

Note: *p<0.1; **p<0.05; ***p<0.01 GPA is measured in GPA points (0-4). Own and friend HS study and expected study are measured in hours/week. Own and friend study are measured in hours/day. The variable "Friend z" for student i in period t is the average of the variable z across i's friends in period t.

of college.[68] Column (2) runs a similar probit, substituting student characteristics in for estimated human capital type. Both statistical models show a strong link between performance during the first year and whether or not a student graduates from college within ten years of starting. The results are calculated using the covariates in the first column; the results are very similar when we to use those in the second column instead. For each of our simulated pairs of networks, we compare the predicted probability of graduation for each student under the baseline network to the predicted probability of graduation after achievement has changed under the simulated counterfactual network.

[68]Note that this specification is consistent with the separable manner in which human capital type and own study time enter the human capital production function.

Table A10: Probit of graduating on average of first year achievement and student characteristics

	Dependent variable: Graduate within 10 years	
	(1)	(2)
$\widehat{\mu}_y$	-0.044	
	(0.495)	
Black		0.441
		(0.281)
Male		-0.282
		(0.210)
HS GPA		0.279
		(0.403)
ACT		-0.045
		(0.039)
HS study		-0.013
		(0.008)
Expected study		0.003
		(0.009)
Avg. achievement$^{\heartsuit}$	1.361^{***}	1.447^{*}
	(0.485)	(0.739)
Constant	-3.118^{***}	-3.215^{***}
	(0.652)	(0.981)
Observations	307	307
Log Likelihood	-154.079	-146.092
Akaike Inf. Crit.	314.158	308.184

Note: *p<0.1; **p<0.05; ***p<0.01

$^{\heartsuit}$: Average of model achievement across both semesters

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